



Extending Operations to Fractions



Teacher Guide

$$2 \times 2 = 4$$

$$4 \times 2 = 8$$

$$8 : 4 = 2$$

$$4 - 2 = 2$$



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Extending Operations to Fractions

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Extending Operations to Fractions Teacher Guide

Core Knowledge Mathematics™

Unit 3: Extending Operations to Fractions

At a Glance

Unit 3 is estimated to be completed in 20-22 days including 2 days for assessment.

This unit is divided into three sections including 18 lessons and 2 optional lessons.

- Section A—Equal Groups of Fractions (Lessons 1-6)
- Section B—Addition and Subtraction of Fractions (Lessons 7-14)
- Section C—Addition of Tenths (Lessons 15-20)

On pages 8-9 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses six new student centers.

- Rolling for Fractions
- Compare
- Estimate and Measure
- Target Measurements
- Creating Line Plots
- Jump the Line

Unit 3: Extending Operations to Fractions

Unit Learning Goals

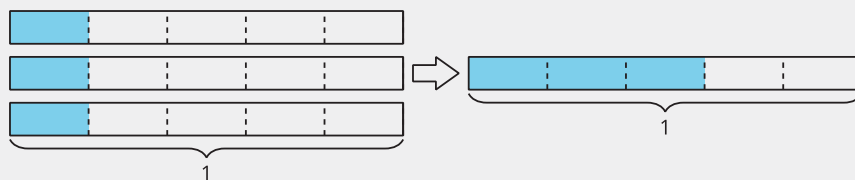
- Students learn that a fraction $\frac{a}{b}$ is a product of a whole number a and a unit fraction $\frac{1}{b}$, or $\frac{a}{b} = a \times \frac{1}{b}$, and that $n \times \frac{a}{b} = \frac{(n \times a)}{b}$. Students learn to add and subtract fractions with like denominators, and to add and subtract tenths and hundredths.

In this unit, students deepen their understanding of how fractions can be composed and decomposed, and learn about operations on fractions.

In grade 3, students partitioned a whole into equal parts and identified one of the parts as a unit fraction. They learned that non-unit fractions and whole numbers are composed of unit fractions. They used visual fraction models, including tape diagrams and number lines, to represent and compare fractions. In a previous unit, students extended that work and reasoned about fraction equivalence.

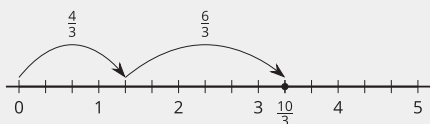
Here, students multiply fractions by whole numbers, add and subtract fractions with the same denominator, and add tenths and hundredths. They rely on familiar concepts and representations to do so. For instance, students had represented multiplication on a tape diagram, with equal-size groups and a whole number in each group. Here, they use a tape diagram that shows a fraction in each group.

$$3 \times \frac{1}{5} = \frac{3}{5}$$

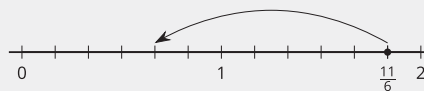


In earlier grades, students used number lines to represent addition and subtraction of whole numbers. Here, they use number lines to represent the decomposition of fractions into sums, and to reason about addition and subtraction of fractions with the same denominator, including mixed numbers.

$$\frac{4}{3} + \frac{6}{3} = \frac{10}{3}$$



$$\frac{11}{6} - \frac{7}{6} = \frac{4}{6}$$



Students then apply these skills in the context of measurement and data. They analyze line plots showing fractional lengths and find sums and differences to answer questions about the data.

Lastly, students use fraction equivalence to find sums of tenths and hundredths. For instance, to find $\frac{3}{10} + \frac{15}{100}$, they reason that $\frac{3}{10}$ is equivalent to $\frac{30}{100}$, so the sum is $\frac{30}{100} + \frac{15}{100}$, which is $\frac{45}{100}$.

Section A: Equal Groups of Fractions

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.a, 4.NF.B.4.b, 4.NF.B.4.c

Building Towards 4.NBT.B.5, 4.NF.B.4, 4.NF.B.4.a

Section Learning Goals

- Recognize that $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.
- Represent and explain that a fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$, namely $a \times \frac{1}{b}$.
- Represent and solve problems involving multiplication of a fraction by a whole number.

In this section, students extend their earlier understanding of multiplication as equal groups of whole numbers of objects to now include equal groups of fractional pieces.

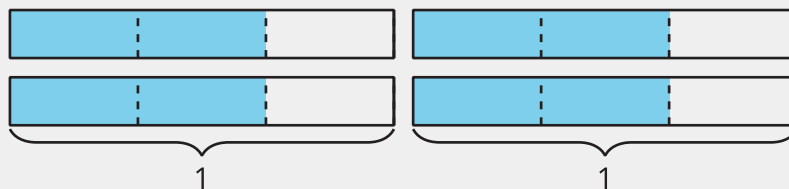
How many do you see? How do you see them?



Students begin by reasoning about groups containing unit fractions. For instance, they interpret the 5 plates with half an orange each as $5 \times \frac{1}{2}$, which is $\frac{5}{2}$. Later, they also reason about groups of non-unit fractions and write expressions to represent the quantities. For instance, 5 groups of $\frac{3}{4}$ can be expressed as $5 \times \frac{3}{4}$ or $\frac{15}{4}$.

Later, students reason with diagrams and equations. Through repeated reasoning, they see regularity in the product of a whole number and a fraction (MP8). The numerator in the resulting fraction is the product of the whole number and the numerator of the fractional factor, and the denominator is the same as in the fractional factor.

$$4 \times \frac{2}{3} = \frac{8}{3}$$



These diagrams also help students see that some fractions can be represented by more than one multiplication expression. Students can reason that $\frac{8}{3}$ is $8 \times \frac{1}{3}$, which is also equivalent to $4 \times 2 \times \frac{1}{3}$

and $2 \times 4 \times \frac{1}{3}$, and is therefore equivalent to $4 \times \frac{2}{3}$ and $2 \times \frac{4}{3}$, respectively.

By circling the diagram in various ways, students can visualize the different combinations of groups, understand their equivalence, and observe the associative property of multiplication. In doing this work, students practice looking for and making use of structure (MP7).

Students then solve problems that involve fraction multiplication, using diagrams and equations to show their reasoning. These diagrams will also be useful in later grades, when students make sense of fractions as quotients.

   PLC: Lesson 2, Activity 1, Card Sort: Expressions and Diagrams

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

Section B: Addition and Subtraction of Fractions

Standards Alignments

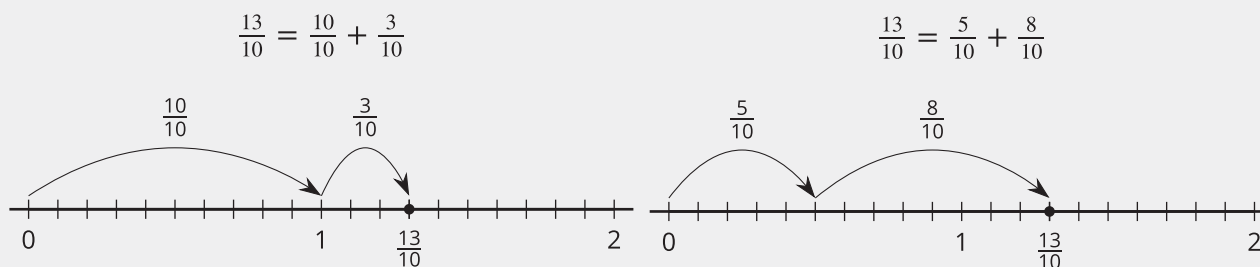
| | |
|------------------|--|
| Building On | 3.MD.B.4, 3.NF.A.1, 4.NF.B.3.a, 4.NF.B.4.b |
| Addressing | 4.MD.B.4, 4.NF.B.3, 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4.c |
| Building Towards | 4.MD.B.4, 4.NF.B.3, 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c |

Section Learning Goals

- Create and analyze line plots that display measurement data in fractions of a unit ($\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$).
- Represent and solve problems that involve the addition and subtraction of fractions and mixed numbers, including measurements presented in line plots.
- Use various strategies to add and subtract fractions and mixed numbers with like denominators.

In this section, students learn to add and subtract fractions by decomposing them into sums of smaller fractions, writing equivalent fractions, and using number lines to support their reasoning.

Students begin by thinking about a fraction as a sum of unit fractions with the same denominator and then as a sum of other smaller fractions. They represent different ways to decompose a fraction by drawing “jumps” on number lines and writing different equations.

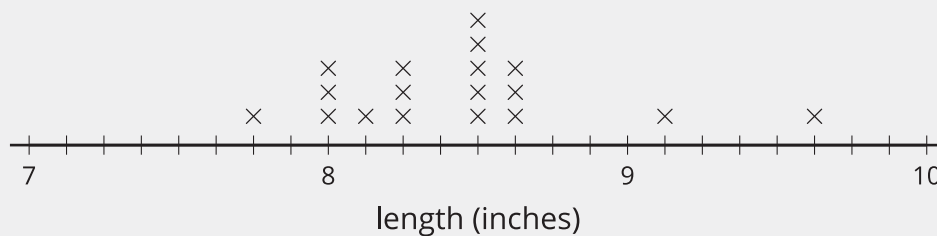


Working with number lines helps students see that a fraction greater than 1 can be decomposed into a whole number and a fraction, and then be expressed as a mixed number. This can in turn help us add and subtract fractions with the same denominator. For example, to find the value of $3 - \frac{2}{5}$, it helps to first decompose the 3 into $2 + \frac{5}{5}$, and then subtract $\frac{2}{5}$ from the $\frac{5}{5}$.

Later in the section, students organize fractional length measurements ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inch) on line plots. They apply their ability to interpret line plots and to add and subtract fractions to solve problems about measurement data.

*What is the difference between the largest and smallest shoe lengths?
Explain or show your reasoning.*

Fourth-grade Shoe Lengths



PLC: Lesson 10, Activity 1, What's Left?

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Creating Line Plots (2–5), Stage 2: Quarter Inches (Supporting)
- Estimate and Measure (1–4), Stage 4: Eighth Inches (Addressing)
- Target Measurements (2–5), Stage 3: Eighth Inches (Addressing)
- Creating Line Plots (2–5), Stage 3: Eighth Inches, Add and Subtract (Addressing)

Section C: Addition of Tenths and Hundredths

Standards Alignments

| | |
|------------------|--|
| Building On | 3.NBT.A.2, 4.NF.A.1, 4.NF.A.2, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4 |
| Addressing | 4.NF.A.1, 4.NF.A.2, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4, 4.NF.B.4.c, 4.NF.C.5 |
| Building Towards | 4.NF.B.3.d, 4.NF.B.4, 4.NF.C.5 |

Section Learning Goals

- Reason about equivalence to add tenths and hundredths.
- Reason about equivalence to solve problems involving addition and subtraction of fractions and mixed numbers.

In this section, students apply their understanding of fraction equivalence to add tenths and hundredths.

In the previous unit, students learned that $\frac{1}{10} = \frac{10}{100}$. They use this reasoning to add tenths and hundredths by generating equivalent fractions. They also apply what they learned in the previous section to strategically use decomposition and the associative and commutative properties to add three or more tenths and hundredths, including mixed numbers.

This section ends with an optional lesson that allows students to apply what they have learned about multiplication, addition, and subtraction of fractions and mixed numbers to solve a design problem.

 ↔  PLC: Lesson 15, Activity 2, Stacks of Blocks

Suggested Centers

- Jump the Line (2–5), Stage 2: Add and Subtract Tenths and Hundredths (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Throughout the Unit

The warm-up activities in this unit allow students to revisit and apply skills learned in previous grades, as well as to practice skills and develop the concepts learned in the unit.

Number Talks that involve multiplication encourage students to use familiar facts and structure in expressions to mentally find the values of related products. For example, the Number Talk in lesson 10

prompts students to notice and generalize patterns in the multiplication of a fraction and a whole number. In the last Number Talk, students revisit the associative property with whole numbers in preparation to strategically add tenths and hundredths when there are three or more addends.

Here is a sampling of Number Talk warm-ups in the unit.

| lesson 2 | lesson 10 | lesson 18 |
|---------------|---------------------------|-----------------------|
| 3×6 | $2 \times \frac{3}{12}$ | $54 + 2 + 18$ |
| 3×9 | $6 \times \frac{3}{12}$ | $61 + 104 + 39$ |
| 6×9 | $12 \times \frac{3}{12}$ | $25 + 63 + 75 + 7$ |
| 12×9 | $12 \times \frac{30}{12}$ | $50 + 106 + 19 + 101$ |

The True or False activities encourage students to look for structure to determine if two expressions are equivalent, rather than to evaluate each expression. Students use their understanding of equal groups of fractions, fraction decomposition, and equivalent fractions to reason about the truth of equations.

Here is a sampling of True or False warm-ups in the unit.

| lesson 6 | lesson 12 | lesson 15 |
|--|--|-------------------------------|
| $\frac{10}{12} = 5 \times \frac{2}{12}$ | $3 = 2 + \frac{3}{3}$ | $\frac{3}{4} = \frac{6}{8}$ |
| $10 \times \frac{10}{12} = 5 \times \frac{2}{12}$ | $3\frac{4}{6} = 2 + 1\frac{3}{6}$ | $\frac{5}{4} = \frac{10}{12}$ |
| $\frac{24}{4} = 8 \times 3 \times \frac{1}{4}$ | $4\frac{2}{8} = 3 + \frac{10}{8}$ | $1\frac{1}{4} = \frac{10}{8}$ |
| $12 \times 2 \times \frac{1}{4} = 8 \times 3 \times \frac{1}{4}$ | $\frac{2}{5} + \frac{5}{5} + 2 = 3\frac{3}{5}$ | $\frac{4}{3} = 1\frac{1}{6}$ |

Materials Needed

| LESSON | GATHER | COPY |
|--------|---|--|
| A.1 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| A.2 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Expressions and Diagrams (groups of 2) |
| A.3 | <ul style="list-style-type: none"> • Paper | <ul style="list-style-type: none"> • none |
| A.4 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| A.5 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| A.6 | <ul style="list-style-type: none"> • Chart paper | <ul style="list-style-type: none"> • none |
| B.7 | <ul style="list-style-type: none"> • Measuring cups | <ul style="list-style-type: none"> • none |
| B.8 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Make Two Jumps (groups of 2) |
| B.9 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Make a Jump, Subtraction Edition (groups of 2) |
| B.10 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Card Sort: Twelfths (groups of 2) |
| B.11 | <ul style="list-style-type: none"> • Tools for creating a visual display | <ul style="list-style-type: none"> • none |
| B.12 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| B.13 | <ul style="list-style-type: none"> • Colored pencils | <ul style="list-style-type: none"> • none |
| B.14 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| C.15 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| C.16 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| C.17 | <ul style="list-style-type: none"> • Sticky notes | <ul style="list-style-type: none"> • Card Sort: Less Than, Equal to, or Greater Than 1 (groups of 2) • Fraction Action: Tenths, Hundredths (groups of 2) |

- C.18
- Chart paper
 - Coins
 - More Than Two Fractions (groups of 15)

- C.19
- Rulers (inches)
 - Sticky notes
 - Tools for creating a visual display
 - Find a Match (groups of 24)

- C.20
- Blank paper
 - Sticky notes
 - none

Center: Rolling for Fractions (3–5)

Stage 1: Equivalent Fractions

Lessons

- Grade4.3.A1 (supporting)
- Grade4.3.A2 (supporting)
- Grade4.3.A3 (supporting)
- Grade4.3.A4 (supporting)

Stage Narrative

One player rolls 6 number cubes and tries to use 4 of them to fill in a statement with 2 equivalent fractions. If the player cannot make a true statement, they can re-roll as many of the cubes as they like. Each player may re-roll twice. If the student can fill in a statement with 2 equivalent fractions, they get a point for the round. Students take turns for 6 rounds and the player with the most points at the end of the game wins.

Standards Alignments

Addressing 3.NF.A.3.b

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 1 Recording Sheet
(groups of 1)

Additional Information

Each group of 2 needs 6 number cubes.

Stage 2: Multiply a Fraction by a Whole Number

Lessons

- Grade4.3.A4 (addressing)
- Grade4.3.A5 (addressing)
- Grade4.3.A6 (addressing)
- Grade4.3.B7 (addressing)
- Grade4.3.B8 (addressing)
- Grade4.3.B9 (addressing)
- Grade4.3.B10 (addressing)
- Grade4.3.B11 (addressing)
- Grade4.3.B12 (addressing)
- Grade4.3.C16 (addressing)
- Grade4.3.C17 (addressing)
- Grade4.3.C18 (addressing)
- Grade4.3.C19 (addressing)
- Grade4.3.C20 (addressing)

Stage Narrative

Students roll 3 number cubes to generate a multiplication expression with a whole number and a fraction and compare the value of the expression to 1 in order to determine how many points are earned. Two recording sheets are provided, one where the fraction is a unit fraction and one where it can be any fraction.

Variation:

Students may choose a different target number to compare the value of their expression to.

Stage Description

Each group of 2 needs 3 number cubes.

Standards Alignments

Addressing 4.NF.B.4

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 2 Recording Sheet
(groups of 1)

Stages used in Grade 3

Stage 1

Addressing

- Grade3.5.C
- Grade3.5.D

Center: Compare (1–5)

Stage 3: Multiply within 100

Lessons

- Grade4.3.C16 (supporting)
- Grade4.3.C17 (supporting)
- Grade4.3.C18 (supporting)
- Grade4.3.C19 (supporting)
- Grade4.3.C20 (supporting)

Stage Narrative

Students use cards with multiplication expressions within 100.

Standards Alignments

Addressing 3.OA.C.7

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 3 Multiplication Cards (groups of 2)

Stage 5: Fractions

Lessons

- Grade4.3.A1 (supporting)
- Grade4.3.A2 (supporting)
- Grade4.3.A3 (supporting)
- Grade4.3.A4 (supporting)

Stage Narrative

Students use cards with fractions. They may use either deck of fraction cards or combine them together to play.

Standards Alignments

Addressing 4.NF.A.2

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2)

Stage 6: Add and Subtract Fractions**Lessons**

- Grade4.3.B10 (addressing)
- Grade4.3.B11 (addressing)
- Grade4.3.B12 (addressing)
- Grade4.3.B14 (addressing)
- Grade4.3.C15 (addressing)
- Grade4.3.C16 (addressing)
- Grade4.3.C17 (addressing)
- Grade4.3.C18 (addressing)
- Grade4.3.C19 (addressing)
- Grade4.3.C20 (addressing)

Stage Narrative

Students use cards with expressions with addition and subtraction of fractions with the same denominator.

Standards Alignments

Addressing 4.NF.B.3

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 6 Cards (groups of 2)

Stages used in Grade 3**Stage 2****Supporting**

- Grade3.4.C

Stage 3

Addressing

- Grade3.4.C

Supporting

- Grade3.6.D

Stage 4

Addressing

- Grade3.4.D

Supporting

- Grade3.7.C
- Grade3.7.D

Center: Estimate and Measure (1–4)

Stage 3: Quarter Inches

Lessons

- Grade4.3.A5 (supporting)
- Grade4.3.A6 (supporting)
- Grade4.3.B7 (supporting)
- Grade4.3.B8 (supporting)
- Grade4.3.B9 (supporting)

Stage Narrative

Students choose and estimate the length of the object and then measure to see the actual length to the nearest $\frac{1}{4}$ inch.

Standards Alignments

Addressing 3.MD.B.4

Materials to Gather

Rulers (inches)

Materials to Copy

Estimate and Measure Stage 3 Recording Sheet
(groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stage 4: Eighth Inches

Lessons

- Grade4.3.B13 (addressing)

Stage Narrative

Students choose and estimate the length of the object and then measure to see the actual length to the nearest $\frac{1}{8}$ Inch.

Standards Alignments

Addressing 4.MD.B.4

Materials to Gather

Rulers (inches)

Materials to Copy

Estimate and Measure Stage 4 Recording Sheet
(groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stages used in Grade 3**Stage 2****Supporting**

- Grade3.6.A

Stage 3**Addressing**

- Grade3.6.A

Center: Target Measurements (2–5)

Stage 2: Quarter Inches

Lessons

- Grade4.3.A5 (supporting)
- Grade4.3.A6 (supporting)
- Grade4.3.B7 (supporting)
- Grade4.3.B8 (supporting)
- Grade4.3.B9 (supporting)

Stage Narrative

Students try to draw a line segment as close as possible to the length of the target measurement (in quarter inches).

Standards Alignments

Addressing 3.MD.B.4

Materials to Gather

Paper, Rulers (inches)

Materials to Copy

Target Measurement Stage 2 Recording Sheet
(groups of 2)

Stage 3: Eighth Inches

Lessons

- Grade4.3.B13 (addressing)

Stage Narrative

Students try to draw a line segment as close as possible to the length of the target measurement (in eighth inches).

Standards Alignments

Addressing 4.MD.B.4

Materials to Gather

Paper, Rulers (inches)

Materials to Copy

Target Measurement Stage 3 Recording Sheet
(groups of 2)

Stages used in Grade 3

Stage 1

Supporting

- Grade3.6.A

Stage 2

Addressing

- Grade3.6.A
- Grade3.6.B
- Grade3.6.C

Center: Creating Line Plots (2–5)

Stage 2: Quarter Inches

Lessons

- Grade4.3.B10 (supporting)
- Grade4.3.B11 (supporting)
- Grade4.3.B12 (supporting)
- Grade4.3.B13 (supporting)

Stage Narrative

Students measure up to eight objects to the nearest quarter inch. They work with a partner to create a line plot to represent their measurement data. Then, they ask their partner two questions that can be answered based on the data in their line plot.

Variation:

If students completed the Estimate and Measure Center, they may choose to use their length measurements to represent on the line plot.

Standards Alignments

Addressing 3.MD.B.4

Materials to Gather

Objects of various lengths, Rulers (inches)

Materials to Copy

Creating Line Plots Stage 2 Recording Sheet
(groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stage 3: Eighth Inches, Add and Subtract

Lessons

- Grade4.3.B14 (addressing)

Stage Narrative

Students measure up to eight objects to the nearest $\frac{1}{8}$ inch. They work with a partner to create a line plot to represent their measurement data. Then, they ask their partner two questions that can be answered based on the data in their line plot that uses addition or subtraction.

Variation:

If students completed the Estimate and Measure Center, they may choose to use their length measurements to represent on the line plot.

Standards Alignments

Addressing 4.MD.B.4

Materials to Gather

Objects of various lengths, Rulers (inches)

Materials to Copy

Creating Line Plots Stage 3 Recording Sheet
(groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stages used in Grade 3

Stage 1

Supporting

- Grade3.6.A

Stage 2

Addressing

- Grade3.6.B
- Grade3.6.C

Center: Jump the Line (2–5)

Stage 2: Add and Subtract Tenths and Hundredths

Lessons

- Grade4.3.C15 (addressing)

Stage Narrative

Both players start at 0 on a number line marked by $\frac{1}{100}$. Spinners show adding or subtracting tenths or hundredths.

Standards Alignments

Addressing 4.NF.C

Materials to Gather

Dry erase markers, Paper clips, Sheet protectors

Materials to Copy

Jump the Line Stage 2 Gameboard (groups of 2),
Jump the Line Stage 2 Spinner (groups of 2)

Additional Information

Each group of 2 needs a sheet protector, a dry erase marker, and a paper clip.

Section A: Equal Groups of Fractions

Lesson 1: Equal Groups of Unit Fractions

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.a

Building Towards 4.NF.B.4

Teacher-facing Learning Goals

- Interpret and relate descriptions, drawings, and expressions that represent situations involving equal groups of fractions.

Student-facing Learning Goals

- Let's look at equal groups of fractions.

Lesson Purpose

The purpose of this lesson is for students to interpret and relate descriptions, drawings, and multiplication expressions that represent equal groups of unit fractions.

In grade 3, students represented multiplication of whole numbers with arrays, equal-group drawings, area diagrams, and expressions. In an earlier unit, students used diagrams to represent and compare fractions. In this unit, they extend their understanding of multiplication to include equal groups of unit fractions while using familiar representations to support their thinking.

Students begin by looking at situations that involve fractional amounts of food items. Students may rely on given images and descriptions, their own drawings, their understanding of fractions, and what they know about writing and evaluating multiplication expressions for equal groups of whole-number objects. In future lessons, student will use more abstract diagrams and generalize the process of multiplying a whole number and a unit fraction.

Access for:



Students with Disabilities

- Representation (Activity 1)



English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Which question did you ask today that best supported students' understanding of multiplication of a fraction by a whole number? What did students say or do that showed the question was effective?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Sandwiches on Plates

Standards Alignments

Addressing 4.NF.B.4

Student-facing Task Statement

Lin has 9 plates. She puts $\frac{1}{4}$ of a sandwich on each plate.

- Which expression represents the sandwiches in this situation?
 - 9×4
 - $9 \times \frac{1}{4}$
 - 4×9
 - $4 \times \frac{1}{9}$
- How many sandwiches did Lin put on plates? Explain or show your reasoning.

Student Responses

- B
- $\frac{9}{4}$ sandwiches or $2\frac{1}{4}$ sandwiches. Sample responses:
 - A diagram showing 9 groups of $\frac{1}{4}$
 - I counted by $\frac{1}{4}$ nine times.
 - I know 4 groups of $\frac{1}{4}$ sandwiches is 1 whole sandwich, so 8 groups of $\frac{1}{4}$ sandwiches

make 2 whole sandwiches. Adding $\frac{1}{4}$ sandwich makes $2\frac{1}{4}$.

Begin Lesson

Warm-up

🕒 10 min

How Many Do You See: Oranges

Standards Alignments

Building Towards 4.NF.B.4

The purpose of this How Many Do You See is to elicit ideas about equal groups of fractional amounts and to prepare students reason about multiplication of a whole number and a fraction. Students may describe the oranges with a whole number without units or without specifying “halves” (for instance, they may say “5”). If this happens, consider asking them to clarify whether they mean “5 oranges” or another amount.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?



Student Responses

- Five plates with $\frac{1}{2}$ orange in each
- Five $\frac{1}{2}$ oranges
- Two whole oranges and half of another orange

Launch

- Groups of 2
- “How many do you see? How do you see them?”
- Display the image.
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “How might you describe this image to a friend?” (There are 5 plates with $\frac{1}{2}$ orange on each plate.)
- “How many groups do you see?” (I see 5 plates or 5 groups.)
- “Besides describing the image in words, how else might you represent the quantity in this image?” (I might write $\frac{1}{2}$ five times, or write an expression with five $\frac{1}{2}$ s being added together. I might write “5 times $\frac{1}{2}$.”)
- “We’ll look at some other situations involving groups and fractional amounts in this lesson.”

Activity 1

🕒 20 min

Crackers, Kiwis, and More

Standards Alignments

Addressing 4.NF.B.4.a

The purpose of this activity is for students to interpret situations involving equal groups of a fractional amount and to connect such situations to multiplication of a whole number by a fraction (MP2).

Students write expressions to represent the number of groups and the size of each group. They reason about the quantity in each situation in any way that makes sense to them. Although images of the food items are given, students may choose to create other diagrams, such as equal-group diagrams used in grade 3, when they learned to multiply whole numbers. This activity enables the teacher to see the representations toward which students gravitate.

Focus the discussions on connecting equal groups with fractions and those with whole numbers.

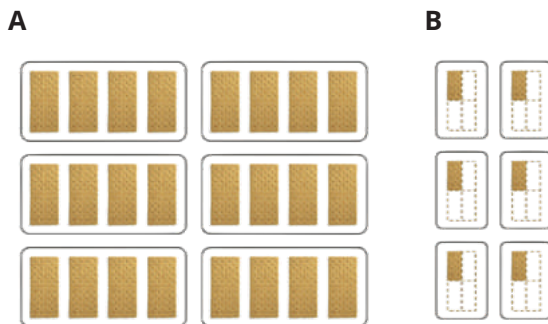
Access for Students with Disabilities

Representation: Access for Perception. Use pictures (or actual crackers, if possible) to represent the situation. Ask students to identify correspondences between this concrete representation and the diagrams they create or see.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

1. Here are images of some crackers.



- How are the crackers in image A like those in B?
 - How are they different?
 - How many crackers are in each image?
 - Write an expression to represent the crackers in each image.
2. Here are more images and descriptions of food items. For each, write a multiplication expression to represent the quantity. Then, answer the question.

- Clare has 3 baskets. She put 4 eggs into each basket. How many eggs did she put in baskets?



- Diego has 5 plates. He put $\frac{1}{2}$ of a kiwi

Launch

- Groups of 2
- “What are some of your favorite snacks?”
- Share responses.
- “What are some snacks that you might break into smaller pieces rather than eating them whole?”
- 1 minutes: partner discussion
- “Let’s look at some food items that we might eat whole or cut or break up into smaller pieces.”

Activity

- “Take a few quiet minutes to think about the first set of problems about crackers. Then, discuss your thinking with your partner.”
- 4 minutes: independent work time
- 2 minutes: partner discussion
- Pause for a whole-class discussion. Invite students to share their responses.
- If no students mention that there are equal groups, ask them to make some observations about the size of the groups in each image.
- Discuss the expressions students wrote:
 - “What expression did you write to represent the crackers in Image A? Why?” (6×4 , because there are 6 groups of 4 full crackers.)

fruit on each plate. How many kiwis did he put on plates?



- c. Priya prepared 7 plates with $\frac{1}{8}$ of a pie on each. How much pie did she put on plates?



- d. Noah scooped $\frac{1}{3}$ cup of brown rice 8 times. How many cups of brown rice did he scoop?



Student Responses

- Alike: They both show 6 groups of crackers.
 - Different: Image A shows 4 crackers in each group. Image B shows $\frac{1}{4}$ crackers in each group.
 - Image A shows 24 crackers. Image B shows $\frac{6}{4}$ crackers.
 - Image A: 6×4 . Image B: $6 \times \frac{1}{4}$.
- 3×4 , 12 eggs
 - $5 \times \frac{1}{2}$, $\frac{5}{2}$ kiwis
 - $7 \times \frac{1}{8}$, $\frac{7}{8}$ of a pie
 - $8 \times \frac{1}{3}$, $\frac{8}{3}$ cups of brown rice

- “What about the crackers in Image B? Why?” ($6 \times \frac{1}{4}$, because there are 6 groups of $\frac{1}{4}$ of a cracker.)

- Ask students to complete the remaining problems.
- 5 minutes: independent or partner work time
- Monitor for students who reason about the quantities in terms of “___ groups of ___” to help them write expressions.

Synthesis

- Select previously identified students to share their expressions and how they reasoned about the amount of food in each image. Record their expressions and supporting diagrams, if any, for all to see.
- If students write addition expressions to represent the quantities, ask if there are other expressions that could be used to describe the equal groups.
- “How is the quantity in Clare's situation different than those in other situations?” (It involves whole numbers of items. Others involve fractional amounts.)
- “How is the expression you wrote for the eggs different than other expressions?” (It shows two whole numbers being multiplied. The others show a whole number and a fraction.)

Advancing Student Thinking

If students are unsure how to name the quantity in the image, consider asking: "How would you describe the amount of the slice of pie on one plate? How would you describe two of the same slices? Three of the same slices?"

Activity 2

🕒 15 min

What Could It Mean?

Standards Alignments

Addressing 4.NF.B.4

In this activity, students start with given multiplication expressions and consider situations or diagrams that they could represent. Situating the expressions in context encourages students to think of the whole number in the expression as the number of groups and the fractional amount as the size of each group, which helps them reason about the value of the expression. When students make explicit connections between multiplication situations, expressions, and drawings they reason abstractly and quantitatively (MP2).

Allow students to use fraction strips, fraction blocks, or other manipulatives that show fractional amounts to support their reasoning.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Student-facing Task Statement

For each expression:

- Write a story that the expression could represent. The story should be about a situation with equal groups.
- Create a drawing to represent the

Launch

- Groups of 2
- "Read the task statement. Then, talk to your partner about what you are asked to do in this activity."
- 1 minute: partner discussion

situation.

- Find the value of the expression. What does this number mean in your story?

1. $8 \times \frac{1}{2}$

2. $7 \times \frac{1}{5}$

Student Responses

Sample responses:

- Jada put half of an apple into each of 8 bags.
 - A drawing of 8 groups of $\frac{1}{2}$
 - $\frac{8}{2}$ or 4. Jada put a total of 4 apples in bags.
- A baker put $\frac{1}{5}$ kilogram of raisins in each of 7 bags.
 - A drawing of 7 groups of $\frac{1}{5}$
 - $\frac{7}{5}$. This tells us the number of kilograms of raisins in bags.

Activity

- “Choose one expression you’d like to start with.”
- “Think of a story that can be represented by the expression. Then, create a drawing or diagram, and find the value of the expression.”
- “If you have extra time you can work on both problems.”
- 7–8 minutes: independent work time
- “Be sure to say what the value of the expression means in your story.”

Synthesis

- Invite students to share their responses. Display their drawings or visual representations for all to see.
- “How did you decide what the numbers in each expression represent in your story?” (It made sense for the whole numbers to represent how many groups there are and the fractions to represent what is in each group.)
- “How did you show the whole number and the fraction in your drawing?” (I drew as many circles as the whole number to show the groups. I drew parts of objects or wrote numbers in each circle to show the fraction.)

Lesson Synthesis

🕒 10 min

“Today we looked at different situations that involved equal-size groups and a fractional amount in each group. We thought about how to find the total amount in each situation.”

“How did we represent these situations?” (We wrote expressions and used drawings or pictures to show the equal groups.)

“What kind of expressions did we write?” (Multiplication expressions with a whole number and a fraction in each)

“What strategies did we use to find the total amount in each situation?” (We counted the number of fractional parts in the drawings. We counted how many parts made 1 whole and saw how many extra fractional parts there were.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not relate the equal-group situation to a multiplication expression.

Next Day Support

- During the warm-up, review the representation used to solve the problem. Consider asking students to identify parts of the problem in the representation.

Lesson 2: Representations of Equal Groups of Fractions

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.a, 4.NF.B.4.c
 Building Towards 4.NBT.B.5, 4.NF.B.4.a

Teacher-facing Learning Goals

- Interpret diagrams and expressions that represent multiplication of a whole number and a unit fraction.
- Use diagrams and expressions to represent and find the product of a whole number and a unit fraction.

Student-facing Learning Goals

- Let's look at diagrams and expressions that can help us multiply a whole number and a fraction.

Lesson Purpose

The purpose of this lesson is for students to interpret and generate diagrams and expressions that represent multiplication of a whole number and a unit fraction in order to find the value of the product.

In this lesson, students interpret and relate multiplication expressions and diagrams that represent products of whole numbers and fractions. After matching expressions and diagrams in a card-sort activity, they practice using diagrams and expressions to find the result of multiplying a whole number and a fraction. They draw a diagram given a multiplication expression, or write an expression given a diagram (MP2).

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Number Talk (Warm-up)

Materials to Copy

- Expressions and Diagrams (groups of 2):

Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 25 min |
| Activity 2 | 10 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Revisit class norms and routines. Are all students contributing to the conversation? Do some students' ideas seem to hold more value in the dynamics of the group? Are there any adjustments you might make so that all students do math tomorrow?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

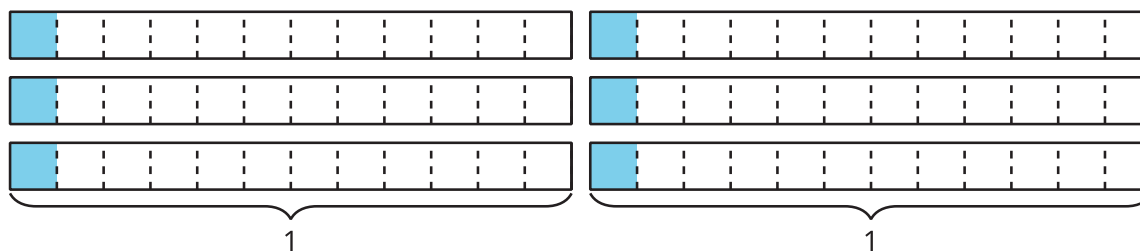
Equal Groups of Fractions

Standards Alignments

Addressing 4.NF.B.4

Student-facing Task Statement

Write a multiplication expression to represent the shaded parts of the diagram. Then, find its value. Explain or show your reasoning.

**Student Responses**

$6 \times \frac{1}{12}$. Its value is $\frac{6}{12}$. Sample response:

- There are 6 equal groups of $\frac{1}{12}$.
- I counted by $\frac{1}{12}$ six times.

- If all the shaded parts are moved to a single rectangle that represents 1 whole, they would take up 6 parts, which represent $\frac{6}{12}$.

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Three, Six, Nine, Twelve

Standards Alignments

Building Towards 4.NBT.B.5

This Number Talk encourages students to use their knowledge of multiplication facts, properties of operations, and the structure of the given expressions to mentally solve problems. The reasoning elicited here will be helpful in upcoming lessons as students find products of whole numbers and non-unit fractions (such as $3 \times \frac{6}{10}$ or $6 \times \frac{9}{4}$).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 3×6
- 3×9
- 6×9
- 12×9

Student Responses

- 18: I just knew it.
- 27: It is 3×3 more than 3×6 , or 9 more than 18, which is 27.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

- 54: I knew 3×9 is 27 and 6×9 is twice that number, so it is 2×27 .
- 108:
 - 12×9 is twice 6×9 , so it is 2×54 , which is 108.
 - 12×10 is 120 and 12×9 is 12 less than 120, which is 108.

Synthesis

- “What did you notice about the factors in all of the expressions?” (They are all multiples of 3.)
- “Did noticing that all the factors are multiples of 3 help you find the values?” (Sample responses:
 - Yes, I was able to think of “3 more groups of something.”
 - Yes, it helped me see how the factors were related, which helped me reason about the products.
 - No, it didn't, but it made me think that the values would be multiples of 3 as well.)

Activity 1

🕒 25 min

Card Sort: Expressions and Diagrams

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 4.NF.B.4
Building Towards 4.NF.B.4.a

In this activity, students interpret multiplication expressions and diagrams as the number of groups and amount in each group and match representations of the same quantity. They then use their insight from the matching activity to generate diagrams for expressions without a match and to find their values (MP2).

Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____, so I matched . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Organization, Conceptual Processing

Instructional Routines

Card Sort

Materials to Copy

Expressions and Diagrams (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards with expressions and diagrams.

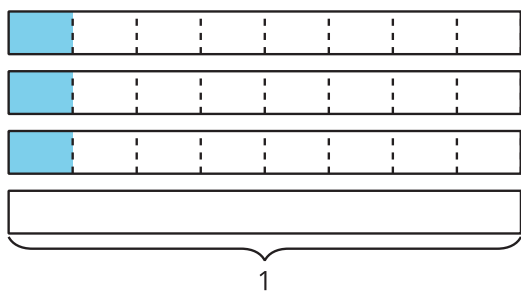
1. Match each expression with a diagram that represents the same quantity.
2. Record each expression without a match.
3. Han started drawing a diagram to represent $7 \times \frac{1}{8}$ and did not finish. Complete his diagram. Be prepared to explain your reasoning.

Launch

- Groups of 2
- Give each group a set of cards from the Instructional master.

Activity

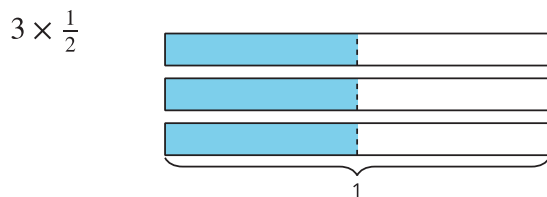
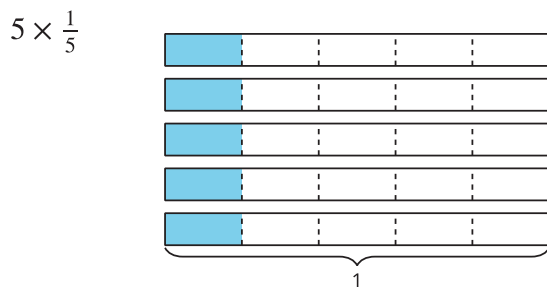
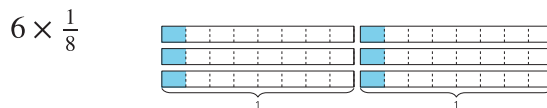
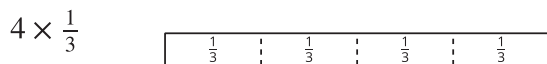
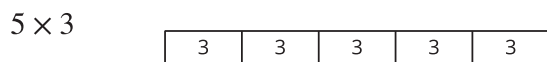
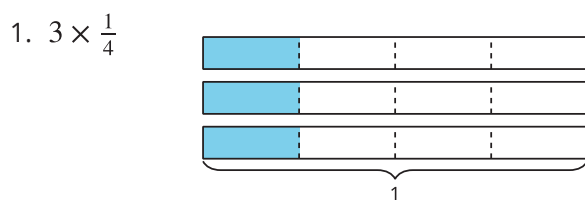
- “Work with your partner to match each expression to a diagram that represents the same equal-group situation and the same amount.”
- “Be prepared to explain how you know the two representations belong together.”
- 5 minutes: partner work time
- Monitor for students who reason about the number of groups and amount in each



4. Choose one expression that you recorded earlier that didn't have a match.

Draw a diagram that can be represented by the expression. What value do the shaded parts of your diagram represent?

Student Responses



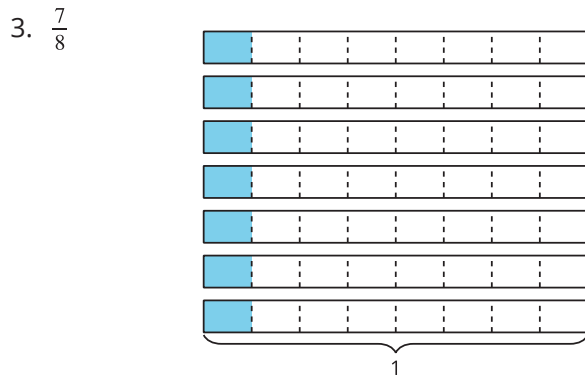
group as they match.

- Pause for a discussion. Select students to share their matches and reasoning.
- Highlight reasoning that clearly connects one factor in the expression to the number of groups and the other factor to the size of each group.
- “Now you will complete an unfinished diagram for $7 \times \frac{1}{8}$ and then draw a new diagram for an expression without a match.”
- 5 minutes: independent work time

Synthesis

- “What was missing from Han’s diagram? How do you know?” (4 more groups of $\frac{1}{8}$ were missing, because $7 \times \frac{1}{8}$ means 7 groups of $\frac{1}{8}$ and there are only 3 in Han’s diagram.)
- “If the expression was for 7 groups of $\frac{1}{3}$ instead of $\frac{1}{8}$ how would Han’s diagram change?” (Each rectangle representing 1 would have 3 equal parts with 1 shaded.)
- Select students to share the diagrams they drew for the expressions without a match. Ask them to point out the number of groups and size of each group in each diagram.

2. $6 \times \frac{1}{6}$ no match
 $8 \times \frac{1}{2}$ no match
 3×4 no match
 $2 \times \frac{1}{12}$ no match



4. Answers vary, but diagrams should show as many groups of the unit fraction in the expression as the whole number in the expression.

Advancing Student Thinking

If students are not yet matching expressions to appropriate diagrams, consider asking them to compare the diagrams for 5×3 and $5 \times \frac{1}{3}$ and reason about the number of groups and the size of each group. Consider asking: “How are these alike? How are they different?”

Activity 2

🕒 10 min

Different Representations

Standards Alignments

Addressing 4.NF.B.4.a, 4.NF.B.4.c

This activity prompts students to use their earlier observations to generate a diagram or expression that represents equal groups of unit fractions when one or the other is given. In one

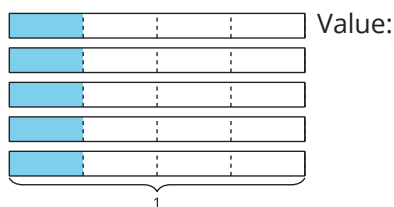
of the problems, only the total quantity ($\frac{7}{2}$) is given, so students need to reason in about the number of groups and the size of each group that could lead to this value. Finally, they analyze two different ways of representing $4 \times \frac{1}{3}$ with a diagram, which further illustrates that the value of the expression is $\frac{4}{3}$.

Student-facing Task Statement

1. a. Write a multiplication expression that represents the shaded parts of the diagram. Then, find the value of the expression.

Diagram:

Expression:



- b. Draw a diagram that the expression $6 \times \frac{1}{3}$ could represent. Then, find the value of the expression.

Diagram:

Expression:
 $6 \times \frac{1}{3}$

Value:

- c. Draw a diagram and write an expression that gives the value $\frac{7}{2}$.

Diagram:

Expression:

Value: $\frac{7}{2}$

2. To represent $4 \times \frac{1}{3}$, Diego drew this diagram:

Launch

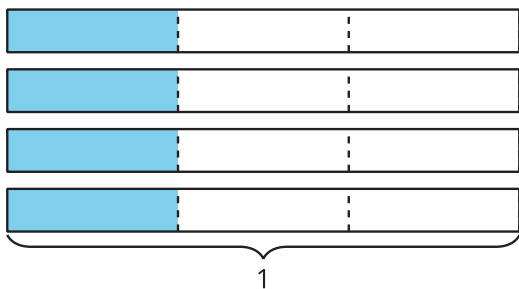
- Groups of 2
- “Turn to a partner and explain what needs to be done to complete the first problem.”

Activity

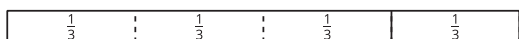
- “Complete the first problem independently. Afterwards, pause for a class discussion.”
- 5 minutes: independent work time
- Pause to discuss the fraction $\frac{7}{2}$ in the first problem.
- “How did you know what diagram and expression would have the value $\frac{7}{2}$?” (Sample response:
 - For the diagram, the numerator, 7, is the number of groups, and the denominator, 2, shows how many parts are in 1 whole.
 - For the expression, I multiplied a whole number and a fraction. The whole number was the same as the number in the numerator of $\frac{7}{2}$ and the fraction has the same number as the denominator of $\frac{7}{2}$.)
- “Work on the last problem with your partner.”
- 5 minutes: partner work time

Synthesis

- See lesson synthesis.



Elena drew this diagram:



Are they representing the same expression and value? Explain or show how you know.

Student Responses

1.
 - a. $5 \times \frac{1}{4}, \frac{5}{4}$
 - b. A diagram showing 6 groups of $\frac{1}{3}, \frac{6}{3}$
 - c. A diagram showing 7 groups of $\frac{1}{2}, 7 \times \frac{1}{2}$
2. Yes, they both show $\frac{4}{3}$. Sample response: In Diego's diagram, the each $\frac{1}{3}$ is shaded in separate 1 wholes. In Elena's, the four $\frac{1}{3}$ s are shown side-by-side in a single row and each part is labeled $\frac{1}{3}$.

Advancing Student Thinking

Students may be unsure about how to begin writing expressions for fractions. Remind students that the fraction will be written as a whole number times a unit fraction. Consider asking: "How might this help to write the expression?"

Lesson Synthesis

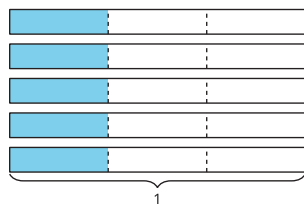
🕒 10 min

"Today we analyzed expressions and diagrams that represent equal groups and created some of these

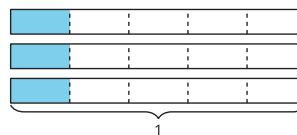
representations.”

Display or sketch these diagrams:

A



B



“How do we know which diagram represents $3 \times \frac{1}{5}$? Where do we see each number in the diagram?” (B represents $3 \times \frac{1}{5}$ because it shows 3 groups of $\frac{1}{5}$.)

“What expression does the other diagram represent?” (A represents $5 \times \frac{1}{5}$, because it shows 5 groups with $\frac{1}{5}$ in each group.)

“What is the value of $3 \times \frac{1}{5}$? How do we know?” ($\frac{3}{5}$. We can count the number of shaded fifths and see that there are 3.)

“What is the value of $5 \times \frac{1}{5}$?” ($\frac{5}{5}$)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students write the number of shaded parts in each diagram rather than the value (for example, they write 6×1 instead of $6 \times \frac{1}{12}$).

Next Day Support

- Before the warm-up, display the diagram and ask students to name the value each shaded part represents. Discuss how knowing that value might help us write a multiplication expression to represent the value of all the shaded parts.

Lesson 3: Patterns in Multiplication

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.a

Teacher-facing Learning Goals

- Evaluate multiplication expressions and recognize that $n \times \frac{1}{b} = \frac{n}{b}$.

Student-facing Learning Goals

- Let's look at patterns in multiplication of a fraction by a whole number.

Lesson Purpose

The purpose of this lesson is for students to understand that every fraction can be written as the product of a whole number and unit fraction.

In this lesson, students analyze two sets of multiplication expressions: one in which the number of groups is kept constant, and another in which the size of each group (a unit fraction) is kept constant. They look for regularity as they reason repeatedly about the expressions and their values (MP8). The patterns that emerge in the series of expressions formalize their prior observations about the value of $a \times \frac{1}{b}$ as $\frac{a}{b}$. They also enable students to see any fraction as a product of a whole number and unit fraction.

Note that students may write either $a \times \frac{1}{b} = \frac{a}{b}$ or $\frac{1}{b} \times a = \frac{a}{b}$ as long as they understand what each factor represents. Teachers can reinforce the meaning of each factor by consistently writing the multiplication in this order: number of groups \times size of each group = total amount. This corresponds to how we tend to express situations with equal groups, which in the case of fractional amounts, is “____ (whole number) groups of ____ (fraction).”

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Choral Count (Warm-up)

Materials to Gather

- Paper: Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

In tomorrow's lesson, students multiply a non-unit fraction by a whole number, such as $5 \times \frac{2}{3}$. How can students apply their understanding from today to reason about these expressions tomorrow?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Fraction Multiplication

Standards Alignments

Addressing 4.NF.B.4

Student-facing Task Statement

1. Complete each equation to make it true. Show your thinking using words or diagrams.

a. $5 \times \frac{1}{8} = \underline{\hspace{2cm}}$

b. $\underline{\hspace{2cm}} \times \frac{1}{3} = \frac{7}{3}$

2. Write each fraction as the product of a whole number and unit fraction.

a. $\frac{8}{9} = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

b. $\frac{6}{5} = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

Student Responses

1. a. $5 \times \frac{1}{8} = \frac{5}{8}$. Sample response: Five groups of 1 eighth make 5 eighths.

b. $7 \times \frac{1}{3} = \frac{7}{3}$. Sample response: A diagram showing 7 groups of $\frac{1}{3}$

2. a. $8 \times \frac{1}{9}$

b. $6 \times \frac{1}{5}$

Begin Lesson

Warm-up

🕒 10 min

Choral Count: $\frac{1}{4}$ and $\frac{1}{8}$ **Standards Alignments**

Addressing 4.NF.B.4

The purpose of this Choral Count is to invite students to practice counting by a unit fraction and notice patterns in the count. These understandings will be helpful later in this lesson when students recognize every fraction can be written as the product of a whole number and unit fraction.

Instructional Routines

Choral Count

Student Responses

- Start recording $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots$ and stop at $\frac{11}{4}$.
- Start recording $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \dots$ and stop at $\frac{15}{8}$.

Launch

- “Count by $\frac{1}{4}$, starting at 0.”

Activity

- Record as students count.
- Stop counting and recording at $\frac{11}{4}$.
- Repeat with $\frac{1}{8}$.
- Stop counting and recording at $\frac{15}{8}$.

Synthesis

- “What patterns do you notice?” (In both counts, the numerators go up by 1, and denominators stay the same.)
- “How many groups of $\frac{1}{4}$ do we have?” (11)
- “Where do you see them?” (Each count represents a new group of $\frac{1}{4}$.)
- “How might we represent 11 groups of $\frac{1}{4}$ with an expression?” ($11 \times \frac{1}{4}$)

- “How many groups of $\frac{1}{8}$ do we have?” (15)
- “How might we represent 15 groups of $\frac{1}{8}$ with an expression?” ($15 \times \frac{1}{8}$)
- “How would our count change if we counted by $\frac{2}{4}$ or $\frac{2}{8}$?” (Each numerator would be a multiple of 2 or an even number.)

Activity 1

🕒 15 min

Describe the Pattern

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.a

Students may have previously noticed a connection between the whole number in a given multiplication expression and the numerator of the fraction that is the resulting product. In this activity, they formalize that observation. Students reason repeatedly about the product of a whole number and a unit fraction, observe regularity in the value of the product, and generalize that the numerator in the product is the same as the whole-number factor (MP8).

🕒 Access for Students with Disabilities

Representation: Develop Language and Symbols. Provide students with access to a chart that shows definitions and examples of the terms that will help them articulate the patterns they see, including whole number, fraction, numerator, denominator, unit fraction, and product.

Supports accessibility for: Language, Memory

Student-facing Task Statement

1. Here are two tables with expressions. Find the value of each expression. Use a diagram if you find it helpful.

Leave the last two rows of each table blank for now.

Launch

- Groups of 2
- “Work with your partner to complete the tables. One person should start with Set A and the other with Set B.”
- “Afterwards, analyze your completed tables together and look for patterns.”

Set A

| expression | value |
|------------------------|-------|
| $1 \times \frac{1}{8}$ | |
| $2 \times \frac{1}{8}$ | |
| $3 \times \frac{1}{8}$ | |
| $4 \times \frac{1}{8}$ | |
| $5 \times \frac{1}{8}$ | |
| $6 \times \frac{1}{8}$ | |
| | |
| | |

Set B

| expression | value |
|------------------------|-------|
| $2 \times \frac{1}{3}$ | |
| $2 \times \frac{1}{4}$ | |
| $2 \times \frac{1}{5}$ | |
| $2 \times \frac{1}{6}$ | |
| $2 \times \frac{1}{7}$ | |
| $2 \times \frac{1}{8}$ | |
| | |
| | |

- Study your completed tables. What patterns do you see in how the expressions and values are related?
- In the last two rows of the table of Set A, write $\frac{11}{8}$ and $\frac{13}{8}$ in the "value" column. Write the expressions with that value.
- In the last two rows of the table of Set B, write $\frac{2}{12}$ and $\frac{2}{15}$ in the "value" column. Write the expressions with that value.

Student Responses

- Set A: $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}$
Set B: $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \frac{2}{7}, \frac{2}{8}$
- Sample responses:
 - In Set A, the value has 8 for the denominator, which is the same as the fraction in the expression. The value goes up by $\frac{1}{8}$ each time because the number of groups increases by 1 each time.
 - In Set B, the value is a fraction with 2 for the numerator because the expression is always 2 groups of a unit fraction. The denominator is the same as the fraction in the expression.

Activity

- 5–7 minutes: partner work time on the first two problems
- Monitor for the language students use to explain patterns:
 - The whole number in each expression is only being multiplied by the numerator of each fraction.
 - Language describing patterns in the denominator of the product (The denominator in the product is the same as the unit fraction each time.)
 - "Groups of" language to justify or explain patterns (The number of groups of each unit fraction is going up each time because it is one more group.)
- "Pause after you've described the patterns in the second problem."
- Select 1–2 students to share the patterns they observed.
- "Now let's apply the patterns you noticed to complete the last two problems."
- 3 minutes: independent or partner work time

Synthesis

- "How did you use the patterns to write expressions for $\frac{11}{8}, \frac{13}{8}, \frac{2}{12},$ and $\frac{2}{15}$?" (I knew that each expression had a whole number and a unit fraction. The whole number is the same as the numerator of the product.)
- Select students to share their multiplication expressions for these four fractions.
- "Can you write any fraction as a multiplication expression using its unit fraction?" (Yes, because the numerator is the number of groups and the denominator represents the size of each group.)

- In both sets, the value is a fraction and its numerator is the result of multiplying the whole number by the numerator of the fraction in the expression.
3. $11 \times \frac{1}{8}$ and $13 \times \frac{1}{8}$
 4. $2 \times \frac{1}{12}$ and $2 \times \frac{1}{15}$
- “What would it look like to write $\frac{3}{10}$ as a multiplication expression using a whole number and a unit fraction?” ($\frac{3}{10} = 3 \times \frac{1}{10}$)

Activity 2

 20 min

What's Missing?

Standards Alignments

Addressing 4.NF.B.4.a

This activity serves two main purposes. The first is to allow students to apply their understanding that the result of $a \times \frac{1}{b}$ is $\frac{a}{b}$. The second is for students to reinforce the idea that any non-unit fraction can be viewed in terms of equal groups of a unit fraction and expressed as a product of a whole number and a unit fraction.

The activity uses a “carousel” structure in which students complete a rotation of steps. Each student writes a non-unit fraction for their group mates to represent in terms of equal groups, using a diagram, and as a multiplication expression. The author of each fraction then verifies that the representations by others indeed show the written fraction. As students discuss and justify their decisions they create viable arguments and critique one another’s reasoning (MP3).

Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support small-group discussion after checking their fraction diagram and equation: “I agree because . . .”, “I disagree because . . .”
Advances: Conversing

Materials to Gather

Paper

Student-facing Task Statement

- Use the patterns you observed earlier to complete each equation so that it's true.
 - $5 \times \frac{1}{10} = \underline{\hspace{2cm}}$
 - $8 \times \frac{1}{6} = \underline{\hspace{2cm}}$
 - $4 \times \underline{\hspace{2cm}} = \frac{4}{5}$
 - $6 \times \underline{\hspace{2cm}} = \frac{6}{10}$
 - $\underline{\hspace{2cm}} \times \frac{1}{4} = \frac{3}{4}$
 - $\underline{\hspace{2cm}} \times \frac{1}{12} = \frac{7}{12}$
- Your teacher will give you a sheet of paper. Work with your group of 3 and complete these steps on the paper. After each step, pass your paper to your right.
 - Step 1: Write a fraction with a numerator other than 1 and a denominator no greater than 12.
 - Step 2: Write the fraction you received as a product of a whole number and a unit fraction.
 - Step 3: Draw a diagram to represent the expression you just received.
 - Step 4: Collect your original paper. If you think the work is correct, explain why the expression and the diagram both represent the fraction that you wrote. If not, discuss what revisions are needed.

Student Responses

- $\frac{5}{10}$
 - $\frac{8}{6}$
 - $\frac{1}{5}$
 - $\frac{1}{10}$

Launch

- Groups of 3
- "Let's now use the patterns we saw earlier to write some true equations showing multiplication of a whole number and a fraction."

Activity

- 3 minutes: independent work time on the first set of problems
- 2 minutes: group discussion
- Select students to explain how they reasoned about the missing numbers in the equations.
- If not mentioned in students' explanations, emphasize that: "We can interpret $\frac{5}{10}$ as 5 groups of $\frac{1}{10}$, $\frac{8}{6}$ as 8 groups of $\frac{1}{6}$, and so on."
- "In an earlier activity, we found that we can write any fraction as a multiplication of a whole number and a unit fraction. You'll now show that this is the case using fractions written by your group mates."
- Demonstrate the 4 steps of the carousel using $\frac{7}{4}$ for the first step.
- Read each step aloud and complete a practice round as a class.
- "What questions do you have about the task before you begin?"
- 5–7 minutes: group work time

Synthesis

- See lesson synthesis.

e. 3

f. 7

2. Answers vary.

Lesson Synthesis

 10 min

“Today we looked at two sets of multiplication expressions. In the first set, the number of groups changed while the unit fraction stayed the same. We found a pattern in their values.”

“Then we looked at expressions in which the unit fraction changed and the number of groups stayed the same. We found a pattern there as well.”

Display the two tables that students completed in the first activity.

“In the first table, why does it make sense that the numerator in the product is the same number as the whole-number factor?” (Because there are as many groups of $\frac{1}{8}$ as the whole-number factor)

“In the second table, why does it make sense that the numerator in the product is always 2?” (Because all the expressions represent 2 groups of a unit fraction.)

“We also discussed how we could write any fraction as a product of a whole number and unit fraction. Tell a partner about how we could write $\frac{8}{3}$ as a product of a whole number and a fraction.” ($\frac{8}{3} = 8 \times \frac{1}{3}$)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may account for only the numerator when writing an expression for a given fraction. For instance, they may reason that there are 2

Next Day Support

- Before the warm-up, pass back the cool-down and work in small groups to make

groups of $\frac{4}{9}$ but write $\frac{8}{9} = 2 \times 4$ instead of $2 \times \frac{4}{9}$.

corrections, being sure to use diagrams to support reasoning.

Lesson 4: Equal Groups of Non-Unit Fractions

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.b

Teacher-facing Learning Goals

- Recognize that $n \times \frac{a}{b} = \frac{n \times a}{b}$
- Use diagrams to represent and evaluate the product of a whole number and a non-unit fraction.

Student-facing Learning Goals

- Let's multiply any fraction by a whole number.

Lesson Purpose

The purpose of this lesson is to apply understandings from previous lessons to multiply a non-unit fraction by a whole number.

Previously, students learned that any unit fraction multiplied by a whole number results in multiples of that unit fraction. They also learned that any fraction can be written as a multiplication expression of a unit fraction by a whole number. In this lesson, students notice that they can multiply any fraction and a whole number by reasoning about the number of groups and amount in each group. They generalize that they can multiply the numerator by the whole number to find the number of parts. They also see that the denominator remains the same because the size of each part is the same. In other words: $n \times \frac{a}{b} = \frac{n \times a}{b}$.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR6 (Activity 1)

Instructional Routines

5 Practices (Activity 1), Notice and Wonder (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |

Teacher Reflection Question

What part of the lesson went really well today in terms of students' learning? What did you do that made that part go well?

| | |
|------------------|--------|
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Cool-down (to be completed at the end of the lesson)

🕒 5 min

What's the Value?

Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.b

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning. Use a diagram if it is helpful.

1. $6 \times \frac{2}{5}$
2. $5 \times \frac{3}{10}$

Student Responses

1. $\frac{12}{5}$. Sample response: Six groups of 2 fifths make 12 fifths.
2. $\frac{15}{10}$. Sample response: Five groups of $\frac{3}{10}$ make $\frac{15}{10}$.

Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Thirds

Standards Alignments

Addressing 4.NF.B.4

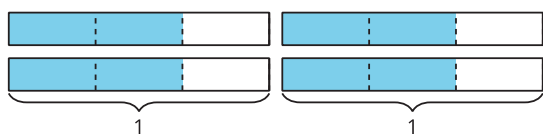
This warm-up prompts students to examine a diagram representing equal groups of non-unit fractions. The understandings elicited here allow students to discuss the relationship between the product of a whole number and a unit fraction and that of a whole number and a non-unit fraction with the same denominator.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 4 wholes.
- 4 rectangles, each partitioned into 3 equal parts
- Each whole has 2 parts shaded.
- There are 4 groups of $\frac{2}{3}$ in each rectangle.

Students may wonder:

- Are we going to work with fractions?
- Will we work with fractions with more than 1 in the numerator?

Launch

- Groups of 2
- Display the image.

Activity

- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- If no students notice or wonder about equal groups, ask, “What groups do you see and how do you see them?” (4 wholes, each whole has $\frac{2}{3}$ shaded)
- “How many thirds do you see?” (8 thirds)
- “How are these diagrams different than those we've seen so far in this unit?” (Previously, each whole has only one shaded part. These have two shaded parts each.)
- “Today, we will think about situations that involve equal groups but now each group has non-unit fractions.”

Activity 1

Jars of Jam

🕒 15 min

Standards Alignments

Addressing 4.NF.B.4.b

In this 5 Practices activity, students reason about a situation that involves finding the product of a whole number and a non-unit fraction. They may rely on what they previously learned about multiplying a whole number and a unit fraction, but can reason in any way that makes sense to them. The goal is to elicit different strategies and help students see the connections between strategies and with their earlier work. Students reason abstractly and quantitatively as they solve the problem (MP2) and construct arguments (MP3) as they share their reasoning during the synthesis.

Monitor for the students who:

- draw a drawing or a diagram to show 5 groups with three $\frac{1}{4}$ s in each group and count the total number of fourths
- reason additively, by finding the value of $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$, or by adding smaller groups of $\frac{3}{4}$ at a time, for instance, 2 groups of $\frac{3}{4}$, another 2 groups, and 1 more group
- reason multiplicatively, for instance, by thinking of $\frac{3}{4}$ as $3 \times \frac{1}{4}$ and then finding $5 \times 3 \times \frac{1}{4}$, or by reasoning about $5 \times \frac{3}{4}$

Students who see the situation as $5 \times \frac{3}{4}$ may, based on their earlier work, generalize that the value is $\frac{5 \times 3}{4}$. Encourage them to clarify how they know this is the case.

During the synthesis, sequence student presentations in the order listed.

Access for English Learners

Reading: MLR6 Three Reads. “We are going to read this 3 times.” After the 1st Read: “Tell your partner what this situation is about.” After the 2nd Read: “List the quantities. What can be counted or measured?” (number of jars, number of friends, number of cups of jam). After the 3rd Read: “What strategies can we use to solve this problem?”

Advances: Reading, Representing.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify details they want to remember. Display the sentence frame: “The next time I need to represent the product of a whole number and a fraction, I will”

Supports accessibility for: Conceptual Processing, Organization, Memory

Instructional Routines

5 Practices

Student-facing Task Statement

Elena fills 5 small jars with homemade jams to share with her friends. Each jar can fit $\frac{3}{4}$ cup of jam. How many cups of jam are in the jars? Explain or show your reasoning.



If you have time: Elena still has some jam left. She takes 2 large jars and puts $\frac{5}{4}$ cups of jam in each jar. How many cups of jam are in the jars?

Student Responses

$\frac{15}{4}$ cups or $3\frac{3}{4}$ cups. Sample reasoning:

- A drawing or diagram showing 5 groups of $\frac{3}{4}$. I counted the shaded fourths and there are 15.
- I know 2 groups of $\frac{3}{4}$ is $\frac{6}{4}$. Another 2 groups of $\frac{6}{4}$ makes $\frac{12}{4}$ or 3. Another $\frac{3}{4}$ makes $\frac{15}{4}$ or $3\frac{3}{4}$.
- $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4}$
- $\frac{3}{4}$ is $3 \times \frac{1}{4}$, so 5 times $\frac{3}{4}$ is $5 \times 3 \times \frac{1}{4}$ or $15 \times \frac{1}{4}$, which is $\frac{15}{4}$.

If you have time: $2 \times \frac{5}{4}$, which is $\frac{10}{4}$ or $2\frac{2}{4}$ cups.

Launch

- Groups of 2
- Read the first problem as a class.
- Invite students to share what they know about homemade jams or any experience in making them.
- If needed, remind students that measuring cups come in different fractional amounts, such as $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

Activity

- “Work independently on the problem. Explain or show your reasoning so that it can be followed by others. Afterwards, share your thinking with your partner.”
- 5 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for the strategies listed in the activity narrative.

Synthesis

- Select previously identified students to share their responses. Display or record their work for all to see.
- “What multiplication expression can represent the amount of jam in the jars? How do you know?” ($5 \times \frac{3}{4}$ or $5 \times 3 \times \frac{1}{4}$, because there are 5 equal groups of $\frac{3}{4}$.)
- “Where do you see the 5 groups in each strategy presented? Where do we see the $\frac{3}{4}$?”
- “How is finding the value of $5 \times \frac{3}{4}$ like finding the value of $5 \times \frac{1}{4}$?” (They're both

about finding the total amount in equal groups. They both involve a whole number of groups and a fraction in each group.)

- “How is it different?” (The amount in each group is a non-unit fraction instead of a unit fraction.)

Advancing Student Thinking

If students are not sure how to represent or reason about 5 groups of $\frac{3}{4}$, consider asking them: “How would you represent (or think about) 5 groups of $\frac{1}{4}$?” and “How can you build on that representation (or strategy) to find how much is in 5 groups of $\frac{3}{4}$?”

Activity 2

🕒 20 min

How Do We Multiply?

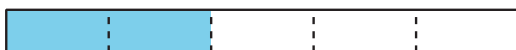
Standards Alignments

Addressing 4.NF.B.4, 4.NF.B.4.b

The purpose of this activity is for students to use diagrams to reason about products of a whole number and a non-unit fraction with diagrams, building on their work with diagrams that represent products of a whole number and a unit fraction. They begin to generalize that the number of shaded parts in a diagram that represents $n \times \frac{a}{b}$ is $n \times a$ and to explain that generalization (MP8).

Student-facing Task Statement

1. This diagram represents $\frac{2}{5}$.



- a. Show how you would use or adjust the

Launch

- Groups of 2
- “Let’s represent some other products of a whole number and a fraction and find their values.”

diagram to represent $4 \times \frac{2}{5}$.

b. What is the value of the shaded parts in your diagram?

2. This diagram represents $\frac{5}{8}$.



a. Show how you would use or adjust the diagram to represent $3 \times \frac{5}{8}$.

b. What is the value of the shaded parts in your diagram?

3. Find the value of each expression. Draw a diagram if you find it helpful. Be prepared to explain your reasoning.

a. $2 \times \frac{1}{6}$

b. $2 \times \frac{4}{6}$

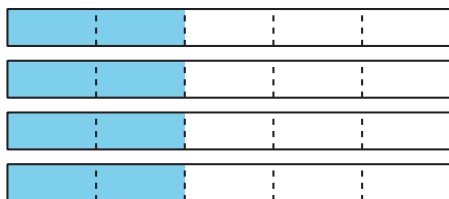
c. $2 \times \frac{5}{6}$

d. $4 \times \frac{5}{6}$

4. Mai said that to multiply any fraction by a whole number, she would multiply the whole number and the numerator of the fraction and keep the same denominator. Do you agree with Mai? Explain your reasoning.

Student Responses

1. a. Sample response:



b. $\frac{8}{5}$

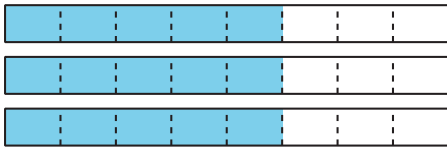
2. a. Sample response:

Activity

- “Take a few quiet minutes to work on the activity. Afterwards, share your responses with your partner.”
- 5–7 minutes: independent work
- 2–3 minutes: partner discussion
- Monitor for the strategies students use to reason about the last two problems.
- Identify students who reason visually (using diagrams), additively, and multiplicatively to share in the synthesis.

Synthesis

- Discuss the four multiplication expressions in the third problem.
- Select 1–2 students who might have drawn diagrams for all expressions.
- “How does your diagram show the value of $2 \times \frac{4}{6}$?” (There are 2 groups of $\frac{4}{6}$, so there are 8 sixths shaded, which is $\frac{8}{6}$.)
- Select 1–2 students who drew a diagram for some expressions and reason numerically for others.
- “Why did you choose to draw a diagram for some expressions and to do something else for others?” (After drawing the first two diagrams, I realized that I’d have to draw a lot of groups or parts, so I thought about the numbers instead.)
- Select 1–2 students who reasoned about all expressions numerically.
- “How did you find the value of the expressions without drawing diagrams at all?” (I saw a pattern in earlier problems, that we can multiply the whole number and the numerator of the fraction and keep the denominator.)
- Discuss the last problem in the lesson synthesis.



- b. $\frac{15}{8}$
3. a. $\frac{2}{6}$
- b. $\frac{8}{6}$
- c. $\frac{10}{6}$
- d. $\frac{20}{6}$
4. Sample response: I agree, because the total number of parts is found by multiplying the numerator by the whole number and the size of the part stays the same.

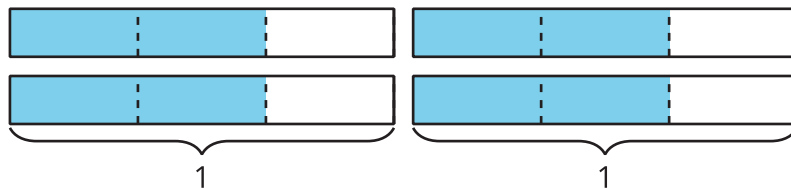
Lesson Synthesis

🕒 10 min

“Mai said she can multiply any fraction by a whole number by multiplying the whole number by the numerator and keeping the denominator.”

Invite students to share whether they agree or disagree with Mai's statement and to explain their reasoning.

“Let's discuss Mai's reasoning using the expression $4 \times \frac{2}{3}$ and the diagram from today's warm-up.”



“Why can we multiply 4×2 to get the numerator of the product?” (We can think in terms of thirds. The diagram shows 4 groups of 2 thirds, or 8 thirds total.)

“Why is the denominator of the product the same as the fraction in the expression?” (The denominator represents the size of the equal parts in each group. The size of the part doesn't change when the number of groups increases.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students multiply the whole number by the numerator of the fraction but do not attend to the denominator (for instance, expressing the value of the first expression as 12 instead of $\frac{12}{5}$).

Next Day Support

- Before the warm up, select a student's cool down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool down.

Lesson 5: Equivalent Multiplication Expressions

Standards Alignments

Addressing 4.NF.B.4.a, 4.NF.B.4.b, 4.NF.B.4.c

Teacher-facing Learning Goals

- Write equivalent expressions for the multiplication of a fraction by a whole number and explain or show that the expressions are equivalent.

Student-facing Learning Goals

- Let's write multiplication expressions in different ways.

Lesson Purpose

The purpose of this lesson is for students to write equivalent expressions for the multiplication of a whole number and a unit fraction and explain the equivalence.

In previous lessons, students multiplied unit and non-unit fractions by a whole number and represented their reasoning with diagrams and expressions.

In this lesson, students apply these understandings to explain how two multiplication expressions are equivalent. Students use what they know about multiple groups of unit fractions to explain how two different expressions result in the same product (MP7). (Students are not expected to use the term "equivalent expressions.")

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |

Teacher Reflection Question

What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day?

Lesson Synthesis 10 min

Cool-down 5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Expressions for Fractions

Standards Alignments

Addressing 4.NF.B.4.a, 4.NF.B.4.b, 4.NF.B.4.c

Student-facing Task Statement

1. Kiran says that the expressions $2 \times \frac{6}{8}$ and $3 \times 4 \times \frac{1}{8}$ both represent the same fraction. Do you agree? Explain or show your reasoning.
2. Write two new expressions that have the same value as $12 \times \frac{1}{9}$. You can use a diagram if it is helpful.

Student Responses

1. Agree. Sample response: $2 \times \frac{6}{8}$ is $\frac{12}{8}$ or 12 groups of $\frac{1}{8}$, and $3 \times 4 \times \frac{1}{8}$ is $12 \times \frac{1}{8}$, which is also 12 groups of $\frac{1}{8}$.
2. Sample responses: $4 \times \frac{3}{9}$, $6 \times \frac{2}{9}$, $2 \times 3 \times \frac{2}{9}$

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

How Many Do You See?

Standards Alignments

Addressing 4.NF.B.4.b, 4.NF.B.4.c

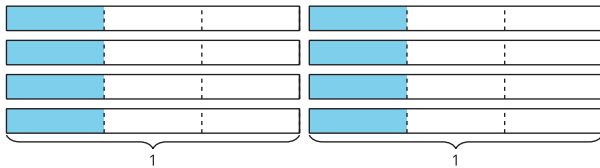
The purpose of this How Many Do You See is for students to use grouping strategies to describe the images they see. Students' descriptions are recorded using equations and expressions to support the goal of creating equivalent expressions. The synthesis encourages students to think about why two expressions can represent the same amount.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many thirds do you see? How do you see them?



Student Responses

- 8 groups of $\frac{1}{3}$, $8 \times \frac{1}{3}$
- 4 groups of $\frac{2}{3}$, $4 \times \frac{2}{3}$
- 2 groups of $\frac{4}{3}$, $2 \times \frac{4}{3}$

Launch

- Groups of 2
- “How many thirds do you see? How do you see them?”

Activity

- Display the image.
- 1 minute: quiet think time

Synthesis

- For each way that students see thirds, ask: “What expression should we use to represent the groups of thirds that ____ saw?”
- If no students suggest “4 groups of $\frac{2}{3}$ ”, ask them how that might be visible in the diagram. (By combining 2 of the thirds from each strip, we can make 4 groups of $\frac{2}{3}$.)
- Write $8 \times \frac{1}{3} = 4 \times \frac{2}{3}$, and ask students if they agree or disagree with the statement.

Activity 1

🕒 15 min

Complete the Equations

Standards Alignments

Addressing 4.NF.B.4.b, 4.NF.B.4.c

The purpose of this activity is for students to think of different ways of using multiplication expressions to represent a non-unit fraction. Students informally use the associative property as they work towards generalizing that $n \times \frac{a}{b} = \frac{n \times a}{b} = (n \times a) \frac{1}{b}$.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to fraction strips or pre-formatted tape diagrams, including sevenths, fifths, and tenths.

Supports accessibility for: Visual-Spatial Processing, Fine Motor Skills

Student-facing Task Statement

- Find the number that makes each equation true. Draw a diagram if it is helpful.

$$\frac{12}{5} = 12 \times \underline{\hspace{2cm}} \quad \frac{12}{5} = 3 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 6 \times \underline{\hspace{2cm}} \quad \frac{12}{5} = 2 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 4 \times \underline{\hspace{2cm}} \quad \frac{12}{5} = 1 \times \underline{\hspace{2cm}}$$

- Here are two sets of numbers:

Set A:

Set B:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$

- Choose a number from set A and a number from set B to complete this equation and make it true:

$$\frac{6}{7} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

- Choose a different number from set A and a number from set B to complete the equation to make it true.

Launch

- Groups of 2

Activity

- “Work with your partner to complete the first problem. Talk about how you know what numbers make the equations true.”
- 3 minutes: partner work time
- Monitor for students who use the factors of 12 to complete the equations.
- “Now take a few minutes to complete the rest of the problems independently. Afterwards, share your responses with your partner.”
- 7 minutes: independent work time
- 3 minutes: partner discussion
- “Did you choose the same numbers as your partner? If not, are both equations correct?”

Synthesis

- “How did you know what fractions to use to complete each equation in the first

$$\frac{6}{7} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

3. Explain or show how you know that the two equations you wrote are both true.

Student Responses

$$1. \quad \frac{12}{5} = 12 \times \frac{1}{5} \qquad \frac{12}{5} = 3 \times \frac{4}{5}$$

$$\frac{12}{5} = 6 \times \frac{2}{5} \qquad \frac{12}{5} = 2 \times \frac{6}{5}$$

$$\frac{12}{5} = 4 \times \frac{3}{5} \qquad \frac{12}{5} = 1 \times \frac{12}{5}$$

2. Sample responses for parts a and b:

- $\frac{6}{7} = 6 \times \frac{1}{7}$
- $\frac{6}{7} = 3 \times \frac{2}{7}$
- $\frac{6}{7} = 2 \times \frac{3}{7}$
- $\frac{6}{7} = 1 \times \frac{6}{7}$

3. Sample response for $\frac{6}{7} = 6 \times \frac{1}{7}$ and $\frac{6}{7} = 2 \times \frac{3}{7}$:

- I know that $6 \times \frac{1}{7}$ means 6 groups of one-seventh, and $2 \times \frac{3}{7}$ means 2 groups of three-sevenths, which also makes 6 groups of one-seventh.
- A diagram showing 6 parts of $\frac{1}{7}$ shaded and the 6 parts put into 2 groups, with 3 parts of $\frac{1}{7}$ in each.

problem?” (We looked for the numbers to multiply to get $\frac{12}{5}$. The denominator stays the same, so we find out what is going to be the whole number and numerator by knowing the factor pairs of 12.)

- Select 2–3 partners to share the following equations from the second problem:
 - $\frac{6}{7} = 6 \times \frac{1}{7}$
 - $\frac{6}{7} = 3 \times \frac{2}{7}$
 - $\frac{6}{7} = 2 \times \frac{3}{7}$
 - $\frac{6}{7} = 1 \times \frac{6}{7}$
- “Why are all these equations true?” (To get $\frac{6}{7}$, we can multiply the whole number by the numerator and then keep the denominator the same. In all of these, multiplying the whole number and the numerator gives 6. There are different ways to multiply to get 6 as a numerator: 6×1 , 3×2 , 2×3 and 1×6 .)

Activity 2

🕒 20 min

Fractions and Matching Expressions

Standards Alignments

Addressing 4.NF.B.4.b, 4.NF.B.4.c

In this activity, students analyze multiplication expressions, match each to one of a given set of fractions, and explain how they know that certain expressions represent the same fraction (MP7).

Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed ____, so I matched . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing

Student-facing Task Statement

Here is a set of expressions.

- | | | |
|----------------------------------|----------------------------------|---------------------------------|
| A. | B. | C. |
| $6 \times \frac{1}{10}$ | $2 \times 4 \times \frac{1}{9}$ | $4 \times \frac{1}{5}$ |
| D. | E. | F. |
| $3 \times 2 \times \frac{1}{10}$ | $5 \times 2 \times \frac{1}{12}$ | $2 \times 2 \times \frac{1}{5}$ |
| G. | H. | I. |
| $4 \times 4 \times \frac{1}{9}$ | $10 \times \frac{1}{12}$ | $4 \times \frac{1}{12}$ |

- Match each expression to one of the following fractions, if possible. Record your matches. Be prepared to explain how you know there is or isn't a match.

| | | | |
|---------------|-----------------|----------------|---------------|
| $\frac{4}{5}$ | $\frac{10}{12}$ | $\frac{6}{10}$ | $\frac{8}{9}$ |
|---------------|-----------------|----------------|---------------|

- Complete each equation to make it true. Try to do so without using unit fractions.

a. $\frac{4}{5} = \underline{\quad} \times \underline{\quad}$ $\frac{4}{5} = \underline{\quad} \times \underline{\quad}$

b. $\frac{10}{12} = \underline{\quad} \times \underline{\quad}$ $\frac{10}{12} = \underline{\quad} \times \underline{\quad}$

c. $\frac{6}{10} = \underline{\quad} \times \underline{\quad}$ $\frac{6}{10} = \underline{\quad} \times \underline{\quad}$

d. $\frac{8}{9} = \underline{\quad} \times \underline{\quad}$ $\frac{8}{9} = \underline{\quad} \times \underline{\quad}$

Student Responses

- $\frac{4}{5}$: C and F

Launch

- Groups of 2

Activity

- “Match the expressions to a fraction that shows its value. Be prepared to explain how you know which expressions match which fraction.”
- “Each fraction may not have the same number of matching expressions.”
- 5–7 minutes: partner work time
- Pause for a discussion before students continue to the second half of the activity.
- Select students to share their matches and their explanations.
- “Are there any expressions without a match? How do you know?” (Yes, expressions G and I. Their values are $\frac{16}{9}$ and $\frac{4}{12}$.)
- “You’ve seen that multiple expressions can represent the same fraction. Some of the expressions have two factors, some have three. All of them show unit fractions.”
- “Now complete each equation in the second problem with two factors that would make the equation true. See if you can use factors that are not unit fractions.”

- $\frac{10}{12}$: E and H
- $\frac{6}{10}$: A and D
- $\frac{8}{9}$: B
- No match: G and I

2. Sample responses:

- a. $2 \times \frac{2}{5}$, $4 \times \frac{1}{5}$
- b. $5 \times \frac{2}{12}$, $2 \times \frac{5}{12}$
- c. $3 \times \frac{2}{10}$, $2 \times \frac{3}{10}$
- d. $8 \times \frac{1}{9}$, $4 \times \frac{2}{9}$, $2 \times \frac{4}{9}$

- 5 minutes: independent work time

Synthesis

- Ask selected students to share expressions for each fraction.
- Display the possible expressions for the final equation:

$$\begin{aligned} &8 \times \frac{1}{9} \\ &2 \times 4 \times \frac{1}{9} \\ &2 \times \frac{4}{9} \\ &4 \times 2 \times \frac{1}{9} \\ &4 \times \frac{2}{9} \end{aligned}$$

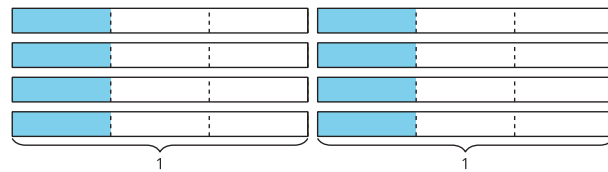
- "How can we explain why these expressions have the value of $\frac{8}{9}$?"

Lesson Synthesis

🕒 10 min

"Today we looked at different expressions to represent the same fraction."

Display the diagram from the warm-up.



Ask students to write as many expressions as they can to describe the value of the shaded parts. Record their responses in a list for all to see. If no students suggest expressions with three factors ($4 \times 2 \times \frac{1}{3}$ or $2 \times 4 \times \frac{1}{3}$), ask them to consider if it's possible to write such expressions.

"Pick two expressions from the list. Talk to your neighbor about how one is related to the other. You can mark up the diagram to support your explanation, if that's helpful."

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)

- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

----- **Complete Cool-Down** -----

Response to Student Thinking

Students generate only one equivalent expression.

Next Day Support

- During the activity syntheses, connect diagrams to expressions or equations.

Lesson 6: Problems with Equal Groups of Fractions

Standards Alignments

Addressing 4.NF.B.4.a, 4.NF.B.4.b, 4.NF.B.4.c

Teacher-facing Learning Goals

- Represent and solve problems involving multiplication of a fraction by a whole number.

Student-facing Learning Goals

- Let's solve problems with fractions.

Lesson Purpose

The purpose of this lesson is for students to apply their understandings about multiplication of a fraction by a whole number to solve problems.

Students may choose to draw diagrams, write equations, or make use of patterns to understand the situations and answer the questions. As students make sense of representations and quantities in context, they practice reasoning quantitatively and abstractly (MP2).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR7 (Activity 2)

Instructional Routines

True or False (Warm-up)

Materials to Gather

- Chart paper: Activity 2

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |

Teacher Reflection Question

What new mathematical connections did you see students make today as they were solving problems about multiplication of fractions? How can those connections be leveraged in

| | | |
|------------------|--------|----------------|
| Activity 2 | 20 min | upcoming work? |
| Lesson Synthesis | 10 min | |
| Cool-down | 5 min | |

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

The Same or Not the Same?

Standards Alignments

Addressing 4.NF.B.4.c

Student-facing Task Statement

1. Tyler bought 5 cartons of milk. Each carton contains $\frac{3}{4}$ liter. How many liters of milk did Tyler buy? Explain or show your reasoning.
2. Han bought 3 cartons of chocolate milk. Each carton contains $\frac{5}{8}$ liter. Did Han buy the same amount of milk as Tyler? Explain or show your reasoning.

Student Responses

1. $\frac{15}{4}$ liters. Sample response: $5 \times \frac{3}{4} = \frac{15}{4}$
2. No, Han bought less milk than Tyler did. Sample response: $3 \times \frac{5}{8} = \frac{15}{8}$, and $\frac{15}{8}$ is less than $\frac{15}{4}$ because an eighth is less than a fourth, so 15 eighths is less than 15 fourths.

----- Begin Lesson -----

Warm-up

⌚ 10 min

True or False: Two and Three Factors

Standards Alignments

Addressing 4.NF.B.4.b

The purpose of this True or False is to elicit strategies and understandings students have for finding products of a whole number and a fraction and identifying equivalent expressions. This work help students deepen their understanding of the properties of operations and will be helpful later when students solve problems with a whole number multiplied by a fraction.

In this activity, students have an opportunity to look for and make use of structure (MP7) as they consider how fractions are decomposed into various factors and multiplied in parts.

Instructional Routines

True or False

Student-facing Task Statement

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $\frac{10}{12} = 5 \times \frac{2}{12}$
- $1 \times \frac{10}{12} = 5 \times \frac{2}{12}$
- $\frac{24}{4} = 6 \times 3 \times \frac{1}{4}$
- $12 \times 2 \times \frac{1}{4} = 8 \times 3 \times \frac{1}{4}$

Student Responses

- True. 5×2 is 10, so 5 groups of $\frac{2}{12}$ is $\frac{10}{12}$.
- True. If $\frac{10}{12} = 5 \times \frac{2}{12}$ was true, and $5 \times \frac{2}{12}$ is the same as $\frac{10}{12}$, then the expressions are equivalent and are also true.
- False. 6×3 is 18, not 24, and $\frac{18}{4}$ is not the same as $\frac{24}{4}$.
- True. Both expressions are equal to $\frac{24}{4}$.

Launch

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- “What strategies did you use to determine if the statements were true or false?”

Activity 1

🕒 15 min

Banana Bread Recipe

Standards Alignments

Addressing 4.NF.B.4.a

The purpose of this activity is for students to use their understanding of multiplication of a unit fraction by a whole number to solve problems. Students use what they know to find a product given the factors and find the factors when given the product. This reinforces the idea that any fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$.

Students may interpret quantities greater than 1 as a combination of whole numbers and fractions (for example, $\frac{4}{3}$ cups as 1 whole cup and $\frac{1}{3}$ cup) or express them as mixed numbers (such as $1\frac{1}{3}$). Both are acceptable. If possible, ask students whether $1\frac{1}{3}$ and $\frac{4}{3}$ express the same amount, but it is not necessary to discuss the term mixed numbers at this point. (Students will be introduced to mixed numbers in upcoming lessons.)

Student-facing Task Statement

A bakery is making banana bread.
Here is the recipe for 1 batch.



Recipe:

- 1 banana
- $\frac{2}{3}$ cup butter
- $\frac{3}{2}$ teaspoons baking soda
- $\frac{5}{8}$ cup sugar
- 2 large eggs
- $\frac{5}{2}$ cups of all-purpose flour

1. The bakery makes 2 batches of banana bread on Monday. Complete the table to show how much of each ingredient is used.

Monday's banana bread

Launch

- Groups of 2
- "Have you followed a recipe to make something before? What is in a recipe?" (A list of ingredients, amounts of each, and instructions for putting the ingredients together)
- "If a recipe is for 5 servings or 5 people, but you need more than that, what would you do?" (Adjust the amount of ingredients.)
- "We often refer to the amounts specified in a recipe as '1 batch'."
- "What might it mean to make 2 batches of a recipe?" (Make twice as much, or need twice as much ingredients)

Activity

- "Take a few quiet minutes on work on the activity. Then, discuss your thinking with your partner."
- 5 minutes: independent work time
- 5 minutes: partner work time

| ingredient | expression | amount of ingredient |
|-------------|------------|----------------------|
| bananas | | _____ |
| butter | | _____ cup(s) |
| baking soda | | _____ teaspoon(s) |
| sugar | | _____ cup(s) |
| eggs | | _____ |
| flour | | _____ cup(s) |

2. On Tuesday, the bakery needs $\frac{8}{3}$ cups of butter to make enough banana bread for the day. How many batches were made? Explain or show your reasoning.

Recipe:

- 1 banana
- $\frac{2}{3}$ cup butter
- $\frac{3}{2}$ teaspoons baking soda
- $\frac{5}{8}$ cup sugar
- 2 large eggs
- $\frac{5}{2}$ cups of all-purpose flour

3. Based on the number of the batches made on Tuesday, complete the table for each ingredient.

Tuesday's banana bread

| ingredient | expression | amount of ingredient |
|-------------|------------|----------------------|
| bananas | | _____ |
| butter | | $\frac{8}{3}$ cups |
| baking soda | | _____ teaspoon(s) |
| sugar | | _____ cup(s) |

- Monitor for students who discuss:
 - that each quantity in Monday's table will be multiplied by 2
 - that each quantity in Tuesday's table need to be multiplied by 4 because the amount of butter tells us that the number of batches is 4

Synthesis

- Display the table for Monday and ask students to share responses. Record their responses for all to see.
- "How is the numerator changing in all of the ingredients?" (It is multiplied by 2 in each problem.)
- "Why is the denominator different in all of them?" (A different unit and unit fraction was used to measure each ingredient.)
- Ask students to share the expressions for the ingredients in the table for Tuesday. Record each expression and its value as an equation:
 - $4 = 4 \times 1$
 - $\frac{8}{3} = 4 \times \frac{2}{3}$
 - $\frac{12}{2} = 4 \times \frac{3}{2}$
 - $\frac{20}{8} = 4 \times \frac{5}{8}$
 - $8 = 4 \times 2$
 - $\frac{20}{2} = 4 \times \frac{5}{2}$
- "Why are two of the ingredients not in fraction form?" (They have whole-number units.)

| ingredient | expression | amount of ingredient |
|------------|------------|----------------------|
| eggs | | _____ |
| flour | | _____ cup(s) |

Student Responses

1. Monday's banana bread

| ingredient | expression | amount of ingredient |
|-------------|------------------------|-------------------------|
| banana | 2×1 | 2 |
| butter | $2 \times \frac{2}{3}$ | $\frac{4}{3}$ cups |
| baking soda | $2 \times \frac{3}{2}$ | $\frac{6}{2}$ teaspoons |
| sugar | $2 \times \frac{5}{8}$ | $\frac{10}{8}$ cups |
| egg | 2×2 | 4 |
| flour | $2 \times \frac{5}{2}$ | $\frac{10}{2}$ cups |

2. Four batches were made. Sample response: There is $\frac{2}{3}$ cup of butter in one batch, and $\frac{8}{3}$ is $4 \times \frac{2}{3}$, which means there are 4 batches.

3. Tuesday's banana bread

| ingredient | expression | amount of ingredient |
|-------------|------------------------|--------------------------|
| banana | 4×1 | 4 |
| butter | $4 \times \frac{2}{3}$ | $\frac{8}{3}$ cups |
| baking soda | $4 \times \frac{3}{2}$ | $\frac{12}{2}$ teaspoons |
| sugar | $4 \times \frac{5}{8}$ | $\frac{20}{8}$ cups |
| egg | 4×2 | 8 |
| flour | $4 \times \frac{5}{2}$ | $\frac{20}{2}$ cups |

Advancing Student Thinking

If students are unsure how to complete the last table, check if they recognize that the given $\frac{8}{3}$ cups represent the amount of butter for 4 batches. If so, ask: "How many bananas are needed in 4 batches?" and "How many eggs are needed?" "How might you use this to help to determine the ingredients needed for 1 batch, 2 batches and so on?"

Activity 2

🕒 20 min

How Much Milk Was Used?

Standards Alignments

Addressing 4.NF.B.4.b

In this activity, students are presented with descriptions of situations and equivalent multiplication expressions. They match each description to an expression that could represent the situation and see that more than one expression can be used, depending on how they interpret the situation. Likewise, students find that one expression can be used to represent different descriptions (MP2).

Students discuss their matching decisions, analyze how the expressions are related, and consider revising the matches they made if appropriate. When students discuss and justify their decisions they are creating viable arguments and critiquing one another's reasoning (MP3).

🌐 Access for English Learners

MLR7 Compare and Connect. Synthesis: Lead a discussion comparing, contrasting, and connecting the different representations. Ask, "How does the situation show up in the representation?", "What do each of these representations have in common?", and "How were they different?"

Advances: Representing, Conversing

♿ Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for the group work in this activity. Record responses on a display and keep visible during the activity.

Supports accessibility for: Social-Emotional Functioning

Materials to Gather

Chart paper

Required Preparation

- Write the 5 expressions from the activity on separate posters and post them around the room:

$$4 \times (2 \times \frac{1}{10})$$

$$4 \times \frac{2}{10}$$

$$8 \times \frac{1}{10}$$

$$2 \times (4 \times \frac{1}{10})$$

$$2 \times \frac{4}{10}$$

Student-facing Task Statement

The bakery that sells banana bread also sells fresh milkshakes. Each serving uses $\frac{1}{10}$ liter of milk.



Here are five descriptions of the milkshakes sold in a week and five expressions that represent the liters of milk used.

Match each description to an expression that represents it.

- | | |
|---|------------------------------------|
| 1. On Monday, the bakery sold 8 servings of milkshake. How much milk was used? | $4 \times (2 \times \frac{1}{10})$ |
| 2. On Tuesday, two customers bought 4 servings of milkshake each. How much milk was used? | $4 \times \frac{2}{10}$ |
| 3. On Wednesday, four customers bought 2 servings of milkshake each. How much milk was used? | $8 \times \frac{1}{10}$ |
| 4. On Thursday, two customers each bought a serving of milkshake. They placed the same order three more times for their friends that day. How much milk was used? | $2 \times (4 \times \frac{1}{10})$ |
| 5. On Saturday, four friends each purchased a serving of milkshake for breakfast. They came back for the same after dinner. How much milk was used? | $2 \times \frac{4}{10}$ |

Student Responses

Sample responses:

- Monday: $8 \times \frac{1}{10}$

Launch

- Groups of 2
- Read the first problem aloud to students.
- “Share the expression you selected with a partner.”
- Students may select any of the expressions because each is equivalent to $\frac{8}{10}$. If this happens, ask, “Does one expression seem to represent what is happening in the situation better than others?” ($8 \times \frac{1}{10}$)

Activity

- 5 minutes: independent work time
- Ask students to stand with the poster showing the expression that they believe represents how much milk was used on Tuesday, and to discuss with others there why they chose this expression.
- Ask students to partner with a student from a different poster to explain why they made a different choice.
- “Does anyone wish to revise their thinking about the expression they selected?”
- “Can you explain why you think that a different expression is a better choice now?”
- Repeat this process for each problem.

Synthesis

- See lesson synthesis.

2. Tuesday: $2 \times \frac{4}{10}$ or $2 \times (4 \times \frac{1}{10})$
3. Wednesday: $4 \times \frac{2}{10}$ or $4 \times (2 \times \frac{1}{10})$
4. Thursday: $2 \times \frac{4}{10}$ or $2 \times (4 \times \frac{1}{10})$
5. Saturday: $4 \times \frac{2}{10}$ or $4 \times (2 \times \frac{1}{10})$

Advancing Student Thinking

If students do not see that each factor in the expressions can be interpreted in different ways, or that different expressions could represent the same quantity, invite them to use diagrams or record their thinking using diagrams to illustrate various groupings of the same quantity. Consider asking: “Where do you see ____ (whole number) groups of ____ (fraction)?”

Lesson Synthesis

🕒 10 min

“Today, we matched expressions to situations. We learned that several expressions can represent the same situation.”

Invite 1–2 students who chose different expressions for the same problem (one of the last two problems in the milkshake activity) to share. Record their ideas for all to see.

“Who can explain how each expression matches the problem?” (On Thursday, there were 4 separate orders of 1 serving each, or $4 \times \frac{1}{10}$, that were made by 2 people, or $2 \times (4 \times \frac{1}{10})$. This is also the same as $2 \times \frac{4}{10}$.)

“Did you notice something about the answers to the problems?” (They are all the same. They are all $\frac{8}{10}$.)

“Why do you think they are all the same?” (They all involve 8 groups of $\frac{1}{10}$.)

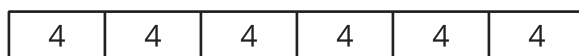
Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

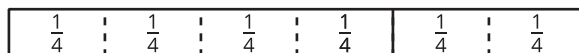
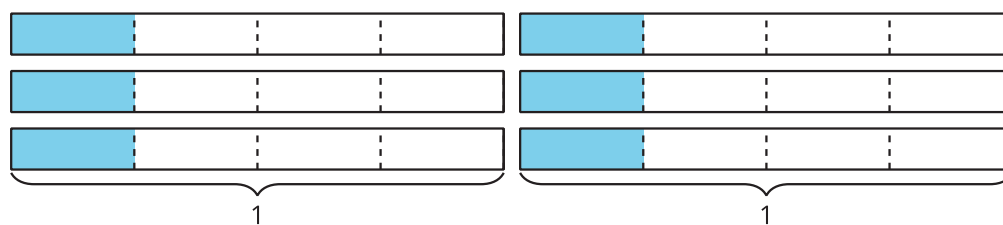
Student Section Summary

In this section, we learned to multiply a whole number and a fraction by thinking about equal-size groups, just as we did when multiplying two whole numbers.

For instance, we can think of 6×4 as 6 groups of 4. A diagram like this can help to show that the product is 24:



Likewise, we can think of $6 \times \frac{1}{4}$ as 6 groups of $\frac{1}{4}$. Diagrams can help us see that the product is $\frac{6}{4}$:



After studying patterns, we saw that when we multiply a whole number and a fraction, the whole number is multiplied only by the numerator of the fraction and the denominator stays the same. For example:

$$6 \times \frac{1}{2} = \frac{6}{2}$$

$$2 \times \frac{4}{5} = \frac{8}{5}$$

We also learned that:

- Every fraction can be written as a product of a whole number and a unit fraction. For example, $\frac{5}{4}$ can be written as $5 \times \frac{1}{4}$.
- We can write different multiplication expressions for the same fraction. For example, $\frac{8}{3}$ can be written as:

$$8 \times \frac{1}{3}$$

$$4 \times 2 \times \frac{1}{3}$$

$$4 \times \frac{2}{3}$$

$$2 \times \frac{4}{3}$$

----- Complete Cool-Down -----

Response to Student Thinking

Students determine the total number of liters of milk to be something other than $\frac{15}{4}$.

Next Day Support

- Before the warm-up, pass back the cool-down and have students discuss strategies they could use to find the product.

Section B: Addition and Subtraction of Fractions

Lesson 7: Fractions as Sums

Standards Alignments

| | |
|------------------|----------------------|
| Building On | 3.NF.A.1 |
| Addressing | 4.NF.B.3, 4.NF.B.3.b |
| Building Towards | 4.NF.B.3 |

Teacher-facing Learning Goals

- Recognize that a fraction can be decomposed into a sum of fractions with the same denominator.
- Write equations to represent fraction decomposition.

Student-facing Learning Goals

- Let's write fractions as sums.

Lesson Purpose

The purpose of this lesson is for students to decompose a fraction into a sum of fractions with the same denominator in more than one way, and to write an equation for each decomposition.

In previous lessons, students expressed a fraction $\frac{a}{b}$ as a product of a unit fraction $\frac{1}{b}$ and a whole number a . In this lesson, students transition to seeing a fraction $\frac{a}{b}$ as a sum of unit fractions and non-unit fractions with the same denominator. Students see that a fraction with a numerator greater than 1 can be decomposed into sums in different ways. They write equations to record the decomposition (for example, $\frac{4}{6} = \frac{3}{6} + \frac{1}{6}$). Later, they write equations to represent addition of fractions with the same denominator.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Choral Count (Warm-up)

Materials to Gather

- Measuring cups: Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Reflect on the times you observed students listening to one another's ideas today in class. What norms would help each student better attend to their classmates' ideas in future lessons?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Make a Sum of $\frac{7}{4}$

Standards Alignments

Addressing 4.NF.B.3.b

Student-facing Task Statement

Find three different ways to use fourths to make a sum of $\frac{7}{4}$.

Write an equation for each.

Student Responses

Sample responses:

- $\frac{1}{4} + \frac{2}{4} + \frac{4}{4} = \frac{7}{4}$
- $\frac{6}{4} + \frac{1}{4} = \frac{7}{4}$
- $\frac{5}{4} + \frac{2}{4} = \frac{7}{4}$

----- Begin Lesson -----

Warm-up

 10 min

Choral Count: Three-fourths at a Time

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.B.3

The purpose of this Choral Count is to invite students to practice counting by $\frac{3}{4}$ and notice patterns in the count. The patterns they recognize here will be helpful when students decompose fractions into sums of $\frac{3}{4}$ s in a problem later in the lesson. The exercise also draws students' attention to multiples of $\frac{3}{4}$ that are equivalent to whole numbers, which will be helpful as students compose and decompose mixed numbers.

Instructional Routines

Choral Count

Student Responses

| | | | |
|----------------|----------------|----------------|----------------|
| $\frac{3}{4}$ | $\frac{6}{4}$ | $\frac{9}{4}$ | $\frac{12}{4}$ |
| $\frac{15}{4}$ | $\frac{18}{4}$ | $\frac{21}{4}$ | $\frac{24}{4}$ |
| $\frac{27}{4}$ | $\frac{30}{4}$ | $\frac{33}{4}$ | $\frac{36}{4}$ |
| $\frac{39}{4}$ | $\frac{42}{4}$ | $\frac{45}{4}$ | $\frac{48}{4}$ |

Launch

- "Count by $\frac{3}{4}$, starting at $\frac{3}{4}$."
- Record as students count.
- Stop counting and recording at $\frac{48}{4}$.

Activity

- "What patterns do you see?" (Sample responses:
 - The numerator is increasing by 3 each time.
 - The numerators are multiples of 3.
 - In every fourth fraction in the list, the numerator is a multiple of 4.)
- 1-2 minutes: quiet think time
- Record responses.

Synthesis

- "Which of these fractions are equivalent to whole numbers?" ($\frac{12}{4}$, $\frac{24}{4}$, $\frac{36}{4}$, $\frac{48}{4}$)

- “To what whole numbers are they equivalent?” (3, 6, 9, 12)
- “Why do you think these patterns are happening?” (The number of parts and the size of each part being added stay the same each time, so the numerator is always increasing by 3 and the denominator always staying the same.)

Activity 1

🕒 20 min

Barley Soup

Standards Alignments

Addressing 4.NF.B.3, 4.NF.B.3.b

Previously, students considered non-unit fractions in terms of equal groups of unit fractions or as a product of a unit fraction and a whole number. This activity prompts students to think about non-unit fractions as being sums of other fractions. The given context—about measuring fractional amounts using measuring cups of certain sizes—allows students to continue thinking in terms of equal groups, but also invites them to consider a fractional quantity as a sum of two or more fractions with the same denominator. For instance, students may see $\frac{5}{4}$ as 5 groups of $\frac{1}{4}$ or as $5 \times \frac{1}{4}$, but they may also see that $\frac{5}{4}$ is equal to $\frac{3}{4} + \frac{1}{4} + \frac{1}{4}$. Students record such a decomposition as an equation. When students connect the quantities in the story problem to an equation, they reason abstractly and quantitatively (MP2).

Students may not be familiar with the use of measuring cups. Consider demonstrating how to use a 1-cup measuring cup to obtain different whole numbers of cups.

Materials to Gather

Measuring cups

Required Preparation

- Gather $\frac{1}{4}$ -cup and $\frac{3}{4}$ -cup measuring cups, if available.

Student-facing Task Statement

Lin is learning to make barley soup using a family recipe. Here are some ingredients in the recipe:



- $\frac{3}{4}$ cup of barley
- $\frac{5}{4}$ cups of chopped celery
- $\frac{6}{4}$ cups of chopped carrots
- 1 cup of chopped onions
- $2\frac{1}{4}$ cups of vegetable broth

1. Lin has only one measuring cup that measures $\frac{1}{4}$ cup. Show how Lin could use the cup to measure the right amount of each ingredient.

- | | |
|------------|--------------------|
| ○ Barley: | ○ Onions: |
| ○ Celery: | ○ Vegetable broth: |
| ○ Carrots: | |

2. Lin later found a $\frac{3}{4}$ -cup measuring cup. Show how she could use the cups to measure the right amount of each ingredient.

- | | |
|------------|--------------------|
| ○ Barley: | ○ Onions: |
| ○ Celery: | ○ Vegetable broth: |
| ○ Carrots: | |

Student Responses

1. Sample response:

- Barley: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
- Celery: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
- Carrots: $6 \times \frac{1}{4}$
- Onions: Fill the cup 4 times
- Broth: Fill the cup 9 times

Launch

- Groups of 2
- "Today we'll look at a soup recipe."
- Ask students to share with a partner:
 - "What is your favorite soup?"
 - "What is in your favorite soup?"
 - "If you were writing a recipe for this soup, what would it say?"
- 2 minutes: partner discussion
- 1 minute: share responses
- "Let's look at a recipe for barley soup. Someone in Lin's family wrote the amounts in the recipe in fourths to make measuring easier."
- If possible, show examples of uncooked barley and make $\frac{1}{4}$ -cup and $\frac{3}{4}$ -cup measuring cups available.

Activity

- "Work independently on the task for a few minutes. Then, share your responses with your partner."
- 6–7 minutes: independent work time
- Monitor for students who express the amounts in terms of:
 - number of times a measuring cup is filled
 - products of a unit fraction and a whole number
 - sums of unit fractions
 - sums of unit and non-unit fractions
- 3–4 minutes: partner discussion

Synthesis

- Select students to share their responses in the order shown in the activity notes (from informal or concrete to formal or symbolic, from multiplication to addition).

2. Sample response:

- Barley: Fill the $\frac{3}{4}$ -cup once
- Celery: $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$
- Carrots: $\frac{3}{4} + \frac{3}{4}$ or $2 \times \frac{3}{4}$
- Onions: $\frac{3}{4} + \frac{1}{4}$
- Broth: $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

- Record the different ways of expressing the quantities in the recipe.
- Consider drawing diagrams and annotating them to help students relate the expressions and the quantities.
- Highlight that each quantity can be written as a product of a whole number and a unit fraction, but it can also be written as a sum of smaller fractions. For example:
 - Barley: $3 \times \frac{1}{4}$, or $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$



- Broth: $9 \times \frac{1}{4}$ or $3 \times \frac{3}{4}$, or $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$



Advancing Student Thinking

If students create drawings to show how they would obtain the correct quantities, ask: “How do your drawings show what Lin could do to get the right amounts?” and “How could you use expressions to show the same information?”

Activity 2

🕒 15 min

Sums in Fifths and Thirds

Standards Alignments

Addressing 4.NF.B.3, 4.NF.B.3.b

In the previous activity, students saw that a fraction can be decomposed into a sum of fractions with the same denominator and that it can be done in more than one way. In this activity, they record such decompositions as equations. The last question prompts students to consider

whether any fraction can be written as a sum of smaller fractions with the same denominator. Students see that only non-unit fractions (with a numerator greater than 1) can be decomposed that way. Students observe regularity in repeated reasoning as they decompose the numerator, 9, into different parts while the denominator in all cases is 5 (MP8).

Access for English Learners

MLR7 Compare and Connect. Synthesis: Lead a discussion comparing, contrasting, and connecting the different representations. Ask, “How do these different representations show the same information?”, “What do each of these representations have in common?”, and “How were they different?”

Advances: Representing, Conversing

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to pre-formatted tape diagrams and colored pencils. Invite students to use different colors for each addend.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Memory

Student-facing Task Statement

- Use different combinations of fifths to make a sum of $\frac{9}{5}$.
 - $\frac{9}{5} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$
 - $\frac{9}{5} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$
 - $\frac{9}{5} = \underline{\quad} + \underline{\quad} + \underline{\quad}$
 - $\frac{9}{5} = \underline{\quad} + \underline{\quad}$
- Write different ways to use thirds to make a sum of $\frac{4}{3}$. How many can you think of? Write an equation for each combination.
- Is it possible to write any fraction with a denominator of 5 as a sum of other fifths? Explain or show your reasoning.

Student Responses

- Sample responses:
 - $\frac{9}{5} = \frac{3}{5} + \frac{2}{5} + \frac{2}{5} + \frac{1}{5} + \frac{1}{5}$

Launch

- “Earlier, we saw different ways to decompose fractions in fourths and write them as sums of smaller fractions.”
- “How can we write the fraction $\frac{9}{5}$ as a sum of unit fractions?”
 $(\frac{9}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5})$
- “Let’s decompose $\frac{9}{5}$ into sums of other fifths and $\frac{4}{3}$ into sums of thirds.”

Activity

- “Take a few quiet minutes to complete the activity. Then, share your responses with your partner.”
- 5–6 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor for different explanations students offer for the last question.

b. $\frac{9}{5} = \frac{2}{5} + \frac{1}{5} + \frac{5}{5} + \frac{1}{5}$

c. $\frac{9}{5} = \frac{2}{5} + \frac{3}{5} + \frac{4}{5}$

d. $\frac{9}{5} = \frac{4}{5} + \frac{5}{5}$

2. Sample responses:

○ $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

○ $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

○ $\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$

○ $\frac{2}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

3. Sample responses:

- No. It's not possible to write $\frac{1}{5}$ as a sum of fifths since it is already the smallest possible fifth.
- Yes, as long as the numerator is 2 or greater, we can write it as a sum of other fifths.

Synthesis

- Invite students to share their equations. Display or record them for all to see.
- Next, discuss students' responses to the last question. Select students with different explanations to share their reasoning.
- If not mentioned by students, highlight that fractions with a numerator of 1 (unit fractions) cannot be further decomposed into smaller fractions with the same denominator because it is already the smallest fractional part. Other fractions with a numerator other than 1 (non-unit fractions) can be decomposed into fractions with the same denominator.

Lesson Synthesis

⌚ 10 min

"In earlier lessons, we saw that a fraction whose numerator is greater than 1 can be written as products. Today, we saw that a fraction whose numerator is greater than 1 can also be seen as sums."

Display:

$$\begin{aligned}\frac{4}{3} &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\ \frac{4}{3} &= \frac{2}{3} + \frac{2}{3} \\ \frac{4}{3} &= \frac{1}{3} + \frac{3}{3} \\ \frac{4}{3} &= \frac{2}{3} + \frac{1}{3} + \frac{1}{3}\end{aligned}$$

"Compare these two ways of thinking about fractions. How are they alike?" (They both involve writing a fraction in terms of smaller parts or smaller fractions. The smaller fractions all have the same denominator.)

"How are they different?" (When writing a fraction as a product, we think of it in terms of equal groups. When writing it as a sum, we decompose it into smaller groups, but they may not be the same size.)

"What are some ways to decompose $\frac{13}{6}$ and write it as a sum?" (Sample

responses: $\frac{10}{6} + \frac{3}{6}$, $\frac{12}{6} + \frac{1}{6}$, $\frac{7}{6} + \frac{1}{6}$)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find three ways to compose $\frac{7}{4}$ from other fractions but do not represent them with expressions.

Next Day Support

- Launch the warm-up or Activity 1 by highlighting important notation from previous lessons.

Lesson 8: Addition of Fractions

Standards Alignments

| | |
|------------------|------------------------------------|
| Addressing | 4.NF.B.3.a, 4.NF.B.3.b |
| Building Towards | 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c |

Teacher-facing Learning Goals

- Decompose fractions greater than 1 into a sum of a whole number and a fraction less than 1.
- Reason about addition of fractions with the same denominator using a number line.

Student-facing Learning Goals

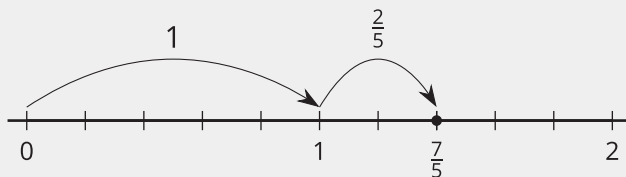
- Let's explore sums of fractions on a number line.

Lesson Purpose

The purpose of this lesson is for students to use a number line to reason about addition of fractions with the same denominator, and to decompose fractions greater than 1 into a whole number and a fraction less than 1.

Previously, students decomposed non-unit fractions into smaller unit and non-unit fractions. Here they continue to think about decomposing fractions, but they now use number line diagrams to support their reasoning.

A number line can illustrate the number of unit fractions that make a whole number. This in turn allows students to see that a fraction greater than 1 can be decomposed into a whole number and a fraction less than 1, preparing them to work with **mixed numbers**. For instance, on a number line partitioned into fifths, the fraction $\frac{7}{5}$ is 7 fifths away from 0 and 2 fifths away from 1, so we can express $\frac{7}{5}$ as 1 and $\frac{2}{5}$. One way to illustrate that sum is by drawing "jumps" on the number line.



Note that in grade 3, students came across mixed numbers in the context of measurement. For instance, they use inch rulers to measure lengths greater than 1 inch to the nearest halves and fourths, express them using numbers such as $2\frac{1}{4}$ and $5\frac{1}{2}$. They did not, however, reason about mixed numbers as sums of smaller fractions or sums of a fraction and a whole number.

Access for:**🇺🇸 Students with Disabilities**

- Action and Expression (Activity 1)

🌐 English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Activity 3 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Materials to Copy

- Make Two Jumps (groups of 2): Activity 3

Teacher Reflection Question

Students have previously represented whole numbers and fractions on the number line. How did you leverage that experience to help them reason about addition of fractions on the number line?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

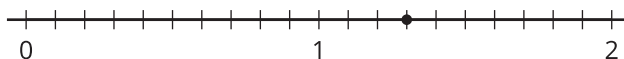
Lucky Thirteen-tenths

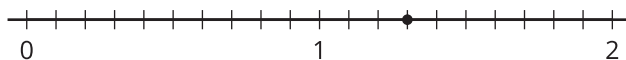
Standards Alignments

Addressing 4.NF.B.3.b

Student-facing Task Statement

1. On each number line, draw two “jumps” to show how to use tenths to make a sum of $\frac{13}{10}$.

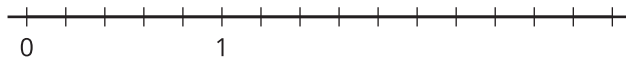




a. Represent each combination of jumps as an equation.

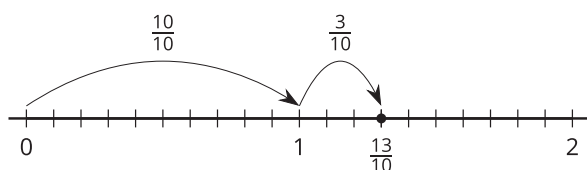
b. Write $\frac{13}{10}$ as a sum of a whole number and a fraction.

2. Find the value of $\frac{8}{5} + \frac{6}{5}$. Use the number line if you find it helpful.



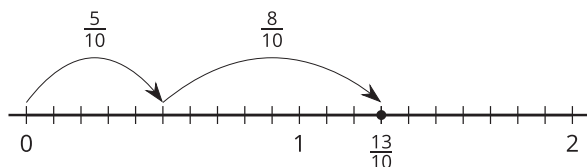
Student Responses

1. Sample response:

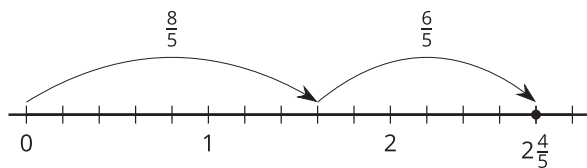


a. $\frac{10}{10} + \frac{3}{10} = \frac{13}{10}$ and $\frac{5}{10} + \frac{8}{10} = \frac{13}{10}$

b. $1 + \frac{3}{10} = \frac{13}{10}$



2. $\frac{14}{5}$ or $2\frac{4}{5}$



----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: A Fraction on a Number Line

Standards Alignments

Building Towards 4.NF.B.3.b

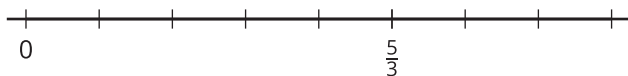
The purpose of this warm-up is to activate what students know about the use of number lines to represent fractional values, preparing them to use number lines to reason about addition of fractions in a later activity. While students may notice and wonder many things about the diagram, be sure to highlight the meaning of each interval and where the numbers 1 and 2 are located on the number line.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 5 tick marks from 0 to $\frac{5}{3}$.
- Each space between tick marks represents $\frac{1}{3}$.
- There are no other numbers on the line.
- There are 5 groups of $\frac{1}{3}$.

Students may wonder:

- Why is $\frac{5}{3}$ labeled?
- Why aren't the other tick marks labeled?
- Where is 1 on the number line? Why isn't 1 shown?

Launch

- Groups of 2
- Display the number line diagram.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What does the space between any two tick marks represent?" (A third) "How do you know?" (If five spaces represent 5 thirds and the spaces are the same size, then each space is 1 third.)
- "Where is 1 on the number line?" (The third tick mark from 0) "Where is 2?" (The sixth tick mark from 0, or 1 tick mark to the right of $\frac{5}{3}$)
- "Today we'll use number lines to help us reason about sums of fractions."

Activity 1

 20 min

Sum of Jumps

Standards Alignments

| | |
|------------------|------------|
| Addressing | 4.NF.B.3.b |
| Building Towards | 4.NF.B.3.a |


This activity prompts students to use number lines to illustrate the decomposition of a fraction into sums of other fractions, reinforcing their work from an earlier lesson. Along the way, students recognize that one way to decompose a fraction greater than 1 is to write it as a sum of a whole number and a fraction less than 1. This insight prepares students to interpret and write mixed numbers in later activities.

Access for Students with Disabilities

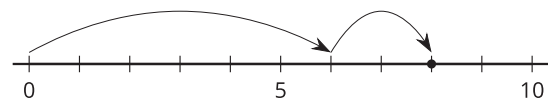
Action and Expression: Develop Expression and Communication. Provide access to colored pencils. Invite students to trace or draw each “jump” with a different color. Then invite students to circle or write out each addend in the corresponding color.

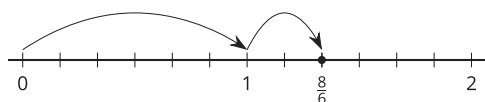
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Organization

Student-facing Task Statement

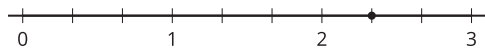
- On each number line, draw two “jumps” to show how to use sixths to make a sum of $\frac{8}{6}$. Then, write an equation to represent each combination of jumps.
 - Noah draws the following diagram and writes: $\frac{8}{6} = \frac{6}{6} + \frac{2}{6}$ and $\frac{8}{6} = 1 + \frac{2}{6}$. Which equation is correct? Explain your reasoning.

Launch

- Display or draw this number line:
 
- “What number does the point describe?” (8)
- “What do you think the ‘jumps’ represent?” (The numbers that are put together to go from 0 to 8)
- “What equations can we write to represent the combination of jumps?” ($6 + 2 = 8$ or $2 + 6 = 8$)
- “Let’s look at the jumps on some other number lines and see what they may represent.”



2. a. On each number line, draw “jumps” to show how to use thirds to make a sum of $\frac{7}{3}$. Then, write an equation to represent each combination of jumps.

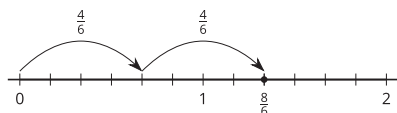


- b. Write $\frac{7}{3}$ as a sum of a whole number and a fraction.

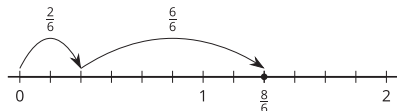
Student Responses

1. a. Sample response:

$$\blacksquare \frac{4}{6} + \frac{4}{6} = \frac{8}{6} \text{ or } \frac{8}{6} = \frac{4}{6} + \frac{4}{6}$$



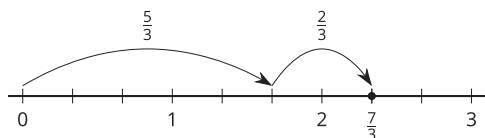
$$\blacksquare \frac{8}{6} = \frac{2}{6} + \frac{6}{6} \text{ (or } \frac{8}{6} = \frac{2}{6} + 1)$$



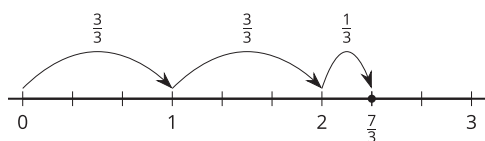
- b. Both are correct. $\frac{6}{6}$ is equivalent to 1.

2. Sample response:

$$\text{a. } \frac{5}{3} + \frac{2}{3} = \frac{7}{3}$$



$$\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$$



Activity

- Groups of 2
- “Work independently on the activity for a few minutes. Afterwards, share your responses with your partner.”
- 5–7 minutes: independent work time
- 2 minutes: partner discussion

Synthesis

- Invite students to share their equations. Record them for all to see.
- Focus the discussion on part b: writing $\frac{7}{3}$ as a sum of a whole number and a fraction. Students are likely to write $1 + \frac{4}{3}$ and $2 + \frac{1}{3}$.
- Explain that $2 + \frac{1}{3}$ can be written as $2\frac{1}{3}$, which we call a **mixed number**. This number is equivalent to $\frac{7}{3}$.
- “Why might it be called a mixed number?” (It is a mix of a whole number and a fraction.)

$$\text{b. } 2 + \frac{1}{3} = \frac{7}{3} \text{ or } 1 + \frac{4}{3} = \frac{7}{3}$$

Advancing Student Thinking

Students may not make a connection between the fractions in the problems and the number lines because the latter show no fractional labels. Consider asking students to label the number line to show fractions or asking, “Where do you think $\frac{1}{6}$ would be labeled on this number line?”

Activity 2

🕒 15 min

What is the Sum?

Standards Alignments

Addressing 4.NF.B.3.a
Building Towards 4.NF.B.3.c

In this activity, students use number lines to represent addition of two fractions and to find the value of the sum. The addends include fractions greater than 1, which can be expressed as a sum of a whole number and a fraction. Students practice constructing a logical argument and critiquing the reasoning of others when they explain which of the strategies they agree with and why (MP3).

🌐 Access for English Learners

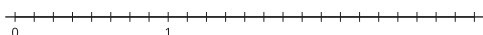
MLR8 Discussion Supports. Synthesis. Display sentence frames to agree or disagree. “I agree because . . .” and “I disagree because . . .”

Advances: Speaking, Conversing

Student-facing Task Statement

- Use a number line to represent each addition expression and to find its value.

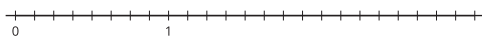
a. $\frac{5}{8} + \frac{2}{8}$



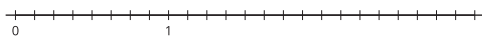
Launch

- Groups of 2
- Draw students' attention to the four addition expressions in the first problem.
- “What do you notice about the numbers?”

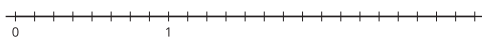
b. $\frac{1}{8} + \frac{9}{8}$



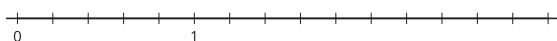
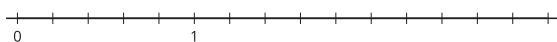
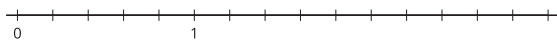
c. $\frac{11}{8} + \frac{9}{8}$



d. $2\frac{1}{8} + \frac{4}{8}$

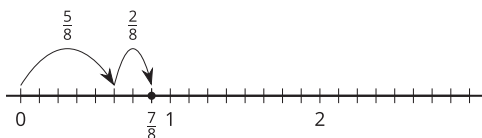


2. Priya says the sum of $1\frac{2}{5}$ and $\frac{4}{5}$ is $1\frac{6}{5}$. Kiran says the sum is $\frac{11}{5}$. Tyler says it is $2\frac{1}{5}$. Do you agree with any of them? Explain or show your reasoning. Use one or more number lines if you find them helpful.

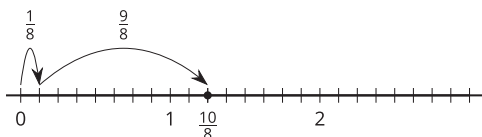


Student Responses

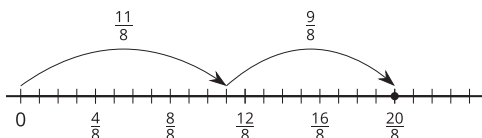
1. a. $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$



b. $\frac{1}{8} + \frac{9}{8} = \frac{10}{8}$



c. $\frac{11}{8} + \frac{9}{8} = \frac{20}{8}$



d. $2\frac{1}{8} + \frac{4}{8} = 2\frac{5}{8}$ or $2\frac{1}{8} + \frac{4}{8} = \frac{21}{8}$

Make some observations." (Sample responses:

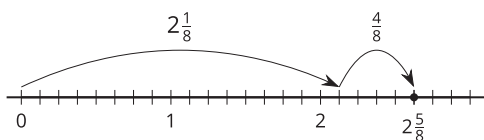
- They all have 8 for the denominator but different numbers for the numerator.
- Some fractions are less than 1 and others are greater than 1.
- There is one mixed number.)
- "What do you notice about the number lines?" (They are identical. They are all partitioned into eighths.)

Activity

- "Take 5–7 minutes to work on the task independently. Then, discuss your responses with your partner."
- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who can explain why Priya, Kiran, and Tyler might each be correct.

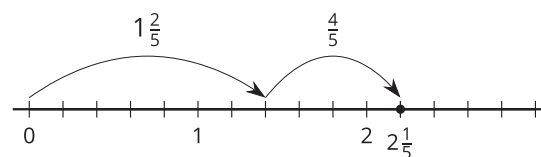
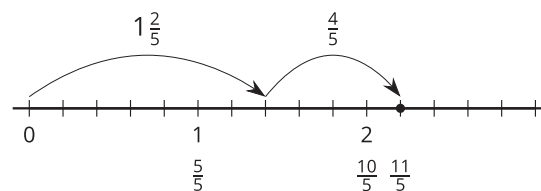
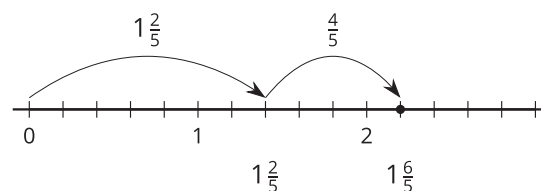
Synthesis

- Invite students to share their responses to the first problem.
- "Look at the sums you found. What do you notice about the numbers in each sum? How do they relate to the numbers in the fractions being added?" (The denominator of the sum is 8. The numerator of the sum is the result of adding the numerators of the addends.)
- Select previously identified students to share their responses to the second problem.
- If no students use number lines to make their case, consider sketching or displaying these diagrams.



2. Sample response: Agree with all of them.

- Agree with Priya because $\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$ and there's another 1 whole, which makes $1\frac{6}{5}$.
- Agree with Kiran because $1\frac{2}{5}$ is 1 whole plus $\frac{2}{5}$, and 1 whole is $\frac{5}{5}$, so $\frac{5}{5} + \frac{2}{5} + \frac{4}{5} = \frac{11}{5}$.
- Agree with Tyler because there are 2 wholes plus a $\frac{1}{5}$ in $\frac{11}{5}$ (or $\frac{11}{5} = \frac{10}{5} + \frac{1}{5} = 2 + \frac{1}{5} = 2\frac{1}{5}$).



- Point out that although it is true that $\frac{11}{5} = 1 + \frac{6}{5}$, we don't usually write $1\frac{6}{5}$ as the mixed number equivalent to $\frac{11}{5}$. Because we can make another 1 whole with $\frac{6}{5}$, or $1 + 1 + \frac{1}{5}$, we'd instead write $2\frac{1}{5}$.

Advancing Student Thinking

Students may not make a connection between the fractions in the problems and the number lines because the latter show no fractional labels. Consider asking: "What do you think the spaces between the tick marks represent?" and "Where would $\frac{1}{2}$ be on the number line?" Encourage students to label every tick mark or every other one as eighths, including those that represent whole numbers or benchmarks such as $\frac{1}{2}$ and $1\frac{1}{2}$.

Activity 3 (optional)

🕒 20 min

Make Two Jumps

Standards Alignments

Addressing 4.NF.B.3.a

This optional activity gives students an additional opportunity to practice using number lines to decompose fractions into sums of other fractions and to record the decompositions as equations.

The fractions on the cards (shown here) contain no whole numbers or mixed numbers, but some students may use them to help them find the second addend (and to avoid counting tick marks on the number line). Some may also choose to label each number line with whole numbers beyond 1 to facilitate their reasoning and equation writing.

$$\frac{1}{3} \quad \frac{2}{3} \quad \frac{3}{3} \quad \frac{4}{3} \quad \frac{5}{3} \quad \frac{6}{3} \quad \frac{7}{3} \quad \frac{8}{3}$$

Materials to Copy

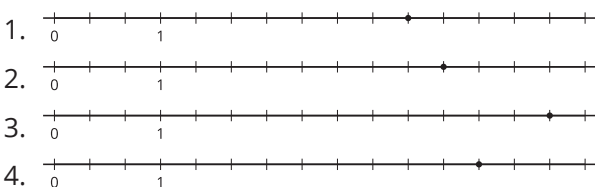
Make Two Jumps (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Here are four number lines, each with a point on it.



For each number line, label the point. This is your target. Make two forward jumps to get from 0 to the target.

- Pick a card from the set given to you. Use the fraction on it for your first jump. Draw the jump and label it with the fraction.
- From there, draw the second jump to reach the target. What fraction do you need to add? Label the jump with the fraction.
- Write an equation to represent the sum of your two fractions.

Launch

- Groups of 2
- Give each group a set of fraction cards from the Instructional master.
- Display the four number lines.
- “What do you notice? What do you wonder?”
- 30 seconds: quiet think time
- 30 seconds: partner discussion

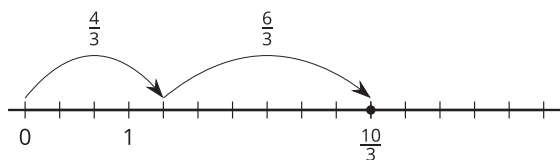
Activity

- “Label each point on the number line with a fraction it represents.”
- 1–2 minutes: independent work time
- “You will make two jumps on the number line to go from 0 to the point and write an equation to represent your moves.”
- Explain how to use the cards and how to complete the task. Consider demonstrating with an example and allowing students to ask clarifying questions before they begin.

Student Responses

Answers vary depending on the cards drawn.

1. Sample responses:



- Point: $\frac{10}{3}$. Card: $\frac{4}{3}$.
The fraction to add is $\frac{6}{3}$.
Equation: $\frac{10}{3} = \frac{4}{3} + \frac{6}{3}$
- Point: $3\frac{1}{3}$. Card: $\frac{4}{3}$, which is $1\frac{1}{3}$.
The number to add is 2 or $\frac{6}{3}$.
Equation: $3\frac{1}{3} = \frac{4}{3} + 2$ or
 $3\frac{1}{3} = \frac{4}{3} + \frac{6}{3}$

2. Point: $\frac{11}{3}$ or $3\frac{2}{3}$

3. Point: $\frac{14}{3}$ or $4\frac{2}{3}$

4. Point: $\frac{12}{3}$ or 4

- Monitor for students who:
 - use whole numbers and mixed numbers in their equations and those who don't
 - label their number lines with whole numbers beyond 1

Synthesis

- Select previously identified students to share their responses to the first couple of diagrams (or more if time permits). Start with students who used no whole numbers or mixed numbers. Ask them to explain why they chose to write the numbers the way they did.
- Consider discussing the merits and challenges (if any) of expressing the fractions as whole or mixed numbers.

Lesson Synthesis

🕒 10 min

"Today, we used number lines to decompose fractions into sums of smaller fractions, or sums of a whole number and a fraction. We also learned that a fraction greater than 1 can be written as a **mixed number**."

"How would you explain to a classmate who is absent today what a mixed number is?" (It's a number written as a whole number and a fraction.)

"Let's look at some sums you found in the second activity. Which ones can be written as mixed numbers and why?" ($\frac{10}{8}$ and $\frac{20}{8}$, because they are greater than 1.)

"What mixed number is equivalent to each of those fractions? How do you know?" ($1\frac{2}{8}$ and $2\frac{4}{8}$:

- $1\frac{2}{8}$ is equivalent to $\frac{10}{8}$ because $\frac{10}{8} = \frac{8}{8} + \frac{2}{8}$ and $\frac{8}{8}$ is 1.
- $2\frac{4}{8}$ is equivalent to $\frac{20}{8}$ because $\frac{16}{8}$ is 2 wholes and adding $\frac{4}{8}$ more gives $\frac{20}{8}$.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

The numerators in the equations students write for the first problem do not add to 13.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Lesson 9: Differences of Fractions

Standards Alignments

Addressing 4.NF.B.3.a, 4.NF.B.3.c

Teacher-facing Learning Goals

- Reason about subtraction of fractions with the same denominator using a number line.

Student-facing Learning Goals

- Let's explore differences of fractions on a number line.

Lesson Purpose

The purpose of this lesson is for students to use a number line to reason about subtraction of fractions with the same denominator.

Previously, students decomposed fractions into sums of other fractions and wrote equations to record the decompositions. They also used number lines to reason about addition of fractions with the same denominator. In this lesson, students use number lines to think about differences of fractions with the same denominator. They also practice reasoning about equivalence and decomposing fractions (including mixed numbers) mentally to facilitate subtraction.

As in earlier grades, students may think about subtracting as taking away a number from another number, or as finding an unknown addend. The way they represent differences of fractions on a number line may vary accordingly.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

True or False (Warm-up)

Materials to Copy

- Make a Jump, Subtraction Edition (groups of 2): Activity 3

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Activity 3 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Based on students' work today, what did you learn about their understanding of the relationships between addition and subtraction? How will you use these insights to prepare for upcoming work?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Differences of Fifths

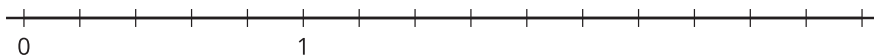
Standards Alignments

Addressing 4.NF.B.3.a, 4.NF.B.3.c

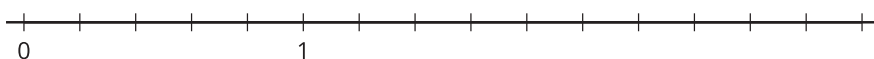
Student-facing Task Statement

Use a number line to represent each difference and to find its value.

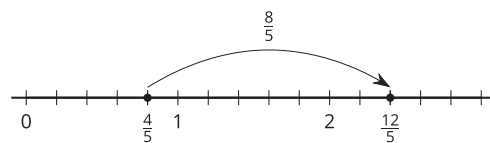
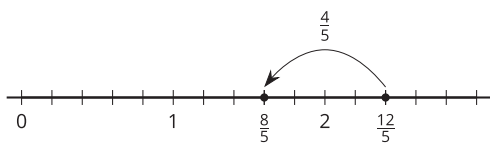
1. $\frac{12}{5} - \frac{4}{5}$



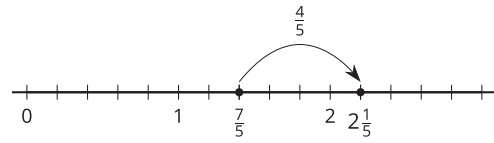
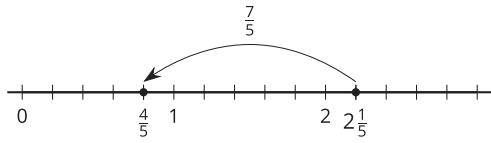
2. $2\frac{1}{5} - \frac{7}{5}$

**Student Responses**

1. $\frac{12}{5} - \frac{4}{5} = \frac{8}{5}$ or $\frac{12}{5} - \frac{4}{5} = 1\frac{3}{5}$. Sample responses:



2. $2\frac{1}{5} - \frac{7}{5} = \frac{4}{5}$. Sample responses:



Begin Lesson

Warm-up

🕒 10 min

True or False: Sums of Tenths

Standards Alignments

Addressing 4.NF.B.3.a

This warm-up prompts students to reason about sums of fractions with the same denominator and to apply their understanding of equivalence, especially of whole numbers and fractions. The reasoning here will be helpful as students explore subtraction of fractions later in the lesson.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} = 1$
- $1 + \frac{7}{10} = \frac{3}{10} + \frac{4}{10} + \frac{10}{10}$
- $\frac{5}{10} + 1 = \frac{6}{10}$
- $\frac{2}{10} + \frac{10}{10} = 1 + \frac{1}{5}$

Launch

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Student Responses

- False: $\frac{1}{10} + \frac{2}{10}$ is $\frac{3}{10}$ and another $\frac{3}{10}$ makes $\frac{6}{10}$, which is less than $\frac{10}{10}$.
- True: 1 is equivalent to $\frac{10}{10}$, so the sum is $\frac{10}{10} + \frac{7}{10}$, which is $\frac{17}{10}$.
- False: Adding $\frac{5}{10}$ and 1 gives a number greater than 1, not $\frac{6}{10}$. The sum of $\frac{5}{10}$ and $\frac{10}{10}$ is $\frac{15}{10}$ or $1\frac{5}{10}$.
- True: $\frac{10}{10}$ is equivalent to 1 and $\frac{2}{10}$ is equivalent to $\frac{1}{5}$, so the two sides have equal values.

Synthesis

- “How can you explain your answer without finding the value of both sides?”
- Consider asking:
 - “Who can restate ___’s reasoning in a different way?”
 - “Does anyone want to add on to _____’s reasoning?”

Activity 1

🕒 20 min

Jump to Subtract

Standards Alignments

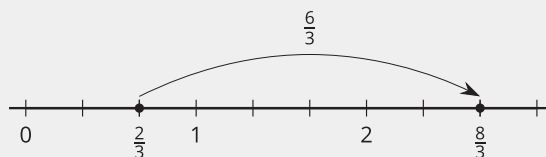
Addressing 4.NF.B.3.a, 4.NF.B.3.c

In this activity, students reason about differences of fractions on a number line and write equations for number line diagrams that represent subtraction. They subtract a fraction from another fraction, as well as a whole number from a fraction, applying what they know about equivalence of whole numbers and fractions to facilitate their reasoning. When students decide whether or not they agree with Noah and explain their reasoning, they critique the reasoning of others (MP3).

In earlier grades, students used number lines to reason about subtraction of whole numbers. To find the value of $42 - 15$, for example, they could start at 42 and jump 15 spaces to the left (or jump 2 spaces to 40, and then 10 spaces to 30, and 3 more to 27). They could also think in terms of addition—“What number added to 15 gives 42?”—and start at 15 and see how many spaces it takes to get to 42.

Students may reason about subtraction of fractions the same way here. For instance, to find $\frac{8}{3} - \frac{2}{3}$, they may:

- Start at $\frac{8}{3}$ and jump 2 thirds to the left. The difference is the point where the arrow lands.
- Start at $\frac{2}{3}$ and see how many thirds are needed to reach $\frac{8}{3}$. The difference is the distance between the two points.



Access for English Learners

MLR8 Discussion Supports. Synthesis: Display sentence frames to support small-group discussion: “___ and ___ are the same because . . .”, and “I wonder if . . .”

Advances: Conversing, Representing

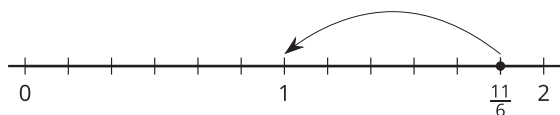
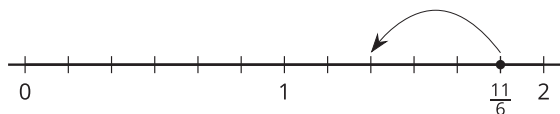
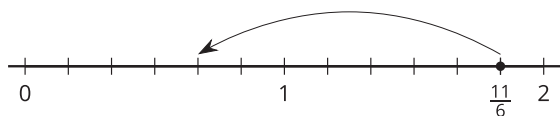
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Differentiate the degree of difficulty or complexity. Some students may benefit from reviewing subtraction on the number line with whole numbers. For example, display a number line with two jumps that represents $11 - 7$. Invite students to write and discuss the equation represented in the diagram.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

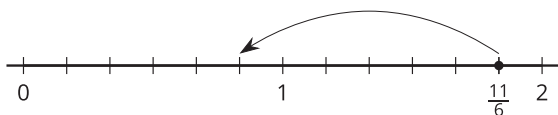
1. To subtract different fractions from $\frac{11}{6}$, Noah draws “jumps” on number lines.



Launch

- Groups of 2
- Display the first three number line diagrams in the activity.
- “How are these diagrams the same as the diagrams we saw in an earlier lesson? How are they different?” (Same: They use jumps to show a change. Each space between tick marks represents a unit fraction. Different: There is only one jump. The arrows point to the left.)
- 1 minute: quiet think time
- Share responses.
- “How do we know that the point represents

- a. The first diagram shows how he finds $\frac{11}{6} - \frac{7}{6}$. What is the value of $\frac{11}{6} - \frac{7}{6}$?
- b. Write an equation to show the difference represented by each of Noah's diagrams.
2. Here is another diagram Noah draws:



Which equations could the diagram represent? Explain your reasoning.

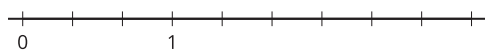
$$\frac{11}{6} - \frac{6}{6} = \frac{5}{6}$$

$$\frac{11}{6} - 1 = \frac{5}{6}$$

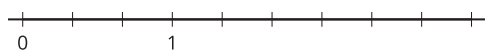
$$1\frac{5}{6} - 1 = \frac{5}{6}$$

3. Use a number line to represent each difference and to find its value.

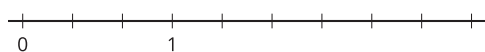
a. $\frac{8}{3} - \frac{2}{3}$



b. $\frac{8}{3} - \frac{4}{3}$



c. $\frac{8}{3} - 1$



Student Responses

1. a. $\frac{4}{6}$
- b. $\frac{11}{6} - \frac{3}{6} = \frac{8}{6}$
 $\frac{11}{6} - \frac{5}{6} = \frac{6}{6}$
 $\frac{11}{6} - \frac{5}{6} = 1$
2. All three equations could represent the diagram. Sample response:
- o $\frac{11}{6} - \frac{6}{6} = \frac{5}{6}$: The jump starts at $\frac{11}{6}$

$\frac{11}{6}$?" (Each space represents 1 sixth. The point is 11 sixths from 0.)

Activity

- "Each of Noah's diagrams represents subtraction from $\frac{11}{6}$. Think about what number is being subtracted and what the result of the subtraction might be."
- "Work with your partner on the first two problems."
- 5–7 minutes: partner work time
- Invite students to share their responses to the first problem.
- "Where do you see the numbers being subtracted?" (The number of spaces jumped)
- "Where do you see the result of the subtraction?" (The point where the arrow lands)
- For the second problem, poll the class on which equations they thought the diagram could represent (for example: only the first, only the second, only the third, the first two, all three, and so on). Invite students from each camp to share their reasoning.
- Make sure students recognize why the diagram can represent all three equations. (See Student Responses.)
- 2 minutes: independent work on the last problem

Synthesis

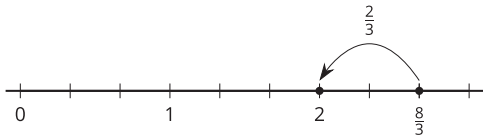
- Focus the discussion on the last expression $\frac{8}{3} - 1$.
- "How did you subtract 1, a whole number, from $\frac{8}{3}$, a fraction?" (Start at $\frac{8}{3}$ and jump to the left 3 thirds, to land at $\frac{5}{3}$. Start at 1 and find out how far to jump to the right to reach $\frac{8}{3}$.)

and goes back 6 spaces (6 sixths), ending up at $\frac{5}{6}$.

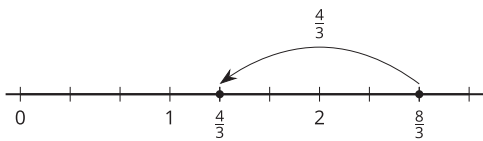
- $\frac{11}{6} - 1 = \frac{5}{6}$: The jump starts at $\frac{11}{6}$ and goes back by $\frac{6}{6}$, which is equivalent to 1, and ends up at $\frac{5}{6}$.
- $1\frac{5}{6} - 1 = \frac{5}{6}$: $\frac{11}{6}$ is equivalent to $1\frac{5}{6}$. Going back 5 spaces (5 sixths) from that point takes us to 1. Going back 1 more space (1 sixth) takes us to $\frac{5}{6}$.

3.

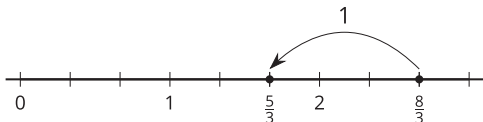
a. $\frac{8}{3} - \frac{2}{3} = \frac{6}{3}$



b. $\frac{8}{3} - \frac{4}{3} = \frac{4}{3}$



c. $\frac{8}{3} - 1 = \frac{5}{3}$



- “How could you subtract 1 from $\frac{8}{3}$ if you didn’t have a number line?” (I could:
 - Think of 1 as $\frac{3}{3}$ and subtract $\frac{3}{3}$ from $\frac{8}{3}$, which gives $\frac{5}{3}$.
 - Think about how many thirds to add to $\frac{3}{3}$ to get $\frac{8}{3}$.
 - Think of $\frac{8}{3}$ as $2\frac{2}{3}$ and subtract 1 from it, which gives $1\frac{2}{3}$.)

Advancing Student Thinking

If students find a value for $\frac{8}{3} - 1$ other than $\frac{5}{3}$ or $1\frac{2}{3}$, consider asking: “How else can we represent 1 whole using thirds?” and “How might we use this to help show the difference on this number line?”

Activity 2

🕒 15 min

What’s the Difference?

Standards Alignments

Addressing 4.NF.B.3.a, 4.NF.B.3.c

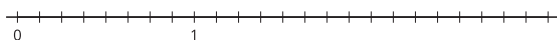
In this activity, students use number lines to represent subtraction of a fraction by another fraction with the same denominator—including a mixed number—and by a whole number. Locating a fraction greater than 1 on the number line prompts students to decompose the fraction mentally into a whole number and a fractional part, rather than to rely on counting tick marks. Representing subtraction of a whole number on the number line encourages students to use their knowledge of whole-number equivalents of fractions and to look for and make use of structure (MP7). For example, when subtracting 1 from $\frac{13}{8}$, it helps to think of 1 as $\frac{8}{8}$, and when subtracting 1 from a mixed number such as $1\frac{5}{8}$, it helps to notice that $1\frac{5}{8}$ is $1 + \frac{5}{8}$.

As before, students may reason about subtraction in terms of removing an amount or finding an unknown addend, resulting in different number line diagrams. While students may rely on number lines to find each difference, the reasoning they do here prompts them to notice patterns and to think flexibly, preparing them to reason numerically in upcoming lessons.

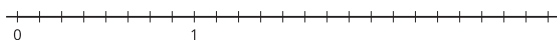
Student-facing Task Statement

Use a number line to represent each difference and to find its value.

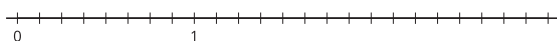
1. $\frac{13}{8} - \frac{2}{8}$



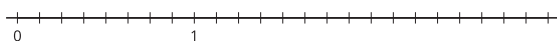
2. $\frac{13}{8} - \frac{6}{8}$



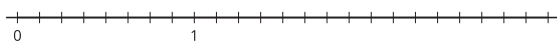
3. $\frac{13}{8} - 1$



4. $1\frac{5}{8} - \frac{7}{8}$



5. $1\frac{5}{8} - 1$



6. $1\frac{5}{8} - 1\frac{4}{8}$

Launch

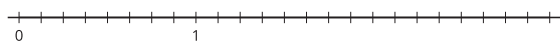
- Groups of 2

Activity

- “Take a few quiet minutes to work on the task. Then, share your responses with your partner.”
- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for the different strategies students use to locate $\frac{13}{8}$ on the number line and to represent subtraction by 1 and $1\frac{4}{8}$.

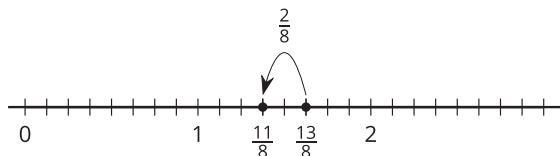
Synthesis

- Select students to share their responses and completed diagrams.
- Focus the discussion on expressions that involve subtraction by a whole number or

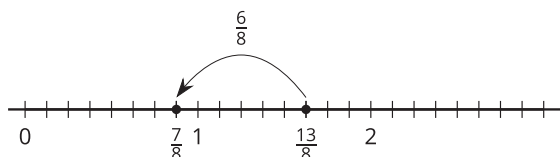


Student Responses

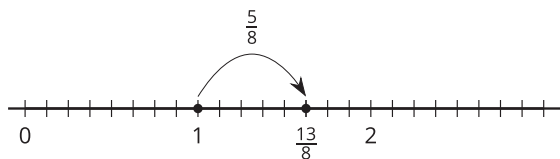
1. $\frac{13}{8} - \frac{2}{8} = \frac{11}{8}$. Sample response:



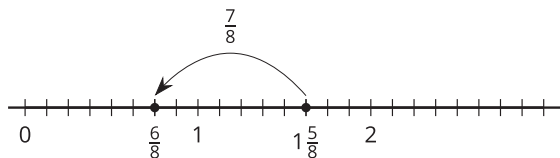
2. $\frac{13}{8} - \frac{6}{8} = \frac{7}{8}$. Sample response:



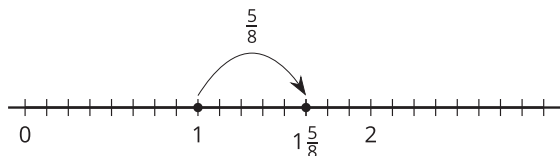
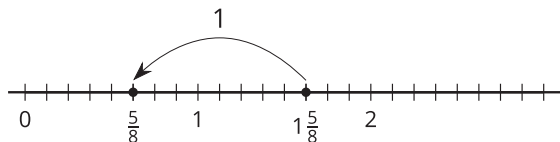
3. $\frac{13}{8} - 1 = \frac{5}{8}$. Sample response:



4. $1\frac{5}{8} - \frac{7}{8} = \frac{6}{8}$. Sample response:



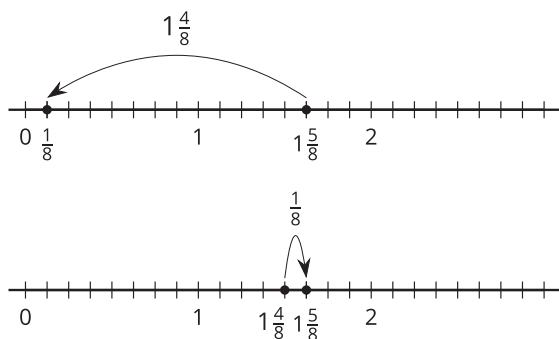
5. $1\frac{5}{8} - 1 = \frac{5}{8}$. Sample responses:



6. $1\frac{5}{8} - 1\frac{4}{8} = \frac{1}{8}$. Sample responses:

by a mixed number.

- Display the third and fifth expressions side by side: $\frac{13}{8} - 1$ and $1\frac{5}{8} - 1$.
- “Did you use the same strategy to represent these expressions on the number line and to find the value? If so, what was your strategy? If not, what was different?”
- Display the fourth and last expressions side by side: $1\frac{5}{8} - \frac{4}{8}$ and $1\frac{5}{8} - 1\frac{4}{8}$. Ask students to compare how they found the value of these expressions.
- See lesson synthesis.



Activity 3 (optional)

🕒 15 min

Make a Jump, Subtraction Edition

Standards Alignments

Addressing 4.NF.B.3.a, 4.NF.B.3.c

This optional activity gives students an additional opportunity to practice using “jumps” on number lines to subtract fractions, decomposing any whole numbers as needed along the way. (It is similar in structure to an optional activity in an earlier lesson on addition.)

Students are given four number lines with a point marked on each. They then draw a card with a fraction on it. All fractions on the cards, shown here, are greater than the values of the points. Students make one or more jumps to find the difference of the two points and then represent it with a subtraction equation.

$$\frac{8}{5} \quad \frac{9}{5} \quad \frac{10}{5} \quad \frac{11}{5} \quad \frac{12}{5} \quad \frac{13}{5} \quad \frac{14}{5} \quad \frac{15}{5}$$

Materials to Copy

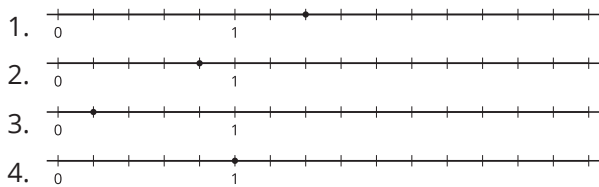
Make a Jump, Subtraction Edition (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Here are four number lines, each with a point on it. Label each point with a fraction it represents.



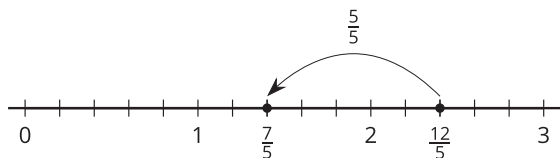
The point you labeled is your target.

- Pick a card from the set given to you. Locate and label the fraction on the number line.
- From that point, draw one or more jumps to reach the target. What do you need to subtract? Label each jump you draw.
- Write an equation to represent the difference of your two fractions.

Student Responses

Answers vary depending on the cards drawn.

1. Sample responses:



- Target: $\frac{7}{5}$ Card: $\frac{12}{5}$
The fraction to subtract is $\frac{5}{5}$.
Equation: $\frac{12}{5} - \frac{5}{5} = \frac{7}{5}$
- Target: $1\frac{2}{5}$ Card: $\frac{12}{5}$, which is $2\frac{2}{5}$
The fraction to subtract is 1 or $\frac{5}{5}$.
Equation: $\frac{12}{5} - \frac{5}{5} = 1\frac{2}{5}$ or
 $2\frac{2}{5} - 1 = 1\frac{2}{5}$

2. Target: $\frac{4}{5}$

3. Target: $\frac{1}{5}$

Launch

- Groups of 2
- Give each group a set of fraction cards from the Instructional master.
- If students did not complete the optional activity in the previous lesson (which had a similar structure):
 - Display the four number lines.
 - “What do you notice? What do you wonder?”
 - 30 seconds: quiet think time
 - 30 seconds: partner discussion

Activity

- “Label each point on the number line with a fraction it represents. This is your target.”
- 1 minute: independent work time
- “You will now make one or more jumps from another fraction to your target and write an equation to represent the difference between the two.”
- Explain how to use the cards and how to complete the task.
- Monitor for students who facilitate subtraction by:
 - rewriting a fraction on their card as a whole number or a mixed number
 - labeling their number lines with whole numbers beyond 1

Synthesis

- Select students to share their responses to the first couple of diagrams (or more if time permits).

4. Target: $\frac{5}{5}$ or 1

Lesson Synthesis

🕒 10 min

“Today we learned to subtract a fraction from a fraction and a whole number from a fraction. We used number lines to help us.”

“How could we find the value of $\frac{11}{8} - \frac{7}{8}$?” (We could:

- Start at $\frac{11}{8}$ on a number line that is partitioned into eighths, and move to the left 7 spaces (7 eighths) to land at $\frac{4}{8}$.
- Subtract the numerators: removing 7 eighths from 11 eighths gives 4 eighths.
- Start at $\frac{7}{8}$ and think about how many eighths to add to reach $\frac{11}{8}$.)

“How could we find the value of $\frac{11}{8} - 1$?” (We could:

- Start at $\frac{11}{8}$ and move to the left 8 spaces—because 1 is 8 eighths—to land at $\frac{3}{8}$.
- Think about how many eighths to add to 1 or $\frac{8}{8}$ to get to $\frac{11}{8}$.
- Find $\frac{11}{8} - \frac{8}{8}$, which is $\frac{3}{8}$.)

“How could we find the value of $\frac{11}{8} - 1\frac{1}{8}$?” (We could:

- Think of $1\frac{1}{8}$ as $\frac{9}{8}$ and move left 9 spaces from $\frac{11}{8}$ on the number line.
- Jump to the left 8 spaces to represent subtraction by 1 and then another 1 space for the $\frac{1}{8}$.
- Write $\frac{11}{8}$ as $1\frac{3}{8}$ and think about how far away it is from $1\frac{1}{8}$.
- Subtract the whole number and the fractional part separately. $1 - 1 = 0$ and $\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Estimate and Measure (1–4), Stage 3: Quarter Inches (Supporting)
- Target Measurements (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

Students incorrectly represent $2\frac{1}{5}$ on the number line or do not see it as equivalent to $\frac{11}{5}$.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Lesson 10: The Numbers in Subtraction

Standards Alignments

Building On 4.NF.B.3.a, 4.NF.B.4.b
Addressing 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4.c

Teacher-facing Learning Goals

- Subtract a fraction from a whole number by decomposing the whole number and reasoning about equivalence.

Student-facing Learning Goals

- Let's subtract fractions from whole numbers.

Lesson Purpose

The purpose of this lesson is for students to recognize that a fraction can be subtracted from a whole number by writing an equivalent fraction for the whole number. It can also be done by decomposing the whole number, the fraction, or both, into a sum of fractions with the same denominator.

In earlier lessons, students explored addition and subtraction of fractions in context and out of context. They used diagrams to represent and find the values of sums and differences, and they also reasoned numerically. Students learned that a non-unit fraction can be decomposed—in at least one way—into a sum of other fractions with the same denominator. They recognized adding and subtracting fractions as joining and removing parts that refer to the same whole.

In this lesson, students apply these insights, as well as their knowledge of equivalent fractions from an earlier unit and from grade 3, to find differences of a whole number and a fraction.

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

5 Practices (Activity 1), Card Sort (Activity 2), MLR2 Collect and Display (Activity 2), Number Talk (Warm-up)

Materials to Copy

- Card Sort: Twelfths (groups of 2): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

How did the student work that you selected impact the direction of class discussion? How would you adjust your selection if you teach the lesson again?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Two Differences

Standards Alignments

Addressing 4.NF.B.3.c

Student-facing Task Statement

Find the value of each difference. Show your reasoning.

- $2 - \frac{5}{6}$
- $4 - \frac{11}{6}$

Student Responses

- $\frac{7}{6}$ or $1\frac{1}{6}$. Sample response:
 - $2 - \frac{5}{6} = \frac{12}{6} - \frac{5}{6} = \frac{7}{6}$
 - $2 - \frac{5}{6} = (1 + \frac{6}{6}) - \frac{5}{6} = 1 + (\frac{6}{6} - \frac{5}{6}) = 1 + \frac{1}{6} = 1\frac{1}{6}$
- $\frac{13}{6}$ or $2\frac{1}{6}$. Sample response:
 - $4 - \frac{11}{6} = \frac{24}{6} - \frac{11}{6} = \frac{13}{6}$
 - $4 - \frac{11}{6} = (2 + \frac{12}{6}) - \frac{11}{6} = 2 + (\frac{12}{6} - \frac{11}{6}) = 2 + \frac{1}{6} = 2\frac{1}{6}$
 - $\frac{11}{6}$ is $\frac{1}{6}$ away from 2. I subtracted 2 from 4, and then add $\frac{1}{6}$ back to get $2\frac{1}{6}$

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Groups of Twelfths

Standards Alignments

Building On 4.NF.B.4.b

This Number Talk encourages students to use what they learned about products of a whole number and a fraction, the relationship between each pair of factors, and the structure in the expressions to mentally solve problems.

Students may write all the products as fractions, including the ones greater than 1. If everyone expresses the last three products only as $\frac{18}{12}$, $\frac{36}{12}$, and $\frac{360}{12}$, during the synthesis, ask if students could write whole-number or mixed-number equivalents for these fractions. The reasoning elicited here will be helpful later in the lesson when students decompose whole numbers in order to subtract fractional amounts.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $2 \times \frac{3}{12}$
- $6 \times \frac{3}{12}$
- $12 \times \frac{3}{12}$
- $12 \times \frac{30}{12}$

Student Responses

Sample response:

- $\frac{6}{12}$, because 2 groups of 3 twelfths makes 6 twelfths.
- $\frac{18}{12}$, because 6×3 is 18 (or $1 \frac{6}{12}$, because two $\frac{6}{12}$ s make 1 whole, and one more $\frac{6}{12}$ makes

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “In which expressions might it be helpful to use 1 whole—or 12 twelfths—to find the product? How?” (The first three. We know 4

$1\frac{6}{12}$).

- 3 (or $\frac{36}{12}$), because every 4 groups of $\frac{3}{12}$ s make $\frac{12}{12}$ or 1 whole, so 8 groups of $\frac{3}{12}$ s makes 2 and 12 groups of $\frac{3}{12}$ s makes 3.
- 30 (or $\frac{360}{12}$), because $\frac{30}{12}$ is 10 times $\frac{3}{12}$, so $12 \times \frac{30}{12}$ is 10 times $12 \times \frac{3}{12}$ or 10×3 .

groups of $\frac{3}{12}$ make 1 whole, so:

- 2 groups of $\frac{3}{12}$ make $\frac{1}{2}$
- 6 groups of $\frac{3}{12}$ make $1\frac{1}{2}$
- 12 groups of $\frac{3}{12}$ make 3
- “Why might it be a little harder to think of the last expression in terms of 12 twelfths?” (The 30 in $\frac{30}{12}$ is not a factor or a multiple of 12.)
- Consider asking:
 - “Who can restate ____'s reasoning in a different way?”
 - “Did anyone have the same strategy but would explain it differently?”
 - “Did anyone approach the expression in a different way?”
 - “Does anyone want to add on to ____'s strategy?”

Activity 1

 20 min

What's Left?

 PLC Activity

Standards Alignments

Building On 4.NF.B.3.a

Addressing 4.NF.B.3.c, 4.NF.B.3.d

This is a 5 Practices activity. Students use any strategy that makes sense to them to reason about subtraction of a fraction from a whole number. They begin by using an image to support their reasoning. Later, when no image is given, students may use a variety of ways to find differences. In the synthesis students share, explain, and relate different strategies for solving the problem (MP3).

Monitor for the following strategies, listed from concrete to abstract, and select students to share during synthesis:

- Draw a diagram showing whole numbers and unit fractions and remove parts of the diagram.

- Create a number line that is partitioned into thirds and draw jumps to the left.
- Reason in terms of addition (for instance, how many thirds to add to $\frac{5}{3}$ to reach 4?).
- Subtract in multiple rounds (for example, to subtract $2\frac{1}{3}$ from 4, first subtract 2 wholes, and then subtract 1 third).
- Decompose or rewrite a whole number or a fraction before subtracting.
 - To subtract $\frac{5}{3}$ from 4, first decompose $\frac{5}{3}$ into $\frac{3}{3} + \frac{2}{3}$, or $1 + \frac{2}{3}$, then subtract.
 - To subtract $2\frac{1}{3}$ from 4, first decompose 4 into $3 + \frac{3}{3}$ and decompose $2\frac{1}{3}$ into $2 + \frac{1}{3}$, and then subtract. Or, rewrite 4 and $2\frac{1}{3}$ as $\frac{12}{3}$ and $\frac{7}{3}$ and then subtract.

At this point, students are not expected to decompose or rewrite numbers using expressions and equations. They may perform the reasoning intuitively and informally. Later, they will formalize the different ways of recording the decomposition of numbers for subtraction.

Access for Students with Disabilities

Representation: Access for Perception. Ask students to identify correspondences between the image of the pitcher and number lines they have worked with in previous lessons. Invite students to use diagrams to solve the task.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Instructional Routines

5 Practices

Student-facing Task Statement

1. A pitcher contains 3 cups of watermelon juice.



How many cups will be left in the pitcher if we pour each of the following amounts from the full amount?

- a. $\frac{1}{4}$ cup
- b. $\frac{5}{4}$ cups
- c. $1\frac{1}{4}$ cups
- d. $2\frac{2}{4}$ cups

Launch

- Groups of 2
- Display the image of the graduated pitcher.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion
- “Let’s now solve some problems about pouring a drink out of a pitcher.”

Activity

- 7–10 minutes: independent work time

2. A second pitcher contains 4 cups of water. How many cups will be left in that pitcher if we pour each of the following amounts from the full amount?

Explain or show your reasoning. Use diagrams or equations, if they are helpful.

- a. $\frac{1}{3}$ cup
- b. $\frac{5}{3}$ cups
- c. $2\frac{2}{3}$ cups

Student Responses

1.
 - a. $2\frac{3}{4}$ cups
 - b. $1\frac{3}{4}$ cups
 - c. $1\frac{3}{4}$ cups
 - d. $\frac{2}{4}$ cups
2.
 - a. $3\frac{2}{3}$ or $\frac{11}{3}$ cups. Sample response: 4 is equivalent to $\frac{12}{3}$ and $\frac{12}{3} - \frac{1}{3} = \frac{11}{3}$.
 - b. $2\frac{1}{3}$ or $\frac{7}{3}$. Sample response: $4 = 2 + \frac{6}{3}$ and $2 + \frac{6}{3} - \frac{5}{3} = 2\frac{1}{3}$.
 - c. $1\frac{1}{3}$ or $\frac{4}{3}$ cups. Sample response: First, I subtracted 2 whole cups from 4, which gives 2 cups. Then, I subtracted $\frac{2}{3}$ cup from those 2 cups, or subtracted $\frac{2}{3}$ from $\frac{6}{3}$ cups, which gives $\frac{4}{3}$ cups.

- 2 minutes: partner discussion
- Monitor for the different strategies used, as noted in the Activity Narrative.

Synthesis

- “How did you find the result of subtracting each fraction from 4, without physically pouring out $\frac{1}{3}$, $\frac{5}{3}$, or $2\frac{2}{3}$ cups of water out of a 4-cup pitcher?”
- Select students who used different strategies in the second problem to share their responses, in the order listed in the Activity Narrative.
- Record all the ways students decomposed or rewrote the numbers in the problem to facilitate subtraction (whether or not they wrote expressions or equations).

Advancing Student Thinking

To support students in visualizing the context, consider revisiting it in several rounds and with a limited scope each time. Consider asking:

- “What is happening in this situation?”
- “What are we trying to find out?” and “What information do we have?”
- “What mathematical operation can help us answer this question?”
- “What expression would we use to represent the amount of liquid?”

Activity 2

 15 min

Card Sort: Twelfths

Standards Alignments

Addressing 4.NF.B.4.c

This activity makes explicit what students may have noticed in earlier activities, namely, that we can subtract a fraction from a whole number by:

- writing an equivalent fraction for the whole number
- decomposing the whole number into a sum of smaller numbers, which could be fractions, or a whole number and a fraction

Students sort a set of cards (with a number or an expression) into two groups and justify the categories. Each card has a value equivalent to either $1 - \frac{5}{12}$ or $2 - \frac{5}{12}$. Though students may sort the cards in a few valid ways, the discussion should focus on the value on the cards. Students then apply their insights from the sorting work to subtract a fraction from whole numbers. This sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

Here is a list of the numbers or expressions on the Instructional master, for reference:

| | | | | |
|--------------------|--|--------------------------------|------------------------|------------------------------------|
| A. | B. | C. | D. | E. |
| $1 - \frac{5}{12}$ | $\frac{12}{12} + \frac{12}{12} - \frac{5}{12}$ | $\frac{24}{12} - \frac{5}{12}$ | $\frac{7}{12}$ | $1\frac{7}{12}$ |
| F. | G. | H. | I. | J. |
| $2 - \frac{5}{12}$ | $\frac{12}{12} - \frac{5}{12}$ | $\frac{19}{12}$ | $1 + 1 - \frac{5}{12}$ | $1 + \frac{12}{12} - \frac{5}{12}$ |

This activity uses *MLR2 Collect and Display*. Advances: conversing, reading, writing

Instructional Routines

Card Sort, *MLR2 Collect and Display*

Materials to Copy

Card Sort: Twelfths (groups of 2)

Required Preparation

- Create a set of cards for each group of 2.

Student-facing Task Statement

1. Sort the cards from your teacher into two groups. Record your sorted expressions. Be prepared to explain why the cards in each group belong together.
2. Find the value of each difference. Show your reasoning.
 - a. $1 - \frac{5}{8}$
 - b. $2 - \frac{7}{8}$
 - c. $3 - \frac{9}{8}$

Student Responses

1. Sample responses:
 - Cards with a value of $\frac{7}{12}$: A, D, and G
Cards with a value of $1\frac{7}{12}$: B, C, E, F, H, I, J
 - Cards showing only a number: D, E, and H
Cards showing an expression: A, B, C, F, G, I, J
 - Cards that show one or more whole numbers: A, F, I, J
Cards that show no whole numbers: B, C, D, E, G, H
2.
 - a. $\frac{3}{8}$. Sample response: 1 is equivalent to $\frac{8}{8}$ and $\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$.
 - b. $\frac{9}{8}$ or $1\frac{1}{8}$. Sample response:
 - 2 is equivalent to $\frac{16}{8}$ and $\frac{16}{8} - \frac{7}{8} = \frac{9}{8}$.
 - 2 is equivalent to $1 + \frac{8}{8}$ and $1 + \frac{8}{8} - \frac{7}{8} = 1\frac{1}{8}$.

Launch

- Groups of 2 or 4
- Give each group one set of cards from the Instructional master.

Activity

- “Work with your group to sort these cards into two groups. Be prepared to explain why the cards in each group belong together.”
- 5 minutes: group work time on the first problem
- Monitor for the ways students sort the cards. Identify those who sort the cards by their value or by their equivalence to $1 - \frac{5}{12}$ and $2 - \frac{5}{12}$.

MLR2 Collect and Display

- Circulate, listen for, and collect the language students use to describe the features of the expressions on the cards or the connections between expressions. Listen for terms such as “equivalent,” or “equivalent fractions,” “equal,” “sum,” “difference,” and “decompose.”
- Record students’ words and phrases on a visual display and update it throughout the lesson.
- Pause before students proceed to the next problem. Select a few groups to share their sorting decisions, ending with a group that sorted the cards by value.
- “Let’s look at that last way of sorting. The cards in each group have the same value, and that value is a result of subtracting a whole number by a fraction.”

c. $\frac{15}{8}$ or $1\frac{7}{8}$. Sample response:

- 3 is equivalent to $\frac{24}{8}$ and $\frac{24}{8} - \frac{9}{8} = \frac{15}{8}$.
- 3 is equivalent to $1 + \frac{16}{8}$ and $1 + \frac{16}{8} - \frac{9}{8} = 1\frac{7}{8}$.

- Display the cards as shown:

| | | |
|--------------------------------------|--|--------------------------------------|
| A. $1 - \frac{5}{12}$ | F. $2 - \frac{5}{12}$ | |
| G. $\frac{12}{12} - \frac{5}{12}$ | I. $1 + 1 - \frac{5}{12}$ | C. $\frac{24}{12} - \frac{5}{12}$ |
| D. $\frac{7}{12}$ | J. $1 + \frac{12}{12} - \frac{5}{12}$ | H. $\frac{19}{12}$ |
| E. $1\frac{7}{12}$ | | |

- "Turn to a partner. Talk about how each card is related to the one below it. Do this for each group of cards."
- 2 minutes: partner discussion
- Select 2–3 students to share the connections between the expressions in each group. Highlight that:
 - In the first group, the 1 can be written as an equivalent fraction, $\frac{12}{12}$, which is helpful for subtracting $\frac{5}{12}$.
 - In the second group, the 2 can be rewritten as an equivalent fraction, $\frac{24}{12}$, or decomposed into a sum of $1 + 1$, which is equivalent to $1 + \frac{12}{12}$. Both strategies help us to subtract $\frac{5}{12}$.
- 5 minutes: independent work on the second problem

Synthesis

- See lesson synthesis.

Lesson Synthesis

🕒 10 min

“Today we learned that we can subtract a fraction from a whole number by either rewriting the whole number as a fraction, or by decomposing the whole number.”

Select students to share how they found the three differences in the last problem in the last activity. As students explain, update the display by adding or replacing language or annotations.

“Are there any other words or phrases that are important to include on our display?”

Highlight explanations that show that:

- The 1 in $1 - \frac{5}{8}$ can be written as $\frac{8}{8}$.
- The 2 in $2 - \frac{7}{8}$ can be written as $\frac{16}{8}$ or decomposed into $1 + \frac{8}{8}$.
- The 3 in $3 - \frac{9}{8}$ can be written as $\frac{24}{8}$ or decomposed into $1 + \frac{16}{8}$ (among other sums).
- Writing an equivalent fraction and decomposing the whole number both make it easier to find each difference.

Remind students to borrow language from the display as needed in future activities.

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Creating Line Plots (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

For the last expression, students rewrite $\frac{11}{6}$ as $1\frac{5}{6}$ and subtract the 1 whole from 4, but leave the fractional part unattended (writing $3\frac{5}{6}$ as a result of the subtraction).

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 11: Subtract Fractions Flexibly

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.3.d

Teacher-facing Learning Goals

- Subtract fractions and mixed numbers by decomposing numbers and reasoning about equivalence.

Student-facing Learning Goals

- Let's find all kinds of differences.

Lesson Purpose

The purpose of this lesson is for students to subtract fractions, including mixed numbers, by decomposing numbers and writing equivalent fractions, and to recognize when these strategies are useful for finding differences.

In the previous lesson, students learned to subtract a fraction from a whole number numerically, by writing an equivalent fraction for the whole number or decomposing the whole number into a sum of fractions with the same denominator. This lesson extends that work to include mixed numbers. It also prompts students to look for structure in subtraction expressions where decomposing one or both numbers makes the expression easier to evaluate (MP7). The work here builds students' ability to subtract fractions flexibly.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Which One Doesn't Belong? (Warm-up)

Required Preparation

- Each group of 4 needs tools for creating a visual display during the lesson synthesis.

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

Reflect on evidence of student thinking that you observed today. Whose thinking was voiced and

| | |
|------------------|--------|
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

heard? Whose thinking was not but could have enriched the conversations? What prompts or structures might better enable the latter to share their voice and reasoning?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

A Shorter Strip, Please

Standards Alignments

Addressing 4.NF.B.3.d

Student-facing Task Statement

Lin has a strip of paper that is $7\frac{1}{4}$ inches long and needs to be trimmed by $2\frac{3}{4}$ inches. What is the length of the paper strip after it is trimmed? Explain or show your reasoning.

Student Responses

$4\frac{2}{4}$ inches. Sample reasoning:

- $7\frac{1}{4}$ is $6 + 1 + \frac{1}{4}$, which is $6 + \frac{4}{4} + \frac{1}{4}$ or $6 + \frac{5}{4}$. I subtracted 2 wholes from 6 wholes, which gives 4 wholes, and then subtracted $\frac{3}{4}$ from $\frac{5}{4}$, which gives $\frac{2}{4}$.
- I know 3 is $\frac{1}{4}$ more than $2\frac{3}{4}$. I subtracted 3 from $7\frac{1}{4}$ to get $4\frac{1}{4}$, and then I added $\frac{1}{4}$ back because I subtracted $\frac{1}{4}$ more than needed earlier.

----- Begin Lesson -----

Warm-up

🕒 10 min

Which One Doesn't Belong: Fractional Values

Standards Alignments

Addressing 4.NF.B.3.c

This warm-up prompts students to carefully analyze subtraction expressions containing two fractions or a whole number and a fraction. To compare the values of the expressions, students need to perform subtraction and apply their knowledge of equivalence. The reasoning here will also be helpful later as students reason about differences of two mixed numbers or a mixed number and a fraction.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A.

$$2 - \frac{3}{5}$$

B.

$$\frac{10}{5} - \frac{3}{5}$$

C.

$$1\frac{3}{5} - \frac{1}{5}$$

D.

$$\frac{10}{5} - 1$$

Student Responses

Sample responses:

- A is the only one where the first number is not written as a fraction.
- B is the only one that doesn't show a whole number.
- C is the only one where the first number is not equivalent to 2.
- D is the only one whose value is not $\frac{7}{5}$ and is not a fraction.

Launch

- Groups of 2
- Display the expressions.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2-3 minutes: partner discussion
- Share and record responses.

Synthesis

Consider asking:

- "Let's find at least one reason why each one doesn't belong."

Activity 1

🕒 15 min

Friendship Bracelets

Standards Alignments

Addressing 4.NF.B.3.d

In this activity, students solve contextual problems that involve subtracting fractions in which at least one value is a mixed number and it is necessary to decompose one or both numbers. Students find differences in any way that makes sense to them. They may use number line diagrams, reason in terms of addition, or perform repeated partial subtractions (without necessarily writing expressions or equations). For instance, to find the difference between $9\frac{4}{8}$ and $\frac{7}{8}$, they may:

- draw a number line, partition it into eighths, locate $9\frac{4}{8}$, and count back 7 spaces to the left to represent subtraction of $\frac{7}{8}$
- reason: “What number plus $\frac{7}{8}$ gives $9\frac{4}{8}$ or _____ + $\frac{7}{8} = 9\frac{4}{8}$?”
- first subtract $\frac{4}{8}$ from $9\frac{4}{8}$, and then subtract another $\frac{3}{8}$
- first subtract 1 from $9\frac{4}{8}$, and then add $\frac{1}{8}$ back

Students may also use the insights they gained previously about rewriting and decomposing whole numbers to facilitate subtraction and consider whether they could be applied to subtraction of a fraction from a mixed number. When students recognize the mathematical features of things in the real world and ask questions that arise from a presented situation, they reason abstractly and quantitatively (MP2).

🕒 Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to a variety of tools, including blank pre-formatted tape diagrams, blank pre-formatted number lines, and colored pencils. Invite students to estimate or solve without these first, then use them to check their work.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Social-Emotional Functioning

Student-facing Task Statement

Launch

- Groups of 2



Clare, Elena, and Andre are making macramé friendship bracelets. They'd like their bracelets to be $9\frac{4}{8}$ inches long. For each question, explain or show your reasoning.

1. Clare started her bracelet first and has only $\frac{7}{8}$ inch left until she finishes it. How long is her bracelet so far?
2. So far, Elena's bracelet is $5\frac{1}{8}$ inches long and Andre's is $3\frac{5}{8}$ inches long. How many more inches do they each need to reach $9\frac{4}{8}$ inches?
3. How much longer is Elena's bracelet than Andre's at the moment?

Student Responses

1. $8\frac{5}{8}$ inches. Sample response: $\frac{7}{8}$ is $\frac{1}{8}$ away from 1, so I subtracted 1 from $9\frac{4}{8}$, which gave $8\frac{4}{8}$. I subtracted $\frac{1}{8}$ too much, so I added $\frac{1}{8}$ back, which gives $8\frac{5}{8}$.
2. Elena: $4\frac{3}{8}$ inches. Andre: $5\frac{7}{8}$ inches. Sample response:
 - Elena: To find $9\frac{4}{8} - 5\frac{1}{8}$, I subtracted the whole numbers first: $9 - 5 = 4$. Next, I subtracted the fractions: $\frac{4}{8} - \frac{1}{8} = \frac{3}{8}$. The difference is $4 + \frac{3}{8}$, which is $4\frac{3}{8}$.
 - Andre: To find $9\frac{4}{8} - 3\frac{5}{8}$, I first subtracted $\frac{5}{8}$ from $9\frac{4}{8}$, which gives $8\frac{7}{8}$, and then subtracted 3 more wholes from $8\frac{7}{8}$ to get $5\frac{7}{8}$.
3. $1\frac{4}{8}$ inches. Sample response: Elena's bracelet is $5\frac{1}{8}$ inches and Andre's is $3\frac{5}{8}$

- Consider asking: "What are some things you have made, given, or received that celebrate your friendships?"
- Explain that macramé is a way of making textile by knotting and is at least a few thousand years old. The name came from the Arabic word "miqramah," with one meaning being "ornamental or decorative fringe."
- "Macramé bracelets are a popular way to celebrate friendships. Have you seen or made one?" (If possible, prepare a couple of bracelets—a finished one and an unfinished one—for students to see, or show images or a video of a bracelet being made.)
- "Let's use what we know about subtracting fractions to solve some problems about friendship bracelets."

Activity

- "Take 5 quiet minutes to work on the task, and then discuss your thinking and complete the rest with your partner."
- 5 minutes: independent work time
- 5 minutes: partner discussion and work time
- Monitor for the different ways students go about finding differences. Identify those who write addition or subtraction expressions or equations.

Synthesis

- Select 2–3 students who use different methods to briefly share their responses. Record subtraction and addition expressions or equations that students wrote or mentioned.
- Students will take a closer look at the same numbers in the next activity.

inches. $3\frac{5}{8}$ is $\frac{5}{8}$ inches away from 4 inches, and 4 inches is $1\frac{1}{8}$ inches away from $5\frac{1}{8}$. So $3\frac{5}{8}$ inches is $\frac{3}{8} + 1\frac{1}{8}$ or $1\frac{4}{8}$ inches away from $5\frac{1}{8}$ inches.

Advancing Student Thinking

Students may be unsure what quantity is unknown in each problem or how it is related to what's given. Consider asking:

- "What do we know about the bracelet?"
- "What do we not know but want to know?"
- "What do we need to do to find out?"

Activity 2

🕒 20 min

Multiple Ways to Subtract

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.3.d

Previously, students found differences of two fractions, including mixed numbers, using any way that made sense to them. This activity formalizes and makes explicit how such differences can be found by writing equivalent fractions and decomposing a whole number or a mixed number. When students share their responses with a partner and revise them based on the feedback they receive, they construct viable arguments and critique the reasoning of others (MP3).

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

Here are four expressions that you may have written about the friendship bracelets.

$$9\frac{4}{8} - \frac{7}{8} \quad 9\frac{4}{8} - 5\frac{1}{8} \quad 9\frac{4}{8} - 3\frac{5}{8} \quad 5\frac{1}{8} - 3\frac{5}{8}$$

- Here is one way to find the value of the first expression. Analyze the calculation. Talk to your partner about why $9\frac{4}{8}$ is written as different sums.

| $9\frac{4}{8} - \frac{7}{8}$ | |
|----------------------------------|---------------|
| first number | second number |
| $9\frac{4}{8}$ | $\frac{7}{8}$ |
| $8 + 1 + \frac{4}{8}$ | |
| $8 + \frac{8}{8} + \frac{4}{8}$ | |
| $8 + \frac{12}{8}$ | |
| $8 + \frac{12}{8} - \frac{7}{8}$ | |
| $8 + \frac{5}{8}$ | |
| $8\frac{5}{8}$ | |

- Here are some unfinished calculations. Complete them to find the value of each difference.

a.

| $9\frac{4}{8} - 5\frac{1}{8}$ | |
|-------------------------------|-------------------|
| first number | second number |
| $9\frac{4}{8}$ | $5\frac{1}{8}$ |
| $9 + \frac{4}{8}$ | $5 + \frac{1}{8}$ |

b.

| $9\frac{4}{8} - 3\frac{5}{8}$ | |
|---------------------------------|-------------------|
| first number | second number |
| $9\frac{4}{8}$ | $3\frac{5}{8}$ |
| $8 + 1 + \frac{4}{8}$ | $3 + \frac{5}{8}$ |
| $8 + \frac{8}{8} + \frac{4}{8}$ | |
| $8 + \frac{12}{8}$ | |

c.

Launch

- Groups of 2
- Display the four expressions in the task statement.
- “What do you notice? What do you wonder?”
- 30 seconds: quiet think time
- Share responses. If not mentioned by students, point out that finding the values of these expressions is a way to answer the questions in the first activity.
- “Let’s take a closer look at how we can subtract fractions like these without using diagrams or a context to help us.”

Activity

- “Take 2 quiet minutes to study the first set of calculations. Then, talk to your partner about what you think is happening in the calculations.”
- 2 minutes: quiet think time
- 1–2 minutes: partner discussion
- Share responses.
- “Why do you think $9\frac{4}{8}$ is decomposed into different sums?” (See Student Responses.)
- “What do the last two expressions show?” (The first one shows the $\frac{7}{8}$ being subtracted from the $\frac{12}{8}$, which is a part of $9\frac{4}{8}$. The second shows the result of that subtraction being rejoined with the 8 from the $9\frac{4}{8}$.)
- “Now it’s your turn to subtract some mixed numbers, by decomposing them, if needed.”
- “Work with your partner to complete the three incomplete calculations in the second problem.”
- 5–7 minutes: partner work time

| $5\frac{1}{8} - 3\frac{5}{8}$ | |
|-------------------------------|-------------------|
| first number | second number |
| $5\frac{1}{8}$ | $3\frac{5}{8}$ |
| $5 + \frac{1}{8}$ | $3 + \frac{5}{8}$ |

Student Responses

1. Sample response: $9\frac{4}{8}$ is rewritten as different sums to make it easier to subtract $\frac{7}{8}$ from $9\frac{4}{8}$. When the 9 is decomposed as $8 + \frac{8}{8}$ and the $\frac{8}{8}$ is added to $\frac{4}{8}$, we have $8 + \frac{12}{8}$. The $\frac{7}{8}$ can then be easily subtracted from $\frac{12}{8}$.

2. Sample response:

$$\begin{array}{l} \text{a. } 9\frac{4}{8} - 5\frac{1}{8} \\ 9 - 5 = 4 \\ \frac{4}{8} - \frac{1}{8} = \frac{3}{8} \\ 4 + \frac{3}{8} = 4\frac{3}{8} \end{array}$$

$$\begin{array}{l} \text{b. } 9\frac{4}{8} - 3\frac{5}{8} \\ 8 - 3 = 5 \\ \frac{12}{8} - \frac{5}{8} = \frac{7}{8} \\ 5 + \frac{7}{8} = 5\frac{7}{8} \end{array}$$

$$\begin{array}{l} \text{c. } 5\frac{1}{8} - 3\frac{5}{8} \\ 5\frac{1}{8} \\ 5 + \frac{1}{8} \\ 4 + 1 + \frac{1}{8} \\ 4 + \frac{8}{8} + \frac{1}{8} \\ 4 + \frac{9}{8} \end{array} \quad \begin{array}{l} 4 - 3 = 1 \\ \frac{9}{8} - \frac{5}{8} = \frac{4}{8} \\ 1 + \frac{4}{8} = 1\frac{4}{8} \end{array}$$

Synthesis

MLR1 Stronger and Clearer Each Time

- Ask each student to find a new partner.
- “Explain your response to part c of the last problem to your new partner.”
- “Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work.”
- 3–4 minutes: structured partner discussion.
- Repeat with at least one other partner.
- “Revise your initial response based on the feedback you got from your partners.”
- 2 minutes: independent work time

Advancing Student Thinking

Students may lose track of the subtraction in each problem because the minuend and subtrahend are expressed as sums. Consider asking students, “What might the first and second numbers represent in a story problem?” Allow students to describe the subtraction in terms of the context to support reasoning.

Lesson Synthesis

🕒 10 min

Arrange students in groups of 4. Give each group tools for creating a visual display.

Assign to each group one expression from the last problem of the last activity. Ask them to record on a visual display the calculations for finding the value of that expression.

Invite groups to share their visual displays for all to see. Ask students to analyze others' calculations and notice how they are like or unlike their own calculations.

Select a student display for each expression or display the calculations from Student Response. Ask students to identify places where mixed numbers are decomposed or equivalent fractions are written to facilitate subtraction.

"When might it be helpful or necessary to decompose a whole number into a sum in order to subtract a fraction?" (When we are subtracting a fraction from a whole number or a mixed number there aren't any or enough fractional parts from which to subtract.)

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Creating Line Plots (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not yet represent a mixed number as the sum of two or more fractions.

Next Day Support

- Launch Activity 1 by reviewing a correct response to the cool-down.

Lesson 12: Sums and Differences of Fractions

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.3.d

Teacher-facing Learning Goals

- Add and subtract fractions (including mixed numbers) with the same denominator.
- Analyze strategies for reasoning about sums and differences of fractions with the same denominator.

Student-facing Learning Goals

- Let's add and subtract fractions and analyze our strategies.

Lesson Purpose

The purpose of this lesson is for students to consider strategies for adding and subtracting fractions with the same denominator, including mixed numbers, and to recognize when it is helpful to decompose numbers or write equivalent fractions when finding sums and differences of fractions.

Previously, students learned to decompose mixed numbers and write equivalent fractions for whole numbers to add and subtract fractions. In this lesson, they practice finding the value of sums and differences of fractions while also taking a closer look at their reasoning strategies. In the first activity, students complete addition and subtraction equations, each with a missing number. They then reflect on the steps they took and consider why they found certain ways of reasoning more productive than others. In the second activity, students examine and explain when it might be useful to decompose numbers when finding sums and differences of fractions. In explaining their answers and strategies, students need to be precise in their word choice and use of language (MP6).

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Which way(s) of reasoning about sums and differences of fractions did most students use today? What suggestions could you offer to improve students flexibility in adding and subtracting fractions?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

How Would You Find the Difference?

Standards Alignments

Addressing 4.NF.B.3.d

Student-facing Task Statement

Consider the expression $\frac{13}{5} - 1\frac{2}{5}$.

1. What would be your first step for finding the value of the expression?
2. Find the value of the expression. Show your reasoning.

Student Responses

1. Sample responses:
 - I would decompose $\frac{13}{5}$ into a whole number and a fraction and write it as a mixed number.
 - I would write $1\frac{2}{5}$ as a fraction without a whole number.
2. $1\frac{1}{5}$ or $\frac{6}{5}$. Sample response:
 - $\frac{13}{5} = \frac{10}{5} + \frac{3}{5} = 2 + \frac{3}{5} = 2\frac{3}{5}$ and $2\frac{3}{5} - 1\frac{2}{5} = 1\frac{1}{5}$
 - $1\frac{2}{5} = \frac{5}{5} + \frac{2}{5} = \frac{7}{5}$ and $\frac{13}{5} - \frac{7}{5} = \frac{6}{5}$

----- **Begin Lesson** -----

Warm-up

🕒 10 min

Number Talk: Subtract Some Eighths

Standards Alignments

Addressing 4.NF.B.3.c

This Number Talk encourages students to rely on what they know about fractions to mentally find the value of differences with mixed numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $2\frac{3}{8} - \frac{3}{8}$
- $2\frac{3}{8} - \frac{5}{8}$
- $2\frac{3}{8} - 2$
- $2\frac{3}{8} - 1\frac{7}{8}$

Student Responses

- 2: The fraction being subtracted, $\frac{3}{8}$, is the same as the fraction in the mixed number, so what's left is the whole number, 2.
- $1\frac{6}{8}$: I know that $\frac{5}{8}$ is $\frac{2}{8}$ more than $\frac{3}{8}$, so I subtracted another $\frac{2}{8}$ from 2, which gives $1\frac{6}{8}$.
- $\frac{3}{8}$: I subtracted 2 from the whole number in $2\frac{3}{8}$.
- $\frac{4}{8}$: $1\frac{7}{8}$ is $\frac{1}{8}$ less than 2, so I added $\frac{1}{8}$ back to the value of $2\frac{3}{8} - 2$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the first few expressions help you find the value of the last expression?"
- "When subtracting $1\frac{7}{8}$, why might it be helpful to first think about subtracting 2?" ($2\frac{3}{8}$ has a whole number and a fraction, so we can easily subtract 2 from the whole number and then put back the extra $\frac{1}{8}$ that we took out.)
- Consider asking:
 - "Did anyone have the same strategy but would explain it differently?"
 - "Did anyone approach the expression in a different way?"

Activity 1

 20 min

Make It True

Standards Alignments

Addressing 4.NF.B.3.c

In this activity, students find the number that makes addition and subtraction equations with mixed numbers true without a context. The equations are designed to encourage students to decompose or write equivalent fractions for one or more numbers to find the unknown value, but students may choose to reason without doing either. When students share their strategies with their group they construct viable arguments (MP3).

Access for English Learners

MLR8 Discussion Supports. Synthesis. Display sentence frames to support partner discussions: “First, I ____ because”, “I noticed ____ so I”

Advances: Speaking, Conversing

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide alternatives to writing on paper. Invite students to share their first steps orally.

Supports accessibility for: Organization, Attention

Student-facing Task Statement

- Find the number that makes each equation true. Show your reasoning.

a. _____ + $\frac{2}{6}$ = $1\frac{1}{6}$

b. $2\frac{4}{5}$ + _____ = $7\frac{1}{5}$

c. $3 - 2\frac{1}{3}$ = _____

d. $4\frac{1}{12} - 2\frac{5}{12}$ = _____

- Write a sentence to describe your first step for finding the missing number in each equation in the first problem.

Launch

- Groups of 2–4

Activity

- “Work independently to find the number that should go in the blank to make each equation true.”
- 5 minutes: independent work time on the first problem
- “Now think about how you started finding each missing number. Write a sentence to describe your first step in completing each

- a. First step:
 - b. First step:
 - c. First step:
 - d. First step:
3. Compare and reflect on your first steps with your group. Did you make the same moves?

Discuss why you might have chosen the same way or different ways to start finding the missing numbers.

Student Responses

1.
 - a. $\frac{5}{6}$. Sample response: $1\frac{1}{6}$ is equivalent to $\frac{7}{6}$, and $\frac{2}{6} + \frac{5}{6} = \frac{7}{6}$.
 - b. $4\frac{2}{5}$. Sample response: Adding $\frac{1}{5}$ to $2\frac{4}{5}$ makes 3. To get to $7\frac{1}{5}$, we need to add $4\frac{1}{5}$ more. $\frac{1}{5} + 4\frac{1}{5} = 4\frac{2}{5}$.
 - c. $\frac{2}{3}$. Sample response:
 $3 - 2\frac{1}{3} = \frac{9}{3} - \frac{7}{3} = \frac{2}{3}$
 - d. $1\frac{8}{12}$. Sample response:
 $4\frac{1}{12} = 3 + \frac{12}{12} + \frac{1}{12}$. Subtracting 2 wholes from 3 wholes gives 1, and subtracting $\frac{5}{12}$ from $\frac{13}{12}$ gives $\frac{8}{12}$. Adding 1 and $\frac{8}{12}$ gives $1\frac{8}{12}$.
2.
 - a. I wrote $1\frac{1}{6}$ as $\frac{7}{6}$.
 - b. I added $\frac{1}{5}$ to $2\frac{4}{5}$ to make 3.
 - c. I decomposed 3 and $2\frac{1}{3}$ into thirds and rewrote them as $\frac{9}{3}$ and $\frac{7}{3}$.
 - d. I decomposed $4\frac{1}{12}$ into $3 + \frac{12}{12}$.
3. Sample response: For the first addition equation, I rewrote the sum of $1\frac{1}{6}$ as $\frac{7}{6}$ so I could see how many sixths need to be added to $\frac{2}{6}$ to get to it. For the second, adding $\frac{1}{5}$ to $2\frac{4}{5}$ to make it a whole number made it easier to see how much more needs

equation.”

- 5 minutes: independent work time on the second problem
- “Share your first steps with your group. For each equation, did you start to find the missing number the same way? Discuss why or why not.”
- 5 minutes: group discussion

Synthesis

- Invite students to share how they went about finding the missing numbers. Display or record their reasoning for all to see.
- To invite others into a discussion, consider asking:
 - “Did anyone find the missing number in this equation the same way?”
 - “Who used the same or a similar strategy but would explain it differently?”

to be added to get to $7\frac{1}{5}$.

Activity 2

🕒 15 min

To Decompose or Not to Decompose

Standards Alignments

Addressing 4.NF.B.3.c

In this activity, students analyze a set of addition and subtraction expressions and consider whether it is helpful or necessary to decompose a number in order to find the value of the expressions. In doing so, they practice looking for structure in expressions, which they can in turn use to find sums or differences more effectively (MP7). Note that there is more than one way to sort the expressions, as students may have different ways of reasoning about these expressions.

Student-facing Task Statement

- Here are some addition and subtraction expressions. Sort them into two groups based on whether you think it would be helpful to decompose a number to find the value of the expression. Be prepared to explain your reasoning.

A. $\frac{18}{5} - \frac{7}{5}$

B. $\frac{1}{6} + \frac{9}{6}$

C. $7 - 1\frac{3}{8}$

D. $\frac{102}{100} + 5\frac{27}{100}$

E. $2\frac{5}{12} + \frac{6}{12}$

F. $6\frac{1}{10} - \frac{6}{10}$

G. $3\frac{8}{100} + 4\frac{93}{100}$

Launch

- Groups of 3–4
- “In earlier activities, we saw it was sometimes useful to decompose a whole number or a mixed number before subtracting a fraction from it.”
- “Can you tell—by looking at an addition or subtraction expression—whether it would be helpful (or necessary) to decompose a number or rewrite it before we could add or subtract? Let’s find out!”

Activity

- “Work with your group to sort the expressions into two categories.”
- 5 minutes: group work time
- “When you are done, choose at least one expression from each category and work

H. $5 - \frac{17}{12}$

I. $1\frac{3}{10} + \frac{6}{10}$

J. $\frac{17}{8} - 1\frac{7}{8}$

- Not necessary or not helpful to decompose any number:
- Necessary or helpful to decompose one or more numbers:

2. Choose at least one expression from each group and find their values. Show your reasoning.

Student Responses

1. Not necessary or not helpful to decompose: A, B, I, E
Necessary or helpful to decompose one or more numbers: C, D, F, G, H, J
2. Answers vary. Values of expressions:
 - A. $\frac{11}{5}$
 - B. $\frac{10}{6}$
 - C. $5\frac{5}{8}$
 - D. $6\frac{29}{100}$
 - E. $2\frac{11}{12}$
 - F. $5\frac{5}{10}$
 - G. $8\frac{1}{100}$
 - H. $3\frac{7}{12}$
 - I. $1\frac{9}{10}$
 - J. $\frac{2}{8}$

independently to find its value.”

- “Every group member should choose a different expression from each category.”
- 5 minutes: independent work time

Synthesis

- Select groups to share the expressions they put in each category and invite others to agree or disagree.
- Ask other students to choose 1–2 expressions that they evaluated from each category and show that it is or it is not helpful or necessary to decompose a number. (For example, if they put expression A in the 'not necessary to decompose' category, their work should show that the difference can be found without decomposing.)

Lesson Synthesis

🕒 10 min

“Today we thought about different ways to find the value of sums and differences of fractions and mixed numbers and whether it is helpful to decompose one of the numbers or write equivalent fractions.”

“In the last activity, how did you sort the expressions? How did you know, without doing any computation, whether it would be necessary or helpful to decompose a number?” (Sample responses:

- For subtraction expressions: We looked at the numerators of the first and second numbers. If the first one is greater, there is no need to decompose. If the first number is a whole number, it is helpful to decompose it.
- For addition expressions: We looked at whether the fractional part of each number would add up to more than 1. If so, it may be necessary to decompose the sum to write a mixed number.)

Highlight that there are numerous ways to start adding and subtracting fractions. Depending on the numbers at hand, it might make sense to decompose or write an equivalent fraction for one or both numbers, to count up or count down, to add or subtract in parts, and so on.

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Creating Line Plots (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find the value of the difference without explaining their first step.

Next Day Support

- Before the first activity, pair students up to discuss their responses.

Lesson 13: Fractional Measurements on Line Plots

Standards Alignments

| | |
|------------------|----------------------|
| Building On | 3.MD.B.4 |
| Addressing | 4.MD.B.4, 4.NF.B.3.d |
| Building Towards | 4.MD.B.4 |

Teacher-facing Learning Goals

- Analyze and interpret fractional measurement data on line plots.
- Organize measurement data in fractions of a unit ($\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$) onto line plots.

Student-facing Learning Goals

- Let's create line plots and analyze the data.

Lesson Purpose

The purpose of this lesson is for students to display a set of measurements in fractions of a unit ($\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$) on a line plot and interpret the data. Students also add and subtract fractions to answer questions about data presented in line plots.

In grade 3, students generated measurement data in nearest $\frac{1}{2}$ inch or $\frac{1}{4}$ inch and represented such data on line plots. Earlier in the course, students learned about equivalent fractions and about sums and differences of fractions with the same denominator. In this lesson, students plot data involving lengths measured in $\frac{1}{8}$ inch and analyze the data. They use the line plots and their knowledge of equivalence and fraction operations to answer questions about situations.

An optional measuring activity is included in the lesson. While grade 4 standards do not require students to measure lengths or generate measurement data, measuring reinforces student understanding of the relative size of fractions and gives meaning to the context used in subsequent activities.

The activities in this lesson call for used colored pencils. If colored pencils are unavailable, substitute with regular pencils.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR7 (Activity 3)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Colored pencils: Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 25 min |
| Activity 2 | 20 min |
| Activity 3 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Today's lesson encouraged small-group collaboration. How did students interact with each other's ideas today in the work? How can you ensure in future small-group collaborations that all students' voices are heard?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

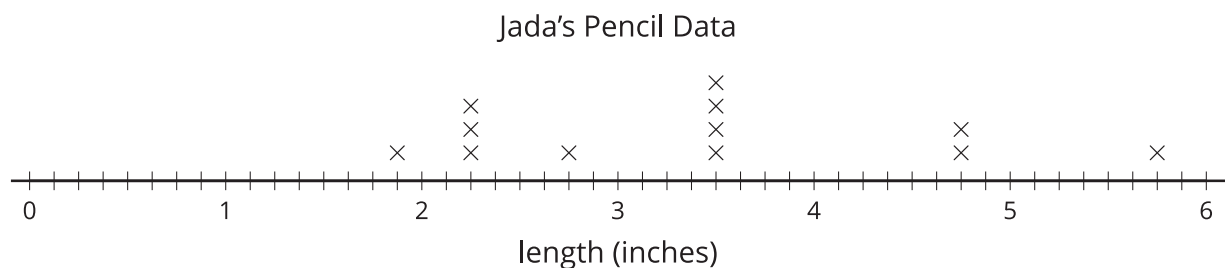
Jada's Pencil Data

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.d

Student-facing Task Statement

Jada measured the lengths of her pencils and displayed her data on a line plot.



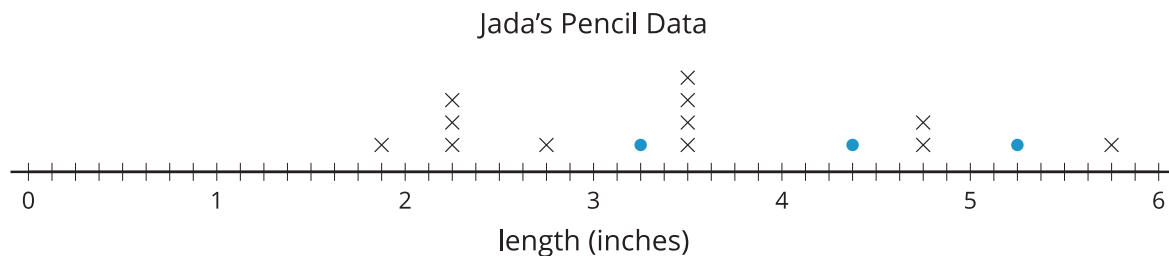
- The last three pencils in her collection are not yet plotted. Their lengths are: $3\frac{1}{4}$, $4\frac{3}{8}$, and $5\frac{1}{4}$.

Plot them on the line plot.

2. What is the difference in the length of the shortest and the longest pencil in her collection?
Show your reasoning.

Student Responses

1.



2. $3\frac{7}{8}$ inches. Sample response: $5\frac{6}{8} - 1\frac{7}{8} = 4\frac{14}{8} - 1\frac{7}{8} = 3\frac{7}{8}$

Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Which Ruler?

Standards Alignments

Building On 3.MD.B.4
Building Towards 4.MD.B.4

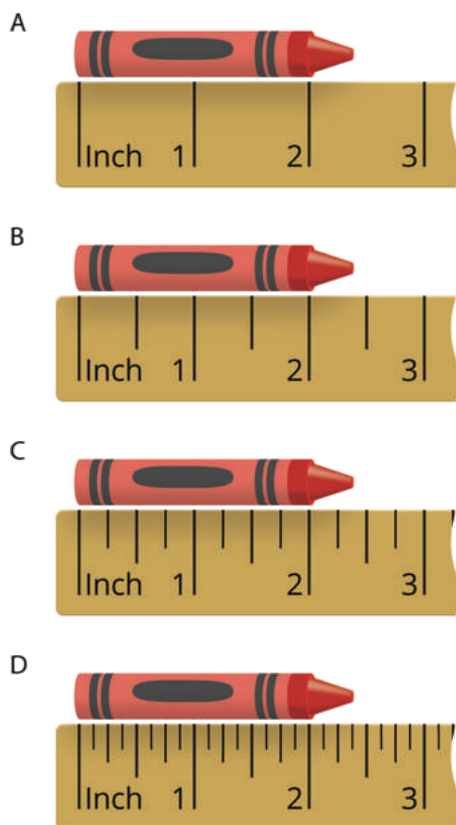
The purpose of this warm-up is to elicit ideas that students have about rulers and measurements of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, which will be useful when students measure objects and generate and analyze line plots in a later activity. While students may notice and wonder many things about the images, focus the discussion on how to name the fractional measures in each image and the progression of the different levels of precision.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are four different rulers.
- Each ruler has more marks.
- Some marks have no numbers beside them.

Students may wonder:

- What do the marks represent?
- Is the crayon getting shorter?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “Why does the crayon measurement change each time?” (Sample responses: The crayon is not changing. The measurements on the ruler are becoming more precise. We’re using smaller and smaller pieces to measure the crayon.)
- “What do you think the tick marks on each ruler represent?” (They represent inches, halves, fourths, and eighths of an inch.)
- “What are some things that you would measure with the first ruler? What about things you would measure with the last ruler? Why might that be?” (Sample response: I would use the first ruler if the measurement doesn’t need to be exact and use the last ruler if it needs to be pretty exact.)

Activity 1 (optional)

🕒 25 min

Measure to the Nearest $\frac{1}{4}$ and $\frac{1}{8}$ Inch

Standards Alignments

Building On 3.MD.B.4
 Building Towards 4.MD.B.4

In this activity, students first measure colored pencils to the nearest $\frac{1}{4}$ inch, collect a set of data in a table, and then plot them on a line plot. Then, they measure the colored pencils again, but this time to the nearest $\frac{1}{8}$ inch. They plot their data on a new number line and attend to a greater level of precision as they do so. Students then reflect on the difference in the measuring process and in the measurements on the same set of pencils. Students attend to precision when they measure the pencils to the appropriate fractional unit (MP6).

Materials to Gather

Colored pencils

Required Preparation

- Each student needs a used colored pencil.

Student-facing Task Statement

Your teacher will give your group a set of colored pencils.

- Work with your group to measure each colored pencil to the nearest $\frac{1}{4}$ inch. Check each other's measurements. Record each measurement in the table.

| group members | pencil length (inches) |
|---------------|------------------------|
| | |
| | |
| | |
| | |

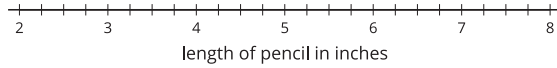


- Create a line plot to represent the data your group collected.

Launch

- Groups of 5
- Give a used colored pencil to each student.
- Prepare a number line on a poster and display it for students to see. Label the tick marks with whole numbers.
- "If we wanted to make a line plot and show measurements to the nearest $\frac{1}{4}$ inch, what else might we do that would be helpful?" (Partition the space between two consecutive whole numbers into 4 equal parts.)
- Partition the number line in increments of $\frac{1}{4}$.
- "What if we wanted to show measurements that include $\frac{1}{8}$ inch?" (Partition the space between two whole numbers into 8 equal parts.)

Colored-Pencil Data



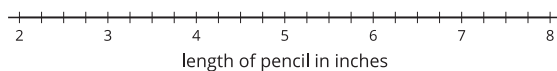
3. Work with your group to measure each colored pencil to the nearest $\frac{1}{8}$ inch.

Check one another's measurements. Record each measurement in the table.

| group members | pencil length (inches) |
|---------------|------------------------|
| | |
| | |
| | |
| | |

4. Create a line plot to represent your new data.

Colored-Pencil Data



5. How was measuring to the nearest $\frac{1}{4}$ inch different from measuring to the nearest $\frac{1}{8}$ inch?

Student Responses

1. Sample response:

| group members | pencil length (inches) |
|---------------|------------------------|
| student 1 | $4\frac{3}{4}$ |
| student 2 | $2\frac{1}{4}$ |
| student 3 | $5\frac{1}{4}$ |
| student 4 | $4\frac{2}{4}$ |
| 2. student 5 | $6\frac{1}{4}$ |

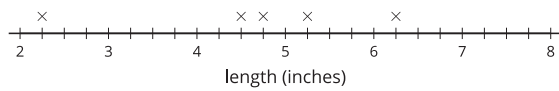
Activity

- "Work with your group to measure colored pencils to the nearest $\frac{1}{4}$ inch. Record your measurements in the first table and then plot them on the first line plot."
- 5–7 minutes: group work time
- "Now measure the pencils again, but this time to the nearest $\frac{1}{8}$ inch. Record your measurements in the second table and then plot the new data on the second line plot."
- 5–7 minutes: group work time

Synthesis

- Allow students to record their two sets of data on two different class line plots. (If dot stickers are available, consider using them—one sticker for each data point.)
- "How did your data and line plots change when you measured colored pencils to the nearest $\frac{1}{8}$ inch?" (Sample responses:
 - We got different numbers.
 - The marks or points on the line plots are distributed differently. The points for some of the same pencils show up as different lengths in the second line plot.)
- "What is challenging about measuring to the nearest $\frac{1}{8}$ inch?" (The tick marks are smaller and harder to see on the ruler.)
- "Why do you think we measure to the nearest $\frac{1}{8}$ inch?" (We measure to be more accurate.)
- "Let's look at some other length data with measurements in halves, fourths and eighths of an inch."

Colored-Pencil Data

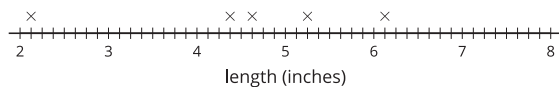


3. Sample response:

| group members | pencil length (inches) |
|---------------|------------------------|
| student 1 | $4\frac{5}{8}$ |
| student 2 | $2\frac{1}{8}$ |
| student 3 | $5\frac{2}{8}$ |
| student 4 | $4\frac{3}{8}$ |
| student 5 | $6\frac{1}{8}$ |

4.

Colored-Pencil Data



5. Sample responses: Measurements became more precise. We estimated less.

Advancing Student Thinking

Students may identify the nearest inch and half inch but need support identifying the quarter or eighth inch. Consider asking students to think about what the halfway points between $\frac{1}{2}$ -inch increments on the ruler represent, and then asking them again about the halfway points between $\frac{1}{4}$ -inch increments.

Activity 2

🕒 20 min

Colored-pencil Measurements

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.d

In this activity, students create a line plot using measurements to the nearest $\frac{1}{4}$ and $\frac{1}{8}$ inch. This task prompts students to use their understanding of fraction equivalence to plot and partition the horizontal axis.

Access for Students with Disabilities

Representation: Access for Perception. Provide access to fraction strips that show fourths and eighths, and invite students to use them to answer question 4. Ask students to identify correspondences between the fraction strips and the horizontal axis of the line plot.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

- Andre's class measured the length of some colored pencils to the nearest $\frac{1}{4}$ inch. The data are shown here:

$$1\frac{3}{4} \quad 2\frac{1}{4}$$

$$5\frac{1}{4} \quad 5\frac{1}{4}$$

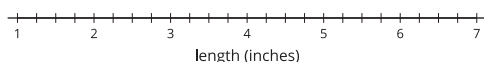
$$4\frac{2}{4} \quad 4\frac{2}{4}$$

$$6\frac{1}{4} \quad 6\frac{3}{4}$$

$$6\frac{3}{4} \quad 6\frac{3}{4}$$



- Plot the colored-pencil data on the line plot.



- Which colored-pencil length is the most common in the data set?
 - Write 2 new questions that could be answered using the line plot data.
- Next, Andre's class measured their colored pencils to the nearest $\frac{1}{8}$ inch. The data are shown here:

Launch

- Groups of 2
- "The table lists many different lengths."
- "What do you notice about the pencil lengths?" (Sample responses:
 - Some repeat more than one time.
 - The numbers are mixed numbers.
 - There are no whole numbers.)
- 1 minute: quiet think time
- "There are some lengths that are more common or occur more often than others."
- "Tell a partner the length that is most common." ($6\frac{3}{4}$)

Activity

- Groups of 2
- 5 minutes: independent work time
- Monitor for students who use equivalence to plot measurements to the nearest eighth inch.
- "Share your line plots with your partner and make revisions as needed."
- 2 minutes: partner discussion

$$1\frac{6}{8} \quad 2\frac{2}{8}$$

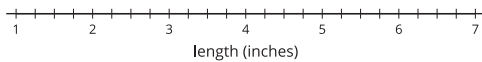
$$5\frac{2}{8} \quad 5\frac{4}{8}$$

$$4\frac{4}{8} \quad 4\frac{4}{8}$$

$$6\frac{6}{8} \quad 6\frac{6}{8}$$

$$6\frac{6}{8} \quad 6\frac{4}{8}$$

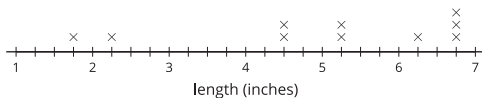
- a. Plot the colored-pencil data on the line plot.



- b. Which colored-pencil length is the most common in the line plot?
- c. Why did some colored-pencil lengths change on this line plot?
- d. What is the difference between the length of the longest colored pencil and the shortest colored pencil? Show your reasoning.

Student Responses

1. a. Sample response:



- b. $6\frac{3}{4}$ inches
- c. Which data point was the least common?

How many colored pencils were more than $3\frac{1}{2}$ inches?

2. a. Sample response:



- b. $6\frac{6}{8}$ inches

Synthesis

- “How many colored pencils were measured in Andre’s class?” (10, because there were ten data points and each represented a pencil that was measured.)
- “What was the most common measurement in the first set of data? In the second set of data?” (In the first set of data: $6\frac{3}{4}$ inches. In the second set of data: $6\frac{6}{8}$ inches.)
- “How did you use equivalence to help as you plotted measurement data in eighths of an inch?” (I know that two eighths are equivalent to one fourth and this helped to find eighths on the line plots.)
- Use the line plot image to clearly label $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ with help from students.

- c. The pencil was measured to the nearest eighth, so it was not rounded as much as it had been before. The measurement is more accurate.
- d. 5 inches. Sample response:
 $6\frac{6}{8} - 1\frac{6}{8} = 5$

Advancing Student Thinking

If students do not yet identify “most common” length as the length with the most data points on the line plot, consider asking: “Which measurement occurred more often than others?” and “Which pencil measurement was plotted most often?”

Activity 3

 15 min

Noah's Colored Pencils

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.d

In this activity, students continue to analyze line plot data and use the data to answer questions. Each data set involves lengths measured to the nearest $\frac{1}{4}$ and $\frac{1}{8}$ inch. As students organize and analyze data, they revisit ideas about fraction equivalence and addition and subtraction of fractions. When students relate the data to the context it represents and carefully interpret the elements of a graph, they reason abstractly and quantitatively and attend to precision (MP2, MP6).

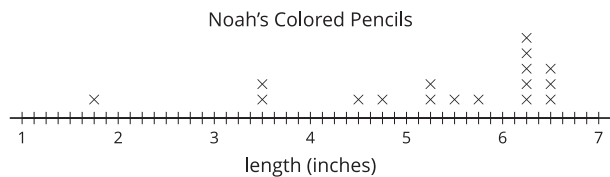
Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, “Why did the different approaches lead to the same (or different) outcome(s)?”, “What did the strategies have in common?”, and “How were they different?”

Advances: Representing, Conversing

Student-facing Task Statement

The line plot shows the data Noah collected on a set of colored pencils.



Use the line plot to tell if each of the following statements is true or false. Be prepared to explain or show how you know. For each false statement, correct it so that it is true.

- Noah measured the colored pencils to the nearest $\frac{1}{2}$ inch.
- There are five pencils that are $6\frac{1}{4}$ inches long.
- The shortest pencil is $1\frac{3}{4}$ inches long.
- The three longest pencils are exactly 5 inches longer than the shortest one.
- If Noah removed the shortest pencil from the collection, the difference between the longest and shortest pencils would be 3 inches.

If You Have Time

Noah wants to create a collection of at least 10 pencils where the difference between the longest and shortest colored pencils is no more than $1\frac{1}{2}$ inches.

Is that possible? If so, which pencils should he remove from his collection?

Student Responses

- False. He measured to the nearest $\frac{1}{8}$ inch. The line plot shows data points in eighths of an inch.
- True. There are five pencils at $6\frac{2}{8}$, which is

Launch

- Groups of 2
- “Take a look at the line plot. Think of a couple of things that you know to be true about Noah’s colored pencils based on the data you see.”
- 1 minute: quiet think time
- Invite 2–3 students to share their responses.

Activity

- “Take a few minutes to work on your own before sharing ideas with your partner.”
- 5 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who:
 - decompose the mixed numbers to find the difference between the longest and shortest points of data
 - recognize and label eighths on the number line as the halfway points between consecutive fourths

Synthesis

- Select students to share strategies for finding the difference between the longest and shortest lengths?
- Make connections between strategies, being sure to emphasize strategies that involve decomposing fractions in different ways.
- Consider asking: “What are the similarities between these strategies?”

equivalent to $6\frac{1}{4}$.

3. True. The shortest pencil is marked at $1\frac{6}{8}$, which is equivalent to $1\frac{3}{4}$.
4. False. The longest pencil is $4\frac{6}{8}$ or $4\frac{3}{4}$ inches longer than the shortest. $6\frac{4}{8} - 1\frac{6}{8} = 4\frac{6}{8}$
5. True. If he removed the $1\frac{6}{8}$ -inch pencil, the two shortest pencils are $3\frac{4}{8}$ inches, which are 3 inches shorter than the longest pencils at $6\frac{4}{8}$ inches.

If You Have Time

Yes, it is possible. If Noah removed the five shortest pencils in the collection, the shortest pencil would be $5\frac{2}{8}$ inches and the longest would still be $6\frac{4}{8}$ inches. The difference between them is $1\frac{1}{8}$ inches, which is less than $1\frac{1}{2}$ inches.

Lesson Synthesis

 10 min

“Today we organized data on line plots and answered questions about the data.”

“How did you compare the data points or use them to answer questions when the data were fractions with different denominators?” (We used equivalence to relate them. We knew the relationship between halves, fourths, and eighths.)

“How could we find the difference between the longest and shortest colored pencils from the last line plot?” (The leftmost point represents the shortest pencil, the rightmost point represents the longest. We could use the marks on the number line to count up or down to find the difference, or we can subtract the two fractions.)

Suggested Centers

- Estimate and Measure (1–4), Stage 4: Eighth Inches (Addressing)
- Target Measurements (2–5), Stage 3: Eighth Inches (Addressing)
- Creating Line Plots (2–5), Stage 2: Quarter Inches (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find a difference between the shortest and longest pencils other than $3\frac{7}{8}$ inches.

The work in this lesson builds from the measurement and data concepts developed in a prior unit.

Next Day Support

- Before the warm-up, have students work in groups of 2 to discuss a correct response to one of the problems of this cool-down.

Prior Unit Support

Grade 3, Unit 6, Section A: Measurement Data on Line Plots

Lesson 14: Problems about Fractional Measurement Data

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.c
 Building Towards 4.MD.B.4

Teacher-facing Learning Goals

- Use information on line plots to solve problems involving addition and subtraction of fractions and mixed numbers.

Student-facing Learning Goals

- Let's solve problems involving measurement data on line plots.

Lesson Purpose

The purpose of this lesson is for students to solve problems using information presented in line plots.

Previously, students organized and analyzed measurement data on a line plot. They also learned to express equivalent fractions (for example, they expressed 3 fourths as 6 eighths). In this lesson, they continue to use these skills, along with their knowledge of addition and subtraction of fractions with the same denominator, to solve problems involving fractional measurements.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |

Teacher Reflection Question

Which question asked during this lesson generated the most discourse? What was it about this question to motivate student

| | |
|------------------|--------|
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

thinking? How might you use the structure of this question moving forward in upcoming lessons?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

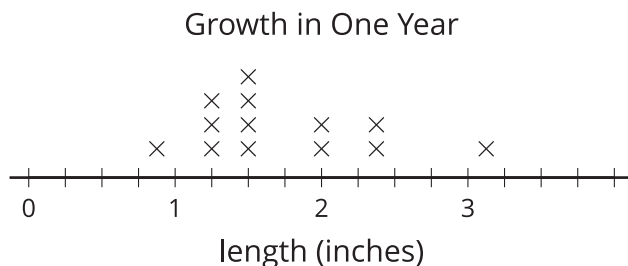
Fourth-grade Height Data

Standards Alignments

Addressing 4.MD.B.4

Student-facing Task Statement

The students in a fourth-grade class keep track of their height all year long. The line plot shows the number of inches each student in the class has grown this year.



- How many students grew more than $1\frac{3}{8}$ inches? Explain your reasoning.
- What is the difference between the greatest amount of growth and the least amount of growth in inches?

Student Responses

- Nine students grew more than $1\frac{3}{8}$ inches. Sample response: $\frac{3}{8}$ is located between $1\frac{1}{4}$ and $1\frac{2}{4}$ and there are 9 points to the right of $1\frac{3}{8}$.
- $2\frac{2}{8}$ inches. Sample response: $3\frac{1}{8} - \frac{7}{8} = 2\frac{9}{8} - \frac{7}{8} = 2\frac{2}{8}$.

----- Begin Lesson -----

Warm-up

 10 min

Notice and Wonder: Shoe Sizes

Standards Alignments

Building Towards 4.MD.B.4

This warm-up prompts students to make sense of data and quantities before using them to solve problems, by familiarizing themselves with a context and the mathematics that might be involved. Students may be familiar with shoe sizes but may not recognize that each size is associated with a particular measurement.

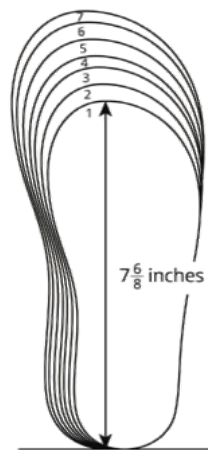
Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

| US youth shoe size | insole length in inches |
|--------------------|-------------------------|
| 1 | $7\frac{6}{8}$ |
| 1.5 | 8 |
| 2 | $8\frac{1}{8}$ |
| 2.5 | $8\frac{2}{8}$ |
| 3 | $8\frac{4}{8}$ |
| 3.5 | |
| 4 | $8\frac{6}{8}$ |
| 4.5 | 9 |
| 5 | $9\frac{1}{8}$ |
| 5.5 | |
| 6 | $9\frac{4}{8}$ |
| 6.5 | $9\frac{5}{8}$ |
| 7 | $9\frac{6}{8}$ |



Launch

- Display the shoe-size chart and diagram of insoles.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- Explain that an “insole” is the inside part of a shoe, underneath the foot. Its length is approximately the length of the foot.
- Check students’ interpretation of the data and diagram:
 - “What information do the table and diagram show?”
 - “If someone’s shoe size is 5, what’s the length of the insole?”
 - “What shoe size do you wear? What’s the length of the insole?”

Student Responses

Students may notice:

- There is a chart with sizes in whole numbers and numbers with a point followed by a 5.
- There are measurements in whole numbers and mixed numbers, measured in inches.
- The diagram shows a length of $7\frac{6}{8}$ for size 1 and how other sizes compare.
- The fractions are all eighths.
- The lengths for sizes 3.5 and 5.5 are missing.
- “What do you think the missing lengths might be for sizes 3.5 and 5.5?” (For 3.5, the missing length would be $8\frac{5}{8}$, as there are no other fractions in eighths between $8\frac{4}{8}$ and $8\frac{6}{8}$. For size 5.5, it could be $9\frac{2}{8}$ or $9\frac{3}{8}$.)
- “Today we’ll look at some data and solve some problems related to shoe lengths. The sizing chart here gives us an idea of where the numbers come from and what they mean.”

Students may wonder:

- What are the numbers with a point and a 5?
- Why are most of the lengths fractions?
- What does “insole” mean?
- Why is the shortest length $7\frac{6}{8}$ inches? What about shoes for toddlers?
- What are the lengths for sizes 3.5 and 5.5?

Activity 1

🕒 20 min

Shoe Lengths

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.c

This activity allows students to integrate several concepts and skills on data analysis and fraction operations. Students plot fractional measurements on a line plot, interpret the data, and find sums or differences of fractions to solve problems in context (MP2).

To find the difference between the longest and shortest shoe lengths, students can reason in a number of ways, using visual representations or more abstract reasoning. For example, they may:

- use the tick marks on the line plot and count up by eighths from $7\frac{6}{8}$ to $9\frac{5}{8}$
- count up from $7\frac{6}{8}$ to 8, then from 8 to 9, and from 9 to $9\frac{5}{8}$
- subtract $7\frac{6}{8}$ from $9\frac{5}{8}$ in parts: first subtract $\frac{5}{8}$, then 7, and then another $\frac{1}{8}$

- find $9\frac{5}{8} - 7\frac{6}{8}$ by first decomposing the 9 into $8 + \frac{8}{8}$ and then subtracting $7\frac{6}{8}$ from it

To complete the activity, students will need to make sense of the data and the questions, identify relevant numbers or quantities, and persevere in solving problems (MP1).

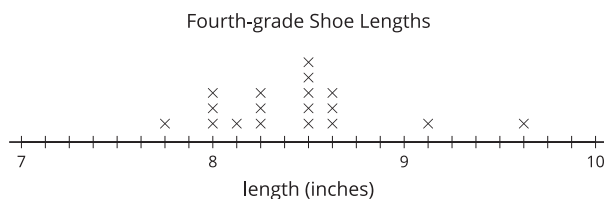
Access for Students with Disabilities

Representation: Access for Perception. Read directions and the first problem aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Attention, Language

Student-facing Task Statement

Students in a fourth-grade class collected data on their shoe sizes and lengths. They plotted the shoe lengths on a line plot.



The line plot is missing the shoe lengths of six students:

$$\begin{array}{ccc} 9 & 9\frac{1}{8} & 8\frac{6}{8} \\ 7\frac{6}{8} & 9\frac{2}{8} & 8\frac{1}{8} \end{array}$$

1. Complete the line plot with the missing data.
2. Use the completed line plot to answer the following questions:
 - a. What is the largest shoe length?
 - b. What is the smallest shoe length?
 - c. What is the difference between the largest and smallest shoe lengths? Explain or show your reasoning.

Launch

- Groups of 2
- “Take a minute to read the opening paragraphs and the first problem of the activity. Afterwards, explain to a partner the directions to the first problem.”
- 1 minute: quiet think time
- 1 minute: partner discussion

Activity

- “Take a few quiet minutes to work on the activity. Then, discuss your responses with your partner.”
- 5–6 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor for the different ways students find the difference between $9\frac{5}{8}$ and $7\frac{6}{8}$ as described in the Activity Narrative.
- Identify students who use different strategies to share during synthesis.

Synthesis

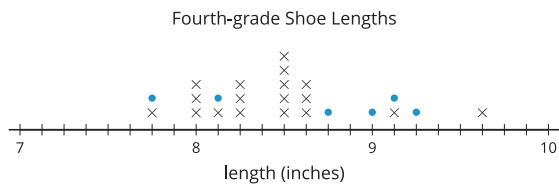
- Invite students to share their completed line plot and their responses.
- Focus the discussion on the last two questions about the differences in the

- d. The student who recorded 9 inches for her shoe length made a mistake when reading the shoe chart. Her actual shoe length is $\frac{7}{8}$ inches shorter.

What's her shoe length? Plot her corrected data point on the line plot.

Student Responses

1.



2. a. $9\frac{5}{8}$ inches
 b. $7\frac{6}{8}$ inches or $7\frac{3}{4}$ inches
 c. $1\frac{7}{8}$ inches. Sample reasoning: It's $\frac{2}{8}$ from $7\frac{6}{8}$ to 8, and then $1\frac{5}{8}$ from there to $9\frac{5}{8}$, so the difference is $\frac{2}{8} + 1\frac{5}{8}$ or $1\frac{7}{8}$.
 d. $8\frac{1}{8}$ inches. $9 - \frac{7}{8} = 8\frac{1}{8}$. The corrected line plot should show the point at 9 moved to $8\frac{1}{8}$.

longest and shortest shoe lengths. Select previously identified students to share their reasoning strategies, in the order shown in the Activity Narrative.

- As students explain, record and display their reasoning for all to see.
- “Did you use the same strategy to solve the last problem? How did you find out what shoe length of the student who made an error?” (Sample responses: I started at 9 on the number line and moved by eighths 7 times to the left to land at $8\frac{1}{8}$. I subtracted $\frac{7}{8}$ from 9.)

Activity 2

🕒 15 min

Larger Shoes, Anyone?

Standards Alignments

Addressing 4.MD.B.4, 4.NF.B.3.c

In this activity, students analyze a line plot that is incomplete. They relate the list of given fractions

to the data on the line plot and use their understanding of equivalence to determine the missing data points. Students also continue to interpret the data and add and subtract fractions to solve problems in context (MP2).

Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support small-group discussion: “First, I _____ because . . .”, and “I noticed _____ so I . . .”

Advances: Conversing, Representing

Student-facing Task Statement

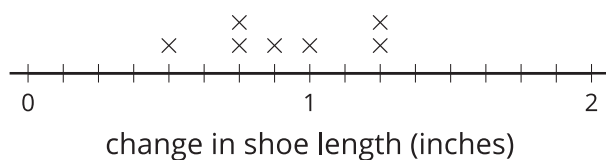
Ten students recorded their shoe lengths in third grade and then again in fourth grade.

They found how much their feet have grown over a year and organized the data in a table and on a line plot.

| student | change in shoe length (inches) |
|---------|--------------------------------|
| Jada | $\frac{5}{4}$ |
| Priya | $\frac{7}{8}$ |
| Andre | $\frac{3}{4}$ |
| Elena | $\frac{1}{2}$ |
| Han | $1\frac{2}{8}$ |

| student | change in shoe length (inches) |
|---------|--------------------------------|
| Clare | 1 |
| Tyler | $1\frac{1}{8}$ |
| Kiran | $\frac{6}{8}$ |
| Diego | $1\frac{1}{4}$ |
| Lin | $\frac{5}{8}$ |

How Much Have Our Feet Grown?



- The line plot shows only seven points. Whose information is missing? Add the

Launch

- Groups of 2
- “How much do you think your feet have grown in the past year? Have you changed to a larger shoe size since third grade?”
- Invite students to share responses.
- “Let’s look at some problems about the change in shoe lengths from third grade to fourth grade.”

Activity

- “Take a few quiet minutes to work on the activity. Afterwards, share your thinking with your partner.”
- 5–7 minutes: independent work time
- 4–5 minutes: partner work
- “Be sure to discuss how you know whose data points are missing from the line plot.”

Synthesis

- Select students to display their completed line plot and to share how they decided which data points didn’t get plotted.
- Highlight that one point that is missing could be Jada, Diego, or Han’s, as the fractions that represent their change in shoe length, $1\frac{1}{4}$, are equivalent.

three missing points to the line plot.

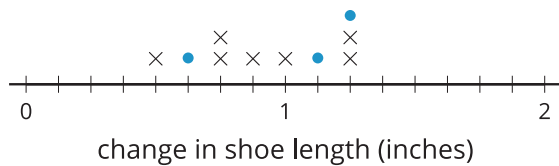
2. If Han's shoe length now is $9\frac{1}{8}$ inches, what was his shoe length in third grade?
3. If Priya's shoe length was $7\frac{6}{8}$ inches last year, what's her shoe length this year?
4. Tyler made a calculation error. What he recorded, $1\frac{1}{8}$ inches, was $\frac{3}{8}$ inches off from the actual change in shoe length.
 - a. What could be the actual change in his shoe length? Explain or show your reasoning.
 - b. How does his error affect the line plot? Explain your reasoning.

- Select other students to share their responses to questions about Han and Priya's shoe lengths.
- Then, focus the discussion on the last question about Tyler's error, his actual change in shoe length, and how the corrected value might affect the line plot.

Student Responses

1. Tyler and Lin's information is missing, along with a length that is equivalent to $1\frac{2}{8}$ inches, which could be Jada, Han, or Diego's.

How Much Have Our Feet Grown?



2. $7\frac{7}{8}$ inches
3. $8\frac{5}{8}$ inches
4.
 - a. It could be $1\frac{4}{8}$ inches or $\frac{6}{8}$ inches.
 Sample reasoning: $1\frac{1}{8} - \frac{3}{8} = \frac{6}{8}$ and
 $1\frac{1}{8} + \frac{3}{8} = 1\frac{4}{8}$
 - b. The point at $1\frac{1}{8}$ would go to $\frac{3}{4}$ or $1\frac{2}{4}$ instead.

Lesson Synthesis

🕒 10 min

“Today we used our understanding of fractions to plot and analyze data on line plots. We also added and subtracted fractions to answer questions about measurement data.”

“How was plotting fractional data in halves, fourths, and eighths on a line plot different from plotting whole numbers?” (With whole-number data, we could just count up or down from the labeled tick marks to know where to put a number. With fractions, sometimes it's necessary to think about equivalent fractions first to know where to put a data point. For example, the number line might be partitioned into fourths, but the data might be in eighths or halves.)

“The problems we saw today involved finding differences of two fractions. Did you find the line plots helpful for subtracting two fractions? Why or why not?” (Sample responses: Yes, because I could use the number line and tick marks to help us count up or down, or to know roughly what the difference would be. No, because I could reason about the difference mentally or figure it out on paper.)

Suggested Centers

- Creating Line Plots (2–5), Stage 3: Eighth Inches, Add and Subtract (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)

Student Section Summary

In this section, we added and subtracted fractions with the same denominator, using number lines to help with our reasoning.

First, we learned that a fraction can be decomposed into a sum of smaller fractions. For example, here are a few ways to write $\frac{6}{10}$:

$$\frac{6}{10} = \frac{4}{10} + \frac{2}{10}$$

$$\frac{6}{10} = \frac{5}{10} + \frac{1}{10}$$

$$\frac{6}{10} = \frac{2}{10} + \frac{2}{10} + \frac{2}{10}$$

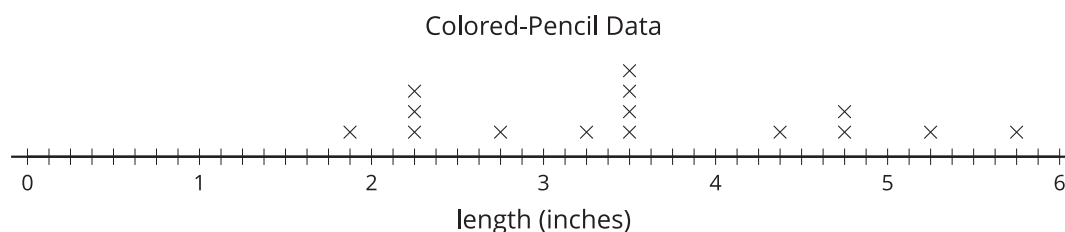
If the fraction is greater than 1, it can be decomposed into a whole number and a fraction less than 1. For instance, we can decompose $\frac{17}{10}$ and rewrite it as $1\frac{7}{10}$. A number such as $1\frac{7}{10}$ is called a **mixed number**.

$$\begin{array}{r} \frac{10}{10} + \frac{7}{10} \\ 1 + \frac{7}{10} \\ 1\frac{7}{10} \end{array}$$

Later, we decomposed fractions into sums and wrote equivalent fractions to help us add and subtract fractions. For example, to find the value of $1\frac{2}{5} - \frac{3}{5}$, we can:

- Decompose $1\frac{2}{5}$ into $1 + \frac{2}{5}$ or $\frac{5}{5} + \frac{2}{5}$, which is $\frac{7}{5}$.
- Find the value of $\frac{7}{5} - \frac{3}{5}$, which is $\frac{4}{5}$.

Finally, we organized and analyzed measurement data on line plots. The data were lengths measured to the nearest inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{1}{8}$ inch.



Because the measurements have different denominators, we used equivalent fractions to plot them. Then, we used the line plots and what we know about addition and subtraction of fractions to solve problems about the data.

----- Complete Cool-Down -----

Response to Student Thinking

Students say that there are five data points that are greater than $1\frac{3}{8}$ because they mistake the third tick mark between 1 and 2 to be $1\frac{3}{8}$, while it should be $1\frac{3}{4}$.

Next Day Support

- Before the warm-up, have students work in groups of 2 to discuss a correct response to this cool-down.

Section C: Addition of Tenths and Hundredths

Lesson 15: An Assortment of Fractions

Standards Alignments

| | |
|------------------|--|
| Building On | 4.NF.A.1, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4 |
| Addressing | 4.NF.A.1, 4.NF.A.2, 4.NF.B.3.d |
| Building Towards | 4.NF.C.5 |

Teacher-facing Learning Goals

- Use equivalence to reason about addition and subtraction problems.

Student-facing Learning Goals

- Let's find the heights of some stacked objects.

Lesson Purpose

The purpose of this lesson is for students to use equivalence to reason about problems that involve combining or removing fractional amounts.

In a previous unit, students learned to recognize and generate equivalent fractions. Earlier in this unit, they learned to add and subtract fractions with the same denominator, seeing these operations as joining and separating parts of the same whole. In this lesson, students encounter situations that involve combining and removing fractions with different denominators (limited to 2, 3, 4, 6, and 8), prompting them to rely on their understanding about equivalence to reason about the problems. This work prepares students to use equivalent fractions to join tenths and hundredths in upcoming lessons.

Students are not expected to reason symbolically, or to write fractional expressions with different denominators and then rewrite them with a common denominator. Instead, they reason using their intuitive understanding of equivalence, which they have begun to build since grade 3, and with the support of visual representations as needed.

Access for:



Students with Disabilities

- Engagement (Activity 1)



English Learners

- MLR1 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

How did the activities in today's lesson prepare students to add tenths and hundredths in the next lesson? How will you connect the ideas around equivalent fractions to upcoming work?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Which Stack is Taller?

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2, 4.NF.B.3.d

Student-facing Task Statement

Which stack of foam blocks is taller:

- Two $\frac{1}{3}$ -foot blocks and one $\frac{1}{6}$ -foot block, or
- One $\frac{1}{2}$ -foot block and two $\frac{1}{6}$ -foot blocks?

Explain or show your reasoning.

Student Responses

They are the same height. Sample reasoning: First stack: $2 \times \frac{1}{3} = \frac{2}{3}$, which is equivalent to $\frac{4}{6}$. Adding another $\frac{1}{6}$ makes $\frac{5}{6}$. Second stack: $\frac{1}{2}$ is equivalent to $\frac{3}{6}$. Adding another $\frac{2}{6}$ makes $\frac{5}{6}$.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Which One Doesn't Belong: Halves, Fourths, Sixths, and Eighths

Standards Alignments

Building On 4.NF.A.1, 4.NF.B.3.c

This warm-up prompts students to carefully analyze and compare fractions or expressions containing fractions, relying on what they know about the size of fractions, equivalence, mixed numbers, and addition of fractions. The reasoning also helps students to recall familiar relationships between fractions where one denominator is a factor or a multiple of the other. This awareness will be helpful later when students solve problems that involve combining quantities with different fractional parts.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A

$$1\frac{1}{2}$$

B

$$\frac{4}{4} + \frac{2}{4}$$

C

$$\frac{12}{8}$$

D

$$\frac{4}{6}$$

Student Responses

- A is the only fraction that isn't written only with a numerator and denominator. (It's the only mixed number.)
- B is the only fraction that isn't just a number. (It's the only expression.)
- C is the only fraction that doesn't have a single-digit numerator.
- D is the only fraction that isn't greater than 1 or equivalent to $\frac{6}{4}$.

Launch

- Groups of 2
- Display the numbers and expressions.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "Let's find at least one reason why each one doesn't belong."
- "What are some fractions that are equivalent to the numbers or expressions in A, B, and C?"
($\frac{3}{2}$, $\frac{6}{4}$, $\frac{9}{6}$, $\frac{15}{10}$, $\frac{18}{12}$)
- "What are some ways to decide if two fractions are equivalent?" (Sample responses:
 - Think about the relationship of unit fractions—for instance, how many one eighths are in one fourth?
 - Compare the fractions to a benchmark—for instance, if one

fraction is greater than 1 and the other less than 1, then they're not equivalent.

- See if the numerator and denominator of one fraction could be multiplied by the same factor to get the other fraction.)

Activity 1

🕒 15 min

All the Way to the Top

Standards Alignments

Building On 4.NF.A.1, 4.NF.B.3.d

Building Towards 4.NF.C.5

In earlier lessons, students found sums and differences of fractions (including mixed numbers) with the same denominator. In this activity, they reason about problems that involve combining or removing fractional amounts with different denominators—2, 4, and 8—in the context of stacking playing bricks. Because the denominators are familiar and are multiples or factors of one another, students can rely on what they know about the relationships of halves, fourths, and eighths to compare amounts (how much more or less one amount than another) or to combine them.

The last question asks students to reason about the height of a tower of bricks created by combining three shorter stacks. Students may arrive at two different answers depending on their familiarity with playing bricks and attention to precision. Some students may notice that each playing brick has studs that disappear into the bottom of another brick when stacked, so the combined height of two stacks will be less than the sum of the heights of individual stacks (MP6). Both answers are acceptable as long as they are supported.

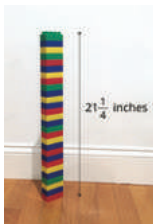
♿ Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. For example, direct students' focus to 1a. First, invite them to write an equation to represent the problem. Then, invite them to plan a strategy, including the tools they will use (for example number lines or fraction strips) to solve the equation. Finally, invite them to solve the equation, then move on to 1b. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Organization, Memory, Attention

Student-facing Task Statement

Priya, Kiran, and Lin are using large playing bricks to make towers. Here are the heights of their towers so far:



- Priya: $21\frac{1}{4}$ inches
- Kiran: $32\frac{3}{8}$ inches
- Lin : $55\frac{1}{2}$ inches

For each question, show your reasoning.

1. How much taller is Lin's tower compared to:
 - a. Priya's tower?
 - b. Kiran's tower?
2. They are playing in a room that is 109 inches tall. Priya says that if they combine their towers to make a super tall tower, it would be too tall for the room and they'll have to remove one brick.

Do you agree with Priya? Explain your reasoning.

Student Responses

1.
 - a. Lin's tower is $34\frac{1}{4}$ inches taller than Priya's. Sample reasoning:
 $55 - 21 = 34$, and $\frac{1}{2}$ is the same size as $\frac{2}{4}$, so $\frac{1}{2} - \frac{1}{4}$ is $\frac{1}{4}$.
 - b. Lin's tower is $23\frac{1}{8}$ inches taller than Kiran's. Sample reasoning:
 $55 - 32 = 23$, and $\frac{1}{2}$ is the same size as $\frac{4}{8}$, so $\frac{1}{2} - \frac{3}{8}$ is $\frac{1}{8}$.
2. Sample responses and reasoning:
 - Agree. $21 + 32 + 55 = 108$, and 108 is 1 away from 109. I know that $\frac{1}{2}$

Launch

- Groups of 2
- Display the image of stacked playing bricks. Read the task statement (including the heights of the three stacks) together.
- "The picture shows Priya's tower. Try visualizing Kiran and Lin's towers in the same picture. How tall would they be?" Invite students to try sketching or describing where the top of each tower would reach.
- Consider asking: "How tall was the tallest tower of playing bricks you have built?"

Activity

- "Work independently on the task for a few minutes. Then, share your thinking with your partner."
- 5–6 minutes: independent work time
- 4–5 minutes: partner discussion
- Monitor for students who:
 - use the fact that there are 2 fourths or 4 eighths in 1 half to reason about sums or differences of the fractions
 - compare the fractional parts of the mixed number to the benchmark of $\frac{1}{2}$ or 1 (especially in the last problem)
 - found differences and sums by writing equivalent fractions in fourths or eighths

Synthesis

- Select students to share their responses and reasoning.
- Focus the discussion on how students found the fractional differences between

and $\frac{1}{4}$ make $\frac{3}{4}$, so now we're only $\frac{1}{4}$ or $\frac{2}{8}$ from 109, but we still have $\frac{3}{8}$ to add.

- Disagree. The sum of all the whole numbers is 108, which is 1 away from 109. I know that $\frac{1}{2} = \frac{4}{8}$ and $\frac{1}{4} = \frac{2}{8}$, so if we add $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$, we'd end up with $\frac{9}{8}$, which is more than 1. But, when we stack any two towers, the studs of the bottom tower would overlap with the top tower, shrinking the combined height a little bit.

the measurements. Highlight explanations about the relative sizes of halves, fourths, and eighths, and about equivalence. (See Student Responses.)

Advancing Student Thinking

Students may be inclined to add all the mixed numbers but may be unsure about how to proceed given the different fractional parts. Consider asking: "Are there other ways to see if the tower would fit?" and "Would it help to think about the whole number measurements and fractional parts separately?"

Activity 2

Stacks of Blocks

 20 min

 PLC Activity

Standards Alignments

Building On 4.NF.A.1, 4.NF.B.3.d, 4.NF.B.4

Building Towards 4.NF.C.5

Previously, students used their knowledge of equivalence to reason about the sums and differences of fractions with denominators 2, 4, or 8. In this activity, they do the same with fractions with denominators 2, 3, and 6. As before, students are not expected to write addition expressions in which the fractions are written with a common denominator (though some students may choose to do so). Instead, they rely on what they know about the relationship between $\frac{1}{3}$ and $\frac{1}{6}$, and between $\frac{1}{2}$ and $\frac{1}{6}$, to solve the problems. Students may choose to use visual representations to support their reasoning. When students create and compare their own

representations for the context, they develop ways to model the mathematics of a situation and strategies for making sense of and persevering to solve problems (MP1, MP4).

The measurements in the task— $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ —are given in feet. Because each of them has a whole-number equivalent in inches, some students may choose to reason entirely in inches, which is a valid strategy. Ask these students to think about how they'd approach the problems if the given unit is an unfamiliar one, or one that doesn't convert handily to whole numbers in another unit.

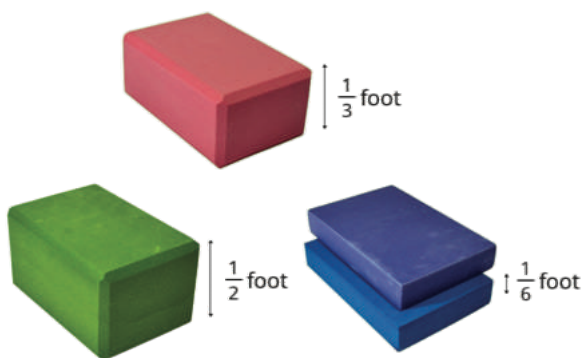
Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “whether Andre can use the fraction ___ to make a stack that is ___ feet tall”. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student-facing Task Statement

Andre is building a tower out of foam blocks. The blocks come in three different thicknesses: $\frac{1}{2}$ foot, $\frac{1}{3}$ foot, and $\frac{1}{6}$ foot.



1. Andre stacks one block of each size. Will that stack be more than 1 foot tall? Explain or show how you know.
2. Can Andre use only the $\frac{1}{6}$ -foot and $\frac{1}{3}$ -foot blocks to make a stack that is $1\frac{1}{2}$ feet tall? If you think so, show one or more ways. If not, explain why not.

Launch

- Groups of 2
- Display the image of the three foam blocks.
- “What do you notice? What do you wonder?”
- 30 seconds: quiet think time
- 30 seconds: partner discussion

Activity

- “Take a few quiet minutes to work on the activity. Then, share your thinking with your partner.”
- 7–8 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor for the different reasoning strategies students use to combine different fractional parts, including use of diagrams, descriptions, and expressions or equations.

3. Can Andre use only the $\frac{1}{6}$ -foot and $\frac{1}{2}$ -foot blocks to make a stack that is $1\frac{1}{3}$ feet tall? If so, show one or more ways. If not, explain why not.

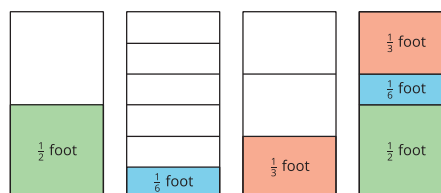
Student Responses

- No, it will be exactly 1 foot tall. Sample reasoning: I know $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, so adding one $\frac{1}{3}$ -foot block and one $\frac{1}{6}$ -foot block would give $\frac{3}{6}$ foot, which is $\frac{1}{2}$ foot. Adding another $\frac{1}{2}$ -foot block would make the stack 1 foot tall.
- Yes. Sample responses:
 - Three $\frac{1}{3}$ -foot blocks and three $\frac{1}{6}$ -foot blocks. $3 \times \frac{1}{3} = 1$ and $3 \times \frac{1}{6} = \frac{3}{6}$, which is $\frac{1}{2}$. The total height is $1 + \frac{1}{2}$ or $1\frac{1}{2}$.
 - Four $\frac{1}{3}$ -foot blocks and one $\frac{1}{6}$ -foot block. $4 \times \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}$, which is equivalent to $1\frac{2}{6}$. Another $\frac{1}{6}$ would make $1\frac{3}{6}$, which is $1\frac{1}{2}$.
- Yes. Sample responses:
 - Two $\frac{1}{2}$ -foot blocks and two $\frac{1}{6}$ -foot blocks. $2 \times \frac{1}{2} = 1$ and $2 \times \frac{1}{6} = \frac{2}{6}$, which is $\frac{1}{3}$. The total height is $1 + \frac{1}{3}$ or $1\frac{1}{3}$.
 - One $\frac{1}{2}$ -foot block and five $\frac{1}{6}$ -foot blocks. $\frac{1}{2} = \frac{3}{6}$ and $5 \times \frac{1}{6} = \frac{5}{6}$, so the combined height is $\frac{8}{6}$ or $1\frac{2}{6}$, which is $1\frac{1}{3}$.

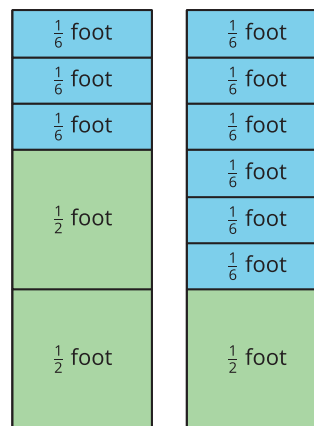
Synthesis

- Invite students to share their strategies for determining whether or how certain combinations of blocks would make a specified height. Record and display their reasoning.
- To highlight different ways to combine different-size fractional parts, consider sketching or displaying diagrams as shown:

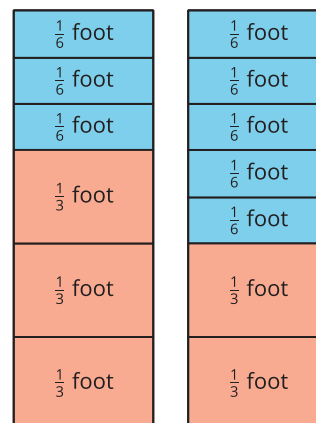
- Making 1 foot with all blocks:



- Making $1\frac{1}{2}$ foot with $\frac{1}{2}$ and $\frac{1}{6}$ blocks:



- Making $1\frac{1}{2}$ foot with $\frac{1}{2}$ and $\frac{1}{3}$ blocks:



Advancing Student Thinking

Students may not notice a relationship between the fractions in the task. Consider asking “What do you know about the relationship between thirds and sixths?” and “What about between halves and sixths?” If needed, consider referring to fraction strip diagrams from an earlier unit.

Lesson Synthesis

🕒 10 min

“Today we solved problems where we had to combine halves, fourths, and eighths, or remove one of those fractions from another. We also combined halves, thirds, and sixths.”

“How would you find the combined lengths of $\frac{1}{2}$ inch and $\frac{3}{8}$ inch? How would you find the difference of the two lengths?”

Consider asking students to record their response in writing, or to turn and talk to a partner after some quiet think time.

“In upcoming lessons, we’ll use some of the strategies we used today to combine tenths and hundredths.”

Suggested Centers

- Jump the Line (2–5), Stage 2: Add and Subtract Tenths and Hundredths (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)

Response to Student Thinking

Students correctly recognize the heights of the two stacks as $\frac{1}{3} + \frac{1}{3} + \frac{1}{6}$ and $\frac{1}{2} + \frac{1}{6} + \frac{1}{6}$, but do not use equivalence to find each height. (They may, for instance, add the numerators of each expression, as they had done when adding fractions with the same denominator.)

Students do not recognize equivalence of 1 third and 2 sixths, or 1 half and 3 sixths.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Prior Unit Support

Grade 4, Unit 2, Section B: Equivalent Fractions

Lesson 16: Tenths and Hundredths, Together

Standards Alignments

| | |
|------------------|--------------------|
| Building On | 4.NF.A.1, 4.NF.A.2 |
| Addressing | 4.NF.A.1, 4.NF.C.5 |
| Building Towards | 4.NF.C.5 |

Teacher-facing Learning Goals

- Use equivalent fractions to add tenths and hundredths, up to a sum of 1.

Student-facing Learning Goals

- Let's add some tenths and hundredths.

Lesson Purpose

The purpose of this lesson is for students to write equivalent fractions to add tenths and hundredths, up to a sum of 1.

Prior to this lesson, students refreshed their understanding of equivalence. They used it to reason about sums and differences of fractions whose denominators are different but are factors or multiples of one another (2, 4, and 8, and 2, 3, and 6). This lesson extends that work to include fractions with denominators of 10 and 100. Students revisit how to write equivalent fractions in tenths and hundredths, and then use that understanding to add tenths and hundredths, up to a sum of 1.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR1 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |

Teacher Reflection Question

Reflect on a time recently when your thinking about students' understanding or reasoning changed. What led to the change in perspective? How will you alter your teaching practice to incorporate your new understanding?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Some Sums

Standards Alignments

Addressing 4.NF.C.5

Student-facing Task Statement

Find the value of each sum. Show your reasoning. Use number lines if you find them helpful.

1. $\frac{1}{10} + \frac{50}{100}$
2. $\frac{20}{100} + \frac{4}{10}$
3. $\frac{6}{10} + \frac{3}{100}$
4. $\frac{18}{100} + \frac{7}{10}$

Student Responses

1. $\frac{6}{10}$ or $\frac{60}{100}$
2. $\frac{6}{10}$ or $\frac{60}{100}$
3. $\frac{63}{100}$
4. $\frac{88}{100}$

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Shaded Rectangles and Squares

Standards Alignments

Addressing 4.NF.A.1

The purpose of this warm-up is to elicit observations about fractions in tenths and hundredths and about equivalence, which will be useful when students find sums of tenths and hundredths later in the lesson. While students may notice and wonder many things about these diagrams, focus the discussion on the relationship between tenths and hundredths and how we might express equivalent amounts.

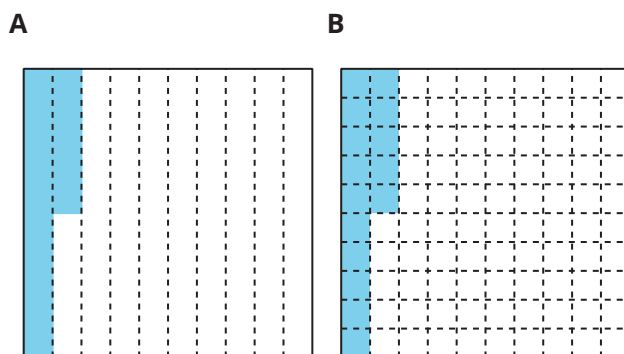
Instructional Routines

Notice and Wonder

Student-facing Task Statement

Each large square represents 1.

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are two square diagrams. One is partitioned into tenths. The other is partitioned into hundredths.
- The amount shaded on each diagram is the same.
- Each part in the first diagram is a long rectangle. The first rectangle is shaded, and half of the second rectangle is shaded.
- Each part in the second is a small square. Fifteen squares are shaded.

Launch

- Groups of 2
- Display the diagrams.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “What fraction does each part in the first diagram represent?” (One tenth or $\frac{1}{10}$) “What about in the second diagram?” (One hundredth or $\frac{1}{100}$)
- “Can you see tenths in both diagrams? Where?” (Yes. Each rectangle in A is a tenth. Each group of small squares in B is a tenth.)
- “Can you see hundredths in both diagrams? Where?” (No, only in B. Each square is a hundredth.)
- “Do you think the shaded parts of the two diagrams represent the same fraction or different fractions? Which fraction(s)?” (The

- The shaded parts in the second diagram represents $\frac{15}{100}$.

Students may wonder:

- Why is the second rectangle in the first diagram shaded halfway?
- What fraction does the shaded parts in the first diagram represent?

same fraction, $\frac{15}{100}$. Different fractions: The second one represents $\frac{15}{100}$, but I'm not sure about the first one.)

Activity 1

🕒 15 min

Tenths and Hundredths

Standards Alignments

Building On 4.NF.A.1, 4.NF.A.2

Building Towards 4.NF.C.5

In this activity, students refresh what they know about equivalent fractions in tenths and hundredths. Students are given fractions in tenths and are to write equivalent fractions in hundredths, and vice versa. In one case, they encounter a fraction in hundredths that cannot be written as tenths and consider why this might be. The work here reminds students of the relative sizes of tenths and hundredths and prepares students to add such fractions in upcoming activities.

♿ Access for Students with Disabilities

Representation: Access for Perception. Invite students to examine a meter stick, and identify correspondences between this and the number line: One centimeter is one hundredth of a meter and ten centimeters is one tenth of a meter (called a decimeter). Clearly mark decimeters on the meter stick, and invite students to come back to reference this concrete representation as they work on the task.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

1. Complete the table with equivalent fractions in tenths or hundredths. In the last row,

Launch

- Groups of 2
- Consider asking students some of these

write a new pair of equivalent fractions.

| | tenths | hundredths |
|----|-----------------|-------------------|
| a. | $\frac{1}{10}$ | |
| b. | $\frac{4}{10}$ | |
| c. | $\frac{6}{10}$ | |
| d. | | $\frac{50}{100}$ |
| e. | | $\frac{90}{100}$ |
| f. | $\frac{12}{10}$ | |
| g. | | $\frac{200}{100}$ |
| h. | $2\frac{3}{10}$ | |
| i. | | $\frac{125}{100}$ |
| j. | | |

2. Name some fractions that are:

- between $\frac{50}{100}$ and $\frac{60}{100}$
- between $\frac{3}{10}$ and $\frac{4}{10}$

Be prepared to explain your reasoning.

Student Responses

- $\frac{10}{100}$
 - $\frac{40}{100}$
 - $\frac{60}{100}$
 - $\frac{5}{10}$
 - $\frac{9}{10}$
 - $\frac{120}{100}$
 - $\frac{20}{10}$
 - $\frac{230}{100}$
 - No equivalent in tenths
 - Answers vary.
- Sample responses:
 - $\frac{52}{100}$, $\frac{55}{100}$ (or any fractions greater than

questions:

- “What do you know about 1 tenth? What about 1 hundredth?”
- “Which is greater: 1 tenth or 1 hundredth?”
- “How many hundredths are in 1 tenth?”

Activity

- “Work independently on the activity for 5 minutes. Then, share your responses with your partner.”
- 5 minutes: independent work time
- 2–3 minutes: partner discussion

Synthesis

- Select students to share their responses and reasoning for the first set of problems. Display or record their responses.
- Discuss the fraction $\frac{125}{100}$ and what students wrote for its equivalent in tenths.
- Invite students to share the fractions they thought of for the last set of problems. Focus the discussion on how they know what fractions are between $\frac{3}{10}$ and $\frac{4}{10}$.
- If not mentioned in students' explanations, ask: “Could that number be expressed in tenths? Why or why not?” (No, because there is not a whole number between 3 and 4.)
- Highlight explanations that show how expressing the $\frac{3}{10}$ and $\frac{4}{10}$ in hundredths would allow us to name the fractions between these given two fractions.

- $\frac{50}{100}$ and less than $\frac{60}{100}$)
- b. $\frac{32}{100}$, $\frac{35}{100}$ (or any fractions greater than $\frac{30}{100}$ and less than $\frac{40}{100}$)

Activity 2

🕒 20 min

Walk, Stop, and Sip

Standards Alignments

Building On 4.NF.A.1, 4.NF.A.2
Addressing 4.NF.C.5

In this activity, students use jumps on number lines to visualize addition of tenths and hundredths and find the values of such sums. Using diagrams helps to reinforce the relative sizes of tenths and hundredths. It provides a visual reminder that all tenths can be expressed in terms of hundredths, and that some hundredths can be written in tenths, which can in turn help with addition of these fractions.

This is the first activity in which students are to write expressions and equations to represent sums of fractions with different denominators. Initially, students will likely find it helpful to write equivalent fractions in the same denominator. Later, as students become more fluent in expressing tenths in hundredths and vice versa, they may perform the rewriting mentally rather than on paper. When students create and compare their own representations for the context, they reason abstractly and quantitatively (MP2).

Access for English Learners

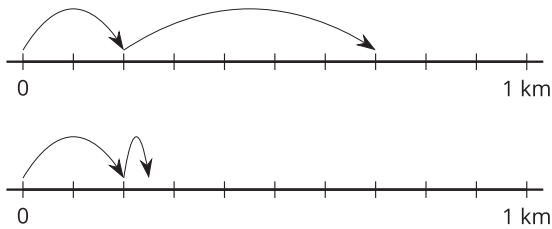
MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “the total distance Noah has walked”. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student-facing Task Statement

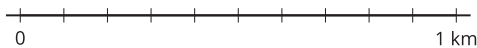
Noah walks $\frac{2}{10}$ kilometer (km), stops for a drink of water, walks $\frac{5}{100}$ kilometer, and stops for another sip.

- Which number line diagram represents the distance Noah has walked? Explain how you know.

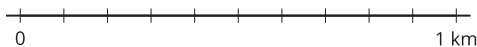


- The diagram that you didn't choose represents Jada's walk. Write an equation to represent:
 - the total distance Jada has walked
 - the total distance Noah has walked
- Find the value of each of the following sums. Show your reasoning. Use number lines if you find them helpful.

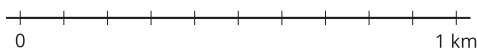
a. $\frac{5}{10} + \frac{1}{10}$



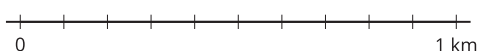
b. $\frac{50}{100} + \frac{10}{100}$



c. $\frac{5}{10} + \frac{30}{100}$



d. $\frac{15}{100} + \frac{4}{10}$



Student Responses

- The second number line represents Noah's

Launch

- Groups of 2
- Ask students what they know about kilometers.
- If not mentioned by students, explain that, just like mile (which may be more familiar), it is a unit of length, used to measure long distances.
- If students wonder how it is related to meter, explain that 1 kilometer is 1,000 meters. (Kilometers will be explored more closely in a future unit.)

Activity

- "Take a few quiet minutes to work on the first two problems. Then, share your responses with your partner."
- 5 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for the ways students think about the total distance Noah has walked (third problem) given a fraction in tenths and one in hundredths.
- "Now try finding the values of the sums in the last problem."
- 5 minutes: independent or partner work time

Synthesis

- Invite students to share how they know which diagram represents Noah's walk and their equations for the distances Noah and Jada walked.
- Given the number line diagram for support, students are likely to write $\frac{2}{10} + \frac{5}{100} = \frac{25}{100}$. Discuss why this is true.
- "How do you know that the sum of $\frac{2}{10}$ and $\frac{5}{100}$ is $\frac{25}{100}$?"

walk. Sample reasoning:

- $\frac{5}{100}$ is less than $\frac{2}{10}$, so the second “jump” in the diagram should be smaller than the first.
- In the first diagram, the second jump shows $\frac{5}{10}$, which is $\frac{50}{100}$, not $\frac{5}{100}$.
- Each space between the tick marks represents $\frac{1}{10}$, which is $\frac{10}{100}$, so $\frac{5}{100}$ would be half of the space between tick marks.

2. a. $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

b. $\frac{2}{10} + \frac{5}{100} = \frac{25}{100}$

3. Sample reasoning:

a. $\frac{5}{10} + \frac{1}{10} = \frac{6}{10}$

b. $\frac{50}{100} + \frac{10}{100} = \frac{60}{100}$

c. $\frac{5}{10} + \frac{30}{100} = \frac{50}{100} + \frac{30}{100} = \frac{80}{100}$

d. $\frac{15}{100} + \frac{4}{10} = \frac{15}{100} + \frac{40}{100} = \frac{55}{100}$

- Highlight that $\frac{2}{10}$ is equivalent to $\frac{20}{100}$ and another $\frac{5}{100}$ makes $\frac{25}{100}$.
- Consider displaying a number line that is partitioned into tenths and hundredths and shows $\frac{25}{100}$ as $\frac{20}{100} + \frac{5}{100}$.



Lesson Synthesis

🕒 10 min

“Today we learned to find the sum of tenths and hundredths. We used what we know about equivalent fractions and what we know about adding fractions with the same denominator.”

“How do we find the sums of tenths and hundredths when the denominators are different?” (Either think about tenths in terms of hundredths or hundredths in terms of tenths. Then, add them together.)

Discuss the last two sums: $\frac{5}{10} + \frac{30}{100}$ and $\frac{15}{100} + \frac{4}{10}$.

“In each case, how do we know whether to rewrite the tenths as hundredths, or to write hundredths as tenths?” (Sample response:

- For $\frac{5}{10} + \frac{30}{100}$, either way works. $\frac{5}{10}$ is equivalent to $\frac{50}{100}$ and $\frac{30}{100}$ is equivalent to $\frac{3}{10}$.
- For $\frac{15}{100} + \frac{4}{10}$, we’d write in hundredths, because $\frac{4}{10}$ is equivalent to $\frac{40}{100}$ but $\frac{15}{100}$ has no equivalence in tenths.)

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students write an equivalent fraction for the tenths but also multiply the numerator of the hundredths by 10, changing the value of the hundredths.

The work in this lesson builds from the equivalent fraction concepts developed in a prior unit.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Prior Unit Support

Grade 4, Unit 2, Section B: Equivalent Fractions

Lesson 17: Sums of Tenths and Hundredths

Standards Alignments

Building On 4.NF.A.1, 4.NF.A.2

Addressing 4.NF.C.5

Teacher-facing Learning Goals

- Use equivalent fractions to add tenths and hundredths, where the sum is greater than 1.

Student-facing Learning Goals

- Let's add more tenths and hundredths.

Lesson Purpose

The purpose of this lesson is for students to use equivalent fractions to add tenths and hundredths, where the sum is greater than 1.

Prior to this lesson, students learned to combine tenths and hundredths up to a sum of 1. In this lesson, they extend that work to include larger fractions and continue to build their ability to identify equivalent fractions that are helpful for finding sums.

They also encounter some equations involving unknown addends. While subtracting tenths and hundredths is not an expectation at this point, students can reason about the unknown addends by relying on their understanding of addition, their experience with decomposing a fraction into a sum, and their knowledge of equivalence.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Sticky notes: Activity 1

Materials to Copy

- Card Sort: Less Than, Equal to, or Greater Than 1 (groups of 2): Activity 1
- Fraction Action: Tenths, Hundredths

(groups of 2): Activity 3

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Activity 3 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What questions that you asked supported students' thinking about equivalence and addition of fractions today? What did students say or do that showed they were effective?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Missing Fractions

Standards Alignments

Addressing 4.NF.C.5

Student-facing Task Statement

Each equation is missing a fraction in tenths or hundredths. Find the fraction that makes each equation true.

- $\frac{26}{100} + \frac{8}{10} = \underline{\hspace{2cm}}$
- $\frac{7}{10} + \underline{\hspace{2cm}} = \frac{92}{100}$
- $\underline{\hspace{2cm}} + \frac{8}{100} = \frac{128}{100}$
- $\frac{12}{100} + \frac{12}{10} = \underline{\hspace{2cm}}$

Student Responses

- $\frac{106}{100}$ or $1\frac{6}{100}$
- $\frac{22}{100}$
- $\frac{12}{10}$ or $\frac{120}{100}$

4. $\frac{132}{100}$ or $1\frac{32}{100}$

----- Begin Lesson -----

Warm-up

🕒 10 min

Which One Doesn't Belong: Tenths and Hundredths

Standards Alignments

Building On 4.NF.A.1

This warm-up prompts students to carefully analyze and compare the features of four fractions. They may consider size (of the fraction, the numerator, or the denominator), equivalence, relationship to benchmark numbers, and more. The reasoning here will be helpful later in the lesson, as students classify sums of fractions by their size and relationship to 1.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

- | | | | |
|----|-------------------|----|------------------|
| A. | $\frac{48}{100}$ | B. | $\frac{8}{10}$ |
| C. | $\frac{120}{100}$ | D. | $\frac{70}{100}$ |

Student Responses

Sample response:

- A is the only one that doesn't have an

Launch

- Groups of 2
- Display the four expressions.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2-3 minutes: partner discussion
- Share and record responses.

equivalent fraction in tenths.

- B is the only one that is not in hundredths.
- C is the only one that is not less than 1.
- D is the only fraction with a numerator that is not a multiple of 4 or 8.

Synthesis

- Consider asking: "Let's find at least one reason why each one doesn't belong."
- "Are any of these equal to 1?" (No)
- "Which of these fractions are greater than 1? How do you know?" ($\frac{120}{100}$, because it is greater than $\frac{100}{100}$.)

Activity 1

🕒 20 min

Card Sort: Less Than, Equal to, or Greater Than 1?

Standards Alignments

Building On 4.NF.A.2

Addressing 4.NF.C.5

The purpose of this activity is for students to practice adding tenths and hundredths, by sorting a set of expressions based on whether their values are less than, equal to, or greater than 1. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP7). They decide whether it is necessary to write equivalent fractions, and if so, whether to use tenths or hundredths.

Through repeated reasoning, students build their ability to compare the size of hundredths to tenths and to 1 (MP8). They also have an opportunity to look for and make use of structure (MP7). For instance, students may conclude that certain expressions are greater than 1 by noticing that one of the addends is greater than 1.

Here is a list of the expressions on the Instructional master, for reference:

A.

$$\frac{10}{100} + \frac{8}{10}$$

B.

$$\frac{80}{100} + \frac{2}{10}$$

C.

$$\frac{20}{10} + \frac{30}{100}$$

D.

$$\frac{7}{10} + \frac{8}{100}$$

E.

$$\frac{22}{100} + \frac{8}{100}$$

F.

$$\frac{12}{10} + \frac{8}{100}$$

G.

$$\frac{12}{100} + \frac{12}{10}$$

H.

$$\frac{73}{100} + \frac{3}{10}$$

I.

$$\frac{150}{100} + \frac{1}{10}$$

J.

$$\frac{9}{10} + \frac{11}{100}$$

K.

$$\frac{10}{100} + \frac{9}{10}$$

L.

$$\frac{6}{10} + \frac{39}{100}$$

Access for English Learners

MLR8 Discussion Supports. Students should take turns deciding where to place their card and explaining their reasoning to their group. Display the following sentence frame for all to see: “I noticed ___, so I placed the card . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Writing, Conversing, Representing

Instructional Routines

Card Sort

Materials to Gather

Sticky notes

Materials to Copy

Card Sort: Less Than, Equal to, or Greater Than 1 (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2-4 students.

Student-facing Task Statement

1. Sort the cards from your teacher based on whether the value of the expression is less than 1, equal to 1, or greater than 1.

When done, make a quick list of which expressions you have in each group.

2. Visit the sorted collection of another group.
 - Did they sort the cards the same way?
 - Select 1–2 cards that you have a question about or whose placement you disagree with.
 - Leave a note for the group members to discuss.
3. Return to your collection.
 - Discuss any notes that are left for your group, or revise your sorting decision based on what you learned

Launch

- Groups of 2–4
- Give each group one set of cards from the Instructional master and a couple of sticky notes.

Activity

- “Work with your group to sort the cards into three groups, based on whether the expressions are less than 1, equal to 1, or greater than 1.”
- “Be prepared to explain or show how you know where each sum belongs.”
- 8–10 minutes: group work time
- Monitor for:
 - the ways students decide whether to write equivalent fractions in tenths or hundredths
 - students who estimate the value of

from another group.

- Record the expressions here.

| less than 1 | equal to 1 | greater than 1 |
|-------------|------------|----------------|
| | | |

Student Responses

| less than 1 | equal to 1 | greater than 1 |
|-------------------------------------|------------------------------------|-------------------------------------|
| A. $\frac{10}{100} + \frac{8}{10}$ | | C. $\frac{20}{10} + \frac{30}{100}$ |
| D. $\frac{7}{10} + \frac{8}{100}$ | B. $\frac{80}{100} + \frac{2}{10}$ | F. $\frac{12}{10} + \frac{8}{100}$ |
| E. $\frac{22}{100} + \frac{8}{100}$ | K. $\frac{10}{100} + \frac{9}{10}$ | G. $\frac{12}{100} + \frac{12}{10}$ |
| L. $\frac{6}{10} + \frac{39}{100}$ | | H. $\frac{73}{100} + \frac{3}{10}$ |
| | | I. $\frac{150}{100} + \frac{1}{10}$ |
| | | J. $\frac{9}{10} + \frac{11}{100}$ |

the expressions by looking at the relative size of the addends, without finding the sum

- “When you finish, visit another group’s sorted collection. Examine it and leave a note about any questions you have.”
- 3–4 minutes: Visit another group.
- “Return to your collection. Address any questions left for you or revise your thinking. Then, record what’s in each group.”

Synthesis

- Discuss questions such as:
 - “How did you decide where each expression should go? Did you always write an equivalent fraction?”
 - “When was it necessary to write an equivalent fraction? When was it not?”
 - “Were there expressions you were able to sort without rewriting any fractions or adding anything? What was it about those expressions that made that possible?”

Advancing Student Thinking

Some students may have a better intuition for how tenths or hundredths relate to $\frac{1}{2}$ than how they relate to one another, or how they relate to 1. Consider asking: “Which fraction in the pair is close to $\frac{1}{2}$?” and “Can this help you determine how this expression relates to 1?”

Activity 2

🕒 15 min

What’s Missing?

Standards Alignments

Addressing 4.NF.C.5

In previous activities, students learned to combine tenths and hundredths. In this activity, students complete addition equations to make them true. To do so, they rely on a range of understandings and skills: how to write equivalent fractions, how to add fractions, and how to decompose a fraction into a sum. Though many of the equations involve an unknown addend, students are not expected to find them by subtraction.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Differentiate the degree of difficulty or complexity. Some students may benefit from starting with whole numbers to recall how they might decompose a sum to find a missing addend. For example, invite students to articulate strategies to find the missing number in $10 + \underline{\hspace{2cm}} = 30$. Make sure to elicit responses that do not require subtraction.

Supports accessibility for: Conceptual Processing, Organization, Social-Emotional Functioning

Student-facing Task Statement

- Each equation is missing a fraction in hundredths. Find the fraction that makes each equation true.

a. $\frac{10}{100} + \underline{\hspace{2cm}} = \frac{30}{100}$

b. $\underline{\hspace{2cm}} + \frac{2}{10} = \frac{80}{100}$

c. $\frac{7}{10} + \underline{\hspace{2cm}} = \frac{94}{100}$

d. $\frac{9}{100} + \underline{\hspace{2cm}} = \frac{8}{10}$

e. $\frac{16}{100} + \frac{4}{10} = \underline{\hspace{2cm}}$

f. $\underline{\hspace{2cm}} + \frac{14}{10} = \frac{172}{100}$

- Each equation is missing a fraction in tenths or hundredths. Find the fraction that makes each equation true.

Launch

- Groups of 2
- Display these equations:
 - $\frac{1}{2} + \frac{4}{2} = 2$
 - $\frac{9}{10} + \underline{\hspace{2cm}} = 1$
- “Are these equations true? Take a minute to think about it.”
- 1 minute: quiet think time
- Discuss responses.
- “Why is the first equation not true?” (The sum of the fractions on the left is $\frac{5}{2}$, which does not equal 2.)
- “Why are $\frac{1}{10}$ and $\frac{10}{100}$ both true for the last equation?” (They are equivalent, so when added to $\frac{9}{10}$ both result in 1.)
- “Let’s find some other fractions that would make equations true.”

a. $\frac{20}{100} + \underline{\hspace{2cm}} = \frac{28}{100}$

b. $\frac{110}{100} + \underline{\hspace{2cm}} = \frac{15}{10}$

c. $\frac{61}{100} + \frac{3}{10} = \underline{\hspace{2cm}}$

d. $\frac{9}{10} + \underline{\hspace{2cm}} = \frac{170}{100}$

e. $\underline{\hspace{2cm}} + \frac{72}{100} = \frac{102}{100}$

f. $\frac{15}{100} + \underline{\hspace{2cm}} = 1 \frac{55}{100}$

Student Responses

Fractions that are equivalent to the ones shown are also acceptable.

1.
 - a. $\frac{20}{100}$
 - b. $\frac{60}{100}$
 - c. $\frac{24}{100}$
 - d. $\frac{71}{100}$
 - e. $\frac{56}{100}$
 - f. $\frac{32}{100}$
2.
 - a. $\frac{8}{100}$
 - b. $\frac{4}{10}$
 - c. $\frac{91}{100}$
 - d. $\frac{8}{10}$
 - e. $\frac{30}{100}$
 - f. $1 \frac{40}{100}$

Activity

- “Work independently to complete at least three equations from the first problem and three from the second before discussing with your partner.”
- 6–7 minutes: independent work time
- 3–4 minutes: partner work time
- Monitor for the equations that seem to be challenging to many students or to be prone to errors. Discuss them during synthesis.

Synthesis

- “Which equations were difficult to complete? What about the given fractions made it hard to find the missing numbers?”
- “Which did you find more challenging: finding missing tenths or missing hundredths? Why might that be?”

Activity 3 (optional)

 20 min

Fraction Action: Tenths, Hundredths

Standards Alignments

Addressing 4.NF.C.5

This optional activity allows students to practice adding tenths and hundredths (and to reinforce their ability to compare fractions) through a game. Students use fraction cards to play a game in groups of 2, 3, or 4. To win the game is to draw pairs of cards with the greater (or greatest) sum, as many times as possible.

Consider arranging students in groups of 2 for the first game or two (so that students would need to compare only 2 sums at a time), and arranging groups of 3 or 4 for subsequent games. Before students begin playing, ask them to keep track of and record pairs of fractions that they find challenging to add.

Materials to Copy

Fraction Action: Tenths, Hundredths (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Play Fraction Action with 2 players:

- Shuffle the cards from your teacher. Place the cards in a stack, face down.
- Each player turns 2 cards over and adds the fractions on the cards.
- Compare the sums. The player with the greater sum wins that round and keeps all four cards.
- If the sums are equivalent, each player turns one more card over and adds the value to their sum. The player with the greater new sum keeps all cards.
- The player with the most cards wins the game.

Play Fraction Action with 3 or 4 players:

Launch

- Groups of 2 for the first game or two, then groups of 3–4 for subsequent games, if time permits
- Give each group one set of fraction cards from the Instructional master.
- Tell students that they will play one or more games of Fraction Action.
- Demonstrate how to play the game. Invite a student to be your opponent in the demonstration game.
- Read the rules as a class and clarify any questions students might have.

Activity

- “Play one game with your partner.”
- “As you play, you may come across one or

- The player with the greatest sum of fractions wins the round.
- If 2 or more players have the greatest sum, those players turn two more cards over and find their sum. The player with the greatest sum keeps all the cards.

Record any pair of fractions whose sum is challenging to find here.

_____ and _____ _____ and _____
 _____ and _____ _____ and _____

Student Responses

No response required.

more pairs of fractions whose sums are hard to find. Record those fractions. Be prepared to explain how you eventually figured out which sum is greater.”

- “If you finish before time is up, play another game with the same partner, or play a game with the players from another group.”
- 15 minutes: group play time

Synthesis

- Invite groups to share some of the challenging expressions they recorded and how they eventually determined the sums.
- As one group shares, ask others if they have other ideas about how the fractions could be added.

Lesson Synthesis

🕒 10 min

“Today we practiced adding fractions and finding missing fractions that would make equations true.”

“What strategies did you find helpful when adding tenths and hundredths and writing true equations with both tenths and hundredths?”

“Was there an error that you made or something that was missed multiple times? What was it? Why might it be an easy error to make or an easy thing to miss?”

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students add or subtract only the numerators of the fractions when finding sums or differences of tenths and hundredths.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 18: Lots of Fractions to Add

Standards Alignments

Building On 3.NBT.A.2
Addressing 4.NF.B.4.c, 4.NF.C.5

Teacher-facing Learning Goals

- Find the sum of three or more tenths and hundredths, using the commutative and associative properties strategically.

Student-facing Learning Goals

- Let's add tenths and hundredths again, more than two at a time.

Lesson Purpose

The purpose of this lesson is for students to find the sum of three or more tenths and hundredths.

Previously, students learned to find sums of fractions with the same denominator and sums of tenths and hundredths. They added two or more tenths and hundredths, applying the commutative and associative properties along the way. This lesson prompts students to apply their understanding and skills to solve problems in context, and to practice finding sums of three or more tenths and hundredths (including mixed numbers).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Chart paper: Activity 2
- Coins: Activity 1

Materials to Copy

- More Than Two Fractions (groups of 15): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 25 min |
| Activity 2 | 10 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List the ways you have seen each student grow as a young mathematician throughout this work. List the ways you have seen yourself grow as a teacher. What will you continue to do and what will you improve upon in Unit 4?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

U.S. Coins

Standards Alignments

Addressing 4.NF.C.5

Student-facing Task Statement

The table shows the thicknesses of U.S. coins.

| coin | thickness (cm) |
|-------------|------------------|
| penny | $\frac{15}{100}$ |
| nickel | $\frac{2}{10}$ |
| dime | $\frac{14}{100}$ |
| quarter | $\frac{18}{100}$ |
| half dollar | $\frac{22}{100}$ |
| dollar | $\frac{2}{10}$ |

Find the combined thickness of:

1. a penny, a nickel, a quarter
2. a dollar, a half dollar, a quarter, and a dime

Student Responses

- $\frac{53}{100}$. Sample reasoning: $\frac{15}{100} + \frac{2}{10} + \frac{18}{100} = \frac{33}{100} + \frac{20}{100} = \frac{53}{100}$
- $\frac{74}{100}$. Sample reasoning: $\frac{2}{10} + \frac{22}{100} + \frac{18}{100} + \frac{14}{100} = \frac{20}{100} + \frac{54}{100} = \frac{74}{100}$

Begin Lesson

Warm-up

🕒 10 min

Number Talk: A Bunch of Numbers

Standards Alignments

Building On 3.NBT.A.2

The purpose of this Number Talk is to encourage students to apply properties of operations (especially the commutative and associative properties of addition) to mentally find sums of three or more whole numbers. The reasoning elicited here will be helpful later in the lesson when students are to add three or more tenths and hundredths.

To mentally add several two- and three-digit numbers, students need to look for and make use of structure (MP7), such as finding pairs of numbers that add up to 10 or 100, or numbers that end in 0 or 5.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $54 + 2 + 18$
- $61 + 104 + 39$
- $25 + 63 + 75 + 7$
- $50 + 106 + 19 + 101$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Student Responses

Sample reasoning:

- 74: $18 + 2$ is 20, and $54 + 20$ is 74.
- 204: $61 + 39$ is 100, and $104 + 100$ is 204.
- 170: $25 + 75$ is 100, and $63 + 7$ is 70.
- 276: $101 + 19$ is 120, $120 + 50$ is 170, and $170 + 106$ is 276.

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “What strategies were helpful for adding multiple numbers?” (Sample responses:
 - Find two or three numbers that add up to 10 or 100.
 - Add numbers that end with 5 or 0 first. Add familiar numbers first.
 - Put the numbers into groups of two, and then add what’s in each group before adding the groups.)
- Consider asking:
 - “Who can restate _____’s reasoning in a different way?”
 - “Did anyone have the same strategy but would explain it differently?”
 - “Did anyone approach the expression in a different way?”
 - “Does anyone want to add on to ___’s strategy?”

Activity 1

🕒 25 min

Stack Centavos and Pesos

Standards Alignments

Addressing 4.NF.B.4.c, 4.NF.C.5

In this activity, students solve problems involving tenths and hundredths in a context about coins. Given information about the thickness of some Mexican coins, students compare the heights of different combinations of stacked coins. To complete the task, students need to write equivalent

fractions, add tenths and hundredths, and compare fractions. Some students may choose to use multiplication to reason about the problems. Though the mathematics here is not new, the context and given information may be novel to students. Students have a wide variety of approaches available for these problems with no solution approach suggested (MP1). For example, to compare the peso coins of Diego and Lin, students could reason that they each have a 5 peso and a 20 peso coin and then compare the remaining coins, a 1 peso coin and 2 peso coin on the one hand and a 20 peso coin on the other. This method would require minimal calculations. Other students may add the thicknesses of Lin's coins and Diego's coins and compare these values.

To help students visualize stacked coins, prepare some coins of different thicknesses or include an image of stacked coins. (Access to the Mexican coins would be interesting to students but is not essential.) Some students may be curious about the equivalents of centavos or pesos in U.S. dollars. Consider checking the exchange rates before the lesson.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Display sentence frames to agree or disagree. "I agree because . . ." "I disagree because . . ."

Advances: Speaking, Conversing

Access for Students with Disabilities

Representation: Access for Perception. Invite students to model the situation using sticky notes or scrap paper to represent the coins. Students can label each sticky note with the thickness, value, and owner of the coins, then move the sticky notes around as they model and solve each problem. Encourage students to use the sticky notes to solve strategically. For example, they might group like denominators before adding or layer repeated fractions to represent multiplication.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Memory

Materials to Gather

Coins

Required Preparation

- Gather a few coins of different thicknesses for display.

Student-facing Task Statement

Diego and Lin each have a small collection of Mexican coins.

Launch

- Display the image of Mexican coins.
- "What do you notice? What do you

The table shows the thickness of different coins in centimeters (cm) and how many of each Diego and Lin have.



| coin value | thickness in cm | Diego | Lin |
|-------------|------------------|-------|-----|
| 1 centavo | $\frac{12}{100}$ | 3 | 1 |
| 10 centavos | $\frac{22}{100}$ | 0 | 1 |
| 1 peso | $\frac{16}{100}$ | 0 | 1 |
| 2 pesos | $\frac{14}{100}$ | 0 | 1 |
| 5 pesos | $\frac{2}{10}$ | 1 | 1 |
| 20 pesos | $\frac{25}{100}$ | 2 | 1 |

1. If Diego and Lin each stack their centavo coins, whose stack would be taller? Show your reasoning.
2. If they each stack their peso coins, whose stack would be taller? Show your reasoning.
3. If they each stack all their coins, whose stack would be taller? Show your reasoning.
4. If they combine their coins to make a single stack, would it be more than 2 centimeters tall? Show your reasoning.

Student Responses

1. Diego's stack of centavos is taller. Sample reasoning:
 - Diego: $3 \times \frac{12}{100} = \frac{36}{100}$
 - Lin: $\frac{22}{100} + \frac{12}{100} = \frac{34}{100}$
2. Lin's stack of pesos is taller. Sample reasoning:
 - Diego: $\frac{2}{10} + (2 \times \frac{25}{100}) = \frac{2}{10} + \frac{50}{100} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$
 - Lin: $\frac{16}{100} + \frac{14}{100} + \frac{2}{10} + \frac{25}{100} = \frac{30}{100} + \frac{20}{100} + \frac{25}{100} = \frac{75}{100}$
3. Lin's coin stack is taller. Sample reasoning:

wonder?"

- 1 minute: quiet think time
- Share responses.
- Explain that pesos and centavos are units of Mexican money, just as dollars and cents are units of American money.
- "What are the values of the American coins we use today?" (1 cent, 5 cents, 10 cents, 25 cents, half a dollar, and 1 dollar.)
- Explain that Mexico uses many more types of coins than the U.S. does.
- Display coins of different thicknesses.
- "Just like pennies, dimes, nickels, and quarters, centavo and peso coins have different weights and thicknesses."
- Refer to the table in the task, showing the thicknesses in tenths or hundredths of a centimeter.

Activity

- "Work with your partner on the first three problems."
- 7–8 minutes: partner work time
- "Find another group of classmates and share your responses. Discuss any disagreement you might have."
- 3–4 minutes: group discussion
- "Take a few quiet minutes to answer the last question."
- 3 minutes: independent work time
- Consider asking each combined group to share their response to one assigned problem.

Synthesis

- Invite groups to share their responses and reasoning. Highlight the different approaches students took to solve the same problems.

- Diego: $\frac{36}{100} + \frac{7}{10} = \frac{36}{100} + \frac{70}{100} = \frac{106}{100}$
 - Lin: $\frac{34}{100} + \frac{75}{100} = \frac{109}{100}$
 - Diego's centavo stack is $\frac{2}{100}$ cm taller than Lin's centavo stack, but his peso stack is $\frac{5}{100}$ shorter than Lin's, so Diego's combined stack must be shorter than Lin's.
4. More than 2 centimeters tall. Sample reasoning:
- Each person's stack is more than 1 cm (or more than $\frac{100}{100}$ cm), so the combined stack would be more than 2 cm.
 - $\frac{106}{100} + \frac{109}{100} = \frac{215}{100}$, which is greater than $\frac{200}{100}$.
- If no students mentioned using multiplication to find the height of Diego and Lin's centavo stacks, ask if any of the heights could be found using multiplication.

Activity 2

🕒 10 min

More Than Two Fractions

Standards Alignments

Addressing 4.NF.C.5

The purpose of this activity is for students to practice finding sums of three or more fractions in tenths and hundredths and applying properties of operations to facilitate that addition. (Students are not expected to use the terms “commutative property” or “associative property,” but should recognize from the work in earlier grades that numbers can be added in different orders and in different groups.)

This activity can be done in the format of a gallery walk. Ask students to visit at least three of six posters (or as many as time permits). The last three expressions include one or more mixed numbers. In the last expression, the fractional parts add up to a sum greater than 1, which would need to be decomposed into a mixed number and a fraction before being added to the whole number. Consider assigning this as a starting expression for students who could use an extra challenge.

Materials to Gather

Chart paper

Materials to Copy

More Than Two Fractions (groups of 15)

Required Preparation

- Create six posters with an addition expression from the activity on each one.

Student-facing Task Statement

Find the value of at least 3 of the expressions.
Show your reasoning.

1. $\frac{2}{100} + \frac{13}{10} + \frac{1}{10} + \frac{8}{100}$
2. $\frac{50}{10} + \frac{16}{100} + \frac{2}{10}$
3. $\frac{3}{10} + \frac{4}{100} + \frac{7}{10} + \frac{26}{100}$
4. $\frac{4}{100} + 3\frac{2}{10} + 1\frac{5}{10}$
5. $1\frac{1}{10} + 5\frac{2}{100} + \frac{78}{100}$
6. $2\frac{7}{10} + \frac{2}{100} + \frac{8}{10}$

Student Responses

1. $\frac{15}{10}$ or $1\frac{5}{10}$. Sample reasoning:

$$\frac{2}{100} + \frac{1}{10} + \frac{8}{100} + \frac{13}{10}$$

$$= \frac{2}{100} + \frac{8}{100} + \frac{1}{10} + \frac{13}{10}$$

$$= \frac{10}{100} + \frac{14}{10}$$

$$= \frac{1}{10} + \frac{14}{10}$$

$$= \frac{15}{10}$$
2. $\frac{536}{100}$ or $5\frac{36}{100}$. Sample reasoning:

$$\frac{50}{10} + \frac{16}{100} + \frac{2}{10}$$

$$= \frac{50}{10} + \frac{2}{10} + \frac{16}{100}$$

$$= \frac{52}{10} + \frac{16}{100}$$

$$= \frac{520}{100} + \frac{16}{100}$$

$$= \frac{536}{100}$$
3. $\frac{13}{10}$ or $1\frac{3}{10}$. Sample reasoning:

Launch

- Groups of 2

If done as a gallery walk:

- Consider assigning each group a starting poster and giving directions for rotation.
- "You'll find six posters with addition expressions on each one. Visit at least three posters and find the value of the expressions."

Activity

- "Work with your partner on the first two expressions and independently on at least one of them. Show your reasoning."
- 10 minutes: group work or gallery walk

Synthesis

- See lesson synthesis.

$$\begin{aligned} & \frac{3}{10} + \frac{4}{100} + \frac{7}{10} + \frac{26}{100} \\ &= \frac{3}{10} + \frac{7}{10} + \frac{4}{100} + \frac{26}{100} \\ &= \frac{10}{10} + \frac{30}{100} \\ &= \frac{10}{10} + \frac{3}{10} \\ &= \frac{13}{10} \end{aligned}$$

4. $4\frac{74}{100}$. Sample reasoning:

$$\begin{aligned} & \frac{4}{100} + 3\frac{2}{10} + 1\frac{5}{10} \\ &= \frac{4}{100} + 4\frac{7}{10} \\ &= \frac{4}{100} + 4\frac{70}{100} \\ &= 4\frac{74}{100} \end{aligned}$$

5. $6\frac{90}{100}$ or $6\frac{9}{10}$. Sample reasoning:

$$\begin{aligned} & 1\frac{1}{10} + 5\frac{2}{100} + \frac{78}{100} \\ &= 1\frac{1}{10} + 5\frac{80}{100} \\ &= 1\frac{1}{10} + 5\frac{8}{10} \\ &= 6\frac{9}{10} \end{aligned}$$

6. $3\frac{52}{100}$. Sample reasoning:

$$\begin{aligned} & 2\frac{7}{10} + \frac{2}{100} + \frac{8}{10} \\ &= 2\frac{7}{10} + \frac{8}{10} + \frac{2}{100} \\ &= 2\frac{15}{10} + \frac{2}{100} \\ &= 2 + 1\frac{50}{100} + \frac{2}{100} \\ &= 3\frac{52}{100} \end{aligned}$$

Lesson Synthesis

🕒 10 min

Select a group to share their response and reasoning for finding the value of each expression. Focus the discussion on whether there are other possible solution paths and on the last expression.

“Today we used what we know about equivalent fractions and addition of fractions to solve problems.”

Invite students to reflect on how their ability to find sums of fractions have improved and any areas of struggles. Consider asking:

- “In what ways has your ability to add fractions improved? What might still be challenging?”
- “Was there a kind of error you made multiple times? What was the error and why might that be?”

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Student Section Summary

In this section, we learned more ways to add fractions and to solve problems that involve adding, subtracting, and multiplying fractions.

We started by adding tenths and hundredths, using what we know about equivalent fractions. For example, to find the sum of $\frac{4}{10}$ and $\frac{30}{100}$, we can:

- Write $\frac{4}{10}$ as $\frac{40}{100}$, and then find $\frac{40}{100} + \frac{30}{100}$, or
- Write $\frac{30}{100}$ as $\frac{3}{10}$, and then find $\frac{4}{10} + \frac{3}{10}$.

We learned that when adding a few fractions, it may help to rearrange or group them. For instance:

- $\frac{6}{100} + \frac{2}{10} + \frac{74}{100}$ can be rearranged as $\frac{6}{100} + \frac{74}{100} + \frac{2}{10}$.
- Next, the hundredths can be added first, giving $\frac{80}{100} + \frac{2}{10}$.
- Then, we can write an equivalent fraction for $\frac{80}{100}$ and find $\frac{8}{10} + \frac{2}{10}$, or write an equivalent fraction for $\frac{2}{10}$ and find $\frac{80}{100} + \frac{20}{100}$.

----- Complete Cool-Down -----

Response to Student Thinking

Students do not accurately add tenths and hundredths.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 19: Flexible with Fractions (Optional)

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4, 4.NF.C.5

Building Towards 4.NF.B.3.d, 4.NF.B.4

Teacher-facing Learning Goals

- Interpret and solve problems that involve the addition, subtraction, and multiplication of fractions.

Student-facing Learning Goals

- Let's solve all kinds of problems involving fractions.

Lesson Purpose

The purpose of this lesson is for students to interpret and solve problems that involve adding, subtracting, and multiplying fractions.

This optional lesson gives students additional opportunities to integrate and apply the work from this unit to solve novel contextual problems. All three activities prompt students to make sense of and persevere in solving problems that involve adding, subtracting, and multiplying fractions. In the first two activities, students think abstractly and quantitatively to relate their calculations to a situation (MP2). The last activity encourages students to identify structure in expressions with many different operations involving fractions (MP7).

Completing all three activities will take more than 60 minutes. Consider expanding the lesson across 2 days or selecting one or two activities based on students' needs or interests and time constraints.

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR6 Three Reads (Activity 2), Notice and Wonder (Warm-up)

Materials to Gather

- Rulers (inches): Activity 1
- Sticky notes: Activity 1

Materials to Copy

- Find a Match (groups of 24): Activity 3

- Tools for creating a visual display: Activity 3

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 25 min |
| Activity 2 | 20 min |
| Activity 3 | 25 min |
| Lesson Synthesis | 5 min |
| Cool-down | 5 min |

Teacher Reflection Question

What evidence did you see of students thinking flexibly and choosing a method strategically as they worked to solve problems? For students who chose a fixed way of reasoning about fractional amounts, what questions could you ask to prompt them to be more strategic?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

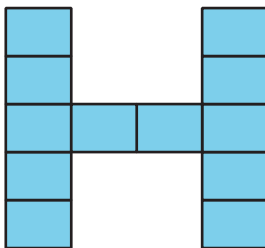
Han's Design

Standards Alignments

Addressing 4.NF.B.3.d, 4.NF.B.4

Student-facing Task Statement

Han is using small sticky notes to make an H shape to decorate a notebook that is 6 inches wide and 9 inches tall. His design is shown here.



The longer side of the sticky note is $\frac{15}{8}$ inches. The shorter side is $\frac{11}{8}$ inches.

Is the notebook tall enough for his design? Show your reasoning.

Student Responses

Yes. Sample response: The H shape is $5 \times \frac{11}{8}$ or $\frac{55}{8}$ inches tall. The notebook is $9 \times \frac{8}{8}$ or $\frac{72}{8}$ inches tall.

Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Sticky Notes

Standards Alignments

Building Towards 4.NF.B.3.d, 4.NF.B.4

This warm-up prompts students to make sense of a problem before solving it, by familiarizing themselves with a context and the mathematics that might be involved. Students observe images that show three ways of making a T shape using sticky notes, a context they will see in the first activity.

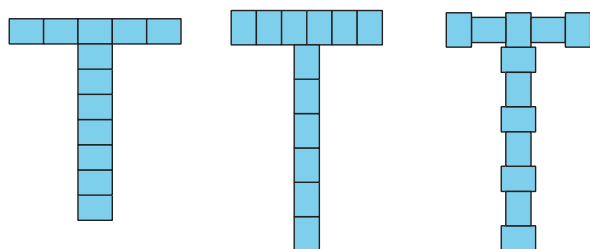
This prompt gives students opportunities to look for structure (MP7)—specifically, the number and orientation of sticky notes of which each T shape is composed—and make use of it to solve problems later.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

**Student Responses**

Students may notice:

- There are three T shapes made of little rectangles (or sticky notes).
- Some rectangles are oriented vertically and some horizontally.
- There are 12 rectangles in each T.

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “All three Ts are made of the same number of sticky notes. Do the Ts have the same width and height?” (No)
- “Why might that be?” (The sticky notes have a longer side and a shorter side, and are not all

- In the first T, all the rectangles are horizontal. In the second T, all of them are vertical.

oriented the same way.)

Students may wonder:

- Are the rectangles sticky notes?
- What size are the sticky notes? Are they all the same size?
- Why are the rectangles (or sticky notes) in the last T oriented in different ways?

Activity 1

🕒 25 min

Sticky-note Designs

Standards Alignments

Addressing 4.NF.B.3.d, 4.NF.B.4

This optional activity prompts students to analyze a design problem that involves fractional measurements. Students determine which of the three designs they saw in the warm-up would fit on a folder that is 9 inches wide and 12 inches tall. To do so, they find the heights and widths of each design using addition, subtraction, multiplication, or a combination of operations.

🕒 Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Invite students to discuss the skills and concepts they have learned during this unit. Record their answers with examples or pictures on a visual display. Encourage students to reference the display as they approach their tasks today.

Supports accessibility for: Conceptual Processing, Organization, Memory

Materials to Gather

Rulers (inches), Sticky notes

Required Preparation

- Each group needs 12 small sticky notes measuring $1\frac{7}{8}$ by $1\frac{3}{8}$ inches.

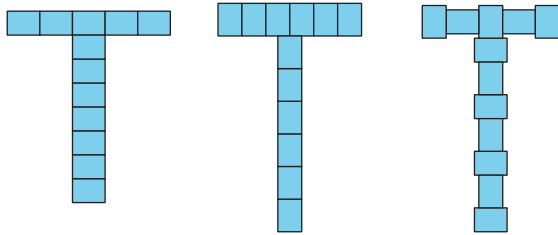
Student-facing Task Statement



Tyler is using small sticky notes to make a T shape to decorate a folder.

The longer side of the sticky note is $\frac{15}{8}$ inches. The shorter side is $\frac{11}{8}$ inches. The folder is 9 inches wide and 12 inches tall.

Here are three ways he could arrange the sticky notes.



Is the folder tall enough and wide enough for his designs? If so, which design(s) would fit? Show your reasoning.

Student Responses

None of them would fit. Sample response: Nine is equivalent to $\frac{72}{8}$ and 12 is equivalent to $\frac{96}{8}$.

- A: Too wide
 - Width of the T: $5 \times \frac{15}{8} = \frac{75}{8}$
 - Height of the T: $8 \times \frac{11}{8} = \frac{88}{8}$
- B: Too tall
 - Width of the T: $6 \times \frac{11}{8} = \frac{66}{8}$
 - Height of the T: $7 \times \frac{15}{8} = \frac{105}{8}$
- C: Too tall
 - Width of the T: $(3 \times \frac{11}{8}) + (2 \times \frac{15}{8})$
 - $3 \times \frac{11}{8} = \frac{33}{8}$

Launch

- Groups of 2–4
- Read the task together as a class.
- “What do you think the problem is asking? What questions do you need answered before working on it?”
- 1 minute: group discussion
- Share responses. Clarify any confusion before students begin the task.

Activity

- “Work independently on the task for about 8–10 minutes. Then, discuss your thinking with your group.”
- 10 minutes: independent work time
- 5 minutes: group discussion

Synthesis

- Invite a group to share their response and reasoning on each design. Display their reasoning or record it for all to see.
- Ask others if they agree and if approached it the same way.
- Explain to students that they will now verify their responses. Assign one design for each group to verify.
- Give 12 small sticky notes (measuring $1\frac{7}{8}$ by $1\frac{3}{8}$) to each group. Ask students to use the sticky notes to create the design.
- Next, give each group an inch ruler and ask them to measure if their design is less than 9 inches wide and less than 12 inches tall.

- $2 \times \frac{15}{8} = \frac{30}{8}$
- $\frac{33}{8} + \frac{30}{8} = \frac{63}{8}$
- Height of the T:
 $(4 \times \frac{11}{8}) + (4 \times \frac{15}{8})$
 - $4 \times \frac{11}{8} = \frac{44}{8}$
 - $4 \times \frac{15}{8} = \frac{60}{8}$
 - $\frac{44}{8} + \frac{60}{8} = \frac{104}{8}$

Advancing Student Thinking

Students may perform correct calculations and obtain fractions greater than 1 for the results but are not sure how to compare them to 9 and 12. Consider asking them how many eighths are in 1, 2, 3, and so on, and to extend the pattern to find out how many eighths are equivalent to the width and height of the folder.

Activity 2

🕒 20 min

Hiking Trails

Standards Alignments

Addressing 4.NF.B.3.d, 4.NF.B.4, 4.NF.C.5

This optional activity offers students to interpret and solve problems involving fractional measurements and operations of fractions in the context of distances on a map. First, students examine the measurements on the map and use them to answer questions. Next, they interpret given expressions and consider what the expressions might represent in the situation. Finally, they write a new problem based on the given quantities and information. The work here prompts students to reason quantitatively and abstractly (MP2).

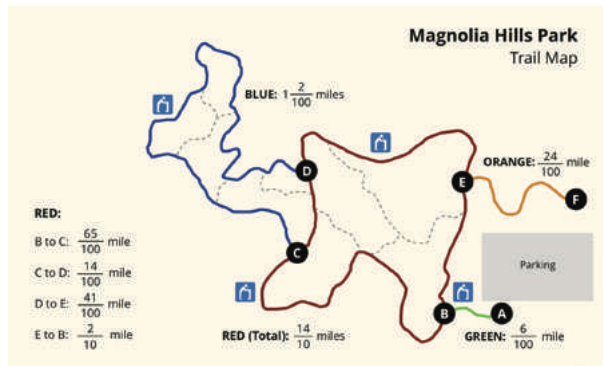
This activity uses *MLR6 Three Reads*. *Advances: reading, listening, representing*

Instructional Routines

MLR6 Three Reads

Student-facing Task Statement

Jada and Noah's class are hiking at a park. Here is a map of the trails. The length of each trail is shown.



- Jada and Noah hike the orange trail from point F to point E, make one full loop on the red trail back to point E, and then hike from E back to F.

How many miles do they hike? Show your reasoning.

- Here are two expressions that represent some hiking situations and can help to answer two questions. What question might each expression help to answer? Write the question and the answer.

a. $\frac{6}{100} + \frac{65}{100} + 1\frac{2}{100} + \frac{41}{100} + \frac{24}{100}$

b. $(2 \times \frac{14}{10}) + (2 \times \frac{6}{100})$

- Use the distances on the map to write a new question and find its answer. Then, trade questions with a partner and answer one another's question.

Launch

- Groups of 2

MLR6 Three Reads

- Display only the problem stem, without revealing the question(s).
- "We are going to read this problem 3 times."
- 1st Read: "Jada and Noah's class are hiking at a park. Here is a map of the trails. The length of each trail is shown."
- "What is this situation about?"
- 1 minute: partner discussion
- Listen for and clarify any questions about the context.
- 2nd Read: "Jada and Noah's class are hiking at a park. Here is a map of the trails. The length of each trail is shown." (Display the trail map.)
- "Name the quantities. What can we count or measure in this situation?"
- 30 seconds: quiet think time
- 2 minutes: partner discussion
- Share and record all quantities.
- Reveal the question(s).
- 3rd Read: Read the entire problem, including question(s) aloud.
- "What are some strategies we can use to solve this problem?"
- 30 seconds: quiet think time
- 1–2 minutes: partner discussion

Student Responses

- $\frac{188}{100}$ or $1\frac{88}{100}$. Sample reasoning:

$$\frac{24}{100} + \frac{14}{10} + \frac{24}{100}$$

$$= \frac{24}{100} + \frac{140}{100} + \frac{24}{100}$$

$$= \frac{188}{100}$$

$$= 1\frac{88}{100}$$
- Sample response:
 - Question: Jada hikes all of the trails except the segments between C and D and between B and E, how many miles does she hike? Answer: $\frac{138}{100}$ or $1\frac{38}{100}$
 - Question: Noah walks from the parking lot (marker A) toward marker B and then hikes the Red Trail twice before returning to the parking lot. How many miles does he hike?
Answer: $2\frac{92}{100}$ or $\frac{292}{100}$
- Answers vary.

Activity

- “Work independently on the task for 10 minutes. Then, discuss your responses and complete the last problem with your partner.”
- 10 minutes: independent work time
- 3–4 minutes: group work time

Synthesis

- Invite 2–3 students to share their responses to the last two problems.

Activity 3

🕒 25 min

Find a Match

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.4, 4.NF.C.5

In this optional activity, students hone the skills they have learned in this unit: multiplying a fraction by a whole number, adding and subtracting fractions with the same denominator (including mixed numbers), and adding tenths and hundredths. Students are each given a fractional expression. They evaluate the expression, find a classmate whose expression is different but has the same value (verifying that this is indeed the case), and write a new expression that also has the same value. (See Student Responses for the matched expressions.)

In addition to evaluating expressions, students who have cards J, K, and L will also need to think

about fractions that are equivalent to the value of their expression in order to find their matches. For instance, a student may reason that the value of card K is $\frac{8}{10}$ or $\frac{80}{100}$, but the match—card 2—shows $\frac{4}{5}$. Consider using these expressions to differentiate for students who could use an extra challenge.

Materials to Gather

Tools for creating a visual display

Materials to Copy

Find a Match (groups of 24)

Required Preparation

- Create one set of Match Cards for each group of 24 students.

Student-facing Task Statement

Your teacher will give you one card with an expression on it.

1. Find the value of the expression.
2. Find a classmate whose card also has the same value. Prove to each other that you're a match.
3. Work with your partner to find at least two features that your expressions share (other than the fact that they have the same value.)
4. Write one more expression that has the same value but uses a different operation.

Student Responses

Cards in the same row are a match.

| | | | |
|----|----------------------------------|-----|--|
| A. | $11 \times \frac{8}{6}$ | 9. | $4 \times \frac{11}{6} \times 2$ |
| B. | $8 \times \frac{7}{10}$ | 12. | $\frac{4}{10} \times 2 \times 7$ |
| C. | $3\frac{5}{12} + \frac{7}{12}$ | 4. | $5\frac{1}{12} - \frac{13}{12}$ |
| D. | $9 \times \frac{21}{5}$ | 7. | $3 \times \frac{7}{5} \times 3 \times 3$ |
| E. | $8 - 1\frac{4}{6}$ | 10. | $5\frac{3}{6} + \frac{5}{6}$ |
| F. | $3\frac{4}{10} + 1\frac{7}{100}$ | 5. | $\frac{44}{10} + \frac{7}{100}$ |
| G. | $4 \times \frac{20}{3}$ | 8. | $5 \times 2 \times \frac{8}{3}$ |

Launch

- Give one card from the Instructional master to each student.
- Tell students that they are to find the value of the expression, and then find a classmate in the class whose expression has the same value.
- “If your expression is labeled with a letter, your match is someone whose expression is labeled with a number. And vice versa.”
- “Once you’ve found your match, complete the rest of the task as directed in the task statement.”

Activity

- 7–8 minutes: independent work time on the first problem and then matching time
- 7–8 minutes: partner work time
- Give each group tools for creating a visual display. Ask them to show that their two expressions are a match, and that their new expressions also have the same value.

Synthesis

- Ask students to display their work around the room.

| | | | |
|----|---|-----|--|
| H. | $10\frac{7}{8} + \frac{9}{8}$ | 11. | $\frac{15}{4} + \frac{25}{4} + 1\frac{3}{4} + \frac{1}{4}$ |
| I. | $9\frac{3}{10} - 8\frac{3}{100}$ | 6. | $1\frac{4}{10} - \frac{13}{100}$ |
| J. | $3\frac{5}{12} - \frac{7}{12}$ | 1. | $1\frac{2}{6} + \frac{9}{6}$ |
| K. | $\frac{61}{100} + \frac{9}{100} + \frac{1}{10}$ | 2. | $2 \times \frac{1}{5} \times 2$ |
| L. | $5 - \frac{11}{3}$ | 3. | $3\frac{4}{6} - \frac{14}{6}$ |

- “Take a few minutes to walk around and look at the work of at least 3 other groups.”
- “As you study others’ work, pay attention to how the work is like and unlike yours.”
- 6–7 minutes: gallery walk
- “What is the same about the calculations that you saw? What is different?”
- 1 minute: quiet think time
- Discuss responses.

Advancing Student Thinking

Students may not find a match for their expression because they are looking only for expressions with the same denominator. Remind them that fractions with different denominators can be equivalent.

Lesson Synthesis

 5 min

“In the past few lessons, we solved a variety of problems that involve fractions and operations of fractions. We saw problems about situations and those that are not about situations.”

“What were some helpful ways to get started when solving problems with fractions?” (Make sense of the problem and what it is asking. Read any word descriptions carefully and more than one time. Make sense of the quantities.)

“What were some helpful ways to prevent making common errors?” (Check the numbers, including numerators and denominators, carefully. Think about what the numbers mean in the situation.)

“How did you know if your answers make sense?” (Check to see if the result makes sense in the situation. Discuss with a partner. Work backwards from the solution toward the problem.)

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students determine the notebook is not tall enough for Han's design.

Next Day Support

- Before the warm-up, review strategies and solutions for the cool-down.

Lesson 20: Sticky Notes (Optional)

Standards Alignments

Addressing 4.NF.B.3.d, 4.NF.B.4

Teacher-facing Learning Goals

- Use addition, subtraction, and multiplication of fractions to model and solve a design problem.

Student-facing Learning Goals

- Let's make a design using sticky notes.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of multiplication of a whole number by a fraction to create sticky-note letter designs.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In previous lessons, students used diagrams, expressions, and equations to represent multiplication of a fraction by a whole number.

In this lesson, students apply their knowledge of fraction by whole number multiplication to create sticky note designs. They create a design given a set of constraints. Students describe their design to others before gaining access to the supplies to make their design.

When students make decisions and choices, analyze real-world situations with mathematical ideas, translate a mathematical answer back into the context of a (real-world) situation, and adhere to constraints, they model with mathematics (MP4).

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Blank paper: Activity 1

- Sticky notes: Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 30 min |
| Lesson Synthesis | 10 min |

Teacher Reflection Question

How comfortable were the students in making choices? Were your students able to explain their thinking and convince others that their design fit the given constraints?

Begin Lesson

Warm-up

🕒 10 min

Which One Doesn't Belong: Sticky Notes

This warm-up prompts students to compare four images of letters made with sticky notes and attend to the ways the sticky-note units are composed to make the letters. To identify reasons that one or more of them don't belong, students need to pay attention to the length and width of the overall designs as well as the orientation of the sticky notes.

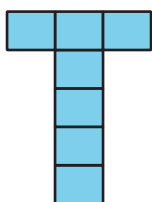
Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

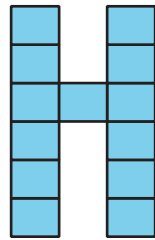
A



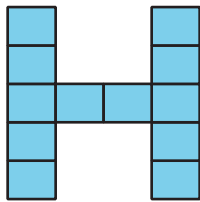
B

Launch

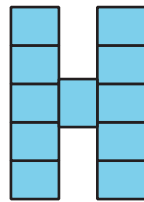
- Groups of 2
- Display image.
- "Each rectangle represents a sticky note."
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time



C



D



Student Responses

Sample responses:

- A is the only one that doesn't show the letter H and doesn't use more than 10 sticky notes.
- B is the only one that is not 5 sticky-notes tall (or uses more than 5 sticky notes for the height).
- C is the only one that is not 3 sticky-notes wide (or uses more than 3 sticky notes for the width).
- D is the only one that doesn't have all sticky notes oriented horizontally (or has a sticky note oriented vertically).

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "What makes the three 'H's in options B, C, and D look different?"
- "Can you think of another way to make a letter H that is different than these three? What might you change?" (Sample responses: the number of sticky notes to use for the vertical segments or for the horizontal segment, the orientation of the sticky notes)
- Consider asking: "Let's find at least one reason why each one doesn't belong."

Activity 1

🕒 15 min

Estimation Exploration: Sticky Notes

Standards Alignments

Addressing 4.NF.B.4

The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it. In this activity, students estimate how many sticky notes are needed to make a row or column along a piece of paper. There are 4 possible ways to interpret the question: the two orientations of the paper and two orientations of the sticky notes. Students use their understanding of multiplication of a fraction by a whole number to make and represent their estimate (MP2).

Access for English Learners

MLR8 Discussion Supports. Encourage students to begin partner discussions by reading aloud their expression representing their estimate. If time allows, invite students to revise or add to their responses based on the conversation that follows.

Advances: Conversing, Speaking

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for making an original design with sticky notes before they begin. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Materials to Gather

Blank paper, Sticky notes

Required Preparation

- Gather rectangular sticky notes with fractional lengths. If this is not possible then cut rectangles from card stock with fractional lengths.

Student-facing Task Statement

1. How many sticky notes will fit across the top or the side of the page?

Record an estimate that is:

| too low | about right | too high |
|---------|-------------|----------|
| | | |

2. What information do you need to help you make a better estimate?

Launch

- Groups of 2
- Distribute one sticky note and paper to each student.
- “How many sticky notes will fit across the top or the side of the page? What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

3. With the new information you have now, make a better estimate. Show or explain your reasoning.
4. Write an expression that represents your estimate that shows how many sticky notes fit across or on the side of the paper.

Student Responses

1. Answers vary based on the size and orientation of sticky notes.
2. Measurements for the sides of the paper and sticky note. The orientation of the sticky note.
3. Answers vary based on the size and orientation of sticky notes. For sticky notes with side lengths $\frac{11}{8}$ inches and $\frac{15}{8}$ inches:
 - Either 8 or 5 sticky notes are needed to cover the 11 inch side, depending on the orientation of the sticky notes. If using the shorter side of sticky notes ($\frac{11}{8}$ inches), then 11 are needed, because $8 \times \frac{11}{8} = \frac{88}{8}$ and $\frac{88}{8} = 11$. If using the longer side ($\frac{15}{8}$ inches), then 5 are needed, but there will be a gap because $5 \times \frac{15}{8} = \frac{75}{8}$ or $9\frac{3}{8}$ inches, which is less than 11 inches. Six sticky notes would be too many, because $6 \times \frac{15}{8} = \frac{90}{8} = 11\frac{2}{8}$, which is greater than 11 inches.
 - Either 6 or 4 sticky notes to cover most of the $8\frac{1}{2}$ inch side. If using the shorter side, 6 are needed: $6 \times \frac{11}{8} = \frac{66}{8}$ and $\frac{66}{8} = 8\frac{2}{8} = 8\frac{1}{4}$, which is less than $8\frac{1}{2}$ inches. If using the longer side, 4 are needed: $4 \times \frac{15}{8} = \frac{60}{8}$, which is less than $8\frac{1}{2}$ inches. Five sticky notes would be too many,

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.
- “What information do you need to help you make a better estimate?” (measurement of the paper, measurement of the sticky note, orientation of the note and/or paper)
- Provide measurements. If students mention orientation, have them consider one based on their choice.
- 5 minutes: partner work time
- Monitor that student that consider the different orientations of the sticky note.

Synthesis

- Invite previously selected students to share.
- If it doesn't come up, highlight the different orientations of the sticky note.
- Consider asking:
 - “What are some other ways you could have arranged the sticky notes?”
- Invite a few students to share their expressions.
- “How does this expression connect to the estimate?” (The whole number represents the number of sticky notes.)
- “How can we use the expression to show if the estimate is too high, low, or about right?” (We can compare the product of the expressions to the side lengths of the paper.)

because $5 \times \frac{15}{8} = \frac{75}{8}$ which is greater than $8\frac{1}{2}$ inches.

4. $5 \times \frac{11}{8}$

Activity 2

🕒 30 min

Design Your Initial

Standards Alignments

Addressing 4.NF.B.3.d, 4.NF.B.4

In this activity, students use their understanding of multiplication of fractions to make an original design with sticky notes. They may choose:

- the letter to make
- the orientation of the letter
- the orientation of the page
- the arrangement of the sticky notes next to each other

Before making their design, students determine if their design will fit on the given paper. While not a part of the task, students can be asked to estimate how many sticky notes they need before they make a calculation.

Alternatively, instead of asking students to choose their own letter, the class can choose a joint project, such as making the name of the school or a club. In that case, groups should each get one letter of the project to design and work together.

Student-facing Task Statement

Design your initial with sticky notes.

1. Plan your design and determine the number of sticky notes that you need.
2. Write at least two equations that show your design will fit on a piece of paper.

Launch

- Groups of 2
- “In this activity you will make a plan to design your initial with sticky notes. Before you get the supplies, figure out if your design will fit on the paper and how many sticky notes you need.”

3. Take turns sharing your design with your partner.
4. Get the supplies and make your design.

Student Responses

Answers vary based on letter design. Sample response:

1. For the letter B, I need 17 notes if all sticky notes are horizontally oriented and the paper is vertically oriented.
2. Height of letter B is $7 \times \frac{11}{8} = \frac{77}{8}$, which is less than 11 inches. The width of the letter is $4 \times \frac{15}{8} = \frac{60}{8}$, which is less than $8\frac{1}{2}$ inches.

Activity

- 10 minutes: independent work time
- 5 minutes: partner share
- As students work, monitor for designs that use different orientations and arrangements.

Synthesis

- "If we arranged all your designs around the classroom, how much space would we need?"

Lesson Synthesis

 10 min

"Today we made sticky-note designs."

"What was the most challenging part of your experience today? What would you do differently to make it less challenging?"

"What was the easiest part of your experience today? How would you make it more challenging?"

Suggested Centers

- Compare (1–5), Stage 6: Add and Subtract Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

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Family Support
Materials

Family Support Materials

Extending Operations to Fractions

In this unit, students think about how fractions can be composed (put together) and decomposed (taken apart). They also learn about fraction operations: multiplying fractions and whole numbers, adding and subtracting fractions with the same denominator, and adding tenths and hundredths.

Section A: Equal Groups of Fractions

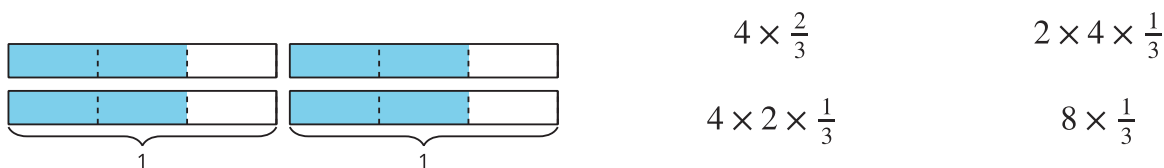
Previously, students thought about multiplication as equal groups of whole numbers of objects, such as 5 bags with 2 oranges in each bag. In this section, they think about equal groups of fractional pieces, such as 5 plates with $\frac{1}{2}$ orange on each plate. They see that the amount can be represented by $5 \times \frac{1}{2}$, which is $\frac{5}{2}$.



Students then make sense of diagrams and equations that represent the multiplication of a whole number and a fraction, such as $4 \times \frac{2}{3} = \frac{8}{3}$.

They learn that the numerator in the resulting fraction is the product of the whole number (the 4) and the numerator of the fractional factor (the 2 in $\frac{2}{3}$), and the denominator is the same as in the fractional factor (the 3 in $\frac{2}{3}$).

Diagrams can help students see that some fractions can be represented by more than one multiplication expression. For example, the diagram shows that the following expressions all have the value of $\frac{8}{3}$.

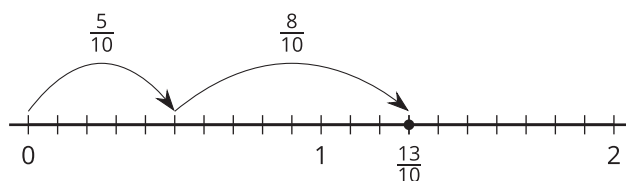


Section B: Addition and Subtraction of Fractions

In this section, students learn to add and subtract fractions by decomposing them into sums of smaller fractions, writing equivalent fractions, and using number lines.

Students first think about a fraction as a sum of other smaller fractions. They represent different ways to decompose a fraction by drawing “jumps” on number lines and writing different equations. Later, they use number lines to represent subtraction of fractions.

$$\frac{13}{10} = \frac{5}{10} + \frac{8}{10}$$



Working with number lines helps students see that a fraction greater than 1 can be decomposed into a whole number and a fraction, and then written as a mixed number. For example, to find the value of $3 - \frac{2}{5}$, it helps to first decompose the 3 into $2 + \frac{5}{5}$, and then subtract $\frac{2}{5}$ from the $\frac{5}{5}$ to get $2\frac{3}{5}$.

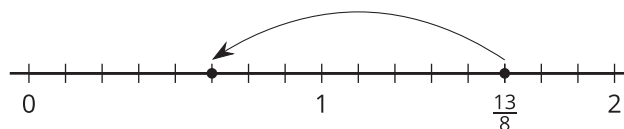
Section C: Adding Tenths and Hundredths

In this section, students learn to add tenths and hundredths. Previously, students learned that $\frac{1}{10} = \frac{10}{100}$. They use this reasoning to find equivalent fractions that can help them add tenths and hundredths.

Try it at home!

Near the end of the unit, ask your student to solve the following problems:

What equation is represented by the jump on the number line?



Find the value of $\frac{8}{10} + \frac{29}{100}$.

Questions that may be helpful as they work:

- How did you know those fractions were needed for the equation?
- How did you find your answer?
- How could you solve your problem in a different way?

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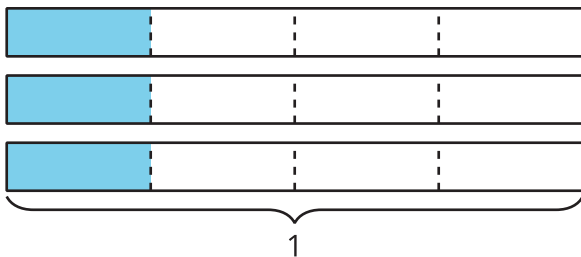
Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment

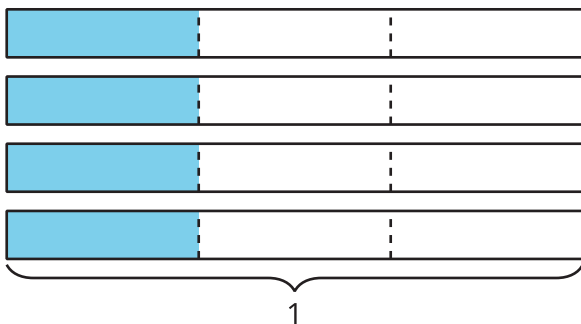
Extending Operations to Fractions: Section A Checkpoint

1. Select all diagrams that show $4 \times \frac{1}{3}$.

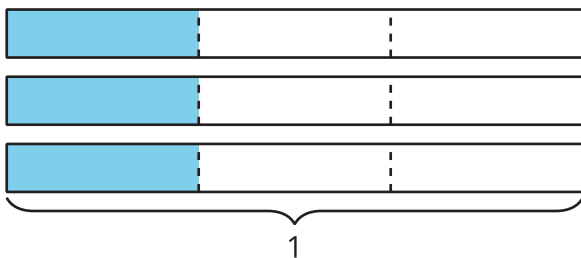
A.



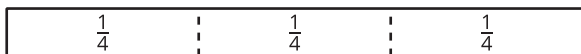
B.



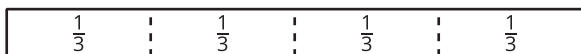
C.



D.



E.



2. Select **all** the expressions that are equivalent to $\frac{8}{5}$.

A. $5 \times \frac{1}{8}$

B. $8 \times \frac{1}{5}$

C. $2 \times \frac{4}{5}$

D. $4 \times \frac{2}{5}$

E. $2 \times \frac{6}{5}$

3. a. Draw a diagram showing $4 \times \frac{2}{3}$.

b. Use the diagram to calculate $4 \times \frac{2}{3}$.

Extending Operations to Fractions: Section B Checkpoint

1. Select all expressions that are equivalent to $\frac{5}{8}$.

A. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

B. $\frac{3}{8} + \frac{2}{8} + \frac{2}{8}$

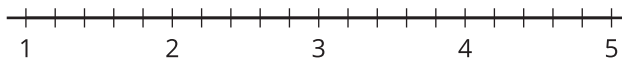
C. $\frac{2}{8} + \frac{2}{8} + \frac{1}{8}$

D. $\frac{4}{8} + \frac{1}{8}$

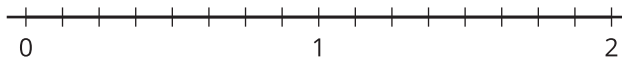
E. $\frac{3}{5} + \frac{2}{3}$

2. Find the value of each expression. Explain or show your reasoning. Use the number line if it is helpful.

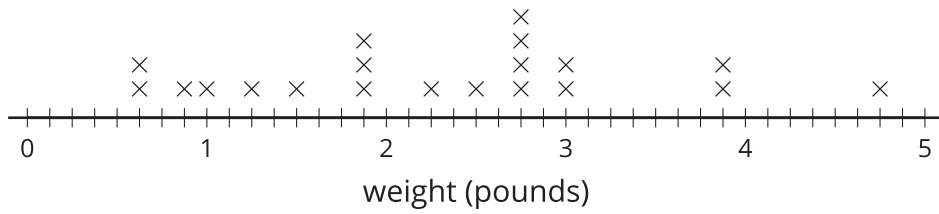
a. $3\frac{2}{5} + \frac{4}{5}$



b. $2 - \frac{1}{8}$



3. The line plot shows the weights of some puppies at a pet store.



a. What is the difference between the weights of the heaviest puppy and the lightest puppy? Explain or show your reasoning.

b. How much did the 5 heaviest puppies weigh all together? Explain or show your reasoning.

Extending Operations to Fractions: Section C Checkpoint

1. Select all expressions that are equivalent to $\frac{53}{100}$.

A. $\frac{3}{10} + \frac{5}{100}$

B. $\frac{50}{100} + \frac{3}{10}$

C. $\frac{5}{10} + \frac{3}{100}$

D. $\frac{1}{10} + \frac{4}{10} + \frac{3}{100}$

E. $\frac{31}{100} + \frac{12}{100} + \frac{1}{10}$

2. Find the value of each expression. Explain or show your reasoning.

a. $\frac{19}{100} + \frac{26}{100} + \frac{1}{100}$

b. $\frac{4}{10} + \frac{3}{10} + \frac{18}{100}$

3. If we combine each person's times for the two races, who finished in less time? Explain or show your reasoning.

| | Lin | Tyler |
|-------------|-------------------|-------------------|
| first race | $6\frac{5}{10}$ | $6\frac{72}{100}$ |
| second race | $6\frac{41}{100}$ | $6\frac{26}{100}$ |

Extending Operations to Fractions: End-of-Unit Assessment

1. Select all the expressions equivalent to $\frac{5}{4}$.

A. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

B. $\frac{2}{4} + \frac{1}{4} + \frac{2}{4}$

C. $\frac{3}{4} + \frac{1}{4} + \frac{2}{4}$

D. $1 + \frac{1}{4}$

E. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

2. Select all expressions with a value larger than 1.

A. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

B. $\frac{2}{4} + \frac{1}{4} + \frac{2}{4}$

C. $2\frac{3}{12} - 1\frac{5}{12}$

D. $3 \times \frac{2}{5}$

E. $9 \times \frac{1}{10}$

3. Select all the expressions that are equivalent to $\frac{8}{12}$.

A. $8 \times \frac{1}{12}$

B. $2 \times \frac{4}{12}$

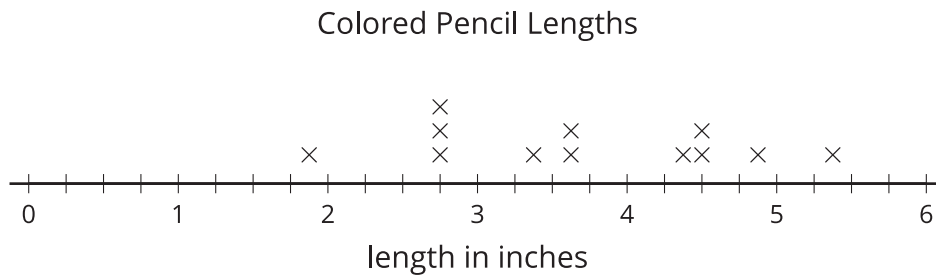
C. $4 \times \frac{4}{12}$

D. $4 \times 2 \times \frac{1}{12}$

E. $12 \times \frac{1}{8}$

4. Jada needs 2 pounds of walnuts for a trail mix. She has 3 packages of walnuts that each weigh $\frac{3}{4}$ pound. Does Jada have enough walnuts to make the trail mix? Explain or show your reasoning.

5. The line plot shows the lengths of some colored pencils.



a. What is the difference between the longest pencil and the shortest pencil shown in this line plot? Show your reasoning.

b. How many pencils measured $4\frac{1}{2}$ inches or more?

c. Two more colored pencils measure $2\frac{1}{4}$ inches and $5\frac{1}{8}$ inches. Plot these measurements on the line plot.

6. Find the value of each expression.

a. $\frac{5}{6} + \frac{2}{6} + \frac{3}{6}$

b. $3 - \frac{7}{8}$

c. $4\frac{3}{5} + 3\frac{4}{5}$

d. $8 - \frac{8}{10}$

e. $5 \times \frac{3}{8}$

7. Find the value of each expression.

a. $\frac{7}{100} + \frac{8}{100} + \frac{1}{10}$

b. $\frac{3}{10} + \frac{17}{100} + \frac{2}{10}$

c. $\frac{14}{100} + \frac{5}{10} + \frac{26}{100}$

8. Noah's favorite song is $5\frac{1}{6}$ minutes long. Jada's favorite song is $2\frac{5}{6}$ minutes long.

a. How many minutes does it take to listen to Noah's favorite song and Jada's favorite song? Explain or show your reasoning.

b. How many minutes longer is Noah's favorite song than Jada's favorite song? Explain or show your reasoning.

c. Jada says she can listen to her song two times in the same amount of time it takes Noah to listen to his song once. Do you agree with Jada? Explain or show your reasoning.

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Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment

Assessment Answer Keys

Assessment: Section A Checkpoint

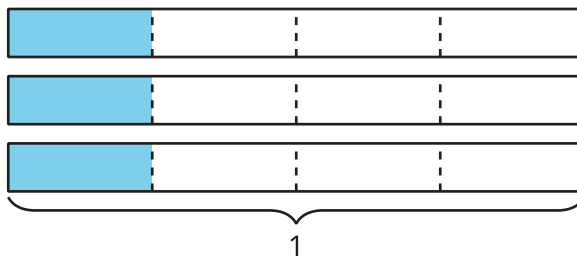
Problem 1

Goals Assessed

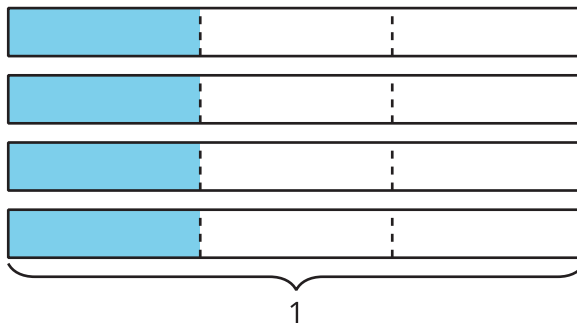
- Recognize that $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.
- Represent and explain that a fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$, namely $a \times \frac{1}{b}$.

Select **all** diagrams that show $4 \times \frac{1}{3}$.

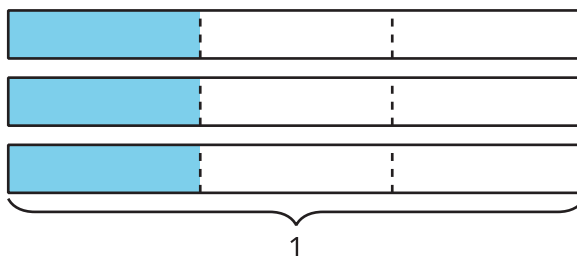
A.



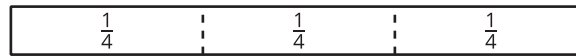
B.



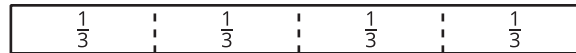
C.



D.



E.



Solution

["B", "E"]

Problem 2

Goals Assessed

- Recognize that $n \times \frac{a}{b} = \frac{(n \times a)}{b}$.

Select **all** the expressions that are equivalent to $\frac{8}{5}$.

- A. $5 \times \frac{1}{8}$
- B. $8 \times \frac{1}{5}$
- C. $2 \times \frac{4}{5}$
- D. $4 \times \frac{2}{5}$
- E. $2 \times \frac{6}{5}$

Solution

["B", "C", "D"]

Problem 3

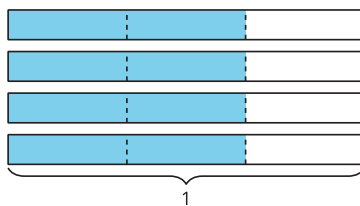
Goals Assessed

- Represent and solve problems involving multiplication of a fraction by a whole number.

- Draw a diagram showing $4 \times \frac{2}{3}$.
- Use the diagram to calculate $4 \times \frac{2}{3}$.

Solution

- Sample response:



- $\frac{8}{3}$, as there are 4×2 or 8 shaded parts and each one is $\frac{1}{3}$ of a full rectangle.

Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Use various strategies to add and subtract fractions and mixed numbers with like denominators.

Select **all** expressions that are equivalent to $\frac{5}{8}$.

A. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

B. $\frac{3}{8} + \frac{2}{8} + \frac{2}{8}$

C. $\frac{2}{8} + \frac{2}{8} + \frac{1}{8}$

D. $\frac{4}{8} + \frac{1}{8}$

E. $\frac{3}{5} + \frac{2}{3}$

Solution

["A", "C", "D"]

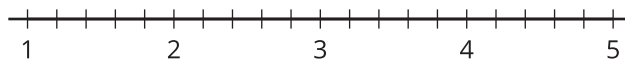
Problem 2

Goals Assessed

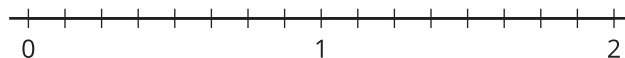
- Use various strategies to add and subtract fractions and mixed numbers with like denominators.

Find the value of each expression. Explain or show your reasoning. Use the number line if it is helpful.

a. $3\frac{2}{5} + \frac{4}{5}$



b. $2 - \frac{1}{8}$



Solution

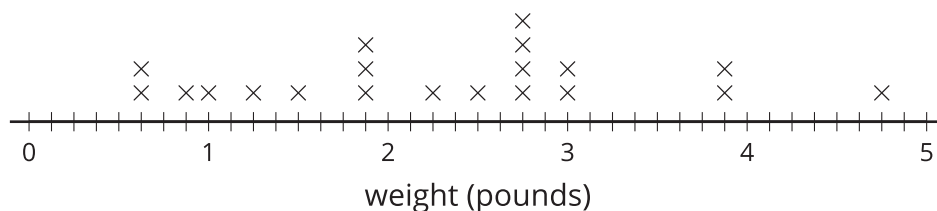
- a. $\frac{21}{5}$. Sample reasoning: Each whole is $\frac{5}{5}$ so there are $\frac{15}{5}$ in 3 and $\frac{17}{5}$ in $3\frac{2}{5}$. Then $\frac{4}{5}$ more makes $\frac{21}{5}$.
- b. $\frac{15}{8}$. Sample reasoning: Each whole is $\frac{8}{8}$ so 2 is equivalent to $\frac{16}{8}$ and subtracting $\frac{1}{8}$ gives $\frac{15}{8}$.

Problem 3

Goals Assessed

- Create and analyze line plots that display measurement data in fractions of a unit ($\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$).
- Represent and solve problems that involve the addition and subtraction of fractions and mixed numbers.

The line plot shows the weights of some puppies at a pet store.



- a. What is the difference between the weights of the heaviest puppy and the lightest puppy? Explain or show your reasoning.
- b. How much did the 5 heaviest puppies weigh all together? Explain or show your reasoning.

Solution

- a. $4\frac{1}{8}$ pounds. Sample response: $4\frac{6}{8} - \frac{5}{8} = 4\frac{1}{8}$
- b. $18\frac{4}{8}$ pounds. Sample response: I first added the whole number of pounds which was $4 + 3 + 3 + 3 + 3$ or 16. Then I added the eighths which was $\frac{6}{8} + \frac{7}{8} + \frac{7}{8} = \frac{20}{8}$ which is the same as $2\frac{4}{8}$.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Reason about equivalence to add tenths and hundredths.

Select **all** expressions that are equivalent to $\frac{53}{100}$.

A. $\frac{3}{10} + \frac{5}{100}$

B. $\frac{50}{100} + \frac{3}{10}$

C. $\frac{5}{10} + \frac{3}{100}$

D. $\frac{1}{10} + \frac{4}{10} + \frac{3}{100}$

E. $\frac{31}{100} + \frac{12}{100} + \frac{1}{10}$

Solution

["C", "D", "E"]

Problem 2

Goals Assessed

- Reason about equivalence to add tenths and hundredths.

Find the value of each expression. Explain or show your reasoning.

a. $\frac{19}{100} + \frac{26}{100} + \frac{1}{100}$

b. $\frac{4}{10} + \frac{3}{10} + \frac{18}{100}$

Solution

- a. $\frac{46}{100}$. Sample response: I first added $\frac{19}{100}$ and $\frac{1}{100}$ to get $\frac{20}{100}$ and then added $\frac{26}{100}$.
- b. $\frac{88}{100}$. Sample response: I first added $\frac{4}{10}$ and $\frac{3}{10}$ to get $\frac{7}{10}$ and then made this $\frac{70}{100}$ to put together with $\frac{18}{100}$.

Problem 3

Goals Assessed

- Reason about equivalence to solve problems involving addition and subtraction of fractions and mixed numbers.

If we combine each person's times for the two races, who finished in less time? Explain or show your reasoning.

| | Lin | Tyler |
|-------------|-------------------|-------------------|
| first race | $6\frac{5}{10}$ | $6\frac{72}{100}$ |
| second race | $6\frac{41}{100}$ | $6\frac{26}{100}$ |

Solution

Lin finished in less time. Her two times add to $12\frac{91}{100}$ and Tyler's times add to $12\frac{98}{100}$.

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments

Addressing 4.NF.B.3.a, 4.NF.B.3.b

Narrative

Students identify expressions that are equivalent to $\frac{5}{4}$. Students who fail to select A do not understand how to decompose a fraction into unit fractions. Failure to select B or D means more work is needed with adding non-unit fractions or whole numbers and fractions.

Students may select C if they do not pay close attention to the numerators or add them incorrectly. Students who select E are likely confused about the meaning of the numerator and denominator in a fraction.

Select **all** the expressions equivalent to $\frac{5}{4}$.

A. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

B. $\frac{2}{4} + \frac{1}{4} + \frac{2}{4}$

C. $\frac{3}{4} + \frac{1}{4} + \frac{2}{4}$

D. $1 + \frac{1}{4}$

E. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

Solution

["A", "B", "D"]

Problem 2

Standards Alignments

Addressing 4.NF.B.3.c, 4.NF.B.4.a, 4.NF.B.4.b

Narrative

Students compare the value of expressions with 1. The expressions involve sums of fractions with the same denominator or products of a whole number and a fraction. Students may select A if they identify the expression as equivalent to 1 and do not read the question carefully. Students may not select B if they do not pay close attention to the numerators. Students may select C if they convert incorrectly to fractions or only subtract 1 or $\frac{5}{12}$. Students who fail to select D or select E need further review of multiplication of a whole number and a fraction.

Select **all** expressions with a value larger than 1.

A. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

B. $\frac{2}{4} + \frac{1}{4} + \frac{2}{4}$

C. $2\frac{3}{12} - 1\frac{5}{12}$

D. $3 \times \frac{2}{5}$

E. $9 \times \frac{1}{10}$

Solution

["B", "D"]

Problem 3

Standards Alignments

Addressing 4.NF.B.4.a, 4.NF.B.4.b

Narrative

Students identify products of whole numbers and fractions that are equivalent to a given fraction. They need to understand that the numerator gives the number of parts and multiplying by a whole number increases the number of parts by that factor while not changing the size of the parts.

Students may choose response C if they add 4 and 4 to get 8 instead of multiplying them. They may select response E if they see the 12 and 8 but do not pay close attention to the meaning of the numerator and denominator of $\frac{8}{12}$.

Select **all** the expressions that are equivalent to $\frac{8}{12}$.

- A. $8 \times \frac{1}{12}$
- B. $2 \times \frac{4}{12}$
- C. $4 \times \frac{4}{12}$
- D. $4 \times 2 \times \frac{1}{12}$
- E. $12 \times \frac{1}{8}$

Solution

["A", "B", "D"]

Problem 4

Standards Alignments

Addressing 4.MD.A.2, 4.NF.B.4.c

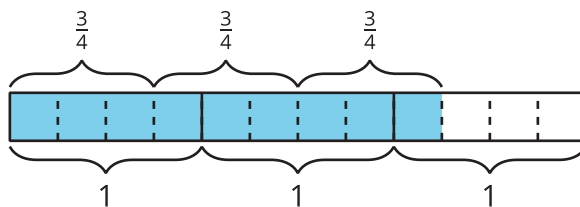
Narrative

Students solve a contextual problem which requires multiplying a whole number and a fraction. Watch for tape diagrams and number line representations. Tape diagrams need to be labeled carefully in order to identify the whole. For the number line representation, it should be clear that the unit on the number line corresponds to 1 pound in the context. Students may also write and find the value of the expression $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ instead of using a drawing.

Jada needs 2 pounds of walnuts for a trail mix. She has 3 packages of walnuts that each weigh $\frac{3}{4}$ pound. Does Jada have enough walnuts to make the trail mix? Explain or show your reasoning.

Solution

Jada has $2\frac{1}{4}$ pounds or $\frac{9}{4}$ pounds of walnuts and that is more than 2 pounds so she has enough walnuts.



Problem 5

Standards Alignments

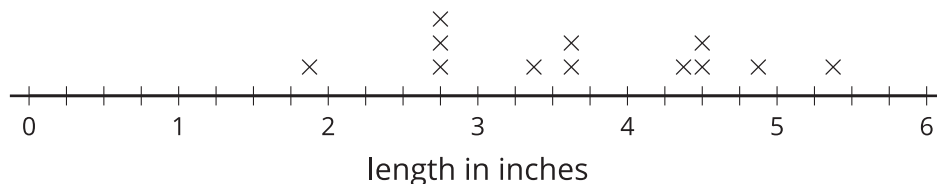
Addressing 4.MD.B.4

Narrative

Students interpret the measurement data on the line plot to answer questions and use the data to subtract fractions. For the first question, students may use the numbers on the line plot to help find the difference or they may reason more abstractly as in the provided solution.

The line plot shows the lengths of some colored pencils.

Colored Pencil Lengths

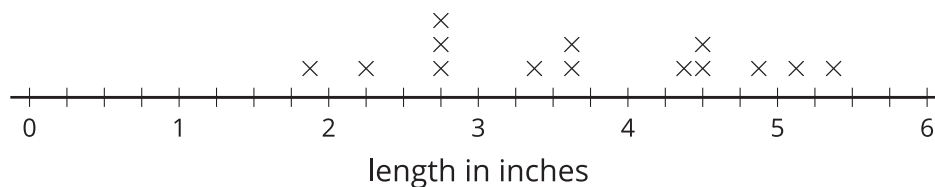


- What is the difference between the longest pencil and the shortest pencil shown in this line plot? Show your reasoning.
- How many pencils measured $4\frac{1}{2}$ inches or more?
- Two more colored pencils measure $2\frac{1}{4}$ inches and $5\frac{1}{8}$ inches. Plot these measurements on the line plot.

Solution

- $5\frac{3}{8} - 1\frac{7}{8} = 3\frac{4}{8}$. First I subtracted 1 and then $\frac{3}{8}$ to get 4. Then, I subtracted $\frac{4}{8}$ more to get $3\frac{4}{8}$.
- 4 pencils

Colored Pencil Lengths



Problem 6

Standards Alignments

Addressing 4.NF.B.3, 4.NF.B.4

Narrative

Students find sums, differences, and products of fractions without context. The numbers (for sums and differences) are presented both as fractions and as mixed numbers. No reasoning is requested and this item and the next assess grade level skills calculating with fractions.

Find the value of each expression.

- $\frac{5}{6} + \frac{2}{6} + \frac{3}{6}$
- $3 - \frac{7}{8}$
- $4\frac{3}{5} + 3\frac{4}{5}$
- $8 - \frac{8}{10}$
- $5 \times \frac{3}{8}$

Solution

- $\frac{10}{6}$ (or equivalent)
- $2\frac{1}{8}$ (or equivalent)
- $8\frac{2}{5}$ (or equivalent)
- $7\frac{2}{10}$ (or equivalent)
- $\frac{15}{8}$ (or equivalent)

Problem 7

Standards Alignments

Addressing 4.NF.C.5

Narrative

Students add fractions with denominators 10 and 100. They may use the commutative and associative properties of addition in order to make the calculations more efficiently.

Find the value of each expression.

- a. $\frac{7}{100} + \frac{8}{100} + \frac{1}{10}$
- b. $\frac{3}{10} + \frac{17}{100} + \frac{2}{10}$
- c. $\frac{14}{100} + \frac{5}{10} + \frac{26}{100}$

Solution

- a. $\frac{25}{100}$ (or equivalent)
- b. $\frac{67}{100}$ (or equivalent)
- c. $\frac{90}{100}$ (or equivalent)

Problem 8

Standards Alignments

Addressing 4.MD.A.2, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4.c

Narrative

Students find sums and differences of fractions with denominator 6 in context. In some cases the answer can be listed in terms of sixths or it can be written in terms of thirds, or for the first problem, a whole number. All of these answers are correct and students should choose the form that makes most sense to them. Some students may know that there are 60 seconds in a minute and that $\frac{1}{6}$ of 60 is 10. It is possible to solve some or all of the problems using minutes and seconds though this reasoning is not within the standards for 4th grade work with fractions.

Noah's favorite song is $5\frac{1}{6}$ minutes long. Jada's favorite song is $2\frac{5}{6}$ minutes long.

- How many minutes does it take to listen to Noah's favorite song and Jada's favorite song? Explain or show your reasoning.
- How many minutes longer is Noah's favorite song than Jada's favorite song? Explain or show your reasoning.
- Jada says she can listen to her song two times in the same amount of time it takes Noah to listen to his song once. Do you agree with Jada? Explain or show your reasoning.

Solution

- 8 or equivalent: Noah's song takes $5\frac{1}{6}$ and Jada's song is another $2\frac{5}{6}$ minutes so that's 7 minutes and another $\frac{6}{6}$ or 1 minute.
- $2\frac{2}{6}$ or equivalent: $5\frac{1}{6}$ is 2 more than $3\frac{1}{6}$ and then that's $\frac{1}{6}$ more than 3 which is $\frac{1}{6}$ more than $2\frac{5}{6}$. Adding that up gives $2\frac{2}{6}$ minutes.
- Sample responses:
 - No, Noah's song is only $2\frac{2}{6}$ minutes longer than Jada's so that is not long enough for Jada to listen to her song again.
 - No, adding $2\frac{5}{6}$ and $2\frac{5}{6}$ gives $2 + 2$ which is 4 and then $\frac{5}{6} + \frac{5}{6}$ more. That's $\frac{10}{6}$ which is $\frac{6}{6} + \frac{4}{6}$ or 1 and $\frac{4}{6}$ more. So Jada's song played twice takes $5\frac{4}{6}$ minutes and that's longer than Noah's song.

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Lesson
Cool Downs

Lesson 1: Equal Groups of Unit Fractions

Cool Down: Sandwiches on Plates

Lin has 9 plates. She puts $\frac{1}{4}$ of a sandwich on each plate.

1. Which expression represents the sandwiches in this situation?

A. 9×4

B. $9 \times \frac{1}{4}$

C. 4×9

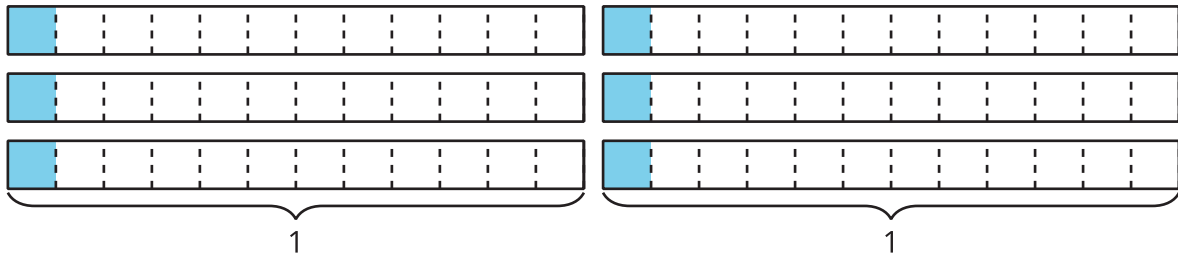
D. $4 \times \frac{1}{9}$

2. How many sandwiches did Lin put on plates? Explain or show your reasoning.

Lesson 2: Representations of Equal Groups of Fractions

Cool Down: Equal Groups of Fractions

Write a multiplication expression to represent the shaded parts of the diagram. Then, find its value. Explain or show your reasoning.



Lesson 3: Patterns in Multiplication

Cool Down: Fraction Multiplication

1. Complete each equation to make it true. Show your thinking using words or diagrams.

a. $5 \times \frac{1}{8} = \underline{\hspace{2cm}}$

b. $\underline{\hspace{2cm}} \times \frac{1}{3} = \frac{7}{3}$

2. Write each fraction as the product of a whole number and unit fraction.

a. $\frac{8}{9} = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

b. $\frac{6}{5} = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

Lesson 4: Equal Groups of Non-Unit Fractions

Cool Down: What's the Value?

Find the value of each expression. Explain or show your reasoning. Use a diagram if it is helpful.

1. $6 \times \frac{2}{5}$

2. $5 \times \frac{3}{10}$

Lesson 6: Problems with Equal Groups of Fractions

Cool Down: The Same or Not the Same?

1. Tyler bought 5 cartons of milk. Each carton contains $\frac{3}{4}$ liter. How many liters of milk did Tyler buy? Explain or show your reasoning.

2. Han bought 3 cartons of chocolate milk. Each carton contains $\frac{5}{8}$ liter. Did Han buy the same amount of milk as Tyler? Explain or show your reasoning.

Lesson 7: Fractions as Sums

Cool Down: Make a Sum of $\frac{7}{4}$

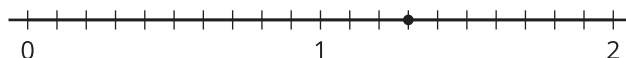
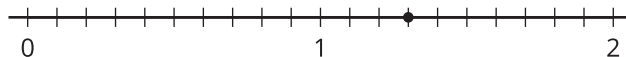
Find three different ways to use fourths to make a sum of $\frac{7}{4}$.

Write an equation for each.

Lesson 8: Addition of Fractions

Cool Down: Lucky Thirteen-tenths

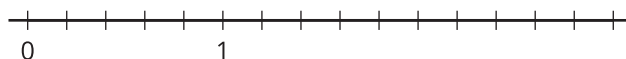
1. On each number line, draw two “jumps” to show how to use tenths to make a sum of $\frac{13}{10}$.



a. Represent each combination of jumps as an equation.

b. Write $\frac{13}{10}$ as a sum of a whole number and a fraction.

2. Find the value of $\frac{8}{5} + \frac{6}{5}$. Use the number line if you find it helpful.

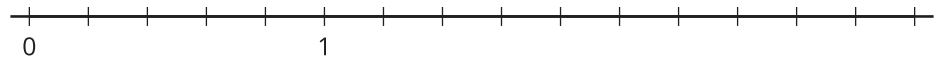


Lesson 9: Differences of Fractions

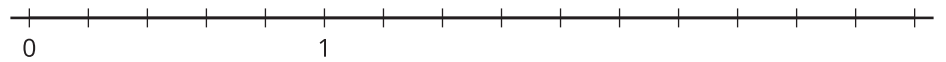
Cool Down: Differences of Fifths

Use a number line to represent each difference and to find its value.

1. $\frac{12}{5} - \frac{4}{5}$



2. $2\frac{1}{5} - \frac{7}{5}$



Lesson 10: The Numbers in Subtraction

Cool Down: Two Differences

Find the value of each difference. Show your reasoning.

1. $2 - \frac{5}{6}$

2. $4 - \frac{11}{6}$

Lesson 11: Subtract Fractions Flexibly

Cool Down: A Shorter Strip, Please

Lin has a strip of paper that is $7\frac{1}{4}$ inches long and needs to be trimmed by $2\frac{3}{4}$ inches. What is the length of the paper strip after it is trimmed? Explain or show your reasoning.

Lesson 12: Sums and Differences of Fractions

Cool Down: How Would You Find the Difference?

Consider the expression $\frac{13}{5} - 1\frac{2}{5}$.

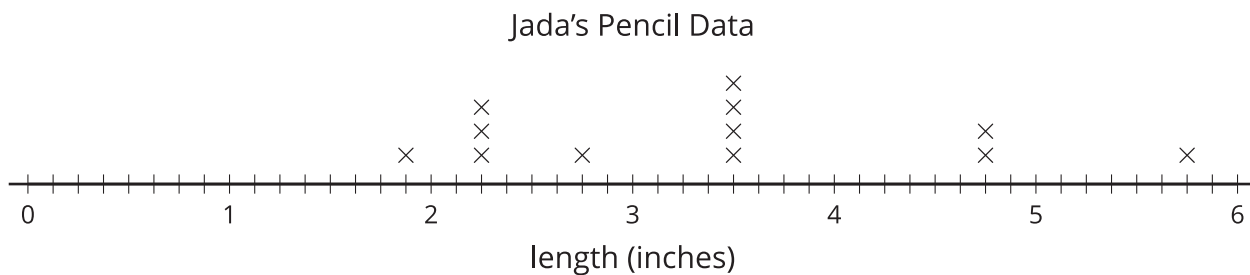
1. What would be your first step for finding the value of the expression?

2. Find the value of the expression. Show your reasoning.

Lesson 13: Fractional Measurements on Line Plots

Cool Down: Jada's Pencil Data

Jada measured the lengths of her pencils and displayed her data on a line plot.

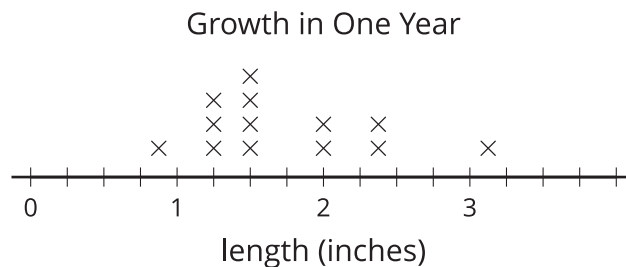


1. The last three pencils in her collection are not yet plotted. Their lengths are: $3\frac{1}{4}$, $4\frac{3}{8}$, and $5\frac{1}{4}$. Plot them on the line plot.
2. What is the difference in the length of the shortest and the longest pencil in her collection? Show your reasoning.

Lesson 14: Problems about Fractional Measurement Data

Cool Down: Fourth-grade Height Data

The students in a fourth-grade class keep track of their height all year long. The line plot shows the number of inches each student in the class has grown this year.



1. How many students grew more than $1\frac{3}{8}$ inches? Explain your reasoning.

2. What is the difference between the greatest amount of growth and the least amount of growth in inches?

Lesson 15: An Assortment of Fractions

Cool Down: Which Stack is Taller?

Which stack of foam blocks is taller:

- Two $\frac{1}{3}$ -foot blocks and one $\frac{1}{6}$ -foot block, or
- One $\frac{1}{2}$ -foot block and two $\frac{1}{6}$ -foot blocks?

Explain or show your reasoning.

Lesson 16: Tenths and Hundredths, Together

Cool Down: Some Sums

Find the value of each sum. Show your reasoning. Use number lines if you find them helpful.

1. $\frac{1}{10} + \frac{50}{100}$

2. $\frac{20}{100} + \frac{4}{10}$

3. $\frac{6}{10} + \frac{3}{100}$

4. $\frac{18}{100} + \frac{7}{10}$

Lesson 17: Sums of Tenths and Hundredths

Cool Down: Missing Fractions

Each equation is missing a fraction in tenths or hundredths. Find the fraction that makes each equation true.

1. $\frac{26}{100} + \frac{8}{10} = \underline{\hspace{2cm}}$

2. $\frac{7}{10} + \underline{\hspace{2cm}} = \frac{92}{100}$

3. $\underline{\hspace{2cm}} + \frac{8}{100} = \frac{128}{100}$

4. $\frac{12}{100} + \frac{12}{10} = \underline{\hspace{2cm}}$

Lesson 18: Lots of Fractions to Add

Cool Down: U.S. Coins

The table shows the thicknesses of U.S. coins.

| coin | thickness (cm) |
|-------------|------------------|
| penny | $\frac{15}{100}$ |
| nickel | $\frac{2}{10}$ |
| dime | $\frac{14}{100}$ |
| quarter | $\frac{18}{100}$ |
| half dollar | $\frac{22}{100}$ |
| dollar | $\frac{2}{10}$ |

Find the combined thickness of:

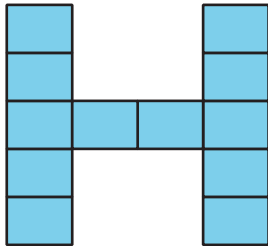
1. a penny, a nickel, a quarter

2. a dollar, a half dollar, a quarter, and a dime

Lesson 19: Flexible with Fractions

Cool Down: Han's Design


Han is using small sticky notes to make an H shape to decorate a notebook that is 6 inches wide and 9 inches tall. His design is shown here.



The longer side of the sticky note is $\frac{15}{8}$ inches. The shorter side is $\frac{11}{8}$ inches.

Is the notebook tall enough for his design? Show your reasoning.

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Instructional
Masters

Instructional Masters for Extending Operations to Fractions

| address | title | students per copy | written on? | requires cutting? | card stock recommended? | color paper recommended? |
|------------------------|---|-------------------|-------------|-------------------|-------------------------|--------------------------|
| Activity Grade4.3.8.3 | Make Two Jumps | 2 | no | yes | no | no |
| Activity Grade4.3.2.1 | Expressions and Diagrams | 2 | no | yes | no | no |
| Activity Grade4.3.18.2 | More Than Two Fractions | 15 | no | yes | no | no |
| Activity Grade4.3.17.3 | Fraction Action: Tenths, Hundredths | 2 | no | yes | no | no |
| Activity Grade4.3.19.3 | Find a Match | 24 | no | yes | no | no |
| Activity Grade4.3.10.2 | Card Sort: Twelfths | 2 | no | yes | no | no |
| Activity Grade4.3.9.3 | Make a Jump, Subtraction Edition | 2 | no | yes | no | no |
| Activity Grade4.3.17.1 | Card Sort: Less Than, Equal to, or Greater Than 1 | 2 | no | yes | no | no |
| Center | Rolling for Fractions Stage 1 Recording Sheet | 1 | yes | no | no | no |
| Center | Fraction Cards Grade 3 | 2 | no | no | yes | no |
| Center | Fraction Cards Grade 4 | 2 | no | no | yes | no |
| Center | Compare Stage 3-8 Directions | 2 | yes | no | no | no |
| Center | Compare Stage 3-8 Directions | 2 | yes | no | no | no |

| | | | | | | |
|--------|---|---|-----|-----|----|----|
| Center | Compare Stage 3-8 Directions | 2 | yes | no | no | no |
| Center | Rolling for Fractions Stage 2 Recording Sheet | 1 | yes | no | no | no |
| Center | Estimate and Measure Stage 3 Recording Sheet | 1 | yes | no | no | no |
| Center | Target Measurement Stage 2 Recording Sheet | 2 | yes | no | no | no |
| Center | Compare Stage 6 Cards | 2 | no | yes | no | no |
| Center | Creating Line Plots Stage 2 Recording Sheet | 1 | yes | no | no | no |
| Center | Estimate and Measure Stage 4 Recording Sheet | 1 | yes | no | no | no |
| Center | Target Measurement Stage 3 Recording Sheet | 2 | yes | no | no | no |
| Center | Creating Line Plots Stage 3 Recording Sheet | 1 | yes | no | no | no |
| Center | Jump the Line Stage 2 Spinner | 2 | no | no | no | no |
| Center | Jump the Line Stage 2 Gameboard | 2 | yes | no | no | no |
| Center | Compare Stage 3 Multiplication Cards | 2 | no | yes | no | no |

Make Two Jumps

Make Two Jumps

$$1 + 3$$

Make Two Jumps

$$2 + 3$$

Make Two Jumps

$$3 + 3$$

Make Two Jumps

$$4 + 3$$

Make Two Jumps

$$5 + 3$$

Make Two Jumps

$$6 + 3$$

Make Two Jumps

$$7 + 3$$

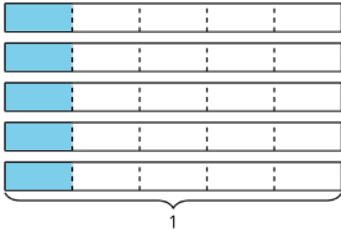
Make Two Jumps

$$8 + 3$$

Expressions and Diagrams

Expressions and Diagrams

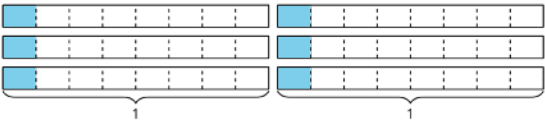
$$3 \times \frac{1}{4}$$



Expressions and Diagrams

Expressions and Diagrams

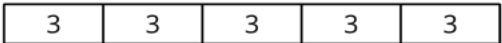
$$5 \times 3$$



Expressions and Diagrams

Expressions and Diagrams

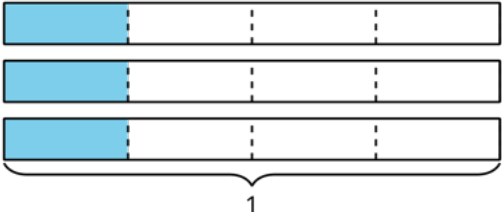
$$4 \times \frac{1}{3}$$



Expressions and Diagrams

Expressions and Diagrams

$$6 \times \frac{1}{8}$$



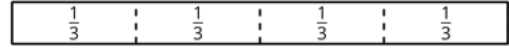
Expressions and Diagrams

Expressions and Diagrams

Expressions and Diagrams

$$5 \times \frac{1}{5}$$

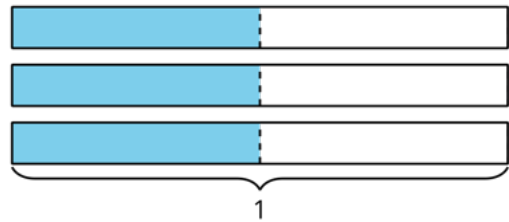
Expressions and Diagrams



Expressions and Diagrams

$$3 \times \frac{1}{2}$$

Expressions and Diagrams



Expressions and Diagrams

$$6 \times \frac{1}{6}$$

Expressions and Diagrams

$$2 \times \frac{1}{12}$$

Expressions and Diagrams

$$8 \times \frac{1}{2}$$

Expressions and Diagrams

$$3 \times 4$$

More Than Two Fractions

A

$$\frac{2}{100} + \frac{1}{10} + \frac{8}{100} + \frac{13}{10}$$

More Than Two Fractions

B

$$\frac{50}{10} + \frac{16}{100} + \frac{2}{10}$$

More Than Two Fractions

C

$$\frac{3}{10} + \frac{4}{100} + \frac{8}{10} + \frac{26}{100}$$

More Than Two Fractions

D

$$\frac{4}{100} + 3\frac{2}{10} + 1\frac{5}{10}$$

More Than Two Fractions

E

$$1\frac{1}{10} + 5\frac{2}{10} + \frac{78}{100}$$

More Than Two Fractions

F

$$2\frac{7}{10} + \frac{2}{100} + \frac{8}{10}$$

Fraction Action: Tenths, Hundredths

Fraction Action: Tenths,
Hundredths

$$\frac{1}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{2}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{3}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{4}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{11}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{10}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{9}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{8}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{12}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{15}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{18}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{20}{10}$$

Fraction Action: Tenths, Hundredths

Fraction Action: Tenths,
Hundredths

$$\frac{35}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{48}{10}$$

Fraction Action: Tenths,
Hundredths

$$\frac{1}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{2}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{5}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{8}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{9}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{10}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{12}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{14}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{16}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{17}{100}$$

Fraction Action: Tenths, Hundredths

Fraction Action: Tenths,
Hundredths

$$\frac{25}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{30}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{36}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{40}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{41}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{45}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{50}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{62}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{68}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{75}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{80}{100}$$

Fraction Action: Tenths,
Hundredths

$$\frac{91}{100}$$

Find a Match

Find a Match

A

$$11 \times \frac{8}{6}$$

Find a Match

B

$$8 \times \frac{7}{10}$$

Find a Match

C

$$3\frac{5}{12} + \frac{7}{12}$$

Find a Match

D

$$9 \times \frac{21}{5}$$

Find a Match

E

$$8 - 1\frac{4}{6}$$

Find a Match

F

$$3\frac{4}{10} + 1\frac{7}{100}$$

Find a Match

G

$$4 \times \frac{20}{3}$$

Find a Match

H

$$10\frac{7}{8} + \frac{9}{8}$$

Find a Match

I

$$9\frac{3}{10} - 8\frac{3}{100}$$

Find a Match

Find a Match

J

$$3\frac{5}{12} - \frac{7}{12}$$

Find a Match

K

$$\frac{61}{100} + \frac{9}{100} + \frac{1}{10}$$

Find a Match

L

$$5 - \frac{11}{3}$$

Find a Match

1

$$1\frac{2}{6} + \frac{9}{6}$$

Find a Match

2

$$2 \times \frac{1}{5} \times 2$$

Find a Match

3

$$3\frac{4}{6} - \frac{14}{6}$$

Find a Match

4

$$5\frac{1}{12} - \frac{13}{12}$$

Find a Match

5

$$\frac{44}{10} + \frac{7}{100}$$

Find a Match

6

$$1\frac{4}{10} - \frac{13}{100}$$

Find a Match

Find a Match

7

$$3 \times \frac{7}{5} \times 3 \times 3$$

Find a Match

8

$$5 \times 2 \times \frac{8}{3}$$

Find a Match

9

$$4 \times \frac{11}{6} \times 2$$

Find a Match

10

$$5\frac{3}{6} + \frac{5}{6}$$

Find a Match

11

$$\frac{15}{4} + \frac{25}{4} + 1\frac{3}{4} + \frac{1}{4}$$

Find a Match

12

$$\frac{4}{10} \times 2 \times 7$$

Card Sort: Twelfths

Card Sort: Twelfths

A

$$1 - \frac{5}{12}$$

Card Sort: Twelfths

B

$$\frac{12}{12} + \frac{12}{12} - \frac{5}{12}$$

Card Sort: Twelfths

C

$$\frac{24}{12} - \frac{5}{12}$$

Card Sort: Twelfths

D

$$\frac{7}{12}$$

Card Sort: Twelfths

E

$$1\frac{7}{12}$$

Card Sort: Twelfths

F

$$2 - \frac{5}{12}$$

Card Sort: Twelfths

G

$$\frac{12}{12} - \frac{5}{12}$$

Card Sort: Twelfths

H

$$\frac{19}{12}$$

Card Sort: Twelfths

I

$$1 + 1 - \frac{5}{12}$$

Card Sort: Twelfths

J

$$1 + \frac{12}{12} - \frac{5}{12}$$

Make a Jump, Subtraction Edition

Make a Jump, Subtraction Edition

$$8 - 5$$

Make a Jump, Subtraction Edition

$$9 - 5$$

Make a Jump, Subtraction Edition

$$10 - 5$$

Make a Jump, Subtraction Edition

$$11 - 5$$

Make a Jump, Subtraction Edition

$$12 - 5$$

Make a Jump, Subtraction Edition

$$13 - 5$$

Make a Jump, Subtraction Edition

$$14 - 5$$

Make a Jump, Subtraction Edition

$$15 - 5$$

Card Sort: Less Than, Equal to, or Greater Than 1

Card Sort: Less Than, Equal to, or Greater Than 1?

A

$$\frac{10}{100} + \frac{8}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

B

$$\frac{80}{100} + \frac{2}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

C

$$\frac{20}{10} + \frac{30}{100}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

D

$$\frac{7}{10} + \frac{8}{100}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

E

$$\frac{22}{100} + \frac{8}{100}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

F

$$\frac{12}{10} + \frac{8}{100}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

G

$$\frac{12}{100} + \frac{12}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

H

$$\frac{73}{100} + \frac{3}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

I

$$\frac{150}{100} + \frac{1}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

J

$$\frac{9}{10} + \frac{11}{100}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

K

$$\frac{10}{100} + \frac{9}{10}$$

Card Sort: Less Than, Equal to, or Greater Than 1?

L

$$\frac{6}{10} + \frac{39}{100}$$

Rolling for Fractions Stage 1 Recording Sheet

- Each partner:
 - Roll 6 number cubes. If you roll any fives, they count as a wild and can be any number you'd like.
 - See if you can fill in a statement to show equivalent fractions.
 - If you cannot make equivalent fractions, re-roll as many cubes as you'd like.
 - If you can make equivalent fractions, record your statement and show or explain how you know the fractions are equivalent. You get 1 point for each pair of equivalent fractions you write.
- Take turns. The partner who has the most points once the recording sheet is full wins the game.

| | |
|---|---|
| $\frac{\square}{\square} = \frac{\square}{\square}$ | $\frac{\square}{\square} = \frac{\square}{\square}$ |
| $\frac{\square}{\square} = \frac{\square}{\square}$ | $\frac{\square}{\square} = \frac{\square}{\square}$ |

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

$$\frac{2}{4}$$

Fraction Cards Grade 3

$$\frac{3}{4}$$

Fraction Cards Grade 3

$$\frac{4}{4}$$

Fraction Cards Grade 3

$$\frac{5}{4}$$

Fraction Cards Grade 3

$$\frac{1}{6}$$

Fraction Cards Grade 3

$$\frac{2}{6}$$

Fraction Cards Grade 3

$$\frac{3}{6}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{4}{6}$$

Fraction Cards Grade 3

$$\frac{5}{6}$$

Fraction Cards Grade 3

$$\frac{6}{6}$$

Fraction Cards Grade 3

$$\frac{7}{6}$$

Fraction Cards Grade 3

$$\frac{1}{2}$$

Fraction Cards Grade 3

$$\frac{2}{2}$$

Fraction Cards Grade 3

$$\frac{1}{3}$$

Fraction Cards Grade 3

$$\frac{2}{3}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{3}{3}$$

Fraction Cards Grade 3

$$\frac{6}{3}$$

Fraction Cards Grade 3

$$\frac{4}{2}$$

Fraction Cards Grade 3

$$\frac{16}{6}$$

Fraction Cards Grade 3

$$\frac{6}{2}$$

Fraction Cards Grade 3

$$\frac{8}{2}$$

Fraction Cards Grade 3

$$\frac{5}{3}$$

Fraction Cards Grade 3

$$\frac{13}{4}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{1}{8}$$

Fraction Cards Grade 4

$$\frac{2}{8}$$

Fraction Cards Grade 4

$$\frac{3}{8}$$

Fraction Cards Grade 4

$$\frac{4}{8}$$

Fraction Cards Grade 4

$$\frac{5}{8}$$

Fraction Cards Grade 4

$$\frac{6}{8}$$

Fraction Cards Grade 4

$$\frac{7}{8}$$

Fraction Cards Grade 4

$$\frac{8}{8}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{1}{5}$$

Fraction Cards Grade 4

$$\frac{2}{5}$$

Fraction Cards Grade 4

$$\frac{3}{5}$$

Fraction Cards Grade 4

$$\frac{4}{5}$$

Fraction Cards Grade 4

$$\frac{5}{5}$$

Fraction Cards Grade 4

$$\frac{6}{5}$$

Fraction Cards Grade 4

$$\frac{1}{10}$$

Fraction Cards Grade 4

$$\frac{2}{10}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{3}{10}$$

Fraction Cards Grade 4

$$\frac{4}{10}$$

Fraction Cards Grade 4

$$\frac{5}{10}$$

Fraction Cards Grade 4

$$\frac{6}{10}$$

Fraction Cards Grade 4

$$\frac{7}{10}$$

Fraction Cards Grade 4

$$\frac{8}{10}$$

Fraction Cards Grade 4

$$\frac{9}{10}$$

Fraction Cards Grade 4

$$\frac{10}{10}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{11}{10}$$

Fraction Cards Grade 4

$$\frac{19}{10}$$

Fraction Cards Grade 4

$$\frac{1}{12}$$

Fraction Cards Grade 4

$$\frac{3}{12}$$

Fraction Cards Grade 4

$$\frac{4}{12}$$

Fraction Cards Grade 4

$$\frac{7}{12}$$

Fraction Cards Grade 4

$$\frac{9}{12}$$

Fraction Cards Grade 4

$$\frac{10}{12}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{13}{12}$$

Fraction Cards Grade 4

$$\frac{15}{12}$$

Fraction Cards Grade 4

$$\frac{1}{100}$$

Fraction Cards Grade 4

$$\frac{5}{100}$$

Fraction Cards Grade 4

$$\frac{10}{100}$$

Fraction Cards Grade 4

$$\frac{20}{100}$$

Fraction Cards Grade 4

$$\frac{49}{100}$$

Fraction Cards Grade 4

$$\frac{50}{100}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{51}{100}$$

Fraction Cards Grade 4

$$\frac{75}{100}$$

Fraction Cards Grade 4

$$\frac{99}{100}$$

Fraction Cards Grade 4

$$\frac{200}{100}$$

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

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- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
 - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
 - Check your partner's work to make sure you agree.
 - Determine the number of points each partner gets:
 - 2 points for creating an expression less than 1
 - 5 points for creating an expression greater than 1
 - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

| round | equation | points |
|-------|--|--------|
| 1 | $\square \times \frac{\square}{\square} =$ | |
| 2 | $\square \times \frac{\square}{\square} =$ | |
| 3 | $\square \times \frac{\square}{\square} =$ | |
| 4 | $\square \times \frac{\square}{\square} =$ | |
| 5 | $\square \times \frac{\square}{\square} =$ | |
| 6 | $\square \times \frac{\square}{\square} =$ | |

Rolling for Fractions Stage 2 Recording Sheet

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 - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
 - Check your partner's work to make sure you agree.
 - Determine the number of points each partner gets:
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- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

| round | equation | points |
|-------|--|--------|
| 1 | $\square \times \frac{\square}{\square} =$ | |
| 2 | $\square \times \frac{\square}{\square} =$ | |
| 3 | $\square \times \frac{\square}{\square} =$ | |
| 4 | $\square \times \frac{\square}{\square} =$ | |
| 5 | $\square \times \frac{\square}{\square} =$ | |
| 6 | $\square \times \frac{\square}{\square} =$ | |

Target Measurement Stage 2 Recording Sheet

Directions:

- Partner A:
 - Choose a target length in quarter inches (up to 10).
 - Begin to draw a line with a straightedge.
- Partner B:
 - Say "Stop!" when you think the length of the line is equal to the target measurement.
- Both partners measure the line and find the difference between its length and the target measurement. The difference is Partner B's score for the round.
- Take turns. After 8 rounds, the player with the lowest total score wins.

| round | Partner A | | | Partner B | | |
|-------|---------------|---------------|--------|---------------|---------------|--------|
| | target length | actual length | points | target length | actual length | points |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |

Compare Stage 6 Cards

Compare Stage 6

$$\frac{4}{6} + \frac{1}{6}$$

Compare Stage 6

$$\frac{2}{4} - \frac{1}{4}$$

Compare Stage 6

$$\frac{2}{5} + \frac{4}{10}$$

Compare Stage 6

$$\frac{3}{6} - \frac{1}{3}$$

Compare Stage 6

$$\frac{4}{6} + \frac{4}{12}$$

Compare Stage 6

$$\frac{5}{8} - \frac{1}{2}$$

Compare Stage 6

$$\frac{7}{10} + \frac{35}{100}$$

Compare Stage 6

$$\frac{8}{10} - \frac{64}{100}$$

Compare Stage 6 Cards

Compare Stage 6

$$\frac{8}{10} + \frac{26}{100}$$

Compare Stage 6

$$\frac{7}{10} - \frac{59}{100}$$

Compare Stage 6

$$\frac{5}{10} + \frac{43}{100}$$

Compare Stage 6

$$\frac{9}{10} - \frac{72}{100}$$

Compare Stage 6

$$2\frac{2}{5} + 3\frac{3}{5}$$

Compare Stage 6

$$1\frac{4}{6} + 4\frac{1}{6}$$

Compare Stage 6

$$3\frac{1}{4} - \frac{2}{4}$$

Compare Stage 6

$$4\frac{3}{5} - 2\frac{4}{5}$$

Compare Stage 6 Cards

Compare Stage 6

$$5\frac{2}{12} - \frac{7}{12}$$

Compare Stage 6

$$2 + \frac{3}{6} + \frac{4}{6}$$

Compare Stage 6

$$3\frac{1}{4} - 1\frac{3}{4}$$

Compare Stage 6

$$5\frac{2}{7} + \frac{4}{7} + \frac{3}{7}$$

Compare Stage 6

$$1 - \frac{3}{12} - \frac{5}{12}$$

Compare Stage 6

$$1 - \frac{5}{8}$$

Compare Stage 6

$$\frac{9}{10} + \frac{4}{10} + \frac{5}{10}$$

Compare Stage 6

$$8 + \frac{6}{9}$$

Compare Stage 6 Cards

Compare Stage 6

$$\frac{3}{4} + \frac{6}{4} + 1$$

Compare Stage 6

$$1 - \frac{2}{9} - \frac{6}{9}$$

Compare Stage 6

$$1\frac{3}{8} - \frac{6}{8}$$

Compare Stage 6

$$6\frac{5}{8} - 2\frac{7}{8}$$

Creating Line Plots Stage 2 Recording Sheet

Directions:

- Measure up to 8 objects to the nearest quarter inch.
- Create a line plot of your measurement data. Don't forget to add a title and label.
- Ask your partner 2 questions that can be answered based on the data in your line plot.



Target Measurement Stage 3 Recording Sheet

Directions:

- Partner A:
 - Choose a target length in eighth inches (up to 10).
 - Begin to draw a line with a straightedge.
- Partner B:
 - Say "Stop!" when you think the length of the line is equal to the target measurement.
- Both partners measure the line and find the difference between its length and the target measurement. The difference is Partner B's score for the round.
- Take turns. After 8 rounds, the player with the lowest total score wins.

| round | Partner A | | | Partner B | | |
|-------|---------------|---------------|--------|---------------|---------------|--------|
| | target length | actual length | points | target length | actual length | points |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |

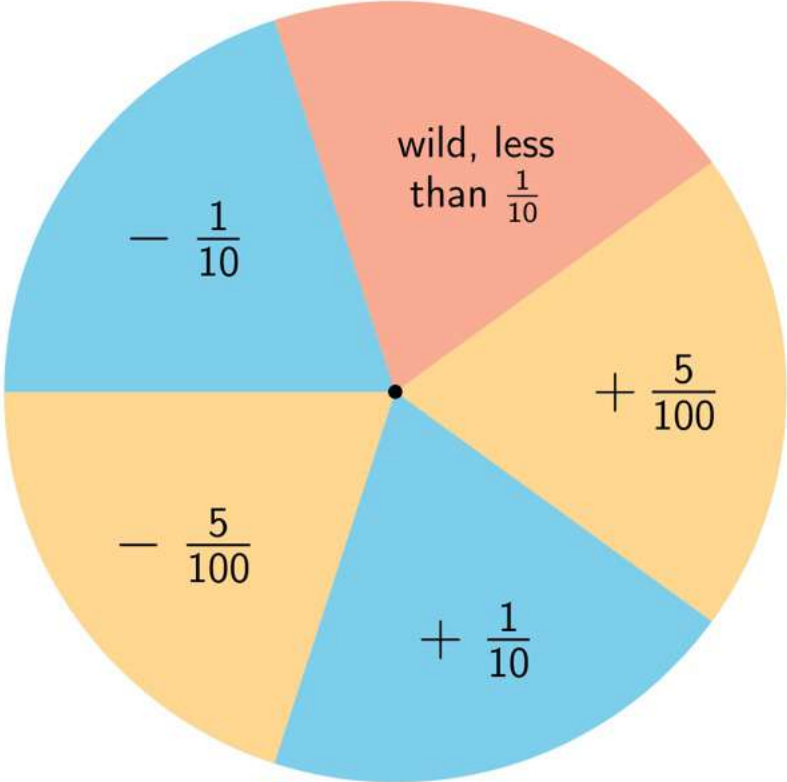
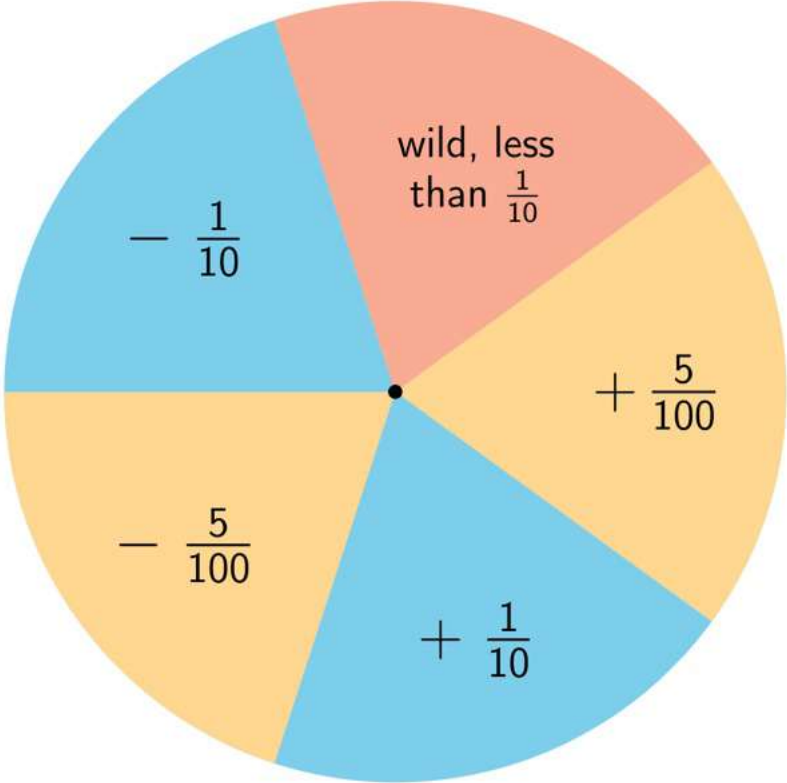
Creating Line Plots Stage 3 Recording Sheet

Directions:

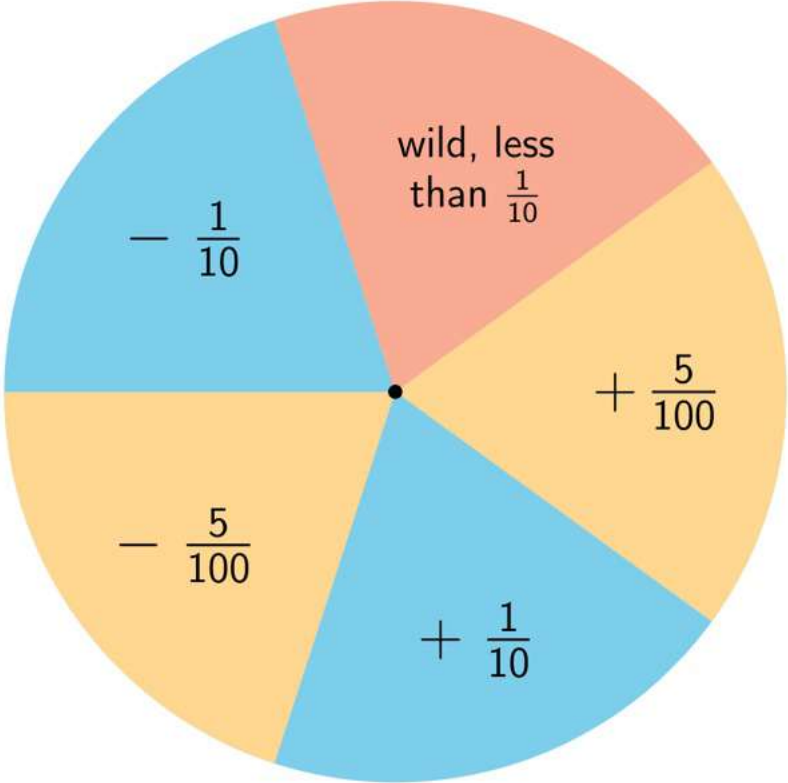
- Measure up to 8 objects to the nearest $\frac{1}{8}$ inch.
- Create a line plot of your measurement data. Don't forget to add a title and label.
- Ask your partner 2 questions that can be answered based on the data in your line plot using addition or subtraction.



Jump the Line Stage 2 Spinner



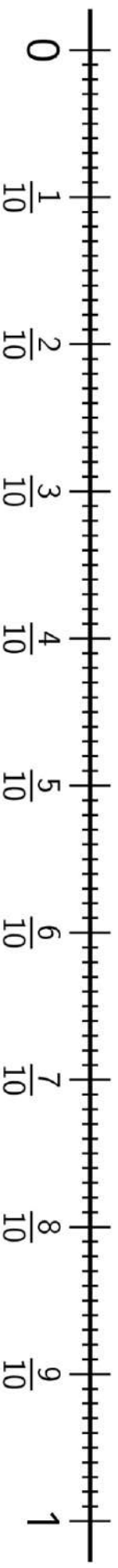
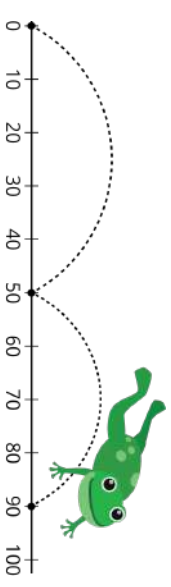
Jump the Line Stage 2 Spinner



Jump the Line Stage 2 Gameboard

Directions:

- Together with your partner, decide on 3 target numbers, mark them on your number line.
- On your turn:
 - Spin all 3 spinners. Decide which moves you want to use on your turn.
 - Mark where you ended up on the number line.
- Take turns spinning and moving on the number line. The first partner to land on 2 of the target numbers wins.



Compare Stage 3 Multiplication Cards

Compare Stage 3

$$12 \times 9$$

Compare Stage 3

$$12 \times 7$$

Compare Stage 3

$$13 \times 7$$

Compare Stage 3

$$14 \times 6$$

Compare Stage 3

$$15 \times 6$$

Compare Stage 3

$$10 \times 20$$

Compare Stage 3

$$21 \times 4$$

Compare Stage 3

$$19 \times 5$$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$18 \times 5$$

Compare Stage 3

$$17 \times 4$$

Compare Stage 3

$$16 \times 6$$

Compare Stage 3

$$14 \times 7$$

Compare Stage 3

$$31 \times 3$$

Compare Stage 3

$$20 \times 4$$

Compare Stage 3

$$8 \times 9$$

Compare Stage 3

$$9 \times 7$$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$12 \times 5$$

Compare Stage 3

$$13 \times 4$$

Compare Stage 3

$$15 \times 3$$

Compare Stage 3

$$9 \times 5$$

Credits

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