



Core Knowledge<sup>®</sup> MATHEMATICS

# Extending Operations to Fractions



Student Workbook

$$2 \times 2 = 4$$

$$4 \times 2 = 8$$

$$8 : 4 = 2$$

$$4 - 2 = 2$$



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# Extending Operations to Fractions

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## **Extending Operations to Fractions Student Workbook**

Core Knowledge Mathematics™



# Lesson 1: Equal Groups of Unit Fractions

- Let's look at equal groups of fractions.

## Warm-up: How Many Do You See: Oranges

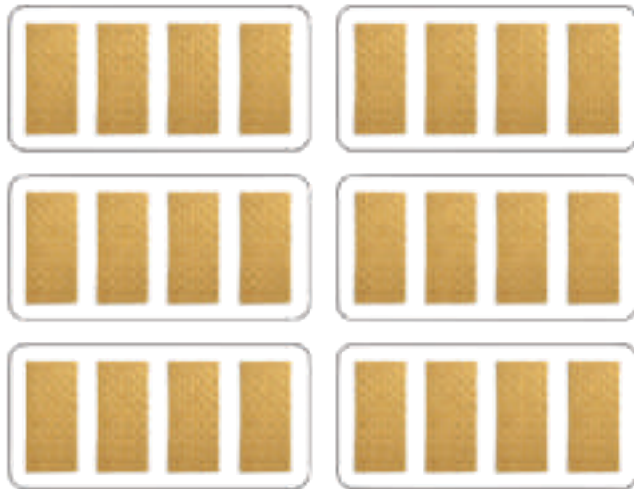
How many do you see? How do you see them?



## 1.1: Crackers, Kiwis, and More

1. Here are images of some crackers.

A



B



a. How are the crackers in image A like those in B?

b. How are they different?

c. How many crackers are in each image?

d. Write an expression to represent the crackers in each image.

2. Here are more images and descriptions of food items. For each, write a multiplication expression to represent the quantity. Then, answer the question.

a. Clare has 3 baskets. She put 4 eggs into each basket. How many eggs did she put in baskets?



b. Diego has 5 plates. He put  $\frac{1}{2}$  of a kiwi fruit on each plate. How many kiwis did he put on plates?



c. Priya prepared 7 plates with  $\frac{1}{8}$  of a pie on each. How much pie did she put on plates?



d. Noah scooped  $\frac{1}{3}$  cup of brown rice 8 times. How many cups of brown rice did he scoop?



## 1.2: What Could It Mean?

For each expression:

- Write a story that the expression could represent. The story should be about a situation with equal groups.
- Create a drawing to represent the situation.
- Find the value of the expression. What does this number mean in your story?

1.  $8 \times \frac{1}{2}$

2.  $7 \times \frac{1}{5}$

# Lesson 2: Representations of Equal Groups of Fractions

- Let's look at diagrams and expressions that can help us multiply a whole number and a fraction.

## Warm-up: Number Talk: Three, Six, Nine, Twelve

Find the value of each expression mentally.

- $3 \times 6$

- $3 \times 9$

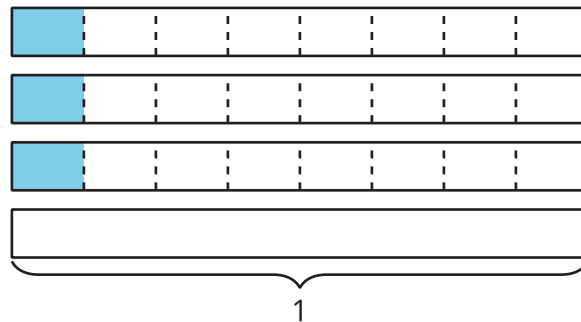
- $6 \times 9$

- $12 \times 9$

## 2.1: Card Sort: Expressions and Diagrams

Your teacher will give you a set of cards with expressions and diagrams.

1. Match each expression with a diagram that represents the same quantity.
2. Record each expression without a match.
3. Han started drawing a diagram to represent  $7 \times \frac{1}{8}$  and did not finish. Complete his diagram. Be prepared to explain your reasoning.



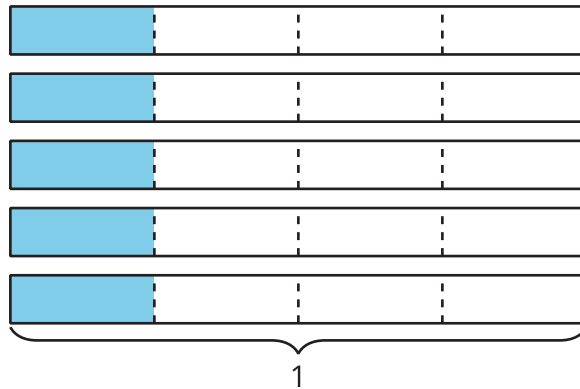
4. Choose one expression that you recorded earlier that didn't have a match.

Draw a diagram that can be represented by the expression. What value do the shaded parts of your diagram represent?

## 2.2: Different Representations

1. a. Write a multiplication expression that represents the shaded parts of the diagram. Then, find the value of the expression.

Diagram:



Expression:

Value:

- b. Draw a diagram that the expression  $6 \times \frac{1}{3}$  could represent. Then, find the value of the expression.

Diagram:

Expression:  $6 \times \frac{1}{3}$

Value:

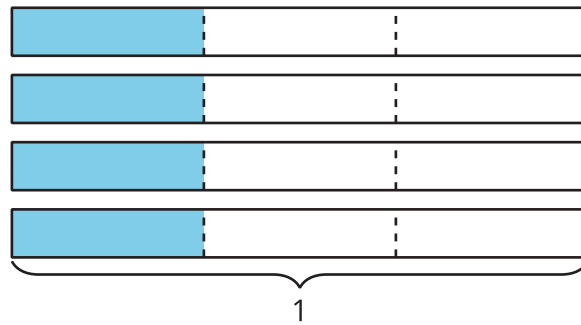
- c. Draw a diagram and write an expression that gives the value  $\frac{7}{2}$ .

Diagram:

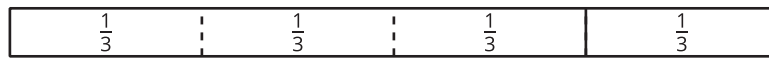
Expression:

Value:  $\frac{7}{2}$

2. To represent  $4 \times \frac{1}{3}$ , Diego drew this diagram:



Elena drew this diagram:



Are they representing the same expression and value? Explain or show how you know.

## Lesson 3: Patterns in Multiplication

- Let's look at patterns in multiplication of a fraction by a whole number.

### 3.1: Describe the Pattern

1. Here are two tables with expressions. Find the value of each expression. Use a diagram if you find it helpful.

Leave the last two rows of each table blank for now.

Set A

expression	value
$1 \times \frac{1}{8}$	
$2 \times \frac{1}{8}$	
$3 \times \frac{1}{8}$	
$4 \times \frac{1}{8}$	
$5 \times \frac{1}{8}$	
$6 \times \frac{1}{8}$	

Set B

expression	value
$2 \times \frac{1}{3}$	
$2 \times \frac{1}{4}$	
$2 \times \frac{1}{5}$	
$2 \times \frac{1}{6}$	
$2 \times \frac{1}{7}$	
$2 \times \frac{1}{8}$	

2. Study your completed tables. What patterns do you see in how the expressions and values are related?

3. In the last two rows of the table of Set A, write  $\frac{11}{8}$  and  $\frac{13}{8}$  in the “value” column. Write the expressions with that value.

4. In the last two rows of the table of Set B, write  $\frac{2}{12}$  and  $\frac{2}{15}$  in the “value” column. Write the expressions with that value.

## 3.2: What's Missing?

1. Use the patterns you observed earlier to complete each equation so that it's true.

a.  $5 \times \frac{1}{10} = \underline{\hspace{2cm}}$

b.  $8 \times \frac{1}{6} = \underline{\hspace{2cm}}$

c.  $4 \times \underline{\hspace{2cm}} = \frac{4}{5}$

d.  $6 \times \underline{\hspace{2cm}} = \frac{6}{10}$

e.  $\underline{\hspace{2cm}} \times \frac{1}{4} = \frac{3}{4}$

f.  $\underline{\hspace{2cm}} \times \frac{1}{12} = \frac{7}{12}$

2. Your teacher will give you a sheet of paper. Work with your group of 3 and complete these steps on the paper. After each step, pass your paper to your right.

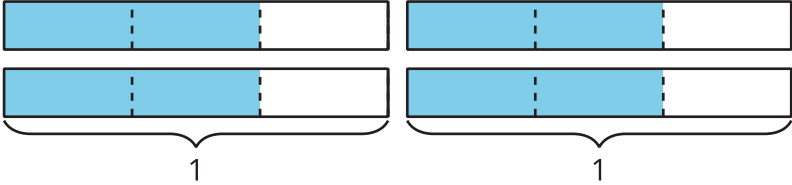
- Step 1: Write a fraction with a numerator other than 1 and a denominator no greater than 12.
- Step 2: Write the fraction you received as a product of a whole number and a unit fraction.
- Step 3: Draw a diagram to represent the expression you just received.
- Step 4: Collect your original paper. If you think the work is correct, explain why the expression and the diagram both represent the fraction that you wrote. If not, discuss what revisions are needed.

# Lesson 4: Equal Groups of Non-Unit Fractions

- Let's multiply any fraction by a whole number.

## Warm-up: Notice and Wonder: Thirds

What do you notice? What do you wonder?



## 4.1: Jars of Jam

Elena fills 5 small jars with homemade jams to share with her friends. Each jar can fit  $\frac{3}{4}$  cup of jam. How many cups of jam are in the jars? Explain or show your reasoning.



If you have time: Elena still has some jam left. She takes 2 large jars and puts  $\frac{5}{4}$  cups of jam in each jar. How many cups of jam are in the jars?

## 4.2: How Do We Multiply?

1. This diagram represents  $\frac{2}{5}$ .



- Show how you would use or adjust the diagram to represent  $4 \times \frac{2}{5}$ .
- What is the value of the shaded parts in your diagram?

2. This diagram represents  $\frac{5}{8}$ .



- Show how you would use or adjust the diagram to represent  $3 \times \frac{5}{8}$ .
- What is the value of the shaded parts in your diagram?

3. Find the value of each expression. Draw a diagram if you find it helpful. Be prepared to explain your reasoning.

a.  $2 \times \frac{1}{6}$

b.  $2 \times \frac{4}{6}$

c.  $2 \times \frac{5}{6}$

d.  $4 \times \frac{5}{6}$

4. Mai said that to multiply any fraction by a whole number, she would multiply the whole number and the numerator of the fraction and keep the same denominator. Do you agree with Mai? Explain your reasoning.

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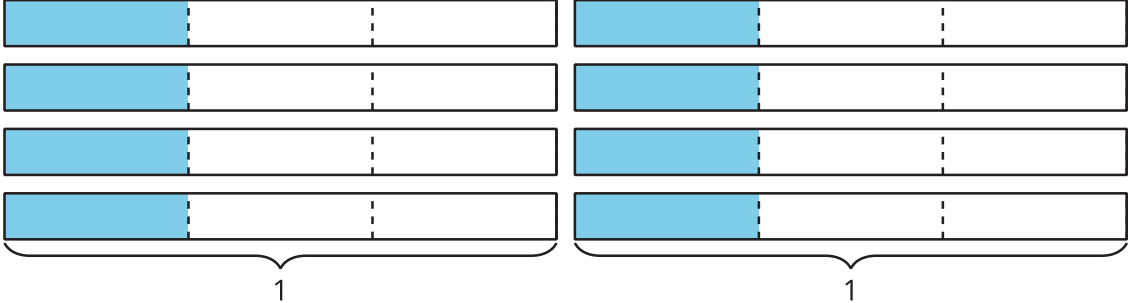
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# Lesson 5: Equivalent Multiplication Expressions

- Let's write multiplication expressions in different ways.

## Warm-up: How Many Do You See?

How many thirds do you see? How do you see them?



## 5.1: Complete the Equations

1. Find the number that makes each equation true. Draw a diagram if it is helpful.

$$\frac{12}{5} = 12 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 3 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 6 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 2 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 4 \times \underline{\hspace{2cm}}$$

$$\frac{12}{5} = 1 \times \underline{\hspace{2cm}}$$

2. Here are two sets of numbers:

Set A:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Set B:

$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$

a. Choose a number from set A and a number from set B to complete this equation and make it true:

$$\frac{6}{7} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

b. Choose a different number from set A and a number from set B to complete the equation to make it true.

$$\frac{6}{7} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

3. Explain or show how you know that the two equations you wrote are both true.

## 5.2: Fractions and Matching Expressions

Here is a set of expressions.

A.

$$6 \times \frac{1}{10}$$

B.

$$2 \times 4 \times \frac{1}{9}$$

C.

$$4 \times \frac{1}{5}$$

D.

$$3 \times 2 \times \frac{1}{10}$$

E.

$$5 \times 2 \times \frac{1}{12}$$

F.

$$2 \times 2 \times \frac{1}{5}$$

G.

$$4 \times 4 \times \frac{1}{9}$$

H.

$$10 \times \frac{1}{12}$$

I.

$$4 \times \frac{1}{12}$$

1. Match each expression to one of the following fractions, if possible. Record your matches. Be prepared to explain how you know there is or isn't a match.

$$\frac{4}{5}$$

$$\frac{10}{12}$$

$$\frac{6}{10}$$

$$\frac{8}{9}$$

2. Complete each equation to make it true. Try to do so without using unit fractions.

a.  $\frac{4}{5} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$$\frac{4}{5} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

b.  $\frac{10}{12} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$$\frac{10}{12} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

c.  $\frac{6}{10} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$$\frac{6}{10} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

d.  $\frac{8}{9} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$$\frac{8}{9} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

## Lesson 6: Problems with Equal Groups of Fractions

- Let's solve problems with fractions.

### Warm-up: True or False: Two and Three Factors

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $\frac{10}{12} = 5 \times \frac{2}{12}$

- $1 \times \frac{10}{12} = 5 \times \frac{2}{12}$

- $\frac{24}{4} = 6 \times 3 \times \frac{1}{4}$

- $12 \times 2 \times \frac{1}{4} = 8 \times 3 \times \frac{1}{4}$

## 6.1: Banana Bread Recipe

A bakery is making banana bread. Here is the recipe for 1 batch.



Recipe:

- 1 banana
- $\frac{2}{3}$  cup butter
- $\frac{3}{2}$  teaspoons baking soda
- $\frac{5}{8}$  cup sugar
- 2 large eggs
- $\frac{5}{2}$  cups of all-purpose flour

1. The bakery makes 2 batches of banana bread on Monday. Complete the table to show how much of each ingredient is used.

Monday's banana bread

ingredient	expression	amount of ingredient
bananas		_____
butter		_____ cup(s)
baking soda		_____ teaspoon(s)
sugar		_____ cup(s)
eggs		_____
flour		_____ cup(s)

2. On Tuesday, the bakery needs  $\frac{8}{3}$  cups of butter to make enough banana bread for the day. How many batches were made? Explain or show your reasoning.

Recipe:

- 1 banana
- $\frac{2}{3}$  cup butter
- $\frac{3}{2}$  teaspoons baking soda
- $\frac{5}{8}$  cup sugar
- 2 large eggs
- $\frac{5}{2}$  cups of all-purpose flour

3. Based on the number of the batches made on Tuesday, complete the table for each ingredient.

Tuesday's banana bread

ingredient	expression	amount of ingredient
bananas		_____
butter		$\frac{8}{3}$ cups
baking soda		_____ teaspoon(s)
sugar		_____ cup(s)
eggs		_____
flour		_____ cup(s)

## 6.2: How Much Milk Was Used?

The bakery that sells banana bread also sells fresh milkshakes. Each serving uses  $\frac{1}{10}$  liter of milk.

Here are five descriptions of the milkshakes sold in a week and five expressions that represent the liters of milk used.



Match each description to an expression that represents it.

1. On Monday, the bakery sold 8 servings of milkshake. How much milk was used?

$$4 \times (2 \times \frac{1}{10})$$

2. On Tuesday, two customers bought 4 servings of milkshake each. How much milk was used?

$$4 \times \frac{2}{10}$$

3. On Wednesday, four customers bought 2 servings of milkshake each. How much milk was used?

$$8 \times \frac{1}{10}$$

4. On Thursday, two customers each bought a serving of milkshake. They placed the same order three more times for their friends that day. How much milk was used?

$$2 \times (4 \times \frac{1}{10})$$

5. On Saturday, four friends each purchased a serving of milkshake for breakfast. They came back for the same after dinner. How much milk was used?

$$2 \times \frac{4}{10}$$

## Section Summary

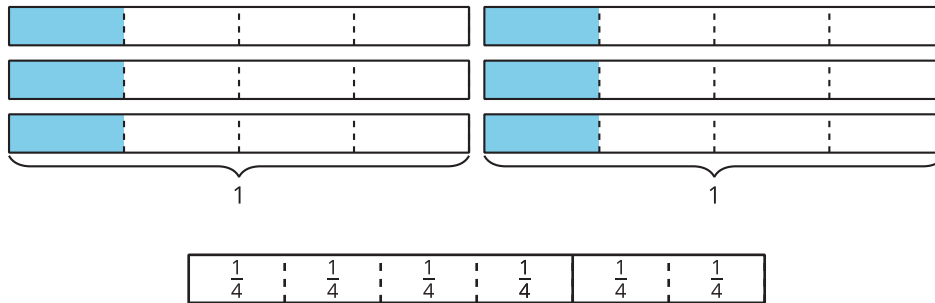
### Section Summary

In this section, we learned to multiply a whole number and a fraction by thinking about equal-size groups, just as we did when multiplying two whole numbers.

For instance, we can think of  $6 \times 4$  as 6 groups of 4. A diagram like this can help to show that the product is 24:



Likewise, we can think of  $6 \times \frac{1}{4}$  as 6 groups of  $\frac{1}{4}$ . Diagrams can help us see that the product is  $\frac{6}{4}$ :



After studying patterns, we saw that when we multiply a whole number and a fraction, the whole number is multiplied only by the numerator of the fraction and the denominator stays the same. For example:

$$6 \times \frac{1}{2} = \frac{6}{2}$$

$$2 \times \frac{4}{5} = \frac{8}{5}$$

We also learned that:

- Every fraction can be written as a product of a whole number and a unit fraction. For example,  $\frac{5}{4}$  can be written as  $5 \times \frac{1}{4}$ .
- We can write different multiplication expressions for the same fraction. For example,  $\frac{8}{3}$  can be written as:

$$8 \times \frac{1}{3}$$

$$4 \times 2 \times \frac{1}{3}$$

$$4 \times \frac{2}{3}$$

$$2 \times \frac{4}{3}$$

# Lesson 7: Fractions as Sums

- Let's write fractions as sums.

## 7.1: Barley Soup

Lin is learning to make barley soup using a family recipe. Here are some ingredients in the recipe:



- $\frac{3}{4}$  cup of barley
- $\frac{5}{4}$  cups of chopped celery
- $\frac{6}{4}$  cups of chopped carrots
- 1 cup of chopped onions
- $2\frac{1}{4}$  cups of vegetable broth

1. Lin has only one measuring cup that measures  $\frac{1}{4}$  cup. Show how Lin could use the cup to measure the right amount of each ingredient.

- Barley:
- Celery:
- Carrots:
- Onions:
- Vegetable broth:

2. Lin later found a  $\frac{3}{4}$ -cup measuring cup. Show how she could use the cups to measure the right amount of each ingredient.

- Barley:
- Celery:
- Carrots:
- Onions:
- Vegetable broth:

## 7.2: Sums in Fifths and Thirds

1. Use different combinations of fifths to make a sum of  $\frac{9}{5}$ .

a.  $\frac{9}{5} = \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad}$

b.  $\frac{9}{5} = \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad}$

c.  $\frac{9}{5} = \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad}$

d.  $\frac{9}{5} = \underline{\quad\quad} + \underline{\quad\quad}$

2. Write different ways to use thirds to make a sum of  $\frac{4}{3}$ . How many can you think of? Write an equation for each combination.

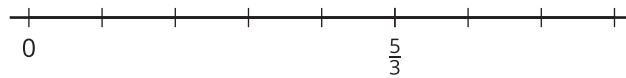
3. Is it possible to write any fraction with a denominator of 5 as a sum of other fifths? Explain or show your reasoning.

# Lesson 8: Addition of Fractions

- Let's explore sums of fractions on a number line.

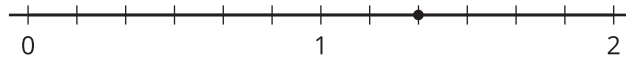
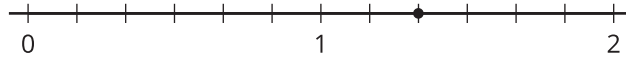
## Warm-up: Notice and Wonder: A Fraction on a Number Line

What do you notice? What do you wonder?

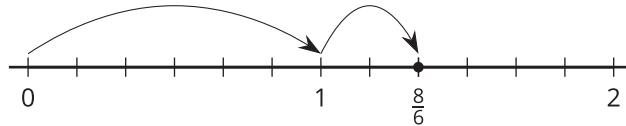


## 8.1: Sum of Jumps

1. a. On each number line, draw two “jumps” to show how to use sixths to make a sum of  $\frac{8}{6}$ . Then, write an equation to represent each combination of jumps.



- b. Noah draws the following diagram and writes:  $\frac{8}{6} = \frac{6}{6} + \frac{2}{6}$  and  $\frac{8}{6} = 1 + \frac{2}{6}$ . Which equation is correct? Explain your reasoning.

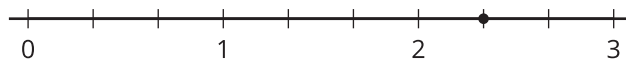
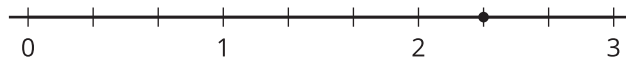



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2. a. On each number line, draw “jumps” to show how to use thirds to make a sum of  $\frac{7}{3}$ . Then, write an equation to represent each combination of jumps.

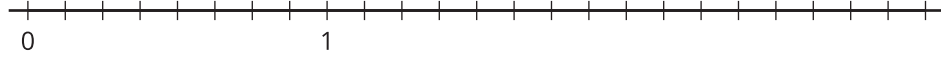


- b. Write  $\frac{7}{3}$  as a sum of a whole number and a fraction.

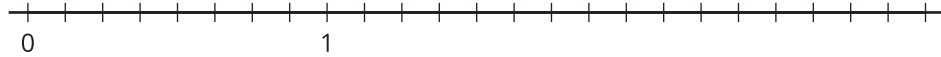
## 8.2: What is the Sum?

1. Use a number line to represent each addition expression and to find its value.

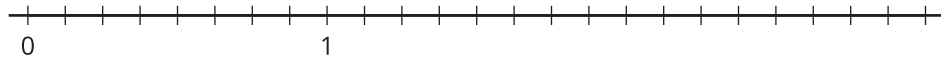
a.  $\frac{5}{8} + \frac{2}{8}$



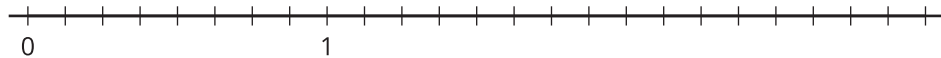
b.  $\frac{1}{8} + \frac{9}{8}$



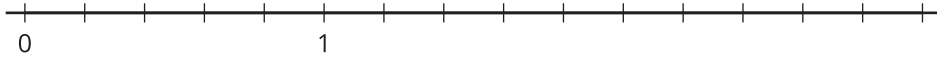
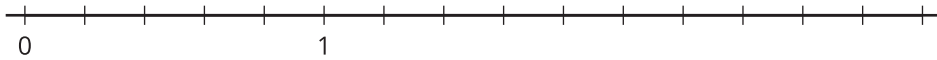
c.  $\frac{11}{8} + \frac{9}{8}$



d.  $2\frac{1}{8} + \frac{4}{8}$

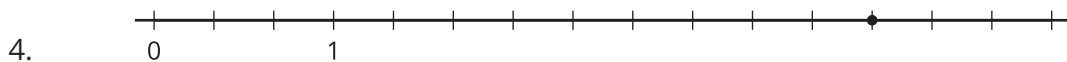
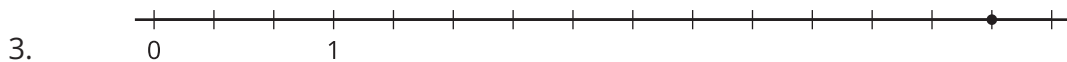
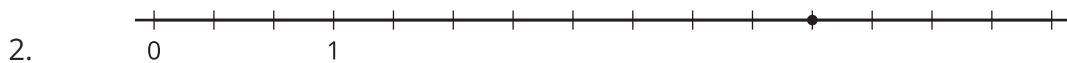
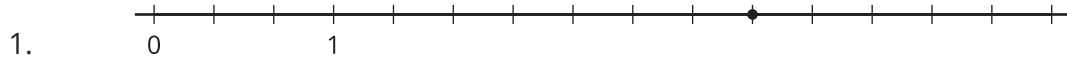


2. Priya says the sum of  $1\frac{2}{5}$  and  $\frac{4}{5}$  is  $1\frac{6}{5}$ . Kiran says the sum is  $\frac{11}{5}$ . Tyler says it is  $2\frac{1}{5}$ . Do you agree with any of them? Explain or show your reasoning. Use one or more number lines if you find them helpful.



## 8.3: Make Two Jumps

Here are four number lines, each with a point on it.



For each number line, label the point. This is your target. Make two forward jumps to get from 0 to the target.

- Pick a card from the set given to you. Use the fraction on it for your first jump. Draw the jump and label it with the fraction.
- From there, draw the second jump to reach the target. What fraction do you need to add? Label the jump with the fraction.
- Write an equation to represent the sum of your two fractions.

## Lesson 9: Differences of Fractions

- Let's explore differences of fractions on a number line.

### Warm-up: True or False: Sums of Tenths

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} = 1$

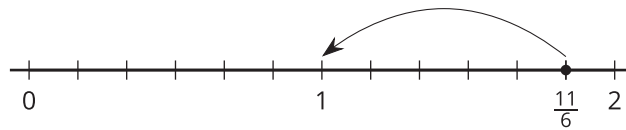
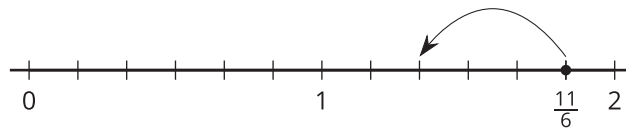
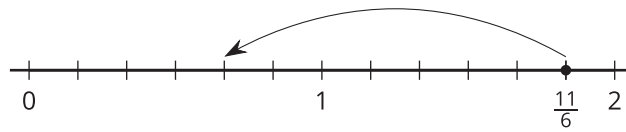
- $1 + \frac{7}{10} = \frac{3}{10} + \frac{4}{10} + \frac{10}{10}$

- $\frac{5}{10} + 1 = \frac{6}{10}$

- $\frac{2}{10} + \frac{10}{10} = 1 + \frac{1}{5}$

## 9.1: Jump to Subtract

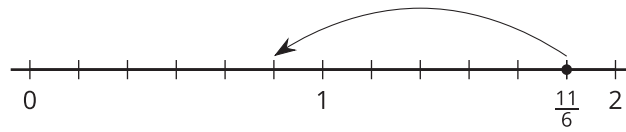
1. To subtract different fractions from  $\frac{11}{6}$ , Noah draws “jumps” on number lines.



a. The first diagram shows how he finds  $\frac{11}{6} - \frac{7}{6}$ . What is the value of  $\frac{11}{6} - \frac{7}{6}$ ?

b. Write an equation to show the difference represented by each of Noah's diagrams.

2. Here is another diagram Noah draws:



Which equations could the diagram represent? Explain your reasoning.

$$\frac{11}{6} - \frac{6}{6} = \frac{5}{6}$$

$$\frac{11}{6} - 1 = \frac{5}{6}$$

$$1\frac{5}{6} - 1 = \frac{5}{6}$$

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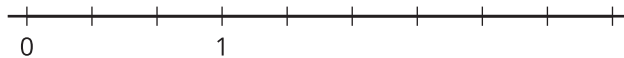
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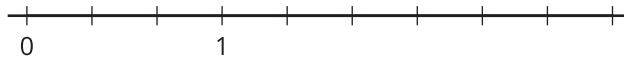
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3. Use a number line to represent each difference and to find its value.

a.  $\frac{8}{3} - \frac{2}{3}$



b.  $\frac{8}{3} - \frac{4}{3}$



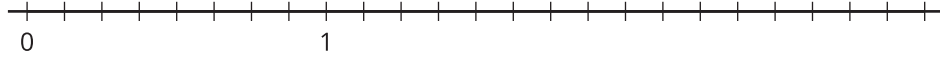
c.  $\frac{8}{3} - 1$



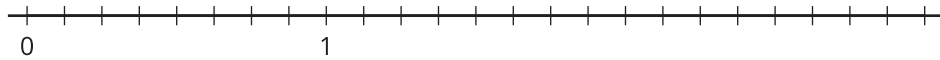
## 9.2: What's the Difference?

Use a number line to represent each difference and to find its value.

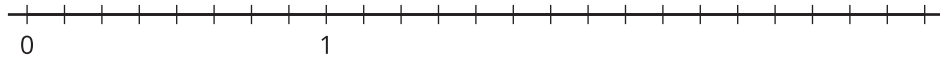
1.  $\frac{13}{8} - \frac{2}{8}$



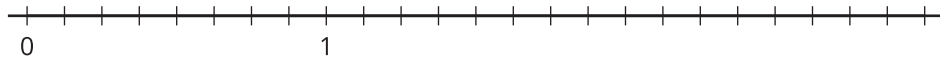
2.  $\frac{13}{8} - \frac{6}{8}$



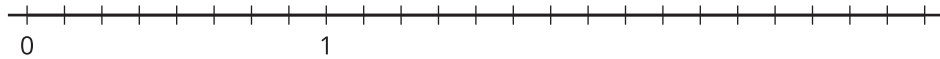
3.  $\frac{13}{8} - 1$



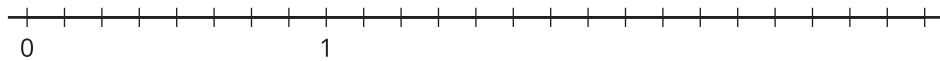
4.  $1\frac{5}{8} - \frac{7}{8}$



5.  $1\frac{5}{8} - 1$

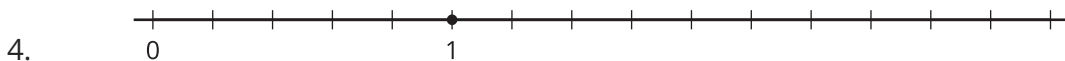
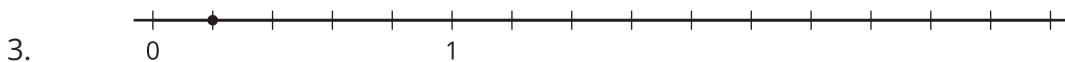
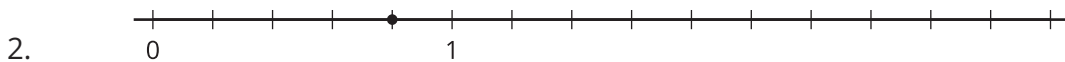
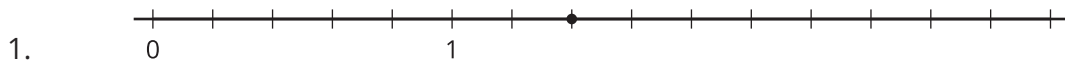


6.  $1\frac{5}{8} - 1\frac{4}{8}$



### 9.3: Make a Jump, Subtraction Edition

Here are four number lines, each with a point on it. Label each point with a fraction it represents.



The point you labeled is your target.

- Pick a card from the set given to you. Locate and label the fraction on the number line.
- From that point, draw one or more jumps to reach the target. What do you need to subtract? Label each jump you draw.
- Write an equation to represent the difference of your two fractions.

# Lesson 10: The Numbers in Subtraction

- Let's subtract fractions from whole numbers.

## Warm-up: Number Talk: Groups of Twelfths

Find the value of each expression mentally.

- $2 \times \frac{3}{12}$

- $6 \times \frac{3}{12}$

- $12 \times \frac{3}{12}$

- $12 \times \frac{30}{12}$

## 10.1: What's Left?

1. A pitcher contains 3 cups of watermelon juice.

How many cups will be left in the pitcher if we pour each of the following amounts from the full amount?



a.  $\frac{1}{4}$  cup

b.  $\frac{5}{4}$  cups

c.  $1\frac{1}{4}$  cups

d.  $2\frac{2}{4}$  cups

2. A second pitcher contains 4 cups of water. How many cups will be left in that pitcher if we pour each of the following amounts from the full amount?  
Explain or show your reasoning. Use diagrams or equations, if they are helpful.

a.  $\frac{1}{3}$  cup

b.  $\frac{5}{3}$  cups

c.  $2\frac{2}{3}$  cups

## 10.2: Card Sort: Twelfths

1. Sort the cards from your teacher into two groups. Record your sorted expressions. Be prepared to explain why the cards in each group belong together.

2. Find the value of each difference. Show your reasoning.

a.  $1 - \frac{5}{8}$

b.  $2 - \frac{7}{8}$

c.  $3 - \frac{9}{8}$

# Lesson 11: Subtract Fractions Flexibly

- Let's find all kinds of differences.

## Warm-up: Which One Doesn't Belong: Fractional Values

Which one doesn't belong?

A.

$$2 - \frac{3}{5}$$

B.

$$\frac{10}{5} - \frac{3}{5}$$

C.

$$1\frac{3}{5} - \frac{1}{5}$$

D.

$$\frac{10}{5} - 1$$



## 11.2: Multiple Ways to Subtract

Here are four expressions that you may have written about the friendship bracelets.

$$9\frac{4}{8} - \frac{7}{8}$$

$$9\frac{4}{8} - 5\frac{1}{8}$$

$$9\frac{4}{8} - 3\frac{5}{8}$$

$$5\frac{1}{8} - 3\frac{5}{8}$$

- Here is one way to find the value of the first expression. Analyze the calculation. Talk to your partner about why  $9\frac{4}{8}$  is written as different sums.

$9\frac{4}{8} - \frac{7}{8}$	
first number	second number
$9\frac{4}{8}$ $8 + 1 + \frac{4}{8}$ $8 + \frac{8}{8} + \frac{4}{8}$ $8 + \frac{12}{8}$	$\frac{7}{8}$
$8 + \frac{12}{8} - \frac{7}{8}$ $8 + \frac{5}{8}$ $8\frac{5}{8}$	

- Here are some unfinished calculations. Complete them to find the value of each difference.

a.

$9\frac{4}{8} - 5\frac{1}{8}$	
first number	second number
$9\frac{4}{8}$ $9 + \frac{4}{8}$	$5\frac{1}{8}$ $5 + \frac{1}{8}$

b.

$9\frac{4}{8} - 3\frac{5}{8}$	
first number	second number
$9\frac{4}{8}$	$3\frac{5}{8}$
$8 + 1 + \frac{4}{8}$	$3 + \frac{5}{8}$
$8 + \frac{8}{8} + \frac{4}{8}$	
$8 + \frac{12}{8}$	

c.

$5\frac{1}{8} - 3\frac{5}{8}$	
first number	second number
$5\frac{1}{8}$	$3\frac{5}{8}$
$5 + \frac{1}{8}$	$3 + \frac{5}{8}$

# Lesson 12: Sums and Differences of Fractions

- Let's add and subtract fractions and analyze our strategies.

## Warm-up: Number Talk: Subtract Some Eighths

Find the value of each expression mentally.

- $2\frac{3}{8} - \frac{3}{8}$

- $2\frac{3}{8} - \frac{5}{8}$

- $2\frac{3}{8} - 2$

- $2\frac{3}{8} - 1\frac{7}{8}$

## 12.1: Make It True

1. Find the number that makes each equation true. Show your reasoning.

a. \_\_\_\_\_ +  $\frac{2}{6}$  =  $1\frac{1}{6}$

b.  $2\frac{4}{5}$  + \_\_\_\_\_ =  $7\frac{1}{5}$

c.  $3 - 2\frac{1}{3}$  = \_\_\_\_\_

d.  $4\frac{1}{12} - 2\frac{5}{12}$  = \_\_\_\_\_

2. Write a sentence to describe your first step for finding the missing number in each equation in the first problem.

a. First step:

b. First step:

c. First step:

d. First step:

3. Compare and reflect on your first steps with your group. Did you make the same moves?

Discuss why you might have chosen the same way or different ways to start finding the missing numbers.

## 12.2: To Decompose or Not to Decompose

1. Here are some addition and subtraction expressions. Sort them into two groups based on whether you think it would be helpful to decompose a number to find the value of the expression. Be prepared to explain your reasoning.

A.  $\frac{18}{5} - \frac{7}{5}$

B.  $\frac{1}{6} + \frac{9}{6}$

C.  $7 - 1\frac{3}{8}$

D.  $\frac{102}{100} + 5\frac{27}{100}$

E.  $2\frac{5}{12} + \frac{6}{12}$

F.  $6\frac{1}{10} - \frac{6}{10}$

G.  $3\frac{8}{100} + 4\frac{93}{100}$

H.  $5 - \frac{17}{12}$

I.  $1\frac{3}{10} + \frac{6}{10}$

J.  $\frac{17}{8} - 1\frac{7}{8}$

○ Not necessary or not helpful to decompose any number:

○ Necessary or helpful to decompose one or more numbers:

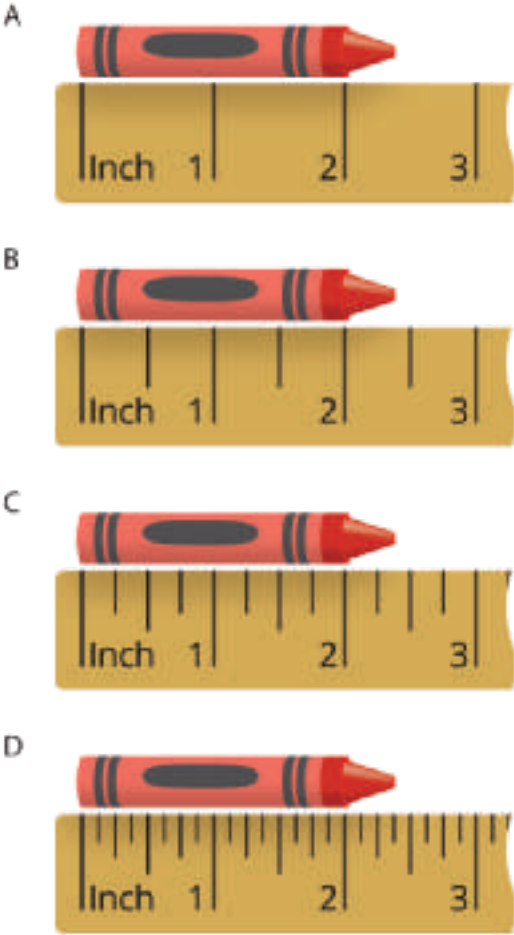
2. Choose at least one expression from each group and find their values. Show your reasoning.

# Lesson 13: Fractional Measurements on Line Plots

- Let's create line plots and analyze the data.

## Warm-up: Notice and Wonder: Which Ruler?

What do you notice? What do you wonder?



## 13.1: Measure to the Nearest $\frac{1}{4}$ and $\frac{1}{8}$ Inch

Your teacher will give your group a set of colored pencils.

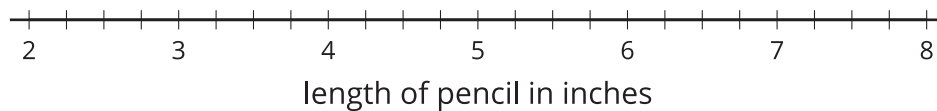
1. Work with your group to measure each colored pencil to the nearest  $\frac{1}{4}$  inch. Check each other's measurements. Record each measurement in the table.

group members	pencil length (inches)



2. Create a line plot to represent the data your group collected.

Colored-Pencil Data



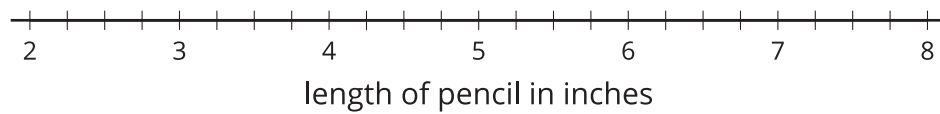
3. Work with your group to measure each colored pencil to the nearest  $\frac{1}{8}$  inch.

Check one another's measurements. Record each measurement in the table.

group members	pencil length (inches)

4. Create a line plot to represent your new data.

Colored-Pencil Data



5. How was measuring to the nearest  $\frac{1}{4}$  inch different from measuring to the nearest  $\frac{1}{8}$  inch?

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## 13.2: Colored-pencil Measurements

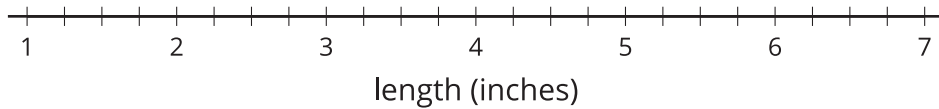
1. Andre's class measured the length of some colored pencils to the nearest  $\frac{1}{4}$  inch.

The data are shown here:

$1\frac{3}{4}$      $2\frac{1}{4}$      $5\frac{1}{4}$      $5\frac{1}{4}$      $4\frac{2}{4}$   
 $4\frac{2}{4}$      $6\frac{1}{4}$      $6\frac{3}{4}$      $6\frac{3}{4}$      $6\frac{3}{4}$



a. Plot the colored-pencil data on the line plot.



b. Which colored-pencil length is the most common in the data set?

c. Write 2 new questions that could be answered using the line plot data.

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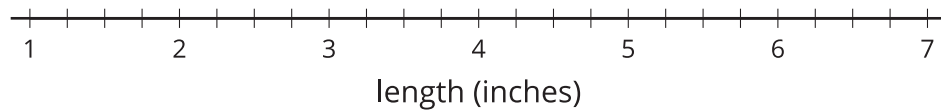
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2. Next, Andre's class measured their colored pencils to the nearest  $\frac{1}{8}$  inch. The data are shown here:

$1\frac{6}{8}$        $2\frac{2}{8}$        $5\frac{2}{8}$        $5\frac{4}{8}$        $4\frac{4}{8}$   
 $4\frac{4}{8}$        $6\frac{6}{8}$        $6\frac{6}{8}$        $6\frac{6}{8}$        $6\frac{4}{8}$

a. Plot the colored-pencil data on the line plot.



b. Which colored-pencil length is the most common in the line plot?

c. Why did some colored-pencil lengths change on this line plot?

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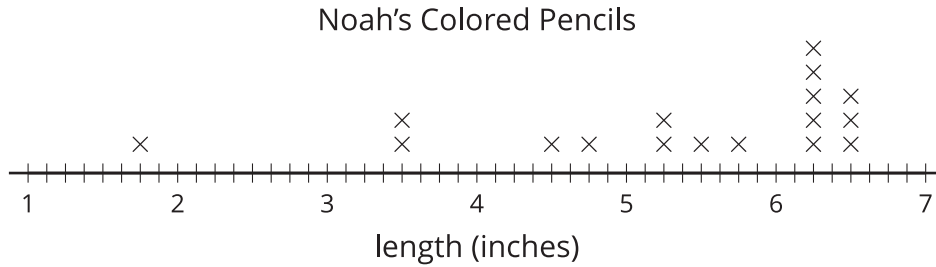
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d. What is the difference between the length of the longest colored pencil and the shortest colored pencil? Show your reasoning.

### 13.3: Noah's Colored Pencils

The line plot shows the data Noah collected on a set of colored pencils.



Use the line plot to tell if each of the following statements is true or false. Be prepared to explain or show how you know. For each false statement, correct it so that it is true.

1. Noah measured the colored pencils to the nearest  $\frac{1}{2}$  inch.
2. There are five pencils that are  $6\frac{1}{4}$  inches long.
3. The shortest pencil is  $1\frac{3}{4}$  inches long.
4. The three longest pencils are exactly 5 inches longer than the shortest one.
5. If Noah removed the shortest pencil from the collection, the difference between the longest and shortest pencils would be 3 inches.

#### If You Have Time

Noah wants to create a collection of at least 10 pencils where the difference between the longest and shortest colored pencils is no more than  $1\frac{1}{2}$  inches.

Is that possible? If so, which pencils should he remove from his collection?

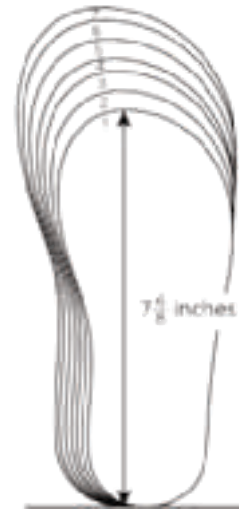
# Lesson 14: Problems about Fractional Measurement Data

- Let's solve problems involving measurement data on line plots.

## Warm-up: Notice and Wonder: Shoe Sizes

What do you notice? What do you wonder?

US youth shoe size	insole length in inches
1	$7\frac{6}{8}$
1.5	8
2	$8\frac{1}{8}$
2.5	$8\frac{2}{8}$
3	$8\frac{4}{8}$
3.5	
4	$8\frac{6}{8}$
4.5	9
5	$9\frac{1}{8}$
5.5	
6	$9\frac{4}{8}$
6.5	$9\frac{5}{8}$
7	$9\frac{6}{8}$

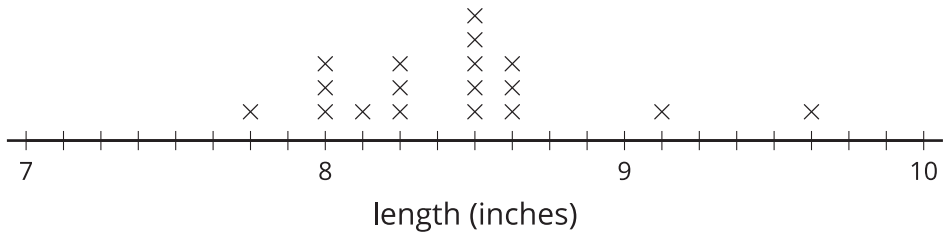


# 14.1: Shoe Lengths

Students in a fourth-grade class collected data on their shoe sizes and lengths. They plotted the shoe lengths on a line plot.



Fourth-grade Shoe Lengths



The line plot is missing the shoe lengths of six students:

- 9       $9\frac{1}{8}$        $8\frac{6}{8}$        $7\frac{6}{8}$        $9\frac{2}{8}$        $8\frac{1}{8}$

1. Complete the line plot with the missing data.
2. Use the completed line plot to answer the following questions:
  - a. What is the largest shoe length?
  - b. What is the smallest shoe length?
  - c. What is the difference between the largest and smallest shoe lengths?  
Explain or show your reasoning.
  - d. The student who recorded 9 inches for her shoe length made a mistake when reading the shoe chart. Her actual shoe length is  $\frac{7}{8}$  inches shorter.  
What's her shoe length? Plot her corrected data point on the line plot.

## 14.2: Larger Shoes, Anyone?

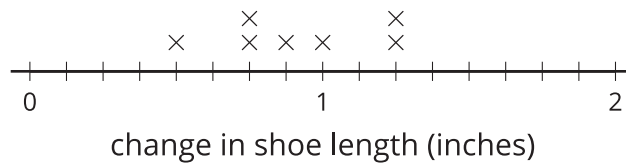
Ten students recorded their shoe lengths in third grade and then again in fourth grade.

They found how much their feet have grown over a year and organized the data in a table and on a line plot.

student	change in shoe length (inches)
Jada	$\frac{5}{4}$
Priya	$\frac{7}{8}$
Andre	$\frac{3}{4}$
Elena	$\frac{1}{2}$
Han	$1\frac{2}{8}$

student	change in shoe length (inches)
Clare	1
Tyler	$1\frac{1}{8}$
Kiran	$\frac{6}{8}$
Diego	$1\frac{1}{4}$
Lin	$\frac{5}{8}$

How Much Have Our Feet Grown?



1. The line plot shows only seven points. Whose information is missing? Add the three missing points to the line plot.

2. If Han's shoe length now is  $9\frac{1}{8}$  inches, what was his shoe length in third grade?

3. If Priya's shoe length was  $7\frac{6}{8}$  inches last year, what's her shoe length this year?

4. Tyler made a calculation error. What he recorded,  $1\frac{1}{8}$  inches, was  $\frac{3}{8}$  inches off from the actual change in shoe length.

a. What could be the actual change in his shoe length? Explain or show your reasoning.

b. How does his error affect the line plot? Explain your reasoning.

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## Section Summary

### Section Summary

In this section, we added and subtracted fractions with the same denominator, using number lines to help with our reasoning.

First, we learned that a fraction can be decomposed into a sum of smaller fractions. For example, here are a few ways to write  $\frac{6}{10}$ :

$$\frac{6}{10} = \frac{4}{10} + \frac{2}{10}$$

$$\frac{6}{10} = \frac{5}{10} + \frac{1}{10}$$

$$\frac{6}{10} = \frac{2}{10} + \frac{2}{10} + \frac{2}{10}$$

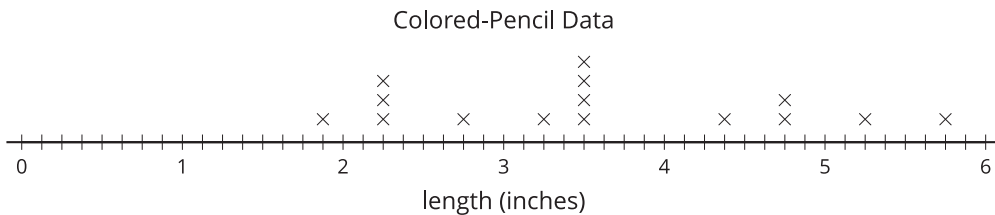
If the fraction is greater than 1, it can be decomposed into a whole number and a fraction less than 1. For instance, we can decompose  $\frac{17}{10}$  and rewrite it as  $1\frac{7}{10}$ . A number such as  $1\frac{7}{10}$  is called a **mixed number**.

$$\begin{array}{r} \frac{10}{10} + \frac{7}{10} \\ 1 + \frac{7}{10} \\ 1\frac{7}{10} \end{array}$$

Later, we decomposed fractions into sums and wrote equivalent fractions to help us add and subtract fractions. For example, to find the value of  $1\frac{2}{5} - \frac{3}{5}$ , we can:

- Decompose  $1\frac{2}{5}$  into  $1 + \frac{2}{5}$  or  $\frac{5}{5} + \frac{2}{5}$ , which is  $\frac{7}{5}$ .
- Find the value of  $\frac{7}{5} - \frac{3}{5}$ , which is  $\frac{4}{5}$ .

Finally, we organized and analyzed measurement data on line plots. The data were lengths measured to the nearest inch,  $\frac{1}{2}$  inch,  $\frac{1}{4}$  inch, and  $\frac{1}{8}$  inch.



Because the measurements have different denominators, we used equivalent fractions to plot them. Then, we used the line plots and what we know about addition and subtraction of fractions to solve problems about the data.

# Lesson 15: An Assortment of Fractions

- Let's find the heights of some stacked objects.

## Warm-up: Which One Doesn't Belong: Halves, Fourths, Sixths, and Eighths

Which one doesn't belong?

A

$$1\frac{1}{2}$$

B

$$\frac{4}{4} + \frac{2}{4}$$

C

$$\frac{12}{8}$$

D

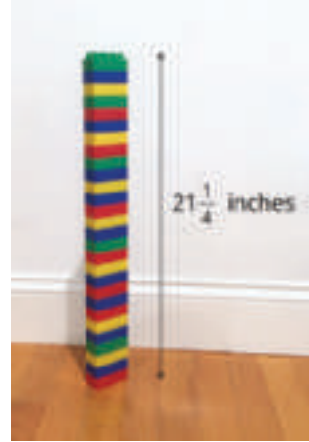
$$\frac{4}{6}$$

## 15.1: All the Way to the Top

Priya, Kiran, and Lin are using large playing bricks to make towers. Here are the heights of their towers so far:

- Priya:  $21\frac{1}{4}$  inches
- Kiran:  $32\frac{3}{8}$  inches
- Lin :  $55\frac{1}{2}$  inches

For each question, show your reasoning.



1. How much taller is Lin's tower compared to:

a. Priya's tower?

b. Kiran's tower?

2. They are playing in a room that is 109 inches tall. Priya says that if they combine their towers to make a super tall tower, it would be too tall for the room and they'll have to remove one brick.

Do you agree with Priya? Explain your reasoning.

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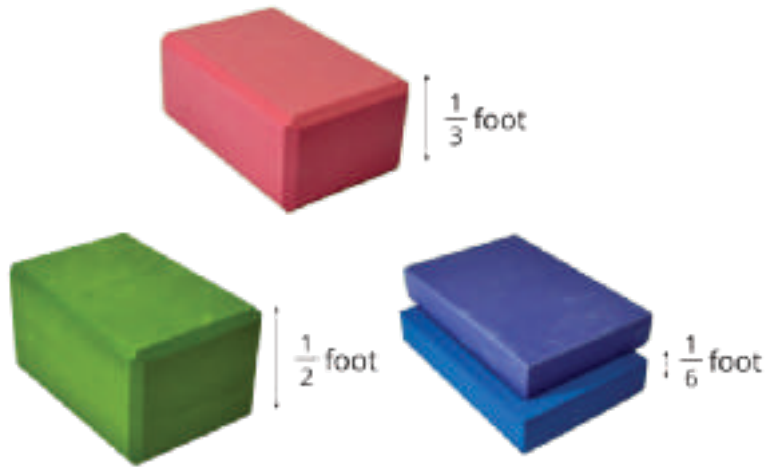
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## 15.2: Stacks of Blocks

Andre is building a tower out of foam blocks. The blocks come in three different thicknesses:  $\frac{1}{2}$  foot,  $\frac{1}{3}$  foot, and  $\frac{1}{6}$  foot.



1. Andre stacks one block of each size. Will that stack be more than 1 foot tall? Explain or show how you know.
  
  
  
  
  
  
  
  
  
  
2. Can Andre use only the  $\frac{1}{6}$ -foot and  $\frac{1}{3}$ -foot blocks to make a stack that is  $1\frac{1}{2}$  feet tall? If you think so, show one or more ways. If not, explain why not.
  
  
  
  
  
  
  
  
  
  
3. Can Andre use only the  $\frac{1}{6}$ -foot and  $\frac{1}{2}$ -foot blocks to make a stack that is  $1\frac{1}{3}$  feet tall? If so, show one or more ways. If not, explain why not.

# Lesson 16: Tenths and Hundredths, Together

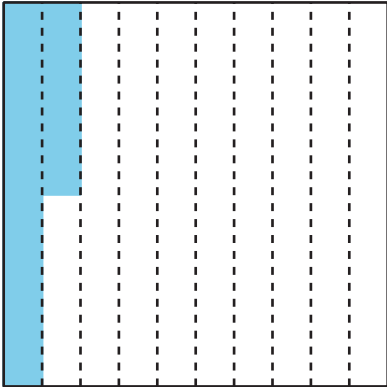
- Let's add some tenths and hundredths.

## Warm-up: Notice and Wonder: Shaded Rectangles and Squares

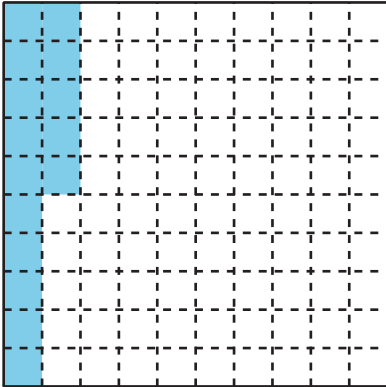
Each large square represents 1.

What do you notice? What do you wonder?

A



B



## 16.1: Tenths and Hundredths

1. Complete the table with equivalent fractions in tenths or hundredths. In the last row, write a new pair of equivalent fractions.

	tenths	hundredths
a.	$\frac{1}{10}$	
b.	$\frac{4}{10}$	
c.	$\frac{6}{10}$	
d.		$\frac{50}{100}$
e.		$\frac{90}{100}$
f.	$\frac{12}{10}$	
g.		$\frac{200}{100}$
h.	$2\frac{3}{10}$	
i.		$\frac{125}{100}$
j.		

2. Name some fractions that are:

a. between  $\frac{50}{100}$  and  $\frac{60}{100}$

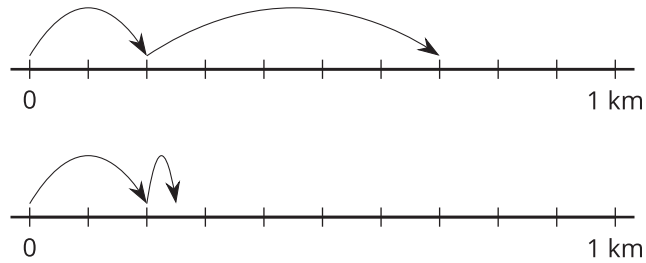
b. between  $\frac{3}{10}$  and  $\frac{4}{10}$

Be prepared to explain your reasoning.

## 16.2: Walk, Stop, and Sip

Noah walks  $\frac{2}{10}$  kilometer (km), stops for a drink of water, walks  $\frac{5}{100}$  kilometer, and stops for another sip.

1. Which number line diagram represents the distance Noah has walked? Explain how you know.



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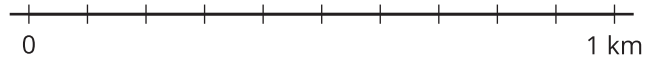
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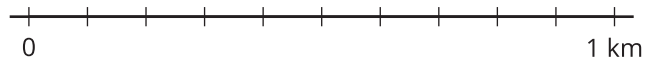
2. The diagram that you didn't choose represents Jada's walk. Write an equation to represent:
- the total distance Jada has walked
  - the total distance Noah has walked

3. Find the value of each of the following sums. Show your reasoning. Use number lines if you find them helpful.

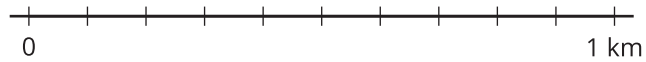
a.  $\frac{5}{10} + \frac{1}{10}$



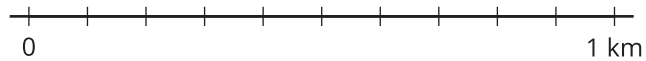
b.  $\frac{50}{100} + \frac{10}{100}$



c.  $\frac{5}{10} + \frac{30}{100}$



d.  $\frac{15}{100} + \frac{4}{10}$



# Lesson 17: Sums of Tenths and Hundredths

- Let's add more tenths and hundredths.

## Warm-up: Which One Doesn't Belong: Tenths and Hundredths

Which one doesn't belong?

A.

$$\frac{48}{100}$$

B.

$$\frac{8}{10}$$

C.

$$\frac{120}{100}$$

D.

$$\frac{70}{100}$$

## 17.1: Card Sort: Less Than, Equal to, or Greater Than 1?

1. Sort the cards from your teacher based on whether the value of the expression is less than 1, equal to 1, or greater than 1.

When done, make a quick list of which expressions you have in each group.

2. Visit the sorted collection of another group.
  - Did they sort the cards the same way?
  - Select 1–2 cards that you have a question about or whose placement you disagree with.
  - Leave a note for the group members to discuss.
3. Return to your collection.
  - Discuss any notes that are left for your group, or revise your sorting decision based on what you learned from another group.
  - Record the expressions here.

less than 1	equal to 1	greater than 1

## 17.2: What's Missing?

1. Each equation is missing a fraction in hundredths. Find the fraction that makes each equation true.

a.  $\frac{10}{100} + \underline{\hspace{2cm}} = \frac{30}{100}$

b.  $\underline{\hspace{2cm}} + \frac{2}{10} = \frac{80}{100}$

c.  $\frac{7}{10} + \underline{\hspace{2cm}} = \frac{94}{100}$

d.  $\frac{9}{100} + \underline{\hspace{2cm}} = \frac{8}{10}$

e.  $\frac{16}{100} + \frac{4}{10} = \underline{\hspace{2cm}}$

f.  $\underline{\hspace{2cm}} + \frac{14}{10} = \frac{172}{100}$

2. Each equation is missing a fraction in tenths or hundredths. Find the fraction that makes each equation true.

a.  $\frac{20}{100} + \underline{\hspace{2cm}} = \frac{28}{100}$

b.  $\frac{110}{100} + \underline{\hspace{2cm}} = \frac{15}{10}$

c.  $\frac{61}{100} + \frac{3}{10} = \underline{\hspace{2cm}}$

d.  $\frac{9}{10} + \underline{\hspace{2cm}} = \frac{170}{100}$

e.  $\underline{\hspace{2cm}} + \frac{72}{100} = \frac{102}{100}$

f.  $\frac{15}{100} + \underline{\hspace{2cm}} = 1\frac{55}{100}$

### 17.3: Fraction Action: Tenths, Hundredths

Play Fraction Action with 2 players:

- Shuffle the cards from your teacher. Place the cards in a stack, face down.
- Each player turns 2 cards over and adds the fractions on the cards.
- Compare the sums. The player with the greater sum wins that round and keeps all four cards.
- If the sums are equivalent, each player turns one more card over and adds the value to their sum. The player with the greater new sum keeps all cards.
- The player with the most cards wins the game.

Play Fraction Action with 3 or 4 players:

- The player with the greatest sum of fractions wins the round.
- If 2 or more players have the greatest sum, those players turn two more cards over and find their sum. The player with the greatest sum keeps all the cards.

Record any pair of fractions whose sum is challenging to find here.

\_\_\_\_\_ and \_\_\_\_\_

\_\_\_\_\_ and \_\_\_\_\_

\_\_\_\_\_ and \_\_\_\_\_

\_\_\_\_\_ and \_\_\_\_\_

## Lesson 18: Lots of Fractions to Add

- Let's add tenths and hundredths again, more than two at a time.

### Warm-up: Number Talk: A Bunch of Numbers

Find the value of each expression mentally.

- $54 + 2 + 18$
  
  
  
  
  
  
  
  
  
  
- $61 + 104 + 39$
  
  
  
  
  
  
  
  
  
  
- $25 + 63 + 75 + 7$
  
  
  
  
  
  
  
  
  
  
- $50 + 106 + 19 + 101$

# 18.1: Stack Centavos and Pesos

Diego and Lin each have a small collection of Mexican coins.

The table shows the thickness of different coins in centimeters (cm) and how many of each Diego and Lin have.



coin value	thickness in cm	Diego	Lin
1 centavo	$\frac{12}{100}$	3	1
10 centavos	$\frac{22}{100}$	0	1
1 peso	$\frac{16}{100}$	0	1
2 pesos	$\frac{14}{100}$	0	1
5 pesos	$\frac{2}{10}$	1	1
20 pesos	$\frac{25}{100}$	2	1

1. If Diego and Lin each stack their centavo coins, whose stack would be taller? Show your reasoning.
  
  
  
  
  
  
  
  
  
  
2. If they each stack their peso coins, whose stack would be taller? Show your reasoning.

3. If they each stack all their coins, whose stack would be taller? Show your reasoning.

4. If they combine their coins to make a single stack, would it be more than 2 centimeters tall? Show your reasoning.

## 18.2: More Than Two Fractions

Find the value of at least 3 of the expressions. Show your reasoning.

1.  $\frac{2}{100} + \frac{13}{10} + \frac{1}{10} + \frac{8}{100}$

2.  $\frac{50}{10} + \frac{16}{100} + \frac{2}{10}$

3.  $\frac{3}{10} + \frac{4}{100} + \frac{7}{10} + \frac{26}{100}$

4.  $\frac{4}{100} + 3\frac{2}{10} + 1\frac{5}{10}$

5.  $1\frac{1}{10} + 5\frac{2}{100} + \frac{78}{100}$

6.  $2\frac{7}{10} + \frac{2}{100} + \frac{8}{10}$

## Section Summary

### Section Summary

In this section, we learned more ways to add fractions and to solve problems that involve adding, subtracting, and multiplying fractions.

We started by adding tenths and hundredths, using what we know about equivalent fractions. For example, to find the sum of  $\frac{4}{10}$  and  $\frac{30}{100}$ , we can:

- Write  $\frac{4}{10}$  as  $\frac{40}{100}$ , and then find  $\frac{40}{100} + \frac{30}{100}$ , or
- Write  $\frac{30}{100}$  as  $\frac{3}{10}$ , and then find  $\frac{4}{10} + \frac{3}{10}$ .

We learned that when adding a few fractions, it may help to rearrange or group them. For instance:

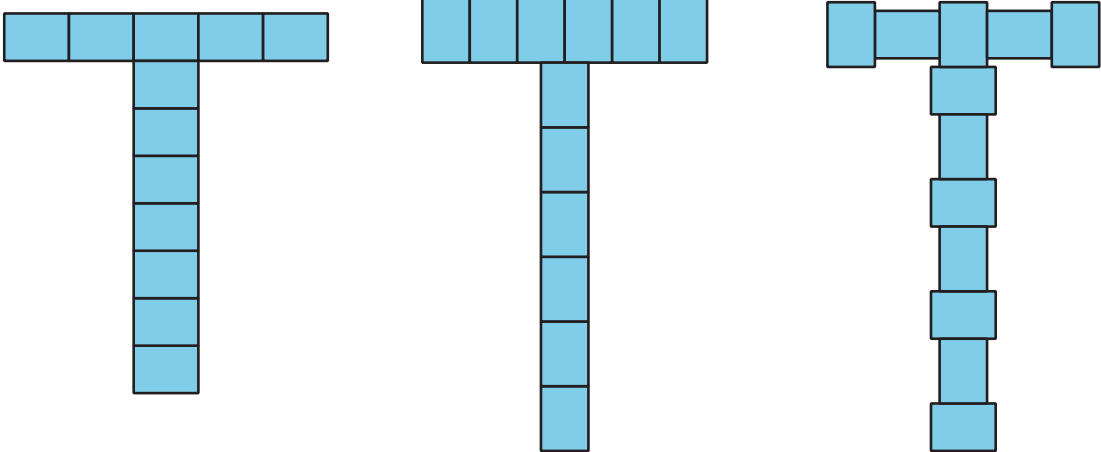
- $\frac{6}{100} + \frac{2}{10} + \frac{74}{100}$  can be rearranged as  $\frac{6}{100} + \frac{74}{100} + \frac{2}{10}$ .
- Next, the hundredths can be added first, giving  $\frac{80}{100} + \frac{2}{10}$ .
- Then, we can write an equivalent fraction for  $\frac{80}{100}$  and find  $\frac{8}{10} + \frac{2}{10}$ , or write an equivalent fraction for  $\frac{2}{10}$  and find  $\frac{80}{100} + \frac{20}{100}$ .

# Lesson 19: Flexible with Fractions

- Let's solve all kinds of problems involving fractions.

## Warm-up: Notice and Wonder: Sticky Notes

What do you notice? What do you wonder?



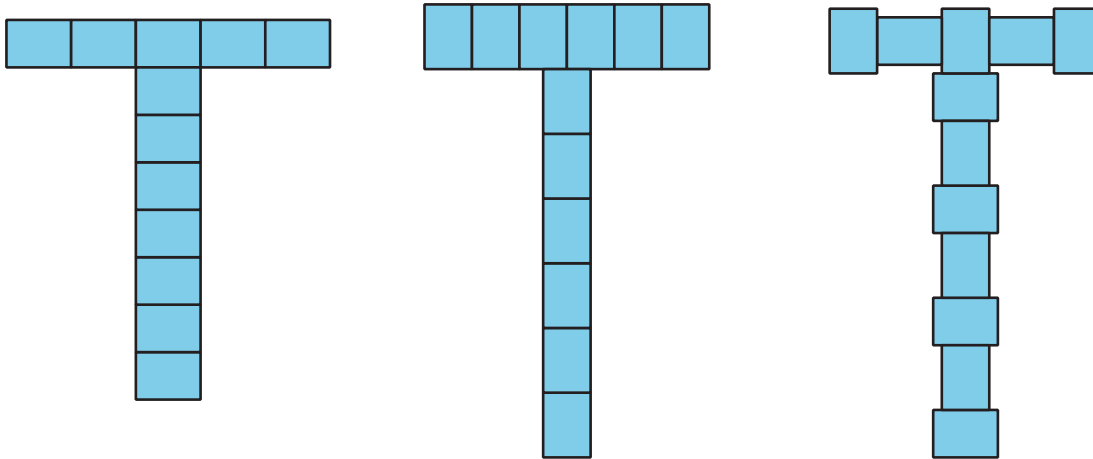
## 19.1: Sticky-note Designs



Tyler is using small sticky notes to make a T shape to decorate a folder.

The longer side of the sticky note is  $\frac{15}{8}$  inches. The shorter side is  $\frac{11}{8}$  inches. The folder is 9 inches wide and 12 inches tall.

Here are three ways he could arrange the sticky notes.



Is the folder tall enough and wide enough for his designs? If so, which design(s) would fit? Show your reasoning.

## 19.2: Hiking Trails

Jada and Noah's class are hiking at a park. Here is a map of the trails. The length of each trail is shown.



1. Jada and Noah hike the orange trail from point F to point E, make one full loop on the red trail back to point E, and then hike from E back to F.

How many miles do they hike? Show your reasoning.

2. Here are two expressions that represent some hiking situations and can help to answer two questions. What question might each expression help to answer? Write the question and the answer.

a.  $\frac{6}{100} + \frac{65}{100} + 1\frac{2}{100} + \frac{41}{100} + \frac{24}{100}$

b.  $(2 \times \frac{14}{10}) + (2 \times \frac{6}{100})$

3. Use the distances on the map to write a new question and find its answer. Then, trade questions with a partner and answer one another's question.

## 19.3: Find a Match

Your teacher will give you one card with an expression on it.

1. Find the value of the expression.
2. Find a classmate whose card also has the same value. Prove to each other that you're a match.
3. Work with your partner to find at least two features that your expressions share (other than the fact that they have the same value.)
4. Write one more expression that has the same value but uses a different operation.

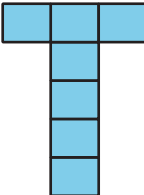
# Lesson 20: Sticky Notes

- Let's make a design using sticky notes.

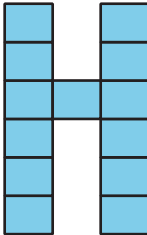
## Warm-up: Which One Doesn't Belong: Sticky Notes

Which one doesn't belong?

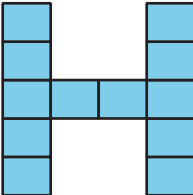
A



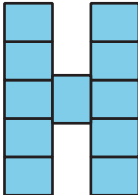
B



C



D



## 20.1: Estimation Exploration: Sticky Notes

1. How many sticky notes will fit across the top or the side of the page?

Record an estimate that is:

too low	about right	too high

2. What information do you need to help you make a better estimate?
3. With the new information you have now, make a better estimate. Show or explain your reasoning.
4. Write an expression that represents your estimate that shows how many sticky notes fit across or on the side of the paper.

## 20.2: Design Your Initial

Design your initial with sticky notes.

1. Plan your design and determine the number of sticky notes that you need.
2. Write at least two equations that show your design will fit on a piece of paper.
3. Take turns sharing your design with your partner.
4. Get the supplies and make your design.

## Section A: Practice Problems

### 1. Pre-unit

What fraction of the rectangle is shaded? Explain how you know.




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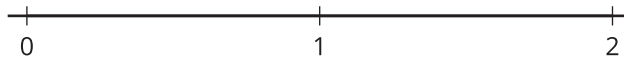
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### 2. Pre-unit

a. Locate and label  $\frac{3}{4}$  and  $\frac{6}{4}$  on the number line.



b. Explain why your points represent  $\frac{3}{4}$  and  $\frac{6}{4}$ .

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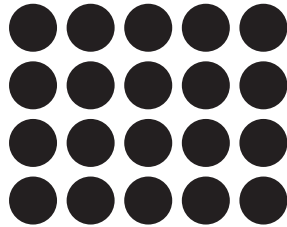


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**3. Pre-unit**

Write a multiplication expression for each image. Explain your reasoning.

a.

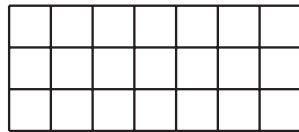



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b.




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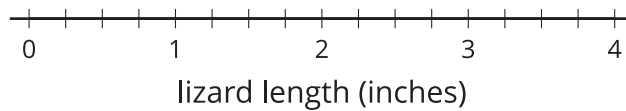
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**4. Pre-unit**

Here are the lengths of some lizards in inches. Use the lengths to complete the line plot.

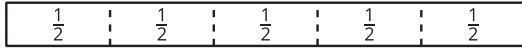
$2\frac{1}{4}$        $1\frac{1}{2}$        $2\frac{2}{4}$       3       $3\frac{2}{4}$       2  
 $2\frac{1}{4}$        $2\frac{1}{4}$        $2\frac{3}{4}$       2       $2\frac{1}{4}$       3

Length of Lizards

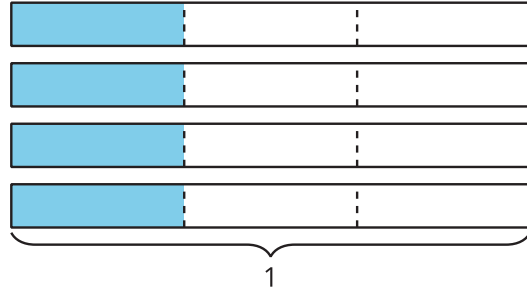


5. Write an expression that matches each diagram. Then, find the value of each expression.

a.



b.

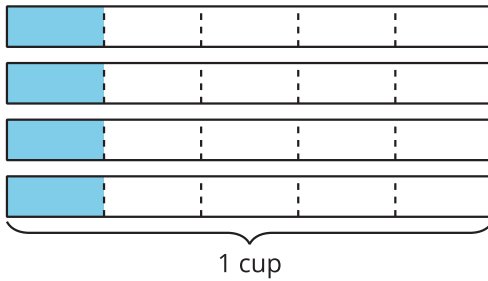


(From Unit 3, Lesson 1.)

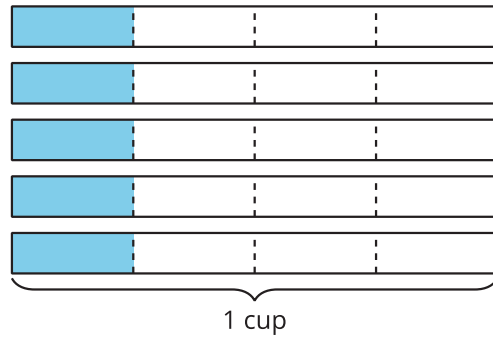
6. Five friends go on a hike. They each bring  $\frac{1}{4}$  cup of nuts.

a. If the shaded parts represent the amount of nuts the friends bring on their hike, which diagram matches the story? Explain your reasoning.

**A**



**B**




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b. How many cups of nuts do the friends bring on the hike?

(From Unit 3, Lesson 2.)

7. Kiran's cat eats  $\frac{1}{2}$  cup of food each day.
- How much food does Kiran's cat eat in a week?
  - Draw a diagram to represent the situation.

(From Unit 3, Lesson 3.)

8. a. Draw a diagram to show  $3 \times \frac{7}{8}$ .

- b. How does the diagram help you find the value of the expression  $3 \times \frac{7}{8}$ ?

(From Unit 3, Lesson 4.)

9. Find the number that makes each equation true. Draw a diagram if it is helpful.
- $\frac{10}{3} = \underline{\hspace{1cm}} \times \frac{1}{3}$
  - $\frac{10}{3} = \underline{\hspace{1cm}} \times \frac{2}{3}$
  - $\frac{10}{3} = \underline{\hspace{1cm}} \times \frac{5}{3}$

(From Unit 3, Lesson 5.)

10. Each bead weighs  $\frac{5}{8}$  gram. How much do 7 beads weigh? Explain or show your reasoning.

(From Unit 3, Lesson 6.)

**11. Exploration**

- a. Measure how thick your workbook is to the nearest  $\frac{1}{8}$  inch.
- b. If all of your classmates stacked their workbooks together, how tall would the stack be? Explain or show your reasoning.

- c. Check your answer by measuring, if possible.

**12. Exploration**

Diego walked the same number of miles to school each day. He says that he walked  $\frac{48}{5}$  miles in total, but does not say how many days that distance includes.

What are some possible number of days Diego counted and the distance he walked each of those days?

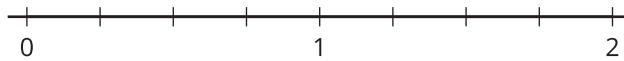
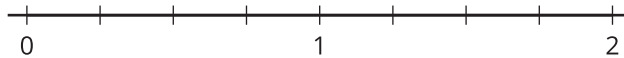
## Section B: Practice Problems

1. a. Write  $\frac{4}{3}$  in as many ways as you can as a sum of fractions.

b. Write  $\frac{9}{8}$  in at least 3 different ways as a sum of fractions.

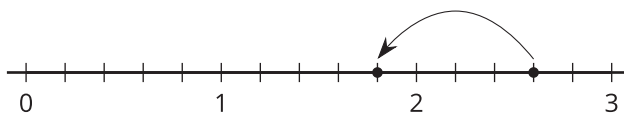
(From Unit 3, Lesson 7.)

2. a. Draw "jumps" on the number lines to show two ways to use fourths to make a sum of  $\frac{7}{4}$ .



b. Represent each combination of jumps as an equation.

(From Unit 3, Lesson 8.)



3. a. Explain how the diagram represents  $\frac{13}{5} - \frac{4}{5}$ .

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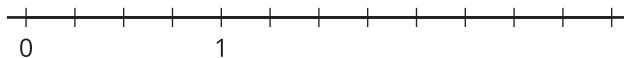
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Use the diagram to find the value of  $\frac{13}{5} - \frac{4}{5}$ .

- b. Use a number line to represent and find the difference  $\frac{9}{4} - \frac{3}{4}$ .



(From Unit 3, Lesson 9.)

4. Show two different ways to find the difference:  $2 - \frac{3}{4}$

(From Unit 3, Lesson 10.)

5. Elena is making friendship necklaces and wants the chain and clasp to be a total of  $18\frac{1}{4}$  inches long. She is going to use a clasp that is  $2\frac{3}{4}$  inches long. How long does her chain need to be? Explain or show your reasoning.

(From Unit 3, Lesson 11.)

6. For each of the expressions, explain whether you think it would be helpful to decompose one or more numbers to find the value of the expression.

a.  $\frac{4}{3} + \frac{5}{3}$

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b.  $5\frac{1}{5} - 2\frac{2}{5}$

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c.  $9\frac{5}{6} - 6\frac{1}{6}$

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(From Unit 3, Lesson 12.)

7. The lengths of the shoes of a dad and his two daughters are shown.



For each question, show your reasoning.

a. How much longer is the older daughter's shoes than her sister's?

b. Which is longer, the dad's shoes or the combined lengths of his daughters' shoes?

(From Unit 3, Lesson 12.)

### 8. Exploration

A chocolate chip cookie recipe calls for  $2\frac{3}{4}$  cups of flour. You only have a  $\frac{1}{4}$ -cup measuring cup and a  $\frac{3}{4}$ -cup measuring cup that you can use.

- a. What are different combinations of the measuring cups that you can use to get a total of  $2\frac{3}{4}$  cups of flour?

- b. Write each of the combinations as an addition equation.

**9. Exploration**

The table shows some lengths of different shoe sizes in inches.

U.S. shoe size	insole length
1	$7\frac{6}{8}$
1.5	8
2	$8\frac{1}{8}$
2.5	$8\frac{2}{8}$
3	$8\frac{4}{8}$
3.5	$8\frac{5}{8}$
4	$8\frac{6}{8}$
4.5	9
5	$9\frac{1}{8}$
5.5	$9\frac{2}{8}$
6	$9\frac{4}{8}$
6.5	$9\frac{5}{8}$
7	$9\frac{6}{8}$

a. What do you notice about the insole lengths as the size increases?

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b. What will the insole length increase be from size 7 to 7.5? What is the insole length of a size 7.5 shoe?

c. Predict the insole length for sizes 9, 10, and 12. Explain your prediction. Then solve to find out if your prediction is true.

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## Section C: Practice Problems

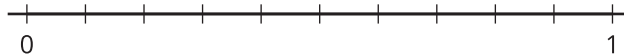
1. Andre is building a tower out of different foam blocks. These blocks come in three different thicknesses:  $\frac{1}{2}$ -foot,  $\frac{1}{4}$ -foot, and  $\frac{1}{8}$ -foot.

Andre stacks two  $\frac{1}{2}$ -foot blocks, two  $\frac{1}{4}$ -foot blocks, and two  $\frac{1}{8}$ -foot blocks to create a tower. What will the height of the tower be in feet? Explain or show how you know.

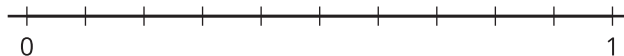
(From Unit 3, Lesson 15.)

2. Find the value of each of the following sums. Show your reasoning. Use number lines if you find them helpful.

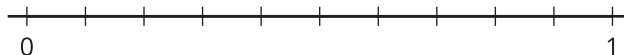
a.  $\frac{1}{10} + \frac{3}{100}$



b.  $\frac{24}{100} + \frac{4}{10}$



c.  $\frac{7}{10} + \frac{13}{100}$



(From Unit 3, Lesson 16.)

3. Is the value of each expression greater than, less than or equal to 1? Explain how you know.

a.  $\frac{3}{10} + \frac{7}{100}$

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b.  $\frac{13}{10} + \frac{7}{100}$

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c.  $\frac{30}{100} + \frac{7}{10}$

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(From Unit 3, Lesson 17.)

4. Diego and Lin continued to play with their coins.

Diego said that he has exactly 3 coins whose thickness adds up to  $\frac{50}{100}$  cm.

What coins does Diego have? Explain or show your reasoning.

coin	thickness in cm
1 centavo	$\frac{12}{100}$
10 centavos	$\frac{22}{100}$
1 peso	$\frac{16}{100}$
2 pesos	$\frac{14}{100}$
5 pesos	$\frac{2}{10}$
20 pesos	$\frac{25}{100}$

(From Unit 3, Lesson 18.)

### 5. Exploration

A chocolate cake recipe calls for 2 cups of flour. You gather your measuring cups and notice you have these sizes:  $\frac{1}{2}$  cup,  $\frac{1}{3}$  cup,  $\frac{1}{4}$  cup, and  $\frac{1}{6}$  cup.

- a. What are the different ways you could use all 4 measuring cups to measure 2 cups of flour?

- b. What are other ways you could use just some of the 4 measuring cups to measure exactly 2 cups of flour?

## 6. Exploration

A dime is worth  $\frac{1}{10}$  of a dollar and a penny is worth  $\frac{1}{100}$  of a dollar.

a. If I have  $\frac{89}{100}$  of a dollar, how many different combinations of dimes and pennies could I have? Use equations to show your reasoning.

b. A nickel is worth  $\frac{5}{100}$  of a dollar. How many different combinations of dimes, nickels and pennies could I have if I still have  $\frac{89}{100}$  of a dollar? Use equations to show your reasoning.

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