



Core Knowledge[®] MATHEMATICS

Fraction Equivalence and Comparison



Teacher Guide

$$2 \times 2 = 4$$

$$4 \times 2 = 8$$

$$8 : 4 = 2$$

$$4 - 2 = 2$$



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Fraction Equivalence and Comparison

Table of Contents

Introduction	i
Unit Overview	1
Section Overview	2
Center Overview	8
Lessons Plans and Student Task Statements:	
Section A: Lessons 1–6 Size and Location of Fractions	18
Section B: Lessons 7–11 Equivalent Fractions	76
Section C: Lessons 12–17 Fraction Comparison	121
Teacher Resources	171

Family Support Materials

Assessments

Cool Downs

Instructional Masters



Fraction Equivalence and Comparison Teacher Guide

Core Knowledge Mathematics™

Unit 2: Fraction Equivalence and Comparison

At a Glance

Unit 2 is estimated to be completed in 18-19 days including 2 days for assessment.

This unit is divided into three sections including 16 lessons and 1 optional lesson.

- Section A—Size and Location of Fractions (Lessons 1-6)
- Section B—Equivalent Fractions (Lessons 7-11)
- Section C—Fraction Comparison (Lessons 12-17)

On pages 6-7 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses five new student centers.

- Get Your Numbers in Order
- Mystery Number
- Number Line Scoot
- Compare
- How Close?

Unit 2: Fraction Equivalence and Comparison

Unit Learning Goals

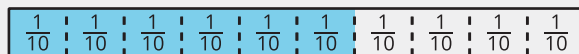
- Students generate and reason about equivalent fractions and compare and order fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.

In this unit, students extend their prior understanding of equivalent fractions and comparison of fractions.

In grade 3, students partitioned shapes into parts with equal area and expressed the area of each part as a unit fraction. They learned that any unit fraction $\frac{1}{b}$ results from a whole partitioned into b equal parts. They used unit fractions to build non-unit fractions, including fractions greater than 1, and represent them on fraction strips and tape diagrams. The denominators of these fractions were limited to 2, 3, 4, 6, and 8. Students also worked with fractions on a number line, establishing the idea of fractions as numbers and equivalent fractions as the same point on the number line.

Here, students follow a similar progression of representations. They use fraction strips, tape diagrams, and number lines to make sense of the size of fractions, generate equivalent fractions, and compare and order fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students generalize that a fraction $\frac{a}{b}$ is equivalent to fraction $\frac{(n \times a)}{(n \times b)}$ because each unit fraction is being broken into n times as many equal parts, making the size of the part n times as small $\frac{1}{(n \times b)}$ and the number of parts in the whole n times as many ($n \times a$). For example, we can see $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ because when each fifth is partitioned into 2 parts, there are 2×3 or 6 shaded parts, twice as many as before, and the size of each part is half as small, $\frac{1}{(2 \times 5)}$ or $\frac{1}{10}$.



As the unit progresses, students use equivalent fractions and benchmarks such as $\frac{1}{2}$ and 1 to reason about the relative location of fractions on a number line, and to compare and order fractions.

Section A: Size and Location of Fractions

Standards Alignments

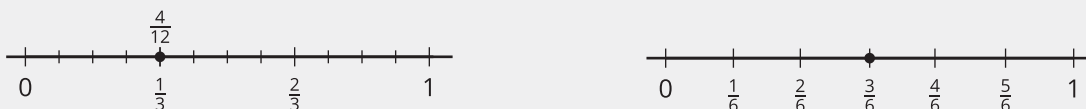
Building On	3.NF.A.1, 3.NF.A.2, 3.NF.A.2.a, 3.NF.A.3, 3.NF.A.3.b, 3.NF.A.3.d, 3.OA.B.5
Addressing	4.NF.A.1, 4.NF.A.2
Building Towards	4.NBT.B.4, 4.NF.A, 4.NF.A.1, 4.NF.A.2

Section Learning Goals

- Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams.
- Reason about the location of fractions on the number line.

In this section, students revisit ideas and representations of fractions from grade 3, working with denominators that now include 5, 10, and 12. They use physical fraction strips, diagrams of fraction strips, tape diagrams, and number lines to make sense of the size of fractions and fractional relationships.

Students reason about the relationship between fractions where one denominator is a multiple of the other denominator (such as $\frac{1}{5}$ and $\frac{1}{10}$, or $\frac{1}{6}$ and $\frac{1}{12}$). They consider different ways to represent these relationships. Students also compare fractions to benchmarks such as $\frac{1}{2}$ and 1.



The work in this section prepares students to reason about equivalence and comparison of fractions in the subsequent lessons.

 PLC: Lesson 4, Activity 2, Fractions on Number Lines

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)

Section B: Equivalent Fractions

Standards Alignments

Building On	3.NF.A.1, 3.NF.A.3.b, 3.OA.B.5
Addressing	4.NF.A.1
Building Towards	4.NBT.B.5, 4.NF.A.1

Section Learning Goals

- Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Use visual representations to reason about fraction equivalence, including using benchmarks such as $\frac{1}{2}$ and 1.

In this section, students develop their ability to reason about and generate equivalent fractions. They begin by using number lines as a tool for finding equivalent fractions and verifying equivalence of two fractions.



Through repeated reasoning, students notice regularity in the visual representations and begin to make sense of a numerical way to determine equivalence and generate equivalent fractions (MP8). They generalize that fraction $\frac{a}{b}$ is equivalent to fraction $\frac{n \times a}{n \times b}$.

Note that students do not need to describe this generalization in algebraic notation. Given their understanding of the size of fractions and relationship between fractions, however, they should be able to explain it with fractions that have denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

As they identify and generate equivalent fractions numerically, students apply their knowledge of factors and multiples from an earlier unit.

   PLC: Lesson 8, Activity 1, Handy Number Lines

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Section C: Fraction Comparison

Standards Alignments

Building On	3.NF.A.1, 4.OA.B.4
Addressing	4.NF.A, 4.NF.A.1, 4.NF.A.2
Building Towards	4.NBT.B.4, 4.NF.A, 4.NF.A.2

Section Learning Goals

- Use visual representations or a numerical process to reason about fraction comparison.

By the time they reach this section, students have an expanded set of understandings and strategies for reasoning about the size of fractions. Here, they further develop these skills and work to compare fractions with different numerators and different denominators.

To make comparisons, students may use visual representations, equivalent fractions, and their understanding of the size of fractions (for instance, relative to benchmarks such as $\frac{1}{2}$ and 1). They may rely on the meaning of the numerator and denominator, and choose a way to compare based on the numbers at hand. Students record the results of comparisons with symbols $<$, $=$, or $>$.

At the end of the section, students learn to write equivalent fractions with a particular denominator as a way to compare any fractions, another opportunity to apply the idea of factors and multiples. Having a numerical strategy notwithstanding, students are still encouraged to use flexible methods to reason about the relative size of fractions.

 PLC: Lesson 12, Activity 1, The Greatest of Them All

Suggested Centers

- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)
- Compare (1–5), Stage 5: Fractions (Addressing)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Throughout the Unit

Students continue to develop strategies for multiplying numbers mentally—building on their fluency from grade 3 and applying the properties of multiplication. The Number Talks in this unit support this goal, focusing on factors 2, 4, 5, 6, 8, 10, and 12. Students engage in strategies such as doubling and halving, relating these the folding of fraction strips and partitioning of tape diagrams into smaller unit

fractions.

Here is a sampling of the Number Talk warm-ups in the unit.

lesson 5	lesson 9	lesson 16
2×12	10×6	5×6
4×12	10×12	5×12
8×12	10×24	6×12
16×12	5×24	11×12

These factors are intentionally chosen to build flexibility with the unit fractions in this unit. As students see the relationship between these factors and their products in Number Talks, they can become more efficient in determining equivalency and comparing fractions with these denominators.

Materials Needed

LESSON	GATHER	COPY
A.1	<ul style="list-style-type: none"> ● Straightedges 	<ul style="list-style-type: none"> ● Fraction Strips (groups of 2)
A.2	<ul style="list-style-type: none"> ● Materials from a previous lesson ● Straightedges 	<ul style="list-style-type: none"> ● none
A.3	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● none
A.4	<ul style="list-style-type: none"> ● Straightedges 	<ul style="list-style-type: none"> ● none
A.5	<ul style="list-style-type: none"> ● Straightedges 	<ul style="list-style-type: none"> ● none
A.6	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● Where Do They Belong (groups of 2)
B.7	<ul style="list-style-type: none"> ● Tools for creating a visual display 	<ul style="list-style-type: none"> ● none
B.8	<ul style="list-style-type: none"> ● Tape (painter's or masking) 	<ul style="list-style-type: none"> ● none
B.9	<ul style="list-style-type: none"> ● Rulers or straightedges ● Sticky notes 	<ul style="list-style-type: none"> ● How Do You Know (groups of 15)
B.10	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● none
B.11	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● Fractions Galore (groups of 3)
C.12	<ul style="list-style-type: none"> ● Colored pencils 	<ul style="list-style-type: none"> ● none
C.13	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● none
C.14	<ul style="list-style-type: none"> ● Tools for creating a visual display 	<ul style="list-style-type: none"> ● none
C.15	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● none
C.16	<ul style="list-style-type: none"> ● none 	<ul style="list-style-type: none"> ● Compare Stage 3-8 Directions (groups of 2) ● Fraction Cards Grade 4 (groups of 2)

C.17

- Markers
 - Paper
 - Paper clips
 - Tape (painter's or masking)
- none

Center: Get Your Numbers in Order (1–5)

Stage 3: Denominators 2, 3, 4, or 6

Lessons

- Grade4.2.A1 (addressing)
- Grade4.2.A2 (addressing)
- Grade4.2.A3 (addressing)
- Grade4.2.A4 (addressing)
- Grade4.2.A5 (addressing)
- Grade4.2.A6 (addressing)

Stage Narrative

Students choose cards with fractions with denominators of 2, 3, 4, or 6. Students write their number in any space on the board, as long as the numbers from left to right go from least to greatest. If students cannot place their number, they get a point. The player with the fewest points when the board is filled is the winner.

Standards Alignments

Addressing 4.NF.A.2

Materials to Gather

Dry erase markers, Sheet protectors

Materials to Copy

Fraction Cards Grade 3 (groups of 2), Get Your Numbers in Order Stage 3 and 4 Gameboard (groups of 2)

Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100

Lessons

- Grade4.2.B7 (addressing)
- Grade4.2.B8 (addressing)
- Grade4.2.B9 (addressing)
- Grade4.2.B10 (addressing)
- Grade4.2.B11 (addressing)

Stage Narrative

Students choose cards with fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students write their number in any space on the board, as long as the numbers from left to right go from least to greatest. If students cannot place their number, they get a point. The player with the fewest points when the board is filled is the winner.

Standards Alignments

Addressing 4.NF.A.2

Materials to Gather

Dry erase markers, Sheet protectors

Materials to Copy

Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2), Get Your Numbers in Order Stage 3 and 4 Gameboard (groups of 2)

Center: Mystery Number (1–4)

Stage 3: Fractions with Denominators 2, 3, 4, 6

Lessons

- Grade4.2.A1 (supporting)
- Grade4.2.A2 (supporting)
- Grade4.2.A3 (supporting)

Stage Narrative

Students choose a mystery fraction (with a denominator of 2, 3, 4, or 6) from the gameboard. Students give clues based on the given vocabulary.

Standards Alignments

Addressing 3.NF.A

Materials to Copy

Mystery Number Stage 3 Gameboard (groups of 2)

Stage 4: Fractions with Denominators 5, 8, 10, 12, 100

Lessons

- Grade4.2.B7 (addressing)
- Grade4.2.B8 (addressing)
- Grade4.2.B9 (addressing)
- Grade4.2.B10 (addressing)
- Grade4.2.B11 (addressing)
- Grade4.2.C12 (addressing)
- Grade4.2.C13 (addressing)

Stage Narrative

Students choose a mystery fraction (with a denominator of 5, 8, 10, 12, or 100) from the gameboard. Students give clues based on the given vocabulary.

Standards Alignments

Addressing 4.NF.A

Materials to Copy

Mystery Number Stage 4 Gameboard (groups of 2)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.5.A

Stage 3

Addressing

- Grade3.5.A
- Grade3.5.B

Center: Number Line Scoot (2–3)

Stage 3: Halves, Thirds, Fourths, Sixths and Eighths

Lessons

- Grade4.2.A4 (supporting)
- Grade4.2.A5 (supporting)
- Grade4.2.A6 (supporting)

Stage Narrative

Students take turns rolling a number cube and using the number as a numerator in a fraction with a denominator of 2, 3, 4, 6, or 8. Students move their centimeter cube that interval on one of the shared number lines. Each time a cube lands exactly on the last tick mark of one of the number lines, the player who moved it keeps the cube and puts a new cube on zero on that number line. The first player to collect five cubes wins.

Standards Alignments

Addressing 3.NF.A.2.b

Materials to Gather

Centimeter cubes, Number cubes

Materials to Copy

Number Line Scoot Stage 3 Directions (groups of 2), Number Line Scoot Stage 3 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 12 centimeter cubes.

Stages used in Grade 3

Stage 1

Supporting

- Grade3.5.A

Stage 2

Addressing

- Grade3.5.A
- Grade3.5.B

Stage 3

Addressing

- Grade3.5.B
- Grade3.5.C
- Grade3.5.D

Center: Compare (1–5)

Stage 3: Multiply within 100

Lessons

- Grade4.2.C12 (supporting)
- Grade4.2.C13 (supporting)
- Grade4.2.C14 (supporting)
- Grade4.2.C15 (supporting)
- Grade4.2.C16 (supporting)
- Grade4.2.C17 (supporting)

Stage Narrative

Students use cards with multiplication expressions within 100.

Standards Alignments

Addressing 3.OA.C.7

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 3 Multiplication Cards (groups of 2)

Stage 5: Fractions

Lessons

- Grade4.2.C14 (addressing)
- Grade4.2.C15 (addressing)
- Grade4.2.C16 (addressing)
- Grade4.2.C17 (addressing)

Stage Narrative

Students use cards with fractions. They may use either deck of fraction cards or combine them together to play.

Standards Alignments

Addressing 4.NF.A.2

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.4.C

Stage 3

Addressing

- Grade3.4.C

Supporting

- Grade3.6.D

Stage 4

Addressing

- Grade3.4.D

Supporting

- Grade3.7.C
- Grade3.7.D

Center: How Close? (1–5)

Stage 6: Multiply to 3,000

Lessons

- Grade4.2.C14 (supporting)
- Grade4.2.C15 (supporting)
- Grade4.2.C16 (supporting)
- Grade4.2.C17 (supporting)

Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 6 cards and chooses 4 of them to create a multiplication expression. Each student multiplies the numbers and the student whose product is closest to 3,000 wins a point for the round. Students pick new cards so that they have 6 cards in their hand and then start the next round.

Variation:

Students can choose a different number as the goal.

Standards Alignments

Addressing 3.OA.B.5

Materials to Gather

Number cards 0–10

Materials to Copy

How Close? Stage 6 Recording Sheet (groups of 1)

Stages used in Grade 3

Stage 4

Addressing

- Grade3.3.B
- Grade3.3.C

Supporting

- Grade3.4.C

Stage 5

Addressing

- Grade3.4.C
- Grade3.4.D

Supporting

- Grade3.6.D
- Grade3.7.C
- Grade3.7.D

Section A: Size and Location of Fractions

Lesson 1: Representations of Fractions (Part 1)

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.1

Teacher-facing Learning Goals

- Make sense of the numerator and denominator of unit fractions that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.
- Use physical and visual representations to reason about fractions.

Student-facing Learning Goals

- Let's name some fractions and represent them visually.

Lesson Purpose

The purpose of this lesson is for students to make sense of unit fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12, using physical and visual representations.

In grade 3, students were introduced to fractions as numbers. They learned to name and represent fractions, to recognize simple equivalent fractions, and to compare fractions with like numerators and denominators (limited to 2, 3, 4, 6, and 8). They used fraction strips, area diagrams, tape diagrams, and number lines to support their reasoning with fractions.

This lesson activates students' prior knowledge of unit fractions and includes fractions with new denominators 5, 6, 10, and 12. Students revisit the meaning of numerator and denominator, name unit fractions, create representations for them, and recall some strategies and tools for reasoning about fractions.

The idea of equivalence may naturally come up (and will help to prepare students for upcoming work), but it is not the focus of this lesson.

Access for:

Students with Disabilities

- Engagement (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), What Do You Know About ____? (Warm-up)

Materials to Gather

- Straightedges: Activity 1, Activity 2

Materials to Copy

- Fraction Strips (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What did you learn about each student and their foundational understanding of fractions based on their work today?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

What Do the Diagrams Show?

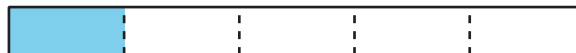
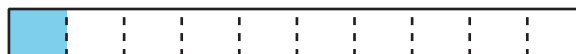
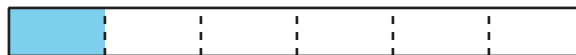
Standards Alignments

Building Towards 4.NF.A.1

Student-facing Task Statement

Each full diagram represents 1.

1. What fraction does each shaded part represent?



2. Explain or show how you could use this diagram to represent sixths.



Student Responses

- $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{5}$.
- Sample response: Split each half into 3 equal parts so there will be a total of 6 parts. Each part is a sixth.

Begin Lesson

Warm-up

🕒 10 min

What Do You Know About $\frac{1}{2}$?

Standards Alignments

Building On 3.NF.A.1
 Building Towards 4.NF.A.1

The purpose of this warm-up is to invite students to share what they know about the number $\frac{1}{2}$ and elicit ways in which it can be represented. It gives the teacher the opportunity to hear students' understandings about and experiences with fractions, $\frac{1}{2}$ in particular. The fraction $\frac{1}{2}$ is familiar to students and will be central in the first activity.

Instructional Routines

What Do You Know About ____?

Student-facing Task Statement

What do you know about $\frac{1}{2}$?

Student Responses

Sample responses:

Launch

- Groups of 2
- Display the number $\frac{1}{2}$.
- "What do you know about this number?"
- 1 minute: quiet think time

- It is a fraction.
- I shared half of my sandwich with my friend.
- It is what we get when we split something into two parts.
- We can “halve” something.
- Dividing by 2.
- It is halfway between 0 and 1 on a number line.
- It is less than 1.
- It is a number.

Activity

- “Discuss your thinking with your partner.”
- 2 minutes: partner discussion
- Share and record responses.

Synthesis

- “What different ways can we represent $\frac{1}{2}$?”
(Cut an object, a rectangle, or another shape into two equal parts, mark the middle point between 0 and 1 on a number line.)

Activity 1

🕒 20 min

Fraction Strips

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.1

The purpose of this activity is for students to use fraction strips to represent halves, fourths, and eighths. The denominators in this activity are familiar from grade 3. The goal is to remind students of the relationships between fractional parts in which one denominator is a multiple of another. Students should notice that each time the unit fractions on a strip are folded in half, there are twice as many equal-size parts on the strip and that each part is half as large.

In the discussion, use the phrases “number of parts” and “size of the parts” to reinforce the meaning of a fraction.

🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to different colored strips of paper students can use to differentiate each fraction.

Supports accessibility for: Organization, Visual-Spatial Processing

Materials to Gather

Straightedges

Materials to Copy

Fraction Strips (groups of 2)

Required Preparation

- Each group of 2 needs 4 strips of equal-size paper (cut lengthwise from letter-size or larger paper or use the provided Instructional master).

Student-facing Task Statement

Your teacher will give you strips of paper. Each strip represents 1.



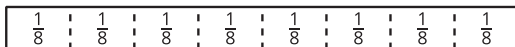
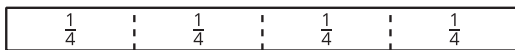
- Use the strips to represent halves, fourths, and eighths.

Use one strip for each fraction and label the parts.

- What do you notice about the number of parts or the size of the parts? Make at least two observations.

Student Responses

1.



2. Sample responses:

- Each time we folded, there were more parts.
- Each time we folded, the parts got smaller.
- The $\frac{1}{8}$ parts are each half the size of the $\frac{1}{4}$ parts.
- The $\frac{1}{4}$ parts are each half of the $\frac{1}{2}$ parts.

Launch

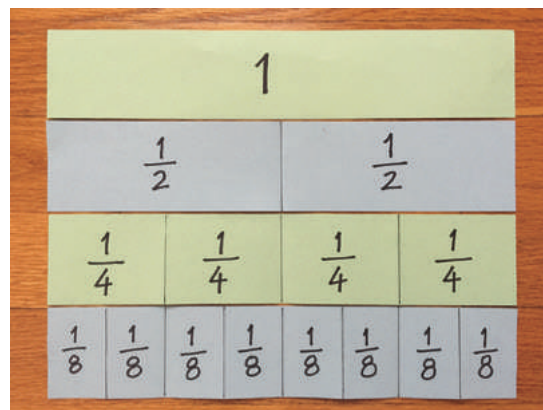
- Groups of 2
- Give each group 4 paper strips and a straightedge.
- Hold up one strip for all to see.
- "Each strip represents 1."
- Label that strip with "1" and tell students to do the same on one of their strips.
- "Take a new strip. How would you fold it to show halves?"
- 30 seconds: partner think time
- "Think about how to show fourths on the next strip and eighths on the last strip."

Activity

- "Work with your partner on the task."
- 10 minutes: partner work time
- Monitor for students who notice that each denominator is twice the next smaller denominator.

Synthesis

- Select a group to share their paper strips and how they found the fractional parts. Ask if others also found them the same way.
- Display one set of completed strips.



- Invite students to share what they noticed about the number and size of the parts on the strips. Highlight the ideas noted in student responses.
- If no students mentions the relationships between the fractions on different strips, encourage them to work with a partner to look for some.
- If the terms “numerator” and “denominator” did not arise during discussion, ask students about them.
- Remind students that the **denominator**, the number at the bottom of a fraction, tells us the number of equal-size parts in 1 whole, and the **numerator**, the number at the top of a fraction, refers to how many of those parts are being described. Consider displaying these terms and their meanings for students to reference.
- Ask students to save the fraction strips for a future lesson.

Advancing Student Thinking

Students may not see the relationships between fractional parts as a result of imprecise folds on fraction strips. Consider asking: “How could we make sure that each part on a strip is equal?” and “What tools might we use to help make precise folds?”

Activity 2

🕒 15 min

Fractions, Represented

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.1

The purpose of this activity is for students to revisit the meaning of unit fractions with familiar and unfamiliar denominators (3, 5, 6, 10, and 12) and recall how to name and represent them.

As they draw tape diagrams to represent these fractions, students have opportunities to look for structure and make use of the relationships between the denominators of the fractions (MP7). For example, to make a diagram with twelfths they can cut each of 6 sixths in half.

To support students in drawing straight lines on the tape diagrams, provide access to a straightedge or ruler. Students should not, however, use rulers to measure the location of a fraction on any diagram.

This activity uses MLR1 Stronger and Clearer Each Time. Advances: reading, writing.

Instructional Routines

MLR1 Stronger and Clearer Each Time

Materials to Gather

Straightedges

Student-facing Task Statement

1. If each full diagram represents 1, what fraction does each shaded part represent?

a.



b.



c.



Launch

- Groups of 2
- Give each student a straightedge.
- “Let’s look at some other fractions and draw diagrams to represent them. Consider using a straightedge when you draw.”

2. Here are four blank diagrams. Each diagram represents 1. Partition each diagram and shade one part so that the shaded part represents the given fraction.

a. $\frac{1}{6}$ 

b. $\frac{1}{8}$ 

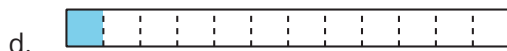
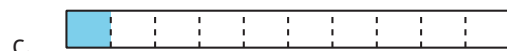
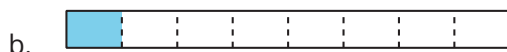
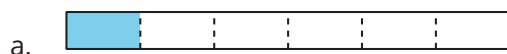
c. $\frac{1}{10}$ 

d. $\frac{1}{12}$ 

3. Suppose you are creating a representation of $\frac{1}{20}$ using the same blank diagram. Would the shaded part be larger or smaller than the shaded part in the diagram of $\frac{1}{10}$? Explain how you know.

Student Responses

1. a. $\frac{1}{2}$
b. $\frac{1}{3}$
c. $\frac{1}{5}$
2. Sample response:



3. It will be smaller, because the whole will be split into 20 equal parts and each part will be smaller (half the size) of the parts representing tenths.

Activity

- 7–8 minutes: independent work time
- “Discuss your responses with your partner. Be sure to talk about how you created diagrams for $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{12}$.”
- 2–3 minutes: partner discussion
- Monitor for students who:
 - notice the relationship of thirds, sixths, and twelfths, and of fifths and tenths
 - use the given diagrams to help partition the other diagrams

Synthesis

- “How did you know how to partition the diagrams in the second question?”
- Select students who use the given diagrams or the relationships between denominators to display their diagrams and share their reasoning.
- “What relationships do you see between the fractions in this activity?” (Sample responses:
 - As the denominator gets larger, each fractional part gets smaller.
 - A fifth is twice the size of a tenth, or a tenth is half as big as a fifth.
 - Thirds, sixths, and twelfths are related in that a third is 2 sixths and a sixth is 2 twelfths. Fifths and tenths are related in the same way.)

MLR1 Stronger and Clearer Each Time

- “Share your response to the last question with your partner. Take turns being the speaker and the listener. If you are the speaker, share your response. If you are the listener, ask questions and give feedback to help your partner improve their work.”

- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.
- “Revise your initial response based on the feedback from your partners.”
- 2–3 minutes: independent work time

Lesson Synthesis

🕒 10 min

“Today we refreshed our memory about fractions. We used fraction strips and diagrams to represent some familiar and some new fractions.”

Based on students’ work during the lesson, choose the questions that need more discussion:

- “In general, what does the denominator in a fraction represent?” (The number of equal parts in 1 whole.)
- “What does the fraction $\frac{1}{5}$ tell us?” (The size of one part if 1 whole is split into 5 equal parts.)
- “What did you notice about the size of a fraction as the denominator gets larger?” (The size of the fraction gets smaller.) “Why might that be?” (There are more equal parts in 1 whole, so each part gets smaller.)
- “What relationships did we see between the fractions that we studied today?” (The denominators of some fractions are multiples of other fractions. A representation of one fraction can be split into two or three parts to represent another fraction.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students describe what the second tape diagram shows but not how it could be used to show sixths.

The work in this lesson builds from concepts of fractions addressed in a prior unit.

Next Day Support

- At the start of the next lesson, invite students to share how they would show sixths on a blank diagram and on a diagram already partitioned into halves.

Prior Unit Support

Grade 3, Unit 5, Section A: Introduction to Fractions

Lesson 2: Representations of Fractions (Part 2)

Standards Alignments

Building On 3.NF.A.1
 Building Towards 4.NF.A, 4.NF.A.1

Teacher-facing Learning Goals

- Make sense of the numerator and denominator of unit fractions that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.
- Use diagrams to represent fractions.

Student-facing Learning Goals

- Let's name some other fractions and represent them with diagrams.

Lesson Purpose

The purpose of this lesson is for students to make sense of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.

In the previous lesson, students made sense of the meaning of numerator and denominator in unit fractions. They identified fractions represented by diagrams, and partitioned diagrams to represent given fractions. In this lesson, they reason in similar ways—using numerical and visual representations—about non-unit fractions and fractions that are greater than 1.

Students are reminded of what they learned in grade 3: that a non-unit fraction $\frac{a}{b}$ can be understood as a parts of a unit fraction $\frac{1}{b}$, and that fractions with different numerators and denominators can be equivalent. Unlike in grade 3, the denominators they see here now include 5, 10, and 12.

As in the previous lesson, rulers can be provided to help students draw, extend, or align partition lines, but should not be used to measure the location of a fraction on any diagram.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 2
- Straightedges: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

What Do the Diagrams Show?

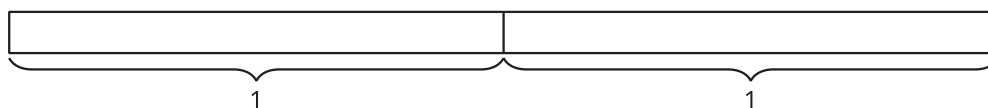
Standards Alignments

Building Towards 4.NF.A

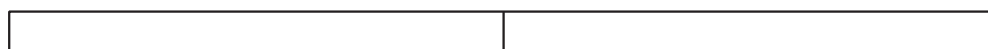
Student-facing Task Statement

Use a blank diagram to create a representation for each fraction. Both blank diagrams represent the same quantity.

1. $\frac{5}{8}$



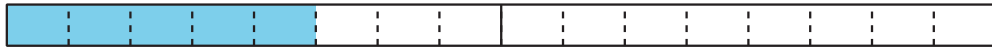
2. $\frac{9}{8}$



Student Responses

Sample response:

1. $\frac{5}{8}$



2. $\frac{9}{8}$



Begin Lesson

Warm-up

⌚ 10 min

Which One Doesn't Belong: All Cut Up

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.1

This warm-up prompts students to carefully analyze and compare the features of four partitioned shapes. It allows the teacher to hear the terminologies students use to talk about fractions and fractional parts. In making comparisons, students have a reason to use language precisely (MP6).

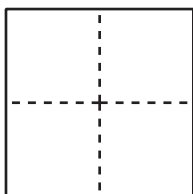
Instructional Routines

Which One Doesn't Belong?

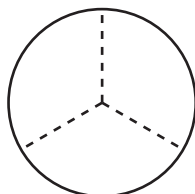
Student-facing Task Statement

Which one doesn't belong?

A



B



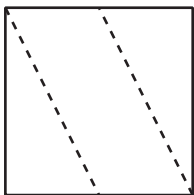
Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

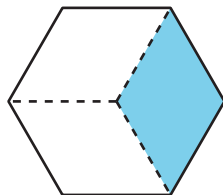
Activity

- "Discuss your thinking with your partner."

C



D



Student Responses

Sample response:

- A is the only one not partitioned into 3 parts.
- B is the only one that does not have straight edges.
- C is the only one not partitioned into equal parts.
- D is the only one whose parts are not all clear or unshaded.

- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- “What does the shaded part in D represent?” ($\frac{1}{3}$ or one-third of the shape).
- Shade one part of B and C.
- “Is each shaded part one-third of the shape as well?” (Yes for B, no for C.)
- “Why is the shaded part not one-third of the square in C?” (The parts aren’t equal in size.)
- Shade one part of A. “Is it a third of the square?” (No, it is $\frac{1}{4}$ or one-fourth.)

Activity 1

🕒 20 min

A Diagram for Each Fraction

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.1

The purpose of this activity is to activate what students know about the meaning and size of non-unit fractions. Students match a set of fractions with diagrams that represent them. There are a 3 sets of equivalent fractions to prompt students to share what they know about equivalent fractions.

To add movement to the activity, students can check their matches with other groups in the room before the synthesis.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use visual details such as color or arrows to illustrate connections between representations. For example, use the same color for the numerator and the shaded portion of the corresponding diagram.

Supports accessibility for: Visual-Spatial Processing

Materials to Gather

Straightedges

Student-facing Task Statement


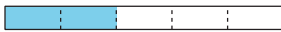



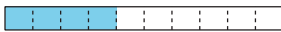



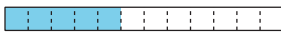

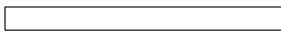


Each full diagram represents 1. Match each fraction to a diagram whose shaded parts represents it.

Two of the fractions are not represented. Create a representation for each of them.

$$\frac{2}{3} : \underline{\hspace{1cm}} \quad \frac{3}{8} : \underline{\hspace{1cm}} \quad \frac{4}{10} : \underline{\hspace{1cm}} \quad \frac{4}{6} : \underline{\hspace{1cm}} \quad \frac{6}{6} : \underline{\hspace{1cm}}$$

$$\frac{3}{5} : \underline{\hspace{1cm}} \quad \frac{4}{8} : \underline{\hspace{1cm}} \quad \frac{6}{12} : \underline{\hspace{1cm}} \quad \frac{6}{10} : \underline{\hspace{1cm}} \quad \frac{3}{4} : \underline{\hspace{1cm}}$$

$$\frac{5}{6} : \underline{\hspace{1cm}} \quad \frac{2}{5} : \underline{\hspace{1cm}} \quad \frac{5}{12} : \underline{\hspace{1cm}} \quad \frac{7}{10} : \underline{\hspace{1cm}} \quad \frac{7}{8} : \underline{\hspace{1cm}}$$

<p>A</p> 	<p>I</p> 
<p>B</p> 	<p>J</p> 
<p>C</p> 	<p>K</p> 
<p>D</p> 	<p>L</p> 
<p>E</p> 	<p>M</p> 
<p>F</p> 	<p>N</p> 
<p>G</p> 	<p>O</p> 

Launch

- Groups of 2
- Give each student a straightedge
- Record and display the fraction $\frac{1}{4}$.
- “Describe to your partner what the diagram would look like for this fraction.”
- 30 seconds: partner discussion
- Record and display the fraction $\frac{2}{4}$.
- “Describe what the diagram would look like for this fraction.”
- 30 seconds: partner discussion
- Share responses.
- “In an earlier lesson, we looked at fractions with 1 for the numerator. Now let’s look at fractions with other numbers for the numerator.”
- As a class, read aloud the word name of each fraction in the task.

Activity

- “Take a minute to think quietly about how you might match each fraction with a diagram that represents it.”
- 1 minute: quiet think time
- “Work with a partner to match each fraction with a diagram. Two of the



H



Student Responses

- | | | | | |
|-------------------|-------------------|--------------------|--------------------|-------------------|
| $\frac{2}{3}$: B | $\frac{3}{8}$: G | $\frac{4}{10}$: K | $\frac{4}{6}$: H | $\frac{6}{6}$: N |
| $\frac{3}{5}$: C | $\frac{4}{8}$: A | $\frac{6}{12}$: L | $\frac{6}{10}$: E | $\frac{3}{4}$: J |
| $\frac{5}{6}$: F | $\frac{2}{5}$: I | $\frac{5}{12}$: M | $\frac{7}{10}$: O | $\frac{7}{8}$: D |

The missing diagrams for $\frac{6}{6}$ and $\frac{7}{10}$:

- N



- O



fractions have no matching diagrams. Use the blank diagrams to create representations for them."

- 10 minutes: group work time

Synthesis

- Invite students to share how they went about making the matches.
- Highlight explanations that emphasize the meaning of numerator and denominator in a fraction.
- Ask students if they noticed that some diagrams have the same amount shaded but the fractions they represent have different numbers. "Which diagrams show this?" (A and L, B and H, C and E, I and K)
- "What does it mean that the diagrams representing those fractions are the same?" (The fractions are the same size. The term "equivalent" may or may not come up at this point.)

Activity 2

🕒 15 min

Diagrams for Some Other Fractions

Standards Alignments

- | | |
|------------------|----------|
| Building On | 3.NF.A.1 |
| Building Towards | 4.NF.A.1 |

This activity extends students' reasoning about the meaning of numerator and denominator and the size of non-unit fractions to include fractions greater than 1. Students see the size of 1 whole marked in a couple of diagrams and learn that the same size applies to all diagrams. They are prompted to both interpret diagrams and create them: they write a fraction to represent the shaded part of a diagram and partition a diagram to represent a given fraction.

Some students may benefit from having physical manipulatives to help them conceptualize fractions that are greater than 1. Consider using fraction strips to support them, for instance, by asking them to fold as many strips as needed to represent, say, 4 halves or 5 fourths.

Access for English Learners

MLR2 Collect and Display. Collect the language students use to reason about fractions greater than one. Display words and phrases such as: fraction, numerator, denominator, 1 whole, greater than, equal-size parts, etc. During the activity, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Materials to Gather

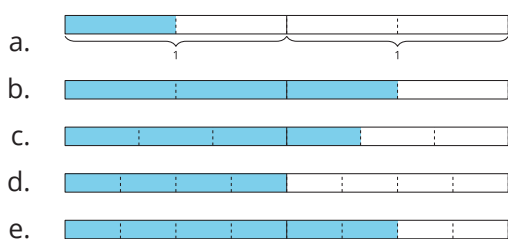
Materials from a previous lesson,
Straightedges

Required Preparation

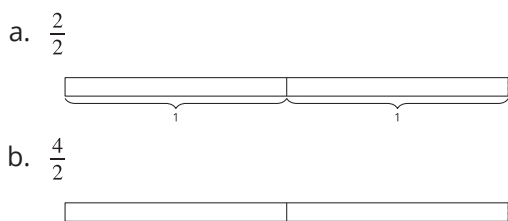
- Each student needs access to their fraction strips from a previous lesson.

Student-facing Task Statement

- What fraction do the shaded parts represent?



- Here are four fractions and four blank diagrams. Partition each diagram and shade the parts to represent the fraction.



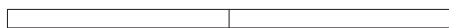
Launch

- Groups of 2
- Give each student a straightedge and access to their fraction strips from a previous lesson.
- “How can you show $\frac{3}{4}$ with fraction strips?” (Find the strip showing fourths, highlight 3 parts of fourths.)
- “How can you show $\frac{8}{4}$?”
- 1 minute: partner discussion
- If students say that they don’t have enough strips to show 8 fourths, ask them to combine their strips with another group’s.
- Invite groups to share their representations of $\frac{8}{4}$. Students may use different fractional parts (fourths and halves, or fourths and eighths).
- Display two strips that show fourths side by

c. $\frac{5}{4}$



d. $\frac{10}{8}$



Student Responses

1. a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. $\frac{4}{3}$ d. $\frac{4}{4}$ e. $\frac{6}{4}$

2. Sample response:



side, as shown.



- Count the fourths: " $\frac{1}{4}, \frac{2}{4}, \dots, \frac{8}{4}$."
- "How many fourths did we count?" (8) "How many wholes was that?" (2)

Activity

- "Each bracket in the first diagram shows 1 whole. The size of 1 whole is the same in all the diagrams."
- "Take a moment to work on the first question. Then, discuss your responses with your partner."
- "Be prepared to explain how you know what fractions the diagrams represent."
- 2–3 minutes: independent work time on the first question
- 2 minutes: partner discussion
- Pause for a discussion.
- "How did you determine what fractions the diagrams represent?" (Count the number of parts in 1, and then count the number of shaded parts.)
- Display students' work, or display and annotate the tape diagrams as they explain.
- Consider labeling each part with the unit fraction and counting each shaded part aloud ("1 half, 2 halves, 3 halves," or "1 third, 2 thirds, 3 thirds, 4 thirds") before writing the represented fractions ($\frac{3}{2}$ or $\frac{4}{3}$).
- "Work with your partner on the second question. You may use a straightedge to help you draw your diagrams."
- 5–7 minutes: partner work time

Synthesis

- Select students to share their completed diagrams.
- “How did you know how many parts to partition each diagram and how many parts to shade?” (Cut each 1 whole portion into as many equal parts as the number in the denominator. Shade as many parts as the number in the numerator.)
- “How did you partition a diagram into 4 equal parts?” (Split each 1 whole into 2, and then split each half into 2 parts again.)
- “How did you partition a diagram into 8 equal parts?” (Split each fourth into 2 parts.)

Advancing Student Thinking

Students may not attend to the size of 1 whole as they partition the blank diagrams. Consider asking: “What does each part in your diagram represent? How do you know?” and “Where is 1 whole in your diagram?”

Lesson Synthesis

🕒 10 min

“Today we made sense of and created diagrams that represent fractions, including fractions greater than 1.”

“Did you notice anything interesting about the last two diagrams you created and the fractions they represent?” (Students may or may not refer to equivalence. Sample responses:

- They are both greater than 1.
- The shaded parts are the same size. They have the same amount of shading.
- The numerator and denominator in one fraction are twice the numerator and denominator in the other.
- The fractions are equivalent.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not attend to the size of 1 whole in representing the fractions, or do not recognize $\frac{9}{8}$ as greater than 1.

The work in this lesson builds from concepts of fractions greater than 1 addressed in a prior unit.

Next Day Support

- Add this cool-down to Activity 1 to review how to represent fractions greater than 1.

Prior Unit Support

Grade 3, Unit 5, Section B: Fractions on the Number Line

Lesson 3: Same Denominator or Numerator

Standards Alignments

Building On	3.NF.A.3.d
Addressing	4.NF.A.2
Building Towards	4.NBT.B.4, 4.NF.A.2

Teacher-facing Learning Goals

- Compare fractions with the same numerator or the same denominator using physical or visual representations.
- Use the meaning of numerator and denominator to reason about the size of fractions.

Student-facing Learning Goals

- Let's compare fractions with the same numerator or the same denominator.

Lesson Purpose

The purpose of this lesson is for students to use the meaning of numerator and denominator and to compare fractions with the same numerator or the same denominator.

In this lesson, students reason about the relative size of fractions based on the meaning of numerator and denominator, and use fraction strips to support their reasoning.

Students first compare pairs of fractions with the same denominator. They recall that fractions with the same denominator are composed of the same unit fractions or have parts that are the same size, so the numerators can tell us how the fractions compare: the greater the numerator, the greater the fraction.

Next, students compare fractions with the same numerator. They recognize that we cannot simply look at the denominators and see which is greater. Because the denominator tells us the number of parts in 1 whole, the greater that number, the smaller the fractional part.

Access for:

Students with Disabilities

- Engagement (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Most students may find it more intuitive to compare fractions with a common denominator than those with a common numerator. Did you see students who grasp both equally well? How did they conceptualize the latter?

Cool-down (to be completed at the end of the lesson)

 5 min

Sizing Up Fractions

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

In each pair of fractions, which one is greater? Explain or show your reasoning.

- $\frac{7}{8}$ or $\frac{10}{8}$
- $\frac{4}{10}$ or $\frac{4}{5}$

Student Responses

- $\frac{10}{8}$ is greater. Sample reasoning: The two fractions have the same fractional parts (eighths). There are more eighths in $\frac{10}{8}$ than in $\frac{7}{8}$.
- $\frac{4}{5}$ is greater. A fifth is greater than a tenth, so 4 fifths are greater than 4 tenths.

----- Begin Lesson -----

Warm-up

 10 min

Number Talk: Hundreds More

Standards Alignments

Building Towards 4.NBT.B.4

The purpose of this Number Talk is to elicit strategies and understandings students have for adding and subtracting multi-digit numbers. These understandings help students develop fluency and will be helpful in later units as students add and subtract multi-digit numbers fluently using the standard algorithm.

When students decompose addends to support mental addition they are looking for and making use of the base-ten structure of numbers (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $136 + 100$
- $136 + 300$
- $136 + 370$
- $136 + 378$

Student Responses

- 236: $136 + 100 = 236$
- 436: $136 + 300 = 436$
- 506: This is 70 more than 436. I know $430 + 70 = 500$, so $436 + 70 = 506$.
- 514: $300 + 100 = 400$, $30 + 70 = 100$, and $6 + 8 = 14$, and $400 + 100 + 14 = 514$.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How did the first couple of expressions help you reason about last two expressions?”
- Consider asking:
 - “Who can restate ____’s reasoning in a different way?”
 - “Did anyone have the same strategy but would explain it differently?”
 - “Did anyone approach the expression

in a different way?"

- "Does anyone want to add on to ____'s strategy?"

Activity 1

🕒 15 min

Fractions with the Same Denominator

Standards Alignments

Building On 3.NF.A.3.d

Building Towards 4.NF.A.2

The purpose of this activity is to prompt students to reason about the relative sizes of two fractions with the same numerator and articulate how they know which one is greater. Students have done similar reasoning work (and used similar tools to support their reasoning) in grade 3, but here the fractions include those with denominators 5 and 10. When students observe that 5 equal parts are greater than 3 of the same equal part, regardless of the size of those parts, they see regularity in repeated reasoning (MP8).

To add movement to this activity and if time permits, assign each group a pair of fractions in the second question and ask them to create a visual display showing their reasoning. Then, allow a few minutes for a gallery walk. Ask students to identify any patterns they notice on the displays.

🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to colored pencils students can use to label each rectangle.

Supports accessibility for: Attention, Organization

Student-facing Task Statement

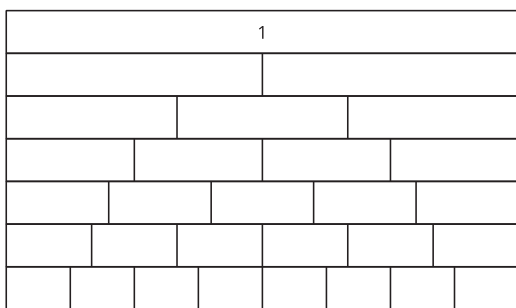
1. This diagram shows a set of fraction strips. Label each rectangle with the fraction it represents.

Launch

- Groups of 2

Activity

- "Take a few quiet minutes to answer the first three questions. Discuss your work with your partner before moving on to the



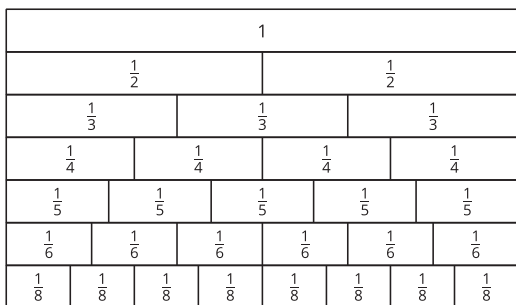
2. Circle the greater fraction in each of the following pairs. If helpful, use the diagram of fraction strips.

- a. $\frac{3}{4}$ or $\frac{5}{4}$
 b. $\frac{3}{5}$ or $\frac{5}{5}$
 c. $\frac{3}{6}$ or $\frac{5}{6}$
 d. $\frac{3}{8}$ or $\frac{5}{8}$
 e. $\frac{3}{10}$ or $\frac{5}{10}$

3. What pattern do you notice about the circled fractions? How can you explain the pattern?
 4. Which one is greater: $\frac{7}{3}$ or $\frac{10}{3}$? Explain your reasoning.

Student Responses

1.



2. The greater fractions are: $\frac{5}{4}$, $\frac{5}{5}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{5}{10}$.

3. Sample responses:

- In each pair, the fraction with 5 for the numerator is greater than the

last question.”

- 5–7 minutes: independent work time
- 3–4 minutes: partner discussion
- 2–3 minutes: independent work time on the last question

Synthesis

- “What do you notice about each pair of fractions in the second question?” (They all have 3 and 5 for the numerators, and they have the same denominator.)
- “What does it mean when two fractions, say $\frac{3}{8}$ and $\frac{5}{8}$, have the same denominator?” (They are made up of the same fractional part—eighths in this case.)
- “How can we tell which fraction is greater?” (Because the fractional parts are the same size, we can compare the numerators. The fraction with the greater numerator is greater.)

- one with 3 for the numerator. The denominator in each pair is the same, say, 4, so 5 fourths will be greater than 3 fourths.
- Five fractional parts will always be greater than 3 of the same fractional parts.
4. Sample response: $\frac{10}{3}$ is greater. Ten $\frac{1}{3}$ s are greater than seven $\frac{1}{3}$ s.

Activity 2

🕒 20 min

Fractions with the Same Numerator

Standards Alignments

Building On 3.NF.A.3.d

Building Towards 4.NF.A.2

The purpose of this activity is for students to reason about the relative sizes of two fractions with the same numerator. As before, a diagram of fraction strips can be used to help students visualize the sizes of various fractional parts. When students discuss and improve their explanation for why $\frac{70}{20}$ is greater than $\frac{70}{100}$ they develop their mathematical communication skills (MP3).

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: *Reading, Writing*

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

1. Circle the greater fraction in each of the following pairs. If helpful, use the diagram of fraction strips.

Launch

- Groups of 2
- “What do you notice about the fractions in the first question?” (Each pair has the same numerator and has 3 and 5 for the

- a. $\frac{1}{3}$ or $\frac{1}{5}$
 b. $\frac{2}{3}$ or $\frac{2}{5}$
 c. $\frac{3}{3}$ or $\frac{3}{5}$
 d. $\frac{4}{3}$ or $\frac{4}{5}$
 e. $\frac{9}{3}$ or $\frac{9}{5}$

2. What pattern do you notice about the circled fractions? How can you explain the pattern?
3. Which one is greater: $\frac{70}{100}$ or $\frac{70}{20}$? Explain your reasoning.
4. Tyler is comparing $\frac{4}{10}$ and $\frac{4}{6}$. He says, "Ten is greater than 6, so $\frac{4}{10}$ is greater than $\frac{4}{6}$." Explain or show why Tyler's conclusion is incorrect.

Student Responses

1. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, and $\frac{9}{3}$ are greater than the fractions with 5 in the denominator.
2. Sample response: All the fractions that are greater have 3 in the denominator. A third is greater than a fifth, so 2 thirds will be greater than 2 fifths, 3 thirds greater than 3 fifths, and so on.
3. $\frac{70}{20}$ is greater than $\frac{70}{100}$. Sample reasoning: One twentieth is greater than one hundredth, so 70 twentieths are greater than 70 hundredths.
4. Sample response: The 6 and 10 mean 1 whole is divided into 6 and 10 equal parts. Each sixth is greater than each tenth, so 4 sixths are greater than 4 tenths.

denominators.)

Activity

- "Think quietly for a moment about how you can find out which fraction in each pair is greater. Then, share your thinking with your partner."
- 1 minute: quiet think time
- 2 minutes: partner discussion
- Monitor for students who use the size of a unit fraction or one fractional part to help them make comparisons.
- "Take a few quiet minutes to work on the questions."
- 7–8 minutes: independent work time

Synthesis

- Select students to share their responses to the first two questions.
- "In the set of fractions you saw, why are the fractions with 3 for the denominator always greater than fractions with 5 for the denominator?" (A third is always greater than a fifth, so some number of thirds will always be greater than the same number of fifths.)

MLR1 Stronger and Clearer Each Time

- "Share your response to the third question with your partner. Take turns being the speaker and the listener. If you are the speaker, share your explanation. If you are the listener, ask questions and give feedback to help your partner improve their explanation."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- "Revise your initial explanation based on the feedback from your partners."
- 2–3 minutes: independent work time

Lesson Synthesis

🕒 10 min

“Today we looked at fractions with the same denominator and those with the same numerator.”

Select students to share their explanations on the last question in the second activity. “What might have Tyler misunderstood? What would you say to help clear it up for him?”

“Based on your work today, how would you complete these sentence starters?”

Display and read aloud:

- “If two fractions have the same denominator, I can tell which one is greater by”
(looking at which one has a greater numerator, because it would mean more of the same fractional parts)
- “If two fractions have the same numerator, I can tell which one is greater by”
(looking at which denominator is smaller, because the smaller denominator would mean a larger fractional part)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students who used tape diagrams to reason about $\frac{4}{10}$ and $\frac{4}{5}$ may reason correctly about each fraction but draw different lengths to represent 1 whole (for instance, a longer tape to show tenths and a much shorter one for fifths), leading to the wrong conclusion about which one is greater.

The work in this lesson builds from comparing fractions with the same numerator or denominator addressed in a prior unit.

Next Day Support

- Before the warm-up, display and discuss tape diagrams for $\frac{4}{10}$ and $\frac{4}{5}$. Ask students to notice and wonder. Explain that to compare two fractions, the 1 whole must be the same size.

Prior Unit Support

Grade 3, Unit 5, Section D: Fraction Comparisons

Lesson 4: Same Size, Related Sizes

Standards Alignments

Building On	3.NF.A.2, 3.NF.A.2.a, 3.NF.A.3.b
Addressing	4.NF.A.1
Building Towards	4.NF.A.1

Teacher-facing Learning Goals

- Use the relationships between fractions whose denominators are multiples of one another (for instance $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$) to locate fractions on the number line.
- Use visual representations to reason about fractions that have the same size. Recall that these fractions are equivalent.

Student-facing Learning Goals

- Let's find some fractions that are the same size.

Lesson Purpose

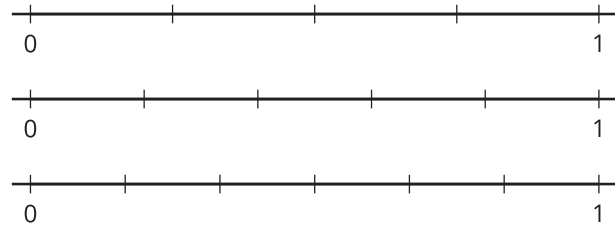
The purpose of this lesson is for students to use visual representations to reason about the fractions that have the same size and to locate them on the number line.

In grade 3, students reasoned about equivalent fractions, using fraction strips, tape diagrams, and number lines. Here, they begin to revisit the idea of equivalence. Students examine fractions that have the same size but are expressed with different numerators and denominators. They use diagrams of fraction strips, now expanded to include fractions with denominator 10 and 12, and then transition to using number lines to support their reasoning.

The relationships between fractions such as $\frac{1}{4}$ and $\frac{1}{8}$, $\frac{1}{5}$ and $\frac{1}{10}$, and $\frac{1}{6}$ and $\frac{1}{12}$, in which one denominator is a multiple of the other, continue to be highlighted and offer many opportunities for students to look for and make use of structure (MP7).

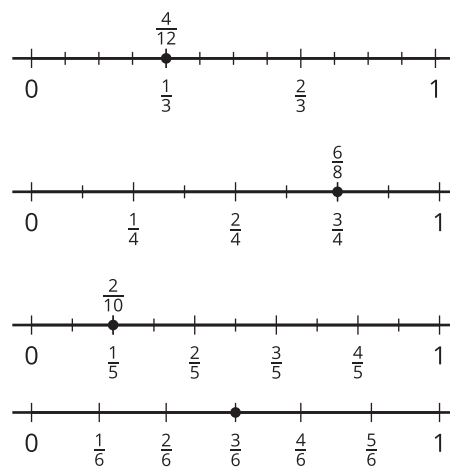
Later in the unit, students will take a closer look at equivalence and investigate new ways to reason about equivalence.

As in earlier activities, rulers can be provided to help students draw, extend, or align partition lines, but should not be used to measure the location of a fraction.



Student Responses

Sample response:



Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: A Fraction Strip and a Number Line

Standards Alignments

Building On 3.NF.A.2

Building Towards 4.NF.A.1

The purpose of this warm-up is to revisit the idea from grade 3 that tape diagrams and number lines are related, which will be useful later in the lesson, when students transition from using fraction strips to using the number line to represent fractions and reason about their size.

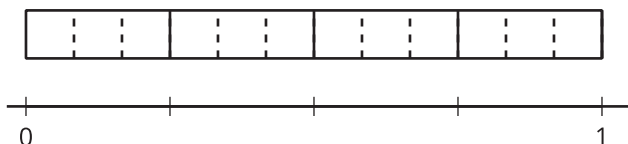
While students may notice and wonder many things about these representations, the connections between the tape diagram and number line (the number and size of the parts in relation to 1) are important to note.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- The number line is partitioned into fourths.
- There are 12 parts (or 12 twelfths) in the entire fraction strip.
- The entire length of the fraction strip is 1.
- There are 3 parts of $\frac{1}{12}$ for each tick mark on the number line.

Students may wonder:

- Why is the number line only partitioned into fourths?
- Why is the fraction strip partitioned into twelfths but the number line is not?
- Where is 0 on the fraction strip?
- Can we add tick marks to make twelfths?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “How are these representations alike? How are they different?”
- “Some tick marks on the number line are not labeled. What labels do you think would be appropriate for them?” ($\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$, or $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, or $\frac{3}{12}, \frac{6}{12}, \frac{9}{12}$)

Activity 1

🕒 20 min

Same Size, Different Numbers

Standards Alignments

Building On 3.NF.A.3.b
Addressing 4.NF.A.1

This activity serves two main goals: to revisit the idea of equivalence from grade 3, and to represent non-unit fractions with denominator 10 and 12. Students use diagrams of fraction strips, which allow them to see and reason about fractions that are the same size. In the next activity, students will apply a similar process of partitioning to represent these fractional parts on number lines.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to colored pencils students can use to label each rectangle.

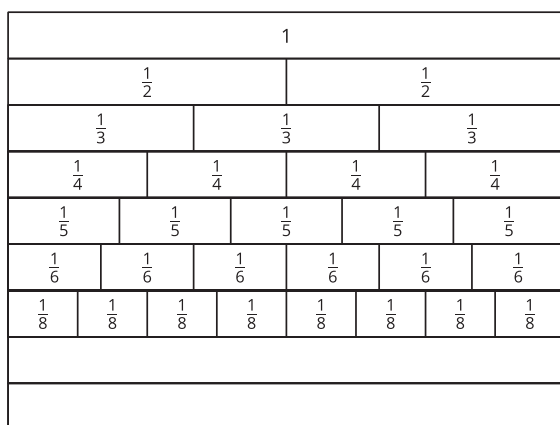
Supports accessibility for: Attention, Organization

Materials to Gather

Straightedges

Student-facing Task Statement

Here's a diagram of fraction strips, with two strips added for tenths and twelfths.



1. Use a blank strip to show tenths. Label the parts. How did you partition the strip?
2. Use a blank strip to show twelfths. Label the parts. How did you partition the strip?

Launch

- Groups of 2
- Give each student a straightedge.
- "Here is a diagram of fraction strips you saw before, with two new rows added."
- "How can we show tenths and twelfths in the two rows? Think quietly for a minute."
- 1 minute: quiet think time

Activity

- "Work on the first two questions on your own. Afterward, discuss your responses with your partner."
- "Use a straightedge when drawing your diagram."
- 5–6 minutes: independent work time
- 2–3 minutes: partner discussion

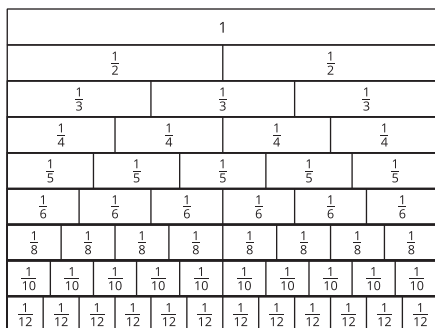
3. Jada says, "I noticed that one part of $\frac{1}{2}$ is the same size as two parts of $\frac{1}{4}$ and three parts of $\frac{1}{6}$. So $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be **equivalent**."

Find a fraction that is equivalent to each of the following fractions. Be prepared to explain your reasoning.

- $\frac{1}{6}$
- $\frac{2}{10}$
- $\frac{3}{3}$

Student Responses

- See diagram. Sample reasoning: We can split each fifth into 2 equal parts so that there are 10 equal parts in the entire row. Each part is a tenth.
- See diagram. Sample reasoning:
 - Each sixth can be split into 2 equal parts, for a total of 12 equal parts. Each part is a twelfth.
 - Each fourth can be split into 3 equal parts, for a total of 12 parts.



- Sample response:
 - $\frac{2}{12}$
 - $\frac{1}{5}$
 - $\frac{2}{2}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{8}{8}$, $\frac{10}{10}$, and $\frac{12}{12}$

- Monitor for students who found the size of tenths and twelfths as noted in student responses.
- Pause for a brief discussion. Select students who used different strategies to find tenths and twelfths to share.
- After each person shares, ask if others in the class did it the same way or if they had anything to add to the explanation.
- "Look at your completed diagram. What can you say about the relationship between $\frac{1}{5}$ and $\frac{1}{10}$?" (There are two $\frac{1}{10}$ s in every $\frac{1}{5}$. One fifth is twice one tenth. One fifth is the same size as 2 tenths.)
- "What can you say about the relationship between $\frac{1}{6}$ and $\frac{1}{12}$?" (There are two $\frac{1}{12}$ s in every $\frac{1}{6}$. One sixth is twice one twelfth. One sixth is the same size as 2 twelfths.)
- "Take 2 minutes to answer the last question."
- 2 minutes: independent or group work time

Synthesis

- Invite students to share their response to the last question and how they found **equivalent fractions**.
- Highlight the idea that two fractions that are the same size are equivalent, even if they have different numbers for the numerators and denominators.
- If needed, "How many fractions that are equivalent to $\frac{3}{3}$ do you see on the diagram?" (Every strip on the diagram shows a fraction equivalent to $\frac{3}{3}$.)

Activity 2

🕒 15 min

Fractions on Number Lines

👤 ↔ 👤 PLC Activity

Standards Alignments

Building On 3.NF.A.2.a

Building Towards 4.NF.A.1

The purpose of this activity is to remind students of their work in grade 3 using number lines as a way to reason about fractions. Students see that they can partition number lines in a similar way as they partitioned fraction strips and diagrams.

The activity gives students another opportunity to notice the relationship between two fractions whose denominator is a multiple or a factor of each other, and then use this relationship to locate fractions on a number line. In doing so, students practice looking for and making use of structure (MP7).

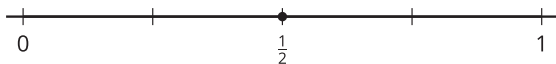
The work prepares students to use number lines to think about equivalent fractions in the next lesson.

Instructional Routines

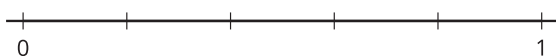
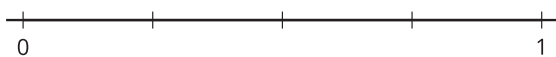
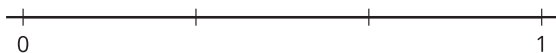
MLR5 Co-craft Questions

Student-facing Task Statement

- Here are some number lines. The point on this number line shows the fraction $\frac{1}{2}$.



Label the tick marks on each number line.



Launch

- Groups of 2

MLR5 Co-craft Questions

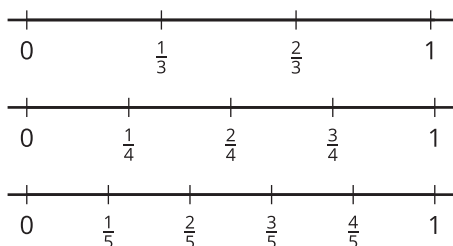
- “Keep your books closed.”
- Display only the four number lines without revealing the question(s).
- “Write a list of mathematical questions that could be asked about this situation.”
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one

2. Suppose you are to locate $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$ on one of the number lines.
- Which number line would you use for each fraction? Be prepared to explain your reasoning.
 - Locate and label each fraction ($\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$) on a different number line.
3. Locate and label each of the following fractions on one of the number lines.

$\frac{2}{3}$	$\frac{2}{8}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{6}$
$\frac{4}{8}$	$\frac{4}{10}$	$\frac{6}{6}$	$\frac{6}{10}$	$\frac{8}{8}$

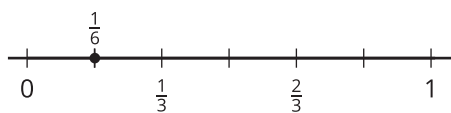
Student Responses

1.



2. a. Sample response:
- For $\frac{1}{6}$, I'd use the number line that shows thirds, and partition each third into 2 equal parts.
 - For $\frac{1}{8}$, I'd use the number line that shows fourths, and partition each fourth into 2 equal parts.
 - For $\frac{1}{10}$, I'd use the number line that shows fifths, and partition each fifth into 2 equal parts.

b.



question with the class. Record responses.

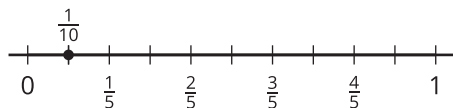
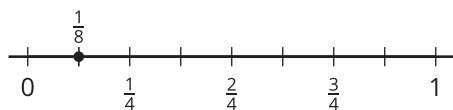
- "What do these questions have in common? How are they different?"
- Reveal the task (students open books), and invite additional connections.

Activity

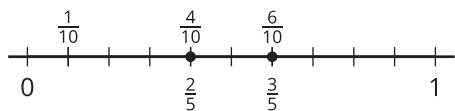
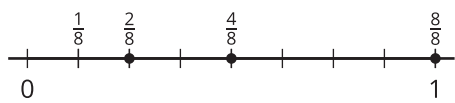
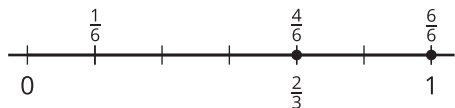
- "Take 5 minutes to complete the first two questions."
- 5 minutes: independent work time
- "Discuss your work with a partner. Make sure you and your partner agree on the labels for the number lines and can explain how you know before moving on to the last question."
- 2 minutes: partner discussion
- Monitor for students who use the tick marks for $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ on the given number lines to locate $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$.

Synthesis

- See lesson synthesis.



3.



Advancing Student Thinking

Students may place the labels between the tick marks on the number line, rather than at or below the tick marks. Clarify that the numbers on a number line represent distance from 0. Consider asking: “If we start from 0 and move halfway toward 1, where should we put the $\frac{1}{2}$ to mark the halfway point? Why?”

Lesson Synthesis

🕒 10 min

Select 1–2 students to share their completed number lines from the last activity, with points marked on the lines to represent the given fractions.

Consider asking:

- “How did you know which number line to choose for $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$?” (We could locate $\frac{1}{6}$, for example, on any of the number lines. But since we know that $3 \times 2 = 6$, we can split each part in the number line that shows thirds into 2 to make 6 parts, which makes it easiest to locate $\frac{1}{6}$.)
- “How did you know where to put a point for, say, $\frac{4}{10}$?” (Starting from 0, count as many tick marks as the number in the numerator. For $\frac{4}{10}$, count 4 tick marks on the number line that show tenths.)

Display a completed diagram of fraction strips from an earlier activity.

“How is representing a fraction like $\frac{6}{10}$ on a number line like representing it on a fraction strip? How is it different?” (Sample responses:

- Alike: They both involve identifying the right fractional parts—by looking at the denominator—and then counting as many parts as the numerator of the fraction.
- Different: One involves the size of parts that are folded and the other involves a specific place on the number line.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)

Complete Cool-Down

Response to Student Thinking

Students place the labels between the tick marks on the number line, rather than at or below the tick marks.

The work in this lesson builds from locating fractions on a number line addressed in a prior unit.

Next Day Support

- Consider explaining that, on a fraction strip or a tape diagram, a label like $\frac{1}{2}$ is not placed at the partition line because the number refers to a portion of the tape, rather than to the distance from 0.

Prior Unit Support

Grade 3, Unit 5, Section B: Fractions on the Number Line

Lesson 5: Fractions on Number Lines

Standards Alignments

Building On 3.OA.B.5
Addressing 4.NF.A.1

Teacher-facing Learning Goals

- Identify equivalent fractions on a number line.
- Recognize that fractions that describe the same point on the number line are equivalent.

Student-facing Learning Goals

- Let's investigate equivalent fractions on a number line.

Lesson Purpose

The purpose of this lesson is for students to recognize that equivalent fractions describe the same point on the number line and to identify such fractions on the number line.

Prior to this lesson, students used fraction strips and tape diagrams to visualize and represent fractions that are the same size. Here they use number lines to do so. Students are reminded that equivalent fractions describe the same point on the number line, or are the same distance from 0.

To determine whether two fractions are equivalent, students rely on their understanding of fractions with related denominators (in which one denominator is a multiple of another). They practice thinking of certain fractions in terms of other fractions (for instance, thinking that they can split $\frac{1}{3}$ into 2 sixths, or $\frac{1}{5}$ into 2 tenths).

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR3 Clarify, Critique, Correct (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Straightedges: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In the next lesson, students will be comparing fractions to $\frac{1}{2}$ and 1, applying what they know about equivalence and distance on a number line. How did today's work prepare them for that lesson?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Two of the Same

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

Show $\frac{5}{6}$ on the number line. Be sure to include labels. Then, explain or show that the fraction $\frac{10}{12}$ is equivalent to $\frac{5}{6}$.

**Student Responses**

Sample response:



Each third can be partitioned into 2 sixths, and each sixth into 2 twelfths. There are 10 twelfths in 5 sixths.

----- **Begin Lesson** -----

Warm-up

 10 min

Number Talk: A Number Times Twelve

Standards Alignments

Building On 3.OA.B.5

The purpose of this warm-up is to remind students of doubling as a strategy for multiplication in which a factor in one product is twice a factor in another product. The reasoning that students do here with the factors 2, 4, 8, and 16 will support them as they reason about equivalent fractions and find multiples of numerators and denominators.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 2×12
- 4×12
- 8×12
- 16×12

Student Responses

- 24. I just know it.
- 48, because 4 is twice 2, so 4×12 is twice 2×12 . I doubled 24 to get 48.
- 96, because 8 is twice 4, so 8×12 is twice 4×12 . I doubled 48 to get 96.
- 192, because 16 is twice 8, so 16×12 is twice 8×12 . I doubled 96 to get 192.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each expression.

Synthesis

- “How did the first three expressions help you find the value of the last expression?”

Activity 1

 20 min

All Lined Up

Standards Alignments

Addressing 4.NF.A.1

The purpose of this activity is to remind students of a key insight from grade 3—that the same point on the number line can be named with fractions that don't look alike. Students see that those fractions are equivalent, even though their numerators and denominators may be different.

Students have multiple opportunities to look for regularity in repeated reasoning (MP8). For instance, they are likely to notice that:

- Fractions that have the same number for the numerator and denominator all represent 1.
- In fractions that describe the halfway point between 0 and 1, the numerator is always half the denominator, or the denominator twice the numerator.
- In fractions that describe $\frac{1}{4}$, the denominator is 4 times the numerator.

These observations will help students to identify and generate equivalent fractions later in the unit.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most important to solve the problem. Display the sentence frame, “The next time points are in the same place on different number lines, I will”

Supports accessibility for: Language, Attention

Instructional Routines

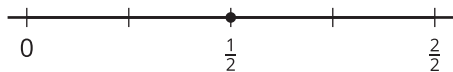
MLR3 Clarify, Critique, Correct

Materials to Gather

Straightedges

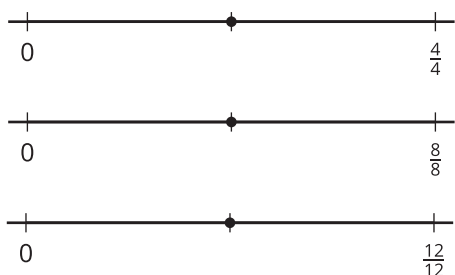
Student-facing Task Statement

1. These number lines have different labels for the tick mark on the far right.

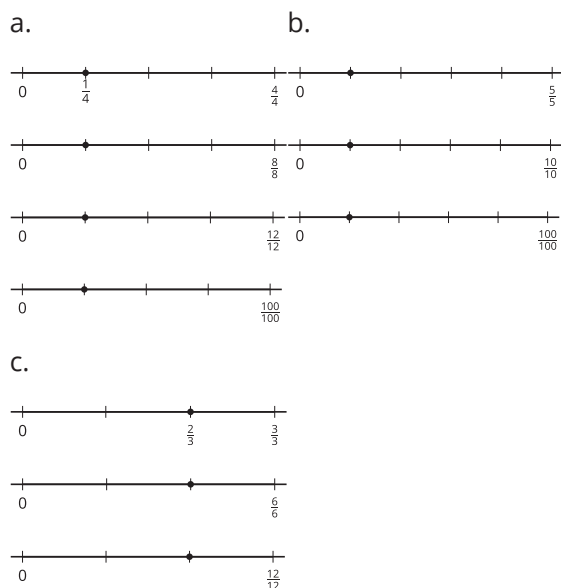


Launch

- Groups of 2
- Give students access to straightedges. Display the first set of number lines.
- “What do you notice? What do you wonder?” (I notice each number line has



- Explain to your partner why the tick mark on the far right can be labeled with fractions with different numbers.
 - Label each point with a number it represents (other than $\frac{1}{2}$).
 - Explain to your partner why the fractions you wrote are equivalent.
2. Label the point on each number line with a number it represents. Be prepared to explain your reasoning.



Student Responses

- Sample responses:
 - Each fraction has a value of 1 and there are different ways to represent 1 with a fraction.
 - The labels should say $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{6}{12}$.

different fractions represented. The first number line has a point that is half-way between 0 and 1 labeled $\frac{1}{2}$, but if you label all the tick marks you won't have 2 in the denominator for all of them. I wonder what fraction goes on each mark? Can you have a number line with both halves and fourths? How many fourths are at the $\frac{1}{2}$ line?)

- 1 minute: quiet think time
- "Share what you noticed and wondered with your partner."
- 1 minute: partner discussion

Activity

- "Take a moment to work independently on the task. Then, discuss your work with your partner."
- "The labels that you write for the points on different number lines should be different."
- 7–8 minutes: independent work time
- Monitor for students who:
 - partition each number line into as many parts as the denominator before naming a fraction for the point on the number line
 - use multiplicative relationships between denominators to name a fraction (for instance, $4 \times 3 = 12$, so the line showing twelfths has 3 times as many parts as the one showing fourths)

Synthesis

- Select students to share their responses and reasoning for the first set of questions. Highlight explanations that convey that:
 - Any fraction with the same number for the numerator and denominator has a value of 1.

- c. They are all halfway between 0 and 1 on the number line, so they are the same size. They all represent one-half. They are the same point on the number line.
2. a. $\frac{2}{8}$, $\frac{3}{12}$, and $\frac{25}{100}$
 b. $\frac{1}{5}$, $\frac{2}{10}$, and $\frac{20}{100}$
 c. $\frac{4}{6}$ and $\frac{8}{12}$

- Equivalent fractions share the same location or are the same distance from 0 on the number line.
- Select students to share their responses for the second set of questions.

MLR3 Clarify, Critique, Correct

- If students show the following partially correct idea, display this explanation: "To know what fraction a point represents, I counted the tick marks from 0. Then, I used the denominator of the fraction for 1. For example, for question 2 part b, the point is on the first tick mark from 0 and the label for 1 says $\frac{10}{10}$, so I'd label the point $\frac{1}{10}$."
- Read the explanation aloud.
- "What do you think the student understands well? What do you think they might have misunderstood?"
- 1 minute: quiet think time
- 2 minutes: partner discussion
- "With your partner, work together to write a revised explanation."
- Display and review the following criteria:
 - Explain: How would one know what numerator and denominator the fraction can have?
 - Write in complete sentences.
 - Use words such as: "first," "next," or "then."
 - Include the number line diagram.
- 3–5 minutes: partner work time
- Select 1–2 groups to share their revised explanation with the class. To facilitate their explanation, display blank number lines for students to annotate. Record responses.
- "What is the same and what is different about the explanations?"

Activity 2

🕒 15 min

How Far to Run?

Standards Alignments

Addressing 4.NF.A.1

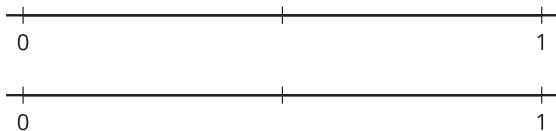
In this activity, students reason about whether two fractions are equivalent in the context of distance. To support their reasoning, students use number lines and their understanding of fractions with related denominators (where one number is a multiple of the other). The given number lines each have only one tick mark between 0 and 1, so students need to partition each line strategically to represent two fractions with different denominators on the diagram.

To help students intuit the distance of 1 mile, consider preparing a neighborhood map that shows the school and some points that are a mile away. Display the map during the launch.

Student-facing Task Statement

- Han and Kiran plan to go for a run after school. They are deciding how far to run.
 - Han says, "Let's run $\frac{3}{4}$ of a mile. That's how far I run to my soccer practice."
 - Kiran says, "I can only run $\frac{9}{12}$ of a mile."

Which distance should they run? Explain your reasoning. Use one or more number lines to show your reasoning.



- Tyler wants to join Han and Kiran on their run. He says, "How about we run $\frac{7}{8}$ of a mile?"



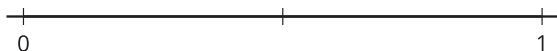
Launch

- Groups of 2
- "Who has walked a mile? Who has run a mile?"
- "How far is 1 mile? How would you describe it?"
- Consider showing a map of the school and some landmarks or points on the map that are a mile away.

Activity

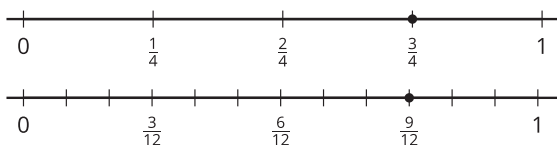
- 6–8 minutes: independent work time
- Monitor for the different ways students reason about the equivalence of $\frac{9}{12}$ and $\frac{3}{4}$. For instance, they may:
 - know that 1 fourth is equivalent to 3 twelfths and reason that 3 fourths must be 9 twelfths
 - note that $\frac{3}{4}$ and $\frac{9}{12}$ are both halfway

Is the distance Tyler suggested the same as what his friends wanted to run? Explain or show your reasoning.

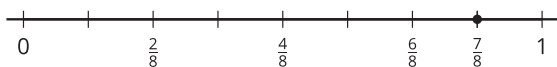


Student Responses

- The distances they propose are the same so either choice works. $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent. Sample response:



- No, $\frac{7}{8}$ is not the same as what Kiran and Han suggested. Sample response: $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, not $\frac{7}{8}$. The distance Tyler suggested is greater.



between $\frac{1}{2}$ and 1 on the number line

- locate $\frac{3}{4}$ and $\frac{9}{12}$ on the same number line (or separate ones) and show that they are in the same location
- 2-3 minutes: partner discussion

Synthesis

- Select students to share their responses and how they knew that $\frac{9}{12}$ is equivalent to $\frac{3}{4}$ but $\frac{7}{8}$ is not.
- To facilitate their explanation, ask them to display their work, or display blank number lines for them to annotate.
- Consider asking:
 - “Who reasoned the same way but would explain it differently?”
 - “Who thought about it differently but arrived at the same conclusion?”

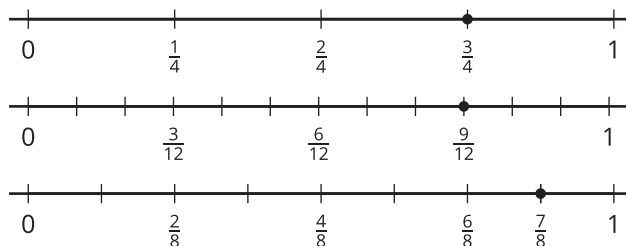
Lesson Synthesis

🕒 10 min

“Today we represented fractions on number lines and reasoned about equivalent fractions.”

Display a labeled diagram of fraction strips and the labeled number lines from today's activity.

1											
$\frac{1}{2}$						$\frac{1}{2}$					
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$			
$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$		
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$



“Where in the diagram of fraction strips do we see equivalent fractions?” (Parts that have the same length are equivalent.)

“Where on the number lines do we see equivalent fractions?” (Points that are in the same location on the number line, or are the same distance from 0, are equivalent.)

“Suppose you’d like to help someone see that $\frac{1}{5}$ is equivalent to $\frac{10}{50}$. Would you use a number line or a fraction strip? Why?” (Sample response: Use a number line, because it’s not necessary to show all the tick marks. If using fraction strips, it would mean partitioning each fifth into 10 fiftieths, which is cumbersome.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)

Complete Cool-Down

Response to Student Thinking

Students label $\frac{5}{6}$ but do not explain or show the equivalence of $\frac{5}{6}$ and $\frac{10}{12}$.

Next Day Support

- Before the warm-up, ask students to work with a partner to discuss a correct response to this cool-down. Encourage them to use both fraction strips and a number line to support their reasoning.

Lesson 6: Relate Fractions to Benchmarks

Standards Alignments

Building On	3.NF.A.2, 3.NF.A.3
Addressing	4.NF.A.2
Building Towards	4.NF.A.2

Teacher-facing Learning Goals

- Locate fractions on the number line and compare their size to $\frac{1}{2}$ and to 1.

Student-facing Learning Goals

- Let's compare the size of fractions to $\frac{1}{2}$ and to 1.

Lesson Purpose

The purpose of this lesson is for students to locate fractions on the number line and compare their size to $\frac{1}{2}$ and to 1.

In this lesson, students continue to identify fractions on the number line by reasoning about known distances or intervals. They also consider the size of fractions in relation to $\frac{1}{2}$ and 1, by examining the position and distance of fractions from these benchmarks on the number line.

Although students consider the distance between a point on the number line and either $\frac{1}{2}$ or 1, finding differences of two fractions is not the focus of this lesson. (That mathematical work will take place in a future unit.) What is important is for students to reason about the relative sizes of fractions using number lines, their knowledge of equivalent fractions and familiar benchmarks, and the meaning of numerator and denominator. Activity 2 is optional and allows an opportunity for students to use the relationships between numerator and denominator in a fraction and between different denominators without requiring them to use the number line.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 3)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 2), Notice and Wonder (Warm-up)

Materials to Copy

- Where Do They Belong (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	20 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What question asked today seemed to promote students reasoning about benchmarks to compare fractions?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Greater Than or Less Than . . . ?

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

For each question, explain or show your reasoning. Use a number line if it is helpful.

1. Is $\frac{6}{10}$ more or less than $\frac{1}{2}$?
2. Is $\frac{11}{12}$ more or less than 1?

Student Responses

1. More than $\frac{1}{2}$. Sample reasoning: I know that $\frac{5}{10}$ is equivalent to $\frac{1}{2}$ and $\frac{6}{10}$ is more than $\frac{5}{10}$.
2. Less than 1. Sample reasoning: I know that $\frac{12}{12}$ is 1 and $\frac{11}{12}$ is less than $\frac{12}{12}$.

----- **Begin Lesson** -----

Warm-up

 10 min

Notice and Wonder: A Point on a Number Line

Standards Alignments

Building On 3.NF.A.2

The purpose of this warm-up is for students to recognize that two values of reference are needed to determine the number that a point on the number line represents. The numbers 0 and 1 are commonly used when the numbers of interest are small. With only one number shown (for example, only a 0 or a 1), we can't tell what number a point represents, though we can tell if the number is greater or less than the given number. These understandings will be helpful later in the lesson, as students determine the size of fractions relative to $\frac{1}{2}$ and 1.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- A number line has a point on it.
- The point is on the sixth tick mark from 0.
- There are no other numbers on the line other than 0.
- There are 10 tick marks on the line.
- The point shows a number greater than 0.

Students may wonder:

- Why isn't there a label for 1?
- Is it possible to tell what number the point represents if there are no other labels?
- Can we assume that the last tick mark represents 1?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How would we know what number the point represents? What's missing and needs to be there?" (A label for one of the tick marks so that we'd know what each interval represents.)

Activity 1

🕒 20 min

Greater Than or Less Than 1?

Standards Alignments

Building On 3.NF.A.2

Building Towards 4.NF.A.2

The purpose of this activity is for students to identify fractions using known benchmarks on the number line and to compare them to 1. Given a point on a number line, the location of 0, and one other benchmark value, students decide if the point represents a number greater or less than 1. They also quantify the distance of that number from 1. Students do so by relying on what they know about the number of fractional parts in 1 whole, as well as by looking for and making use of structure (MP7).

The work here also develops students' ability and flexibility in using number lines to reason about fractions. In later lessons, students will work with number lines that are increasingly more abstract to help them reason about fractions in more sophisticated ways.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

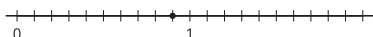
Advances: Listening, Speaking

Student-facing Task Statement

For each diagram:

- Name a fraction the point represents.
- Is that fraction greater than or less than 1?
- How far is it from 1?

1.



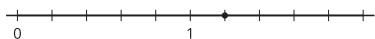
a.

b.

Launch

- Groups of 2–4
- “Tell your partner a fraction that is greater than 1 and a fraction that is less than 1. Explain how you know.”
- 1 minute: partner discussion
- Share responses and ask how they used 1 whole to choose their fractions.
- Read the task statement as a class. Make sure students understand that they are to do three things for each number line diagram.

2.



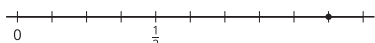
c.

a.

b.

c.

3.

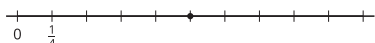


a.

b.

c.

4.



a.

b.

c.

Student Responses

1.
 - a. $\frac{9}{10}$
 - b. less
 - c. $\frac{1}{10}$
2.
 - a. $\frac{6}{5}$
 - b. greater
 - c. $\frac{1}{5}$
3.
 - a. $\frac{9}{8}$
 - b. greater
 - c. $\frac{1}{8}$
4.
 - a. $\frac{5}{4}$
 - b. greater
 - c. $\frac{1}{4}$

Activity

- “Take a few minutes to work independently on at least two diagrams before discussing with your group.”
- 5 minutes: independent work time
- 5–7 minutes: group work time
- Monitor for students who:
 - label one or more tick marks with unit fractions
 - locate the number 1 on the number line when it is not given

Synthesis

- Select students to share their responses. Display their work, or display the number lines from the task for them to annotate as they explain.
- “How did you know what fraction each point represents?” (Figure out what one interval between tick marks represents, and then count the number of intervals.)
- “How did you know if it’s more or less than 1?” (It is more than 1 if the point is to the right of 1, or if the numerator is greater than the denominator.)

Activity 2 (optional)

🕒 20 min

Card Sort: Where Do They Belong?

Standards Alignments

Building On 3.NF.A.3

Building Towards 4.NF.A.2

In this optional activity, students sort a set of fractions into groups based on whether they are less than, equal to, or greater than $\frac{1}{2}$. Sorting enables students to estimate or to reason informally about the size of fractions relative to this benchmark before they go on to do so more precisely. In the next activity, students reason about fractions represented by unlabeled points on the number line and their distance from $\frac{1}{2}$.

As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).

This activity is optional because it asks students to reason about fractions without the support of the number line.

Instructional Routines

Card Sort

Materials to Copy

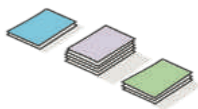
Where Do They Belong (groups of 2)

Required Preparation

- Create a set of fraction cards from the Instructional master for each group.

Student-facing Task Statement

Sort the cards from your teacher into three groups: less than $\frac{1}{2}$, equal to $\frac{1}{2}$, and greater than $\frac{1}{2}$. Be prepared to explain how you know.



Launch

- Groups of 2–4
- Give each group one set of fraction cards

Activity

- “Work with your group to sort the fraction

Record your sorting results here after you have discussed them with another group.

less than $\frac{1}{2}$	equal to $\frac{1}{2}$	greater than $\frac{1}{2}$

Complete the following sentences after class discussion:

- A fraction is less than $\frac{1}{2}$ when . . .
- A fraction is greater than $\frac{1}{2}$ when . . .
- A fraction is between $\frac{1}{2}$ and 1 when . . .

Student Responses

less than $\frac{1}{2}$	equal to $\frac{1}{2}$	greater than $\frac{1}{2}$
$\frac{1}{3}, \frac{1}{5}, \frac{1}{12},$ $\frac{2}{10}, \frac{3}{8}, \frac{3}{12},$ $\frac{4}{12}, \frac{5}{12}$	$\frac{2}{4}, \frac{3}{6}, \frac{4}{8},$ $\frac{5}{10}, \frac{6}{12}$	$\frac{2}{3}, \frac{3}{5}, \frac{4}{2}, \frac{4}{5},$ $\frac{5}{6}, \frac{5}{8}, \frac{6}{3}, \frac{7}{10},$ $\frac{8}{4}, \frac{9}{10}, \frac{11}{12}$

cards into three groups: less than $\frac{1}{2}$, equal to $\frac{1}{2}$, or greater than $\frac{1}{2}$. Be prepared to explain how you know.”

- “When you are done, compare your sorting results with another group.”
- “If the two groups disagree about where a fraction belongs, discuss your thinking until you reach an agreement.”
- 7–8 minutes: group work time
- 3–4 minutes: Discuss results with another group.
- “Record your sorting results after you have discussed them.”

Synthesis

- Invite groups to share how they sorted the fractions.
- “How did the numerator and denominator of each fraction tell you how a fraction relates to $\frac{1}{2}$?” (Sample responses:
 - We already know fractions that are equivalent to $\frac{1}{2}$, so we could compare any fraction to one of those equivalent fractions that has the same denominator.
 - A fraction that is equal to $\frac{1}{2}$ has a denominator that is twice the numerator.
 - If a numerator is less than half of the denominator, the fraction is less than $\frac{1}{2}$. If it is more than half of the denominator, it is more than $\frac{1}{2}$.
 - If a numerator is 1 or is much less than the denominator, then the fraction is small and less than $\frac{1}{2}$.
 - If a numerator is really close to the denominator, then the fraction is close to 1, which means it is more

than $\frac{1}{2}$.)

- Give students 2–3 minutes of quiet time to complete the sentence frames in the activity.

Activity 3

🕒 15 min

Greater Than or Less Than $\frac{1}{2}$?

Standards Alignments

Building On 3.NF.A.2, 3.NF.A.3

Building Towards 4.NF.A.2

Previously, students located fractions on number lines and considered their distance and relative position to 1. Here, they think about fractions in relation to $\frac{1}{2}$. The purpose of this activity is to prompt students to use another benchmark value to determine the size of a fraction.

While students may be able to visually tell if a point on the number line is more or less than $\frac{1}{2}$, finding its distance to $\frac{1}{2}$ is less straightforward than finding its distance to 1. The former requires thinking about $\frac{1}{2}$ in terms of equivalent fractions.

In three cases, the fraction $\frac{1}{2}$ and the point of interest are each on a tick mark on the number line. This makes it possible for students to quantify the distance without further partitioning the number line. In the last diagram, $\frac{1}{2}$ is not on a tick mark, prompting students to subdivide the given intervals, relying on their understanding of equivalence and relationships between fractions.

The work here encourages students to look for and make use of structure (MP7) and will be helpful later in the unit when students compare fractions by reasoning about their distance from benchmark values.

🕒 Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were necessary to solve the problem. Display the sentence frame, “The next time I compare a fraction to $\frac{1}{2}$, I will look for”

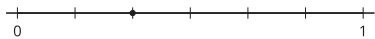
Supports accessibility for: Language, Attention, Conceptual Processing

Student-facing Task Statement

For each diagram:

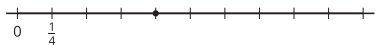
- Name a fraction the point represents.
- Is that fraction greater than or less than $\frac{1}{2}$?
- How far is it from $\frac{1}{2}$?

1.



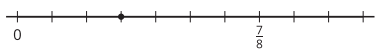
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2.



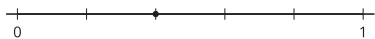
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-
-

3.



-
-
-

4.



-
-
-

Student Responses

- $\frac{2}{6}$
 - less
 - $\frac{1}{6}$
- $\frac{4}{4}$

Launch

- Groups of 2–4
- “Let’s identify a few more fractions on number lines, but this time, let’s find out how they relate to $\frac{1}{2}$.”

Activity

- “Work independently for a few minutes. Work through at least two diagrams before discussing with your group.”
- 5 minutes: independent work time
- 5 minutes: group work time
- Monitor for students who:
 - locate 1 and $\frac{1}{2}$ on the number line.
 - label the point for $\frac{1}{2}$ with an equivalent fraction whose denominator matches the number of intervals between 0 and 1. (for example, labeling the middle tick mark on the first number line with $\frac{3}{6}$.)
 - on the last number line, subdivide the intervals of fifths into tenths in order to locate $\frac{1}{2}$.

Synthesis

- “How did you know where $\frac{1}{2}$ is on the number line?” (Find out where 1 is, and then locate the halfway point. Use fractions that are equivalent to $\frac{1}{2}$, such as $\frac{3}{6}$, $\frac{4}{8}$, and so on.)
- “What was different about the last number line compared to the others?” (There was no tick mark to represent $\frac{1}{2}$ on the number line. The number line had an odd number of intervals.)

- b. greater
c. $\frac{2}{4}$
3. a. $\frac{3}{8}$
b. less
c. $\frac{1}{8}$
4. a. $\frac{2}{5}$
b. less
c. $\frac{1}{10}$

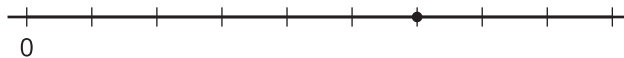
- “What did you have to do differently to figure out how far away the fraction is from $\frac{1}{2}$?” (First split each fifth into tenths and then locate $\frac{5}{10}$.)

Lesson Synthesis

🕒 10 min

“Today we identified fractions on a number line and compared them to $\frac{1}{2}$ and 1.”

Display the number line from the warm-up (or ask students to refer to the diagram there).



Label one of the tick marks (other than the one with the point) with “ $\frac{1}{2}$ ”.

“Suppose a classmate is absent today, and you are asked to explain how to figure out the fraction that the point represents and how far away it is from $\frac{1}{2}$. What would you say?” (I’d see how far away $\frac{1}{2}$ is from 0 and then double that distance to know where 1 is, which would tell me the size of each space between tick marks. If $\frac{1}{2}$ is 4 spaces away from 0, then 1 must be 8 spaces away, and each space must represent $\frac{1}{8}$. I’d count the spaces from 0 to know the fraction. I’d count the spaces between the point and $\frac{1}{2}$ to know its distance from $\frac{1}{2}$.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, or 6 (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Supporting)

Student Section Summary

In this section, we used fraction strips to represent fractions with denominators of 2, 3, 4, 5, 6, 8, 10,

and 12. We also used the strips to reason about relationships between fifths and tenths, and between sixths and twelfths.

1										
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$						
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

1											
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

We learned that 2 tenths are equivalent to 1 fifth, or that splitting 5 fifths into two will produce 10 equal parts or tenths. When the denominator is larger, there are more parts in a whole.

We used what we learned about fraction strips to partition number lines and represent different fractions.



----- Complete Cool-Down -----

Response to Student Thinking

Students respond that $\frac{6}{10}$ is less than $\frac{1}{2}$ or $\frac{11}{12}$ is more than 1 whole.

Next Day Support

- In a small group, give students access to pre-made fraction strips. Ask them to list fractions that have the same size as $\frac{1}{2}$ and to notice patterns in the numbers in those fractions.

Section B: Equivalent Fractions

Lesson 7: Equivalent Fractions

Standards Alignments

Building On	3.NF.A.3.b
Addressing	4.NF.A.1
Building Towards	4.NF.A.1

Teacher-facing Learning Goals

- Generate equivalent fractions using a representation that makes sense to students.

Student-facing Learning Goals

- Let's find some equivalent fractions.

Lesson Purpose

The purpose of this lesson is for students to generate equivalent fractions using a representation that makes sense to them.

In grade 3, students learned to recognize and generate simple equivalent fractions. In earlier lessons, they reasoned about the size of fractions and identified some equivalent fractions. Throughout those experiences, they used fraction strips, tape diagrams, number lines, and benchmark fractions to support their reasoning.

In this lesson, students continue to rely on different representations and reasoning strategies to generate equivalent fractions (including those with denominators 5, 10, and 12, and fractions greater than 1). They also hone their ability to communicate their reasoning clearly.

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR7 Compare and Connect (Activity 2), True or False (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Whose ideas and voices were heard, valued, and accepted today? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Two Equivalent Fractions

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

Name two fractions that are equivalent to $\frac{5}{3}$. Explain or show your reasoning.

Student Responses

Sample response: $\frac{10}{6}$ and $\frac{20}{12}$. Sample reasoning: I drew a tape diagram to show 5 thirds. Then:

I partitioned each 1 third into 2 parts, so in 5 thirds there are 10 parts. Each part is 1 sixth, so in 5 thirds there are 10 sixths.

I partitioned each 1 sixth into 2 parts again, so now there are 20 parts. Each part is 1 twelfth, so in 10 sixths there are 20 twelfths.

----- Begin Lesson -----

Warm-up

🕒 10 min

True or False: Equivalence

Standards Alignments

Building Towards 4.NF.A.1

The purpose of this warm-up is to elicit students' prior understanding of equivalence and strategies for comparing fractions. To determine equivalence, students may rely on familiarity with benchmark fractions, use fraction strips, or think about the relative sizes of the fractional parts. They may also use their knowledge about fractions with the same numerator or denominator. In any case, students have opportunities to look for and make use of structure (MP7).

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{4}{8} = \frac{7}{8}$
- $\frac{3}{4} = \frac{6}{8}$
- $\frac{2}{6} = \frac{2}{8}$
- $\frac{6}{3} = \frac{4}{2}$

Student Responses

- False. Four eighths can't be the same size as 7 eighths.
- True. In each 1 fourth there are 2 eighths, so in 3 fourths there are 6 eighths.
- False. A sixth is greater than an eighth, so 2 sixths is greater than 2 eighths.
- True. Six thirds make 2, and four halves also make 2.

Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- If no students refer to a visual representation (a tape diagram or number line) to explain an equation such as $\frac{3}{4} = \frac{6}{8}$, ask how one of these representations could help with their explanation.
- "For the pair of fractions that you know are not equal, can you tell which fraction is greater? How?"

Activity 1

 20 min

Two or More Fractions

Standards Alignments

Building On 3.NF.A.3.b

Addressing 4.NF.A.1

The purpose of this activity is to elicit strategies for finding equivalent fractions when the fractions are represented by tape diagrams or points on the number line. Students may reason in various ways, but here are two likely approaches:

- partition given fractional parts into smaller equal-size parts and count the new parts (for instance, partitioning each 1 fourth into 3 parts and then counting the twelfths).
- bundle given fractional parts into larger equal-size groups and count the new groups (for instance, bundling every 2 tenths to make 5 fifths in 1 whole and then counting the fifths).

During this and upcoming activity syntheses, help students recognize regularity in their moves to find equivalent fractions. In future lessons, students will connect more explicitly how diagrams of equivalent fractions relate to a numerical process for generating them. They will relate the subdividing or grouping fractional parts to the idea of using multiples and factors to find equivalent fractions.

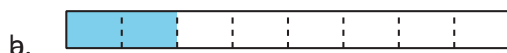
Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were required to solve the problem. Display the sentence frame, “The next time I find equivalent fractions, I will follow the steps of. . . .”

Supports accessibility for: Conceptual Processing, Language

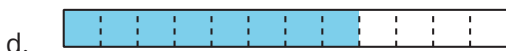
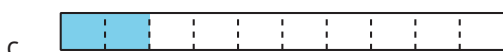
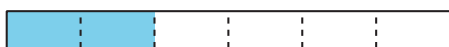
Student-facing Task Statement

- Each entire diagram represents 1 whole. Write two or more fractions that the shaded part of each diagram represents. Be prepared to explain your reasoning.

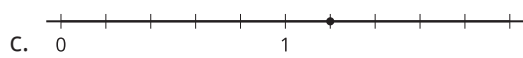
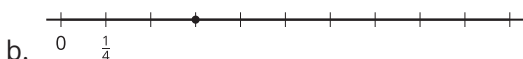


Launch

- Groups of 2
- “In this activity, you’ll see diagrams and number lines that represent fractions.”
- “Find at least two fractions to describe the shaded part of each diagram, and two fractions for the point on each number



2. Write two or more fractions that the point on each number line represents. Be prepared to explain your reasoning.



3. Place a new point on a tick mark on one of the last two number lines (in part c or d). Then, write two fractions that the point represents.

Student Responses

1. a. $\frac{2}{8}, \frac{1}{4}$
 b. $\frac{2}{6}, \frac{1}{3}, \frac{4}{12}$
 c. $\frac{2}{10}, \frac{1}{5}$
 d. $\frac{8}{12}, \frac{2}{3}, \frac{4}{6}$
2. a. $\frac{2}{5}, \frac{4}{10}$
 b. $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}$
 c. $\frac{6}{5}, \frac{12}{10}$
 d. $\frac{5}{4}, \frac{10}{8}$
3. Answers vary.

line.”

Activity

- “Take 5 minutes to work independently on the first problem. Afterwards, share your thinking with your partner.”
- 5 minutes: independent work time for the first problem
- 2 minutes: partner discussion
- “Now take a few minutes to work independently on the second problem.”
- 5 minutes: independent work time for the remaining problems
- 2 minutes: partner discussion
- Monitor for students who:
 - partition each unit fraction (a single section on a tape diagram or an interval on a number line) into smaller parts
 - bundle multiple unit fractions into larger groups
- Identify 2–3 students who reason differently on tape diagrams and 2–3 who reason differently on number lines.

Synthesis

- Select previously identified students to share how they found multiple equivalent fractions on the two kinds of representations. Display their work, or display the diagrams in the activity for them to annotate as they explain.
- “How is the process of finding equivalent fractions using diagrams like the process of using number lines?” (They both involve partitioning given parts into smaller parts, or bundling the given parts into larger parts.)
- “How are they different?” (The length of a diagram usually is 1 whole or another

whole number. A number line doesn't always show 1 whole, so we may have to figure out where it is first.)

- If time permits: "Can you write other equivalent fractions for diagram ____?" (Sample response for the last number line diagram: $\frac{15}{12}$, $\frac{20}{16}$)
- "How many fractions do you think you could write for that diagram?" (This prompts students to begin to realize that there are infinite equivalent fractions as the whole is partitioned into smaller parts.)

Advancing Student Thinking

Students label number lines using tick marks alone. For example if 4 marks are visible (including zero) each line would be labeled as fourths instead of thirds. If this happens, consider using the idea of movement from 0 to 1. Ask: "Where is 1 on the number line?" "If we are moving from 0 toward 1, what does this tick mark between 0 and 1 mean?" Ask students to review the labels on their number lines and decide if revisions are needed before continuing to work on the next activity.

Activity 2

🕒 15 min

Equivalent for Sure?

Standards Alignments

Addressing 4.NF.A.1

In this activity, students find equivalent fractions for fractions given numerically. They also work to clearly convey their thinking to a partner, which involves choosing and using words, numbers, or other representations with care. In doing so, students practice attending to precision (MP6) as they communicate about mathematics.

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

For each fraction, find two equivalent fractions.

Partner A

1. $\frac{3}{2}$

2. $\frac{10}{6}$

Partner B

1. $\frac{4}{3}$

2. $\frac{14}{10}$

Next, show or explain to your partner how you know that the fractions you wrote are equivalent to the original. Use any representation that you think is helpful.

Student Responses

Students may use fraction strips, tape diagrams, or number lines to show their reasoning.

Sample responses:

Partner A

1. $\frac{6}{4}$ and $\frac{12}{8}$

2. $\frac{20}{12}$ and $\frac{5}{3}$

Partner B

1. $\frac{8}{6}$ and $\frac{16}{12}$

2. $\frac{28}{20}$ and $\frac{7}{5}$

Launch

- Groups of 2
- “Work with a partner on this activity. One person is partner A and the other is B.”
- “Your task is to find two equivalent fractions for each fraction listed under A or B, and then convince your partner that your fractions are equivalent.”

Activity

- “Take 5 to 6 quiet minutes to find equivalent fractions and to plan your explanation.”
- 5–6 minutes: independent work time
- “Take turns sharing your fractions and explanation with your partner.”
- “When your partner explains, listen carefully to their reasoning and ask them to clarify if something is unclear.”
- 5–6 minutes: partner discussion

Synthesis

MLR7 Compare and Connect

- “Create a visual display that shows how you found two equivalent fractions for the second fraction on your list: $\frac{10}{6}$ for Partner A, and $\frac{14}{10}$ for Partner B.”
- “Include diagrams, notes, and any descriptions that might help others understand your thinking.”
- See lesson synthesis.

Lesson Synthesis

🕒 10 min

Ask students to display their work around the room.

“Take a few minutes to walk around and look at the work of at least 4 classmates. Make sure to look at the work by both partners, A and B.”

“As you study others’ work, pay attention to how the reasoning is alike and how it is different.”

“What is the same about the diagrams, words, or explanations that you saw? What is different?”

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Complete Cool-Down

Response to Student Thinking

The work in this lesson builds from equivalence of simple fractions addressed in a prior unit.

Prior Unit Support

Grade 3, Unit 5, Section C: Equivalent Fractions

Lesson 8: Equivalent Fractions on the Number Line

Standards Alignments

Building On 3.NF.A.1

Addressing 4.NF.A.1

Teacher-facing Learning Goals

- Reason about and generate equivalent fractions on the number line.

Student-facing Learning Goals

- Let's use number lines to reason about equivalent fractions.

Lesson Purpose

The purpose of this lesson is for students to reason about and generate equivalent fractions on the number line.

Previously, students generated equivalent fractions in any way that was intuitive to them. In this lesson, students use number lines to reason about and generate equivalent fractions. In particular, they experiment with partitioning a fractional part on the number line into smaller equal-size parts. Through repeated reasoning, students begin to notice regularity in the numerator and denominator of the equivalent fractions—namely, that the numbers are multiples of those in the original fraction. The experience of sub-partitioning number lines prepares students to formalize their observation and reason numerically about equivalent fractions in upcoming lessons.

In this lesson, students take a closer look at the relationships between fractions with denominator 5, 10, and other multiples of 5. They begin to consider the meaning of fractions with denominator 100.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Tape (painter's or masking): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In past lessons and in grade 3, students partitioned unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ (on fraction strips, tape diagrams, and number lines) into smaller fractional parts such as $\frac{1}{6}$ and $\frac{1}{8}$. How readily did students transfer those insights to work with fractions with larger denominators on the number line? What was intuitive to them and what wasn't?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

In Search of Equivalence

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

For each problem, explain or show your reasoning. Use a number line, if it helps.

1. Name a fraction that is equivalent to $\frac{9}{10}$.



2. Is $\frac{8}{5}$ equivalent to $\frac{15}{10}$?

**Student Responses**

Sample responses:

- $\frac{18}{20}$, $\frac{36}{40}$, or $\frac{90}{100}$. (If using a number line, students may partition it into tenths, and further partition each tenth into 2 parts to get twentieths or into 10 parts to get hundredths. Or they may group every 2 tenths to make 5 fifths.)
- No. Sample reasoning: One fifth is 2 tenths, so 8 fifths must be 8×2 or 16 tenths, not 15 tenths. (If using a number line, students may show fifths, partition into 2 tenths each, and see 8 fifths as equal to 16 tenths.)

Begin Lesson

Warm-up

🕒 10 min

Estimation Exploration: A Shaded Portion

Standards Alignments

Building On 3.NF.A.1

The purpose of an Estimation Exploration is to practice estimating a reasonable answer based on experience and known information. Students can identify fractions represented by the shaded portions in tape diagrams in which unit or non-unit fractions are marked. To estimate the shaded parts in an unmarked tape, students may rely on the size of benchmark fractions— $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ —and partition those parts mentally until it approximates the size of the shaded parts. They may also estimate how many copies of the shaded part could fit in the entire diagram.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

If the entire diagram represents 1 whole, about what fraction is shaded?



Make an estimate that is:

too low	about right	too high

Student Responses

Sample response:

- Too low: $\frac{1}{25}$ to $\frac{1}{20}$
- About right: $\frac{1}{12}$ to $\frac{1}{10}$
- Too high: $\frac{1}{5}$ to $\frac{1}{4}$

Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high? Too low? About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

Consider asking:

- “Is anyone’s estimate less than $\frac{1}{20}$? Is anyone’s estimate greater than $\frac{1}{4}$?”
- “Based on this discussion does anyone want

to revise their estimate?"

Activity 1

🕒 20 min

Handy Number Lines

👤 ↔ 👤 PLC Activity

Standards Alignments

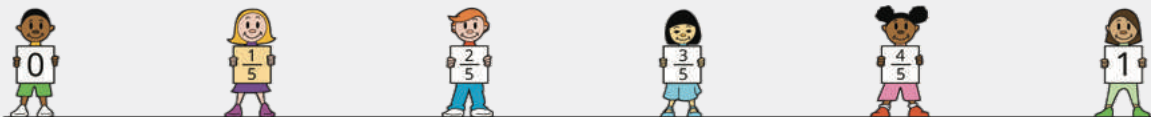
Addressing 4.NF.A.1

In this activity, students examine number lines that have been partitioned into smaller and smaller parts. They see that this strategy can be used to generate many equivalent fractions and to verify if two fractions are equivalent.

Students encounter fractions with 5, 10, 15, and 20 for the denominator. Working with multiples of a number (in this case, 5) allows students to notice structure in how the partitioning of a part on a number line relates to equivalent fractions (MP7). (Students will not be assessed on fractions with denominator 15 or 20.)

If desired and logistically feasible, consider enacting Andre's reasoning with one or more human number lines.

- Place a strip of masking tape or painter's tape, at least 25 feet long, on the floor of the classroom or a hallway.
- Ask a student to stand on each end of the tape. They represent 0 and 1.
- "How can we partition this line into fifths?" (Position 4 students on the tape, spaced apart equally between 0 and 1.)
- Ask each student on the line to say the number they represent ($0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$). Give each student a sign with their fraction. (Consider distinguishing the sign for $\frac{1}{5}$ with a different color.)



- Invite 5 students to join the line, each person standing exactly in the middle of two others.
- "What fraction do you represent now?" Ask every student to say the number they represent now ($0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \dots, 1$). Give the student representing $\frac{1}{5}$ another sign showing $\frac{2}{10}$.
- Repeat a couple more times. Each time:
 - Ask 5 additional students to each join a space between two students representing

fifths. (The students representing fifths should stay in place, but, to maintain equal intervals, those representing smaller fractional parts may need to shift when others join the line.)

- Ask students to say aloud the number they represent.
- Give the student representing $\frac{1}{5}$ another sign showing a new fraction they represent.
- “Can you explain why the student representing $\frac{1}{5}$ also ends up representing $\frac{2}{10}$, $\frac{3}{15}$, and $\frac{4}{20}$?”

Access for English Learners

MLR8 Discussion Supports. During partner work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say . . .” Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.

Supports accessibility for: Social-Emotional Functioning, Attention

Materials to Gather

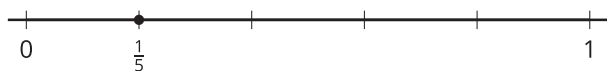
Tape (painter's or masking)

Required Preparation

- Consider creating a human number line by placing a strip of masking tape or painter's tape, at least 25 feet long, on the floor of the classroom or a hallway.

Student-facing Task Statement

Andre used number lines to find fractions that are equivalent to $\frac{1}{5}$. He drew this number line:

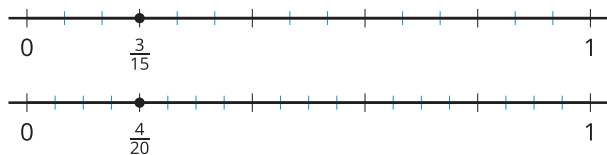


Then, he drew three more lines and wrote a fraction for the point on each line:



Launch

- Groups of 2
- Read the opening paragraphs of the activity as a class.
- If possible, consider creating a human number line as outlined in the Activity Narrative.



- How did Andre use the number lines to find fractions equivalent to $\frac{1}{5}$? Explain your thinking to a partner.
- How can number lines be used to show whether the following fractions are equivalent?
 - $\frac{8}{10}$ and $\frac{4}{5}$
 - $\frac{14}{20}$ and $\frac{4}{5}$
- Find three fractions that are equivalent to $\frac{6}{5}$. Explain or show how Andre's number lines can help.

Student Responses

- Sample response: Andre split each fifth into different numbers of equal parts: first it was 2 equal parts, then it was 3, and then 4 equal parts. The 10, 15, and 20 are the new number of parts in 1 whole. The 2, 3, and 4 are how many of each fractional parts are in the original 1 fifth.
- Sample response:
 - A number line would show that every 2 tenths is equivalent to 1 fifth and that it takes 4×2 or 8 tenths to make $\frac{4}{5}$ so they are equivalent.
 - It takes 4 twentieths to make $\frac{1}{5}$ so $\frac{4}{5}$ would be 16 twentieths not 14.
- Sample response: $\frac{12}{10}$, $\frac{18}{15}$, and $\frac{24}{20}$
 - If 1 fifth is split into 2 parts, each part being 1 tenth, then 6 fifths has 6×2 or 12 tenths.
 - If 1 fifth is split into 3 parts, each part being 1 fifteenth, then 6 fifths has 6×3 or 18 fifteenths.

Activity

- “Think quietly for a minute about the first problem. Then, discuss your thinking with a partner and work on the second and third problems together.”
- 1 minute: quiet think time
- 7–8 minutes: partner work time
- Monitor for students who:
 - extend each number line past 1 whole to show $\frac{6}{5}$, $\frac{12}{10}$, $\frac{18}{15}$, and $\frac{24}{20}$
 - skip-count by the number in each numerator 6 times—by 2 to get to $\frac{12}{10}$, by 3 to get to $\frac{18}{15}$, and by 4 to get to $\frac{24}{20}$.
 - multiply the numerator (6) and the denominator (5) by 2, 3, and 4 to get $\frac{12}{10}$, $\frac{18}{15}$, and $\frac{24}{20}$.

Synthesis

- Invite a student to share their explanation of Andre's strategy (first problem). Ask the class if they agree with the explanation or if they would amend it or explain it differently.
- Ask another student to show how the number lines could help us see if two fractions are equivalent (second problem).
- Select previously identified students to share their reasoning for the last problem. Record their reasoning.
- Solicit some initial impressions on how the strategies are alike and different, but save further comparisons for future lessons.

- If 1 fifth is split into 4 parts, each one being 1 twentieth, then 6 fifths has 6×4 or 24 twentieths.

Activity 2

🕒 15 min

Can It Be Done?

Standards Alignments

Addressing 4.NF.A.1

In this activity, students continue to use the idea of partitioning a number line into smaller increments to reason about and generate equivalent fractions. Through repeated reasoning, students begin to see regularity in how the process of decomposing parts on a number line produces the numbers in the equivalent fractions (MP8). The task encourages students to think of the relationship between one denominator and the other in terms of factors or multiples (even if they don't use those terms which connect to work in a previous unit).

Partitioning a number line into smaller parts becomes increasingly inconvenient when the denominator gets larger. As students begin to think about the relationship between tenths and hundredths, they see some practical limitations to using a number line to find equivalent fractions and are prompted to generalize the process of partitioning. (Students are not expected to draw a full number line with 100 parts.)

Student-facing Task Statement

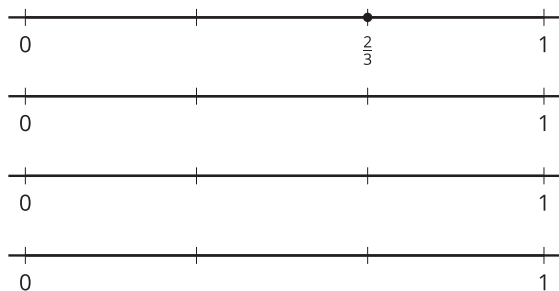
1. Priya wants to find fractions that are equivalent to $\frac{2}{3}$, other than $\frac{4}{6}$. She wonders if she can find equivalent fractions with denominator 9, 10, and 12.

$$\frac{\quad}{9} \quad \frac{\quad}{10} \quad \frac{\quad}{12}$$

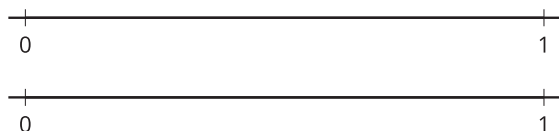
Can it be done? Use number lines to show your reasoning.

Launch

- Groups of 2
- Read the first problem as a class. Ask students to think quietly for a moment about whether what Priya wants to do can be done.
- 30 seconds: quiet think time
- 1 minute: partner discussion



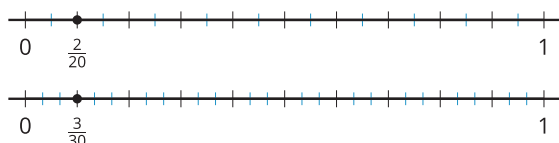
2. Represent $\frac{1}{10}$ on a number line. Then, find two fractions that are equivalent to $\frac{1}{10}$. How would you use the number line to show that they are equivalent to $\frac{1}{10}$?



3. Can you find an equivalent fraction for $\frac{1}{10}$ with 100 for the denominator? Explain or show your reasoning.

Student Responses

1. Yes for denominator 9 and 12, but not for 10: $\frac{6}{9}$ and $\frac{8}{12}$. Reasoning should show each third in each number line partitioned into 3 and 4 sub-parts to get 9 and 12 parts, respectively, in 1 whole. Each of the original thirds can't be partitioned into the same number of parts to get 10 equal parts in 1 whole. (Or 3 is not a factor of 10.)
2. $\frac{2}{20}$ and $\frac{3}{30}$. Sample reasoning: Each tenth can be broken into 2 parts to get twentieths, and into 3 parts to get thirtieths.



3. Yes. Sample reasoning: I know that $10 \times 10 = 100$. If we use a number line, each tenth would be split into 10 smaller parts, giving 100 parts in 1 whole. One tenth

Activity

- "Take about 7–8 quiet minutes to work on the task. Afterwards, discuss your responses with your partner."
- 7–8 minutes: independent work time
- 2–3 minutes: partner discussion
- For the first problem, monitor for students who partition the lines by:
 - guessing and checking
 - reasoning multiplicatively (3 times what number gives 9, 10, or 12?) or in terms of multiples (Is 9, 10, or 12 a multiple of 3?)
 - reasoning in terms of division (9 divided by 3 is what number?) or in terms of factors (Is 3 a factor of 9, 10, or 12?)
- For the second and third problem, monitor for students who find equivalent fractions for $\frac{1}{10}$ by:
 - partitioning the number lines into smaller increments quantifying the new number of parts
 - finding multiples of 1 and 10, and using this strategy to write an equivalent fraction with denominator 100

Synthesis

- Invite previously identified students to share their strategies for answering the first two problems.
- If not done by students in their explanations, consider asking students to revoice their reasoning in terms of factors and multiples.
- See lesson synthesis.

is the same size as 10 of those hundredths.

Advancing Student Thinking

If students conclude that the answer to the last question is “no, it can’t be done” because they find it impractical to partition each tenth on the number line into so many parts, ask them to visualize the line and think about how they’d partition it. Consider asking: “Into how many parts should each one-tenth section on the line be split to get hundredths? How do you know?”

Lesson Synthesis

🕒 10 min

“Today we used number lines and partitioning to help us write equivalent fractions and to tell if two fractions are equivalent.”

“How can number lines help us find equivalent fractions for, say, $\frac{1}{10}$?” (We can draw a number line showing tenths, and then partition the tenths into 2 parts, 3 parts, 4 parts, and so on.)

“There were times in the lesson when some of you chose not to use the number lines to find equivalent fractions or to tell if two fractions were equivalent. Why was that?” (Sample responses: It was not necessary. It’d take too long to draw all the tick marks. We could skip count, reason about the numbers mentally, or find multiples of the numbers in the fraction.)

“In upcoming lessons, we’ll continue to develop our strategies for finding equivalent fractions and checking if two fractions are equivalent.”

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

----- Complete Cool-Down -----

Response to Student Thinking

When reasoning about the second problem, students labeled the right end of the number line with 1 and plotted $\frac{8}{5}$ or $\frac{15}{10}$ between 0 and 1.

Next Day Support

- Before the warm-up, ask students to discuss with a partner how to label a number line to show fractions greater than 1, such as $\frac{8}{5}$ or $\frac{13}{10}$.

Lesson 9: Explain Equivalence

Standards Alignments

Building On	3.OA.B.5
Addressing	4.NF.A.1
Building Towards	4.NBT.B.5

Teacher-facing Learning Goals

- Determine if given fractions are equivalent in a way that makes sense to them.
- Given a pair of equivalent fractions, explain why they are equivalent.

Student-facing Learning Goals

- Let's talk about how we know whether two fractions are equivalent.

Lesson Purpose

The purpose of this lesson is for students to determine if two fractions are equivalent, and if they are, explain why they are equivalent.

In earlier lessons, students developed their ability to use different representations and strategies to reason about equivalence and generate equivalent fractions. This lesson enables them to consolidate the work so far and communicate their understanding conceptually, before they move on to reason about equivalent fractions numerically in the next lesson.

Students work with some fractions in the hundredths. Although students might try to partition a number line into 100 parts, they are not expected to do so. The idea is to motivate students to look for another way—one that is less tedious and more general—to generate equivalent fractions.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Rulers or straightedges: Activity 1

Materials to Copy

- How Do You Know (groups of 15): Activity 2

- Sticky notes: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

This lesson centers on explanations for equivalence. What representations and strategies did most students rely on to justify equivalence? What aspects of the explanation was manageable for them? What was more challenging than anticipated?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

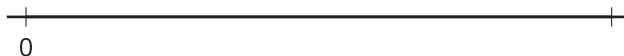
To Be or Not to Be (Equivalent)

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

1. Explain or show why this statement is true: $\frac{5}{4}$ is equivalent to $\frac{15}{12}$. Use a number line, if it helps.



2. Diego wrote $\frac{11}{5}$ and $\frac{55}{10}$ as equivalent fractions. Are those fractions equivalent? Explain or show how you know. Use a number line, if it helps.



Student Responses

Students may use number lines to show their reasoning. Sample responses:

1. If I split each fourth into 3 equal parts, then I can see that $3 \times 5 = 15$.
2. No. Sample reasoning: One fifth can be partitioned into 2 parts to get tenths, so 11 fifths has 11×2 or 22 tenths, not 55 tenths.

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Familiar Numbers

Standards Alignments

Building On	3.OA.B.5
Building Towards	4.NBT.B.5

This Number Talk encourages students to use the relationship between related numbers (5 and 10, and 6, 12, and 24) and properties of operations to find products. The strategies of doubling and halving elicited here will be helpful later in the lesson when students generate equivalent fractions. In describing strategies, students need to be precise in their word choice and use of language (MP6).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 10×6
- 10×12
- 10×24
- 5×24

Student Responses

- 60. I just know.
- 120. Twelve is twice 6, so 10×12 is twice 10×6 , or 2×60 .
- 240. Twenty-four is twice 12, so 10×24 is twice 10×12 or 2×120 .
- 120. Five is half of 10, so 5×24 is half of 10×24 or half of 240.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How did the first three expressions help you find the value of the last one?”

Activity 1

🕒 20 min

Pointed Discussion

Standards Alignments

Addressing 4.NF.A.1

In this activity, students look closely at the relationships of fractions with denominator 5, 10, and 100. They use their observations and understanding to identify equivalent fractions and to explain why two fractions are or are not equivalent. When students analyze and criticize the reasoning presented in the activity statements and when they discuss their work with classmates, they are critiquing the reasoning of others and improving their arguments (MP3).

🕒 Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community.

Supports accessibility for: Social-Emotional Functioning

Instructional Routines

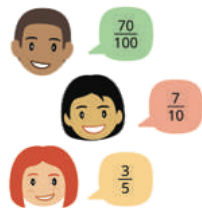
MLR1 Stronger and Clearer Each Time

Materials to Gather

Rulers or straightedges

Student-facing Task Statement

Andre, Lin, and Clare are representing $\frac{70}{100}$ on a number line.



- Andre said, "Oh, no! We'll need to partition the line into 100 equal parts and count 70 parts just to mark one point!"
- Lin said, "What if we mark $\frac{7}{10}$ instead? We could

Launch

- Groups of 2
- Give students access to rulers or straightedges.
- Ask students to keep their materials closed.

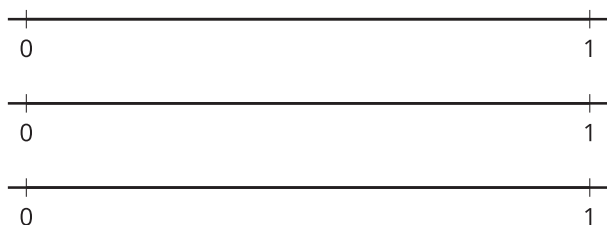
Activity

- "Take 5 quiet minutes to answer the problem. Afterwards, discuss your thinking with your partner."
- 5 minutes: independent work time

partition the line into just 10 parts and count 7 parts.”

- Clare said, “What if we partition the line into 5 parts and mark $\frac{3}{5}$?”

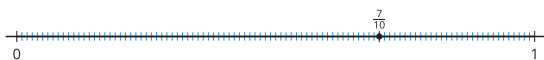
Do you agree with any of them? Explain or show your reasoning.



Student Responses

Agree with Andre and Lin, and disagree with Clare. Sample reasoning:

- Agree with Andre, because $\frac{70}{100}$ means 70 hundredths or 70 one-hundredth parts.



- Agree with Lin, because if we put every 10 hundredths into a group, we’d have 10 equal groups in 1 whole, each group being 1 tenth. 70 hundredths is the same size as 7 tenths.



- Disagree with Clare, because if we put the hundredths into 5 equal groups, each group would have 20 hundredths, and 3 groups would mean 60 hundredths, not 70 hundredths.



- 5 minutes: partner discussion

Synthesis

MLR1 Stronger and Clearer Each Time

- “Share your reasoning and number lines with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work.”
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- “Revise your initial draft based on the feedback you got from your partners.”
- 2–3 minutes: independent work time

Activity 2

 15 min

How Do You Know?

Standards Alignments

Addressing 4.NF.A.1

This activity gives students opportunities to practice explaining or showing whether two fractions are equivalent. Students may do so using a visual representation, by reasoning about the number and size of the fractional parts in each fraction, or by thinking about multiplicative relationships between the numbers in the given fractions.

Students participate in a gallery walk in which they generate equivalent fractions for the numbers on the posters. Students visit at least two of six posters (or as many as time permits)—at least one poster with two fractions (posters A–C) and one poster with three fractions (posters D–F). Here are the sets shown on the Instructional master:

A	B	C	D	E	F
$\frac{2}{10}, \frac{20}{100}$	$\frac{6}{4}, \frac{18}{12}$	$\frac{3}{5}, \frac{60}{100}$	$\frac{1}{4}, \frac{3}{12}, \frac{30}{100}$	$\frac{15}{6}, \frac{7}{4}, \frac{30}{12}$	$\frac{7}{3}, \frac{21}{10}, \frac{28}{12}$

Because at the posters with two fractions (A–C) students would need to generate an equivalent fraction that hasn't already been written by others, generating equivalent fractions becomes more difficult as the activity goes on. Consider using this to differentiate for students who may need an additional challenge: start them at the posters with three fractions (D–F).

Access for English Learners

MLR8 Discussion Supports. Synthesis. Display sentence frames to support whole-class discussion: "I agree because . . ." and "I disagree because . . ."

Advances: Speaking, Conversing

Materials to Gather

Sticky notes

Materials to Copy

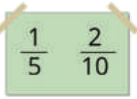
How Do You Know (groups of 15)

Required Preparation

- Each group needs 4 sticky notes.

Student-facing Task Statement

Around the room you will find six posters, each showing either two or three fractions.



With your group, visit at least two posters: one with two fractions and one with three fractions.

For the set of 2 fractions:

- Explain or show how you know the fractions are equivalent.
- Write a new equivalent fraction on a sticky note and add it to the poster. Think of a fraction that hasn't already been written by someone else.

We visited poster _____, which shows _____ and _____.

New equivalent fraction: _____

For the set of 3 fractions:

- Identify 2 fractions that are equivalent. Explain your reasoning.

We visited poster _____, which shows _____, _____, and _____.

Student Responses

- A: Sample response: One tenth can be split into 10 equal parts, each being 1 hundredth. Two tenths is then 2×10 or 20 hundredths. Sample equivalent fractions: $\frac{1}{5}$, $\frac{10}{50}$
- B: Sample response: One fourth is equal to 3 twelfths. Six fourths is 6×3 or 18 twelfths. Sample equivalent fractions: $\frac{12}{8}$, $\frac{24}{16}$
- C: Sample response: One fifth can be partitioned into 20 equal parts to make 20

Launch

- Groups of 3–4
- Give each group 4 sticky notes
- Read the task statement as a class. Solicit clarifying questions from students.
- Invite a couple of students to recap the directions in their own words or to demonstrate the process, if helpful.
- Consider assigning each group a starting poster and giving directions for rotation.

Activity

- 10 minutes: gallery walk
- Tell students who are visiting posters A–C that they could leave feedback about the fraction on a sticky note if they disagree that it is equivalent to the fractions on the poster. They should include their name and be prepared to explain how they know.

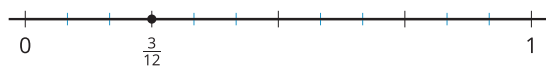
Synthesis

- See lesson synthesis.

hundredths, so 3 fifths is 3×20 or 60 hundredths. Sample equivalent fractions:

$$\frac{6}{10}, \frac{12}{20}$$

- D: $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent. $\frac{30}{100}$ is not equivalent to the other two. Sample reasoning:



- If each 1 fourth is split into 30 parts, there are 120 parts in 1 whole, so 1 fourth is 30 one-hundred-twentieths, not 30 hundredths.
- If each 1 fourth is split into 25 parts, there are 100 parts in 1 whole, so 1 fourth is 25 hundredths, not 30 hundredths.
- E: $\frac{15}{6}$ and $\frac{30}{12}$ are equivalent. $\frac{7}{4}$ is not equivalent to the other two. Sample reasoning: Each sixth can be partitioned into 2 parts to make 2 twelfths, so 15 sixths is 15×2 or 30 twelfths.
 - If we partition the fourths into twelfths, $\frac{7}{4}$ will be the same size as $\frac{21}{12}$, not $\frac{30}{12}$.
 - $\frac{7}{4}$ is less than 2, while $\frac{15}{6}$ and $\frac{30}{12}$ are both greater than 2.
- F: $\frac{7}{3}$ and $\frac{28}{12}$ are equivalent. $\frac{21}{10}$ is not equivalent to the other two. Sample reasoning: If we partition 1 third into 4 parts, we will have 4 twelfths. Seven thirds will be 7×4 or 28 twelfths.
 - We can't partition a third into a whole number to get tenths, or 10 is not a multiple of 3.
 - $\frac{21}{10}$ is 2 wholes and $\frac{1}{10}$, while $\frac{7}{3}$ is 2 wholes and $\frac{1}{3}$. A tenth is less than a third.

Lesson Synthesis

🕒 10 min

Select a group to share their response and reasoning for each poster.

Highlight visual diagrams or verbal explanations that clearly show how the number and size of the parts of two fractions can differ even though the fractions are the same size.

When students explain their work on posters D–F, ask about the non-equivalent fraction. For instance: “How did you know that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent but $\frac{30}{100}$ is not equivalent to them?”

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Complete Cool-Down

Response to Student Thinking

The work in this lesson builds from equivalence of simple fractions addressed in a prior unit.

Prior Unit Support

Grade 3, Unit 5, Section C: Equivalent Fractions

Lesson 10: Use Multiples to Find Equivalent Fractions

Standards Alignments

Addressing 4.NF.A.1

Teacher-facing Learning Goals

- Make sense of a way to generate equivalent fractions by using multiples of the numerator and denominator.

Student-facing Learning Goals

- Let's look at a way to find equivalent fractions without using diagrams.

Lesson Purpose

The purpose of this lesson is for students to make sense of a way to identify and generate equivalent fractions by using multiples of the numerator and denominator.

Up until this point, students have used visual representations or other strategies to reason about and generate equivalent fractions. Along the way, they are likely to have noticed patterns in the numerator and denominator of equivalent fractions. While some students may have generalized and applied those observations intuitively, this is the first lesson in which students are prompted to reason numerically about the numbers in equivalent fractions.

Students notice that a fraction $\frac{a}{b}$ has the same location on the number line as a fraction $\frac{n \times a}{n \times b}$, so we can generate fractions that are equivalent to $\frac{a}{b}$ by multiplying both a and b by n . In other words, they can use multiples of a and b to generate fractions that are equivalent to $\frac{a}{b}$. Sample responses are shown in the form $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ but students do not need to use this notation.

In an upcoming lesson, students will reason in the other direction: using factors that are common to a and b to write equivalent fractions. They will see that dividing a and b by the same factor n gives a fraction equivalent to $\frac{a}{b}$.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

To reason numerically we hope students begin to describe number relationships without visual representations. Did it seem that students were doing this in today's lesson? Which diagrams are they still holding on to?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Fractions of the Same Size

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

- Find two fractions that are equivalent to $\frac{3}{8}$. Explain or show your reasoning.
- Decide if each of the following fractions are equivalent to $\frac{9}{4}$.
 - $\frac{10}{8}$
 - $\frac{16}{10}$
 - $\frac{18}{8}$
 - $\frac{27}{12}$

Student Responses

- Sample response: $\frac{6}{16}$ and $\frac{9}{24}$. $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$ and $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$
- No
 - No
 - Yes
 - Yes

Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Four Equations

Standards Alignments

Addressing 4.NF.A.1

The purpose of this warm-up is to draw students' attention to the multiplicative relationships between the numerators and denominators of two equivalent fractions. These observations will be helpful later as students use the idea of multiples to generate equivalent fractions.

While students may notice and wonder many things about these equations, highlight observations about a factor relating the numbers in the two sides of each equation.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

- $\frac{1}{3} = \frac{2}{6}$
- $\frac{2}{3} = \frac{4}{6}$
- $\frac{3}{3} = \frac{6}{6}$
- $\frac{4}{3} = \frac{8}{6}$

Student Responses

Students may notice:

- There are four equations with a fraction on each side of the equal sign.
- The fraction on the left side has 3 for the denominator. The fraction on the right each has 6 for the denominator.
- The numerators on the left side are 1, 2, 3, and 4. The ones on the right are 2, 4, 6, and 8. They each follow a pattern.

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How are the numbers on the right side of each equal sign related to the numbers on the left?" (Each number on the right is twice the number on the left.)
- "Are the fractions on the right twice the size of the fractions on the left?" (No, they are the

- Each pair of fractions are equivalent.
- The first two pairs of fractions are less than 1. The third is 1. The last pair is more than 1.

same size.)

Students may wonder:

- What might the equations look like if we continued the pattern?
- Why do the numerators change but the denominators don't?
- Are each pair of fractions really equivalent?
- Are the fractions on the right side of the equal sign twice the size of those on the left?

Activity 1

🕒 20 min

Elena's Way

Standards Alignments

Addressing 4.NF.A.1

In an earlier lesson, students used visual representations to generate equivalent fractions. They did so by partitioning each increment on a number line into smaller equal-size parts. In this activity, they connect that action to a numerical process—one that involves multiplying both the numerator and denominator by the same factor. When students notice that they can multiply the numerator and denominator of a fraction by any whole number to get an equivalent fraction they observe regularity in repeated reasoning (MP8).

🌐 Access for English Learners

MLR2 Collect and Display. Collect the language students use to reason about how to find equivalent fractions. Display words and phrases such as: equivalent fraction, equation, number line, numerator, denominator, multiply, multiples, etc. During the activity, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student-facing Task Statement

Elena thought of another way to find equivalent fractions. She wrote:

$$\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

$$\frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

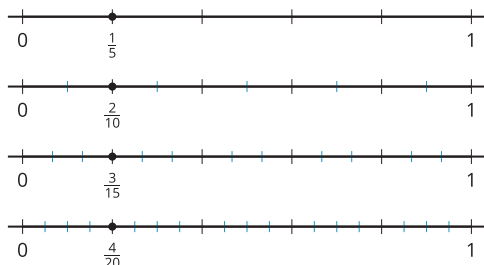
$$\frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

$$\frac{1 \times 5}{5 \times 5} = \frac{5}{25}$$

$$\frac{1 \times 10}{5 \times 10} = \frac{10}{50}$$

- Analyze Elena's work. Then, discuss with a partner:

- How are Elena's equations related to Andre's number lines?



- How might Elena find other fractions that are equivalent to $\frac{1}{5}$? Show a couple of examples.

- Use Elena's strategy to find five fractions that are equivalent to $\frac{1}{8}$. Use number lines to check your thinking, if they help.

Student Responses

- Sample response:
 - Andre partitioned the $\frac{1}{5}$ segment on the number line into different numbers of parts (2, 3, 4, and 5), to get twice, 3 times, 4 times, and 5 times as many parts in each segment. Elena showed the same idea by multiplying

Launch

- Groups of 2
- "Take a look at Andre's number lines you worked with in a previous lesson."

Activity

- "Think quietly for a couple of minutes about what Elena did and how it relates to Andre's number lines."
- 1-2 minutes: quiet think time for the first problem
- 3-4 minutes: partner discussion on the first problem
- Pause for a brief whole-class discussion. Invite students to share their ideas about Elena's work and how it is related to Andre's number lines.
- 4-5 minutes: independent work time for the last problem
- Monitor for students who find equivalent fractions for $\frac{1}{8}$ by multiplying by a factor other than 2, 3 or 4.

Synthesis

- Select 1-2 students to share their equivalent fractions for $\frac{1}{8}$ and their reasoning. Display their equivalent fractions as equations (for example, $\frac{1}{8} = \frac{3}{24}$, $\frac{1}{8} = \frac{4}{32}$, $\frac{1}{8} = \frac{7}{56}$, and $\frac{1}{8} = \frac{8}{64}$).
- "Do these equations show the same patterns as the equations in the warm-up? How are they alike or different?" (Alike: In all of the equations the numerators and denominators are multiplied by the same amount. Different: In the warm up each numerator and denominator was multiplied by 2. In these problems each the numerator and denominator of the fraction is multiplied by different numbers each time.)

1 and 5 by 2, 3, 4, and 5.

b. She'd multiply 1 and 5 by other numbers, for example: $\frac{1 \times 6}{5 \times 6} = \frac{6}{30}$ and

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}.$$

2. Sample response:

$$\frac{1 \times 3}{8 \times 3} = \frac{3}{24}, \frac{1 \times 4}{8 \times 4} = \frac{4}{32}, \frac{1 \times 5}{8 \times 5} = \frac{5}{40},$$

$$\frac{1 \times 7}{8 \times 7} = \frac{7}{56}, \text{ and } \frac{1 \times 8}{8 \times 8} = \frac{8}{64}$$

Activity 2

🕒 15 min

Equivalence Hunting

Standards Alignments

Addressing 4.NF.A.1

In this activity, students identify equivalent fractions. In the first problem, they use the numerical strategy they learned earlier to determine if two fractions are equivalent. In the second problem, they can use any strategy in their toolkit—which now includes a numerical method—to identify equivalent fractions.

Students encounter some fractions with unfamiliar denominators such as 9, 16, 32, 40, and 80, but they will not be assessed on such fractions. These denominators are multiples of familiar denominators such as 2, 3, 4, 5, 8, or 10, and are included to give students opportunities to generalize their reasoning about equivalence.

🕒 Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Synthesis: Check for understanding by inviting students to rephrase directions in their own words. Keep a display of directions visible throughout the activity.

Supports accessibility for: Memory, Organization

Student-facing Task Statement

Look at Elena's strategy from an earlier activity.

Launch

- Groups of 2

1. Could her strategy help us know whether two fractions are equivalent? Try using it to check the equivalence of these fractions:

a. $\frac{5}{2}$ and $\frac{10}{8}$

b. $\frac{2}{6}$ and $\frac{4}{12}$

For any two fractions that are equivalent, write an equation.

2. Find all fractions in the list that are equivalent to $\frac{3}{4}$. Be prepared to explain or show how you know.

$\frac{2}{10}$	$\frac{6}{8}$	$\frac{12}{15}$	$\frac{30}{40}$
$\frac{8}{9}$	$\frac{12}{20}$	$\frac{12}{16}$	$\frac{15}{20}$
$\frac{8}{10}$	$\frac{24}{32}$	$\frac{75}{100}$	$\frac{60}{80}$

Student Responses

1. Sample response:

a. No, $\frac{5}{2}$ is not equivalent to $\frac{10}{8}$.

$$\frac{5 \times 4}{2 \times 4} = \frac{20}{8}$$

b. Yes, $\frac{2}{6}$ is equivalent to $\frac{4}{12}$. $\frac{2 \times 2}{6 \times 2} = \frac{4}{12}$,

or $\frac{2}{6} = \frac{4}{12}$

2. $\frac{6}{8}$, $\frac{12}{16}$, $\frac{15}{20}$, $\frac{30}{40}$, $\frac{24}{32}$, $\frac{60}{80}$, and $\frac{75}{100}$

- “Now you are going to see whether you can use Elena’s method to see if fractions are equivalent.”

Activity

- 3–4 minutes: independent time to work on the first problem
- Pause for a brief whole-class discussion.
- “How did you know what number to multiply to the numerator and denominator to check equivalence?” (Sample responses:
 - See if there’s a whole number that can be multiplied by 5 to get 10, multiplied by 2 to get 8, and so on.
 - Divide 10 by 5, or 8 by 2, and so on, and see if what the result is and whether it’s a whole number.)
- “Work with your partner to identify all fractions on the list that are equivalent to $\frac{3}{4}$. Be prepared to show how you know.”
- 6–7 minutes: group work time for the second problem

Synthesis

- “Check your list of equivalent fractions with another group.”
- “Discuss any disagreement about a fraction until both groups agree whether or not it is equivalent to $\frac{3}{4}$.”
- 3 minutes: Check list with another group.

Lesson Synthesis

🕒 10 min

“Today we used a numerical strategy for finding equivalent fractions and for checking if fractions are equivalent.”

“Suppose a classmate was absent today. They later saw some examples of how to find equivalent fractions for $\frac{1}{3}$ using this strategy, but they don’t fully follow the examples.”

Display: $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$ and $\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$

“What would you say to help your classmate understand what is happening in the equations? How would you explain the multiplication by 4 or by 6?”

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students multiplied the numerator by a number and the denominator by a different number, thinking that as long as both are being multiplied by a number, an equivalent fraction could be generated.

Next Day Support

- Encourage students to use visual representations to justify the equivalence of fractions they generate.

Lesson 11: Use Factors to Find Equivalent Fractions

Standards Alignments

Addressing 4.NF.A.1

Teacher-facing Learning Goals

- Generate equivalent fractions by using factors of the numerator and denominator.
- Reason about fraction equivalence numerically, by using multiples or factors of the numerator and denominator.

Student-facing Learning Goals

- Let's find equivalent fractions by working with numerators and denominators.

Lesson Purpose

The purpose of this lesson is for students to generate equivalent fractions numerically, by using factors and multiples of the numerator and denominator.

In earlier lessons, students saw that one way to generate equivalent fractions is by grouping unit fractions on a number line into larger units. For instance, 12 twelfths could be put in groups of 3 to make 4 equal parts, each part being a fourth. Or they could be put into groups of 2 to make 6 equal parts, each part being a sixth, which means that $\frac{12}{12} = \frac{4}{4} = \frac{6}{6}$. Some students may have related these observations to the fact that $12 \div 4 = 3$ and $12 \div 2 = 6$. These insights are formalized and generalized in this lesson.

Students have also generated equivalent fractions and verified equivalence by multiplying the numerator and denominator by the same number. In this lesson, they find equivalent fractions by dividing a and b by a factor n that is common to both numbers.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 3), Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Fractions Galore (groups of 3): Activity 3

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence did you see of students choosing a method strategically as they generated equivalent fractions? For students who chose a fixed way regardless of the given fractions, what questions could you ask them to prompt them to be more strategic?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Find Three or More

Standards Alignments

Addressing 4.NF.A.1

Student-facing Task Statement

Name at least 3 fractions that are equivalent to $\frac{20}{100}$. Explain or show your reasoning.

Student Responses

Sample responses: $\frac{2}{10}$, $\frac{4}{20}$, $\frac{10}{50}$, $\frac{40}{200}$

$$\frac{20 \div 2}{100 \div 2} = \frac{10}{50}, \quad \frac{20 \div 5}{100 \div 5} = \frac{4}{20}, \quad \frac{20 \div 10}{100 \div 10} = \frac{2}{10}, \quad \frac{20 \times 2}{100 \times 2} = \frac{40}{200}$$

----- Begin Lesson -----

Warm-up

🕒 10 min

Which One Doesn't Belong: Four Representations

Standards Alignments

Addressing 4.NF.A.1

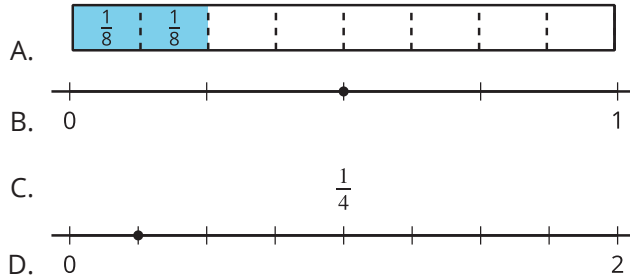
This warm-up prompts students to carefully analyze and compare representations of fractions. To make comparisons, students need to draw on their knowledge about fractional parts, the size of fractions, and equivalent fractions.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?



Student Responses

Sample response:

- A is the only one that doesn't show fourths as the unit fraction or the smallest fractional part. (It shows eighths.)
- B is the only one that doesn't represent $\frac{1}{4}$ or is not equivalent to $\frac{1}{4}$.
- C is the only one that doesn't show a visual representation of $\frac{1}{4}$.
- D is the only representation that shows fractions greater than 1.

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

Consider asking:

- "Let's find at least one reason why each one doesn't belong."

Activity 1

🕒 20 min

The Other Way Around

Standards Alignments

Addressing 4.NF.A.1

In this activity, students see that they can find equivalent fractions by dividing the numerator and denominator by a common factor. They connect this strategy to the process of grouping unit fractions on a number line into larger equal-size parts. The result is a fewer number of parts, and smaller numbers for the numerator and denominator of equivalent fractions.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: At the appropriate time, give students 2–3 minutes to make sure that everyone in their group can explain the strategies used in the given examples. Invite groups to rehearse what they will say when they share with the whole class.

Advances: Speaking, Conversing, Representing

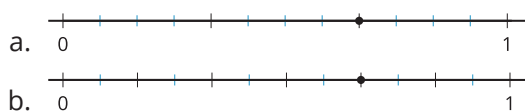
Student-facing Task Statement

- Andre drew a number line and marked a point on it. Label the point with the fraction it represents.



- To find other fractions that the point represents, Andre made copies of the number line. He drew darker marks on some of the existing tick marks.

Label the darker tick marks Andre made on each number line.



- Kiran wrote the same fractions for the points but used a different strategy, as

Launch

- Groups of 2

Activity

- “Work with your partner to answer the first three problems.”
- “Be prepared to explain how you think Andre’s and Kiran’s strategies are related.”
- 7–8 minutes: partner work time
- “Take a few minutes to answer the last problem independently.”
- 2–3 minutes: independent work time for the last problem

Synthesis

- “What did Andre do with the number lines?”

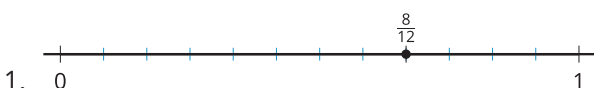
shown. Analyze his reasoning.

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3} \quad \text{How do you think Andre's and Kiran's strategies are related?}$$

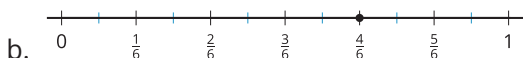
$$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

4. Try using Kiran's strategy to find one or more fractions that are equivalent to $\frac{10}{12}$ and $\frac{18}{12}$.

Student Responses



2. Labeled number lines:



3. Sample reasoning:

- In diagram A, Andre first grouped the 12 parts in 1 whole into groups of 4 so we now see 3 equal parts (or 3 thirds). The 8 twelfths are equal to 2 of the thirds. In diagram B, he grouped the twelfths into groups of 2, which results in 6 parts or 6 sixths. The 8 twelfths is equal to 4 of the sixths.
- Kiran's reasoning is similar in that instead of grouping the parts—first by 4 and then by 2—and counting the new parts, he divided the numerator and denominator—first by 4 and then by 2—to get new numerators and denominators.

4. Equivalent to $\frac{10}{12}$: $\frac{10 \div 2}{12 \div 2} = \frac{5}{6}$

$$\text{Equivalent to } \frac{18}{12}: \frac{18 \div 2}{12 \div 2} = \frac{9}{6}, \frac{18 \div 3}{12 \div 3} = \frac{6}{4},$$

$$\text{and } \frac{18 \div 6}{12 \div 6} = \frac{3}{2}$$

How would it help him find equivalent fractions?" (Andre grouped the 12 parts into equal groups of different sizes—2s, 4s—to make bigger parts. Then, he counted the number of those new parts.)

- "What did Kiran do? How is his strategy related to Andre's?" (Kiran divided 12 by 4 and then by 2, similar to how Andre put 12 parts into groups of 4 and then of 2.)
- "Notice that the equivalent fractions $\frac{2}{3}$ and $\frac{4}{6}$ both have smaller numbers for the numerator and denominator than the original fraction. Can you use Andre's number line to show why this might be?" (The size of the parts are bigger, so there are fewer parts in 1 whole.)

If time permits, ask students:

- "How many equivalent fractions can you find for $\frac{10}{12}$ using Kiran's way?" (One) "How many can you find for $\frac{18}{12}$?" (Three)
- "What might be a reason that you could find more equivalent fractions for $\frac{18}{12}$ than for $\frac{10}{12}$?" (18 and 12 have more factors in common than 10 and 12.)
- "How would you show $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$ on the number line?" (Put the original 12 parts into groups of 3 to get 4 parts, each being a fourth. Mark 3 of those 4 parts to show $\frac{3}{4}$.)

Activity 2

🕒 15 min

How Would You Find Them?

Standards Alignments

Addressing 4.NF.A.1

In this activity, students generate equivalent fractions by applying the numerical strategies they learned. (Students might opt to use other strategies, but most of the given fractions have numbers that would make visual representation and reasoning inconvenient.) Depending on the given fractions, students need to decide whether it makes sense to multiply or divide the numerator and denominator by a common number.

🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Leverage choice around perceived challenge. Invite students to select at least 3 of the 5 given fractions for this activity.

Supports accessibility for: Organization, Attention, Social-Emotional Functioning

Student-facing Task Statement

Find at least two fractions that are equivalent to each fraction. Show your reasoning.

1. $\frac{16}{8}$
2. $\frac{40}{10}$
3. $\frac{7}{6}$
4. $\frac{90}{100}$
5. $\frac{5}{4}$

Student Responses

Sample responses:

1. $\frac{16 \div 2}{8 \div 2} = \frac{8}{4}$ and $\frac{16 \div 4}{8 \div 4} = \frac{4}{2}$
2. $\frac{40 \div 2}{10 \div 2} = \frac{20}{5}$ and $\frac{40 \div 5}{10 \div 5} = \frac{8}{2}$

Launch

- Groups of 2

Activity

- “Work on the activity independently. Then, share your responses with your partner and check each other’s work.”
- 8–10 minutes: independent work time
- 3–5 minutes: partner discussion

Synthesis

- See lesson synthesis.

3. $\frac{7 \times 2}{6 \times 2} = \frac{14}{12}$ and $\frac{7 \times 3}{6 \times 3} = \frac{21}{18}$
4. $\frac{90 \div 2}{100 \div 2} = \frac{45}{50}$ and $\frac{90 \div 10}{100 \div 10} = \frac{9}{10}$
5. $\frac{5 \times 2}{4 \times 2} = \frac{10}{8}$ and $\frac{5 \times 3}{4 \times 3} = \frac{15}{12}$

Advancing Student Thinking

If students attempt to partition the number line into 30, 60, or 90 parts, consider asking: “How can we use the patterns from the previous activity to help us here?”

Activity 3 (optional)

🕒 15 min

Card Sort: Fractions Galore

Standards Alignments

Addressing 4.NF.A.1

This activity is optional because it provides an opportunity for students to apply concepts from previous activities that not all classes may need. It allows students to practice using numerical strategies to find equivalent fractions by sorting a set of 36 cards. Students are not expected to find all equivalent fractions in the set. When students look for equivalent fractions they use their understanding of multiples and the meaning of fractions (MP7).

Instructional Routines

Card Sort

Materials to Copy

Fractions Galore (groups of 3)

Required Preparation

- Create a set of Fraction Galore cards from the Instructional for each group of 3.

Student-facing Task Statement

Your teacher will give you a set of cards. Find as many sets of equivalent fractions as you can. Be prepared to explain or show your reasoning.

Record the sets of equivalent fractions here.

Record fractions that do not have an equivalent fraction here.

Student Responses

Sets of equivalent fractions:

Launch

- Groups of 3–4
- Give each group one set of cards created from the Instructional master.

Activity

- “Work with your group to sort the cards by equivalence. Find as many sets of equivalent fractions as you can. Some fractions have no equivalent fractions.”
- 8–10 minutes: small group work time

Synthesis

- “What strategy did your group use to find equivalent fractions? How well did the strategy work? How efficient was it?” (We looked at the numerators and denominators to see if they were multiples or factors we recognized.)
- “Did you notice any new patterns in the fractions that are equivalent?”

$\frac{4}{3}, \frac{8}{6}, \frac{16}{12}$	$\frac{1}{4}, \frac{25}{100}$	$\frac{3}{4}, \frac{75}{100}$	$\frac{4}{5}, \frac{8}{10}, \frac{80}{100}$
$\frac{5}{5}, \frac{100}{100}$	$\frac{9}{5}, \frac{18}{10}$	$\frac{10}{5}, \frac{12}{6}, \frac{16}{8}, \frac{100}{50}$	$\frac{1}{20}, \frac{5}{100}$
$\frac{1}{10}, \frac{5}{50}, \frac{10}{100}$	$\frac{5}{10}, \frac{50}{100}$	$\frac{6}{20}, \frac{30}{100}$	$\frac{16}{100}, \frac{4}{25}$

Fractions that don't have an equivalent fraction:

$$\frac{5}{4}, \frac{5}{6}, \frac{12}{8}, \frac{7}{12}, \frac{1}{100}, \frac{49}{100}, \frac{20}{100}$$

Lesson Synthesis

🕒 10 min

"Today we looked at another way to find equivalent fractions. We divided the numerator and denominator of a fraction by a factor they have in common."

"How did you decide whether to use multiplication or division to write an equivalent fraction?" (Sample response: It depends on the numbers in the fraction. When the numbers are large to start with and both have a factor in common, we'd divide by that factor. When the numbers are small and have no shared factors, we'd multiply.)

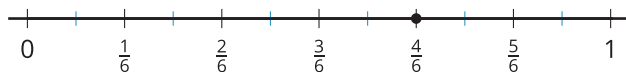
Suggested Centers

- Get Your Numbers in Order (1–5), Stage 4: Denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100 (Addressing)
- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

Student Section Summary

In this section, we learned to identify and write equivalent fractions. We placed fractions on number lines and saw that two fractions that occupy the same spot on a number line are equivalent.





We also looked at strategies for finding equivalent fractions and learned that multiplying or dividing the numerator and denominator by the same number will result in an equivalent fraction. Here are some examples:

$$\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

$$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

$$\frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

$\frac{1}{5}$ is equivalent to $\frac{2}{10}$ and $\frac{4}{20}$.

$\frac{8}{12}$ is equivalent to $\frac{4}{6}$ and $\frac{2}{3}$.

Complete Cool-Down

Response to Student Thinking

Students multiply or divide the numerator and the denominator by two different numbers resulting in fractions that are not equivalent.

Next Day Support

- Present this approach as a warm up for the next lesson. Ask students to analyze the approach and discuss student reasoning.

Section C: Fraction Comparison

Lesson 12: Ways to Compare Fractions

Standards Alignments

Building On	3.NF.A.1
Addressing	4.NF.A.2
Building Towards	4.NF.A.2

Teacher-facing Learning Goals

- Compare fractions using methods that make sense to them.

Student-facing Learning Goals

- Let's compare some fractions.

Lesson Purpose

The purpose of this lesson is for students to compare fractions in a way that makes sense to them, including by reasoning about the size of fractional parts, common numerators or denominators, or relationships to benchmarks such as $\frac{1}{2}$ and 1.

Previously, students have investigated the relative sizes of fractions with the same numerator or denominator. They have also compared fractions to $\frac{1}{2}$ and 1. In this lesson, they apply those understandings to compare a wider range of fractions.

Some students may make comparisons by writing equivalent fractions, which shows they are applying learning from earlier in the unit. It is not necessary to highlight this approach at this point, however. In the next lesson, students will take a closer look at how equivalence can be used to compare fractions.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Colored pencils: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Which questions did you ask today that were effective in prompting students to compare the size of fractions strategically or structurally? Which ones might have pushed them toward a particular method or process?

Cool-down (to be completed at the end of the lesson)

 5 min

Pick the Greater Fraction

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

In each pair of fractions, which fraction is greater? Explain or show your reasoning.

1. $\frac{5}{12}$ and $\frac{5}{8}$
2. $\frac{11}{10}$ and $\frac{18}{100}$
3. $\frac{6}{10}$ and $\frac{7}{12}$

Student Responses

1. $\frac{5}{8}$. Sample reasoning: 1 eighth is greater than 1 twelfth, so 5 eighths is greater than 5 twelfths.
2. $\frac{11}{10}$. Sample reasoning: $\frac{11}{10}$ is greater than 1, and $\frac{18}{100}$ is less than 1.
3. $\frac{6}{10}$. Sample reasoning: $\frac{7}{12}$ is $\frac{1}{12}$ more than $\frac{1}{2}$, and $\frac{6}{10}$ is $\frac{1}{10}$ more than $\frac{1}{2}$. $\frac{1}{10}$ is greater than $\frac{1}{12}$, so $\frac{6}{10}$ is greater.

Begin Lesson

Warm-up

🕒 10 min

Estimation Exploration: What's That Point?

Standards Alignments

Building On 3.NF.A.1

Building Towards 4.NF.A.2

The purpose of this warm-up is for students to practice estimating a reasonable fractional value on a number line. The reasoning here prepares students to use these benchmarks as a way to compare fractions later in the lesson.

The warm-up gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it (MP3).

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What is the value represented by the point on the number line?



Make an estimate that is:

too low	about right	too high

Student Responses

Sample response:

- Too low: $\frac{1}{5}$ to $\frac{1}{4}$
- About right: $\frac{2}{5}$ to $\frac{1}{2}$
- Too high: $\frac{3}{5}$ to $\frac{3}{4}$

Launch

- Groups of 2
- Display the number line.
- “What is an estimate that’s too high? Too low? About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “How did you decide what fraction would be ‘about right?’” (The point is a little to the left of the middle point, so the fraction must be a little less than $\frac{1}{2}$.)

- “Would writing the label ‘1’ as $\frac{10}{10}$, or as $\frac{100}{100}$, help us make better estimates? Why or why not?” (Sample response: It could, because it would help us mentally partition the number line into 10 or 100 parts, which makes it possible to estimate more precisely.)

Activity 1

🕒 15 min

The Greatest of Them All

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 4.NF.A.2

In earlier lessons, students compared two fractions that share the same denominator or the same numerator. In this activity, students use that understanding to compare a large set of fractions that are arranged into rows and columns. The fractions in each row share the same numerator and those in each column share the same denominator.

🕒 Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Invite students to look at column A first, then column B, then row 1. Provide access to pre-made fraction strips for thirds and fifths to help them get started. Check in with students to provide feedback and encouragement after each chunk, particularly in terms of looking for and making use of structure.

Supports accessibility for: Conceptual Processing, Organization, Social-Emotional Functioning

Student-facing Task Statement

Here are 25 fractions in a table.

	A	B	C	D	E
1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{10}$	$\frac{2}{12}$	$\frac{2}{100}$
2	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{4}{10}$	$\frac{4}{12}$	$\frac{4}{100}$
3	$\frac{7}{3}$	$\frac{7}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{7}{100}$

Launch

- Groups of 2
- Display the table of fractions.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion

	A	B	C	D	E
4	$\frac{11}{3}$	$\frac{11}{5}$	$\frac{11}{10}$	$\frac{11}{12}$	$\frac{11}{100}$
5	$\frac{26}{3}$	$\frac{26}{5}$	$\frac{26}{10}$	$\frac{26}{12}$	$\frac{26}{100}$

For each question, be prepared to explain your reasoning.

1. Identify the greatest fraction in each column (A, B, C, D, and E).
2. Identify the greatest fraction in each row (1, 2, 3, 4, and 5).
3. Which fraction is the greatest fraction in the entire table?

Student Responses

1. The last fraction of each column (in row 5) is the greatest (A: $\frac{26}{3}$, B: $\frac{26}{5}$, C: $\frac{26}{10}$, D: $\frac{26}{12}$, E: $\frac{26}{100}$).
2. The first fraction of each row (in column A) is the greatest (1: $\frac{2}{3}$, 2: $\frac{4}{3}$, 3: $\frac{7}{3}$, 4: $\frac{11}{3}$, 5: $\frac{26}{3}$).
3. $\frac{26}{3}$

Activity

- “Take a few quiet minutes to complete the problems. Afterward, discuss your responses with your partner.”
- 6–7 minutes: independent work time
- “When discussing with your partner, explain how you know which fraction is the greatest in each row, each column, and the entire table.”
- 3–4 minutes: partner discussion

Synthesis

- Invite students to share their responses and reasoning. Highlight responses that clarify that:
 - In each column, the fraction in row 5 is the greatest because it has the greatest numerator of all fractions with the same denominator (with fractional parts of the same size).
 - In each row, the fraction in column A is greater than others to its right because it has the greatest fractional part of all fractions with the same numerator ($\frac{1}{3}$ is greater than $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$, and $\frac{1}{100}$).
- “How did you know that $\frac{26}{3}$ is the greatest fraction in the entire table?” (Sample responses:
 - It is the greatest fraction in row 5 and in column A.
 - It is more than 8 wholes. All the other fractions are less than that.)

Advancing Student Thinking

Students may decide $\frac{26}{3}$ and $\frac{26}{5}$ are less than $\frac{26}{12}$ and $\frac{26}{100}$ because the former two involve smaller numbers than the latter two. Suggest that students compare fractions with the same numerator, but one that is more familiar (such as those in row 1). Consider asking: “Which is greater, $\frac{2}{5}$ or $\frac{2}{10}$?”

$\frac{8}{5}$ or $\frac{8}{10}$?" Refer them to the diagram of fraction strips to make a similar comparison, if helpful.

Activity 2

🕒 20 min

Relative to $\frac{1}{2}$ and 1

Standards Alignments

Addressing 4.NF.A.2

In this activity, students apply previous reasoning about the size of fractions and their knowledge about fractions that are equivalent to $\frac{1}{2}$ to classify and compare fractions. Along the way, students have opportunities to make new observations about the structure of fractions that are less than $\frac{1}{2}$, greater than $\frac{1}{2}$ but less than 1, and greater than 1 (MP7).

The activity calls for the use of colors as a way to code fractions in different groups. If colored pencils are not available, students can code the fractions by putting circles, triangles, and squares around the fractions. In either case, a key or legend should be created.

🌐 Access for English Learners

MLR8 Discussion Supports. Encourage students to begin partner discussions by reading their written responses aloud. If time allows, invite students to revise or add to their responses based on the conversation that follows.

Advances: Conversing, Speaking

Materials to Gather

Colored pencils

Required Preparation

- Each group of 2 needs 3 colored pencils (3 different colors).

Student-facing Task Statement

Here is the same table you saw earlier.

Launch

- Groups of 2
- Give each group 3 colored pencils.

	A	B	C	D	E
1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{10}$	$\frac{2}{12}$	$\frac{2}{100}$
2	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{4}{10}$	$\frac{4}{12}$	$\frac{4}{100}$
3	$\frac{7}{3}$	$\frac{7}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{7}{100}$
4	$\frac{11}{3}$	$\frac{11}{5}$	$\frac{11}{10}$	$\frac{11}{12}$	$\frac{11}{100}$
5	$\frac{26}{3}$	$\frac{26}{5}$	$\frac{26}{10}$	$\frac{26}{12}$	$\frac{26}{100}$

1. Which fractions are less than $\frac{1}{2}$? Circle each one of them. Then, complete this sentence:

I know a fraction is less than $\frac{1}{2}$ when . . .

2. Which are greater than $\frac{1}{2}$ but less than 1? Circle each of them with a pencil of a different color (or draw a triangle around each one). Then, complete this sentence:

I know a fraction is greater than $\frac{1}{2}$ but less than 1 when . . .

3. Circle the remaining fractions with a pencil of a third color (or draw a square around each one). How would you describe the size of these fractions?
4. Next to the table, create a legend or key to show what each color (or each shape) represents.
5. Here are some pairs of fractions from the table. In each pair, which fraction is greater?

- a. $\frac{2}{5}$ or $\frac{7}{10}$
 b. $\frac{4}{10}$ or $\frac{7}{12}$
 c. $\frac{11}{100}$ or $\frac{4}{3}$
 d. $\frac{26}{10}$ or $\frac{11}{12}$

Student Responses

1. $\frac{2}{5}$, $\frac{2}{10}$, $\frac{2}{12}$, $\frac{2}{100}$, $\frac{4}{10}$, $\frac{4}{12}$, $\frac{4}{100}$, $\frac{7}{100}$, $\frac{11}{100}$, $\frac{26}{100}$.

Sample response: I know a fraction is less than $\frac{1}{2}$ when the numerator is less than half

Activity

- “Take a few quiet minutes to answer the first 4 questions. Afterward, discuss your responses with your partner.”
- “You will need to code the fractions by color or by shape.”
- 7–8 minutes: individual work time
- 5 minutes: partner discussion
- Pause for a whole-class discussion before proceeding to the last question.
- Invite students to share how they knew if a fraction is less than $\frac{1}{2}$, or if it is greater than $\frac{1}{2}$ but less than 1. Record their responses.
- “How did you describe the last group of fractions that don’t fall into the first two groups?” (fractions greater than 1)
- “Now compare some fractions in the last problem.”
- 5 minutes: individual work time

Synthesis

- See lesson synthesis.

of the denominator (or the denominator is more than twice the numerator).

2. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{11}{12}$. Sample response: I know a fraction is greater than $\frac{1}{2}$ but less than 1 when doubling the numerator makes the fraction greater than 1.
3. The remaining fractions are fractions greater than 1.
4. Sample response:
 - yellow: less than $\frac{1}{2}$
 - green: more than $\frac{1}{2}$, less than 1
 - purple: more than 1
5.
 - a. $\frac{7}{10}$
 - b. $\frac{7}{12}$
 - c. $\frac{4}{3}$
 - d. $\frac{26}{10}$

Lesson Synthesis

🕒 10 min

Invite students to share their strategies for comparing fractions in the last question of the last activity.

“How did you compare fractions in which neither the numerator nor the denominator are the same?” (Sample response: We compared them to $\frac{1}{2}$ or 1.)

“How did the color coding (or comparison to $\frac{1}{2}$ or 1) help?” (Sample responses:

- Knowing whether a fraction is more or less than 1, or more or less than $\frac{1}{2}$, can help us tell which one is greater.
- All the fractions circled in yellow [less than $\frac{1}{2}$] are less than all the numbers in green [greater than $\frac{1}{2}$ but less than 1]. All fractions in green are less than all fractions in purple [greater than 1].)

Suggested Centers

- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

When responding to the last problem, students may say that $\frac{6}{10}$ and $\frac{7}{12}$ were the same size because they are both 1 unit fraction greater than $\frac{1}{2}$ ($\frac{6}{10}$ is $\frac{1}{10}$ away from $\frac{5}{10}$, and $\frac{7}{12}$ is $\frac{1}{12}$ away from $\frac{6}{12}$).

Next Day Support

- Before the warm-up, ask students to think about these questions with a partner: “Are $\frac{6}{10}$ and $\frac{7}{12}$ greater or less than $\frac{1}{2}$? Are they the same distance from $\frac{1}{2}$? How do you know?”

Lesson 13: Use Equivalent Fractions to Compare

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

Teacher-facing Learning Goals

- Compare two fractions by rewriting one of them into an equivalent fraction with the same denominator as the other.

Student-facing Learning Goals

- Let's compare fractions by writing an equivalent fraction.

Lesson Purpose

The purpose of this lesson is for students to compare two fractions by rewriting one of them as an equivalent fraction with the same denominator as the other.

Previously, students used various strategies and representations to reason about the relative size of fractions. In this lesson, they focus on writing equivalent fractions as a way to compare fractions. Here the denominator of one fraction is a factor or a multiple of the denominator of the other fraction, making it likely for students to see one fraction in terms of the fractional part of the other. In a future lesson, students will compare fractions in which the denominators have no common factors.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR7 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

How readily did students grasp the idea of writing equivalent fractions with a common denominator as a way to compare fractions? What evidence did you see of students connecting it to the reasoning they did about equivalent fractions on number lines? How could the connections be made more explicit?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Make It True

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

Compare each pair of fractions. Use the symbols $<$, $=$, and $>$ to make each statement true. Explain or show your reasoning.

- $\frac{15}{8}$ _____ $\frac{7}{4}$
- $\frac{2}{5}$ _____ $\frac{30}{100}$

Student Responses

- $\frac{15}{8} > \frac{7}{4}$. Sample reasoning: $\frac{7}{4}$ is equivalent to $\frac{14}{8}$, so it is less than $\frac{15}{8}$.
- $\frac{2}{5} > \frac{30}{100}$. Sample reasoning: $\frac{2}{5}$ is equivalent to $\frac{40}{100}$, so it is greater than $\frac{30}{100}$.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Notice and Wonder: Pairs of Numbers

Standards Alignments

Addressing 4.NF.A.2

The purpose of this warm-up is to draw students' attention to inequality statements. It reminds them of the meaning of inequality symbols and how to read the statements, which will be useful when students compare fractions later in the lesson. The warm-up also elicits observations that an equation

or inequality can be true or false. While students may notice and wonder many things, highlight observations about comparison and about the meaning of the symbols and statements.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

$$5 < 8 \quad \frac{9}{2} > 4\frac{1}{2} \quad 4 = \frac{3}{2} \quad \frac{1}{3} < \frac{1}{2}$$

Student Responses

Students may notice:

- The statements use $<$, $=$, and $>$ symbols.
- There are fractions, whole numbers, and mixed numbers.
- Some fractions are greater than 1 and some are less than 1.
- Some statements are true and some are false.

Students may wonder:

- What do the $<$ and $>$ symbols mean?
- How can I compare $\frac{9}{2}$ and $4\frac{1}{2}$?
- How can we correct statements that are not true?

Launch

- Groups of 2
- Display the four statements.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “What does each statement say?”
- “Which of these statements are true? Which ones are not?” (The first and last are true. The second and third are false.)
- “Why are they false?” ($\frac{9}{2}$ is equal to, not greater than, 4 wholes and $\frac{1}{2}$. Four is greater than $\frac{3}{2}$.)

Activity 1

🕒 20 min

Pairs to Compare

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

Previously, students classified fractions based on their relationship to $\frac{1}{2}$ and 1 (whether they are less than or more than these benchmarks). They used these classifications to compare fractions. In this activity, students are presented with fractions that are in the same group (for example, both less than $\frac{1}{2}$, or both greater than $\frac{1}{2}$ but less than 1), so they need to reason in other ways to make comparisons.

Students can reason in a number of ways—by thinking about size and number of parts, drawing a diagram or number line, or reasoning numerically, but in most cases, they need to also rely on the idea of equivalence.

Access for English Learners

MLR7 Compare and Connect. Synthesis: After each strategy has been presented, lead a whole-class discussion comparing, contrasting, and connecting the different approaches. Ask, “Did anyone solve the problem the same way, but would explain it differently?” and “Why did the different approaches lead to the same outcome?”

Advances: Representing, Conversing

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate background knowledge. Invite students to review the strategies they know for comparing fractions (reasoning about denominators or numerators, comparing to a benchmark, and writing equivalent fractions). Record students’ strategies on a visible display, including details (words or pictures) that will help them remember how to use the strategy.

Supports accessibility for: Conceptual Processing, Memory, Attention

Student-facing Task Statement

Here are some pairs of fractions sorted into three groups. Circle the greater fraction in each pair. Explain or show your reasoning.

1. Group 1:

a. $\frac{2}{10}$ or $\frac{26}{100}$

b. $\frac{2}{5}$ or $\frac{11}{100}$

2. Group 2:

a. $\frac{2}{3}$ or $\frac{7}{12}$

b. $\frac{4}{5}$ or $\frac{7}{10}$

Launch

- Groups of 2
- “Here are some fractions you’ve sorted in an earlier lesson. We compared them to $\frac{1}{2}$ and 1.”
- “What do the fractions in group 3 have in common? Why might they be in the same group?” (They are all greater than 1.)
- “How are the fractions in group 1 different than those in group 2?” (Those in group 1 are less than $\frac{1}{2}$, and those in group 2 are greater than $\frac{1}{2}$ but less than 1.)

3. Group 3:

- a. $\frac{11}{5}$ or $\frac{26}{10}$
 b. $\frac{11}{3}$ or $\frac{26}{12}$

Student Responses

1. Group 1:

- a. $\frac{26}{100}$. Sample reasoning: $\frac{2 \times 10}{10 \times 10} = \frac{20}{100}$
 and $\frac{20}{100}$ is less than $\frac{26}{100}$.
 b. $\frac{2}{5}$. Sample reasoning: $\frac{2 \times 20}{5 \times 20} = \frac{40}{100}$
 and $\frac{40}{100}$ is greater than $\frac{11}{100}$.

2. Group 2:

- a. $\frac{2}{3}$. Sample reasoning: $\frac{2}{3}$ is $\frac{1}{3}$ or $\frac{4}{12}$ less than 1, while $\frac{7}{12}$ is $\frac{5}{12}$ less than 1.
 b. $\frac{4}{5}$. Sample reasoning: $\frac{1}{5}$ is $\frac{2}{10}$, so $\frac{4}{5}$ is $\frac{8}{10}$, which is greater than $\frac{7}{10}$.

3. Group 3:

- a. $\frac{26}{10}$. Sample reasoning: If I partition $\frac{1}{5}$ into 2 parts, each part is $\frac{1}{10}$. In $\frac{11}{5}$ there will be $\frac{22}{10}$, which is less than $\frac{26}{10}$.
 b. $\frac{11}{3}$. Sample reasoning: $\frac{1}{3}$ is equivalent to $\frac{4}{12}$, so $\frac{11}{3}$ are equivalent to $\frac{44}{12}$, which is greater than $\frac{26}{12}$.

- “We can tell that the fractions in group 2 are greater than those in group 1, and the fractions in group 3 are greater than those in the other groups.”
- “Now compare the fractions in each group.”

Activity

- “Take a few quiet minutes to work on the problems. Afterward, share your responses with your partner.”
- 7–8 minutes: independent work time
- 5 minutes: partner discussion
- Monitor for students who:
 - reason by drawing number lines or tape diagrams
 - reason about the distance of each fraction from 0, $\frac{1}{2}$, or 1
 - reason about equivalent fractions (even if they don’t write out the multiplication numerically)
 - reason about equivalent fractions numerically by writing out the multiplication

Synthesis

- Select students who used different strategies to share their responses.
- “How do we compare two fractions that are in the same group—say, both less than $\frac{1}{2}$ or both greater than 1?” (We can think about how close or far away from $\frac{1}{2}$ each fraction is.)
- Highlight how equivalent fractions came into play in each strategy. For example, ask, “When comparing $\frac{2}{3}$ and $\frac{7}{12}$, why was it helpful to think of the $\frac{2}{3}$ as $\frac{8}{12}$?” Or, “When comparing $\frac{7}{10}$ and $\frac{4}{5}$, why did you think of $\frac{4}{5}$ as $\frac{8}{10}$?”

- If no students mention that it is often easier to compare two fractions when they have the same denominator, ask them about it.

Activity 2

🕒 15 min

New Pairs to Compare

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

The purpose of this activity is for students to compare pairs of fractions by writing one or more equivalent fractions. In all pairs of fractions given here, one denominator is a factor or a multiple of the other, which encourages students to convert one into an equivalent fraction with the same denominator as the other fraction. On repeated reasoning, students see that writing an equivalent fraction can facilitate the comparison (though in some cases, students may still find it efficient to reason in other ways).

This is the first time in grade 4 that students use the symbols $<$ and $>$ to express comparison, so some supports for reading aloud inequality statements are suggested in the launch.

Student-facing Task Statement

1. Decide whether each statement is true or false. Be prepared to show how you know.
 - a. $\frac{5}{12} = \frac{2}{6}$
 - b. $\frac{10}{3} < \frac{44}{12}$
 - c. $\frac{1}{4} > \frac{25}{100}$
 - d. $\frac{8}{15} < \frac{3}{5}$
2. Compare each pair of fractions. Use the symbols $<$, $=$, and $>$ to make each statement true.

Launch

- Groups of 2
- Read together the four statements in the first question.
- Consider writing out in words the meaning of the symbols $<$ and $>$ (“is greater than” and “is less than”) and display them for students’ reference.

Activity

- 7–8 minutes: independent work time
- 2–3 minutes: partner discussion

- a. $\frac{6}{12}$ _____ $\frac{4}{6}$
 b. $\frac{4}{3}$ _____ $\frac{7}{6}$
 c. $\frac{8}{5}$ _____ $\frac{400}{100}$
 d. $\frac{12}{10}$ _____ $\frac{35}{5}$
 e. $\frac{11}{4}$ _____ $\frac{17}{8}$
 f. $\frac{7}{12}$ _____ $\frac{4}{3}$

Student Responses

1. a. false
b. true
c. false
d. true
2. a. <
b. >
c. <
d. <
e. >
f. <

- Monitor for students who make comparisons by:
 - using the relationship and distance to benchmark numbers
 - writing an equivalent fraction either by dividing or multiplying the numerator and denominator by a number

Synthesis

- Select students to share their responses and how they reasoned about them.

Lesson Synthesis

🕒 10 min

“Today we compared fractions by writing equivalent fractions and by using some other ways.”

Ask students to find an example of a pair of fractions in today’s activity that it was helpful to compare by:

- reasoning about the denominators and numerators
- seeing where the fractions are in relation to $\frac{1}{2}$, 1, or another benchmark
- writing an equivalent fraction for one of the fractions

Suggested Centers

- Mystery Number (1–4), Stage 4: Fractions with Denominators 5, 8, 10, 12, 100 (Addressing)

- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students tried to write equivalent fractions by dividing both numbers in each fraction, but the numerator doesn't divide equally (or has a remainder).

Next Day Support

- Before the warm-up, display $\frac{15}{8}$ and $\frac{7}{4}$. Discuss with students why it would be helpful to rewrite $\frac{7}{4}$ with a denominator 8 to compare these fractions.

Lesson 14: Fraction Comparison Problems

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2
 Building Towards 4.NBT.B.4

Teacher-facing Learning Goals

- Solve fraction comparison problems in and out of context.

Student-facing Learning Goals

- Let's solve different kinds of fraction comparison problems.

Lesson Purpose

The purpose of this lesson is for students to compare fractions to solve problems in and out of context.

In the previous lesson, students wrote equivalent fractions to help them compare pairs of fractions with different denominators. Here, they include this newly developed strategy in their toolkit for comparing fractions.

In the first activity, students compare sets of fractions with like and unlike denominators. They do so by using benchmarks, writing equivalent fractions, or reasoning about the numerators and denominators. In the second activity, students interpret and solve problems involving fractional measurements in context. Both activities present a new setup, structure, or context, requiring students to make sense of the given information and the problems, and to persevere in solving them (MP1).

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Required Preparation

- Each group of 3–4 needs tools for creating a visual display during the lesson synthesis.

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Were there students with unique approaches who didn't get air time? If so, what might be some possible reasons? How can their thinking be made visible in upcoming lessons?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Who Ran the Farthest?

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

Jada, Kiran, and Lin tried to run as far as possible before they had to stop and rest.

- Jada ran $\frac{3}{4}$ mile.
- Kiran ran $\frac{7}{12}$ mile.
- Lin ran $\frac{4}{6}$ mile.

Who ran the farthest before stopping? Explain or show your reasoning.

Student Responses

Jada ran the farthest. Sample reasoning:

- Comparing $\frac{7}{12}$ and $\frac{4}{6}$: $\frac{4}{6}$ is equivalent to $\frac{8}{12}$ and greater than $\frac{7}{12}$, so Lin ran farther than Kiran.
- Comparing $\frac{8}{12}$ and $\frac{3}{4}$: $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$, so $\frac{3}{4}$ is greater than $\frac{8}{12}$.

----- **Begin Lesson** -----

Warm-up

 10 min

Number Talk: Multiples of Ten

Standards Alignments

Building Towards 4.NBT.B.4

The purpose of this Number Talk is to elicit strategies and understandings students have for adding and subtracting multi-digit numbers. These understandings help students develop fluency and will be helpful in later units as students will need to be able to add and subtract multi-digit numbers fluently using the standard algorithm.

When students make adjustments and create multiples of ten for mental addition they are looking for and making use of the base ten structure of numbers (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $119 + 119$
- $139 + 139$
- $159 + 159$
- $199 + 199$

Student Responses

- 238. $119 + 1 = 120$, and $120 + 120 = 240$, I added an extra 1 to each addend so $240 - 2 = 238$
- 278. $139 + 1 = 140$, and $140 + 140 = 280$, I added an extra 1 to each addend so $280 - 2 = 278$
- 318. $150 + 150 = 300$, and $9 + 9 = 18$ and $300 + 18 = 318$
- 398. $200 + 200 = 400$, I added 1 to each addend so $400 - 2 = 398$.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

How did you use multiples of ten, for example 20, 40, and 60 to help add these numbers mentally? (I changed the addends by adding one more to each addend and the subtracting the extra two from the final sum.)

Consider asking:

- “Who can restate ____’s reasoning in a different way?”

- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone approach the expression in a different way?”
- “Does anyone want to add on to ____’s strategy?”

Activity 1

🕒 20 min

Mystery Fractions

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

In this activity, students are given several sets of fractions and some clues about the size of a particular fraction in each set. To identify a fraction that meets certain size requirements or falls within a specified range, students need to use multiple comparison strategies they have learned. For example, they can use comparisons to benchmarks such as $\frac{1}{2}$ and 1 to eliminate some fractions, and then use equivalent fractions to compare the remaining ones.

Student-facing Task Statement

Six friends are each given a list of 5 fractions. They each chose one fraction quietly and wrote clues about their choice. Use their clues to identify the fractions they chose.

Andre:	$\frac{8}{12}$	$\frac{3}{6}$	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{2}{12}$
	●	less than 1			
	●	greater than $\frac{1}{3}$			
	●	less than $\frac{2}{3}$			

Tyler:	$\frac{2}{6}$	$\frac{2}{2}$	$\frac{2}{4}$	$\frac{2}{3}$	$\frac{2}{5}$
	●	greater than $\frac{1}{3}$			
	●	less than 1			
	●	less than $\frac{1}{2}$			

Launch

- Groups of 3–4
- Read the opening paragraph of the task as a class.
- “There are six sets of fractions in the activity. Each set comes with some clues. Your task is to find one fraction that meets all three clues in each set.”

Activity

- “Work with your group to find the mystery fractions.”
- “Each group member should start with a different set, and should find at least two

Clare:

$$\frac{4}{3} \quad \frac{4}{2} \quad \frac{3}{4} \quad \frac{1}{4} \quad \frac{2}{10}$$

- greater than $\frac{2}{8}$
- less than $\frac{11}{6}$
- greater than 1

Diego:

$$\frac{2}{8} \quad \frac{6}{12} \quad \frac{6}{8} \quad \frac{12}{10} \quad \frac{11}{12}$$

- greater than $\frac{1}{2}$
- less than 1
- greater than $\frac{3}{4}$

Elena:

$$\frac{2}{12} \quad \frac{50}{100} \quad \frac{4}{10} \quad \frac{3}{5} \quad \frac{7}{5}$$

- greater than $\frac{2}{10}$
- less than 1
- greater than $\frac{3}{6}$

Noah:

$$\frac{18}{10} \quad \frac{7}{8} \quad \frac{2}{5} \quad \frac{18}{5} \quad \frac{150}{100}$$

- greater than $\frac{1}{2}$
- less than $\frac{25}{10}$
- greater than $\frac{8}{5}$

mystery fractions before discussing their responses with the group.”

- 5–6 minutes: independent work time
- 7–8 minutes: group work time

Synthesis

- “Which clues helped you eliminate fractions the fastest?” (clues about size relative to 1)
- “What strategies did you use to compare fractions?” (compare fractions to $\frac{1}{2}$, 1, or another benchmark, write equivalent fractions to compare two fractions, compare fractions with the same numerator or denominator)
- “Did you ever have to use more than one strategy to compare fractions?” (Yes, two or three were often needed to find the mystery fraction.)

Student Responses

Andre: $\frac{3}{6}$, Tyler: $\frac{2}{5}$, Clare: $\frac{4}{3}$

Diego: $\frac{11}{12}$, Elena: $\frac{3}{5}$, Noah: $\frac{18}{10}$

Activity 2

 10 min

Distances on Foot

Standards Alignments

Addressing 4.NF.A.2

This activity has two purposes: to give students an opportunity to solve fraction comparison problems in context, and to reinforce the idea that two fractions can be compared only if they refer to the same whole. To serve the former, students compare fractional distance measurements. To serve the latter, they investigate fractional measurements in two different

units of distance: Chinese “li” and kilometer.

When comparing the distances in the first question, students can rely on a number of familiar strategies. Two of the fractional values are close to 2. Some students are likely to use that benchmark for efficient comparison. For example, they may note that the school and the market are both a little over 1 li from home, the library is more than 2 li, and the badminton club is a little under 2 li.

Focus the synthesis on the last two questions about interpreting fractional measurements in two different units.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Display sentence frames to support whole-class discussion: “I agree because . . .” and “I disagree because . . .”

Advances: Speaking, Conversing

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Optimize meaning and value. Share information about the unit “li” and help students understand the size of one li by referencing a common point of interest. Invite students to share this knowledge with family members and other teachers.

Supports accessibility for: Visual-Spatial Processing, Social-Emotional Functioning

Student-facing Task Statement

In China and some East Asian countries, the unit “li” is used for measuring distance.

Here are the walking distances between the home of a student in China and the places he visits regularly.

- school: $\frac{7}{5}$ li
- market: $\frac{7}{4}$ li
- library: $\frac{23}{10}$ li
- badminton club: $\frac{23}{12}$ li



1. Which is a shorter distance from the student’s home:

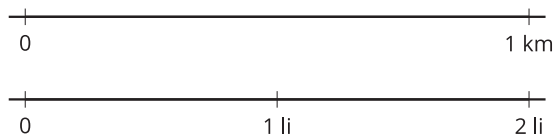
Launch

- Groups of 3–4
- “What are some units that we use for measuring distance? Let’s name as many as we can think of.” (Sample responses: inches, feet, miles, meters, kilometers)
- Share and record responses.
- “Today we’ll look at distances measured in ‘li,’ a unit commonly used in China.”

Activity

- “Take a few quiet minutes to work on the questions. Be prepared to explain your reasoning.”
- “Afterward, discuss your responses with your group and work together to complete

- a. His school or the library?
 - b. The market or the badminton club?
 - c. The library or the market?
2. A student in America walks $\frac{4}{5}$ kilometer (km) to school. These number lines show how 1 kilometer compares to 1 li.



Which student walks a longer distance to school? Use the number lines to show your reasoning.

3. Explain why we can't just compare the fractions $\frac{4}{5}$ and $\frac{7}{5}$ to see which student walks a longer distance.

Student Responses

1.
 - a. his school
 - b. the market
 - c. the market
2. The student in America. Sample reasoning:
3. Sample response: The size of 1 whole is different in the two cases. One kilometer is a different size than 1 li.

the activity.”

- 4 minutes: independent work time
- 4 minutes: group work time
- Monitor for students who:
 - attend to the location of 1 whole when representing $\frac{4}{5}$ and $\frac{7}{5}$ on the two number lines (rather than partitioning both number lines into the same 5 parts)
 - recognize that $\frac{4}{5}$ and $\frac{7}{5}$ in the two cases refer to different wholes and can articulate their reasoning

Synthesis

- Ask previously selected students to share their responses to the last two questions. Display their number lines, or display the number lines from the activity for them to annotate while they explain.
- Emphasize that just as 1 km is not the same distance as 1 li, $\frac{4}{5}$ km is not the same distance as $\frac{4}{5}$ li. We can't compare two fractions that refer to different wholes.

Lesson Synthesis

🕒 10 min

“Today we used a combination of strategies to help us compare fractions. We also solved fraction comparison problems in a situation about distance.”

Keep students in groups of 3–4. Give tools for creating a visual display to each group.

Assign each group one set of fractions in Activity 1 or the first set of questions in Activity 2.

- For the former: “Create a visual display that explains how you found the mystery fraction for your assigned set of fractions from Activity 1. Your display should list the five fractions, the three clues, and how the chosen fraction satisfies all the clues.”
- For the latter: “Create a visual display that shows your responses to the first set of questions in Activity 2. Your display should show the four walking distances and how you compared them.”

“Include diagrams, notes, and any descriptions that might help others understand your thinking.”

Ask students to display their work around the room.

“Visit the display of at least 2 other groups.”

“At each display, check to see if the reasoning strategies make sense to you. Think about how the reasoning in different displays is alike and how it is different.”

“How are the diagrams, explanations, or calculations that you saw alike? How are they different?”

Share and record responses. Reference the displays that students created to show similarities and differences in their reasoning strategies.

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students conclude that Kiran ran the furthest because both the numerator and denominator are larger.

Next Day Support

- Launch Activity 1 with a discussion about what each fraction would look like on a number line and using this representation to compare fractions.

Lesson 15: Common Denominators to Compare

Standards Alignments

Building On	4.OA.B.4
Addressing	4.NF.A.1, 4.NF.A.2
Building Towards	4.NF.A.2

Teacher-facing Learning Goals

- Compare two fractions with different denominators by rewriting both into equivalent fractions with a common denominator.

Student-facing Learning Goals

- Let's compare fractions by writing equivalent fractions with the same denominator.

Lesson Purpose

The purpose of this lesson is for students to compare two fractions with different denominators by rewriting both into an equivalent fraction with a common denominator.

Previously, students worked with fractions whose features encouraged different comparison strategies. For instance, the fractions might:

- have a common numerator or a common denominator
- be noticeably greater or less than a familiar benchmark, and their distance from a benchmark could be discerned
- have related denominators, in which one denominator is a factor or a multiple of the denominator of the other, making it intuitive to rewrite one fraction into an equivalent fraction with the same denominator as the second fraction

In this lesson, students encounter pairs of fractions with different denominators, in which neither denominator is a factor or multiple of the other, and for which other means of comparison are not feasible or intuitive. These fractions motivate students to find another way to compare: by rewriting both fractions into equivalent fractions with a shared denominator.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

What Do You Know About ____? (Warm-up)

Lesson Timeline

Warm-up	5 min
Activity 1	20 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did students' earlier work on factors and multiples support their work in this lesson? What surprised you about the insights students brought forth to help them find common denominators? What challenges did you not anticipate seeing?

Cool-down (to be completed at the end of the lesson)

 5 min

Which is Greater?

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

In each pair of fractions, which fraction is greater? Explain or show your reasoning.

- $\frac{3}{10}$ or $\frac{2}{6}$
- $\frac{99}{100}$ or $\frac{9}{10}$

Student Responses

- $\frac{2}{6}$. Sample reasoning: $\frac{3 \times 6}{10 \times 6} = \frac{18}{60}$ and $\frac{2 \times 10}{6 \times 10} = \frac{20}{60}$.
- $\frac{99}{100}$. Sample reasoning: $\frac{99}{100}$ is $\frac{1}{100}$ less than 1. $\frac{9}{10}$ is $\frac{1}{10}$ less than 1, so it is farther away from 1 and less than $\frac{99}{100}$.

----- Begin Lesson -----

Warm-up

 5 min

What Do You Know about 15 and 30?

Standards Alignments

Building On 4.OA.B.4

Building Towards 4.NF.A.2

The purpose of this warm-up is to elicit what students know about the numbers 15 and 30, preparing them to work with fractions whose denominators are factors of 15 and 30 later in the lesson. While students may bring up many things about these numbers, highlight responses that relate the two numbers by their factors and multiples.

Instructional Routines

What Do You Know About ____?

Student-facing Task Statement

What do you know about 15 and 30?

Student Responses

Sample responses:

- They are greater than 10.
- $5 \times 3 = 15$ and $5 \times 6 = 30$
- One is twice the other, or $15 \times 2 = 30$.
- They are both multiples of 3 and 5.
- One is 15 more than the other.
- Every other multiple of 15 is a multiple of 30.

Launch

- Display the numbers.
- “What do you know about 15 and 30?”
- 1 minute: quiet think time

Activity

- Record responses.

Synthesis

- If no students mentioned factors of 15 and 30, ask them about it.
- “What are the factors of 15?” (1, 3, 5, 15)
- “What are the factors of 30?” (1, 2, 3, 5, 6, 10, 15, 30)
- “What factors do they have in common?” (1, 3, 5, 15)
- “Do 15 and 30 have any common multiples? What are some of them?” (30, 60, 90)

Activity 1

 20 min

Tricky Fractions?

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

In earlier lessons, students compared fractions by rewriting one fraction as an equivalent fraction with the same denominator as the second fraction. In this activity, students see that—although it’s still possible to compare the fractions—this particular strategy doesn’t work if neither of the denominators of the two fractions is a factor or multiple of each other. Students learn that in such a case, both fractions can be expressed as equivalent fractions with a **common denominator**, and the denominator is a different number that is a multiple of both of the original denominators.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Display sentence frames to support partner discussions: “First, I ____ because . . .”, “I noticed ____ so I . . .”

Advances: Speaking, Conversing

Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate background knowledge. Provide students with access to a visible display that shows definitions or reminders of the terms “factor” and “multiple.”

Supports accessibility for: Memory, Language

Student-facing Task Statement

- In each pair of fractions, which fraction is greater? Explain or show your reasoning.
 - $\frac{4}{3}$ or $\frac{13}{12}$
 - $\frac{4}{3}$ or $\frac{7}{5}$
- Han says he can compare $\frac{4}{3}$ and $\frac{13}{12}$ by writing an equivalent fraction for $\frac{4}{3}$. He says he can’t use that strategy to compare $\frac{4}{3}$ and

Launch

- Groups of 2

Activity

- “Take a few quiet minutes to work on the first two questions.”
- 6–7 minutes: independent work time
- “Share your responses to both questions with your partner. Be sure to explain how you compared the fractions in the first

$\frac{7}{5}$. Do you agree? Explain your reasoning.

3. Priya and Lin showed different ways for comparing $\frac{4}{3}$ and $\frac{7}{5}$. Make sense of what they did. How are their strategies alike? How are they different?

Priya: $\frac{4 \times 5}{3 \times 5} = \frac{20}{15}$ $\frac{7 \times 3}{5 \times 3} = \frac{21}{15}$

$\frac{21}{15}$ is greater than $\frac{20}{15}$, so $\frac{7}{5}$ is greater than $\frac{4}{3}$.

Lin: $\frac{4 \times 10}{3 \times 10} = \frac{40}{30}$ $\frac{7 \times 6}{5 \times 6} = \frac{42}{30}$

$\frac{42}{30}$ is greater than $\frac{40}{30}$, so $\frac{7}{5}$ is greater than $\frac{4}{3}$.

Student Responses

- $\frac{4}{3}$ is greater than $\frac{13}{12}$. Sample reasoning: $\frac{4 \times 4}{3 \times 4} = \frac{16}{12}$.
 - $\frac{7}{5}$ is greater than $\frac{4}{3}$. Sample reasoning: Both fractions are greater than 1. $\frac{4}{3}$ is $\frac{1}{3}$ greater than 1 and $\frac{7}{5}$ is $\frac{2}{5}$ greater than 1. On the fraction strips, I see that two $\frac{1}{5}$ s are greater than $\frac{1}{3}$.
- Sample responses:
 - Agree. Neither denominator is a factor or a multiple of the other: 3 is not a factor of 5, and 5 is not a multiple of 3.
 - Disagree. We can't partition thirds into equal parts to get fifths, but we can partition both thirds and fifths to get fifteenths. If we write an equivalent fraction for both $\frac{4}{3}$ and $\frac{7}{5}$ in fifteenths, they can be compared.
- Sample response: Priya and Lin both multiplied the numerator and denominator

question."

- 3–4 minutes: partner discussion
- Pause for a brief whole-class discussion. Invite students to share their responses for the first two questions.
- If no students suggested that the second pair of fractions are hard to compare because their denominators have no factors in common (or one does not multiply or divide to make the other), ask them about it.
- "Now work with your partner on the last question."
- 3–4 minutes: group work time

Synthesis

- Display Priya and Lin's reasoning for all to see. Select students to share their observations on how the two are alike and how they are different.
- Highlight the fact that both students rewrote the two fractions so that they have a **common denominator**.
- "Why might it be helpful to write equivalent fractions with the same denominator?" (It is easier to compare the fractions when the fractional part is the same size.)
- "Can we choose any number to be the common denominator?" (No, it must be a multiple of both of the original denominators.)
- "Does it matter if we choose a smaller or a larger common multiple?" (No, but it could work better to multiply by a smaller number.)

by a number that would give equivalent fractions with the same denominator, but they chose different numbers for the denominator.

Activity 2

🕒 20 min

Use a Common Denominator, or Not

Standards Alignments

Addressing 4.NF.A.1, 4.NF.A.2

This activity serves two main goals: to prompt students to rewrite pairs of fractions into equivalent fractions with a common denominator, and to consider this newly developed skill as a possible way to compare fractions.

To write equivalent fractions, many students are likely to reason numerically (by multiplying or dividing the numerator and denominator by a common number). Some may, however, find equivalent fractions effectively by continuing to reason about how many of this fractional part is in that fractional part.

To compare the fractions in the second question, students may choose to write equivalent fractions with a common denominator because they were just learning to do so. The fractions, however, were chosen so that students have opportunities to choose an approach strategically, rather than writing equivalent fractions each time. For instance, students may notice that:

- In part a, one fraction is $\frac{1}{12}$ away from $\frac{1}{2}$ and the other is $\frac{1}{8}$ from $\frac{1}{2}$.
- In part b, one fraction is greater than 2 and the other is greater than 1.
- In part c, writing an equivalent fraction for only one given fraction (rather than for both) is sufficient for comparing.
- In part d, one fraction is less than $\frac{1}{2}$, and the other is greater than $\frac{1}{2}$.

Student-facing Task Statement

1. For each pair of fractions, write a pair of

Launch

- Groups of 2

equivalent fractions with a common denominator.

- a. $\frac{5}{6}$ and $\frac{3}{4}$
- b. $\frac{2}{3}$ and $\frac{5}{8}$
- c. $\frac{2}{6}$ and $\frac{4}{10}$
- d. $\frac{7}{4}$ and $\frac{17}{10}$

2. For each pair of fractions, decide which fraction is greater. Be prepared to explain your reasoning.

- a. $\frac{5}{12}$ or $\frac{3}{8}$
- b. $\frac{13}{5}$ or $\frac{11}{6}$
- c. $\frac{71}{10}$ or $\frac{34}{5}$
- d. $\frac{7}{12}$ or $\frac{49}{100}$

Student Responses

1. Sample responses:

- a. $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ and $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$
- b. $\frac{2 \times 8}{3 \times 8} = \frac{16}{24}$ and $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$
- c. $\frac{2 \times 5}{6 \times 5} = \frac{10}{30}$ and $\frac{4 \times 3}{10 \times 3} = \frac{12}{30}$
- d. $\frac{7 \times 10}{4 \times 10} = \frac{70}{40}$ and $\frac{17 \times 4}{10 \times 4} = \frac{68}{40}$

2. a. $\frac{5}{12}$. Sample reasoning: $\frac{5 \times 2}{12 \times 2} = \frac{10}{24}$ and $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$.
- b. $\frac{13}{5}$. Sample reasoning: $\frac{13}{5}$ is greater than 2, while $\frac{11}{6}$ is less than 2.
- c. $\frac{71}{10}$. Sample reasoning: $\frac{34 \times 2}{5 \times 2} = \frac{68}{10}$
- d. $\frac{7}{12}$. Sample reasoning: $\frac{7}{12}$ is greater than $\frac{1}{2}$, while $\frac{49}{100}$ is less than $\frac{1}{2}$.

Activity

- “Work with your partner to write equivalent fractions for the first set of questions.”
- 7–8 minutes: group work time on the first set of questions
- Pause for a brief whole-class discussion.
- Poll the class on the common denominator they chose for each pair of fractions. Record their responses. (Likely denominators for each part:
 - a. 24 or 12
 - b. 24
 - c. 60 or 30
 - d. 40 or 20)
- Some students are likely to suggest multiplying one denominator by the other. Discuss whether there are other ways to find a common denominator.
- “Work independently to compare the fractions in the second set of questions. Be prepared to explain how you know which fraction is greater.”
- 7–8 minutes: independent work time on the second set of questions
- Monitor for students who are strategic in how they compare the pairs of fractions in the second question (not exclusively writing equivalent fractions).

Synthesis

- See lesson synthesis.

Advancing Student Thinking

When working on the second set of problems, some students might be inclined to immediately

find a common denominator for each pair of fractions. They might get stuck if they don't recognize a common factor or multiple of the denominators (for instance, 12 and 100), or if they aren't sure how to multiply large numbers. Encourage students to consider other strategies they know for gauging the size of fractions and for comparing fractions.

Lesson Synthesis

🕒 10 min

Invite students to share their responses to the last set of questions of Activity 2 and how they went about making comparisons. Record their responses.

Select students who made strategic choices when making comparisons to share their thinking.

Emphasize that, while it is possible to compare every pair of fractions by rewriting them so that they have a common denominator, all the fractions could be compared by reasoning in other ways.

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students attempted to compare $\frac{99}{100}$ and $\frac{9}{10}$ by writing equivalent fractions with 1,000 as the common denominator but didn't manage to complete the multiplication of multi-digit numbers (as the skill is not yet an expectation at this point in their study).

Next Day Support

- Before the warm-up, display $\frac{99}{100}$ and $\frac{9}{10}$ and ask: “What strategies can we use to compare these two fractions? Let's find as many ways as possible.”

Lesson 16: Compare and Order Fractions

Standards Alignments

Building On 4.OA.B.4

Addressing 4.NF.A.2

Teacher-facing Learning Goals

- Compare and order fractions using any strategy.

Student-facing Learning Goals

- Let's put some fractions in order.

Lesson Purpose

The purpose of this lesson is for students to compare and order fractions using any strategy.

Throughout the unit, students have encountered a wide range of fractions and learned a variety of ways to represent and compare fractions. In this lesson students consolidate their understanding and skills and use them to solve new fraction comparison problems strategically and with flexibility.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Copy

- Compare Stage 3-8 Directions (groups of 2): Activity 1
- Fraction Cards Grade 4 (groups of 2): Activity 1

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

As you wrap up this unit, reflect on the norms that have supported your students in learning math. How have you seen each student grow as

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

a learner? How have you seen yourself grow as a teacher?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

All in Order

Standards Alignments

Addressing 4.NF.A.2

Student-facing Task Statement

Put these fractions in order, from least to greatest. Show your reasoning.

$$\frac{5}{12} \quad \frac{8}{6} \quad \frac{4}{10} \quad \frac{7}{5}$$

Student Responses

$\frac{4}{10}$, $\frac{5}{12}$, $\frac{8}{6}$, $\frac{7}{5}$. Sample reasoning:

- $\frac{4}{10}$ and $\frac{5}{12}$ are less than 1. $\frac{8}{6}$ and $\frac{7}{5}$ are greater than 1.
- Comparing $\frac{4}{10}$ and $\frac{5}{12}$: $\frac{4 \times 6}{10 \times 6} = \frac{24}{60}$ and $\frac{5 \times 5}{12 \times 5} = \frac{25}{60}$, so $\frac{5}{12}$ is greater.
- Comparing $\frac{8}{6}$ and $\frac{7}{5}$: $\frac{8 \times 5}{6 \times 5} = \frac{40}{30}$ and $\frac{7 \times 6}{5 \times 6} = \frac{42}{30}$. Or: $\frac{8}{6}$ is $\frac{2}{6}$ more than 1, while $\frac{7}{5}$ is $\frac{2}{5}$ more than 1. Since $\frac{2}{5}$ is greater than $\frac{2}{6}$, $\frac{7}{5}$ is greater than $\frac{8}{6}$.

----- Begin Lesson -----

Warm-up

🕒 10 min

Number Talk: Multiples of 6 and 12

Standards Alignments

Building On 4.OA.B.4

This Number Talk encourages students to think about multiples of 5, 6, and 12—numbers that students will see as denominators later in the lesson. It also prompts students to rely on doubling and on properties of operations to mentally solve multiplication problems. The reasoning elicited here will be helpful later in the lesson when students compare fractions by finding equivalent fractions with a common denominator.

To find products by doubling or by using properties of operations, students need to look for and make use of structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 5×6
- 5×12
- 6×12
- 11×12

Student Responses

Sample reasoning:

- 30. I know the fact by heart.
- 60. I know that 12 is twice 6, so 5×12 is 10×6 .
- 72. Six times 12 is 12 more than 5 times 12.
- 132. I added 5×12 and 6×12 to get 11×12 .

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How do the first three expressions help you find the value of the last one?”

Activity 1

🕒 20 min

Compare Fractions Game

Standards Alignments

Addressing 4.NF.A.2

This activity allows students to practice comparing fractions and apply the comparison strategies they learned through a game. Students use fraction cards from an earlier lesson to play a game in groups of 2, 3, or 4. To win the game is to have the greater (or greatest) fraction of the cards played as many times as possible. This is stage 5 of the Compare center.

Consider arranging students in groups of 2 for the first game or two (so that students would need to compare only 2 fractions at a time), and arranging groups of 3 or 4 for subsequent games. Before students begin playing, ask them to keep track of and record pairs of fractions that they find challenging to compare.

Access for English Learners

MLR8 Discussion Supports. Students should take turns explaining their reasoning to their partner. Display the following sentence frames for all to see: “___ is greater than ___ because . . .”, and “___ and ___ are equivalent because . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing

Materials to Copy

Compare Stage 3-8 Directions (groups of 2),
Fraction Cards Grade 4 (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2–4 students.

Student-facing Task Statement

Play Compare Fractions with 2 players:

- Split the deck between the players.
- Each player turns over a card.
- Compare the fractions. The player with the greater fraction keeps both cards.

**fraction
cards**



Launch

- Groups of 2–4
- Give each group a set of fraction cards.
- Tell students that they will play one or more games of Compare Fractions.
- Demonstrate how to play the game. Invite a student to be your opponent in the demonstration game.
- Read the rules as a class and clarify any questions students might have.

- If the fractions are equivalent, each player turns over one more card. The player with the greater fraction keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Play Compare Fractions with 3 or 4 players:

- The player with the greatest fraction wins the round.
- If 2 or more players have the greatest fraction, those players turn one more card over. The player with the greatest fraction keeps all the cards.

Record any sets of fractions that are challenging to compare here.

_____ and _____ _____ and _____
 _____ and _____ _____ and _____

- Groups of 2 for the first game or two, then groups of 3–4 for subsequent games, if time permits

Activity

- “Play one game with your partner.”
- “As you play, you may come across one or more sets of fractions that are tricky to compare. Record those fractions. Be prepared to explain how you eventually figure out which fraction is greater.”
- “If you finish before time is up, play another game with the same partner, or play a game with the players from another group.”
- 15 minutes: group work time

Synthesis

- Invite groups to share some of the challenging sets of fractions they recorded and how they eventually determined the greater one in each pair.
- As one group shares, ask others if they have other ideas about how the fractions could be compared.

Activity 2

 15 min

Fractions in Order

Standards Alignments

Addressing 4.NF.A.2

This activity prompts students to compare multiple fractions and put them in order by size. The work gives students opportunities to look for and make use of structure (MP7) in each set of

fractions and make comparisons strategically. For instance, rather than comparing two fractions at a time and in the order they are listed, students could first classify the given fractions as greater or less than $\frac{1}{2}$ or 1, look for fractions with a common numerator or denominator, and so on.

If time is limited, consider asking students to choose two sets of fractions to compare and order.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most useful when putting fractions in order. Display the sentence frame, “The next time I put fractions in order, I will pay attention to”

Supports accessibility for: Memory

Student-facing Task Statement

Put each set of fractions in order, from least to greatest. Be prepared to explain your reasoning.

1. $\frac{3}{12}$, $\frac{2}{4}$, $\frac{2}{3}$, $\frac{1}{8}$
2. $\frac{8}{5}$, $\frac{5}{6}$, $\frac{11}{12}$, $\frac{11}{10}$
3. $\frac{21}{20}$, $\frac{9}{10}$, $\frac{6}{5}$, $\frac{101}{100}$
4. $\frac{5}{8}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{3}{6}$

Student Responses

1. $\frac{1}{8}$, $\frac{3}{12}$, $\frac{2}{4}$, $\frac{2}{3}$
2. $\frac{5}{6}$, $\frac{11}{12}$, $\frac{11}{10}$, $\frac{8}{5}$
3. $\frac{9}{10}$, $\frac{101}{100}$, $\frac{21}{20}$, $\frac{6}{5}$
4. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{3}{6}$, $\frac{5}{8}$

Launch

- Groups of 2

Activity

- “Work independently on two sets. Then, discuss your work with your partner and complete the rest together.”
- 10 minutes: independent work time
- Monitor for students who look for and make use of structure. Ask them to share during lesson synthesis.
- 3–4 minutes: partner discussion

Synthesis

- See lesson synthesis.

Advancing Student Thinking

Some students may try to write equivalent fractions with a common denominator for all four fractions in each set before comparing them but may be unable to do so. Encourage them to try reasoning about two fractions at a time, and to use what they know about the fractions to determine how they compare (to one another or to familiar benchmarks).

Lesson Synthesis

🕒 10 min

Invite students to share their strategies for comparing and ordering the fractions in the last activity. Record their responses.

Ask students to reflect on their understanding of fractions in this unit.

“What are some things about writing, representing, or comparing fractions that you didn’t know at the beginning of the unit but you know quite well now? Think of at least two specific things.”

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Student Section Summary

In this section, we compared fractions using what we know about the size of fractions, benchmarks such as $\frac{1}{2}$ and 1, and equivalent fractions. For example, to compare $\frac{3}{8}$ and $\frac{6}{10}$, we can reason that:

- $\frac{4}{8}$ is equivalent to $\frac{1}{2}$, so $\frac{3}{8}$ is less than $\frac{1}{2}$.
- $\frac{5}{10}$ is equivalent to $\frac{1}{2}$, so $\frac{6}{10}$ is more than $\frac{1}{2}$.

This means that $\frac{6}{10}$ is greater than $\frac{3}{8}$ (or $\frac{3}{8}$ is less than $\frac{6}{10}$).

We can also compare by writing equivalent fractions with the same denominator. For example, to compare $\frac{3}{4}$ and $\frac{4}{6}$, we can use 12 as the denominator:

$$\frac{3}{4} = \frac{9}{12} \qquad \frac{4}{6} = \frac{8}{12}$$

Because $\frac{9}{12}$ is greater than $\frac{8}{12}$, we know that $\frac{3}{4}$ is greater than $\frac{4}{6}$.

----- Complete Cool-Down -----

Response to Student Thinking

Students put the fractions in an order other than $\frac{4}{10}, \frac{5}{12}, \frac{8}{6}, \frac{7}{5}$.

Next Day Support

- Before the warm-up, display number lines with the given fractions and ask students to justify comparisons.

Lesson 17: Paper Clip Games (Optional)

Standards Alignments

Addressing	4.NF.A
Building Towards	4.NF.A

Teacher-facing Learning Goals

- Locate and compare fractions on the number line.

Student-facing Learning Goals

- Let's create a game about locating and comparing fractions on the number line.

Lesson Purpose

The mathematical purpose of this lesson is for students create and play a game about locating and comparing fractions on a number line.

This lesson is optional because it does not address any new mathematical content standards. It does offer students an opportunity to model with mathematics to create a game.

In previous lessons, students located and compared fractions on number lines using benchmark fractions and numerical relationships. Here, students work in pairs to create a game based on these skills and concepts. They identify fractions on a number line, decide how numerical results will be interpreted to determine the winner, draft rules for their game, share their game with classmates, and revise rules based on feedback.

When students make choices about their approach, analyze numerical information, interpret results, and describe mathematical procedures, they model with mathematics (MP4). When they label fractions on the number line and revise their procedures to better communicate the rules for the game, they are attending to precision (MP6).

This lesson may take more than 60 minutes, as students may need additional time to create, play, and revise their games. Consider modifying the activities or expanding the lesson across 2 days to meet students' needs and any time constraints.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Warm-up)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Markers: Activity 1, Activity 2
- Paper clips: Activity 1, Activity 2
- Paper: Activity 1, Activity 2
- Tape (painter's or masking): Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	15 min
Activity 3	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

Reflect on whose thinking was heard today. Reflect on whose thinking was not heard but could have enriched the conversations. What prompts or structures might better enable the latter to share their voices and reasoning?

Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Lots of Paper Clips

Standards Alignments

Building Towards 4.NF.A

The purpose of this warm-up is to draw students' attention to the number line, which goes from 0–2, and the locations of tossed paper clips relative to the fractions on the number line.

While students may notice and wonder many things, highlight observations about the locations of the paper clips on the number line—how they are clustered or spread out and how their locations connect to the labeled benchmark fractions.

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they notice and wonder about the diagram. On a visible display, record words and phrases such as: benchmark fractions, equivalent fractions, greater than and less than. Invite students to borrow language from the display as needed, and update it throughout the lesson.

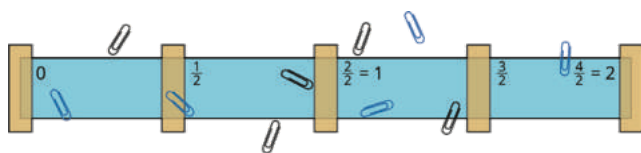
Advances: Conversing, Reading

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- Ten paper clips, 5 blue and 5 black, are on or around a long strip.
- The strip looks like a number line that goes from 0 to 2.
- 0 , $\frac{1}{2}$, 1 , $\frac{3}{2}$, and 2 are labeled on the strip.
- Blue paper clips are more spread out.
- Black paper clips are closer to 1.

Students may wonder:

- Why are the paper clips strewn on a strip with fractions on it?
- What do the paper clips represent?
- Were the paper clips scattered randomly, or placed in specific locations?
- Is this a picture of a game with paper clips?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “Priya and Kiran were playing a paper clip tossing game that uses fractions and a number line. Priya’s paper clips are blue. The image shows the result of one game.”
- “What could be a rule for this game if this picture represents a win for Priya?” (Sample responses: The winner is the player with:
 - more paper clips greater than 1
 - the paper clip closest to 2
 - the most paper clips on the benchmark fractions 0 , $\frac{1}{2}$, 1 , $\frac{3}{2}$, and 2)
- “What if it was a win for Kiran, whose paper clips are black?” (Sample response: The winner is the player with:
 - more paper clips between 0 and 1

- the paper clip closest to 1
- the fewest paper clips on the benchmark fractions 0 , $\frac{1}{2}$, 1 , $\frac{3}{2}$, and 2

Activity 1

🕒 10 min

Paper Clip Tossing Game

Standards Alignments

Addressing 4.NF.A

In this activity, students use their understanding of benchmark fractions and equivalent fractions to play a game that involves fractions on the number line. They toss paper clips on a game board that is a number line, and then write fractions to label the locations where the paper clips land. In previous lessons, students labeled fractions on a number line with premade tick marks. This activity gives students an opportunity to label them independently. Before playing the game, students prepare the game board and materials.

🕒 Access for Students with Disabilities

Representation: Access for Perception. Synthesis: Begin by demonstrating one round of the paper clip tossing game in order to support understanding of the context.

Supports accessibility for: Memory, Social-Emotional Functioning

Materials to Gather

Markers, Paper, Paper clips, Tape (painter's or masking)

Required Preparation

- Each group of 2 needs 1-inch paper strips and 10–12 paper clips.

Student-facing Task Statement

Let's prepare a game board and figure out how to toss paper clips and record the results!

1. Make your game board:

Launch

- Groups of 2
- Give each group strips of paper, markers, paper clips.

- Tape the paper strip to your workspace. Place the tape at these benchmarks: 0 , $\frac{1}{2}$, 1 , $\frac{3}{2}$, and 2 .
 - Label the benchmark fractions 0 , $\frac{1}{2}$, 1 , $\frac{3}{2}$, and 2 on the paper strip.
2. Play the game:
- Take turns tossing the paper clips onto the game board.
 - Label the fraction where each paper clip lands.

Be prepared to share your strategies for tossing the paper clips and for finding out the fractions for their locations.

- “Work with your group to play a version of the paper-clip tossing game. To play the game, you will toss some paper clips and use fractions to label where they land.”
- “First, we’ll make the game board. Fold your paper strip in half and then in half again. Carefully tape it down to your workspace (desk or floor can work) and label the benchmark fractions.”
- 5 minutes: partner work time

Activity

- “As you’re playing, determine the best spot to stand and how to toss the clips.”
- Monitor for students who:
 - use benchmark fraction strategies and numerical strategies for finding equivalent fractions to label the fractions
 - consider what to do with clips that do not land on the strip
- 5 minutes: game playing time

Synthesis

- Invite previously selected students to share their strategies for tossing and labeling the locations of the paper clips.
- “Where was the best spot to stand? What did you think about when you tossed?”
- “Take a look at all the fractions you recorded on your number line. What do you notice?” (Sample responses:
 - They are ordered from smallest to greatest.
 - We used lots of sixths and eighths.
 - There are lots of different denominators.)
- “What did you do if a paper clip landed in a location where it was difficult to name the fraction?” (I found the closest fraction that

was easy to find and used that to help. I used equivalent fractions.)

Activity 2

🕒 15 min

A New Game with New Rules

Standards Alignments

Addressing 4.NF.A

Materials to Gather

Markers, Paper, Paper clips, Tape (painter's or masking)

Student-facing Task Statement

Invent your own game.

1. Make a list with the rules of your game.
2. Play your game, paying close attention to the rules.
3. Revise and clarify your game rules, if necessary.

Student Responses

1. Sample goals for the game:
 - getting closest to 1
 - having the widest spread of fractions
 - hitting close to all the benchmarks
 - getting a clip in each quarter of the game board

Sample rules may include:

- the number of tosses
- the number of clips thrown at once

Launch

- Groups of 2
- “Take a look at the results of your tosses. With your partner, invent a new game.”
- “Think about the goal of the game and decide what someone would need to do to win in a fair game.”
- 1 minute: partner think time
- “What are some things your group needs to think about when you’re making up the rules for the game?”
- 1 minute: whole-class discussion. List should include elements such as:
 - How does someone win the game?
 - Is it possible to have a tie game?
 - How many tosses does a player make?
 - Does a player toss all their clips at once, one after another, or do players take turns?

- whether clips that don't land on the paper would count
2. No response required.
 3. Answers vary.
- What happens if a clip lands off the adding machine tape?
 - Do you measure the front, middle, or back of the paperclip?
 - Do you want to limit the possible fractions that can be on your number line (for example, only $\frac{1}{2}$ s, $\frac{1}{4}$ s, and $\frac{1}{8}$ s)?
 - Is there a penalty if a player miscalculates their location?
 - How far are players away from the game board?
 - Are they standing, crouching, or sitting?
 - Are they in line with or facing the number line when they toss?
- Distribute new paper strips and masking tape.

Activity

- 10 minutes: partner work time
- Monitor for groups that consider multiple aspects of the game or come up with a unique rule.

Synthesis

- "What do you like about the rules you chose?"
- "What parts of the rules make your game fair to both players?"

Activity 3

🕒 15 min

Field Test

The purpose of this activity is for students to share their games with their classmates. This provides pairs the opportunity to articulate their rules and check to see if they are clear to their audience. Based on feedback, they will be able to revise their directions.

Student-facing Task Statement

Let's try out these games!

1. Before playing the game, exchange your game rules with another team. Carefully read the rules. Take turns asking clarifying questions, if you have any.
2. Play each other's games.
3. After playing the game, give feedback to each other about the rules.
 - a. What is one thing you liked about the other team's game?
 - b. What is one thing you might change?

Student Responses

Sample responses for the last question:

- I liked how we could throw all our paper clips at one time.
- It was hard to stand so far away. A lot of my clips weren't on the paper, so it was harder to figure out their locations.
- I didn't know what was meant by "rounding off our fraction to a benchmark."
- I think players should be able to have a tie game if they both have 3 paper clips that landed on the part of the board that is greater than 1.

Launch

- Groups of 4
- "When game makers produce a new game, one of the things they do is test their new product to see if it works and if people like it. This is called doing a 'field test'."
- "Very often, they discover the first time they ask people to try what they made, they need to revise it to make it better. That's what we're going to do."

Activity

- 10 minutes: play time
- Monitor for groups who:
 - discuss the fairness of a rule
 - suggest strategies for winning or tossing the clips a certain way

Synthesis

- "How did you revise your game to make it better? Did your changes have to do with logistics or mathematics or both?"

Lesson Synthesis

🕒 10 min

"Today, we made up a game involving labeling fractions."

"What was your favorite part of the game making experience? What was challenging about the experience?"

Suggested Centers

- Compare (1-5), Stage 5: Fractions (Addressing)
- Compare (1-5), Stage 3: Multiply within 100 (Supporting)
- How Close? (1-5), Stage 6: Multiply to 3,000 (Supporting)

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Family Support
Materials

Family Support Materials

Fraction Equivalence and Comparison

In this unit, students deepen their knowledge of fractions. They explore the size of fractions, write equivalent fractions, and compare and order fractions with the denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Section A: Size and Location of Fractions

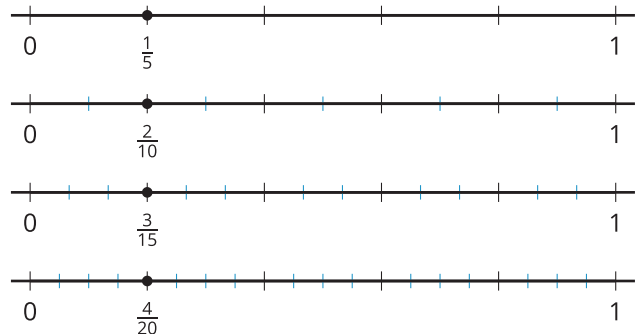
In this section, students revisit the meaning of fractions. They use fraction strips, tape diagrams, and number lines to represent fractions. Students compare fractions with the same numerators or the same denominators, and recall that equivalent fractions have the same size.

Students consider the size of fractions whose denominators are related, such as $\frac{1}{5}$ and $\frac{1}{10}$, or $\frac{1}{6}$ and $\frac{1}{12}$. They also compare fractions to benchmarks such as $\frac{1}{2}$ and 1. (For instance, they see that $\frac{3}{10}$ is less than $\frac{1}{2}$ and $\frac{3}{5}$ is more than $\frac{1}{2}$.)

Section B: Equivalent Fractions

Here, students take a closer look at equivalent fractions and reason using number lines. They show that fractions that are at the same point on the number line are equivalent.

Students then learn to tell if two fractions are equivalent without using number lines.



For example, they can explain that the fraction $\frac{2}{3}$ is equivalent to $\frac{8}{12}$ because the numerator and the denominator of $\frac{2}{3}$ are each multiplied by the same number, 4, to get $\frac{8}{12}$. Students use such observations to identify and write equivalent fractions.

Section C: Fraction Comparison

In this section, students compare fractions with different numerators and denominators using various strategies. For example, they may think about how far each fraction is from 0 on a number line, how each fraction compares to $\frac{1}{2}$ or 1, or think of the fractions in terms of the same denominator.

Students record the results of comparisons with symbols $>$, $=$, or $<$. They then solve problems that involve comparing fractional measurements, such as lengths in fractions of an inch.

Try it at home!

Near the end of the unit, ask your student to compare $\frac{3}{5}$ and $\frac{3}{7}$.

Questions that may be helpful as they work:

- How are the two fractions alike? How are they different?
- What strategy did you use to compare?
- Is there a different strategy that you could use to compare?

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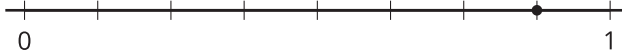
Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment

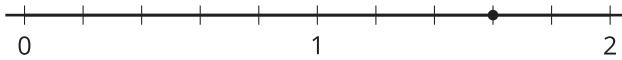
Fraction Equivalence and Comparison: Section A Checkpoint

1. Label the point on each number line with a fraction it represents.

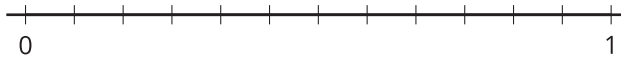
a.



b.



2. Is $\frac{7}{12}$ greater than or less than $\frac{1}{2}$? Explain your reasoning. Use the number line if it is helpful.



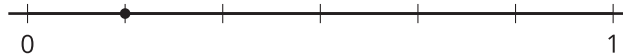
3. Explain why $\frac{4}{12}$ is equivalent to $\frac{1}{3}$. Use the number line if it is helpful.



Fraction Equivalence and Comparison: Section B Checkpoint

1. List two fractions that are equivalent to $\frac{3}{4}$. Explain or show your reasoning.

2. List two equivalent fractions that the point represents. Explain your reasoning.



3. To show that $\frac{7}{12}$ is equivalent to $\frac{14}{24}$, Kiran wrote: $\frac{2 \times 7}{2 \times 12} = \frac{14}{24}$. Do you agree with Kiran? Explain your reasoning.

Fraction Equivalence and Comparison: Section C Checkpoint

1. Use a $<$, $=$, or $>$ symbol to make each statement true. Explain your reasoning.

a. $\frac{4}{12}$ _____ $\frac{1}{3}$

b. $\frac{53}{100}$ _____ $\frac{5}{12}$

c. $\frac{265}{100}$ _____ $\frac{28}{10}$

d. $\frac{13}{8}$ _____ $\frac{7}{5}$

2. Clare walked $\frac{4}{5}$ of the way around a lake. Tyler walked $\frac{7}{12}$ of the way around a different lake. Explain why you don't have enough information to determine who walked farther.

Fraction Equivalence and Comparison: End-of-Unit Assessment

1. Select all fractions that are equivalent to $\frac{3}{12}$.

- A. $\frac{1}{4}$
- B. $\frac{2}{8}$
- C. $\frac{2}{11}$
- D. $\frac{4}{1}$
- E. $\frac{5}{20}$

2. Select all fractions that are greater than $\frac{1}{2}$ but less than 1.

- A. $\frac{4}{5}$
- B. $\frac{1}{3}$
- C. $\frac{5}{4}$
- D. $\frac{4}{7}$
- E. $\frac{5}{10}$

3. Which fraction is less than $\frac{3}{5}$?

- A. $\frac{5}{7}$
- B. $\frac{4}{6}$
- C. $\frac{9}{15}$
- D. $\frac{7}{12}$

4. List three different fractions that are equivalent to $\frac{4}{5}$. Explain or show your reasoning.

5. Elena says that $\frac{3}{5}$ and $\frac{6}{10}$ are not equivalent because there are twice as many parts in $\frac{6}{10}$. Do you agree with Elena? Explain your reasoning.

6. List these fractions from smallest to largest. Explain how you found the order.

$\frac{7}{4}$

$\frac{7}{12}$

$\frac{3}{8}$

$\frac{13}{6}$

$\frac{1}{4}$

7. For each fraction, write an equivalent fraction with the given denominator.

a. $\frac{1}{2} = \frac{\quad}{12}$

b. $\frac{2}{3} = \frac{\quad}{24}$

c. $\frac{6}{5} = \frac{\quad}{35}$

d. $\frac{5}{7} = \frac{\quad}{28}$

e. $\frac{7}{8} = \frac{\quad}{32}$

8. Noah and Lin drew different geometric designs on the same-size rectangular paper and colored the designs.

a. $\frac{4}{10}$ of Noah's design is blue. How can you describe the size of the fraction?

b. $\frac{5}{12}$ of Lin's design is blue.

Sketch an example of what Lin's design could look like.



c. Whose design has more blue, Noah's or Lin's? Explain or show your reasoning.

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Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment

Assessment Answer Keys

Assessment: Section A Checkpoint

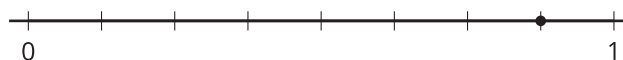
Problem 1

Goals Assessed

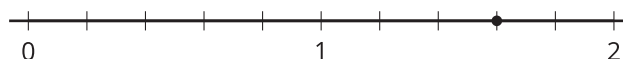
- Reason about the location of fractions on the number line.

Label the point on each number line with a fraction it represents.

a.



b.



Solution

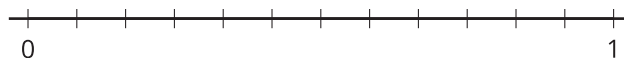
- a. $\frac{7}{8}$, because there are 8 equal parts in 1 and the point is on the 7th tick.
- b. $\frac{8}{5}$, because there are 5 tick marks in each whole and the point is on the 8th tick.

Problem 2

Goals Assessed

- Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams.
- Reason about the location of fractions on the number line.

Is $\frac{7}{12}$ greater than or less than $\frac{1}{2}$? Explain your reasoning. Use the number line if it is helpful.



Solution

$\frac{7}{12}$ is greater than $\frac{1}{2}$ because it's the 7th tick mark and it's more than halfway to 1.

Problem 3

Goals Assessed

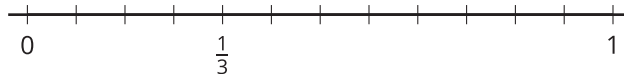
- Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams.
- Reason about the location of fractions on the number line.

Explain why $\frac{4}{12}$ is equivalent to $\frac{1}{3}$. Use the number line if it is helpful.



Solution

If I divide each third into 4 equal pieces those are twelfths and $\frac{1}{3}$ is on the 4th tick mark.



Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.

List two fractions that are equivalent to $\frac{3}{4}$. Explain or show your reasoning.

Solution

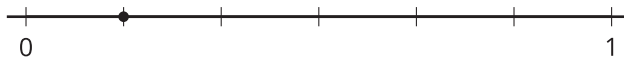
Sample response: $\frac{6}{8}$ and $\frac{9}{12}$. There are two $\frac{1}{8}$ s in $\frac{1}{4}$, so $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. There are three $\frac{1}{12}$ s in $\frac{1}{4}$, so $\frac{9}{12}$ is equivalent to $\frac{3}{4}$.

Problem 2

Goals Assessed

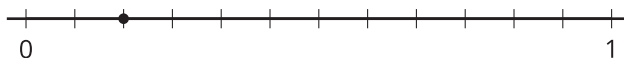
- Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Use visual representations to reason about fraction equivalence, including using benchmarks such as $\frac{1}{2}$ and 1.

List two equivalent fractions that the point represents. Explain your reasoning.



Solution

Sample responses: $\frac{1}{6}$ and $\frac{2}{12}$. There are 6 equal parts on the number line and the point is the first tick mark so that's $\frac{1}{6}$.



The point is the second tick mark if I divide each $\frac{1}{6}$ into two equal parts. Those parts are $\frac{1}{12}$ s since there are 12 in 1. So the point is also $\frac{2}{12}$.

Problem 3

Goals Assessed

- Generate equivalent fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.

To show that $\frac{7}{12}$ is equivalent to $\frac{14}{24}$, Kiran wrote: $\frac{2 \times 7}{2 \times 12} = \frac{14}{24}$. Do you agree with Kiran? Explain your reasoning.

Solution

Kiran is correct. $\frac{7}{12}$ is 7 equal parts where there are 12 of those parts in a whole. If I partition each of those 12 parts into 2 smaller parts, then I get 2×7 of the smaller parts with 2×12 of them in the whole. That's $\frac{2 \times 7}{2 \times 12}$.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Use visual representations or a numerical process to reason about fraction comparison.

Use a $<$, $=$, or $>$ symbol to make each statement true. Explain your reasoning.

a. $\frac{4}{12}$ _____ $\frac{1}{3}$

b. $\frac{53}{100}$ _____ $\frac{5}{12}$

c. $\frac{265}{100}$ _____ $\frac{28}{10}$

d. $\frac{13}{8}$ _____ $\frac{7}{5}$

Solution

- a. $\frac{4}{12} = \frac{1}{3}$, because $\frac{1}{3}$ is the same as $\frac{1 \times 4}{3 \times 4}$, which is $\frac{4}{12}$.
- b. $\frac{53}{100} > \frac{5}{12}$, because $\frac{53}{100}$ is more than $\frac{1}{2}$ and $\frac{5}{12}$ is less than $\frac{1}{2}$.
- c. $\frac{265}{100} < \frac{28}{10}$, because $\frac{28}{10}$ is $\frac{280}{100}$.
- d. $\frac{13}{8} > \frac{7}{5}$, because if I take away 1 from each then I have $\frac{5}{8}$ and $\frac{2}{5}$. I know $\frac{5}{8}$ is more than $\frac{1}{2}$, and $\frac{2}{5}$ is less than $\frac{1}{2}$.

Problem 2

Goals Assessed

- Use visual representations or a numerical process to reason about fraction comparison.

Clare walked $\frac{4}{5}$ of the way around a lake. Tyler walked $\frac{7}{12}$ of the way around a different lake. Explain why you don't have enough information to determine who walked farther.

Solution

I know that $\frac{4}{5}$ is greater than $\frac{7}{12}$ because $\frac{4}{5}$ is close to 1 (or is $\frac{1}{5}$ less than 1) and $\frac{7}{12}$ is just a little over $\frac{1}{2}$ (or is $\frac{1}{12}$ more than $\frac{6}{12}$, which is $\frac{1}{2}$). But I don't know how far it is all the way around each lake. If the lake Tyler is walking around has a much longer distance than the lake Clare is walking around, then Tyler could be walking farther.

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments

Addressing 4.NF.A.1

Narrative

Students identify which fractions are equivalent to $\frac{3}{12}$. They may fail to select A if they make an arithmetic error. They may fail to select B if they try to relate $\frac{2}{8}$ and $\frac{3}{12}$ directly, not thinking that they are both equivalent to $\frac{1}{4}$. They may select C seeing that the numerator and denominator both differ from the numerator and denominator of $\frac{3}{12}$ by 1. They may select D if they do not understand the meaning of the numerator and denominator in a fraction. They may fail to select E if they do not realize that $\frac{5}{20}$ is equivalent to $\frac{1}{4}$.

Select **all** fractions that are equivalent to $\frac{3}{12}$.

- A. $\frac{1}{4}$
- B. $\frac{2}{8}$
- C. $\frac{2}{11}$
- D. $\frac{4}{1}$
- E. $\frac{5}{20}$

Solution

["A", "B", "E"]

Problem 2

Standards Alignments

Addressing 4.NF.A.2

Narrative

Students compare fractions to the benchmarks $\frac{1}{2}$ and 1. Students may select response C and not select the correct responses A and D if they confuse the meaning of the numerator and denominator. They may select E if they do not read the directions carefully or fail to realize that $\frac{5}{10}$ is equivalent to $\frac{1}{2}$.

Select **all** fractions that are greater than $\frac{1}{2}$ but less than 1.

- A. $\frac{4}{5}$
- B. $\frac{1}{3}$
- C. $\frac{5}{4}$
- D. $\frac{4}{7}$
- E. $\frac{5}{10}$

Solution

["A", "D"]

Problem 3

Standards Alignments

Addressing 4.NF.A.2

Narrative

Students identify a fraction that is less than $\frac{3}{5}$. Students can reason that choices A and B are greater than $\frac{3}{5}$ by finding a common denominator or by seeing that they are all 2 parts short of 1 and those parts are smaller for $\frac{5}{7}$ and $\frac{4}{6}$ than for $\frac{3}{5}$. They can see that $\frac{9}{15} = \frac{3}{5}$ by dividing each fifth into 3 equal parts. To see that $\frac{7}{12} < \frac{3}{5}$ they will likely need to find a common denominator or eliminate the other 3 possibilities.

Which fraction is less than $\frac{3}{5}$?

- A. $\frac{5}{7}$
- B. $\frac{4}{6}$
- C. $\frac{9}{15}$
- D. $\frac{7}{12}$

Solution

D

Problem 4

Standards Alignments

Addressing 4.NF.A.1

Narrative

Students find fractions equivalent to a given fraction with no scaffold. They may draw a picture or use a number line or reason abstractly in terms of the number and size of parts. Other items give opportunities for students to demonstrate these skills. The goal here is to check their understanding and fluency with equivalent fractions.

List three different fractions that are equivalent to $\frac{4}{5}$. Explain or show your reasoning.

Solution

Sample response: $\frac{8}{10}$, $\frac{40}{50}$, $\frac{80}{100}$. If each $\frac{1}{5}$ is divided into two equal pieces then they are tenths and there are 8 of them. If each $\frac{1}{5}$ is divided into ten equal pieces, they are fiftieths and there are 40 of them. If each $\frac{1}{50}$ is divided into two equal pieces, they are hundredths and there are 80 of them.

Any equivalent fraction is acceptable, for example $\frac{3 \times 4}{3 \times 5}$ or $\frac{12}{15}$, $\frac{4 \times 4}{4 \times 5}$ or $\frac{16}{20}$, $\frac{5 \times 4}{5 \times 5}$ or $\frac{20}{25}$.

Problem 5

Standards Alignments

Addressing 4.NF.A.1

Narrative

Students address a common misconception about fractions, namely, reasoning that focuses on the numerator without taking into account the meaning of the denominator. Students may draw many different pictures to help explain the equivalence, but the work of the unit supports tape diagrams and number line diagrams. They may also reason using words as in the provided solution.

Elena says that $\frac{3}{5}$ and $\frac{6}{10}$ are not equivalent because there are twice as many parts in $\frac{6}{10}$. Do you agree with Elena? Explain your reasoning.

Solution

No, $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions. There are twice as many parts in $\frac{6}{10}$, but each one is half as large.

Problem 6**Standards Alignments**

Addressing 4.NF.A.1, 4.NF.A.2

Narrative

Students list fractions in terms of increasing size. No method is suggested, but the denominators are large enough that finding a common denominator will not be efficient. The fractions have been selected to encourage other techniques. The benchmarks of $\frac{1}{2}$, 1, and 2 determine the order other than for $\frac{1}{4}$ and $\frac{3}{8}$, and that comparison can be done by finding a common denominator.

List these fractions from smallest to largest. Explain how you found the order.

$\frac{7}{4}$

$\frac{7}{12}$

$\frac{3}{8}$

$\frac{13}{6}$

$\frac{1}{4}$

Solution

$\frac{1}{4}, \frac{3}{8}, \frac{7}{12}, \frac{7}{4}, \frac{13}{6}$. Sample response: $\frac{13}{6}$ is the largest because it is greater than 2, and $\frac{7}{4}$ is less than 2 but greater than 1, so it comes next. $\frac{7}{12}$ is between $\frac{1}{2}$ and 1 so it is the next greatest, and $\frac{1}{4}$ is the same as $\frac{2}{8}$, so $\frac{3}{8}$ is greater than $\frac{1}{4}$.

Problem 7

Standards Alignments

Addressing 4.NF.A.1

Narrative

Students generate equivalent fractions given a fraction and the denominator of the equivalent fraction they are creating.

For each fraction, write an equivalent fraction with the given denominator.

- a. $\frac{1}{2} = \frac{\quad}{12}$
- b. $\frac{2}{3} = \frac{\quad}{24}$
- c. $\frac{6}{5} = \frac{\quad}{35}$
- d. $\frac{5}{7} = \frac{\quad}{28}$
- e. $\frac{7}{8} = \frac{\quad}{32}$

Solution

- a. $\frac{1}{2} = \frac{6}{12}$
- b. $\frac{2}{3} = \frac{16}{24}$
- c. $\frac{6}{5} = \frac{42}{35}$
- d. $\frac{5}{7} = \frac{20}{28}$
- e. $\frac{7}{8} = \frac{28}{32}$

Problem 8

Standards Alignments

Addressing 4.NF.A.2

Narrative

Students compare fractions in context. The first comparisons are with the benchmark fraction $\frac{1}{2}$ and students can make these comparisons by finding a common denominator or comparing the denominator to twice the numerator. Students have different ways they can compare $\frac{4}{10}$ and $\frac{5}{12}$. In an earlier item on the assessment students compared $\frac{3}{5}$ and $\frac{7}{12}$ and the ideas, methods, and numbers here are almost identical.

Noah and Lin drew different geometric designs on the same-size rectangular paper and colored the designs.

- $\frac{4}{10}$ of Noah's design is blue. How can you describe the size of the fraction?
- $\frac{5}{12}$ of Lin's design is blue.

Sketch an example of what Lin's design could look like.

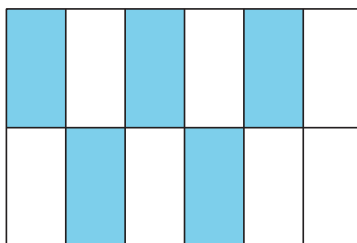


- Whose design has more blue, Noah's or Lin's? Explain or show your reasoning.

Solution

Sample responses:

- If Noah divided the design into 10 equal strips, then 4 of them would be blue. Less than half of the design is blue.
-



- Lin's design has more blue. Sample responses:

- I know that $\frac{4}{10}$ is $\frac{1}{10}$ less than $\frac{1}{2}$ and $\frac{5}{12}$ is $\frac{1}{12}$ less than $\frac{1}{2}$. Since tenths are bigger than twelfths that means $\frac{4}{10}$ is further away from $\frac{1}{2}$ so it is smaller.
- I checked that $\frac{4}{10} = \frac{4 \times 6}{10 \times 6}$, so Noah's design is $\frac{24}{60}$ blue. Then I checked that $\frac{5}{12} = \frac{5 \times 5}{5 \times 12}$, so Lin's design is $\frac{25}{60}$ blue. Since $\frac{25}{60}$ is greater than $\frac{24}{60}$, there is more blue in Lin's design.

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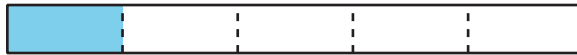
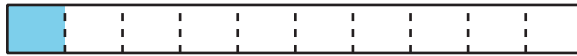
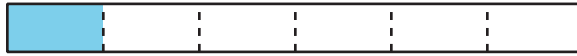
Lesson
Cool Downs

Lesson 1: Representations of Fractions (Part 1)

Cool Down: What Do the Diagrams Show?

Each full diagram represents 1.

1. What fraction does each shaded part represent?



2. Explain or show how you could use this diagram to represent sixths.

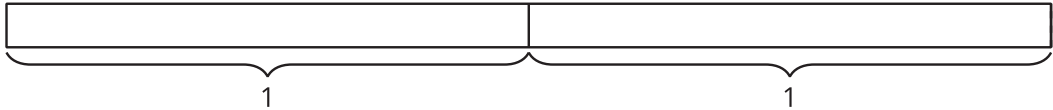


Lesson 2: Representations of Fractions (Part 2)

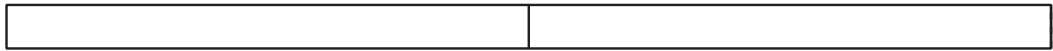
Cool Down: What Do the Diagrams Show?

Use a blank diagram to create a representation for each fraction. Both blank diagrams represent the same quantity.

1. $\frac{5}{8}$



2. $\frac{9}{8}$



Lesson 3: Same Denominator or Numerator

Cool Down: Sizing Up Fractions

In each pair of fractions, which one is greater? Explain or show your reasoning.

1. $\frac{7}{8}$ or $\frac{10}{8}$

2. $\frac{4}{10}$ or $\frac{4}{5}$

Lesson 4: Same Size, Related Sizes

Cool Down: Where on the Number Line?

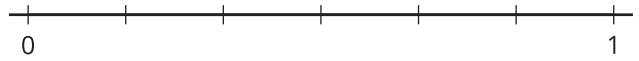
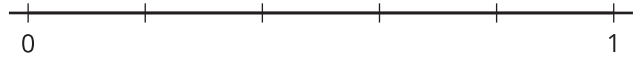
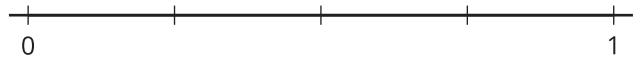
Locate and label each fraction on one of the number lines. Show your reasoning.

$\frac{3}{6}$

$\frac{2}{10}$

$\frac{6}{8}$

$\frac{4}{12}$



Lesson 5: Fractions on Number Lines

Cool Down: Two of the Same

Show $\frac{5}{6}$ on the number line. Be sure to include labels. Then, explain or show that the fraction $\frac{10}{12}$ is equivalent to $\frac{5}{6}$.



Lesson 6: Relate Fractions to Benchmarks

Cool Down: Greater Than or Less Than . . .?

For each question, explain or show your reasoning. Use a number line if it is helpful.

1. Is $\frac{6}{10}$ more or less than $\frac{1}{2}$?

2. Is $\frac{11}{12}$ more or less than 1?

Lesson 7: Equivalent Fractions

Cool Down: Two Equivalent Fractions

Name two fractions that are equivalent to $\frac{5}{3}$. Explain or show your reasoning.

Lesson 8: Equivalent Fractions on the Number Line

Cool Down: In Search of Equivalence

For each problem, explain or show your reasoning. Use a number line, if it helps.

1. Name a fraction that is equivalent to $\frac{9}{10}$.



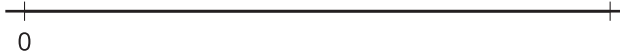
2. Is $\frac{8}{5}$ equivalent to $\frac{15}{10}$?



Lesson 9: Explain Equivalence

Cool Down: To Be or Not to Be (Equivalent)

1. Explain or show why this statement is true: $\frac{5}{4}$ is equivalent to $\frac{15}{12}$. Use a number line, if it helps.



2. Diego wrote $\frac{11}{5}$ and $\frac{55}{10}$ as equivalent fractions. Are those fractions equivalent? Explain or show how you know. Use a number line, if it helps.



Lesson 10: Use Multiples to Find Equivalent Fractions

Cool Down: Fractions of the Same Size

1. Find two fractions that are equivalent to $\frac{3}{8}$. Explain or show your reasoning.

2. Decide if each of the following fractions are equivalent to $\frac{9}{4}$.

a. $\frac{10}{8}$

b. $\frac{16}{10}$

c. $\frac{18}{8}$

d. $\frac{27}{12}$

Lesson 11: Use Factors to Find Equivalent Fractions

Cool Down: Find Three or More

Name at least 3 fractions that are equivalent to $\frac{20}{100}$. Explain or show your reasoning.

Lesson 12: Ways to Compare Fractions

Cool Down: Pick the Greater Fraction

In each pair of fractions, which fraction is greater? Explain or show your reasoning.

1. $\frac{5}{12}$ and $\frac{5}{8}$

2. $\frac{11}{10}$ and $\frac{18}{100}$

3. $\frac{6}{10}$ and $\frac{7}{12}$

Lesson 13: Use Equivalent Fractions to Compare

Cool Down: Make It True

Compare each pair of fractions. Use the symbols $<$, $=$, and $>$ to make each statement true. Explain or show your reasoning.

1. $\frac{15}{8}$ _____ $\frac{7}{4}$

2. $\frac{2}{5}$ _____ $\frac{30}{100}$

Lesson 14: Fraction Comparison Problems

Cool Down: Who Ran the Farthest?

Jada, Kiran, and Lin tried to run as far as possible before they had to stop and rest.

- Jada ran $\frac{3}{4}$ mile.
- Kiran ran $\frac{7}{12}$ mile.
- Lin ran $\frac{4}{6}$ mile.

Who ran the farthest before stopping? Explain or show your reasoning.

Lesson 15: Common Denominators to Compare

Cool Down: Which is Greater?

In each pair of fractions, which fraction is greater? Explain or show your reasoning.

1. $\frac{3}{10}$ or $\frac{2}{6}$

2. $\frac{99}{100}$ or $\frac{9}{10}$

Lesson 16: Compare and Order Fractions

Cool Down: All in Order

Put these fractions in order, from least to greatest. Show your reasoning.

$$\frac{5}{12}$$

$$\frac{8}{6}$$

$$\frac{4}{10}$$

$$\frac{7}{5}$$

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Instructional Masters

Instructional Masters for Fraction Equivalence and Comparison

address	title	students written per copy	on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade4.2.1.1.3	Fractions Galore	3	no	yes	no	no
Activity Grade4.2.9.2	How Do You Know	15	no	yes	no	no
Activity Grade4.2.16.1	Fraction Cards Grade 4	2	no	no	yes	no
Center	Fraction Cards Grade 4	2	no	no	yes	no
Center	Fraction Cards Grade 4	2	no	no	yes	no
Activity Grade4.2.6.2	Where Do They Belong	2	no	yes	no	no
Activity Grade4.2.1.1	Fraction Strips	2	yes	yes	no	no
Activity Grade4.2.16.1	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Get Your Numbers in Order Stage 3 and 4 Gameboard	2	yes	no	no	no
Center	Get Your Numbers in Order Stage 3 and 4 Gameboard	2	yes	no	no	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Mystery Number Stage 3 Gameboard	2	no	no	no	no

Center	Number Line Scoot Stage 3 Gameboard	2	no	no	no	no
Center	Number Line Scoot Stage 3 Directions	2	no	yes	no	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Mystery Number Stage 4 Gameboard	2	no	no	no	no
Center	Compare Stage 3 Multiplication Cards	2	no	yes	no	no
Center	How Close? Stage 6 Recording Sheet	1	yes	no	no	no

Fractions Galore

Fractions Galore

$$4 - \frac{1}{3}$$

Fractions Galore

$$1 - \frac{1}{4}$$

Fractions Galore

$$3 - \frac{1}{4}$$

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$$5 - \frac{1}{4}$$

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$$4 - \frac{1}{5}$$

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$$5 - \frac{1}{5}$$

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$$9 - \frac{1}{5}$$

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$$\frac{10}{5}$$

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$$\frac{8}{6}$$

Fractions Galore

$$\frac{12}{6}$$

Fractions Galore

$$\frac{12}{8}$$

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$$\frac{16}{8}$$

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$$\frac{1}{10}$$

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$$\frac{5}{10}$$

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$$\frac{18}{10}$$

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$$\frac{7}{12}$$

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$$\frac{16}{12}$$

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$$\frac{50}{100}$$

Fractions Galore

$$\frac{75}{100}$$

Fractions Galore

$$\frac{80}{100}$$

Fractions Galore

$$\frac{100}{100}$$

How Do You Know?

A

$$\frac{2}{10}$$

$$\frac{20}{100}$$

How Do You Know?

B

$$\frac{6}{4}$$

$$\frac{18}{12}$$

How Do You Know?

C

$$\frac{3}{5}$$

$$\frac{60}{100}$$

How Do You Know?

D

$$\frac{1}{4}$$

$$\frac{3}{12}$$

$$\frac{30}{100}$$

How Do You Know

How Do You Know?

E

$$\frac{15}{6}$$

$$\frac{7}{4}$$

$$\frac{30}{12}$$

How Do You Know?

F

$$\frac{7}{3}$$

$$\frac{21}{10}$$

$$\frac{28}{12}$$

Fraction Cards Grade 4

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$$\frac{3}{6}$$

Where Do They Belong

Where Do They Belong?

$$3 - 8$$

Where Do They Belong?

$$3 \frac{3}{12}$$

Where Do They Belong?

$$4 - 2$$

Where Do They Belong?

$$4 - 5$$

Where Do They Belong?

$$4 - 8$$

Where Do They Belong?

$$4 \frac{4}{12}$$

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$$5 - 6$$

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$$\frac{7}{10}$$

Where Do They Belong?

$$\frac{8}{4}$$

Where Do They Belong?

$$\frac{9}{10}$$

Where Do They Belong?

$$\frac{11}{12}$$

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Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

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Record any sets of cards that are challenging to compare:

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

$$\frac{2}{4}$$

Fraction Cards Grade 3

$$\frac{3}{4}$$

Fraction Cards Grade 3

$$\frac{4}{4}$$

Fraction Cards Grade 3

$$\frac{5}{4}$$

Fraction Cards Grade 3

$$\frac{1}{6}$$

Fraction Cards Grade 3

$$\frac{2}{6}$$

Fraction Cards Grade 3

$$\frac{3}{6}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{4}{6}$$

Fraction Cards Grade 3

$$\frac{5}{6}$$

Fraction Cards Grade 3

$$\frac{6}{6}$$

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$$\frac{7}{6}$$

Fraction Cards Grade 3

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Fraction Cards Grade 3

$$\frac{3}{3}$$

Fraction Cards Grade 3

$$\frac{6}{3}$$

Fraction Cards Grade 3

$$\frac{4}{2}$$

Fraction Cards Grade 3

$$\frac{16}{6}$$

Fraction Cards Grade 3

$$\frac{6}{2}$$

Fraction Cards Grade 3

$$\frac{8}{2}$$

Fraction Cards Grade 3

$$\frac{5}{3}$$

Fraction Cards Grade 3

$$\frac{13}{4}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

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$$\frac{4}{2}$$

Fraction Cards Grade 3

$$\frac{16}{6}$$

Fraction Cards Grade 3

$$\frac{6}{2}$$

Fraction Cards Grade 3

$$\frac{8}{2}$$

Fraction Cards Grade 3

$$\frac{5}{3}$$

Fraction Cards Grade 3

$$\frac{13}{4}$$

Mystery Number Stage 3 Gameboard

Directions:

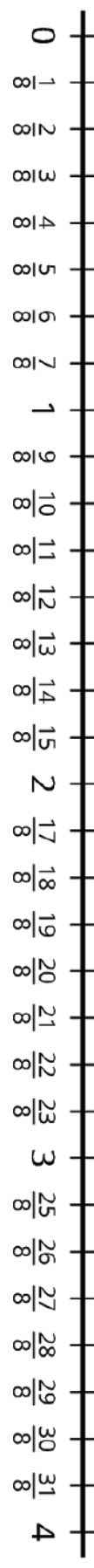
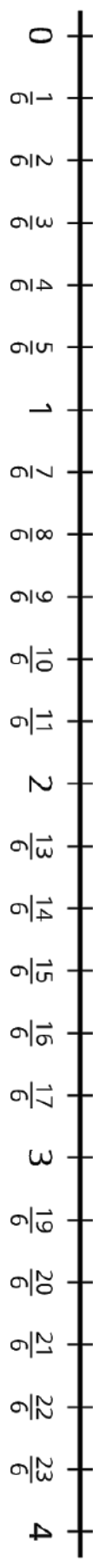
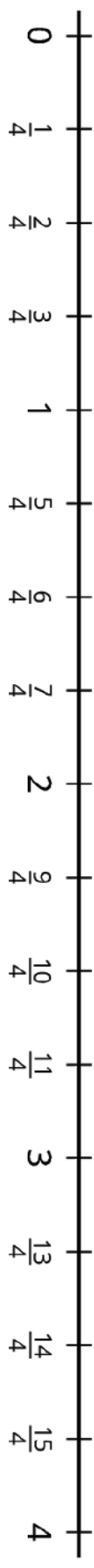
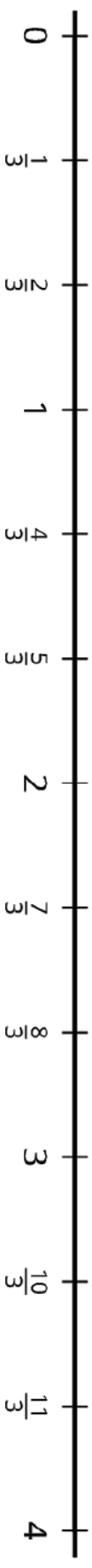
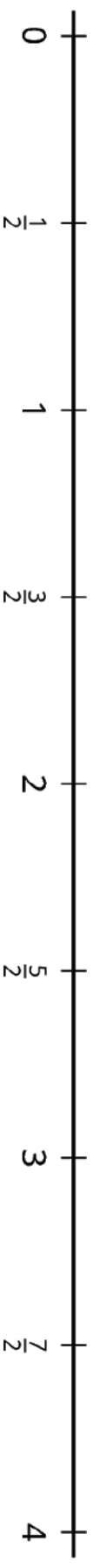
- Partner A:
 - Pick a number on the game board. Don't tell your partner!
 - Give your partner a clue about your mystery number. You can use the vocabulary below to help you give clues, or make up your own.
- Partner B:
 - Guess your partner's mystery number.
- If Partner B guesses the mystery number, switch roles.
- If Partner B does not guess the mystery number, Partner A gives another clue. Go back and forth guessing the number and giving clues until Partner B guesses the mystery number.

Vocabulary:

numerator, denominator, greater than, less than, equivalent, whole, odd, even

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{6}$	$\frac{5}{3}$
$\frac{2}{6}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{5}{6}$
$\frac{4}{6}$	$\frac{5}{2}$	$\frac{6}{6}$	$\frac{18}{6}$
$\frac{3}{3}$	$\frac{1}{3}$	$\frac{2}{2}$	$\frac{1}{4}$
$\frac{13}{4}$	$\frac{15}{6}$	$\frac{2}{3}$	$\frac{7}{4}$

Number Line Scoot Stage 3 Gameboard



Number Line Scoot Stage 3 Directions

Directions:

- Place a small cube on zero on each number line.
 - On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, 4, 6, or 8 as the denominator for your fraction.
 - Count aloud as you move a counter that distance on the appropriate number line.
 - Take turns rolling and moving one cube.
 - If a cube lands *exactly* on the last tick mark of a number line, that partner keeps the cube and puts a new one at 0.
 - The first player to collect 5 cubes wins.
-

Directions:

- Place a small cube on zero on each number line.
- On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, 4, 6, or 8 as the denominator for your fraction.
 - Count aloud as you move a counter that distance on the appropriate number line.
- Take turns rolling and moving one cube.
- If a cube lands *exactly* on the last tick mark of a number line, that partner keeps the cube and puts a new one at 0.
- The first player to collect 5 cubes wins.

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

$$\frac{2}{4}$$

Fraction Cards Grade 3

$$\frac{3}{4}$$

Fraction Cards Grade 3

$$\frac{4}{4}$$

Fraction Cards Grade 3

$$\frac{5}{4}$$

Fraction Cards Grade 3

$$\frac{1}{6}$$

Fraction Cards Grade 3

$$\frac{2}{6}$$

Fraction Cards Grade 3

$$\frac{3}{6}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{4}{6}$$

Fraction Cards Grade 3

$$\frac{5}{6}$$

Fraction Cards Grade 3

$$\frac{6}{6}$$

Fraction Cards Grade 3

$$\frac{7}{6}$$

Fraction Cards Grade 3

$$\frac{1}{2}$$

Fraction Cards Grade 3

$$\frac{2}{2}$$

Fraction Cards Grade 3

$$\frac{1}{3}$$

Fraction Cards Grade 3

$$\frac{2}{3}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{3}{3}$$

Fraction Cards Grade 3

$$\frac{6}{3}$$

Fraction Cards Grade 3

$$\frac{4}{2}$$

Fraction Cards Grade 3

$$\frac{16}{6}$$

Fraction Cards Grade 3

$$\frac{6}{2}$$

Fraction Cards Grade 3

$$\frac{8}{2}$$

Fraction Cards Grade 3

$$\frac{5}{3}$$

Fraction Cards Grade 3

$$\frac{13}{4}$$

Mystery Number Stage 4 Gameboard

Directions:

- Partner A:
 - Pick a number on the game board. Don't tell your partner!
 - Give your partner a clue about your mystery number. You can use the vocabulary below to help you give clues, or make up your own.
- Partner B:
 - Guess your partner's mystery number.
- If Partner B guesses the mystery number, switch roles.
- If Partner B does not guess the mystery number, Partner A gives another clue. Go back and forth guessing the number and giving clues until Partner B guesses the mystery number.

Vocabulary:

numerator, denominator, greater than 1, less than 1, equivalent, factor, multiple, prime, composite, odd, even

$\frac{1}{100}$	$\frac{3}{8}$	$\frac{105}{100}$	$\frac{132}{12}$
$\frac{2}{8}$	$\frac{5}{5}$	$\frac{42}{100}$	$\frac{15}{8}$
$\frac{16}{10}$	$\frac{100}{10}$	$\frac{16}{100}$	$\frac{18}{8}$
$\frac{11}{12}$	$\frac{4}{8}$	$\frac{2}{12}$	$\frac{45}{8}$
$\frac{13}{5}$	$\frac{15}{10}$	$\frac{60}{100}$	$\frac{9}{8}$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$12 \times 9$$

Compare Stage 3

$$12 \times 7$$

Compare Stage 3

$$13 \times 7$$

Compare Stage 3

$$14 \times 6$$

Compare Stage 3

$$15 \times 6$$

Compare Stage 3

$$10 \times 20$$

Compare Stage 3

$$21 \times 4$$

Compare Stage 3

$$19 \times 5$$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$18 \times 5$$

Compare Stage 3

$$17 \times 4$$

Compare Stage 3

$$16 \times 6$$

Compare Stage 3

$$14 \times 7$$

Compare Stage 3

$$31 \times 3$$

Compare Stage 3

$$20 \times 4$$

Compare Stage 3

$$8 \times 9$$

Compare Stage 3

$$9 \times 7$$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$12 \times 5$$

Compare Stage 3

$$13 \times 4$$

Compare Stage 3

$$15 \times 3$$

Compare Stage 3

$$9 \times 5$$

How Close? Stage 6 Recording Sheet

Directions:

- Each partner:
 - Take 6 cards.
 - Choose 4 cards to make a multiplication expression. You can multiply a one-digit number by a three-digit number or a two-digit number by a two-digit number.
 - Write an equation to show the product of the numbers you made.
 - Your score for each round is the difference between your product and 3,000.
- Take new cards so that you have 6 cards to start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.

round	multiplication equation	points for the round
1		
2		
3		
4		
5		
6		
7		
8		

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