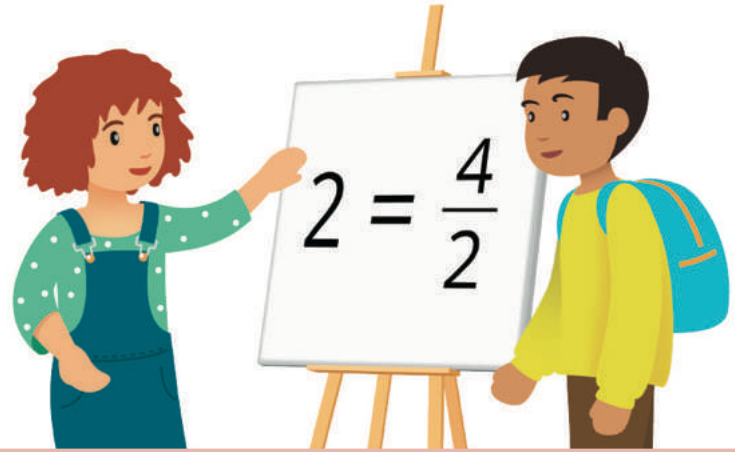




Fractions as Numbers



Teacher Guide



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Fractions as Numbers

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Fractions as Numbers
Teacher Guide
Core Knowledge Mathematics™

Unit 5: Fractions as Numbers

At a Glance

Unit 5 is estimated to be completed in 19-20 days including 2 days for assessment.

This unit is divided into four sections including 17 lessons and 1 optional lesson.

- Section A—Introduction to Fractions (Lessons 1-4)
- Section B—Fractions on the Number Line (Lessons 5-9)
- Section C—Equivalent Fractions (Lessons 10-13)
- Section D—Fraction Comparisons (Lessons 14-18)

On pages 9-10 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses five student centers.

- Mystery Number
- Number Line Scoot
- Secret Fraction
- Rolling for Fractions
- Five in a Row: Multiplication

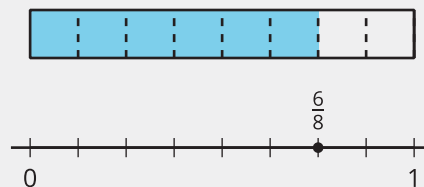
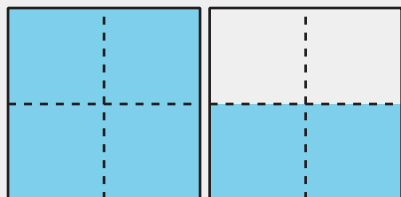
Unit 5: Fractions as Numbers

Unit Learning Goals

- Students develop an understanding of fractions as numbers and of fraction equivalence by representing fractions on diagrams and number lines, generating equivalent fractions, and comparing fractions.

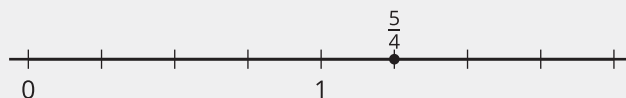
In this unit, students make sense of fractions as numbers, using various diagrams to represent and reason about fractions, compare their size, and relate them to whole numbers. The denominators of the fractions explored here are limited to 2, 3, 4, 6, and 8.

In grade 2, students partitioned circles and rectangles into equal parts and used the language “halves,” “thirds,” and “fourths.” Students begin this unit in a similar way, by reasoning about the size of shaded parts in shapes. Next, they create fraction strips by folding strips of paper into equal parts and later represent the strips as tape diagrams.



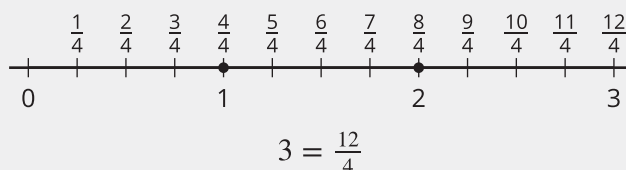
Using fraction strips and tape diagrams to represent fractions prepare students to think about fractions more abstractly: as lengths and locations on the number line. This work builds on students’ prior experience with representing whole numbers on the number line.

In each representation, students take care to identify 1 whole. This helps them reason about the size of the parts and whether a fraction is less or greater than 1. (Fractions greater than 1 are not treated as special cases.)



Students then use these representations to learn about equivalent fractions and to compare fractions.

They see that fractions are equivalent if they are the same size or at the same location on the number line, and that some fractions are the same size as whole numbers.



Later in the unit, students compare fractions with the same denominator and those with the same numerator. They recognize that as the numerator gets larger, more parts are being counted, and as the denominator gets larger, the size of each part in a whole gets smaller.

Section A: Introduction to Fractions

Standards Alignments

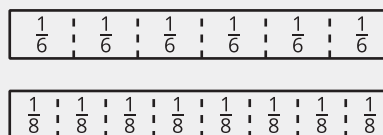
| | |
|------------------|-----------------------------|
| Building On | 2.G.A.3 |
| Addressing | 3.G.A.2, 3.NF.A.1, 3.OA.C.7 |
| Building Towards | 3.G.A.2, 3.NF.A.1, 3.NF.A.2 |

Section Learning Goals

- Understand that fractions are built from unit fractions such that a fraction $\frac{a}{b}$ is the quantity formed by a parts of size $\frac{1}{b}$.
- Understand that unit fractions are formed by partitioning shapes into equal parts.

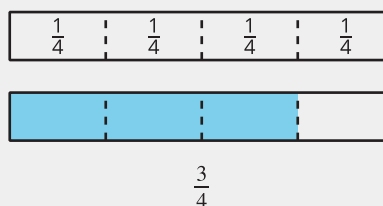
In this section, students use shaded diagrams and fraction strips to learn about fractions, building on their prior knowledge of halves, thirds, and fourths.

Students partition rectangles into 6 or 8 equal parts and describe each part as “a sixth” or “an eighth” and write the notation $\frac{1}{6}$ or $\frac{1}{8}$.



They learn that the notation $\frac{1}{b}$ refers to a unit fraction, or the size of each part if the whole is partitioned into b parts. Working with fraction strips allows students to see non-unit fractions as being composed of unit fractions, so a parts of unit fractions of size $\frac{1}{b}$ gives a non-unit fraction $\frac{a}{b}$.

For example, putting together 3 pieces of fourths or 3 parts of the unit fraction $\frac{1}{4}$ gives $\frac{3}{4}$.



As students develop their understanding, they make connections between the meaning, language, and notation of fractions—between what fractions represent and how they are expressed in words and in numbers. (The terminology “numerator” and “denominator” are not introduced until later so students can focus on meaning making.)

 PLC: Lesson 3, Activity 1, Write and Read Fractions

Suggested Centers

- Mystery Number (1–4), Stage 2: Three-digit Numbers (Supporting)
- Number Line Scoot (2–3), Stage 1: Twos, Fives, and Tens (Supporting)
- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

Section B: Fractions on the Number Line

Standards Alignments

Addressing 3.NF.A.2, 3.NF.A.2.a, 3.NF.A.2.b, 3.NF.A.3.c, 3.OA.C.7

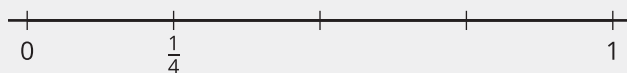
Building Towards 3.NF.A.2

Section Learning Goals

- Understand a fraction as a number and represent fractions on the number line.

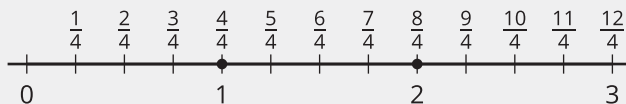
In this section, students reason about fractions on the number line. This work relies on two prior experiences: locating whole numbers on the number line, and partitioning a whole into equal parts.

Students have previously learned that numbers can be represented as distances from 0 on the number line. Here, students learn that the same is true about fractions. Students begin by partitioning the interval between 0 and 1 into equal parts, just as they had done with fraction strips and tape diagrams.



They then mark the first tick mark with a unit fraction $\frac{1}{b}$ and locate non-unit fractions by counting lengths the size of $\frac{1}{b}$. They reason that a tick mark that is a intervals away represents a fraction $\frac{a}{b}$. The terms “numerator” and “denominator” are introduced here.

Students also notice that certain fractions are in the same location as whole numbers on the number line. For example, $\frac{4}{4}$ and $\frac{8}{4}$ are at the same location as 1 and 2, respectively. This observation helps students understand that whole numbers can be represented as fractions.



PLC: Lesson 7, Activity 2, Fractions on the Number Line

Suggested Centers

- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)

Section C: Equivalent Fractions

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b, 3.NF.A.3.c, 3.OA.B.5

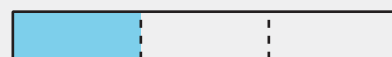
Building Towards 3.NF.A.2, 3.NF.A.3, 3.OA.C.7

Section Learning Goals

- Explain equivalence of fractions in special cases and express whole numbers as fractions and fractions as whole numbers.

In this section, students learn that equivalent fractions are fractions that are the same size.

They first identify equivalent fractions by noticing parts that are of equal length on fraction strips and tape diagrams.



For example, the shaded third in the first diagram is the same size as the two shaded sixths in the second diagram, so $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent.



Students see that they can show equivalence by decomposing each fractional part into smaller parts, or by grouping fractional parts to make larger parts.

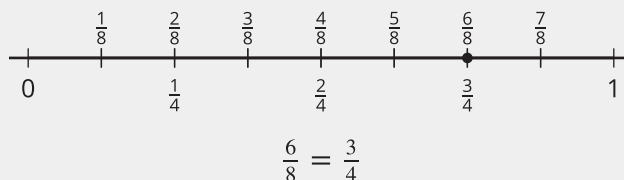
Suppose we want to show that the shaded parts of this diagram represent both $\frac{6}{8}$ and $\frac{3}{4}$.



If we group 2 eighths together, we have 4 equal groups, each being a fourth. We can see that the 6 shaded eighths and 3 shaded fourths are the same size.



Later, students learn that equivalent fractions are the same distance away from 0 and are therefore located at the same point on the number line. They write equations to express equivalence, including for fractions that are equivalent to whole numbers.



 PLC: Lesson 12, Activity 2, Locate and Pair

Suggested Centers

- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)

Section D: Fraction Comparisons

Standards Alignments

| | |
|------------------|--------------------------------------------|
| Building On | 3.NF.A.1, 3.NF.A.2.a |
| Addressing | 3.NF.A.2, 3.NF.A.3, 3.NF.A.3.c, 3.NF.A.3.d |
| Building Towards | 3.MD.B.4 |

Section Learning Goals

- Compare two fractions with the same numerator or denominator, record the results with the symbols $>$, $=$, or $<$, and justify the conclusions.

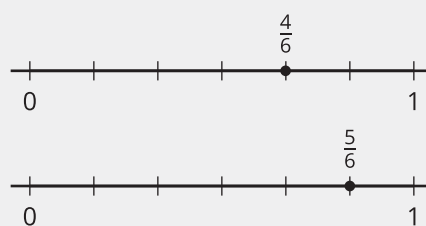
In this section, students compare fractions using any representation or reasoning strategies that make sense to them. They learn that comparisons are only valid if the fractions being compared refer to the same whole.

Students begin by deciding if two fractions are equivalent. They use diagrams, number lines, and the meaning of fractions to support their reasoning.

Next, students compare fractions with the same denominator. They see that these fractions are composed of parts of the same size, so to compare them involves looking at the numerators to see which fraction has more parts.

For example, there are 4 sixths in $\frac{4}{6}$ and 5 sixths in $\frac{5}{6}$, so $\frac{4}{6}$ is less than $\frac{5}{6}$. On the number line, $\frac{4}{6}$ would be to the left of $\frac{5}{6}$, closer to 0.

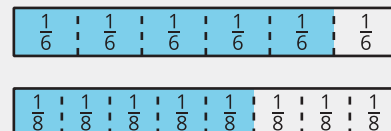
$$\frac{4}{6} < \frac{5}{6}$$



In contrast, fractions with the same numerator have the same number of parts, so to compare them involves looking at the denominators to see which fraction is made up of larger parts.

For instance, 5 sixths is greater than 5 eighths because a sixth is larger than an eighth.

$$\frac{5}{6} > \frac{5}{8}$$



The work here reinforces the idea that as the denominator increases, the size of each part gets smaller.

 PLC: Lesson 17, Activity 1, Comparison Problems

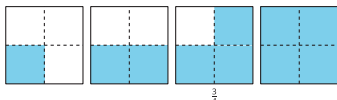



Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Throughout the Unit

The progression of warm-ups in the unit mirrors the development of fraction concepts in the unit. Students work with unit fractions, then learn that non-unit fractions are made of unit fractions. Students learn how to locate fractions on a number line. They identify and generate equivalent fractions before comparing fractions with the same numerator or denominator. Later warm-ups of the unit prepare students for work with fractional lengths in the next unit.

Here is a sampling of the warm-ups in the unit.

| lesson 3 | lesson 5 | lesson 15 | lesson 17 |
|-------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Notice and Wonder</p>  | <p>Notice and Wonder</p>  | <p>Notice and Wonder</p>  | <p>Estimation Exploration</p>  <p>What is the length of this ladybug?</p> |

Materials Needed

| LESSON | GATHER | COPY |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A.1 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Card Sort: Partitions (groups of 2) • Fold and Name (groups of 4) |
| A.2 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Partition the Strips (groups of 2) |
| A.3 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • Fraction Match Part 1 (groups of 2) • Fraction Match Part 2 (groups of 4) |
| A.4 | <ul style="list-style-type: none"> • Colored pencils • Folders • Materials for creating a visual display | <ul style="list-style-type: none"> • Secret Fractions Stage 1 Cards (groups of 2) • Secret Fractions Stage 1 Gameboard (groups of 2) |
| B.5 | <ul style="list-style-type: none"> • Scissors | <ul style="list-style-type: none"> • Card Sort: Number Lines (groups of 2) • Fold and Label Number Lines (groups of 1) |
| B.6 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| B.7 | <ul style="list-style-type: none"> • Base-ten blocks • Number cubes | <ul style="list-style-type: none"> • Number Line Scoot Stage 2 Directions (groups of 2) • Number Line Scoot Stage 2 Gameboard (groups of 2) |
| B.8 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| B.9 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| C.10 | <ul style="list-style-type: none"> • Materials from a previous lesson | <ul style="list-style-type: none"> • none |
| C.11 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |
| C.12 | <ul style="list-style-type: none"> • Number cubes | <ul style="list-style-type: none"> • none |
| C.13 | <ul style="list-style-type: none"> • none | <ul style="list-style-type: none"> • none |

| | | |
|------|-------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| D.14 | <ul style="list-style-type: none">● Materials for creating a visual display | <ul style="list-style-type: none">● none |
| D.15 | <ul style="list-style-type: none">● Colored pencils● Paper clips | <ul style="list-style-type: none">● Spin to Win Recording Sheet (groups of 2)● Spin to Win Spinner (groups of 2) |
| D.16 | <ul style="list-style-type: none">● none | <ul style="list-style-type: none">● none |
| D.17 | <ul style="list-style-type: none">● none | <ul style="list-style-type: none">● none |
| D.18 | <ul style="list-style-type: none">● Paper● Rulers or straightedges | <ul style="list-style-type: none">● none |

Center: Mystery Number (1–4)

Stage 2: Three-digit Numbers

Lessons

- Grade3.5.A1 (supporting)
- Grade3.5.A2 (supporting)
- Grade3.5.A3 (supporting)

Stage Narrative

Students pick three cards and make a mystery three-digit number. Students give clues based on the sentence starters.

Standards Alignments

Addressing 2.NBT.A

Materials to Gather

Number cards 0–10

Materials to Copy

Mystery Number Stage 2 Directions (groups of 2)

Stage 3: Fractions with Denominators 2, 3, 4, 6

Lessons

- Grade3.5.A4 (addressing)
- Grade3.5.B5 (addressing)
- Grade3.5.B6 (addressing)

Stage Narrative

Students choose a mystery fraction (with a denominator of 2, 3, 4, or 6) from the gameboard. Students give clues based on the given vocabulary.

Standards Alignments

Addressing 3.NF.A

Materials to Copy

Mystery Number Stage 3 Gameboard (groups of 2)

Stages used in Grade 2

Stage 1

Supporting

- Grade2.5.A

Stage 2

Addressing

- Grade2.5.A
- Grade2.5.B

Supporting

- Grade2.7.A

Center: Number Line Scoot (2–3)

Stage 1: Twos, Fives, and Tens

Lessons

- Grade3.5.A1 (supporting)
- Grade3.5.A2 (supporting)
- Grade3.5.A3 (supporting)

Stage Narrative

Students take turns spinning a spinner and moving their cube that interval on one of the shared number lines. Students may use their whole spin on one number line or split it between multiple number lines. Each time a cube lands exactly on the last tick mark of one of the number lines, the player who moved it keeps the cube and puts a new cube on zero on that number line. The first player to collect five cubes wins.

Standards Alignments

Addressing 2.MD.B.6, 2.NBT.A.2

Materials to Gather

Centimeter cubes, Paper clips

Materials to Copy

Number Line Scoot Stage 1 Directions (groups of 2), Number Line Scoot Stage 1 Gameboard (groups of 2), Number Line Scoot Stage 1 Spinner (groups of 2)

Additional Information

Each group of 2 needs 12 centimeter cubes.

Stage 2: Halves, Thirds and Fourths

Lessons

- Grade3.5.A4 (addressing)
- Grade3.5.B5 (addressing)
- Grade3.5.B6 (addressing)
- Grade3.5.B7 (addressing)
- Grade3.5.B8 (addressing)

Stage Narrative

Students take turns rolling a number cube and using the number as a numerator in a fraction with a denominator of 2, 3, or 4. Students move their centimeter cube that interval on one of the shared number lines. Each time a cube lands exactly on the last tick mark of one of the number lines, the player who moved it keeps the cube and puts a new cube on zero on that number line. The first player to collect five cubes wins.

Standards Alignments

Addressing 3.NF.A.2.b

Materials to Gather

Centimeter cubes, Number cubes

Materials to Copy

Number Line Scoot Stage 2 Directions (groups of 2), Number Line Scoot Stage 2 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 12 centimeter cubes.

Stage 3: Halves, Thirds, Fourths, Sixths and Eighths

Lessons

- Grade3.5.B9 (addressing)
- Grade3.5.C10 (addressing)
- Grade3.5.C11 (addressing)
- Grade3.5.C12 (addressing)
- Grade3.5.C13 (addressing)
- Grade3.5.D14 (addressing)
- Grade3.5.D15 (addressing)
- Grade3.5.D16 (addressing)
- Grade3.5.D17 (addressing)
- Grade3.5.D18 (addressing)

Stage Narrative

Students take turns rolling a number cube and using the number as a numerator in a fraction with a denominator of 2, 3, 4, 6, or 8. Students move their centimeter cube that interval on one of the shared number lines. Each time a cube lands exactly on the last tick mark of one of the number lines, the player who moved it keeps the cube and puts a new cube on zero on that number line. The first player to collect five cubes wins.

Standards Alignments

Addressing 3.NF.A.2.b

Materials to Gather

Centimeter cubes, Number cubes

Materials to Copy

Number Line Scoot Stage 3 Directions (groups of 2), Number Line Scoot Stage 3 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 12 centimeter cubes.

Stages used in Grade 2**Stage 1****Addressing**

- Grade2.4.A
- Grade2.4.B

Supporting

- Grade2.7.A

Center: Secret Fraction (3)

Stage 1: Building Non-Unit Fractions

Lessons

- Grade3.5.B7 (addressing)
- Grade3.5.B8 (addressing)
- Grade3.5.B9 (addressing)
- Grade3.5.C10 (addressing)
- Grade3.5.C11 (addressing)
- Grade3.5.C12 (addressing)

Stage Narrative

Students take turns trying to build secret fractions. On each turn students can choose to

- Pick up one unit fraction card.
- Trade both of your secret fractions for two new secret fractions from the stack.

Once students have enough unit fractions to make their secret fraction, they fill in the secret fraction on the gameboard.

Standards Alignments

Addressing 3.NF.A.1

Materials to Gather

Folders

Materials to Copy

Secret Fractions Stage 1 Cards (groups of 2), Secret Fractions Stage 1 Gameboard (groups of 2)

Center: Rolling for Fractions (3–5)

Stage 1: Equivalent Fractions

Lessons

- Grade3.5.C13 (addressing)
- Grade3.5.D14 (addressing)
- Grade3.5.D15 (addressing)
- Grade3.5.D16 (addressing)
- Grade3.5.D17 (addressing)
- Grade3.5.D18 (addressing)

Stage Narrative

One player rolls 6 number cubes and tries to use 4 of them to fill in a statement with 2 equivalent fractions. If the player cannot make a true statement, they can re-roll as many of the cubes as they like. Each player may re-roll twice. If the student can fill in a statement with 2 equivalent fractions, they get a point for the round. Students take turns for 6 rounds and the player with the most points at the end of the game wins.

Standards Alignments

Addressing 3.NF.A.3.b

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 1 Recording Sheet
(groups of 1)

Additional Information

Each group of 2 needs 6 number cubes.

Center: Five in a Row: Multiplication (3–5)

Stage 2: Factors 1–9

Lessons

- Grade3.5.D14 (supporting)
- Grade3.5.D15 (supporting)
- Grade3.5.D16 (supporting)
- Grade3.5.D17 (supporting)
- Grade3.5.D18 (supporting)

Stage Narrative

Students multiply using factors of 1–9. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments

Addressing 3.OA.C.7

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Multiplication and Division Stage 2
Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 two-color counters and 2 paper clips.

Section A: Introduction to Fractions

Lesson 1: Name the Parts

Standards Alignments

| | |
|------------------|-------------------|
| Building On | 2.G.A.3 |
| Addressing | 3.G.A.2, 3.NF.A.1 |
| Building Towards | 3.G.A.2 |

Teacher-facing Learning Goals

- Partition shapes into 2, 3, 4, 6, or 8 parts with equal area and name those parts as halves, thirds, fourths, sixths, and eighths.
- Recognize that equal-size parts in a shape can be named with numbers called fractions.

Student-facing Learning Goals

- Let's name parts of a whole.

Lesson Purpose

The purpose of this lesson is for students to be introduced to **fractions** as numbers we write to describe the parts of a whole that has been partitioned into equal parts.

In previous grades, students partitioned circles and rectangles into two, three, or four equal pieces and described the pieces as “halves,” “thirds,” and “fourths.” They used the more concrete term “pieces.”

In this lesson, students extend this understanding to partition rectangles into six or eight equal parts and describe the parts as sixths or eighths. The term “parts” is used in these materials moving forward, but students recognize that “pieces” and “parts” are interchangeable and can use either one.

In the lesson synthesis, students learn the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ as the numbers used to represent the parts described as one half, one third, one fourth, one sixth, and one eighth.

Access for:



Students with Disabilities

- Action and Expression (Activity 2)



English Learners

- MLR2 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Card Sort: Partitions (groups of 2): Activity 1
- Fold and Name (groups of 4): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

In grade 2, students learned to partition rectangles and were introduced to halves, thirds, and fourths. How did they leverage their prior experiences as they were introduced to fractions in this lesson?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Partition a Rectangle

Standards Alignments

Addressing 3.G.A.2

Student-facing Task Statement

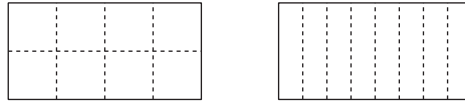
Partition the rectangle into eighths.



Student Responses

Any drawing that shows 8 equal parts is acceptable.

Sample responses:



Begin Lesson

Warm-up

🕒 10 min

Which One Doesn't Belong: Shapes with Parts

Standards Alignments

Building On 2.G.A.3

Building Towards 3.G.A.2

This warm-up prompts students to compare four shapes that have been partitioned and examine the features of the shapes and the partitions. The observations here prepare students to explore fractions later in the lesson and enable the teacher to hear how students describe the features that they see. During the synthesis, ask students to explain the meaning of any terminology they use, such as partition, whole, parts, pieces, equal, and halves.

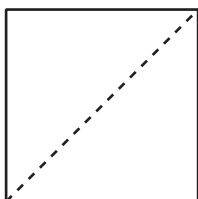
Instructional Routines

Which One Doesn't Belong?

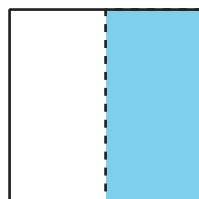
Student-facing Task Statement

Which one doesn't belong?

A



B

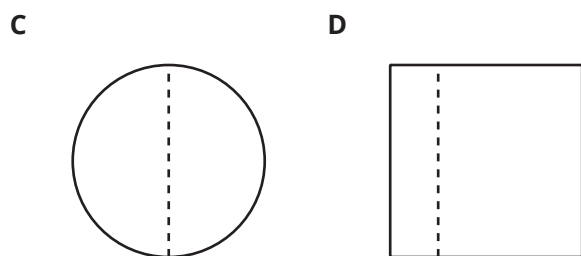


Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."



Student Responses

Sample responses:

- A is the only one that hasn't been split vertically, it's split diagonally.
- B is the only one that is not unshaded.
- C is the only one that isn't a square and doesn't have corners.
- D is the only one that has pieces or parts that aren't the same size.

- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- “Why can’t we say that D is split into halves?” (The pieces or parts aren’t the same size. The pieces have to be equal.)
- “Another word we can use to say something was split into pieces or parts is partition. Partition means to split into parts.”

Activity 1

🕒 15 min

Card Sort: Partitions

Standards Alignments

Building On 2.G.A.3

Building Towards 3.G.A.2

The purpose of this activity is for students to revisit ideas about how to partition shapes into halves, thirds, and fourths. Students sort a set of shapes into categories based on their shared attributes. Monitor for students who distinguish shapes that have been partitioned into equal-size parts and shapes that have not. This distinction will be used to review what it means for a part of a shape to be a half, a third, or a fourth.

Sorting the shapes gives students an opportunity to identify important common characteristics or structures, in this case the number and size of the parts (MP7). When students specify that halves, thirds, and fourths of a shape need to be equal in size, they are attending to precision (MP6).

Students will use the cards again during the lesson synthesis.

Access for English Learners

MLR2 Collect and Display. Collect the language students use while sorting the shapes. Display words and phrases such as: partition, split, parts, equal parts, equal-sized parts, halves, thirds, fourths, whole, and so on. During the synthesis, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Instructional Routines

Card Sort

Materials to Copy

Card Sort: Partitions (groups of 2)

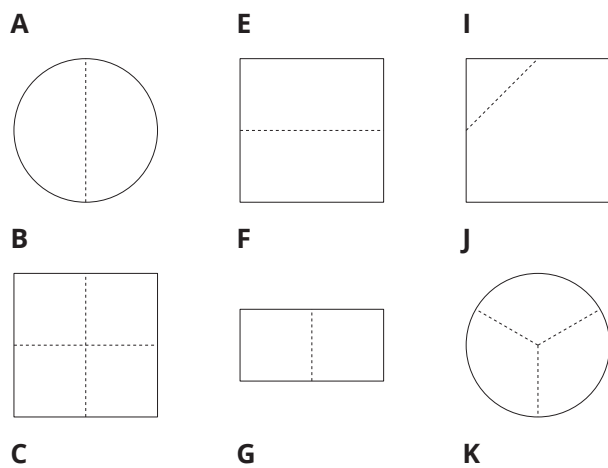
Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards that show some shapes that are partitioned.

Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.

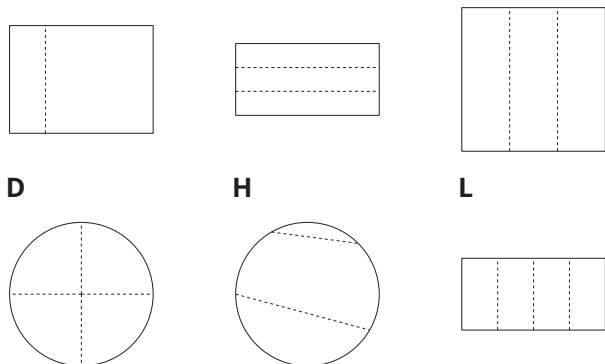


Launch

- Groups of 2
- Give one set of pre-cut cards to each group of students.
- “Take a minute to look at the cards. What do you notice? What do you wonder?”
(Students may notice: There are different shapes. The shapes are split or partitioned. Students may wonder: Why are they partitioned into different numbers of pieces or parts? Why are some of the pieces or parts equal and some are not?)
- 1 minute: quiet think time
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Activity

- “Work with your partner to sort the shapes



Student Responses

Sample responses:

- 2 parts: A, E, F, C, and I
3 parts: G, H, J, and K
4 parts: B, D, and L
- Equal parts: A, B, D, E, F, G, J, K, and L
Non-equal parts: C, H, and I

on the cards into categories. Be prepared to explain your categories and why the shapes in each category belong together.”

- 8 minutes: partner work time
- Monitor for groups that sort shapes based on whether they are partitioned into equal parts.

Synthesis

- Select groups to share their categories and the shapes in the categories.
- Showcase as many different types of categories as time allows, but ensure that one set of categories distinguishes between shapes that were partitioned into equal parts and shapes that were not.
- Attend to the language that students use to describe their categories and shapes, giving them opportunities to describe more precisely how shapes were partitioned.
- Highlight the use of terms such as halves, thirds, fourths, and equal-sized pieces or parts.

Activity 2

🕒 20 min

Fold and Name

Standards Alignments

Addressing 3.G.A.2, 3.NF.A.1

The purpose of this activity is for students to partition rectangles into thirds, sixths, fourths, and eighths before learning the name of sixths and eighths. Students do so by folding rectangular strips of paper into equal-sized parts. While folding, students may notice that thirds can be further partitioned to make sixths and that fourths can be further partitioned to make eighths,

which will be explored more in a future lesson. The focus of the synthesis should be on naming sixths and eighths, as these are new terms for students.

Students will use the partitioned rectangles during the lesson synthesis.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to pre-formatted papers that will be used to fold into rectangles. For example, papers would have dotted lines showing students where to fold for each rectangle.

Supports accessibility for: Social-Emotional Functioning, Visual-Spatial Processing

Materials to Copy

Fold and Name (groups of 4)

Required Preparation

- Each student needs 4 copies of the rectangle from the Instructional master.
- Have extra rectangles available for students who need more than one try to fold the rectangles into equal parts.
- Create poster for synthesis:

| number of equal parts | name of each part |
|-----------------------|-------------------|
| 2 | half |
| 3 | third |
| 4 | fourth |
| 6 | |
| 8 | |

Student-facing Task Statement

Fold each rectangle your teacher gives you into 3, 6, 4, or 8 equal parts. Draw lines where you folded to partition the rectangles. Be prepared to share how you folded your shapes.

Launch

- Groups of 2
- “We just looked at some shapes that were partitioned. Now you’re going to partition some rectangles into equal parts.”
- Give each student 4 rectangles.

Student Responses

Sample response:

- Students fold the rectangle in half, then fold it in half again to make four equal parts. Students fold a rectangle into three equal parts, then fold the thirds in half to make six equal parts.

Activity

- “We are going to fold to partition these rectangles. Fold each rectangle into 3, 6, 4, or 8 equal parts. Draw lines where you folded to partition the rectangles. Be prepared to share how you folded your rectangles.”
- 3–5 minutes: independent work time
- “Now, share how you folded your shapes with your partner.”
- 2–3 minutes: partner discussion
- Monitor for students who used partitioning into 3 or 4 equal parts to partition into 6 or 8 equal parts to highlight during synthesis.

Synthesis

- Select previously identified students to share how they folded the rectangles into 6 and 8 equal parts.
- Consider asking:
 - “How did you use the partitions of ____ equal parts to partition into ____ equal parts?”
 - “Does anyone want to add an observation to the way ____ partitioned?”
- Display poster:

| number of equal parts | name of each part |
|-----------------------|-------------------|
| 2 | half |
| 3 | third |
| 4 | fourth |
| 6 | |
| 8 | |

- “When we partition a shape into 6 equal parts, each part is called a ‘sixth.’ When we partition a shape into 8 equal parts, each part is called an ‘eighth.’”
- Add “sixth” and “eighth” to the chart and keep displayed.

Lesson Synthesis

🕒 10 min

"In the past, we've used the term 'a half' to refer to each part when one whole shape is partitioned into 2 equal parts. We've said 'a third' when there are 3 equal parts, and 'a fourth' when there are 4 equal parts."

"Today, we learned to use 'a sixth' to refer to each part when a whole shape is partitioned into 6 equal parts and 'an eighth' when it is partitioned into 8 equal parts."

"In addition to using words to describe these equal parts, we can also use numbers."

Write each fraction as it is named:

"One half can be written as the number $\frac{1}{2}$."

"One third can be written as the number $\frac{1}{3}$."

"One fourth can be written as the number $\frac{1}{4}$."

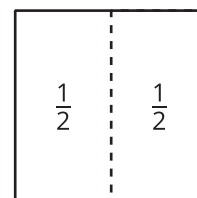
"How would we write one sixth and one eighth as numbers?" ($\frac{1}{6}$ and $\frac{1}{8}$)

"The numbers we use to describe the parts of a whole that has been partitioned into equal parts are called **fractions**. Each fraction has two parts separated by a bar."

"What do you think the part below the bar represents?" (the number of equal parts the whole has been partitioned into)

"What about the 1 above the bar?" (the one in "one half," "one third," and so on)

Display a square partitioned into 2 equal parts with each part labeled with $\frac{1}{2}$, such as:



"We can label the equal parts in a shape with fractions. If this square is the whole shape or 1, each part is one half or $\frac{1}{2}$."

"Find all the cards from the first activity that show a shape partitioned into 2 equal parts. Let's label each half with the fraction $\frac{1}{2}$."

"Let's label the parts in each of your rectangles with fractions."

Suggested Centers

- Mystery Number (1–4), Stage 2: Three-digit Numbers (Supporting)
- Number Line Scoot (2–3), Stage 1: Twos, Fives, and Tens (Supporting)

Complete Cool-Down

Response to Student Thinking

Students partition the rectangle into equal parts, but not eight equal parts.

The work in this lesson builds from the geometry concepts developed in a prior unit.

Next Day Support

- During the launch of the next day's activity, have students discuss the meaning of “sixths” and “eighths.”

Prior Unit Support

Grade 2, Unit 6, Section B: Halves, Thirds, and Fourths

Lesson 2: Name Parts as Fractions

Standards Alignments

| | |
|------------------|-------------------|
| Building On | 2.G.A.3 |
| Addressing | 3.G.A.2, 3.NF.A.1 |
| Building Towards | 3.NF.A.1 |

Teacher-facing Learning Goals

- Express the area of each part as a unit fraction of the whole.
- Partition shapes into halves, thirds, fourths, sixths, and eighths.

Student-facing Learning Goals

- Let's use fractions to describe parts.

Lesson Purpose

The purpose of this lesson is for students to partition shapes into equal parts and express each equal-size part as a unit fraction.

Previously, students partitioned rectangles that each represented 1 into fractional parts by folding. They now draw lines to partition a shape and use the fraction notation they learned to label each part as a unit fraction and describe a shaded part as a unit fraction. This lesson is the first time that students work with fraction strips, which will be used multiple times in the unit.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Partition the Strips (groups of 2): Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What student strategies surprised you in today's lesson? How will you build on those strategies as students develop ideas about fractions?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

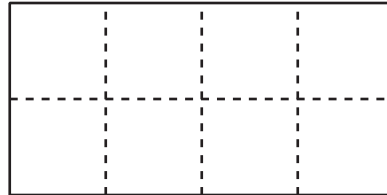
Label the Parts

Standards Alignments

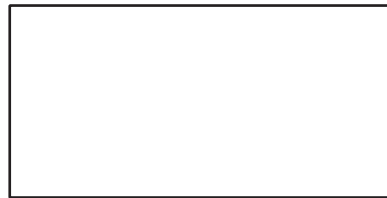
Addressing 3.G.A.2, 3.NF.A.1

Student-facing Task Statement

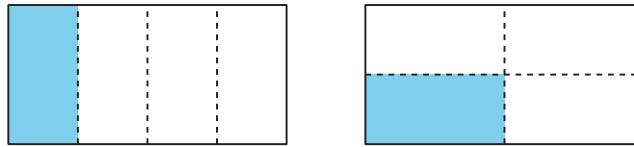
1. Label each part with the correct fraction.



2. Partition and shade the rectangle to show $\frac{1}{4}$.

**Student Responses**

1. Student labels each part with $\frac{1}{8}$.
2. Any drawing that shows 4 equal parts and 1 shaded part is acceptable. Sample responses:



Begin Lesson

Warm-up

🕒 10 min

Which One Doesn't Belong: Shaded Parts

Standards Alignments

Building On 2.G.A.3

Building Towards 3.NF.A.1

This warm-up prompts students to compare four rectangles that have been partitioned and partially shaded. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about the characteristics of the items and the quantities they represent. During the synthesis, ask students to explain the meaning of any terminology they use, such as partition, equal parts, halves, and thirds.

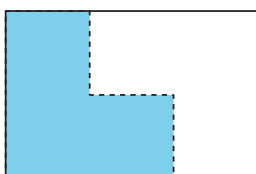
Instructional Routines

Which One Doesn't Belong?

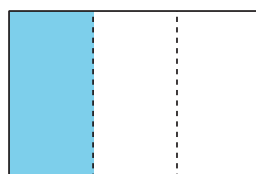
Student-facing Task Statement

Which one doesn't belong?

A

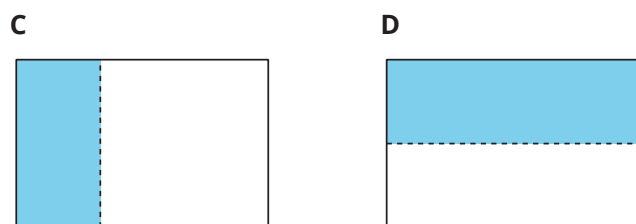


B



Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time



Student Responses

Sample responses:

- A is the only one that isn't partitioned into rectangles. It's the only one where the partition isn't one straight cut.
- B is the only one that isn't partitioned into 2 parts.
- C is the only one that doesn't have equal parts.
- D is the only one that doesn't have a vertical part in the partition.

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "Can we label the parts with fractions? Why or why not?" (We can label the parts in A, B, and D with fractions because they are equal in size, but not in C because the parts aren't the same size.)
- "What do we call the parts in A, B, and D?" ("Halves" in A and D, and "thirds" in B.)
- "What fractions do we use to label the parts in A, B, and D?" ($\frac{1}{2}$ in A and D, and $\frac{1}{3}$ in B.)
- Consider asking: "Let's find at least one reason why each one doesn't belong."

Activity 1

🕒 15 min

Partition the Strips

Standards Alignments

Addressing 3.NF.A.1

The purpose of this activity is for students to practice partitioning and labeling equal-sized parts with unit fractions. This provides students a physical tool they can use throughout the unit to make sense of fractions.

Have students keep their fractions strips to use in future lessons. Consider having students glue the fraction strips in their workbook.

When students make halves, fourths, and eighths they observe regularity in repeated reasoning as each piece is subdivided into 2 equal pieces. They observe the same relationship between thirds and sixths (MP8).

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk. Check in after students fold and label each fraction strip.

Supports accessibility for: Organization, Attention

Materials to Copy

Partition the Strips (groups of 2)

Required Preparation

- Use the Instructional master to create one set of 6 equal-sized strips for each student.

Student-facing Task Statement

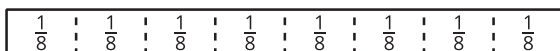
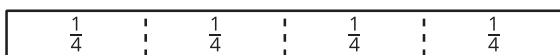
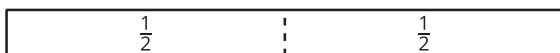
Your teacher will give you some paper strips. Each strip represents 1.

Fold each strip so that the parts represent one of the following fractions. Use one strip for each fraction.

- halves
- fourths
- eighths
- thirds
- sixths

When you finish folding, trace your folding lines with a pencil and then label each part with the correct fraction.

Student Responses

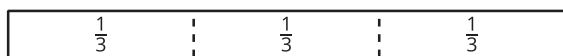


Launch

- Groups of 2
- Give each student one set of 6 equal-sized strips.
- "Today we are going to make fraction strips."
- Demonstrate how to fold a strip into two halves. Emphasize that all the strips should be folded to make vertical partitions as shown in student responses.

Activity

- "Take a few minutes to fold each strip so that the parts represent halves, thirds, fourths, sixths, or eighths. Use one strip for each fraction."
- "Mark your folding lines with a pencil, and then label each part with the correct fraction."
- 5–7 minutes: independent work time
- Monitor for students who fold their strips into fourths, sixths, and eighths by folding halves, thirds, and fourths, respectively, in half.



- “Share how you partitioned your strips and how you labeled the parts with your partner.”
- 2–3 minutes: partner discussion

Synthesis

- Invite students to display their partitioned strips. Keep a full set of fraction strips displayed.
- Ask previously identified students to share how they fold their strips to get 4, 6, and 8 equal parts.
- If not apparent from students' explanations, highlight that fourths, sixths, and eighths can be found by partitioning each half, third, and fourth, respectively, into two equal parts.

Activity 2

🕒 20 min

Partition, Shade, Trade

Standards Alignments

Addressing 3.NF.A.1

Previously, students partitioned rectangular pieces of paper into 2, 3, 4, 6, and 8 equal parts by folding. The purpose of this activity is for students to partition rectangles by drawing and continue to practice naming the parts with the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$. It's important that students try to make the parts as close to equal-sized as they can, but student drawings do not need to be exact. After they practice partitioning, students partition and shade, but don't label, a fraction on a rectangle, then trade with a partner to determine the fraction their partner has shaded. The synthesis focuses on how to name a single equal part, such as one sixth, rather than talking about all the equal parts in a shape, such as sixths. This will be helpful as students use non-unit fractions to name multiple equal parts in the next lesson.

Access for English Learners

MLR8 Discussion Supports. At the appropriate time, give students 2–3 minutes to make sure that everyone in their group can explain their process for partitioning their rectangles and determining how to label each part. Invite groups to rehearse what they will say when they share with the whole class.

Advances: Speaking, Representing

Student-facing Task Statement

1. Partition each rectangle into halves, thirds, fourths, sixths, and eighths. Then label each part with the correct fraction.

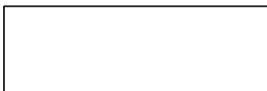
halves



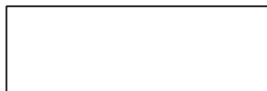
sixths



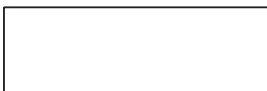
thirds



eighths



fourths



2.
 - a. Partition the rectangle into equal-sized parts. Shade one of the parts.



- b. Trade rectangles with a partner. If the whole rectangle is 1, what number represents the shaded part? Explain your reasoning.

Launch

- Groups of 2

Activity

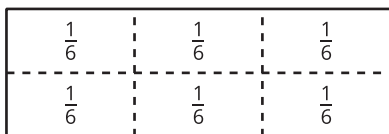
- “Work with your partner to complete the first problem. Partition each rectangle and label each part.”
- 5–7 minutes: partner work time
- For each rectangle, have a group share how they partitioned the rectangle into equal-sized parts and what fraction they used to label each part.
- “Complete part a of the next problem on your own. Partition the rectangle and shade to show a fraction, but don’t label it. Don’t tell your partner how you are partitioning or what number you are showing.”
- 2 minutes: independent work time
- “Now, trade rectangles with your partner and answer the next part of the problem using their rectangle. When you are both finished, share your reasoning.”
- 1–2 minutes: independent work time
- 1–2 minutes: partner work time

Synthesis

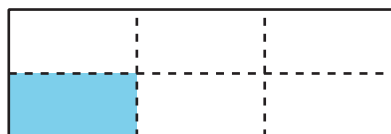
- Have 2–3 students display their shaded rectangles.

Student Responses

1. Students partition each rectangle into 2, 3, 4, 6, or 8 equal parts. Sample response:



2. Sample response:



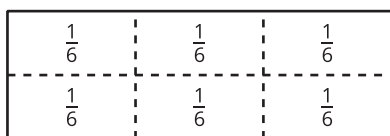
- a.
- b. The shaded part represents one sixth because there are 6 equal parts and one of them is shaded.

- For each rectangle, ask, “How did you know what fraction of the rectangle your partner shaded?” (I counted the equal parts in the rectangle. There were four equal parts, so I knew my partner shaded a fourth.)

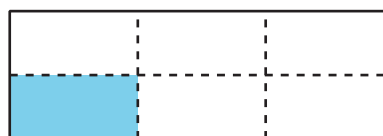
Lesson Synthesis

🕒 10 min

Display a rectangle with each part labeled with the unit fraction and a rectangle shaded to show the unit fraction, such as:



sixths



one sixth or $\frac{1}{6}$

“How do you know the first diagram shows sixths?” (It has six equal parts.)

“Why do you think the second diagram is labeled one sixth?” (Only 1 of the six parts is shaded, so it’s just one of the sixths. We are focusing on one of the sixths.)

“The first diagram shows sixths because the rectangle is partitioned into six equal parts. Each part is one sixth. The second diagram shows one sixth because there are six equal parts and we are describing how many parts are shaded. In this case, one of the parts is shaded.”

Suggested Centers

- Mystery Number (1–4), Stage 2: Three-digit Numbers (Supporting)
- Number Line Scoot (2–3), Stage 1: Twos, Fives, and Tens (Supporting)

Complete Cool-Down

Response to Student Thinking

Students label each part in the first rectangle with “8” or “1.”

Next Day Support

- Use the next day’s warm-up to discuss what each part in a partitioned rectangle represents.

Lesson 3: Non-unit Fractions

Standards Alignments

Addressing 3.NF.A.1

Teacher-facing Learning Goals

- Understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

Student-facing Learning Goals

- Let's learn about non-unit fractions.

Lesson Purpose

The purpose of this lesson is for students to understand non-unit fractions.

Previously, students learned how to write unit fractions, using numbers of the form $\frac{1}{b}$. They also partitioned rectangles and used unit fractions to describe one of the parts. In this lesson, students use diagrams with multiple equal parts shaded to make sense of how non-unit fractions are made of unit fractions.

Students learn that a **unit fraction** is a fraction in which the numerator is 1 because it describes one of the equal-sized parts. They work with fractions that are equal to a whole number and fractions greater than a whole number to see that all non-unit fractions are built from unit fractions in the same way. Students also notice that if all the parts are shaded, then the non-unit fraction is equivalent to a whole number. The terms numerator and denominator are not used until the next section.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Copy

- Fraction Match Part 1 (groups of 2): Activity 2
- Fraction Match Part 2 (groups of 4): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

How did students leverage their knowledge of unit fractions from previous lessons to make sense of non-unit fractions for the first time?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

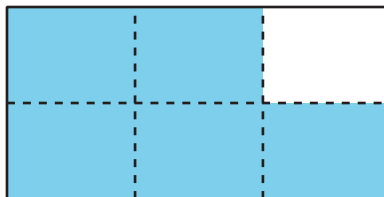
Shaded Fraction

Standards Alignments

Addressing 3.NF.A.1

Student-facing Task Statement

The rectangle represents 1 whole. What fraction is shaded? Explain your reasoning.

**Student Responses**

$\frac{5}{6}$. Sample response: The rectangle is split into 6 equal parts and 5 of the one-sixth parts are shaded.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Notice and Wonder: More than One Part

Standards Alignments

Addressing 3.NF.A.1

The purpose of this warm-up is to elicit the idea that we can think about multiple equal parts in a diagram and use fractions to refer to them, which will be useful when students identify fractions in diagrams and shade diagrams to show a specific fraction in a later activity. While students may notice and wonder many things about these images, the fact that more than one of the equal parts of the square is shaded, there is a fraction underneath the third diagram, and how the shaded parts could be described are the important discussion points.

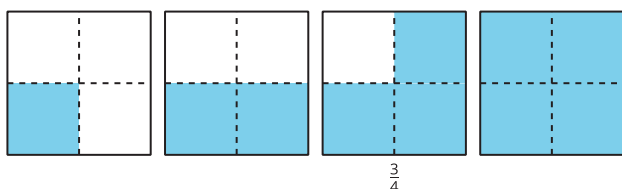
When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see (MP6). They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- Each square shows one more part shaded compared to the square to its left.
- Some of the squares have more than $\frac{1}{4}$ shaded.
- The first square shows $\frac{1}{4}$.
- The second square shows $\frac{1}{2}$.
- There's a fraction below the third square.
- The top part of the fraction has a 3.

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What do you think the 3 and the 4 stand for in the number below the third square?" (The 4 stands for the 4 equal parts in the square. The 3 stands for the number of parts that are shaded.)
- "Sometimes we may want to talk about more than one part and we can describe those parts with a number."

Students may wonder:

- Why is more than one piece shaded sometimes?
- Why are we shading one more piece each time?
- Could each of the big squares be showing a different fraction?
- Does the 3 stand for the 3 shaded parts?
- “What do we call the parts in each square?” (Fourths)
- “How many fourths are shaded in the first square?” (1)
- “How many fourths are shaded in the second square? Third square?” (2 and 3)
- “How many fourths are shaded in the last image?” (4)
- “We can refer to the shaded parts in each image by describing the number of fourths: one fourth, two fourths, three fourths, and four fourths.”
- “We’ll look at how to write these amounts in the next activity.”

Activity 1

🕒 15 min

Write and Read Fractions

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 3.NF.A.1






The purpose of this activity is for students to make sense of non-unit fractions and the notation used to describe them. They learn that the denominator tells the number of equal parts the whole was partitioned into and the numerator tells the number of parts that are being described. Students write non-unit fractions that represent the shaded portions of area diagrams.

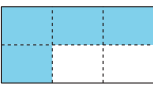
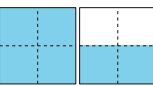
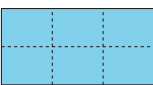

If needed, especially with the fractions greater than one, clarify that each rectangle represents one whole. The activity concludes with students practicing how to read non-unit fractions. The terms “numerator” and “denominator” will be introduced in a later lesson.

When students notice that the bottom part of the fraction stays the same and the top part of the fraction changes, representing the number of equal parts that are shaded, they look for and make use of structure (MP7).

Student-facing Task Statement

Each shape in each row of the table represents 1. Use the shaded parts to complete the missing information in the table. Be prepared to explain your reasoning.

| | number of shaded parts | size of each part | word name for the shaded parts | number name for the shaded parts |
|------------------------------------------------------------------------------------|------------------------|-------------------|--------------------------------|----------------------------------|
|  | 1 | $\frac{1}{4}$ | one-fourth | $\frac{1}{4}$ |
|  | | | | $\frac{3}{4}$ |
|  | | | | |
|  | | | | |
|  | | | | |

| | number of shaded parts | size of each part | word name for the shaded parts | number name for the shaded parts |
|-------------------------------------------------------------------------------------|------------------------|-------------------|--------------------------------|----------------------------------|
|  | | | | |
|  | | | | |
|  | | | | |
|  | | | | |

Student Responses

Launch





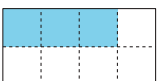
- Groups of 2
- Display the table.
- "Let's look at the first table."
- "The first three images are the squares we saw earlier. Let's name them again." (One-fourth, three-fourths, two-fourths)
- "Let's complete the second row of the table together. This is the square we just worked with in the warm-up and the number that represents the total amount shaded is already in the table. How many of the parts are shaded?" (Three)
- "What is the size of each part?" ($\frac{1}{4}$)
- "Write 'three-fourths' to record how we read this fraction."

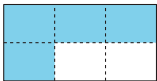
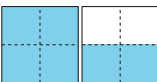

Activity

- "Work with your partner to complete the table."
- 5–7 minutes: partner work time

Synthesis

- Display the completed table.
- "Now that the table is complete, what do you notice? What do you wonder?" (The size of each part always has a 1 in the top part of the fraction. The bottom part of the fraction is the same for each part and the total amount shaded. The top part of the fraction is how many parts are shaded.)
- "Let's think about how we should describe or name the shaded parts in each image."
- 30 seconds: quiet think time
- Invite students to name the shaded parts in each image.
- As students name the shaded part in each image, have them share how they recorded the name using fraction notation.

| | number of shaded parts | size of each part | word name for the shaded parts | number name for the shaded parts |
|-----------------------------------------------------------------------------------|------------------------|-------------------|--------------------------------|----------------------------------|
|  | 1 | $\frac{1}{4}$ | one-fourth | $\frac{1}{4}$ |
|  | 3 | $\frac{1}{4}$ | three-fourths | $\frac{3}{4}$ |
|  | 4 | $\frac{1}{4}$ | four-fourths | $\frac{4}{4}$ |
|  | 2 | $\frac{1}{3}$ | two-thirds | $\frac{2}{3}$ |
|  | 3 | $\frac{1}{8}$ | three-eighths | $\frac{3}{8}$ |

| | number of shaded parts | size of each part | word name for the shaded parts | number name for the shaded parts |
|-------------------------------------------------------------------------------------|------------------------|-------------------|--------------------------------|----------------------------------|
|  | 4 | $\frac{1}{6}$ | four-sixths | $\frac{4}{6}$ |
|  | 6 | $\frac{1}{6}$ | six-fourths | $\frac{6}{4}$ |
|  | 7 | $\frac{1}{6}$ | seven-sixths | $\frac{7}{6}$ |

Activity 2

🕒 20 min

Fraction Match

Standards Alignments

Addressing 3.NF.A.1

The purpose of this activity is for students to match fractions to shaded diagrams. Reiterate that each rectangle represents one whole. After one round of matching, students pause to create 4 new pairs of cards to add to their set. Give students the Fraction Match Part 2 cards when they create their own pairs of cards.

Students observe and use structure as they identify that the top number in the fraction represents the number of shaded pieces while the bottom number represents the number of those pieces in one whole rectangle (MP7).

Access for English Learners

MLR8 Discussion Supports. Students should explain to their partner why the chosen cards match or do not match. Display the following sentence frames for all to see: “I notice _____, so these two cards match/do not match.” Encourage students to challenge each other when they disagree.

Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Access for Perception. Begin by showing a physical demonstration of how to play one round of the game Fraction Match to support understanding of the context.

Supports accessibility for: Memory, Social-Emotional Functioning

Materials to Copy

Fraction Match Part 1 (groups of 2), Fraction Match Part 2 (groups of 4)

Required Preparation

- Create a set of cards from the Fraction Match Part 1 Instructional master for each group of 2.
- 2. Create a set of 8 cards from the Fraction Match Part 2 Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards for playing Fraction Match. Two cards are a match if one is a diagram and the other a number, but they have the same value.

1. To play Fraction Match:
 - Arrange the cards face down in an array.
 - Take turns choosing 2 cards. If the

Launch

- Groups of 2
- “We’re going to play a game in which you match fractions and diagrams. Read the directions to the game with your partner and discuss any questions you have about the game.”
- Answer any questions about the game.
- Give each group one set of cards created from Fraction Match Part 1.

cards match, keep them and go again. If not, return them to where they were, face down. You can't keep more than 2 matches on each turn.

- After all the matches have been found, the player with the most cards wins.
2. Use the cards your teacher gives you to create 4 new pairs of cards to add to the set.
 3. Play another round of Fraction Match using all the cards.

Student Responses

1.
 - a. A and P
 - b. B and I
 - c. C and M
 - d. D and J
 - e. E and N
 - f. F and O
 - g. G and L
 - h. H and K
2. Answers vary.

Activity

- "Play one round of Fraction Match with your partner."
- 5–7 minutes: partner work time
- Give each group one set of Fraction Match Part 2.
- "Before you play another round, work with your partner to create 4 new pairs of cards to add to the set. Partition and shade a diagram to match each fraction."
- 3–5 minutes: partner work time
- "Now, play another round of fraction match with your partner using all the cards."
- 5–7 minutes: partner work time
- Monitor for students who notice that $\frac{3}{3}$ means the whole rectangle is shaded.

Synthesis

- Ask 1–2 groups to display a rectangle they partitioned and shaded to match one of the fractions.
- For each rectangle students share, discuss the fraction that it represents.
- Display a diagram of $\frac{3}{3}$ with all the parts shaded.
- "____ noticed that to show $\frac{3}{3}$, we have to shade the whole rectangle. Look back over both activities for other fractions that are shown by shading the whole rectangle." ($\frac{2}{2}$, $\frac{4}{4}$, and $\frac{6}{6}$)
- Consider asking: "How many ____s would it take to make a whole? How do you know?"
- "These fractions are equivalent to 1 because they represent all the parts of the whole. We'll work with other fractions like this as we learn about fractions."

Lesson Synthesis

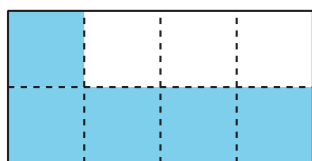
🕒 10 min

Display: $\frac{1}{8}$ and $\frac{5}{8}$

“Today we learned how to build more fractions, like $\frac{5}{8}$, from fractions we already knew, like $\frac{1}{8}$. We call fractions, like $\frac{1}{8}$, that describe one equal-sized part, **unit fractions**. How did you see unit fractions helping us build new fractions today?” (Every fraction we worked with today was made up of unit fractions. If we shade more than one unit fraction in a rectangle, we get a new fraction that doesn’t have a one on the top of the number.)

Display:

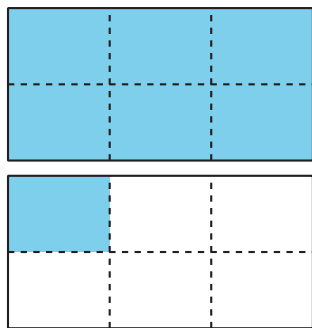
$$\frac{5}{8}$$



“Look at the fraction $\frac{5}{8}$. What does each part of the number tell us?” (The 8 tells us how many parts the rectangle is being split into and what size they are. There are 8 one-eighth parts. The 5 tells us how many of the parts are shaded. Five of the one-eighth parts will be shaded.)

Display:

$$\frac{7}{6}$$



“Look at the fraction $\frac{7}{6}$. What does each part of the number tell us?” (The 6 tells us how many parts each rectangle is being split into and what size they are. There are 6 one-sixth parts in each rectangle. The 7 tells us how many of the parts are shaded. Seven of the one-sixth parts will be shaded.)

Suggested Centers

- Mystery Number (1–4), Stage 2: Three-digit Numbers (Supporting)
- Number Line Scoot (2–3), Stage 1: Twos, Fives, and Tens (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students answer with the unit fraction that represents each part rather than the fraction that represents the entire shaded portion.

Next Day Support

- Before the warm-up, pass back the cool down and have students discuss how $\frac{5}{6}$ represents the shaded portion of the rectangle.

Lesson 4: Build Fractions from Unit Fractions

Standards Alignments

Addressing 3.NF.A.1, 3.OA.C.7
Building Towards 3.NF.A.2

Teacher-facing Learning Goals

- Build non-unit fractions and whole numbers from unit fractions.

Student-facing Learning Goals

- Let's build other fractions from unit fractions.

Lesson Purpose

The purpose of this lesson is for students to build non-unit fractions and whole numbers from unit fractions.

In the previous lesson, students named non-unit fractions and made sense of the notation used to write them. In this lesson, students play a game in which they build non-unit fractions from unit fractions (for example, they try to collect enough cards showing $\frac{1}{6}$ to make $\frac{3}{6}$). They record these fractions on a fraction strip diagram. Then, students partition and shade diagrams to represent situations involving fractional lengths and consider the location of the endpoint of a fractional length. This will be helpful in subsequent lessons, when students represent fractions on a number line.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Colored pencils: Activity 1
- Folders: Activity 1
- Materials for creating a visual display:

Materials to Copy

- Secret Fractions Stage 1 Cards (groups of 2): Activity 1
- Secret Fractions Stage 1 Gameboard (groups of 2): Activity 1

Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

How did having visual representations help students think about building fractions from unit fractions in today's lesson?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Represent a Fraction

Standards Alignments

Addressing 3.NF.A.1

Student-facing Task Statement

This strip represents 1 whole. Partition the diagram and shade it to represent $\frac{6}{8}$.

**Student Responses**

----- Begin Lesson -----

Warm-up

⌚ 10 min

Number Talk: 3 and Another Factor

Standards Alignments

Addressing 3.OA.C.7

This Number Talk encourages students to look for structure in multiplication expressions and rely on properties of operations to mentally solve problems. Reasoning about products of whole numbers helps to develop students' fluency with multiplication within 100.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 3×3
- 7×3
- 10×3
- 3×17

Student Responses

- 9: I just knew it. I know 2×3 is 6, and 3 more is 9.
- 21: Six times 3 is twice 3 times 3, which is 2×9 or 18. Adding 3 more gives 21.
- 30: I know that 3×10 is 30 and 10×3 is also 30. I just knew it.
- 51: It's the sum of 10×3 and 7×3 , which I know is 30 and 21. I know 3×20 is 60 and 3×17 is 3 \times 3 less than 60 or $60 - 9$, which is 51.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the earlier expressions help you find the value of the last expression?"
- Consider asking:
 - "Did anyone have the same strategy but would explain it differently?"
 - "Did anyone approach the problem in a different way?"

Activity 1

 20 min

Introduce Secret Fractions

Standards Alignments

Addressing 3.NF.A.1

The purpose of this activity is for students to learn the Secret Fractions center and build non-unit fractions from unit fractions. Students use unit fractions to build “secret fractions,” which are non-unit fractions. For example, to complete a secret fraction card with $\frac{3}{4}$, students need three cards with $\frac{1}{4}$. After completing each secret fraction, they reveal the fraction they’ve made and shade the gameboard to represent it. The synthesis highlights strategies students used to build their non-unit fractions.

Here are the images of the cards for reference and planning:

Unit Fractions

| | | | |
|---------------|---------------|---------------|---------------|
| $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{8}$ |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

Secret Fractions

| | | | |
|---------------|---------------|---------------|---------------|
| $\frac{2}{2}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{4}$ |
| $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{2}{6}$ | $\frac{4}{6}$ |
| $\frac{5}{6}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ |
| | | | |
| | | | |
| | | | |

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community. For example, check in with partners after the second round of Secret Fractions.

Supports accessibility for: Social-Emotional Functioning

Materials to Gather

Colored pencils, Folders

Materials to Copy

Secret Fractions Stage 1 Cards (groups of 2),
Secret Fractions Stage 1 Gameboard (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.
- Print extra gameboards for the launch and groups that have time for an extra game.
- Students might want a folder or divider so their partner doesn't see their cards.

Student-facing Task Statement

The goal of the game is to be the first to build 2 secret fractions with unit fractions.

1. Make two stacks: one for secret fractions and one for unit fractions. Place all cards face down.
2. Each player draws 2 secret fraction cards. These are the fractions you are trying to make with your unit fractions.
3. On your turn, you can make one of these moves:
 - Pick up 1 unit fraction card.
 - Trade both of your secret fractions for 2 new secret fractions from the stack.
4. When you have enough unit fractions to make one of your secret fractions, shade your gameboard to represent your secret fraction. Then, pick a new secret fraction.
5. The first player to make 2 secret fractions wins.

Gameboard

| | | | | | | | | | | | | | | | |
|---------------|--|--|--|--|--|--|--|---------------|--|--|--|--|--|--|--|
| 1 whole | | | | | | | | | | | | | | | |
| $\frac{1}{2}$ | | | | | | | | $\frac{1}{2}$ | | | | | | | |
| $\frac{1}{3}$ | | | | | | | | $\frac{1}{3}$ | | | | | | | |
| $\frac{1}{4}$ | | | | | | | | $\frac{1}{4}$ | | | | | | | |
| $\frac{1}{6}$ | | | | | | | | $\frac{1}{6}$ | | | | | | | |
| $\frac{1}{8}$ | | | | | | | | $\frac{1}{8}$ | | | | | | | |
| 1 whole | | | | | | | | | | | | | | | |
| $\frac{1}{2}$ | | | | | | | | $\frac{1}{2}$ | | | | | | | |
| $\frac{1}{3}$ | | | | | | | | $\frac{1}{3}$ | | | | | | | |
| $\frac{1}{4}$ | | | | | | | | $\frac{1}{4}$ | | | | | | | |
| $\frac{1}{6}$ | | | | | | | | $\frac{1}{6}$ | | | | | | | |
| $\frac{1}{8}$ | | | | | | | | $\frac{1}{8}$ | | | | | | | |

Launch

- Groups of 2
- Give each group a set of cards and a gameboard.
- "We're going to play a game called Secret Fractions. Let's read through the directions and play 1 round together."
- Read through the directions with the class and play a round against the class, displaying the fractions from the cards and thinking through decisions aloud.
- Show students how to place an object, such as a folder, between them to obstruct the view of the other player.
- Give each group that wants a divider a folder or other divider.

Activity

- "Now, play Secret Fractions with your partner."
- 10–15 minutes: partner work time
- Monitor for students who:
 - trade their secret fractions for new ones because most of their unit fractions have a different denominator
 - keep track of the unit fractions they have as they try to make their non-unit fractions

Synthesis

- "What strategies did you find helpful for building your secret fractions?" (When $\frac{5}{8}$

was my secret fraction, I was keeping track of how many $\frac{1}{8}$ cards I had to make $\frac{5}{8}$. When I had $\frac{3}{4}$ as a secret fraction, I knew I needed $\frac{1}{4}$ cards, but I had a bunch of $\frac{1}{6}$ cards, so I traded for different secret fractions.)

Activity 2

🕒 15 min

Represent Fraction Situations

Standards Alignments

| | |
|------------------|----------|
| Addressing | 3.NF.A.1 |
| Building Towards | 3.NF.A.2 |

The purpose of this activity is for students to use diagrams to represent situations that involve non-unit fractions. The synthesis focuses on how students partition and shade the diagrams and how the end of the shaded portion could represent the location of an object. When students interpret the different situations in terms of the diagrams they reason abstractly and quantitatively (MP2).

Access for English Learners

MLR8 Discussion Supports. Synthesis: During group presentations, invite student(s) who is/are not speaking to follow along and point to the corresponding parts of the display.

Advances: Speaking, Representing

Materials to Gather

Materials for creating a visual display

Student-facing Task Statement

Here are four situations about playing Pilolo and

Launch

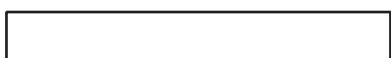
- Groups of 2
- "What are some games that you like to play with friends?"

four diagrams. Each diagram represents the length of a street where the game is played.



Represent each situation on a diagram. Be prepared to explain your reasoning.

1. A student walks $\frac{4}{8}$ the length of the street and hides a rock.



2. A student walks $\frac{2}{3}$ the length of the street and hides a penny.



3. A student walks $\frac{3}{4}$ the length of the street and hides a stick.



4. A student walks $\frac{5}{6}$ the length of the street and hides a penny.



5. This diagram represents the location of a hidden stick.



About what fraction of the length of the street did the student walk to hide it? Be prepared to explain how you know.

Student Responses

1. Students partition the diagram into 8 equal parts and shade 4 of the parts.
2. Students partition the diagram into 3 equal parts and shade 2 of the parts.

- Share responses.
- "Pilolo is a game played in Ghana. One player hides sticks, rocks, or pennies. The other players have to find one of the objects and be the first to reach the finish line to get a point. Look at the picture of some children playing Pilolo and think about some strategies you might use if you played this game." (I would try to hide the objects in a good hiding spot. I would run fast to be the first one to the finish line.)
- 30 seconds: quiet think time
- Share responses.
- "We're going to represent some situations about students playing Pilolo."

Activity

- "In the activity, each strip represents the length of a street where Pilolo is played."
- "Work independently to represent each situation on a diagram."
- 3–5 minutes: independent work time
- "With a partner, choose one of the situations and make a poster to show how you represented the situation with a fraction strip. You may want to include details such as notes, drawings, labels, and so on, to help others understand your thinking."
- Give students materials for creating a visual display.
- 5–7 minutes: partner work time

Synthesis

- Display posters around the room.
- "As you look at each poster, discuss how each situation was shown on the diagram." (The diagram was partitioned into 6 equal parts. 5 of the equal parts were shaded.)
- 2–3 minutes: partner discussion

3. Students partition the diagram into 4 equal parts and shade 3 of the parts.
 4. Students partition the diagram into 6 equal parts and shade 5 of the parts.
 5. Sample responses:
 - It's about $\frac{3}{4}$ the length of the street. I partitioned the diagram into 4 parts and about 3 are shaded.
 - It's about $\frac{7}{8}$ the length of the street. I partitioned the diagram into 8 parts and almost 7 parts are shaded.
- Share responses.
 - "What part of each diagram would represent where the object was hidden?" (The end of the shaded part.)

Advancing Student Thinking

If students partition into sixths or eighths by creating 6 or 8 parts, but the parts aren't equal, consider asking:

- "How did you partition the strip into sixths? Eighths?"
- "How could you use thirds to partition into sixths? Use fourths to partition into eighths?"

Lesson Synthesis

🕒 10 min

Display some completed gameboards from the first activity and one of the diagrams that represents a situation from the second activity.

"How was making the fractions in the game like representing the situations? How was it different?" (The parts had to be equal-sized for both activities. We had to count the parts in both activities. The fractions were made from unit fractions. In the first activity, we had the pieces to build the fraction, but in the second activity, we had to partition and shade in the parts to make the fraction.)

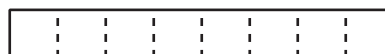
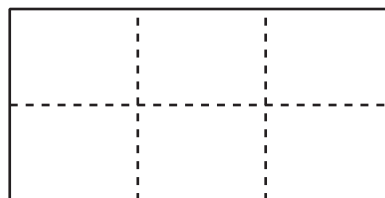
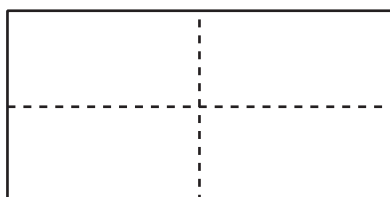
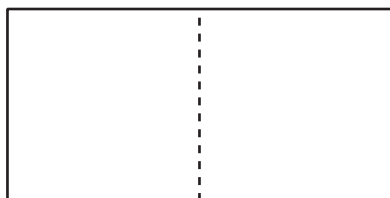
"In both activities we were able to see how unit fractions are used to make other fractions."

Suggested Centers

- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

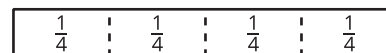
Student Section Summary

In this section, we learned how to partition shapes into halves, thirds, fourths, sixths, and eighths, and how to describe each of those parts in words and using a number.



The numbers we use to describe these equal-sized parts are **fractions**.

A fraction like $\frac{1}{4}$ is read “one-fourth” because it represents one of the 4 equal parts in a whole.



A fraction like $\frac{3}{4}$ is read “three-fourths” because it represents 3 parts that are each one-fourth or $\frac{1}{4}$ in size.



Fractions that refer to only one of the equal parts in a whole—like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$ —are called **unit fractions**.

We learned that the bottom part of the fraction tells us how many equal parts we partitioned the whole into. The top part of the fraction tells us how many of the equal parts are being described.

----- Complete Cool-Down -----

Response to Student Thinking

Students partition the diagram into 8 parts and shade in 6 parts, but the parts aren't equal.

Next Day Support

- Before the warm-up, have students discuss strategies for partitioning the diagram in the cool-down into 8 equal parts.

Section B: Fractions on the Number Line

Lesson 5: To the Number Line

Standards Alignments

Addressing 3.NF.A.2

Teacher-facing Learning Goals

- Extend understanding of whole numbers on the number line to see fractions on a number line.

Student-facing Learning Goals

- Let's learn about fractions on the number line.

Lesson Purpose

The purpose of this lesson is for students to extend their understanding of whole numbers on the number line as they work with number lines partitioned into fractions.

Previously, students used fraction strips to make sense of unit and non-unit fractions. In grade 2, they represented whole numbers on the number line. In this lesson, students learn that quantities that are not whole numbers can also be represented on the number line, an important step toward the understanding that fractions are numbers. To support this transition, students move from representing fractions by shading diagrams (an area representation) to marking their locations on a number line (a linear representation).

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Notice and Wonder (Warm-up)

Materials to Gather

- Scissors: Activity 2

Materials to Copy

- Card Sort: Number Lines (groups of 2): Activity 1

- Fold and Label Number Lines (groups of 1): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 10 min |
| Activity 2 | 25 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflection

Standards Alignments

Addressing 3.NF.A.2

Student-facing Task Statement

Describe something you really understand well after today's lesson or describe something that was confusing or challenging.

Student Responses

Sample responses:

- I understand that fractions show up on the number line in between whole numbers.
- I am confused about where to label fractions on the number line.

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Two Number Lines

Standards Alignments

Addressing 3.NF.A.2

The purpose of this warm-up is to elicit the idea that number lines can be partitioned into intervals smaller than 1, which will be useful when students see number lines partitioned into fractions in a later activity. While students may notice and wonder many things, the idea that fractions can be represented on the number line is the important discussion point. Students do not need to identify the tick mark as showing $\frac{1}{2}$ in the warm-up, as that will be the focus later in the lesson.

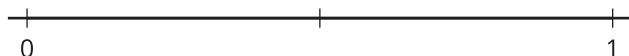
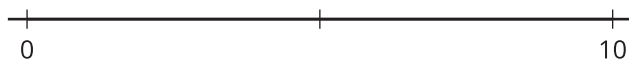
This prompt gives students opportunities to look for and make use of structure (MP7). The specific structure they might notice is that each number line is partitioned in half.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There's a number line that goes from 0 to 10 and one that goes from 0 to 1.
- Both have a tick mark in the middle.
- Only 0 and 10 and 0 and 1 are labeled.

Students may wonder:

- What numbers are at the tick marks?
- Why are the number lines split in half?
- Could we put more numbers on the number lines?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What do you know about the number each tick mark represents?" (On the first number line, it is 5 because it is halfway between 0 and 10. On the second number line, I think it is $\frac{1}{2}$ because it is halfway between 0 and 1.)

Activity 1

🕒 10 min

Card Sort: Number Lines

Standards Alignments

Addressing 3.NF.A.2

The purpose of this activity is for students to further develop the idea that fractional amounts can be represented on a number line. Students sort a given set of cards showing number lines. They first sort in a way of their choice, which might include number of parts or length of the number line. Monitor for different ways groups choose to categorize the number lines, but especially for categories that distinguish between number lines with whole number partitions and fractional partitions.

When students identify common properties of the number lines for their sorts, such as the numbers listed on the tick marks or the total number of tick marks, they look for and make use of structure (MP7).

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they sort the number lines. On a visible display, record words and phrases such as: parts less than one, smaller than one, whole numbers, partitions, partitioned into fractions, and equal parts. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

Instructional Routines

Card Sort

Materials to Copy

Card Sort: Number Lines (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

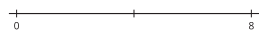
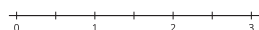
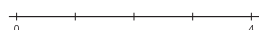
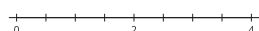
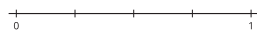
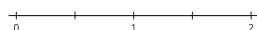
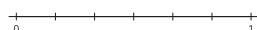
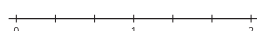
Student-facing Task Statement

Your teacher will give you a set of cards that

Launch

- Groups of 2

show number lines. Sort the cards into categories of your choosing. Be prepared to explain the meaning of your categories.

A**C****E****G****I****B****D****F****H**

Student Responses

Students may sort by:

- The number on the right end of the number line: 1 (B and F) and other than 1 (all others)
- The number of parts from one end of the number line to the other end: 2 (A), 3 (G), 4 (B, D, E), 6 (C, F, H), 8 (I)
- The middle tick mark: labeled (D, H, I) or not labeled (all others)
- What the tick marks represent: whole numbers (A, E, G) or fractions (B, C, D, F, H, I)

- Distribute one set of pre-cut cards to each group of students.

Activity

- “Work with your partner to sort some number lines into categories that you choose. Make sure you have a name for each category.”
- 3-5 minutes: partner work time
- Select groups to share their categories and how they sorted their cards.
- Choose as many different types of categories as time allows. Be sure to highlight categories created based on whether the tick marks represent whole numbers or fractions.
- If not mentioned by students, ask, “Can we sort the number lines based on what the tick marks represent?”
- “Let’s look at B and E. Both are partitioned into 4 parts. What do the unlabeled tick marks in E represent?” (1, 2, 3) “What do you think those in B represent?” ($\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, or amounts less than 1)
- “Take a minute to sort your cards by number lines where the tick marks only represent whole numbers and number lines where the tick marks represent fractions.”
- 1-2 minutes: partner work time

Synthesis

- “How did you know if a number line had tick marks that represent fractions?” (If there is one or more tick marks between two back-to-back whole numbers like 0 and 1, or 1 and 2, then the tick marks between them represent fractions.)
- Attend to the language that students use to describe their number lines, giving them opportunities to describe the number lines

more precisely.

- Highlight the use of phrases like “parts less than 1” or “partitioning one part into smaller parts less than 1.”
- Consider displaying a number line with fractions that are less obvious, such as number line I. Ask students to help identify the fractions on that number line, and label 1 and 3 so that the tick marks between the whole numbers are clear.

Advancing Student Thinking

If students don't identify number lines that would have fractions marked, consider asking:

- “What other numbers can we find on this number line?”
- “What could the marks in between the whole numbers be?”

Activity 2

🕒 25 min

Fold and Label the Number Line

Standards Alignments

Addressing 3.NF.A.2

The purpose of this activity is to transition students from thinking about fractional lengths on fraction strips to thinking about fractions as numbers on the number line. Students build on their experience of folding fraction strips to fold number lines into halves, thirds, fourths, sixths, and eighths and then label unit fractions.

Students begin by considering how the fraction $\frac{1}{2}$ can be labeled on the number line. They learn that each part of the number line has a length of one half, but the endpoint of the first one-half part is the location of the number $\frac{1}{2}$ on the number line. This distinction is important for understanding fractions as numbers that can be represented as points on the number line and for using the number line precisely (MP6).

When folding the number lines, students also need to attend to the fact that it is the interval between 0 and 1 that needs to be partitioned, rather than the length of the entire strip of paper that contains each number line.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Make connections between representations visible. Highlight the similarities and differences in the strategies students used to fold their number lines.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Materials to Gather

Scissors

Materials to Copy

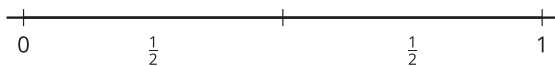
Fold and Label Number Lines (groups of 1)

Required Preparation

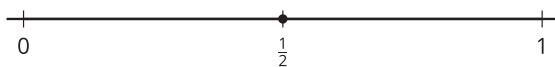
- Each student needs at least 5 number lines from 0 to 1. Each copy of the Instructional master contains a few extra number lines, in case students fold incorrectly at first.
- Create a number line folded into fourths and a fraction strip that shows fourths to display in the synthesis.

Student-facing Task Statement

- Andre and Clare are talking about how to label fractions on the number line.
Andre says $\frac{1}{2}$ can be labeled like this:



Clare says $\frac{1}{2}$ can be labeled like this:



How could each student's labeling make sense?

- Your teacher will give you a set of number lines. Cut your number lines apart so that you can fold each one.

As you fold, discuss your strategies with your partner.

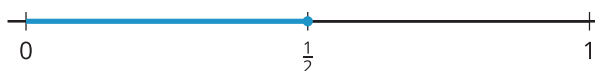
Launch

- Groups of 2
- "We've been thinking about where fractions are located on the number line. Let's take some time to think about how to label fractions on the number line."
- "Take a minute to think about Andre and Clare's number lines."
- 1 minute: quiet think time
- "Talk to your partner about how each student's labeling could make sense."
- 2–3 minutes: partner discussion
- Share responses.
- Display a number line with both the distance to $\frac{1}{2}$ and the number $\frac{1}{2}$ marked in a color, such as:

- Fold one of the number lines into halves. Draw tick marks to show the halves. Label the number $\frac{1}{2}$.
- Fold one of the number lines into thirds. Draw tick marks to show the thirds. Label the number $\frac{1}{3}$.
- Fold one of the number lines into fourths. Draw tick marks to show the fourths. Label the number $\frac{1}{4}$.
- Fold one of the number lines into sixths. Draw tick marks to show the sixths. Label the number $\frac{1}{6}$.
- Fold one of the number lines into eighths. Draw tick marks to show the eighths. Label the number $\frac{1}{8}$.

Student Responses

- Sample response: Andre's reasoning makes sense because that is showing $\frac{1}{2}$ of the length just like we did on our fraction strips. Clare's reasoning makes sense because you can label the point on the number line just like we did with other numbers.
- Students will have 5 number lines. One of each folded into halves, thirds, fourths, sixths, and eighths with the corresponding unit fraction labeled on each one.



- “Andre was thinking about the parts that had length $\frac{1}{2}$, so he labeled the parts from 0 to $\frac{1}{2}$ and $\frac{1}{2}$ to 1 with $\frac{1}{2}$.”
- “To locate and label the number $\frac{1}{2}$, we find the endpoint of the first one-half part from 0 and label it.”
- Give each student a copy of the Instructional master and scissors.

Activity

- “Cut the number lines apart.”
- “Then, fold one into halves, one into thirds, one into fourths, one into sixths, and one into eighths.”
- “As you fold, share your folding strategies with your partner.”
- “Draw tick marks along your folding lines and label the unit fraction on each number line.”
- 3–5 minutes: partner work time
- Monitor for students who need support lining up the 0 and the 1 as they fold. Consider suggesting that they cut off the ends of the number line at 0 and 1, or marking 0 and 1 on both sides of each paper strip to make them easier to see while folding.

Synthesis

- Display a number line folded into fourths and a fraction strip of fourths.
- “How was partitioning these number lines similar to partitioning our fraction strips?” (The number lines were folded just like the strips were, but instead of a rectangle it's just a line. We labeled the location of the unit fractions at the end of the first part on each number line instead of the space.)

Lesson Synthesis

🕒 10 min

"Today we used what we know about fractions to think about where fractions are located on the number line."

"What did you learn about locating and labeling fractions on the number line today?" (Fractions are between the whole numbers on a number line. We can fold number lines just like fraction strips to partition a whole into smaller parts. We can see the fraction as a distance and as a number located on the number line.)

Display a number line from 0 to 1 partitioned into thirds with the distance to $\frac{1}{3}$ marked, such as:



"How could we use this length to locate and label the number $\frac{1}{3}$ on this number line?" (We could label the end of the first part with $\frac{1}{3}$ to show it's a third of the distance to 1 from 0 on the number line.)

"Locate and label $\frac{1}{3}$ on the number line."

Suggested Centers

- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

Complete Cool-Down

Response to Student Thinking

Students have key ideas or lingering questions to discuss with other students.

The work in this lesson builds from number line concepts developed in a prior unit.

Next Day Support

- Before the next day's warm-up, pair students up to discuss their responses.

Prior Unit Support

Grade 2, Unit 4, Section A: The Structure of the Number Line

Lesson 6: Locate Unit Fractions on the Number Line

Standards Alignments

Addressing 3.NF.A.2, 3.NF.A.2.a

Teacher-facing Learning Goals

- Partition the interval from 0 to 1 and locate unit fractions within that interval.

Student-facing Learning Goals

- Let's partition the number line to locate unit fractions.

Lesson Purpose

The purpose of this lesson is for students to partition the interval from 0 to 1 and locate unit fractions within that interval.

In previous lessons, students made sense of number lines that were partitioned into fractions. While this lesson focuses on partitioning and locating unit fractions within the interval from 0 to 1, the number lines in this lesson vary in length. This allows students to consider common misconceptions about partitioning the number line into fractional parts, such as partitioning the whole number line rather than each unit interval, such as from 0 to 1, 1 to 2, and so on, and counting tick marks rather than parts of the interval.

As students partition number lines in the rest of this unit, the most important thing is that they are accurate in relatively locating fractions on the number line and they understand the partitions should be equally spaced (MP6). It is not necessary that they locate fractions exactly or be overly concerned with making sure partitions are exactly the same size.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

How did you see or hear students leverage their prior experiences with fractions to place fractions on the number line?

| | |
|------------------|--------|
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Cool-down (to be completed at the end of the lesson)

🕒 5 min

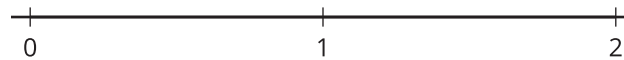
Locate and Label

Standards Alignments

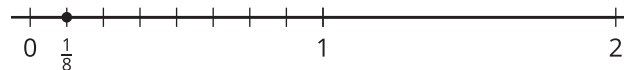
Addressing 3.NF.A.2.a

Student-facing Task Statement

Locate and label $\frac{1}{8}$ on the number line. Explain your reasoning.



Student Responses



I know that 8 one-eighths are in 1, so I partitioned the number line from 0 to 1 into 8 equal parts and labeled the end of the first eighth.

Begin Lesson

Warm-up

🕒 10 min

Which One Doesn't Belong: Fraction Details

Standards Alignments

Addressing 3.NF.A.2

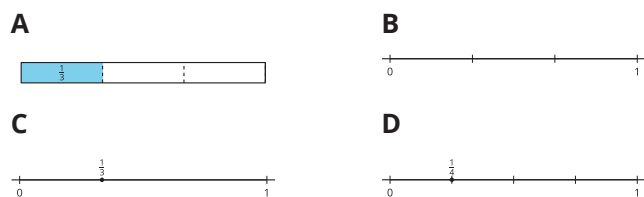
This warm-up prompts students to compare four images. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use, such as parts, partitions, mark, label, thirds, or fourths.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?



Student Responses

Sample responses:

- A is the only one that is not a number line.
- B is the only one not labeled with a fraction.
- C is the only one not partitioned into equal parts.
- D is the only one that doesn't show thirds or $\frac{1}{3}$.

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "To make your reasoning clear while locating and labeling fractions on a number line, what are some important things to include?" (Partitions of the equal parts, a dot and label at the fraction.)
- "We learned in a previous lesson that we label fractions on the number line at the tick marks just like we have labeled whole numbers on the number line."
- "How is this different from how we labeled our fraction strips like in A?" (In diagrams of fraction strips we labeled the part that has size or length $\frac{1}{3}$. On the number line we are labeling the number $\frac{1}{3}$.)

Activity 1

🕒 15 min

Partition Fourths

Standards Alignments

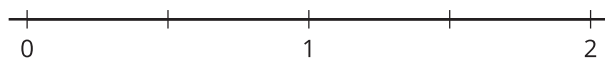
Addressing 3.NF.A.2

The purpose of this activity is for students to make sense of partitioning number lines that extend beyond one. Clare and Diego's work surfaces 2 common misconceptions that students often make while partitioning number lines into fractions. Clare partitions the entire number line into fourths and Diego places 4 tick marks to show fourths. Students analyze these misconceptions (MP3) before they locate and label unit fractions on number lines of various lengths in the next activity.

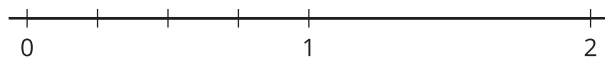
Student-facing Task Statement

Three students are partitioning a number line into fourths. Their work is shown.

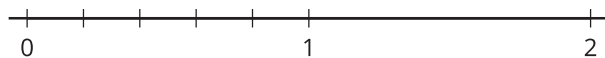
Clare's number line:



Andre's number line:



Diego's number line:



Whose partitioning makes the most sense to you? Explain your reasoning.



Launch

- Groups of 2
- "Today we are going to partition number lines to locate unit fractions. Take a minute to look at how Clare, Andre, and Diego have partitioned their number lines into fourths."
- 1–2 minutes: quiet think time

Activity

- "Work with your partner to decide whose partitioning makes the most sense to you and why."
- 3–5 minutes: partner work time
- Monitor for students who can explain why Andre's partitioning makes sense and why the others do not show fourths.

Synthesis

- Ask students to share why Andre's partitioning makes sense to them.

Student Responses

Sample response: Andre's number line makes the most sense to me because it shows the first part from 0 to 1 in 4 equal parts. I could count up one fourth, two fourths, three fourths, and then four fourths to 1. Diego's number line has 4 tick marks in between 0 and 1, but that is 5 parts. Clare partitioned the whole number line into four parts. That is like fourths, except I know that the first tick mark is at $\frac{1}{2}$ because it is halfway between 0 and 1, so that is showing halves, not fourths.

- Consider asking:
 - "Did anyone think of Andre's reasoning in a different way?"
 - "How do we know that Andre's number line is partitioned into fourths?"
- Ask students to explain why Clare and Diego's partitioning does not show fourths.
- "We learned that when you partition the number line, you have to pay attention to where 0 and 1 are and make sure to partition that into the right number of equal-length parts."

Activity 2

🕒 20 min

Unit Fractions on the Number Line

Standards Alignments

Addressing 3.NF.A.2.a

The purpose of this activity is for students to partition the interval from 0 to 1 into equal parts to locate and label unit fractions. Students see number lines that vary in length, from 1 unit to 4 units, which provides an opportunity for them to practice accurately partitioning the unit on the number line, rather than the entire number line (MP6). Some number lines show numbers greater than one which gives students the opportunity to think about fractions greater than one even though they are not explicitly addressed in this lesson.

🌐 Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . ." Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Access for Students with Disabilities


Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge.


Invite students to select 6 out of 9 number lines to partition and label.

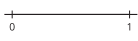
Supports accessibility for: Organization, Attention, Social-emotional skills


Student-facing Task Statement

Partition each number line. Locate and label each fraction.

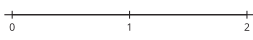
1. $\frac{1}{4}$ 

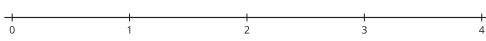
2. $\frac{1}{8}$ 


3. $\frac{1}{3}$ 

4. $\frac{1}{6}$ 

5. $\frac{1}{2}$ 

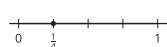
6. $\frac{1}{4}$ 

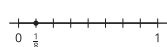
7. $\frac{1}{8}$ 

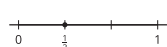
8. $\frac{1}{3}$ 

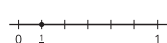
9. $\frac{1}{6}$ 

Student Responses

1. 

2. 

3. 

4. 

Launch

- Groups of 2
- “Now that we’ve thought about some common mistakes about partitioning number lines, you are going to have a chance to partition number lines to locate and label unit fractions.”

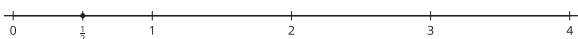
Activity

- “Work independently to partition each number line and locate and label each fraction.”
- 3–5 minutes: independent work time
- “Now, share how you partitioned each number line and where you located and labeled each fraction with your partner.”
- “Be sure to share tips on how you partitioned or ask for tips for any of the partitions that were challenging.”
- 3–5 minutes: partner discussion
- Monitor for students who disagree on how to partition one of the number lines.

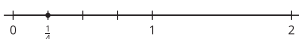
Synthesis

- “Were there any number lines that you and your partner were not sure how to partition or disagreed about? How did you resolve your confusion or disagreement?” (We weren’t sure how to partition the number line that goes up to 4 when we were locating $\frac{1}{2}$. We talked together about

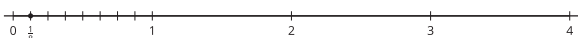
5.



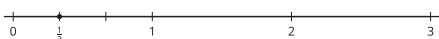
6.



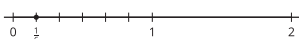
7.



8.

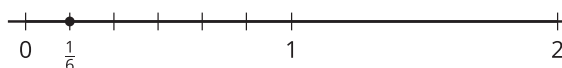
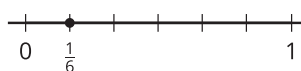


9.



partitioning just the space from 0 to 1 into half.)

- Consider asking: “What was different about partitioning, locating, and labeling fractions on number lines with numbers greater than 1 than on number lines that just go up to 1?” (You have to be careful to just partition the one whole, not the whole number line.)
- Display the same unit fraction on a number line with length 1 and length 2, such as:



- “What do you notice?” (The top number line just has 0 to 1. The bottom number line has the 0 to 2. The number 1 is located in the same place on both number lines. The number $\frac{1}{6}$ is located in the same place on both number lines.)
- “The location of any number on the number line doesn’t change just because we extend the number line. The number $\frac{1}{6}$ is located between 0 and 1 whether the number line goes up to 1 or it goes up to another number.”

Advancing Student Thinking

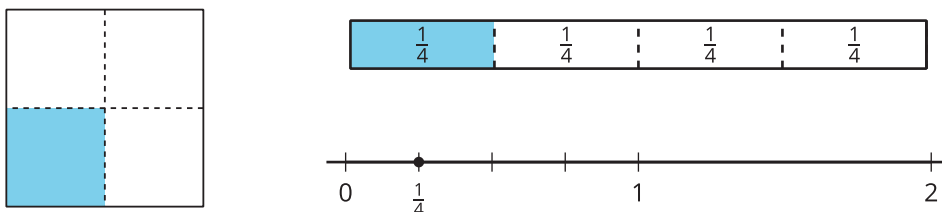
If students create the same number of tick marks as the denominator or partition the entire number line instead of the interval between 0 and 1, consider asking:

- “Tell me about how you partitioned your number line.”
- “What did we learn in the last activity about how to partition number lines?”

Lesson Synthesis

🕒 10 min

Display an example of each of the fraction representations used so far, such as:



“Today we used our knowledge of unit fractions and the number line to locate unit fractions on the number line.”

“We have seen unit fractions represented several ways now. How would you describe a unit fraction to a friend? Use examples from these representations if it helps you.” (When you split a whole into equal parts, a unit fraction is one of those parts. Here we see all these representations show that the whole is split into four equal parts. One fourth is one of those parts. For diagrams, you see the size of one part. On the number line you show the number at the end of the first part.)

“What is particularly helpful for you to remember when you are locating unit fractions on the number line?” (I need to partition the whole, which is the whole shape, the strip, or the space between 0 and 1, into the number of equal parts given by the number on the bottom part of the fraction. Then I can label the end of one of those parts at the unit fraction I am looking for.)

Suggested Centers

- Mystery Number (1–4), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

Complete Cool-Down

Response to Student Thinking

Students do not accurately partition the interval from 0 to 1 into eight parts to locate $\frac{1}{8}$.

The work in this lesson builds from number line concepts developed in a prior unit.

Next Day Support

- Before the next day's warm-up, have students discuss how to partition a number line to show eighths.

Prior Unit Support

Grade 2, Unit 4, Section A: The Structure of the Number Line

Lesson 7: Non-unit Fractions on the Number Line

Standards Alignments

| | |
|------------------|------------|
| Addressing | 3.NF.A.2.b |
| Building Towards | 3.NF.A.2 |

Teacher-facing Learning Goals

- Locate non-unit fractions on the number line (including fractions greater than 1).

Student-facing Learning Goals

- Let's locate non-unit fractions on the number line.

Lesson Purpose

The purpose of this lesson is for students to locate non-unit fractions on the number line.

Previously, students built non-unit fractions from unit fractions with diagrams and fraction strips. Now, students deepen their understanding of fractions on the number line as they locate and label non-unit fractions. Students also discuss how they know when fractions are less than 1 or greater than 1 and are introduced to the terminology **numerator** and **denominator**.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Choral Count (Warm-up)

Materials to Gather

- Base-ten blocks: Activity 1
- Number cubes: Activity 1

Materials to Copy

- Number Line Scoot Stage 2 Directions (groups of 2): Activity 1
- Number Line Scoot Stage 2 Gameboard (groups of 2): Activity 1

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

Who has been sharing their ideas in class lately?
Make a note of students whose ideas have not

| | |
|------------------|--------|
| Activity 1 | 15 min |
| Activity 2 | 10 min |
| Activity 3 | 10 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

been featured in class and look for an opportunity for them to share their thinking in tomorrow's lesson.

Cool-down (to be completed at the end of the lesson)

🕒 5 min

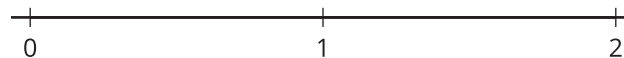
Where is $\frac{5}{3}$?

Standards Alignments

Addressing 3.NF.A.2.b

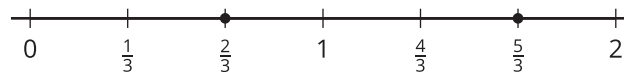
Student-facing Task Statement

Locate and label $\frac{2}{3}$ and $\frac{5}{3}$ on the number line. Explain your reasoning.



Student Responses

I partitioned the number line into thirds, and then I counted 5 one-thirds.



Begin Lesson

Warm-up

🕒 10 min

Choral Count: One-fourths

Standards Alignments

Building Towards 3.NF.A.2

The purpose of this Choral Count is to invite students to practice counting by $\frac{1}{4}$ and notice patterns in the count. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to locate fractions on the number line using their knowledge of unit fractions. Save the recorded count to compare to a count in an upcoming lesson.

Instructional Routines

Choral Count

Student Responses

Sample responses:

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, \frac{12}{4}, \frac{13}{4}, \frac{14}{4}, \frac{15}{4}, \frac{16}{4}$

Launch

- “Count by $\frac{1}{4}$, starting at $\frac{1}{4}$.”
- Record as students count. Record 4 fractions in each row, then start a new row. There will be 4 rows.
- Stop counting and recording at $\frac{16}{4}$.

Activity

- “What patterns do you see?” (The bottom part of the fraction never changes. The top part of the fraction is increasing by 1. The rows end at counts of four in the top like 4, 8, 12, 16.)
- 1–2 minutes: quiet think time
- Record responses.

Synthesis

- “How is counting by fractions the same as counting by whole numbers? How is it different?” (The top part of the fraction is just like counting by whole numbers, going up one. The bottom part is different because it doesn’t change.)
- Consider asking:
 - “Who can restate the pattern in different words?”
 - “Does anyone want to add an observation on why that pattern is happening here?”
 - “Do you agree or disagree? Why?”
- “This is a place where it’s helpful to talk about

the top part of the fraction and the bottom part of the fraction. We have words for those parts. The bottom part of a fraction is called the **denominator**. It tells how many equal parts the whole was partitioned into. The top part of a fraction is called the **numerator**. It tells how many of the equal parts are being described. Look for places in today's lesson where that terminology might help you explain your reasoning."

- Display the terms "denominator" and "numerator" and their definitions, and keep displayed throughout the lesson.

Activity 1

🕒 15 min

Number Line Scoot

Standards Alignments

Addressing 3.NF.A.2.b

The purpose of this activity is for students to practice identifying fractional intervals along a number line. This is Stage 2 of the center activity, Number Line Scoot. This activity encourages students to count by the number of intervals (the numerator). Students have to land exactly on the last tick mark, which represents 4, to encourage them to move along different number lines. While this activity does not focus on equivalence, it gives students exposure to this idea before they work more formally with it in the next section. In the synthesis, students relate counting on a number line marked off in whole numbers to their number lines marked off in fractional-sized intervals.

It may be helpful to play a few rounds with the whole class to be sure students are clear on the rules of the game. Keep the number line game boards for center use.

🕒 Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community. For example, check in after the first round of Number Line Scoot.

Supports accessibility for: Attention, Social-Emotional Functioning

Materials to Gather

Base-ten blocks, Number cubes

Materials to Copy

Number Line Scoot Stage 2 Directions (groups of 2), Number Line Scoot Stage 2 Gameboard (groups of 2)

Required Preparation

- Each group of 2 students needs a number cube.
- Each student needs at least 5 base-ten cubes to use as game pieces.

Launch

- Groups of 2
- Give each group of 2 a gameboard, two recording sheets, a number cube, and at least 10 base-ten cubes.
- “Now you will play a game where you move, by fractions, along different number lines. To start, each player places a small cube on zero on each number line. The goal of the game is to get as many small cubes as you can to the 4 on any of the number lines.”
- Roll the number cube, demonstrate where to record the rolled number and move that fraction along one of the number lines.

Activity

- 10 minutes: partner work time
- As students work, monitor for students who count by the numerator once they have chosen a number line.

Synthesis

- Display a gameboard with a marker on $\frac{3}{4}$.
- “If I rolled a 4, and chose to move $\frac{4}{4}$, how would you count the move?” (I would count 1, 2, 3, 4.)
- “How did you know you have moved $\frac{4}{4}$?”
(Because each space is $\frac{1}{4}$, so I need to move 4

times.)

- Display a number line marked with only 0, 1, 2, 3, 4.
- “How is counting along this number line the same and different than counting along your number lines?” (On the whole number one each space is 1 so we just count 1, 2, 3, 4. On our number lines we still count the jumps, but now each space is smaller than 1 so we need the denominator to tell us the size of each space.)

Activity 2

🕒 10 min

Fractions on the Number Line

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 3.NF.A.2.b

The purpose of this activity is for students to locate a variety of fractions on the number line. Students are given a fraction less than 1 and greater than 1 with the same denominator to locate on each number line. The activity synthesis focuses on counting the number of unit fractions in a fraction to locate it on a number line and how to determine whether fractions are less than 1 or greater than 1. As they locate the fractions on the number lines, students strengthen their understanding of the meaning of the numerator and denominator of a fraction (MP6).

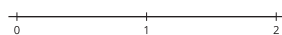
🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Student-facing Task Statement

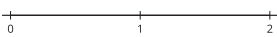
1. Locate and label $\frac{3}{4}$

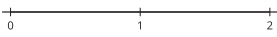


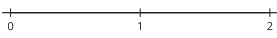
Launch

- Groups of 2
- Display the number line for fourths from

and $\frac{6}{4}$.

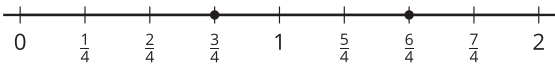
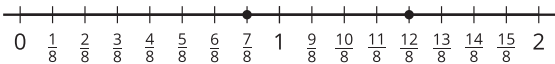
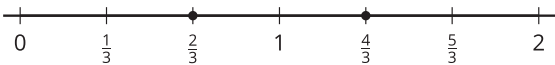
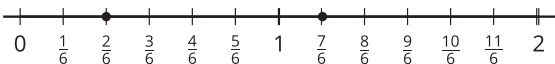
2. Locate and label $\frac{7}{8}$ and $\frac{12}{8}$.
- 

3. Locate and label $\frac{2}{3}$ and $\frac{4}{3}$.
- 

4. Locate and label $\frac{2}{6}$ and $\frac{7}{6}$.
- 

5. How did you partition the number line when you were locating the numbers $\frac{7}{8}$ and $\frac{12}{8}$? Explain your reasoning.
6. What patterns did you notice in the fractions you located?

Student Responses

1. 
2. 
3. 
4. 

5. Into eighths. The 8 on the bottom of the fraction tells us how many parts the whole, or the part from 0 to 1 is split into.
6. The first numbers were all less than 1, the second numbers were all greater than 1. In the first numbers, the top part of the fraction was a smaller number than the bottom part. In the second numbers, the top part was bigger than the bottom part.

Number Line Scoot.

- “What do you know about $\frac{3}{4}$ and $\frac{6}{4}$?” (They both have 4 on the bottom. Three is less than 4 and 6 is greater than 4. They are both numbers of fourths.)
- Share and record responses.

Activity

- “Work with your partner to locate the fractions on each number line and answer the questions about your work.”
- 4–6 minutes: partner work time
- Monitor for students who locate non-unit fractions on the number line by partitioning into equal parts of size $\frac{1}{b}$ and count the number of those parts.

Synthesis

- “What patterns did you notice in the fractions you located?”
- Consider asking:
 - “How do you know when a fraction is less than 1?”
 - “How do you know when a fraction is greater than 1?”
- “How did counting by unit fractions help you locate the other fractions on the number line?” (I counted by $\frac{1}{4}$ three times to find $\frac{3}{4}$. I counted by $\frac{1}{6}$ as I moved along the number line, like $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$, $\frac{7}{6}$ to find $\frac{7}{6}$.)

Advancing Student Thinking

If students partition the interval from 0 to 2 into fourths instead of the interval from 0 to 1 (or a similar error with another fraction), consider asking:

- “Tell me about how you found $\frac{3}{4}$ on the number line?”
- “How does the denominator help us know how to partition the number line?”

Activity 3

🕒 10 min

What’s the Fraction?

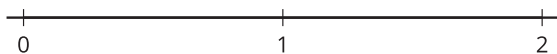
Standards Alignments

Addressing 3.NF.A.2.b

The purpose of this activity is for students to determine how a number line is partitioned and what fraction is marked on it with only 0, 1, and 2 labeled. Students partition and locate and mark, but don’t label, a fraction on a number line and then trade with a partner to determine the fraction their partner has marked. Remind students to mark, but not to label their partitions and their fraction, so that their partner only has the 0, 1, and 2 to use to determine what fraction is on their number line.

Student-facing Task Statement

1. Partition the number line into any number of equal-size parts. Locate and mark, but don’t label, a fraction of your choice.



2. Trade number lines with a partner.
 - a. How did your partner partition their number line?
 - b. What number did your partner mark on their number line? Explain your reasoning.

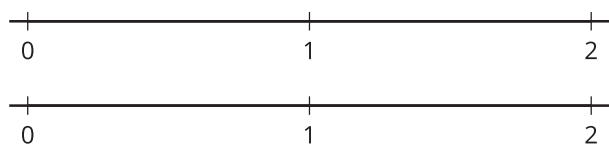
If you have time, play the game again.

Launch

- Groups of 2
- “Complete the first part of the activity on your own. Partition the number line and mark, but don’t label, a fraction on the number line. Don’t tell your partner how you are partitioning or what number you are marking.”
- 2 minutes: independent work time

Activity

- “Now, trade number lines with your partner and answer the questions about their number line. When you are both finished, share your reasoning”



Student Responses

1. Answers vary.
2. Answers vary.

- 1–2 minutes: independent work time
- 1–2 minutes: partner work time

Synthesis

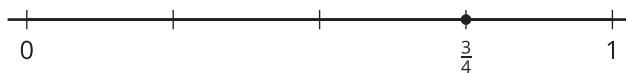
- Display a number line partitioned by a student.
- “Talk to your partner about what fraction is represented.”
- Share and record responses.
- Consider asking:
 - “How did you decide how to partition your number line and what fraction you’d put on your number line?”
 - “How did you decide how your partner’s number line was partitioned and decide what fraction was marked?”

Lesson Synthesis

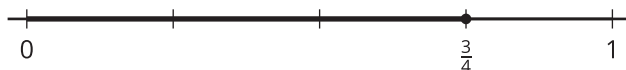
🕒 10 min

“Today we located more fractions on the number line. In an earlier lesson, we learned how fractions are built from unit fractions. How do we see this on the number line?” (I counted the unit fractions, like 3 one-fourths, to get to $\frac{3}{4}$. I partition the number line into unit fractions and then I can count parts up to the fraction I am locating.)

Draw or have students draw a number line with $\frac{3}{4}$ marked, such as:



Trace or have them trace through and count the 3 one-fourths to get to $\frac{3}{4}$, such as:



“Remember, when we are locating a fraction on the number line, it might be helpful to think about or show the 3 one-fourth parts, and then we mark and label the number $\frac{3}{4}$ at the end of those parts. When we locate and label fractions, you don’t have to mark the length, you can just count the unit fractions and then mark and label the point at the end.”

Point to the location of each fraction on the number line and count: " $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$."

Suggested Centers

- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

Complete Cool-Down

Response to Student Thinking

Students locate $\frac{2}{3}$, but don't locate $\frac{5}{3}$.

The work in this lesson builds from number line concepts developed in a prior unit.

Next Day Support

- During the launch of the next day's activity have students discuss how they would locate one of the fractions greater than 1 on the number line.

Prior Unit Support

Grade 2, Unit 4, Section A: The Structure of the Number Line

Lesson 8: Fractions and Whole Numbers

Standards Alignments

Addressing 3.NF.A.2, 3.NF.A.3.c, 3.OA.C.7

Teacher-facing Learning Goals

- Locate whole numbers on the number line given the location of a unit fraction and express them as fractions.
- Recognize that whole numbers can be written as fractions.

Student-facing Learning Goals

- Let's work with fractions and whole numbers on the number line.

Lesson Purpose

The purpose of this lesson is for students to recognize fractions that are equivalent to whole numbers and, given the location of a unit fraction on the number line, to locate whole numbers.

In previous lessons, students learned to partition number lines and located and labeled fractions on the number line. Students deepen their understanding of fractions as they consider which fractions are equivalent to whole numbers and relate that understanding to their knowledge of how many halves, thirds, and fourths, are in one whole. They leverage their understanding of how many halves, thirds, and fourths, are in one whole to locate whole numbers, such as 1 and 2, on the number line when given the location of a unit fraction.

Access for:



Students with Disabilities

- Representation (Activity 2)



English Learners

- MLR1 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |

Teacher Reflection Question

Think about times when students were able to make connections to and build on the ideas of their peers during discussions today. What norms or routines allowed students to engage

| | |
|------------------|--------|
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

with other students' ideas?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Where is 1?

Standards Alignments

Addressing 3.NF.A.2

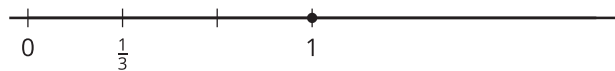
Student-facing Task Statement

Locate and label 1 on the number line.
Explain your reasoning.



Student Responses

I repeated the $\frac{1}{3}$ space 3 times since there are 3 one-thirds in 1.



Begin Lesson

Warm-up

⌚ 10 min

Number Talk: Divide by 4

Standards Alignments

Addressing 3.OA.C.7

This Number Talk encourages students to rely on their knowledge of multiplication, place value, and

properties of operations to mentally solve division problems. The reasoning elicited here helps to develop students' fluency with multiplication and division within 100.

To find the quotients of larger numbers, students need to look for and make use of structure in quotients that are smaller or more familiar, or to rely on the relationship between multiplication and division (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $12 \div 4$
- $24 \div 4$
- $60 \div 4$
- $72 \div 4$

Student Responses

- 3: I just knew it. $3 \times 4 = 12$
- 6: 24 is twice 12, so the value of $24 \div 4$ is twice that of $12 \div 4$ or twice 3, which is 6.
- 15:
 - I knew that 4×10 is 40 and 4×5 is 20, so 4×15 is 60.
 - I knew that $20 \div 4$ is 5 and 60 is 3 times 20, so $60 \div 4$ is 3 times 5.
- 18:
 - $72 = 12 + 60$, so I added the values of $12 \div 4$ and $60 \div 4$, which is $3 + 15$.
 - $4 \times 10 = 40$, $4 \times 8 = 32$, $10 + 8 = 18$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the earlier expressions help you find the value of the later expressions?"
- Consider asking:
 - "Did anyone have the same strategy but would explain it differently?"
 - "Did anyone approach the problem in a different way?"

Activity 1

🕒 15 min

Fractions Located at Whole Numbers

Standards Alignments

Addressing 3.NF.A.3.c

The purpose of this activity is for students to place fractions greater than 1 on the number line and notice how fractions can be written as whole numbers. For example, students will see that for halves, every second half is located at a whole number because it takes 2 halves to make a whole.

Students work in groups. Each member will be assigned a different set of fractions to put on their number line so that the group can look for patterns across halves, thirds, and fourths. Through repeated reasoning, students may notice two types of regularity (MP8):

- It takes 2 halves, 3 thirds, or 4 fourths to make a whole.
- Whole numbers appear regularly (every 2 halves, every 3 thirds).

Student-facing Task Statement

1. Locate and label your assigned fractions on the number line. Be prepared to explain your reasoning.



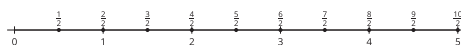
- a. $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{10}{2}$
- b. $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}$
- c. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, \frac{12}{4}$

2. List all the fractions that were located at a whole number in all three number lines that your group labeled.
3. What patterns do you see in all three labeled number lines?



Student Responses

1. a.



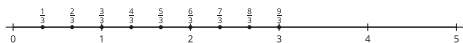
Launch

- Groups of 3
- Assign one set of fractions to each student in the group.

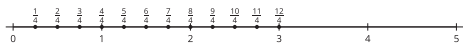
Activity

- "Take a few minutes to locate and label your assigned fractions on the number line."
- 2–3 minutes: independent work time
- "Share your strategy for locating the fractions with your group and look for patterns in the numbers together."
- 4–6 minutes: small-group discussion
- Monitor for students who:
 - Notice that every 2 halves, 3 thirds, and 4 fourths ends up at a whole number.
 - Notice that the numerator is a multiple of the denominator.

b.



c.



2. $\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}, \frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{4}{4}, \frac{8}{4}, \frac{12}{4}$
3. Sample response: For halves, every other fraction on the list ended up at a whole number. For thirds, every third fraction ended up at a whole number, and for fourths every fourth number.

Synthesis

- Display 3 blank number lines from 0 to 5 to label as students share.
- Select previously identified students to share the patterns they noticed in the fractions that share the same location as the whole numbers.
- “Why might it make sense that the fractions show those patterns?” (Sample responses:
 - because it takes 2 halves, 3 thirds, or 4 fourths to make a whole.
 - because there are 2 halves in 1, so there are 2×2 or 4 halves in 2, 3×2 or 6 halves in 3, and so on.)
- Label the number line as students share, particularly the whole numbers, like: $1 = \frac{2}{2}$, $2 = \frac{4}{2}$, $3 = \frac{6}{2}$ to highlight the idea that the number of equal parts (2, 3, or 4) in the fractions affects when you end up at a whole number.

Activity 2

⌚ 20 min

Locate 1 on the Number Line

Standards Alignments

Addressing 3.NF.A.2

The purpose of this activity is for students to use the location of a unit fraction to locate 1 and 2 on a number line. It is likely students will reason about repeating the size of the unit fraction to locate 1. To locate 2 on the number lines, they may continue to count unit fraction size parts or use the location of 1 to locate 2.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “How did you locate 1 when given the location of a unit fraction?” Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation or representation based on the feedback they receive.

Advances: Writing, Representing, Speaking, Listening

Access for Students with Disabilities

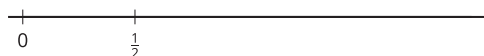
Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were needed or most useful to solve the problem. Display the sentence frame, “The next time I locate 1 on a number line, I will look for/pay attention to . . .”

Supports accessibility for: Conceptual Processing

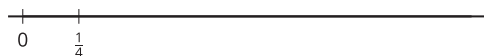
Student-facing Task Statement

1. Locate and label 1 on each number line. Be prepared to explain your reasoning.

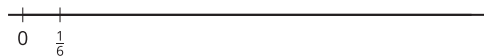
a.



b.



c.



d.



2. How could you locate 2 on the number lines in the previous problem?

Student Responses

1. a.



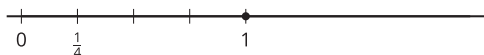
Launch

- Groups of 2
- Display the number line with $\frac{1}{2}$ marked.
- “What do you notice? What do you wonder?” (Students may notice: The number line only has 0 on one end and no whole numbers on the other end. One-half is labeled. Students may wonder: Is the number line partitioned into halves? Where is 1? What other numbers are on the number line? Why is nothing marked after $\frac{1}{2}$?)
- 1 minute: quiet think time
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

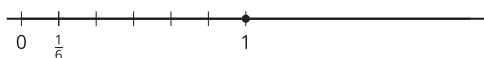
Activity

- “Take a few minutes to locate 1 on these number lines.”
- 3–5 minutes: independent work time

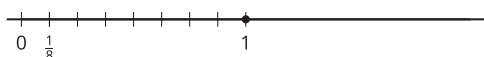
b.



c.



d.



2. I could continue to mark unit fractions past 1 and count up to 2. I could use the location of 1 and double that size to find 2.

- “Share your strategies with your partner and talk about how you might locate 2 on these number lines.”
- 3–5 minutes: partner work time
- Monitor for students who:
 - iterate the size of the unit fraction using tick marks
 - make unit fraction size jumps to count up to 1
 - realize there will be 4 one-fourths in 1, for example, and place the 1 before placing $\frac{2}{4}$ and $\frac{3}{4}$

Synthesis

- Invite students to share a variety of strategies or representations of the number line for locating 1 when given the location of a unit fraction.
- Consider asking:
 - “Did anyone think about it in a similar way?”
 - “Does anyone want to add on to ____’s reasoning?”
- “What did you notice about how different people located 1?” (Sample responses: They marked off the lengths of the unit fraction until reaching 1 whole. They used a multiple of a unit fraction and marked off that length as many times as needed to get 1 whole.)
- “What strategies did you have for locating 2 once you had located 1?”

Advancing Student Thinking

If students don’t locate 1, consider asking:

- “Tell me about what you’ve tried to locate 1.”
- “How many halves (or thirds, fourths, or eighths) are in 1? How could we use that to locate 1?”

Lesson Synthesis

🕒 10 min

"Today we saw that some fractions were located at the same location as whole numbers. What were some examples of fractions that were located at the same location as whole numbers?" ($\frac{2}{2}$, $\frac{6}{3}$, $\frac{8}{4}$)

"How could we explain how fractions and whole numbers were in the same location on the number line?" (Every 2 halves (or 3 thirds or 4 fourths) you are at a whole number, so if you go 2 halves you are at 1. If you moved another 2 halves (or 3 thirds or 4 fourths) you would be at $\frac{4}{2}$ which is at the next whole number, which is 2.)

Suggested Centers

- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)
- Number Line Scoot (2–3), Stage 2: Halves, Thirds and Fourths (Addressing)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not use the length of the given unit fraction to locate 1.

The work in this lesson builds from number line concepts developed in a prior unit.

Next Day Support

- During the launch of the next day's activity, have students complete a choral count by thirds, starting at $\frac{1}{3}$. Have students raise their hand, stop the count, and explain their reasoning when they get to the fraction that is equivalent to 1.

Prior Unit Support

Grade 2, Unit 4, Section A: The Structure of the Number Line

Lesson 9: All Kinds of Numbers on the Number Line

Standards Alignments

Addressing 3.NF.A.2

Teacher-facing Learning Goals

- Locate 1 on the number line given the location of a non-unit fraction.

Student-facing Learning Goals

- Let's locate numbers on the number line when we are given the location of one fraction.

Lesson Purpose

The purpose of this lesson is for students to use their knowledge of fractions on the number line to locate 1 when given a non-unit fraction.

Previously, students have located fractions on the number line, including locating 1 when given a unit fraction. In the first activity, students reinforce their understanding of the numerator and denominator of a fraction as they find 1 given a fraction greater than 1. They use the numerator of the given fraction to identify the size of a unit fraction and then to locate 1. Later, they locate a non-unit fraction given the location of a unit fraction with a different denominator. There, students use their knowledge of locating 1 first and then locating the non-unit fraction from 1.

The second activity in this lesson is optional because it goes beyond the depth of understanding required to address grade 3 standards.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

5 Practices (Activity 2), MLR1 Stronger and Clearer Each Time (Activity 1), Which One Doesn't Belong? (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What unfinished learning or misunderstandings do your students have about fractions? How did you leverage those misconceptions in a positive way to further the understanding of the class?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Where is 1 Now?

Standards Alignments

Addressing 3.NF.A.2

Student-facing Task Statement

Locate and label 1 on the number line. Explain your reasoning.

**Student Responses**

I know there are 7 one-sixths in $\frac{7}{6}$, so I split the space into 7 equal parts. I counted 6 of the parts to get to 1.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Which One Doesn't Belong: Many Number Lines

Standards Alignments

Addressing 3.NF.A.2

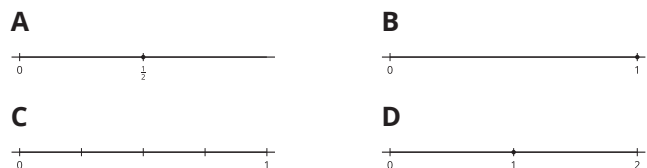
This warm-up prompts students to compare four number lines. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the number lines in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use, such as parts, partitions, mark, label, halves, fourths, or whole.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?



Student Responses

Sample responses:

- A is the only one that doesn't have 1 labeled on it.
- B is the only one that isn't partitioned into multiple parts.
- C is the only one that doesn't have any points marked on it.
- D is the only one that does not end near 1.

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "How did you know that the number line in A stopped at 1?" (The location of $\frac{1}{2}$ shows that the whole number line is split in half right there and I know that 2 halves is 1. I can only fit one more half next to the first one.)
- Consider asking: "Let's find at least one reason why each one doesn't belong."

Activity 1

🕒 20 min

Locate 1 Again

Standards Alignments

Addressing 3.NF.A.2

The purpose of this activity is for students to locate 1 on a number line given the location of a non-unit fraction less than 1 or greater than 1. In either case, it is likely students will reason about unit fractions to locate 1.

In the first problem, students may use the size of thirds to locate 1. In the second problem, they reinforce their knowledge that the denominator of a fraction tells us the number of equal parts in a whole and the size of a unit fraction, and that the numerator gives the number of those parts (MP6). Students typically use the denominator to partition a number line, but here they need to use the numerator.

This activity uses *MLR1 Stronger and Clearer Each Time*.

Advances: reading, writing

🕒 Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Synthesis. Identify connections between strategies that result in the same outcomes but use differing approaches.

Supports accessibility for: Conceptual Processing, Memory

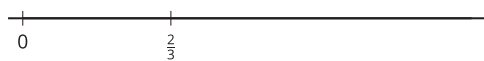
Instructional Routines

MLR1 Stronger and Clearer Each Time

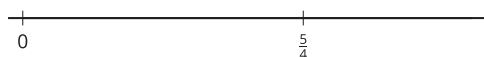
Student-facing Task Statement

1. Locate and label 1 on each number line.

a.



b.



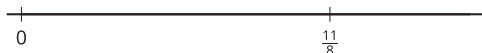
Launch

- Groups of 2

Activity

- “Take a few minutes to locate 1 on these number lines. Then use any of the number lines to explain how you located 1.”
- 5–7 minutes: independent work time

c.



2. Use any of the number lines to explain how you located 1.



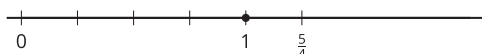
Student Responses

1.

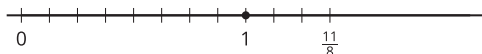
a.



b.



c.



2. Sample response: I know that it takes 5 one-fourths to get to $\frac{5}{4}$, so I partitioned that space on the number line into 5 parts. Then I counted to $\frac{4}{4}$ to label 1.

Synthesis

MLR1 Stronger and Clearer Each Time

- “Share your written reasoning for one of the number lines with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work.”
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- “Revise your initial draft based on the feedback you got from your partners.”
- 2–3 minutes: independent work time
- Invite students to share their revised explanations of how they located 1 on the number lines.

Advancing Student Thinking

If students say they aren't sure how to get started, consider asking:

- “What do we know about this fraction?”
- “How could that help us find 1?”

Activity 2 (optional)

🕒 15 min

Locate $\frac{3}{4}$

Standards Alignments

Addressing 3.NF.A.2

The purpose of this activity is for students to use the location of a unit fraction to locate another fraction with a different denominator on the number line. Students can use their knowledge from the previous activity to place 1 on the number line and then use that to partition the interval from 0 to 1 to find other numbers. Because students have only located fractions with the same denominator on a single number line, they may want to use more than one number line in this activity. They may or may not label the points they find along the way to $\frac{3}{4}$. Encourage them to use whatever strategy makes sense to them.

Monitor for students who use a single number line to show both thirds and fourths and those who use separate number lines. Select them to share during activity synthesis.

This activity is optional because it goes beyond the depth of understanding required to address grade 3 standards.

Access for English Learners

MLR8 Discussion Supports. Synthesis: As students share the similarities and differences between the strategies, use gestures to emphasize what is being described. For example, point to each fraction and show with your fingers the partitions such as thirds and fourths, that are being discussed.

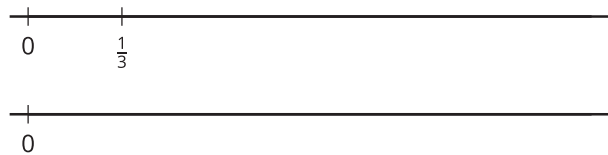
Advances: Listening, Representing

Instructional Routines

5 Practices

Student-facing Task Statement

Locate and label $\frac{3}{4}$ on the number line. Be prepared to explain your reasoning.



Launch

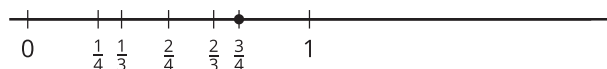
- Groups of 2
- "Now we're going to try something a little bit different. Let's use the location of a unit fraction to find a fraction with a different denominator."

Activity

- "Take a few minutes to locate $\frac{3}{4}$ on this number line. Use any strategy that makes

Student Responses

Sample response:



sense to you.”

- 3–5 minutes: independent work time
- As students work, consider asking:
 - “How did you decide whether to use one or two number lines?”
 - “Did you locate any numbers before locating $\frac{3}{4}$?”
 - “Where is 1 on your number line(s)?”
- “Share your strategy with your partner.”
- 2–3 minutes: partner discussion

Synthesis

- Ask the two selected students to display their work side-by-side for all to see.
- “What do these strategies have in common? How are these representations different?” (They both located 1 before locating $\frac{3}{4}$. In the first strategy, the thirds are one number line and used to find 1. Then, 1 is on the second number line and used to find $\frac{3}{4}$. In the second strategy, the thirds and fourths are marked on the same number line.)
- Consider asking: “What questions do you have about these representations?”

Lesson Synthesis

🕒 10 min

Display fraction strips and a number line.

“Work with your partner to brainstorm all the things you’ve learned about fractions so far. Then, we’ll share and record our ideas.” (The numerator is the top part of a fraction and the denominator is the bottom part. Fractions can be represented with diagrams, fraction strips, and number lines. Number lines can be partitioned to show unit fractions and non-unit fractions, and fractions less than 1 and greater than 1. Non-unit fractions are built from unit fractions.)

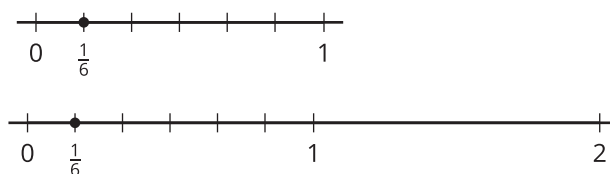
Share and record ideas.

Suggested Centers

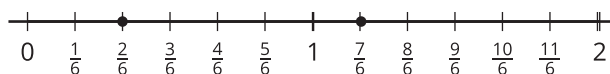
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)

Student Section Summary

In this section, we located and labeled fractions on the number line. We learned how to partition the number line from 0 to 1 to locate unit fractions.



Then we used the location of unit fractions to locate other fractions.



We also learned that some fractions are at the same location as whole numbers on the number line. Here, we can see that $\frac{6}{6}$ shares the same location as 1 and $\frac{12}{6}$ shares the same location as 2.

At the end of the section, we used our understanding of unit fractions to locate 1 on the number line when we only knew the location of a fraction.

----- Complete Cool-Down -----

Response to Student Thinking

Students do not use $\frac{1}{6}$ or partition the interval between 0 and $\frac{7}{6}$ into 7 equal parts to 1.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Section C: Equivalent Fractions

Lesson 10: Equivalent Fractions

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

Building Towards 3.NF.A.2

Teacher-facing Learning Goals

- Identify equivalent fractions.
- Understand two fractions as equivalent if they are the same size and the parts refer to the same whole.

Student-facing Learning Goals

- Let's identify equivalent fractions.

Lesson Purpose

The purpose of this lesson is for students to see that different fractions can be equivalent if they are the same size of the same whole.

Previously, students were introduced to unit fractions and non-unit fractions using area diagrams, fraction strips, and number lines. They began to work with the idea of equivalence by noticing fractions that are also whole numbers. Here, students revisit area diagrams and fraction strips to learn about fraction equivalence. Students learn that fractions that are the same size are **equivalent fractions**. Later, they will identify equivalent fractions as having the same location on a number line.

Access for:



Students with Disabilities

- Representation (Activity 1)



English Learners

- MLR7 (Activity 1)

Instructional Routines

Choral Count (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What ideas do students have about what it means for fractions to be equivalent? How can you build on those ideas in this section?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Find the Equivalent Fractions

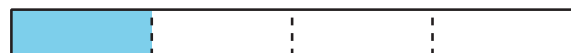
Standards Alignments

Addressing 3.NF.A.3.a

Student-facing Task Statement

Each diagram represents 1.

Select **all** the diagrams whose shaded parts represent equivalent fractions.
Explain your reasoning.

A**B****C****D****E****Student Responses**

C and E. Sample responses: They show different fractions, but are the same size. They are

partitioned into different numbers of parts, but the shaded portions are the same size.

----- Begin Lesson -----

Warm-up

🕒 10 min

Choral Count: One-halves

Standards Alignments

Addressing 3.NF.A.3.b

Building Towards 3.NF.A.2

The purpose of this Choral Count is to invite students to practice counting by $\frac{1}{2}$ and notice patterns in the count. These understandings help students develop fluency and will be helpful later in this lesson when students recognize and generate equivalent fractions. In the synthesis, students have the opportunity to notice that $\frac{2}{2}$ and $\frac{4}{4}$ are both equal to 1 whole.

Instructional Routines

Choral Count

Required Preparation

- Have recording of choral count by one-fourth available, from a previous lesson.

Student Responses

Sample responses:

- The denominator of the fraction never changes.
- The numerator of the fraction is increasing by 1.
- Each row ends with a number you say when you count by 2.

Launch

- "Count by $\frac{1}{2}$, starting at $\frac{1}{2}$."
- Record as students count. Record 2 fractions in each row, then start a new row. There will be 4 rows.
- Stop counting and recording at $\frac{8}{2}$.

Activity

- "What patterns do you see?"

- 1–2 minutes: quiet think time
- Record responses.

Synthesis

- Display count by $\frac{1}{4}$ from the previous lesson. There should be 4 rows and 4 fractions in each row with the count ending at $\frac{16}{4}$.
- “How are these two counts the same? How are they different?” (The denominator stays the same in both counts—4 for the last count, and 2 for today’s count. The numerators change in the same way because they both count by one. They start a new line at $\frac{2}{2}$ and $\frac{4}{4}$, which are both whole numbers.)
- Consider asking:
 - “Who can restate the pattern in different words?”
 - “Does anyone want to add an observation as to why that pattern is happening here?”
 - “Do you agree or disagree? Why?”

Activity 1

🕒 15 min

Equivalent to $\frac{1}{2}$

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to consider equivalent fractions using diagrams. One half has been chosen to introduce equivalent fractions because there are many ways to see and represent fractions that are equivalent to $\frac{1}{2}$. Many students may be familiar with the concept of halves and justify equivalence by saying 2 is half of 4. This reasoning is helpful with 1 half and 2 fourths but may not be generalizable to other cases of equivalence. For this reason, the activity synthesis focuses on justifications about whether or not the shaded parts are the same size. The

idea that $\frac{1}{2}$ and $\frac{2}{4}$ are the same size is used to define **equivalent fractions** as fractions that are the same size.

Students need to use language carefully as they explain why the shaded parts of a shape show $\frac{1}{2}$ (MP6). For example, they may say that 2 of 4 equal parts in shape D are shaded, but if they combine those parts, the total shaded amount is the same as in the shape where 1 of 2 equal parts is shaded.

Access for English Learners

MLR7 Compare and Connect. Synthesis: Lead a discussion comparing, contrasting, and connecting shapes C and D. Ask, "How are shapes C and D the same?", "How are they different?", and "How do these two different representations show $\frac{1}{2}$?"

Advances: Representing, Conversing

Access for Students with Disabilities

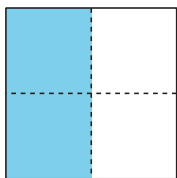
Engagement: Provide Access by Recruiting Interest. Synthesis: Invite students to share connections between finding one-half in fractions with more than two equal parts in this activity and when they might, in their own lives, see one half when there are more than 2 equal parts.

Supports accessibility for: Visual-Spatial Processing

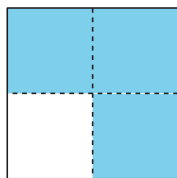
Student-facing Task Statement

- For which shapes is the shaded portion $\frac{1}{2}$ of the shape? Be prepared to share your reasoning.

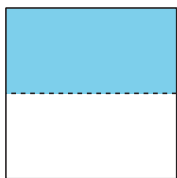
A



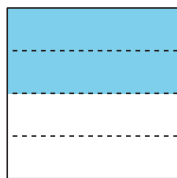
B



C



D

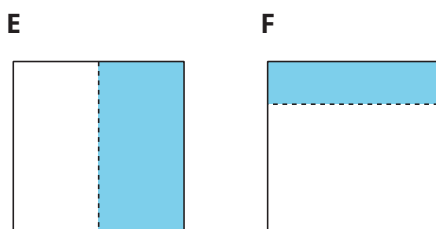


Launch

- Groups of 2
- "What do you know about $\frac{1}{2}$?" (There are 2 equal parts. The parts have to be the same size. One of the parts would be shaded.)
- 1 minute: quiet think time
- Share and record responses.

Activity

- "Now work with your partner to select all the shapes where the shaded portion represents $\frac{1}{2}$ of the shape and explain how there are more than one shape where this is the case."
- 5–7 minutes: partner work time
- Monitor for students who explain that the



2. How can there be more than one way of shading a shape to show $\frac{1}{2}$?

Student Responses

1. A, C, D, E. Sample responses:
 - A is $\frac{1}{2}$ because it's partitioned into four parts and 2 are shaded, but one half of the whole square is shaded.
 - F isn't $\frac{1}{2}$ because the two parts aren't equal.
2. Sample response: C and E both show $\frac{1}{2}$ because they have both been partitioned into 2 equal parts and 1 part is shaded. C and D both show $\frac{1}{2}$ because the same amount is shaded in each square.

shading in A and D both represents $\frac{1}{2}$ of the shape.

Synthesis

- Invite students to share their responses.
- Display C and D.
- “How can the shaded portion in each show $\frac{1}{2}$ when the squares have been partitioned into a different number of equal parts?” (The shaded part is the same size even though they look different. The same amount of the square is shaded.)
- “Even though C is partitioned into halves and D is partitioned into fourths, we can say that $\frac{1}{2}$ of each square is shaded because the same amount is shaded as in squares C and D, which means the two fractions are the same size.”
- “Two numbers that are the same size are equivalent, so the fractions $\frac{2}{4}$ and $\frac{1}{2}$ are **equivalent fractions**.”

Activity 2

🕒 20 min

Find Equivalent Fractions

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to use fraction strips to identify equivalent fractions and explain why they are equivalent. Highlight explanations that make clear that the parts that represent the fractions are the same size and the parts of the fractions refer to the same whole.

Materials to Gather

Materials from a previous lesson

Required Preparation

- Students need the fraction strips they made in a previous lesson.

Student-facing Task Statement

Use your fraction strips from an earlier lesson to find as many equivalent fractions as you can that are equivalent to:

- $\frac{1}{2}$
- $\frac{2}{3}$
- $\frac{6}{6}$
- $\frac{3}{4}$

Be prepared to show how you know the fractions are equivalent.

Student Responses

Sample responses:

- $\frac{1}{2}$ is equivalent to $\frac{3}{6}$ because they are the same size.
- $\frac{2}{3}$ is equivalent to $\frac{4}{6}$ because they are the same size.
- $\frac{6}{6}$ is equivalent to $\frac{4}{4}$ because they are both made with the whole strip.
- $\frac{3}{4}$ is equivalent to $\frac{6}{8}$ because they are the same size.

Launch

- Groups of 2
- Ask students to refer to the fraction strips they made in an earlier lesson.

Activity

- "Use your fraction strips to find as many fractions as you can that are equivalent to the listed fractions."
- 5–7 minutes: independent work time
- If students have extra time, encourage them to use their fraction strips to find other pairs of fractions that are equivalent.
- "Now, share the equivalent fractions you found with your partner. Be sure to share your reasoning."
- 3–5 minutes: partner discussion
- Monitor for students who explain equivalence by saying that the fractions are the same size.

Synthesis

- Invite students to share pairs of equivalent fractions and why they are equivalent. Highlight that the fractions are equivalent because the part of the strips that represent the fractions are the same size.
- Display a set of fraction strip diagram for all to see.
- As students share, mark up the fraction strip diagram to illustrate the equal size of the parts (for example, by drawing lines or

circling the parts). Then, record pairs of equivalent fractions using the equal sign like: $\frac{1}{2} = \frac{3}{6}$.

Advancing Student Thinking

If students don't generate an equivalent fraction for one of the given fractions, consider asking:

- "How did you represent the fraction with the fraction strips?"
- "How could you use the fraction strips to make an equivalent fraction?"

Lesson Synthesis

🕒 10 min

"If you were given two fractions, how could you determine whether they are equivalent?" (I would look at diagrams of them to see if the fractions are the same size. I would use fraction strips to see if the fractions were the same size.)

Suggested Centers

- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)

----- Complete Cool-Down -----

Response to Student Thinking

Students choose two fractions that are close in size, but not equivalent.

Next Day Support

- Add this cool-down to Activity 1 to review.

Lesson 11: Generate Equivalent Fractions

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b, 3.OA.B.5
 Building Towards 3.OA.C.7

Teacher-facing Learning Goals

- Use diagrams to explain or show fraction equivalence.
- Use diagrams to generate equivalent fractions.

Student-facing Learning Goals

- Let's generate equivalent fractions.

Lesson Purpose

The purpose of this lesson is for students to generate equivalent fractions.

In previous lessons, students learned what it means for two fractions to be equivalent. In this lesson, students continue to reason about and show equivalence visually, building on their work with fraction strips. They use shaded diagrams to help them generate equivalent fractions, including fractions greater than 1. The work here prepares students to use number lines to explain fraction equivalence later in the section.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |

Teacher Reflection Question

Which students had opportunities to share their diagrams and thinking during whole-class discussion? How did you select these students?

Lesson Synthesis

10 min

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

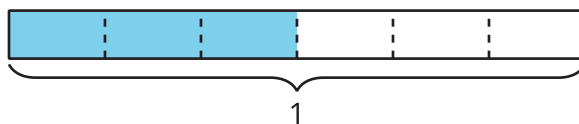
Two Fraction Names for Each Diagram

Standards Alignments

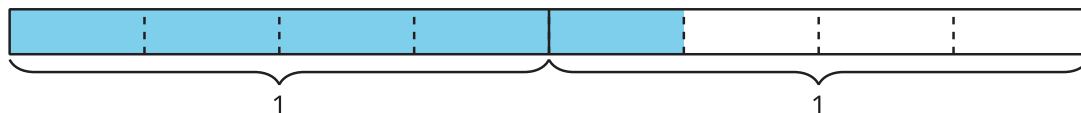
Addressing 3.NF.A.3.b

Student-facing Task Statement

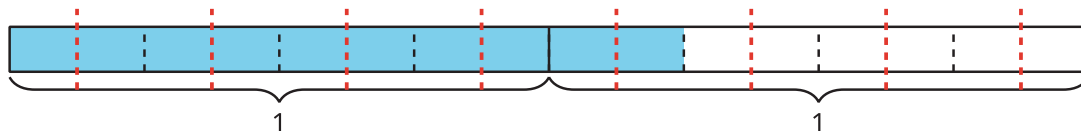
- Write two fractions that the shaded part of this diagram represents.



- Show that the shaded part of this diagram represents both $\frac{5}{4}$ and $\frac{10}{8}$.

**Student Responses**

- $\frac{3}{6}, \frac{1}{2}$
- Sample response: Each 1 whole is partitioned into fourths. Five fourths are shaded, which represents $\frac{5}{4}$. Each fourth can be split into two equal parts, which makes 8 eighths in 1 whole. Ten eighths are shaded, so that's $\frac{10}{8}$.



----- Begin Lesson -----

Warm-up

🕒 10 min

Number Talk: Something Times 8

Standards Alignments

Addressing 3.OA.B.5

Building Towards 3.OA.C.7

This Number Talk encourages students to look for structure in multiplication expressions and rely on properties of operations to mentally solve problems. Reasoning about products of whole numbers helps to develop students' fluency.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 2×8
- 6×8
- 10×8
- 12×8

Student Responses

- 16: I just knew it. It's $8 + 8$, which is 16.
- 48: I knew 3×8 is 24, and 6×8 is twice that number, which is 48. It's 3 groups of 2×8 , so it's two more groups of 16, or $16 + 16 + 16$, which is 48.
- 80: I know 5×8 is 40 and 10 is twice 5, so 10×8 is 80. I just knew it.
- 96: 12 is $10 + 2$, so 12×8 is $(10 \times 8) + (2 \times 8)$ or $80 + 16$, which is 96.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the earlier expressions help you find the value of the last expression?"
- Consider asking:
 - "Did anyone have the same strategy but would explain it differently?"
 - "Did anyone approach the problem in a different way?"

Activity 1

🕒 20 min

Show Equivalence

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to use diagrams to reason about equivalence and reinforce their awareness of the relationship between fractions that are equivalent.

Students show that a shaded diagram can represent two fractions, such as $\frac{1}{2}$ and $\frac{4}{8}$, by further partitioning given parts or composing larger parts from the given parts. Unlike with the fraction strips, where different fractional parts are shown in rows and students could point out where and how they see equivalence, here students need to make additional marks or annotations to show equivalence.

In upcoming lessons, students will extend similar strategies to reason about equivalence on a number line—by partitioning the given intervals on a number line into smaller intervals or by composing larger intervals from the given intervals.

In the first problem, students construct a viable argument in order to convince Tyler that $\frac{4}{8}$ of the rectangle is shaded (MP3).

🕒 Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Synthesis: Identify connections between strategies that result in the same outcomes but use differing approaches.

Supports accessibility for: Memory, Visual-Spatial Processing

Student-facing Task Statement

1. The diagram represents 1.



- What fraction does the shaded part of the diagram represent?
- Jada says it represents $\frac{4}{8}$. Tyler is not so sure.

Launch

- Groups of 2

Activity

- “Work with your partner on the first problem. Discuss whether you agree with Jada and show your reasoning.”
- 3–4 minutes: partner work time

Do you agree with Jada? If so, explain or show how you would convince Tyler that Jada is correct. If not, explain or show your reasoning.

2. Each diagram represents 1.

- a. Show that the shaded part of this diagram represents both $\frac{1}{3}$ and $\frac{2}{6}$.



- b. Show that the shaded part represents both $\frac{6}{8}$ and $\frac{3}{4}$.

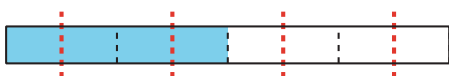


- c. Show that the shaded part represents both $\frac{6}{6}$ and $\frac{2}{2}$.



Student Responses

1.
 - a. $\frac{2}{4}$ or $\frac{1}{2}$
 - b. Agree. Sample response: Each fourth in the diagram can be split into 2 equal parts, making 8 parts. Four of the parts are shaded, which represents $\frac{4}{8}$.



2. Sample response:

- a. One of the 3 parts is shaded, which represents $\frac{1}{3}$. Each third can be split into 2 equal parts, making 6 parts. Two of the parts are shaded, which represents $\frac{2}{6}$.

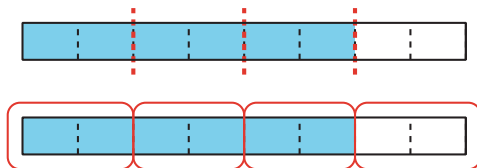


- Pause for a brief discussion. Invite students to share their responses and reasoning.
- “Now, work independently on the rest of the activity.”
- 5 minutes: independent work time
- Monitor for the different strategies students use to show equivalence, such as:
 - drawing circles or brackets to show composing larger parts from the given parts
 - drawing lines to show new partitions
 - labeling parts of the fractions with two names
 - drawing a new diagram with different partitions but the same shaded amount
- Identify students using different strategies to share during synthesis.

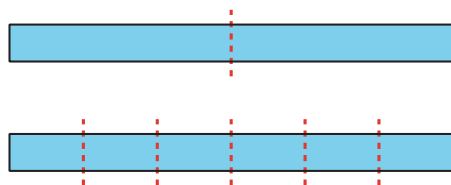
Synthesis

- Select previously identified students to share their responses and reasoning. Display their work for all to see.
- As students explain, describe the strategies students use to show equivalence. Ask if others in the class showed equivalence the same way.

- b. Six of the 8 parts are shaded, which represents $\frac{6}{8}$. If we put 2 eighths together, we'd have 4 larger equal parts. Three of those 4 parts represents $\frac{3}{4}$.



- c. The diagram can be partitioned into 2 equal parts. Both parts are shaded, which represents $\frac{2}{2}$. Each half can be split into 3 equal parts, making 6 parts. All 6 parts are shaded, which represents $\frac{6}{6}$.



Advancing Student Thinking

If students don't explain how the pairs of fractions are equivalent, consider asking:

- "What does it mean for fractions to be equivalent?"
- "How could we show both fractions to determine if they are equivalent?"

Activity 2

🕒 15 min

More Than One Name

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to generate equivalent fractions, including for fractions greater than 1, given partially shaded diagrams. Student may use strategies from an earlier activity—partitioning a diagram into smaller equal parts, or making larger equal parts out of existing parts—or patterns they observed in the numerators and denominators of equivalent fractions (MP7).

Access for English Learners

MLR8 Discussion Supports. Students should take turns naming the equivalent fractions they came up with and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____, so I thought . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Representing

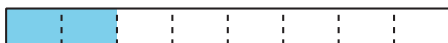
Student-facing Task Statement

- Each diagram represents 1. Write two fractions to represent the shaded part of each diagram.

a.



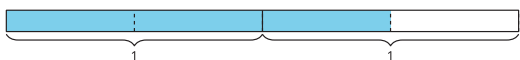
b.



c.



- Here's another diagram.



- What fraction does the shaded part of the diagram represent?
- Write another fraction that it represents.

Student Responses

- a. $\frac{4}{6}$, $\frac{2}{3}$

Launch

- Groups of 2
- Display or draw a diagram with 2 fourths shaded:



- “Notice there's a 1 below the diagram. This is another way to show which part of the diagram represents 1.”
- “What fractions can the shaded parts of the diagram represent?” ($\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$)

Activity

- “Now write two fractions that you think represent the shaded parts of each diagram.”
- 3–5 minutes: independent work time
- “Discuss the names you came up with for each fraction with your partner. Be sure to share your reasoning for each fraction.”
- 2–3 minutes: partner discussion
- Monitor for students who make statements

- b. $\frac{2}{8}, \frac{1}{4}$
 c. $\frac{4}{4}, \frac{8}{8}, \frac{2}{2}$. Students may also say $\frac{3}{3}$ and $\frac{6}{6}$.
2. a. $\frac{3}{2}$
 b. $\frac{6}{4}, \frac{12}{8}, \frac{9}{6}$

like:

- The first diagram is $\frac{4}{6}$, because 4 of the 6 equal parts are shaded. It's also $\frac{2}{3}$ because every 2 sixths is 1 third and there are 3 thirds. Two of the 3 thirds are shaded.
- The second diagram is $\frac{2}{8}$ because 2 of the 8 equal parts are shaded. It's also $\frac{1}{4}$ because every 2 eighths is 1 fourth, and 1 of the 4 fourths is shaded.

Synthesis

- Select students to share their strategies for writing multiple fractions for each diagram. Display the diagrams they marked or annotated.
- "In what ways was the last diagram different than the first three?" (It shows 2 wholes. The shaded parts were greater than 1.)
- "Was your strategy for finding fractions for this diagram different from the first three? Why or why not?" (No, it still involved making smaller equal parts. Yes, I partitioned the first 1 whole and the second 1 whole separately.)
- If no students mention $\frac{12}{8}$ for the last diagram, ask, "Can you name another fraction other than $\frac{3}{2}$ and $\frac{6}{4}$?"

Advancing Student Thinking

If students name a fraction, based only on the given partitions, consider asking:

- "Tell me about how you named the fraction."
- "How could you use the diagram to find another way you could name the fraction?"

Lesson Synthesis

🕒 10 min

"Today, we saw that the shaded parts of a diagram can be represented by multiple equivalent fractions."

Display a diagram of labeled fraction strips from an earlier activity, and a couple of shaded diagrams that show equivalent fractions from this activity.

"How did we use the fraction strips to help us see and name equivalent fractions?" (We could see if some number of parts in one row is the same size as the parts in another row. The labels on the strips help us name the fractions that are equivalent.)

"How did the shaded diagrams in this activity help us see and name equivalent fractions?" (We could either partition the diagram into smaller equal parts, or put the parts together to make larger equal parts.)

Suggested Centers

- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)

Complete Cool-Down

Response to Student Thinking

Students generate two equivalent fractions, but don't show or explain their reasoning around why any of the fractions are equivalent.

Next Day Support

- Before the warm up, select a student's cool down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool down.

Lesson 12: Equivalent Fractions on a Number Line

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

Building Towards 3.NF.A.3

Teacher-facing Learning Goals

- Identify and generate equivalent fractions.
- Understand two fractions as equivalent if they are at the same point on a number line.

Student-facing Learning Goals

- Let's find fractions at the same location.

Lesson Purpose

The purpose of this lesson is for students to use the number line to determine whether fractions are equivalent.

In previous lessons, students learned that two fractions are equivalent if they are the same size. In this lesson, students work with situations that involve lengths to build their understanding that fractions at the same location on a number line are equivalent. Number lines are provided to ensure that if students choose to use two number lines to demonstrate equivalence, they work with the same length interval for 1 unit.

Access for:



Students with Disabilities

- Engagement (Activity 2)



English Learners

- MLR8 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Number cubes: Activity 3

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

In this lesson, students use number lines to

| | |
|------------------|--------|
| Activity 1 | 10 min |
| Activity 2 | 10 min |
| Activity 3 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

show that fractions are equivalent. How did their previous work with diagrams and fraction strips prepare them to do this?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

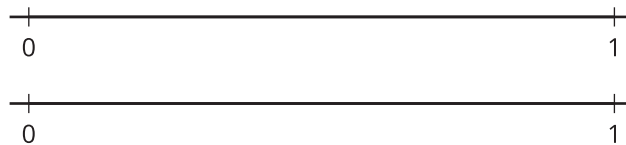
Equivalence on the Number Line

Standards Alignments

Addressing 3.NF.A.3.a

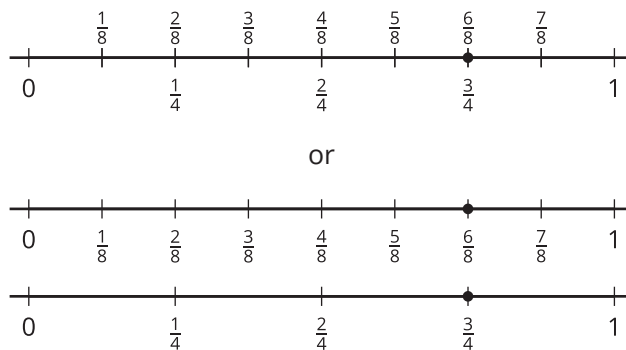
Student-facing Task Statement

Use the number line(s) to decide whether $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent. Explain your reasoning.



Student Responses

$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent because they are at the same point on the number line. Sample responses:



Begin Lesson

Warm-up

🕒 10 min

Notice and Wonder: Running on a Trail

Standards Alignments

Building Towards 3.NF.A.3

The purpose of this warm-up is to elicit the idea that fractions can be used to describe lengths. While students may notice and wonder many things about this statement, the idea that Han and Tyler could have run the same distance or different distances are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Tyler ran part of the length of a trail.
Han ran part of the length of the same trail.

**Student Responses**

Students may notice:

- Neither of them ran the length of the entire trail.
- It doesn't say what part of the trail they ran.

Students may wonder:

- How long is the trail?
- Did they run the same distance?

Launch

- Groups of 2
- Display the statement.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How could fractions give us more information about how far Tyler and Han ran?" (Tyler and Han ran $\frac{1}{2}$ of the field. Tyler ran $\frac{3}{4}$ of the field and Han ran $\frac{7}{8}$ of the field.)
- "What questions could we ask about the situation?" (Did they run the same distance? Who ran farthest? How much farther did one student run than the other?)

- Did one run farther than the other?
- How much of the trail did they run?

Activity 1

🕒 10 min

Running Part of a Trail

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to explain equivalence using a number line. Students are given situations in a measurement context and have to determine whether the distance is the same. Students are encouraged to use a number line to provide an opportunity to explain fraction equivalence as fractions that are at the same location. They may choose to use two number lines for each question (one for each fraction). Choosing to use one number line or two will be discussed in the synthesis of the next activity.

When they identify whether or not two fractions of the same trail represent the same distance, students reason abstractly and quantitatively (MP2).

🌐 Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support whole group discussion. “First, I ____ because . . .”, “I noticed ____ so I . . .”

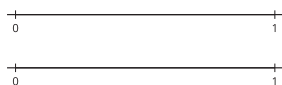
Advances: Speaking, Representing

Student-facing Task Statement

Some students are running on a trail at a park. Decide if each pair of students ran the same distance.

You can use number lines if they are helpful to you.

1. Elena ran $\frac{3}{6}$ of the trail.



Launch

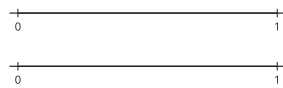
- Groups of 2

Activity

- “Work with your partner to decide whether each pair of students ran the same distance or not. You can use number lines to explain your reasoning if they’re helpful to you.”
- 5–7 minutes: partner work time

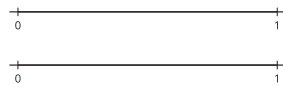
Han ran $\frac{1}{2}$ of the trail.

2. Jada ran $\frac{1}{4}$ of the trail.



Kiran ran $\frac{2}{8}$ of the trail.

3. Lin ran $\frac{2}{3}$ of the trail.

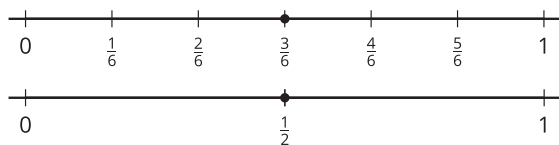


Mai ran $\frac{5}{6}$ of the trail.

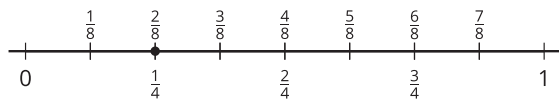
Student Responses

Sample responses:

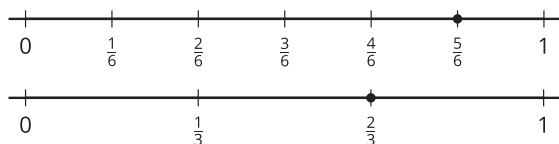
1. Elena and Han ran the same distance. $\frac{3}{6}$ and $\frac{1}{2}$ are the same distance from the starting point.



2. Jada and Kiran ran the same distance. $\frac{1}{4}$ and $\frac{2}{8}$ are the same distance from 0.



3. Lin and Mai didn't run the same distance because $\frac{5}{6}$ is not at the same location on the number line as $\frac{2}{3}$.



- Monitor for students who use the number lines to explain that the students ran the same distance if the fractions are at the same location on the number line.

Synthesis

- Display a student-created number line that shows $\frac{1}{4}$ and $\frac{2}{8}$ at the same location.
- "How does this show that Jada and Kiran ran the same distance?" (The points that represent them are at the same location between 0 and 1 on the number line.)
- "We've learned that two fractions are equivalent if they are the same size. Now we also know that two numbers are equivalent if they are at the same location on a number line. Because $\frac{1}{4}$ and $\frac{2}{8}$ are at the same location, we can say they are equivalent."
- "How could we use the equal sign to record fractions that are equivalent?" ($\frac{3}{6} = \frac{1}{2}$, $\frac{1}{4} = \frac{2}{8}$)
- Share and record responses.

Activity 2

🕒 10 min

Locate and Pair

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.b

The purpose of this activity is for students to locate fractions on the number line, and find pairs of fractions that are equivalent. Students can use a separate number line for each denominator, but they can also place fractions with different denominators on the same number line to show equivalence. Focus explanations about why fractions are equivalent on the fact that they share the same location. In the synthesis, discuss how one number line or two can be used to compare fractions.

♿ Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Organization

Student-facing Task Statement

- Locate and label the following numbers on a number line. You can use more than one number line if you wish.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{6}, \frac{3}{8},$
 $\frac{3}{4}, \frac{4}{6}, \frac{4}{8}, \frac{6}{8}, \frac{7}{8}$

- Find 4 pairs of fractions that are equivalent. Write equations to represent them.

_____ = _____ = _____ = _____ = _____

If you have time:

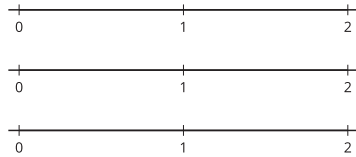
Launch

- Groups of 2

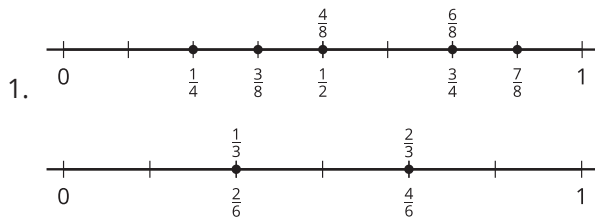
Activity

- “Work independently to locate these numbers on the number line. Then, find 4 pairs of fractions that are equivalent. Be prepared to explain your reasoning.”
- 3–5 minutes: independent work time
- “Now, share the pairs of fractions you wrote with your partner and explain how you know they are equivalent.”
- 2–3 minutes: partner discussion
- Monitor for students who compare fractions on a single number line and those who compare fractions on separate

Use the number lines to generate as many equivalent fractions as you can.



Student Responses



2. Sample responses:

- $\frac{1}{2} = \frac{4}{8}$
- $\frac{1}{3} = \frac{2}{6}$
- $\frac{6}{8} = \frac{3}{4}$
- $\frac{2}{3} = \frac{4}{6}$

If you have time: Sample response: $\frac{3}{2}$ is equivalent to $\frac{6}{4}$ because they are at the same point on the number line.

number lines.

Synthesis

- Select previously identified students to display how a single number line or separate number lines can be used to show equivalent fractions.
- “When might it make sense to use a single number line and when might it be helpful to use two number lines?” (One number line might make sense if one fraction can be partitioned to get to another, like halves to fourths. If a number line is too crowded or has fractions that could be hard to partition together, like halves and thirds, it might be helpful to use two number lines.)

Activity 3

🕒 15 min

Rolling for Equivalent Fractions

Standards Alignments

Addressing 3.NF.A.3.b

The purpose of this activity is for students to practice generating equivalent fractions. The goal of each round is to use the numbers on the number cubes to complete a statement that shows that two fractions are equivalent. Students roll 6 number cubes and try to use 4 of the numbers to create a statement that shows two equivalent fractions. If students roll a 5 (or a blank), they may

choose any number to use. Students may choose to re-roll any of their number cubes up to 2 times. Students get a point for every true statement they make. Students may choose to use fraction strips, diagrams, or number lines to prove that their fractions are equivalent. If students choose to use diagrams, monitor to make sure they are drawing equal-sized wholes.

Materials to Gather

Number cubes

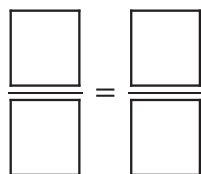
Required Preparation

- Each group of 2 needs 6 number cubes.

Student-facing Task Statement

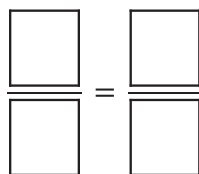
- Roll 6 number cubes. If you roll any fives, they count as a wild card and can be any number you'd like.
- Can you put the numbers you rolled in the boxes to make a statement that shows equivalent fractions? Work with your partner to find out.
- If you cannot, re-roll as many number cubes as you'd like. You can re-roll your number cubes twice.
- If you can make equivalent fractions, record your statement and show or explain how you know the fractions are equivalent. You get 1 point for each pair of equivalent fractions you write.

Round 1:



Show or explain how your fractions are equivalent.

Round 2:



Show or explain how your fractions are equivalent.

Launch

- Groups of 2
- Give each group 6 number cubes.
- "We're going to play a game called Rolling for Equivalent Fractions. Let's read through the directions and play 1 round together."
- Read through the directions with the class and play a round against the class, displaying the fractions from the cards, drawing tape diagrams, and thinking through decisions aloud.

Activity

- "Now, play the game with your partner. See if you can get 5 points."
- 8–10 minutes: partner work time
- Monitor for students to highlight during the synthesis that:
 - create a diagram of their fraction and generate an equivalent fraction with larger parts, such as picturing $\frac{2}{8}$ as $\frac{1}{4}$
 - create a diagram of the fraction they draw and further partition their fraction to make smaller pieces, such as further partitioning $\frac{1}{2}$ to

Round 3:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Round 4:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Round 5:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Round 6:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Round 7:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Round 8:

| | |
|--|--|
| | |
| | |

Show or explain how your fractions are equivalent.

Student Responses

Answers vary.

make $\frac{2}{4}$

- use a pattern to generate equivalent fractions, such as knowing that there are two sixths in each third, so $\frac{2}{3}$ is equivalent to $\frac{4}{6}$

Synthesis

- Display number cubes showing 1, 1, 4, 2, 2, 5
- "If you got these numbers on your last roll, what equivalent fractions could you make?" ($\frac{2}{4} = \frac{1}{2}$ or $\frac{1}{4} = \frac{2}{8}$)
- If needed, ask, "What number should I use for my wild card?"

Advancing Student Thinking

If students say they aren't sure what fractions they can make that would be equivalent, consider asking:

- "What fractions could you make with what you rolled?"
- "How could you use your fraction strips to decide if any of the fractions are equivalent?"

Lesson Synthesis

🕒 10 min

Display a number line that shows two fractions that are at the same location, such as $\frac{3}{2}$ and $\frac{6}{4}$.

“Earlier in the unit, we used fraction strips to see and find equivalent fractions. Here, we use number lines to find equivalent fractions.”

“How are the two ways of showing equivalent fractions alike?” (They both involve partitioning a whole and identifying two or more fractions.)

“How are they different?” (Instead of looking for parts that are the same size, we are looking for the same point or location on the number line.)

“Today, we saw that it can be helpful to use one or two number lines to show that fractions are equivalent. Keep that in mind during the cool-down.”

Suggested Centers

- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Addressing)

----- Complete Cool-Down -----

Response to Student Thinking

Students correctly locate $\frac{3}{4}$ and $\frac{6}{8}$ on each number line, but their lack of precision causes them to say that $\frac{3}{4}$ and $\frac{6}{8}$ are not equal.

Next Day Support

- During the launch of the next day's activity, have students brainstorm ways to partition number lines so that each part is the same size.

Lesson 13: Whole Numbers and Fractions

Standards Alignments

Addressing 3.NF.A.3.c

Teacher-facing Learning Goals

- Express whole numbers as fractions.
- Recognize fractions that are equivalent to whole numbers.

Student-facing Learning Goals

- Let's find fractions and whole numbers that are equivalent.

Lesson Purpose

The purpose of this lesson is for students to recognize fractions that are equivalent to whole numbers and write whole numbers as fractions.

In previous lessons, students noticed fractions at the same location as whole numbers on the number line. In this lesson, students develop more fully the idea that whole numbers can be written as fractions and learn to recognize fractions that are equivalent to whole numbers. Students encounter and make sense of fractions with 1 for the denominator.

Students use their knowledge of parts and wholes and the patterns they observed on number lines to express the numbers 1, 2, and 3 as fractions, then extend the patterns they observed to express larger whole numbers as fractions.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

What question do you wish you had asked today? When and why should you have asked it?

| | |
|------------------|--------|
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Fraction to Whole Number and Whole Number to Fraction

Standards Alignments

Addressing 3.NF.A.3.c

Student-facing Task Statement

1. Is $\frac{18}{4}$ a whole number? Explain or show your reasoning.
2. Write 2 as a fraction. Explain or show your reasoning.

Student Responses

1. No. The fractions that are whole numbers have numbers in the numerator that count by 4, like $\frac{4}{4}$, $\frac{8}{4}$, $\frac{12}{4}$, and 18 isn't in the count.
2. Sample response: $\frac{6}{3}$. I know 3 thirds make 1, so 6 thirds make 2.

----- Begin Lesson -----

Warm-up

⌚ 10 min

Notice and Wonder: Four Number Lines

Standards Alignments

Addressing 3.NF.A.3.c

This warm-up elicits observations about the different ways whole numbers can be expressed as fractions. Students have previously seen number lines where 1, 2, and 3 were labeled with fractions in halves, thirds, fourths, sixths, and eighths. They understand that a denominator of 2 corresponds to 2 equal parts in the length representing 1 whole. The number line marked with $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{3}{1}$ is shown together with those marked with halves, thirds, and fourths to highlight that a denominator of 1 means each whole has 1 part.

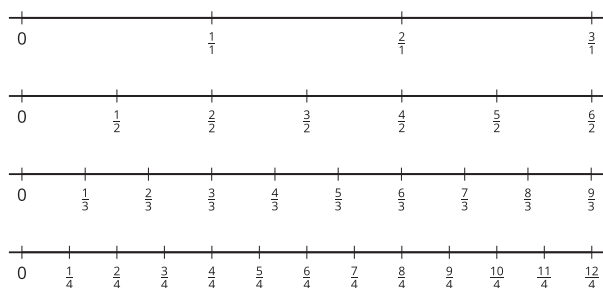
In the synthesis, students learn that fractions with 1 as a denominator can be used to represent whole numbers ($\frac{2}{1} = 2$).

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are four number lines with a different number of tick marks on each.
- The last three number lines are partitioned into halves, thirds, and fourths.
- The first number line has labels with 1 for the denominator.
- Some labels that line up vertically show a pattern (for instance, the first set shows $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$).
- The last fraction on each number line has a numerator that is 3 times the denominator.

Students may wonder:

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “What could it mean to have a denominator of 1?” (The whole hasn’t been partitioned. The whole has been partitioned into 1 part.)
- Have students label the locations of $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{3}{1}$ on the first number line with 1, 2, and 3.



- “The length from 0 to 1 hasn’t been partitioned, so each part has a length of 1. This is what a denominator of 1 means. If we have 1 part of 1, the numerator is 1. If we have 2 parts of 1, the numerator is 2, and so

- Are the fractions in the first number line 1, 2, and 3? Why are they written as fractions with 1 for the denominator?
 - How do we name the fractions with 1 for the denominator?
 - Why are there no whole-number labels?
- on.”
- “What other fractions on these number lines might be equivalent to 1?” ($\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$)

Activity 1

🕒 20 min

Hidden Whole Numbers

Standards Alignments

Addressing 3.NF.A.3.c

In an earlier lesson, students saw that whole numbers could be written as fractions. The purpose of this activity is for students to recognize fractions that are equivalent to whole numbers, using patterns in number lines to support their reasoning. To identify fractions that are equivalent to whole numbers on number lines, students may:

- Use what they know about 2 halves, 3 thirds, and 4 fourths to identify 1, and then circle fractions at the same intervals down each number line.
- Use what they know about 2 halves, 4 halves, and 6 halves to identify 1, 2, and 3 on the first number line, and then circle fractions in the same locations on other number lines.
- Use the relationship between parts and wholes (for instance, 3 thirds make 1, 6 thirds make 2, and 9 thirds make 3).

Students then record equations that show fractions that are equivalent to whole numbers. Finally, given a list of fractions, students determine which ones are equivalent to whole numbers.

When students use patterns to identify fractions that are equivalent to whole numbers, they look for and express regularity in repeated reasoning (MP8).

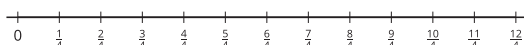
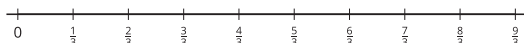
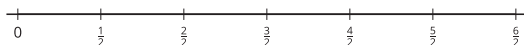
Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select at least 3 of the 5 problems in each question to complete.

Supports accessibility for: Organization, Attention, Social-emotional skills

Student-facing Task Statement

- On each number line, circle the fractions that are equivalent to whole numbers. Explain how you know.



- We can write $\frac{4}{2} = 2$ to show that $\frac{4}{2}$ and 2 are at the same location on the number line, so they are equivalent.

Write 5 other equations that show fractions that are equivalent to whole numbers. Use the number lines if they are helpful.

-
-
-
-
-



- Decide if each fraction is equivalent to a whole number. Use number lines if they are helpful.
 - $\frac{11}{2}$
 - $\frac{5}{1}$
 - $\frac{12}{6}$
 - $\frac{10}{3}$
 - $\frac{12}{8}$
 - $\frac{16}{4}$

Launch

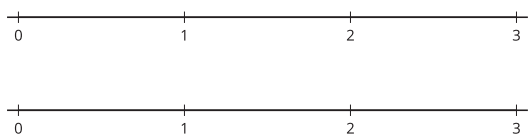
- Groups of 2
- “Work with your partner to circle all the fractions that are equivalent to whole numbers. Be sure to explain your reasoning.”
- 2–3 minutes: partner work time
- “Look at the fractions you circled. How did you know which ones to circle?”
- 1–2 minutes: partner discussion
- Share responses.

Activity

- “We can use equations like $\frac{4}{2} = 2$ to show that $\frac{4}{2}$ and 2 are at the same location on the number line, so they are equivalent.”
- “Work independently to complete the second and third problem.”
- 5–7 minutes: independent work time
- “Share your solutions to the second and third problems with your partner. Be sure to share your reasoning.”
- 2–3 minutes: partner discussion

Synthesis

- Invite students to share their responses and reasoning for the last set of problems.
- As students share, record fractions that are equivalent to whole numbers as equations and highlight that equations can be written starting with the whole number or fraction (for example, $\frac{12}{6} = 2$ or $2 = \frac{12}{6}$).
- “How did you know if each fraction was equivalent to a whole number?”
- “What patterns do you see that could be helpful for finding more fractions that are whole numbers?”
- Highlight strategies that are based on



understanding of the number of equal parts that are in 1 whole and on observed patterns. (For instance, the thirds that are equivalent to whole numbers have a numerator that is a number we get when counting by 3.)

Student Responses

1. Circled fractions:

- Halves: $\frac{2}{2}, \frac{4}{2}, \frac{6}{2}$
- Thirds: $\frac{3}{3}, \frac{6}{3}, \frac{9}{3}$
- Fourths: $\frac{4}{4}, \frac{8}{4}, \frac{12}{4}$

2. Sample responses:

- $\frac{12}{4} = 3$
- $\frac{2}{1} = 2$
- $3 = \frac{9}{3}$
- $\frac{6}{2} = 3$
- $1 = \frac{6}{6}$

3.
 - a. No, because 10 halves make 5, and 2 more halves are needed to make 6.
 - b. Yes, because the 1 whole hasn't been partitioned, so it is 5 lengths of 1.
 - c. Yes, because 6 sixths make 1 whole, so 12 sixths make 2 wholes.
 - d. No, because the whole numbers are $\frac{3}{3}, \frac{6}{3}, \frac{9}{3}$, and $\frac{12}{3}$. The fraction $\frac{10}{3}$ is more than 3 but is not 4.
 - e. No, because 8 eighths make 1 whole and 16 eighths make 2 wholes, so 12 eighths is more than 1 and less than 2.
 - f. Yes, because if I count by 4 in the numerator, 16 is in the count, so it would be a whole number.

Advancing Student Thinking

If students don't locate whole numbers on the number lines, consider asking:

- “Tell me about the fractions on the number line.”
- “How could you use the partitions in 1 whole to find the fractions at 2? At 3?”

Activity 2

🕒 15 min

Write Them as Fractions

Standards Alignments

Addressing 3.NF.A.3.c

The purpose of this activity is for students to write whole numbers as fractions. Students may reason in any way that makes sense to them, including using patterns they noticed previously. When students observe patterns as they write whole numbers as fractions, they look for and make use of structure (MP7).

This activity uses a “carousel” structure in which students complete a rotation of tasks. Consider demonstrating the steps before students begin.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: During group presentations, invite the student(s) who are not speaking to follow along and point to the corresponding parts of the display.

Advances: Speaking, Representing

Student-facing Task Statement

Work with your group to complete the table. In each column, write fractions that are equivalent to the whole number in the top row.

- Step 1: Write two fractions that are equivalent to each whole number (six fractions in all). Pass your paper to your right.
- Step 2: When you receive your neighbor’s

Launch

- Groups of 3
- “Take a moment to look at the table and the numbers in the table. How do you think the table works?”
- 1 minute: quiet think time
- Share responses.
- “In your group, take turns explaining why $\frac{4}{1}$ is equivalent to 4, $\frac{30}{6}$ is equivalent to 5, and

paper, write a new fraction that is equivalent to a whole number.

- Repeat Step 2 until the table is complete.

| 4 | 5 | 6 |
|-------------------|-------------------|-------------------|
| $\frac{4}{1}$ | | |
| | $\frac{\quad}{2}$ | |
| | | $\frac{\quad}{3}$ |
| $\frac{\quad}{4}$ | | |
| | $\frac{30}{6}$ | |
| | | $\frac{48}{8}$ |



Student Responses

Sample completed table:

| 4 | 5 | 6 |
|----------------|----------------|----------------|
| $\frac{4}{1}$ | $\frac{5}{1}$ | $\frac{6}{1}$ |
| $\frac{8}{2}$ | $\frac{10}{2}$ | $\frac{12}{2}$ |
| $\frac{12}{3}$ | $\frac{15}{3}$ | $\frac{18}{3}$ |
| $\frac{16}{4}$ | $\frac{20}{4}$ | $\frac{24}{4}$ |
| $\frac{24}{6}$ | $\frac{30}{6}$ | $\frac{40}{6}$ |
| $\frac{32}{8}$ | $\frac{40}{8}$ | $\frac{48}{8}$ |

$\frac{48}{8}$ is equivalent to 6."

- 2 minutes: group discussion

Activity

- "Work with your group to complete the table. Start by writing two fractions that are equivalent to each whole number: 4, 5, and 6."
- 2 minutes: independent work time
- "Pass your paper to your right. On the paper you receive, write one new fraction that is equivalent to a whole number of your choice."
- "Keep passing and writing one additional fraction for a whole number until the table is complete."
- Encourage students to ask clarifying questions before they begin.
- 7–8 minutes: group work time
- "Be prepared to explain how you know what fractions to write for each whole number."
- Monitor for students who:
 - use a number line (either draw a new one or extending those from an earlier activity)
 - extend a pattern they noticed in fractions that are equivalent to whole numbers
 - use multiplication facts

Synthesis

- Invite a group of students to display their completed table.
- Select previously identified students to share how they knew what fractions to write for each whole number.
- Invite the class to share other strategies for generating equivalent fractions.

- “What new patterns do you notice in the completed table?” (Sample responses:
 - In fractions that are equivalent to 4, you can get the numerator by skip-counting by the denominator 4 times.
 - In the row with thirds, the numerator increases by 3. In the row for fourths, it increases by 4.)

Advancing Student Thinking

If students don't write a whole number as a fraction, consider asking:

- “Tell me about how you would write 1 as a fraction.”
- “How could you use the fraction that's equivalent to 1 to write this number as a fraction?”

Lesson Synthesis

 10 min

“Over the last few lessons, we've learned about equivalent fractions.”

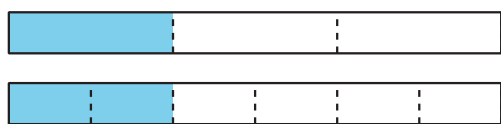
“What are some important things you've learned about fraction equivalence?” (Fractions that are the same size are equivalent. Fractions at the same point on the number line are equivalent. Some fractions are equivalent to whole numbers, but some are not. Whole numbers can be written as fractions.)

Suggested Centers

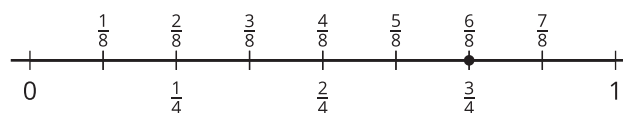
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)

Student Section Summary

In this section, we learned that different fractions can be equivalent. We know fractions are equivalent if they are the same size or located at the same location on the number line.

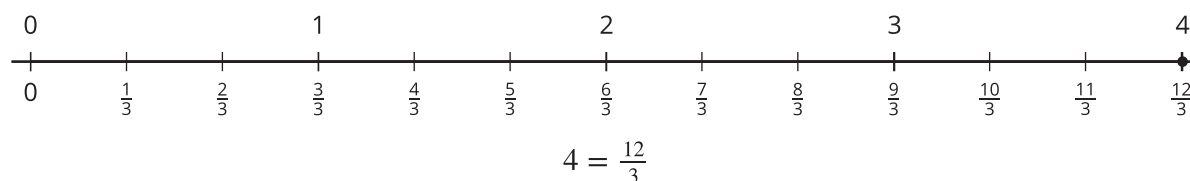


$$\frac{1}{3} = \frac{2}{6}$$



$$\frac{6}{8} = \frac{3}{4}$$

We also learned that some fractions are whole numbers, and that we can write whole numbers as fractions.



$$4 = \frac{12}{3}$$

----- Complete Cool-Down -----

Response to Student Thinking

Students say that $\frac{18}{4}$ is not a whole number and write 2 as a fraction, but don't explain or show their reasoning.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Section D: Fraction Comparisons

Lesson 14: How Do You Compare Fractions?

Standards Alignments

Addressing 3.NF.A.3, 3.NF.A.3.c, 3.NF.A.3.d

Teacher-facing Learning Goals

- Represent and compare fractions in a way that makes sense to them.

Student-facing Learning Goals

- Let's represent and compare fractions.

Lesson Purpose

The purpose of this lesson is for students to represent and compare fractions in a way that makes sense to them.

Previously, students used various representations to make sense of fractions and their size. In this lesson, students consider representations that will be helpful for comparisons, such as diagrams, fraction strips, and number lines. They also learn that comparisons are valid only when the fractions being compared refer to the same size whole. This lesson does not discuss specific strategies for comparing different types of fractions as the intent is to elicit different ways to reason about comparison.

Access for:



Students with Disabilities

- Engagement (Activity 1)



English Learners

- MLR8 (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Materials for creating a visual display:
Activity 1

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 25 min |
| Activity 2 | 10 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

Reflect on a time recently when your thinking about what students understand changed. How will you alter your teaching practice to incorporate your new understanding?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

How Would You Decide?

Standards Alignments

Addressing 3.NF.A.3

Student-facing Task Statement

How would you decide if $\frac{6}{4}$ is equivalent to $\frac{3}{4}$? Explain or show your reasoning.

Student Responses

Sample responses:

- I know $\frac{6}{4}$ is not equivalent to $\frac{3}{4}$ because they are not in the same location on the number line.
- I know $\frac{6}{4}$ is not equivalent to $\frac{3}{4}$ because they aren't the same size.
- I know $\frac{6}{4}$ is not equivalent to $\frac{3}{4}$ because it means 6 fourths, which is more than 3 fourths.

----- **Begin Lesson** -----

Warm-up

⌚ 10 min

Number Talk: Which Whole Numbers?

Standards Alignments

Addressing 3.NF.A.3.c

This Number Talk encourages students to use what they know about the meaning of fractions and about properties of operations to mentally relate fractions that are equivalent to whole numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the whole number that each fraction is equivalent to.

- $\frac{16}{1}$
- $\frac{16}{2}$
- $\frac{16}{4}$
- $\frac{20}{4}$

Student Responses

- 16: It's 16 parts of 1, which is 16.
- 8: I know 2 halves make 1, so I count by 2 halves like 2 halves, 4 halves, and so on, to get to 16 halves, and I counted 8 times.
- 4: There are 4 fourths in 1, so 8 fourths in 2, and 16 fourths in 4.
- 5: If $\frac{16}{4}$ is 4, then 4 more fourths would make 5.

Launch

- Display one fraction.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep fractions and work displayed.
- Repeat with each fraction.

Synthesis

- "How did the earlier fractions help you find the whole number for the last fraction?"
- Consider asking:
 - "Did anyone have the same strategy but would explain it differently?"
 - "Did anyone approach the problem in a different way?"

Activity 1

🕒 25 min

Equivalent or Not?

Standards Alignments

Addressing 3.NF.A.3

The purpose of this activity is for students to analyze pairs of fractions to determine if they are equivalent. Students may use any representation that makes sense to them. Students will create a visual display and have a gallery walk to consider the different ways of looking for equivalence. Highlight representations such as diagrams and number lines, which will support students as they learn to compare fractions with the same numerator or denominator in this section.

This activity uses *MLR7 Compare and Connect*. *Advances: representing, conversing*

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.

Supports accessibility for: Social-Emotional Functioning

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Materials for creating a visual display

Student-facing Task Statement

Are these fractions equivalent? Show your thinking using diagrams, symbols, or other representations.

1. $\frac{1}{2}$ and $\frac{1}{3}$
2. $\frac{4}{6}$ and $\frac{5}{6}$
3. $\frac{3}{4}$ and $\frac{6}{8}$

Student Responses

1. No. Sample response:

Launch

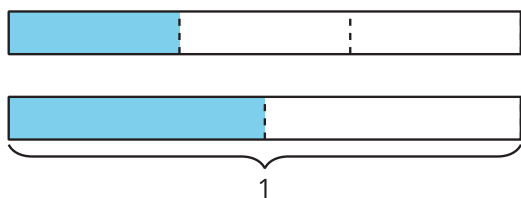
- Groups of 2
- “Decide if these pairs of fractions are equivalent and show your thinking for each one. You can use any representation that makes sense to you.”

Activity

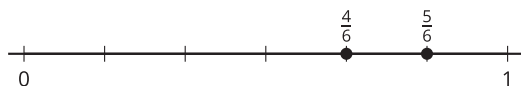
- 3–5 minutes: independent work time
- “Share your ideas with your partner.”
- 2–3 minutes: partner discussion

MLR7 Compare and Connect

- “Work with your partner to create a visual



2. No. Sample response:



3. Yes. Sample response: a number line or diagram that shows the fractions are at the same location or are the same size.

display that shows your thinking about $\frac{4}{6}$ and $\frac{5}{6}$. You may want to include details such as notes, diagrams, drawings, and so on, to help others understand your thinking."

- Give students materials for creating a visual display.
- 2–5 minutes: partner work time
- 5–7 minutes: gallery walk

Synthesis

- "What are the different representations we can use to decide if $\frac{4}{6}$ and $\frac{5}{6}$ are equivalent?" (diagrams, number lines)
- "How did each representation show that $\frac{4}{6}$ and $\frac{5}{6}$ are not equivalent?" (In the diagram, you can see that $\frac{5}{6}$ has more space shaded. On the number line, they are not at the same location.)

Advancing Student Thinking

If students don't determine whether or not fractions are equivalent, consider asking:

- "What do you know about these fractions?"
- "How could you use your fraction strips or diagrams to decide if they are equivalent?"

Activity 2

🕒 10 min

Same Fractions, Different Result?

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to recognize that fraction comparisons are only valid when they refer to the same whole. Previously, students analyzed pairs of fractions to determine if they were equivalent. In doing so, they were likely to have used comparison language, such as “larger or smaller than” or “greater or less than.” In this activity, students encounter a pair of fractions they saw earlier ($\frac{4}{6}$ and $\frac{5}{6}$) and compare them more explicitly. The student work in this activity uses the number line, but this might also come up with student-drawn diagrams.

In order to interpret Lin’s argument that $\frac{4}{6}$ is greater than $\frac{5}{6}$ students will need to articulate the meaning of fractions and highlight the fact that the two wholes Lin is comparing are not equal (MP6).

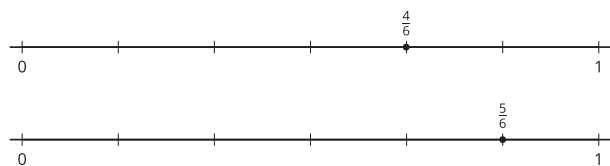
Access for English Learners

MLR8 Discussion Supports. Synthesis: Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement “the number lines are different” as “the size of the whole from 0 to 1 is different.”

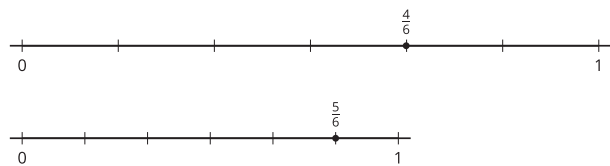
Advances: Speaking, Representing

Student-facing Task Statement

Han says $\frac{4}{6}$ is less than $\frac{5}{6}$. His work is shown.



Lin says $\frac{4}{6}$ is greater than $\frac{5}{6}$. Her work is shown.



Why might Han and Lin make different comparison statements for the same fractions?

Student Responses

Sample response: Lin got a different answer than Han because her number lines are not the

Launch

- Groups of 2
- “Han and Lin are comparing $\frac{4}{6}$ and $\frac{5}{6}$ like you did. Take a minute to look at their work.”
- 1 minute: quiet think time

Activity

- “Talk with your partner about how Han and Lin could get different results.”
- 2–3 minutes: partner discussion
- Monitor for students who notice that the whole is different in Lin’s number lines, which makes her think that $\frac{4}{6}$ is greater.

Synthesis

- Select previously identified students to share how Han and Lin could make different comparison statements for the

same length. Because she drew one number line longer than the other, it makes it look like $\frac{4}{6}$ is greater than $\frac{5}{6}$.

same fractions.

- “It is important to remember when we are comparing fractions that those fractions need to refer to the same whole.”
- “When the whole from 0 to 1 is the same size, we can see that $\frac{4}{6}$ is less than $\frac{5}{6}$.”
- “This is true whether we are drawing a number line, using fraction strips, or drawing a diagram.”
- Consider asking, “What did both Lin and Han understand about representing fractions on a number line?” (The number lines need to be partitioned into equal parts. The denominator tells us then number of parts. The numerator tells us how many parts to count to locate a point. Points farther to the right are greater than those to the left.)

Lesson Synthesis

🕒 10 min

“Today, we studied pairs of fractions to see if they were equivalent or not. How did you decide if two fractions were equivalent?” (Drew diagrams to see if the parts that represent the fractions were the same size. Represent the fractions on number lines and see if they were at the same location.)

“If the fractions were not equivalent, it means that one of the fractions was greater than the other, and one of the fractions was less than the other. We’ll learn more about this in future lessons.”

Draw two number lines (or diagrams) with different lengths (or areas) representing 1 whole. Partition each into 3 parts.

“What might be a problem with comparing $\frac{2}{3}$ and $\frac{3}{3}$ using these number lines (or diagrams)?” (The 1 whole is not the same size, so we can’t use the number lines (or diagrams) to compare the fractions.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)

- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students state that $\frac{6}{4}$ and $\frac{3}{4}$ are equivalent.

Next Day Support

- Before the warm-up, discuss ways to determine that $\frac{6}{4}$ and $\frac{3}{4}$ are not equivalent.

Lesson 15: Compare Fractions with the Same Denominator

Standards Alignments

Addressing 3.NF.A.3, 3.NF.A.3.d

Teacher-facing Learning Goals

- Compare two fractions with the same denominator by reasoning about their size.

Student-facing Learning Goals

- Let's compare two fractions with the same denominator.

Lesson Purpose

The purpose of this lesson is for students to compare two fractions with the same denominator.

In previous lessons, students learned that fractions are built from unit fractions. Here, they reason that fractions with the same denominator are composed of parts that are the same size or length, so the numerator, which describes the number of parts, determines which fraction is greater. It is important that students have the opportunity to develop this understanding rather than learning a rule about comparing fractions with the same denominator.

Students are reminded that they can use the symbols $>$, $=$, or $<$ to record the results of comparison of fractions, just as they did with whole numbers in earlier grades (MP6). This lesson (including the cool-down) enables the teacher to gauge students' familiarity with the symbols, but students are not yet expected to rely on the symbols to express comparison.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR7 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Colored pencils: Activity 2
- Paper clips: Activity 2

Materials to Copy

- Spin to Win Recording Sheet (groups of 2): Activity 2

- Spin to Win Spinner (groups of 2): Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

What prior knowledge about unit and non-unit fractions did you see students use to help them compare fractions?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Same Denominator

Standards Alignments

Addressing 3.NF.A.3.d

Student-facing Task Statement

1. Which is the greater fraction: $\frac{7}{8}$ or $\frac{6}{8}$? Explain or show your reasoning.
2. Use the symbols $>$ or $<$ to make the statement true.

$$\frac{7}{8} \underline{\hspace{1cm}} \frac{6}{8}$$

Student Responses

1. $\frac{7}{8}$ is greater. Sample response: Seven one-eighth parts are more than 6 one-eighth parts.
2. $>$

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Two More Strips

Standards Alignments

Addressing 3.NF.A.3

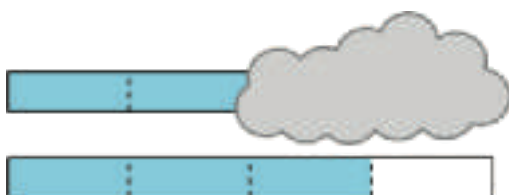
The purpose of this warm-up is to elicit the idea that the size and the number of unit fractions can help us compare fractions. Students can see that the two diagrams have same-size parts but not how much of one diagram is shaded, prompting them to think about the number of shaded parts. While students may notice and wonder many things about these images, what fractions could be represented by the partially hidden strip is the important discussion point.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- The parts on the strips are the same size.
- The bottom strip shows $\frac{3}{4}$.
- Part of the top strip is covered.

Students may wonder:

- How many parts are shaded on the top strip?
- Is more shaded on the top strip than the bottom strip?
- Is less shaded on the top strip than the bottom strip?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “How many parts could be shaded on the top strip? Could less than $\frac{3}{4}$ be shaded? Could more than $\frac{3}{4}$ be shaded?” (If 2 parts are shaded, that's $\frac{2}{4}$, which is less than $\frac{3}{4}$. If 3 parts are shaded, that's $\frac{3}{4}$. If the whole strip is shaded, that's $\frac{4}{4}$, which is more than $\frac{3}{4}$.)

Activity 1

🕒 20 min

Compare Fractions with the Same Denominator

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to compare two fractions with the same denominator. Students may use any representation to reason about how the size or length of the parts in the two fractions are the same because the denominator is the same, but that there are different numbers of those parts because the numerator is different (MP2). Students are also reminded about the meaning of the symbols $>$ and $<$.

🌐 Access for English Learners

MLR7 Compare and Connect. Synthesis: Invite groups to prepare a visual display that shows the strategy they used to compare the fractions. Encourage students to include details that will help others interpret their thinking. For example, specific language, using different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each other's work. During the whole-class discussion, ask students, "What do these representations have in common?" "How are they different?" and "What kinds of additional details or language helped you understand the displays?"

Advances: Representing, Conversing

Student-facing Task Statement

- For each pair of fractions, circle the fraction that is greater. Explain or show your reasoning.

a. $\frac{1}{2}$ and $\frac{3}{2}$

b. $\frac{3}{8}$ and $\frac{2}{8}$

- Use the symbols $>$ or $<$ to make each statement true. Explain or show your reasoning.

a. $\frac{1}{6}$ _____ $\frac{4}{6}$

Launch

- Groups of 2
- Display the first problem.
- "Take a few minutes to decide which fraction is greater for each of these pairs."
- 2–3 minutes: independent work time
- "Share your ideas with your partner."
- 1–2 minutes: partner discussion
- Share and record responses.
- Display: $<$ and $>$
- "Let's refresh our memories about 'less than' and 'greater than' symbols. How do

b. $\frac{4}{4}$ _____ $\frac{5}{4}$

c. $\frac{2}{3}$ _____ $\frac{1}{3}$

d. $\frac{4}{8}$ _____ $\frac{6}{8}$

If you have time: Write in the missing numerator of the fraction to make each statement true. Explain or show your reasoning.

1. $\frac{1}{2} < \frac{\quad}{2}$

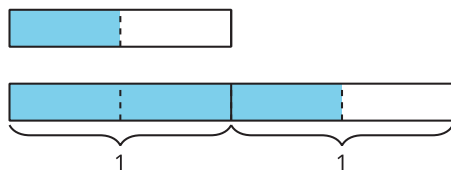
2. $\frac{6}{4} > \frac{\quad}{4}$

3. $\frac{4}{3} < \frac{\quad}{3}$

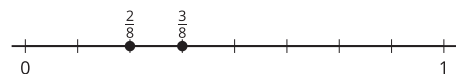
4. $\frac{5}{8} > \frac{\quad}{8}$

Student Responses

1. a. $\frac{3}{2}$. Sample response:



- b. $\frac{3}{8}$. Sample response:



2. a. $<$
b. $<$
c. $>$
d. $<$

If you have time:

- Any numerator greater than 1.
- Any numerator less than 6.
- Any numerator greater than 4.
- Any numerator less than 5.

we read these symbols?" ("is less than" and "is greater than")

- Display: $\frac{1}{2} < 1$ and $\frac{5}{4} > 1$
- "How do we read these statements?" (One-half is less than 1. Five-fourths is greater than 1.)
- "What expressions could you write about $\frac{1}{2}$ and $\frac{3}{2}$ and $\frac{2}{8}$ and $\frac{3}{8}$ using these symbols?" ($\frac{1}{2} < \frac{3}{2}$, $\frac{3}{2} > \frac{1}{2}$, $\frac{2}{8} < \frac{3}{8}$, $\frac{3}{8} > \frac{2}{8}$)
- Share and display responses. Ask students to read aloud each statement that is shared.

Activity

- "Work with your partner to compare the fractions in the next problem and use these symbols. Be sure to explain or show your reasoning."
- 5–7 minutes: partner work time
- Monitor for students who explain their reasoning with:
 - an area diagram
 - a fraction strip diagram
 - a number line
 - written description about the size or length of parts

Synthesis

- Select previously identified students to share different representations for comparing fractions with the same denominator.
- Consider asking: "How are these representations alike? How are they different?"

Activity 2

🕒 15 min

Spin to Win: Same Denominator

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to practice comparing fractions with the same denominator while playing a game. Students spin a spinner for the numerator of their fractions and then locate and label the fractions on a number line to determine which fraction is greater.

♿ Access for Students with Disabilities

Representation: Access for Perception. To support understanding, begin by demonstrating how to play one round of “Spin to Win.”

Supports accessibility for: Memory, Social-Emotional Functioning

Materials to Gather

Colored pencils, Paper clips

Materials to Copy

Spin to Win Recording Sheet (groups of 2),
Spin to Win Spinner (groups of 2)

Required Preparation

- Each group of 2 needs a paper clip for their spinner.

Student-facing Task Statement

In this game, you will record fractions on number lines. Choose a writing utensil in a color different than your partner's so you can tell which fraction is whose on each number line.

- Each player spins the paper clip. The player who spins the highest number is Player 1.
- Player 1 chooses a denominator for the first round: 2, 3, 4, 6, or 8.
- Each player spins for the numerator of their fraction.
- Each player locates and labels their fraction

Launch

- Groups of 2
- Give each group a paper clip, colored pencils, a spinner, and a sheet of number lines.
- “Now you will play a game in which you compare fractions with the same denominator. To start, one player will choose a denominator for the first round.”
- “You will each spin for the numerator of your own fraction with that denominator. Then you’ll locate and label your fractions

on the same number line on the recording sheet.

5. The player with the greater fraction wins and picks the denominator for the next round.
6. Repeat for 10 rounds. The player who wins the most rounds wins the game.

on the same number line and determine whose fraction is greater.”

- “If your fraction is greater, you get to choose the denominator for the next round. The goal of the game is to win as many rounds as you can.”
- Ask the class to choose a denominator (2, 3, 4, 6, or 8). Then spin the spinner and discuss how to represent the fraction on a number line on the recording sheet.

Activity

- 10 minutes: partner work time
- As students work, monitor for students who notice patterns as they play.

Synthesis

- “What kind of number did you want to spin on your turn? Why?” (I wanted to spin a large number because a large numerator means a greater fraction. If you spin a small number, then the small numerator makes a smaller fraction.)

Lesson Synthesis

🕒 10 min

“Today we compared fractions with the same denominator.”

“How do you compare fractions with the same denominator? Does your strategy always work?” (I can just look at the numerators to see which is greater. This always works because the whole is split into the same number of parts that are the same size if the denominator is the same, so we just need to think about how many of those parts we have, which is given by the numerator.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)

- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students' diagrams indicate that they know which fraction is greater, but the symbol in the expression doesn't match their visual representation.

Next Day Support

- Before the next day's warm-up, have students recap the important points of the previous lesson, specifically the meaning of the symbols $<$ and $>$.

Lesson 16: Compare Fractions with the Same Numerator

Standards Alignments

Addressing 3.NF.A.3.d

Teacher-facing Learning Goals

- Compare two fractions with the same numerator by reasoning about their size.

Student-facing Learning Goals

- Let's compare two fractions with the same numerator.

Lesson Purpose

The purpose of this lesson is for students to compare two fractions with the same numerator.

In this lesson, students reason that fractions with the same numerator have the same number of parts, and that the denominator shows the size or length of those parts. Students recognize that as the denominator increases, each part gets smaller. It is important that students develop this understanding rather than learning a rule about comparing fractions with the same numerator.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR1 (Activity 2)

Instructional Routines

True or False (Warm-up)

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

Teacher Reflection Question

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Same Numerator

Standards Alignments

Addressing 3.NF.A.3.d

Student-facing Task Statement

Use the symbols $>$ and $<$ to make the statement true. Explain or show your reasoning.

$$\frac{4}{3} \quad \underline{\hspace{1cm}} \quad \frac{4}{6}$$

Student Responses

$>$. Sample response: Thirds are larger than sixths, so 4 thirds is greater than 4 sixths.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

True or False: Unit Fractions

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this True or False is to elicit insights students have about comparing unit fractions. The reasoning students do helps to deepen their understanding of what the denominator of a fraction means. It will also be helpful later when students compare fractions with the same numerator.

In this activity, students have an opportunity to look for and make use of structure (MP7) because they notice that a larger denominator indicates that a whole is split into more parts. The more parts the whole is split into, the smaller those parts will be.

Instructional Routines

True or False

Student-facing Task Statement

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{2} > \frac{1}{4}$
- $\frac{1}{4} > \frac{1}{3}$
- $\frac{1}{6} > \frac{1}{8}$

Student Responses

- True: I know that one fourth is half the size of one half.
- False: If the whole is split in 3 parts, those parts are bigger than if there were 4 parts, so one fourth is smaller than one third.
- True: Eighths are smaller than sixths because if we split the whole into more parts, each part is smaller.

Launch

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- Consider asking:
 - “Who can restate ____’s reasoning in a different way?”
 - “Does anyone want to add on to ____’s reasoning?”

Activity 1

🕒 20 min

Five Parts of Something

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to represent their thinking visually as they compare pairs of fractions with the same numerator. They also locate fractions with the same numerator on number lines and observe the relative locations of the points. Students see that fractions with larger denominator are smaller in size (or are closer to 0 on the number line). Their reasoning here reinforces the idea that the denominator of a fraction determines how many equal parts are

in a whole, and that the more parts there are, the smaller each part is (MP7).

To compare $\frac{5}{6}$ and $\frac{5}{8}$, students are likely to draw one diagram or number line for sixths and a separate one for eighths. They may use a single diagram or number line, but find it more difficult to partition and represent both denominators.

Student-facing Task Statement

1. Priya says that $\frac{5}{6}$ is greater than $\frac{5}{8}$.

Tyler says that $\frac{5}{8}$ is greater than $\frac{5}{6}$.

Who do you agree with? Show your thinking using diagrams or number lines.

2. For each pair of fractions, which fraction do you think is greater?

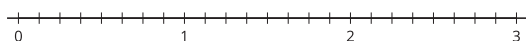
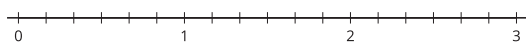
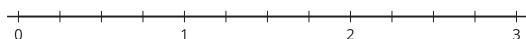
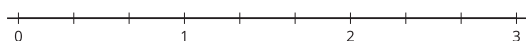
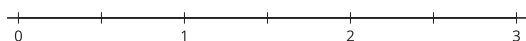
a. $\frac{5}{3}$ or $\frac{5}{4}$

b. $\frac{5}{8}$ or $\frac{5}{2}$

c. $\frac{5}{6}$ or $\frac{5}{4}$



3. Locate and label each fraction on a number line:
line: $\frac{5}{2}$, $\frac{5}{3}$, $\frac{5}{4}$, $\frac{5}{6}$, $\frac{5}{8}$.



What do you notice about the points? Make 1–2 observations.

Student Responses

1. Sample response: I agree with Priya because

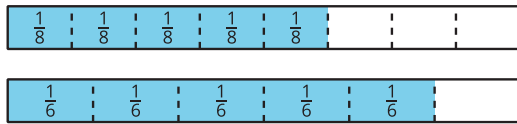
Launch

- Groups of 2
- “Priya and Tyler are comparing two fractions. Read their conclusions and decide who you agree with.”
- 2 minutes: independent work time

Activity

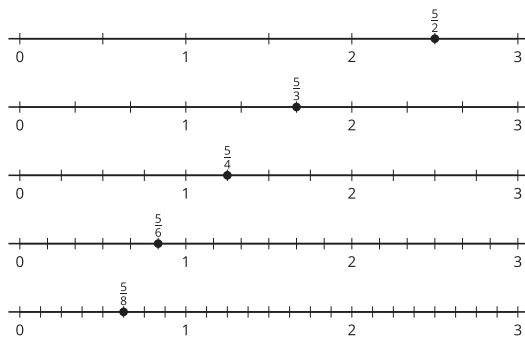
- “Talk to your partner about who you agree with. Use diagrams or number lines to show your thinking.”
- 3–5 minutes: partner discussion
- As students work, consider asking:
 - “How does your representation show which fraction is greater?”
 - “How do you know that eighths are smaller than sixths?”
- Monitor for students who use diagrams and those who use number lines.
- Pause for a discussion.
- Select two students, one who uses each representation, to share. Display their work side-by-side for all to see.
- “How do these representations both show that $\frac{5}{6}$ is greater than $\frac{5}{8}$?” (Both show that we are looking at 5 parts in each fraction and that sixths are larger than eighths. That means that 5 sixths are larger than 5 eighths.)
- “Now complete the last two problems.”

sixths are larger than eighths, so 5 sixths is greater than 5 eighths.



2. a. $\frac{5}{3}$
b. $\frac{5}{2}$
c. $\frac{5}{4}$

3.



Sample responses: The points get closer and closer to 0 as the denominator of the fraction gets larger. The fraction is smaller when the denominator is larger.

- 5–7 minutes: independent or partner work time

Synthesis

- Display a set of number lines that a student completed.
- Invite the class to share their observations about the locations of the points.
- “How can the locations of the points help you decide which is greater, $\frac{5}{3}$ or $\frac{5}{4}$?” (I can see that $\frac{5}{3}$ is located to the right of $\frac{5}{4}$, so $\frac{5}{3}$ is greater.)
- Consider asking: “Why is the fraction with the smallest denominator, 2, the greatest fraction in the set?” (There are only 2 parts in 1 whole. With every other fraction the denominator is larger than 2 so there are more parts in the whole which makes the parts smaller.)

Advancing Student Thinking

If students are not sure how to get started, consider asking: “What do we know about these fractions?” and “How could we represent these fractions?”

Activity 2

🕒 15 min

Fractions with the Same Numerator

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to compare two fractions with the same numerator. Students use any representation to reason about the number and size of the parts of each fraction. In the last problem, students may notice that more than one denominator can sometimes work, but do not need to generalize about all the denominators that make each statement true. There is an opportunity for that kind of generalization in a future lesson.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “Which denominators did you choose and why do they make the statements true?” Invite listeners to ask questions, to press for details, and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Student-facing Task Statement

- For each pair of fractions, circle the fraction that is greater. Explain or show your reasoning.

a. $\frac{1}{4}$ and $\frac{1}{3}$

b. $\frac{3}{4}$ and $\frac{3}{8}$

c. $\frac{5}{3}$ and $\frac{5}{6}$

d. $\frac{9}{8}$ and $\frac{9}{6}$

- Use the symbols $>$ or $<$ to make each statement true. Be prepared to explain your reasoning.

a. $\frac{2}{2}$ _____ $\frac{2}{6}$

b. $\frac{4}{3}$ _____ $\frac{4}{8}$

c. $\frac{8}{8}$ _____ $\frac{8}{4}$

Launch

- Groups of 2
- “We just saw several ways we could represent comparisons of fractions with the same numerator. As you compare fractions with the same numerator in this activity, you can use diagrams or number lines or write to explain your reasoning.”

Activity

- 8–10 minutes: independent work time
- “Share your favorite way to represent your reasoning with your partner.”
- 3–5 minutes: partner discussion
- Monitor for students who choose different denominators in the last problem.

d. $\frac{5}{4}$ _____ $\frac{5}{3}$

3. Write in the missing denominator of the fraction to make each statement true. Be prepared to explain your reasoning.

a. $\frac{1}{3} < \frac{1}{\quad}$

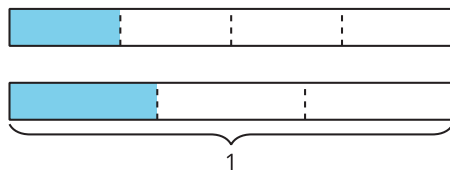
b. $\frac{6}{4} > \frac{6}{\quad}$

c. $\frac{4}{4} < \frac{4}{\quad}$

d. $\frac{2}{6} < \frac{2}{\quad}$

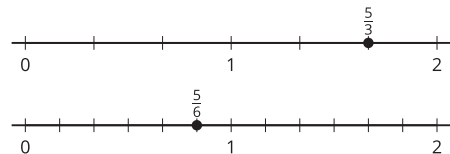
Student Responses

1. a. $\frac{1}{3}$. Sample response:



- b. $\frac{3}{4}$. Sample response: One fourth is twice the size of 1 eighth, so 3 fourths must be twice the size of 3 eighths.

- c. $\frac{5}{3}$. Sample response:



- d. $\frac{9}{6}$. Sample response: One sixth is larger than 1 eighth, so 9 sixths is greater than 9 eighths.

2. a. $>$
 b. $>$
 c. $<$
 d. $<$
3. a. Any denominator less than 3.
 b. Any denominator greater than 4.
 c. Any denominator less than 4.

Synthesis

- Select 3–4 students to share the denominators they chose for one part in the last problem. Ask them to explain why their chosen denominator makes the statement true.

- d. Any denominator less than 6.

Advancing Student Thinking

If students don't determine which fraction is greater, consider asking: "What do we know about these fractions?" and "How could we represent these fractions?"

Lesson Synthesis

🕒 10 min

"Today we compared fractions with the same numerator."

"How would you describe to a friend how to compare fractions with the same numerator?" (We have to think about how big the parts are since the denominators are different. We have the same number of parts, but we need to know which parts are bigger or smaller. If the denominator is larger, there are more parts in the whole, so the parts are smaller than those in a fraction with a smaller denominator.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students explain that $\frac{4}{6}$ is greater than $\frac{4}{3}$ because 6 is greater than 3.

Next Day Support

- During the launch of the next day's activity, have students pair up and discuss what the denominator of a fraction tells us and how it might help us compare two fractions.

Lesson 17: Compare Fractions

Standards Alignments

Addressing 3.NF.A.2, 3.NF.A.3, 3.NF.A.3.d
Building Towards 3.MD.B.4

Teacher-facing Learning Goals

- Compare two fractions with the same numerator or the same denominator.
- Record the results of comparison with the symbols $>$, $=$, or $<$.

Student-facing Learning Goals

- Let's compare more fractions in different situations.

Lesson Purpose

The purpose of this lesson is for students to compare two fractions with the same numerator or the same denominator in and out of context and to justify their conclusions.

In previous lessons, students learned what it means for fractions to be equivalent, and compared two fractions with the same denominator or the same numerator. In this lesson, students apply their knowledge to compare fractions in and out of context and have an opportunity to generalize about what they have learned about fraction comparison.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

| | |
|------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |

Teacher Reflection Question

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. How have you seen each student grow as a young mathematician

| | |
|------------------|--------|
| Activity 2 | 15 min |
| Activity 3 | 10 min |
| Lesson Synthesis | 5 min |
| Cool-down | 5 min |

throughout this work? What will you continue to do and what will you improve upon in the next unit?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

All Kinds of Comparisons

Standards Alignments

Addressing 3.NF.A.3.d

Student-facing Task Statement

1. Use the symbols $>$, $<$, or $=$ to make each statement true.

a. $\frac{4}{6}$ _____ $\frac{2}{6}$

b. $\frac{8}{8}$ _____ $\frac{4}{4}$

2. An ant crawled $\frac{3}{6}$ of the length of a bench. A spider crawled $\frac{3}{4}$ of the length of the same bench.

- Which animal crawled farther? Explain or show your reasoning.
- Write a statement using the symbols $>$, $<$, or $=$ to represent your answer.

Student Responses

- $>$
 - $=$
- The spider crawled farther. Sample response: Fourths are larger than sixths, so 3 fourths is greater than 3 sixths.
 - $\frac{3}{4} > \frac{3}{6}$

----- Begin Lesson -----

Warm-up

🕒 10 min

Estimation Exploration: Ladybug Length

Standards Alignments

Building Towards 3.MD.B.4

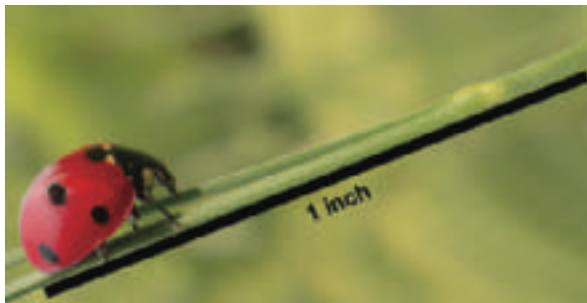
The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. In this warm-up, students apply what they know about fractions to estimate the length of an insect that is less than 1 inch.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What is the length of this ladybug?



Record an estimate that is:

| too low | about right | too high |
|---------|-------------|----------|
| | | |

Student Responses

Sample responses:

- Too low: $\frac{1}{4}$ or less
- About right: between $\frac{1}{4}$ and $\frac{1}{2}$ inch
- Too high: $\frac{3}{4}$ inch or greater

Launch

- Groups of 2
- Display the image.
- "What is an estimate that's too high? Too low? About right?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- Consider asking:
 - "Is anyone's estimate less than ____? Is anyone's estimate greater than ____?"
 - "Based on this discussion, does anyone want to revise their estimate?"

Activity 1

🕒 15 min

Comparison Problems

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to compare two numbers in context, to explain or show their reasoning, and record the results of the comparisons with the symbols $>$, $=$, or $<$ (MP2). The numbers may be fractions with the same numerator or the same denominator, or a fraction and a whole number.

Students are likely to generate different comparison statements for the same situation. For example, they may write $\frac{5}{8} > \frac{3}{8}$ or $\frac{3}{8} < \frac{5}{8}$ to represent $\frac{5}{8}$ being the greater fraction. During synthesis, discuss how both statements capture the comparison and are valid.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: As students share the similarities and differences between the representations and comparison statements, use gestures to emphasize what is being described. For example, show with your fingers the partitions such as fourths or eighths that are the same in the representations that are being compared.

Advances: Listening, Representing

♿ Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Optimize meaning and value. Invite students to share a connection between activity content and their own lives. Ask “How can I use this in my own life?”

Supports accessibility for: Conceptual Processing

Student-facing Task Statement

For each problem:

- Answer the question and explain or show your reasoning.
- Represent your answer with a statement that uses the symbols $>$, $<$, or $=$.

Launch

- Groups of 2
- “Let’s use what we learned about comparing fractions and recognizing equivalent fractions to solve problems about lengths.”

1. A beetle crawled $\frac{2}{8}$ of the length of a log. A caterpillar crawled $\frac{2}{3}$ of the length of the same log. Which insect crawled farther?
2. A grasshopper is 4 centimeters long. A caterpillar is $\frac{12}{3}$ centimeters long. Which insect is longer?
3. A ladybug crawled $\frac{3}{8}$ the length of a branch. An ant crawled $\frac{5}{8}$ the length of the same branch. Which insect crawled farther?
4. A grasshopper jumped $\frac{5}{8}$ the width of the sidewalk. A frog jumped $\frac{5}{6}$ the width of the same sidewalk. Which jumped a longer distance?



Student Responses

1. The caterpillar crawled farther. Sample response:
 - A third is larger than an eighth, so 2 thirds is larger than 2 eighths.
 - $\frac{2}{3} > \frac{2}{8}$ (or $\frac{2}{8} < \frac{2}{3}$)
2. They are the same length. Sample response:
 - Three thirds is equivalent to 1, so 12 thirds is equivalent to 4.
 - $\frac{12}{3} = 4$ (or $4 = \frac{12}{3}$)
3. The ant crawled farther. Sample response:
 - A number line showing that the point for 5 eighths is to the right of the point for 3 eighths
 - $\frac{3}{8} < \frac{5}{8}$ (or $\frac{5}{8} > \frac{3}{8}$)
4. The frog jumped a longer distance. Sample response:
 - Sixths are larger than eighths, so 5 sixths is larger than 5 eighths.

Activity

- “Work independently to solve the problems. For each one, be sure to show your thinking and to write a comparison statement.”
- 6–8 minutes: independent work time
- “Share your responses and reasoning with your partner.”
- 2–3 minutes: partner discussion
- Monitor for:
 - different representations or reasoning strategies used for the same problem, such as diagrams, fraction strips, number lines, or explanations in words
 - different statements written for the same problem, such as $4 = \frac{12}{3}$ and $\frac{12}{3} = 4$, or $\frac{2}{3} > \frac{2}{8}$ and $\frac{2}{8} < \frac{2}{3}$

Synthesis

- Select 2–3 students to share their reasoning strategies or representations for at least one of the situations.
- “How did you use what you’ve learned in earlier lessons to compare fractions?” (If the fractions had the same numerator, I thought about the size of the denominators. If they had the same denominator, I compared the numerators.)
- Select students who wrote different but equally valid comparison statements (for instance, $\frac{2}{8} < \frac{2}{3}$ and $\frac{2}{3} > \frac{2}{8}$) to share.
- Discuss how to read each statement and ask students whether both accurately represent the comparison.
- Emphasize that we can write comparison statements in more than one way, but we need to check that the statements make sense given the numbers we write and the

$$\circ \quad \frac{5}{6} > \frac{5}{8} \text{ (or } \frac{5}{8} < \frac{5}{6} \text{)}$$

symbols we use.

Activity 2

🕒 15 min

What Fraction Makes Sense?

Standards Alignments

Addressing 3.NF.A.3.d

The purpose of this activity is for students to generalize what they have learned about comparing fractions to complete comparison statements and to generate new ones, using the symbols $<$, $>$, or $=$. Students first consider all numbers that could make an incomplete comparison statement true. Then, they find a fraction less than, greater than, and equivalent to a given fraction and write statements to record the comparisons. As in the previous activity, students see that there are different ways to record the same comparison of two numbers.

Student-facing Task Statement

1. Oh, no! Some juice spilled on Noah's fractions. Help him figure out what was written before the juice was spilled.

Find as many numbers as you can to make each statement true. Explain or show your reasoning.

a.

$$\frac{2}{8} < \text{☀}$$

b.

$$\frac{3}{6} = \text{☀}$$

Launch

- Groups of 2
- "Now that we have practiced comparing fractions, let's come up with fractions that are greater than, less than, or equivalent to a given fraction."

Activity

- "Noah was working with fractions when some juice spilled. Now he can't tell what some numbers were. Help him figure out what was written before the juice was spilled."
- 5 minutes: partner work time
- Pause for a discussion and invite students to share the numbers that they think make sense in the first statement ($\frac{2}{8} < \frac{?}{8}$).

c.

$$\frac{4}{3} > \frac{4}{4}$$

2. For each fraction, find a fraction that is less, one that is greater, and one that is equivalent. Then, write a statement that uses the symbols $>$, $<$, or $=$ to record each comparison.

a. Less than $\frac{4}{6}$: Statement:

_____ More than $\frac{4}{6}$: Statement:

_____ Equivalent to $\frac{4}{6}$: Statement:

b. Less than $\frac{3}{4}$: Statement:

_____ More than $\frac{3}{4}$: Statement:

_____ Equivalent to $\frac{3}{4}$: Statement:

Student Responses

1.
 - a. Any numerator greater than 2. Sample response: These numerators all show more than 2 eighths.
 - b. $\frac{1}{2}$, $\frac{2}{4}$, or $\frac{4}{8}$. Sample response: These fractions are all equivalent to $\frac{3}{6}$.
 - c. 4, 6, or 8. Sample response: These denominators have smaller parts than thirds, so 4 parts of these sizes will be smaller than 4 thirds.
2. Sample responses:
 - a. Less: $\frac{2}{6}$, $\frac{2}{6} < \frac{4}{6}$
 More: $\frac{7}{6}$, $\frac{7}{6} > \frac{4}{6}$
 Equivalent: $\frac{2}{3}$, $\frac{2}{3} = \frac{4}{6}$
 - b. Less: $\frac{1}{4}$, $\frac{1}{4} < \frac{3}{4}$

- Display or write the comparison statements using students' numbers.
- "Do all of these statements make sense? How do you know?"
- "Are there any more statements that we could write?"
- If time permits, repeat with the next two parts.
- "Now work independently on the last set of problems."
- 5 minutes: independent work time

Synthesis

- Invite students to share their responses to the last set of problems.
- "How did you find a fraction that was less than (or greater than or equivalent to) the given fraction?"

More: $\frac{5}{4}, \frac{5}{4} > \frac{3}{4}$
 Equivalent: $\frac{6}{8}, \frac{6}{8} = \frac{3}{4}$

Advancing Student Thinking

If students don't find more than one number that would make the statements in the first problem true, consider asking:

- "You found one number that made the statement true. How did you find that number and know that it made the statement true?"
- "How could you use a similar strategy to find another number that would make the statement true?"

Activity 3 (optional)

🕒 10 min

Ultimate Locate and Label

Standards Alignments

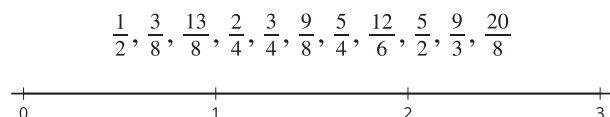
Addressing 3.NF.A.2, 3.NF.A.3

The purpose of this activity is for students to use their knowledge of fractions to locate fractions with different denominators on the number line. Students may use a variety of reasoning to locate the fractions, including their knowledge of equivalence, strategies about the same numerator or denominator, or benchmark numbers they are familiar with. The synthesis focuses on the variety of strategies that make sense, and students should be encouraged to use different strategies for different fractions as needed.

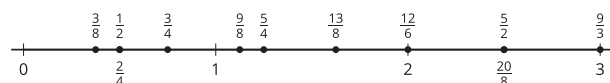
Although students have represented fractions on number lines (including those with two different denominators, when reasoning about equivalence), this activity is optional because representing multiple fractions of different denominators on the same number line involves a deeper understanding than required by the standards.

Student-facing Task Statement

Locate and label each fraction on the number line. Be prepared to share your reasoning.



Student Responses



Launch

- Groups of 2

Activity

- “Work independently to start placing these fractions on the number line.”
- 3–5 minutes: independent work time
- “Share your strategies with your partner and place any fractions you have left together.”
- 5–7 minutes: partner work time
- Monitor for students who:
 - place each fraction separately by partitioning for that single fraction
 - compare the fraction they are placing to others they’ve already placed
 - use equivalent fractions
 - use strategies about same numerators or denominators
 - use benchmarks like whole numbers or halves

Synthesis

- Invite students to share a variety of strategies for placing fractions on the number line.
- Consider asking:
 - “Which fractions were easier to place on the number line?”
 - “Which fractions were more difficult?”
 - “Did anyone have the same strategy but would explain it differently?”

Lesson Synthesis

🕒 5 min

"We have compared a lot of different fractions. Fractions with the same denominator, fractions with the same numerator, and in this lesson, we saw fractions that were equivalent again."

"What do you think would be some of the most important things to tell a friend who wanted to learn about comparing two fractions?" (I would tell my friend to think about whether they can draw a representation, like a number line or a diagram to see which fraction is greater. I think they need to know whether the fractions have the same numerator or the same denominator. They can check to see if the fractions are the same size or are the same location because that means they are equivalent.)

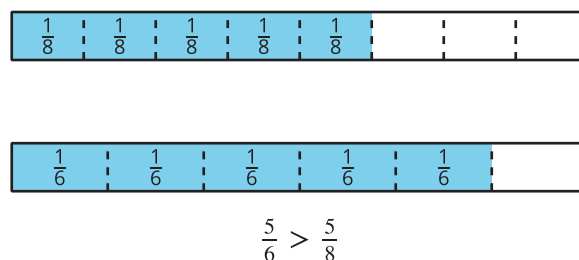
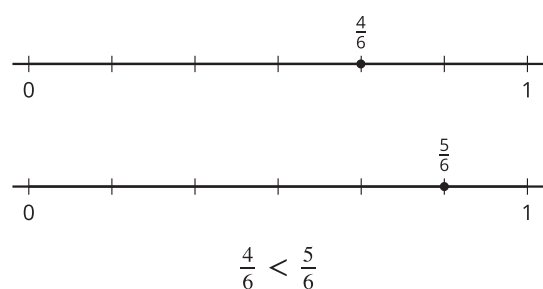
Consider asking: "Does your strategy for comparing fractions change depending on the fractions?"

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Student Section Summary

In this section, we compared fractions with the same numerator or denominator and used the symbols $>$, $=$, or $<$ to record our results. We used diagrams and number lines to represent our thinking.



----- Complete Cool-Down -----

Response to Student Thinking

Students do not complete the statements that compare each pair of fractions.

Next Day Support

- Before the next day's warm-up, pass back the cool down and have students work in small groups to make corrections.

Lesson 18: Design With Fractions (Optional)

Standards Alignments

Building On 3.NF.A.1, 3.NF.A.2.a
Addressing 3.NF.A.2

Teacher-facing Learning Goals

- Apply fraction understanding to create geometric designs.

Student-facing Learning Goals

- Let's use fractions to create a design.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of fractions to create geometric designs.

This lesson is optional because it does not address any new mathematical content standards. It does provide students with an opportunity to apply precursor skills of mathematical modeling.

In this lesson, students apply their understanding of fractions to create geometric designs, starting with a given square. They are tasked with marking a fractional length ($\frac{1}{2}$ or $\frac{1}{4}$) of each side of the square with a point. They then connect the points, which creates a new shape within the square. Students iterate this process of marking a fractional length and connecting points to generate their designs.

To mark a given length, students apply their experience with partitioning a segment into equal parts. To mark a fractional length, they decide which endpoint of each side to use as a starting point, whether to always mark the points in the same direction (clockwise or counterclockwise), how many iterations are practical, and so on (MP4).

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Paper: Activity 1, Activity 2
- Rulers or straightedges: Activity 1, Activity 2

Lesson Timeline

| | |
|------------------|--------|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 25 min |
| Lesson Synthesis | 10 min |

Teacher Reflection Question

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Between 0 and 1

Standards Alignments

Addressing 3.NF.A.2

The purpose of this warm-up is to elicit observations about partitions in number lines of different scales. In subsequent design activities, students will partition the sides of squares and other shapes into unit fractions. That process will be iterative, with the length being partitioned changing each time. The work here familiarizes students with the reasoning they will encounter later in the lesson.

Instructional Routines

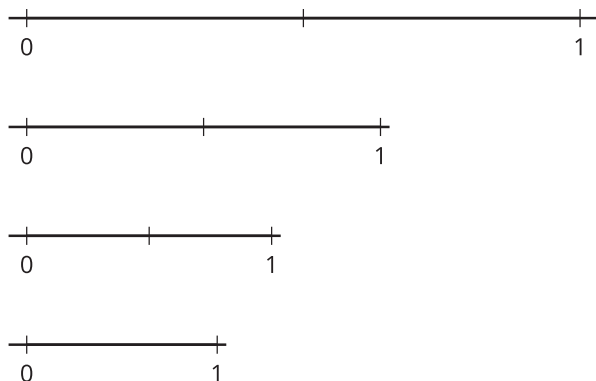
Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Launch

- Groups of 2
- Display the image.



Student Responses

Students may notice:

- Four number lines go from 0 to 1.
- Three of the number lines are partitioned into halves. The last one is not.
- The number lines get shorter and shorter.
- Each number line is not quite half as short as the one before it.

Students may wonder:

- Why are the number lines changing in length?
- Why doesn't the last number have a tick mark between 0 and 1?
- Do the lines get shorter in a certain way?

- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How are all the number lines alike?" (They all go from 0 to 1. They have the labels 0 and 1.)
- "How are they different?" (They have different lengths.)
- "How would you partition the last number line to match the rest?" (Mark the halfway point between 0 and 1.)
- "Suppose we'd like all the number lines to be partitioned into fourths. How would you go about doing so?" (Find the halfway point between 0 and $\frac{1}{2}$ and between $\frac{1}{2}$ and 1.)

Activity 1

🕒 15 min

Design With $\frac{1}{2}$

Standards Alignments

Building On 3.NF.A.1, 3.NF.A.2.a

The purpose of this activity is for students to create a design using the fraction $\frac{1}{2}$ as a constraint for length. Students partition each side of a given square into halves and mark a length of $\frac{1}{2}$ on

each side. They connect those midpoints to form another shape, partition the sides into halves again, and repeat the process to make increasingly smaller shapes. Students notice that the resulting shapes are also squares, and the squares in the pattern alternate between having vertical and horizontal sides and diagonal sides.

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they work in groups. On a visible display, record words and phrases such as: middle, midpoint, point, endpoint, connect, mark, cut, partition, side length. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Representing

Access for Students with Disabilities

Representation: Access for Perception. Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

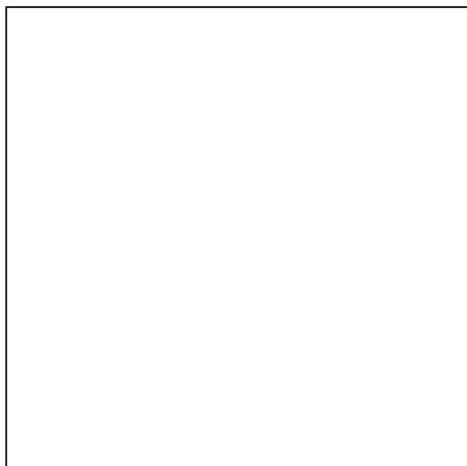
Supports accessibility for: Language, Social-Emotional Functioning

Materials to Gather

Paper, Rulers or straightedges

Student-facing Task Statement

1. Here is a square. On each side, mark a point to show $\frac{1}{2}$ of its length.



Launch

- Groups of 2
- “Let’s create a design using the fraction $\frac{1}{2}$.”
- “Take a minute to read the activity statement. Then, turn and talk to your partner about what you are asked to do.”
- 1 minute: quiet think time
- 1 minute: partner discussion
- Give each student a ruler or a straightedge.
- Provide access to extra paper, in case requested.

Activity

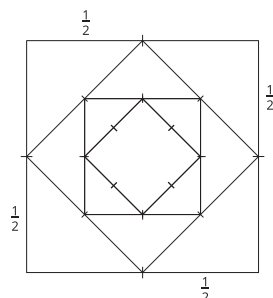
- “Work with your partner to complete the activity. Use a straightedge when you draw lines to connect points.”

Connect each point to the point on the two sides next to it. What shape did you create?

2. Look at the new shape you created. On each side, mark a point to show $\frac{1}{2}$ of its length. Connect the points again. What shape did you create?
3. Repeat the steps you just did at least two more times. Make some observations about the design you just created.

Student Responses

1. A square
2. A square
3. Sample response:



- 10 minutes: partner work time
- Monitor for different strategies and tools students use to partition the sides of the squares, such as:
 - estimating or “eyeballing” the midpoint
 - folding opposite sides of each square in half
 - copying the side length of each square onto another paper, folding it in half, and using it to mark the midpoint of all four sides
 - using a ruler to measure

Synthesis

- Select previously identified students to share their strategies for partitioning the sides of each shape into halves. Ask them to demonstrate their methods as needed.
- “How did you know which endpoint to use as 0 or as a starting point to mark $\frac{1}{2}$ of the length?” (It doesn’t matter. Starting from either end gives the same point.)
- Display one or more completed drawings (showing different numbers of iterations).
- “Why do you think we all ended up with the same design?” (Each time we marked the same set of points—the middle point of each side.)
- “In the next activity, we’ll creating a design with a different fraction.”

Activity 2

Design With $\frac{1}{4}$

🕒 25 min

Standards Alignments

Building On 3.NF.A.1, 3.NF.A.2.a

The purpose of this activity is for students to create a design using the fraction $\frac{1}{4}$ as a constraint for length. The fraction $\frac{1}{4}$ expands the number of possible designs that could be generated.

When the fractional length to be marked on the sides of a square was $\frac{1}{2}$, students could use either end of a side as a starting point and would mark the same point. The shape that resulted from connecting the midpoints was always a square.

When the fractional length to be marked is $\frac{1}{4}$, the location of the point changes depending on the starting point. Consequently, the shapes that result from connecting the points may be a square, another type of quadrilateral, or may vary each time. The shapes in turn determine how many iterations can be done. (For example, if the resulting shapes are narrow parallelograms, students may only be able to do 2 or 3 rounds before further partitioning becomes unfeasible.)

If time permits, encourage students color or decorate their drawings. Some students may also enjoy the challenge of creating another design using new constraints, such as:

- starting with a square of a different size or with another shape
- using another unit fraction or a non-unit fraction to mark the length of each side
- using a different unit fraction for each iteration

Students can observe regularity in repeated reasoning (MP8) in many different ways as the new shapes they make are often smaller versions of the previous shape, but this depends heavily on how they decide to mark off $\frac{1}{4}$ of each side.

Materials to Gather

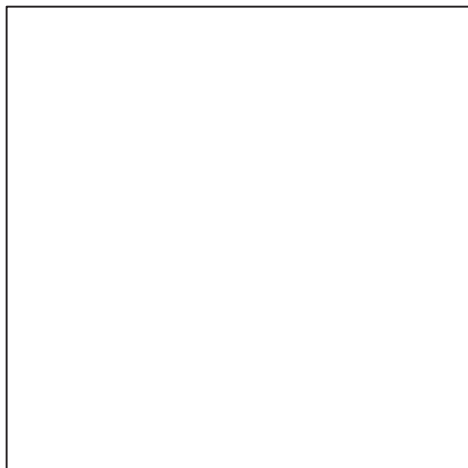
Paper, Rulers or straightedges

Student-facing Task Statement

1. Here is another square. On each side, mark a point to show $\frac{1}{4}$ of its length.

Launch

- Groups of 2–4
- “Let’s now create a design using the fraction $\frac{1}{4}$.”
- “Take a minute to read the activity statement. Then, turn and talk to your partner about how you think the drawing process will be different this time.”



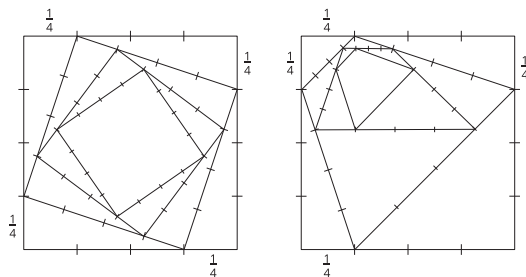
Connect each point to the point on the two sides next to it. What shape did you create?

- Look at the new shape you created. On each side, mark a point to show $\frac{1}{4}$ of its length. Connect the points again. What shape did you create?
- Repeat the steps you just did at least two more times. Make some observations about the design you just created.

Student Responses

Sample responses:

- A square, a trapezoid, a rectangle, a quadrilateral with different side lengths
- A square, a trapezoid, a parallelogram, a quadrilateral with different side lengths
-



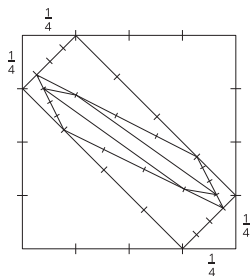
- 1 minute: quiet think time
- 1 minute: partner discussion
- Give each student a ruler or a straightedge.
- Provide access to extra paper, in case requested.

Activity

- "Take a few minutes to work independently on the activity. Afterwards, share your drawing and your process with your group."
- "Use a straightedge when you draw lines to connect points."
- 8–10 minutes: independent work time
- 6–8 minutes: group discussion
- Monitor for different strategies and tools students use to partition the sides of the shapes.
- Also monitor for the starting point students use to identify a length of $\frac{1}{4}$ on each side of the square. For example, they may:
 - always proceed in a clockwise or counterclockwise direction
 - start from the left endpoint for the horizontal sides, and start from the top endpoint for the vertical sides
 - start from either end of each side and in no particular order

Synthesis

- Select students to share their strategies for partitioning each side into fourths. Ask them to demonstrate their methods as needed.
- Invite students who created different designs to share the decisions they made along the way.
- "How did you decide which endpoint to use as a starting point for marking $\frac{1}{4}$ of the



side length?"

- "Did you mark the length the same way each time?"
- "Why do you think we ended up with different designs when the fraction is $\frac{1}{4}$?"
(We didn't mark off $\frac{1}{4}$ from the same starting point or using the same order, so the locations of the points and the shapes from connecting the points were different.)

Lesson Synthesis

🕒 10 min

"Today we used fractions to partition the sides of geometric shapes and create designs."

"How was partitioning the sides of a shape and marking points on them like partitioning and marking points on number lines? How was it different?"

"What did you enjoy about the process of designing with fractions? What was challenging?"

Suggested Centers

- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Addressing)
- Number Line Scoot (2–3), Stage 3: Halves, Thirds, Fourths, Sixths and Eighths (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)



Family Support Materials

Family Support Materials

Fractions as Numbers

In this unit, students develop an understanding of fractions as numbers. They represent fractions with diagrams and number lines and compare and find equivalent fractions.

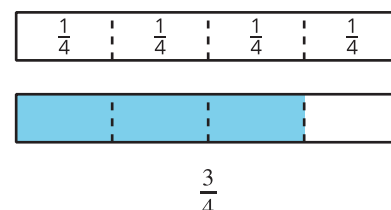
Section A: Introduction to Fractions

In this section, students use diagrams and fraction strips to learn about fractions.

In grade 2, they learned about halves, thirds, and fourths. Now, they partition 1 whole into 6 or 8 parts, describe each part as “a sixth” and “an eighth,” and write the notation $\frac{1}{6}$ and $\frac{1}{8}$.

Students learn that the notation $\frac{1}{b}$ means 1 whole is partitioned into b parts and each part has size $\frac{1}{b}$.

In these diagrams, each part is a unit fraction with the size $\frac{1}{4}$.

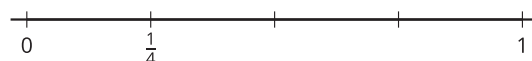


Students see that composing unit fractions create non-unit fractions (fractions with numerators greater than 1). For example, putting together 3 parts of $\frac{1}{4}$ gives $\frac{3}{4}$.

Section B: Fractions on the Number Line

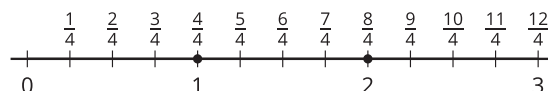
In this section, students locate fractions on the number line. They learn that, just like whole numbers, fractions can be represented as distances from 0 on the number line.

Students partition the interval from 0 to 1 into b equal parts. They label the first tick mark with a unit fraction $\frac{1}{b}$.



Then, students locate non-unit fractions on the number line by counting unit fractions. They notice that certain fractions are in the same location as whole numbers on the number line.

For example, $\frac{4}{4}$ is at the same location as 1 and $\frac{8}{4}$ is at the same location as 2.

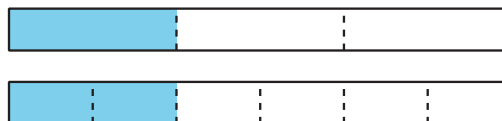


The terms “numerator” and “denominator” are introduced here.

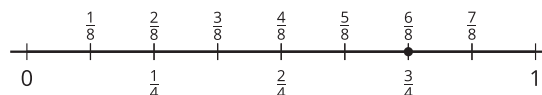
Section C: Equivalent Fractions

In this section, students learn that equivalent fractions are fractions that are the same size. They use fraction strips and diagrams to show and find equivalent fractions.

The shaded parts of the diagrams show that $\frac{1}{3}$ and $\frac{2}{6}$ are the same size, so $\frac{1}{3} = \frac{2}{6}$.



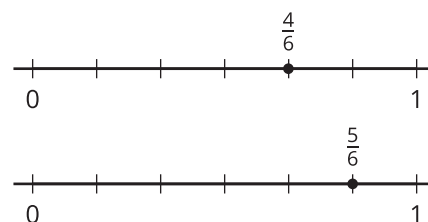
The number line diagram shows that $\frac{6}{8}$ and $\frac{3}{4}$ are at the same location or are the same distance from 0, so $\frac{6}{8} = \frac{3}{4}$.



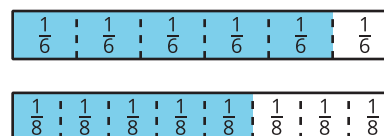
Section D: Fraction Comparisons

In this section, students compare fractions. They learn that comparisons are only valid if the fractions being compared refer to the same whole.

Students first compare fractions with the same denominator (such as $\frac{4}{6}$ and $\frac{5}{6}$).



Then, they compare fractions with the same numerator (such as $\frac{5}{6}$ and $\frac{5}{8}$).



Try it at home!

Near the end of the unit, ask your student to show the fractions $\frac{5}{8}$ and $\frac{6}{4}$ on a fraction strip and a number line.

Questions that may be helpful as they work:

- How did you determine how many partitions needed to be made?
- How did you know how many parts to shade in?
- How did you know where to place the fraction on the number line?
- Which fraction is larger? How do you know?

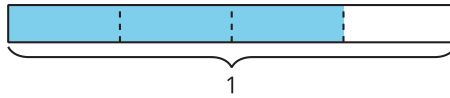
Unit Assessments

Check Your Readiness A, B, C and D
End-of-Unit Assessment

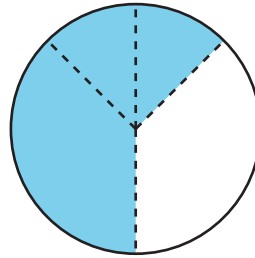
Fractions as Numbers: Section A Checkpoint

1. Select **all** diagrams where $\frac{3}{4}$ of the whole rectangle or circle is shaded.

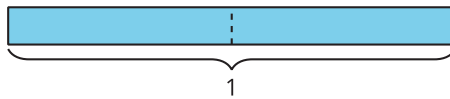
A.



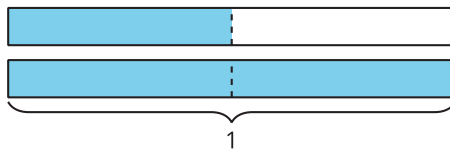
B.



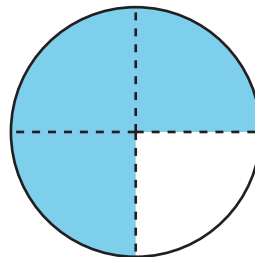
C.



D.



E.

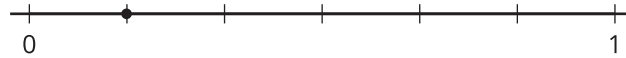


2. Shade $\frac{5}{6}$ of the rectangle.

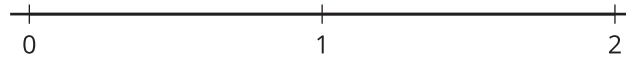


Fractions as Numbers: Section B Checkpoint

1. What fraction is marked on the number line?



2. Locate and label $\frac{2}{3}$ and $\frac{5}{3}$ on the number line:



3. Locate and label the number 1 on the number line. Explain or show your reasoning.



Fractions as Numbers: Section C Checkpoint

1. Select **all** the true equations.

A. $\frac{7}{8} = \frac{3}{4}$

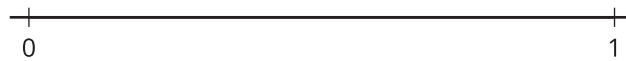
B. $\frac{2}{6} = \frac{1}{3}$

C. $\frac{4}{8} = \frac{3}{6}$

D. $\frac{1}{6} = \frac{2}{8}$

E. $\frac{1}{2} = \frac{2}{3}$

2. Find a fraction that is equivalent to $\frac{4}{6}$. Use the number lines if they are helpful.



3. a. Circle the fraction that is equal to a whole number: $\frac{1}{4}$ $\frac{3}{4}$ $\frac{11}{4}$ $\frac{12}{4}$

b. Write three different fractions that are equal to 2.

Fractions as Numbers: Section D Checkpoint

1. Use $<$, $>$, or $=$ to make each statement true.

a. $\frac{1}{3}$ _____ $\frac{1}{2}$

b. $\frac{5}{8}$ _____ $\frac{4}{8}$

c. $\frac{2}{6}$ _____ $\frac{1}{3}$

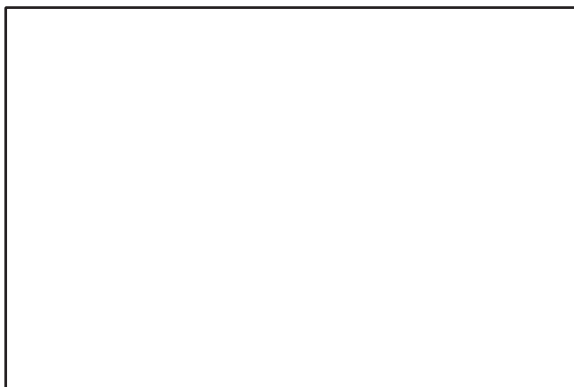
d. $\frac{3}{2}$ _____ $\frac{3}{4}$

2. Elena ate $\frac{1}{3}$ of a loaf of bread while Clare ate $\frac{1}{4}$ of the same loaf of bread. Clare says that she ate more of the bread because 4 is greater than 3.

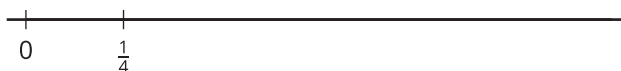
Do you agree with Clare? Explain or show your reasoning.

Fractions as Numbers: End-of-Unit Assessment

1. Shade $\frac{5}{8}$ of the rectangle.



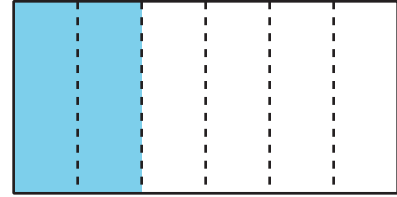
2. a. Locate and label 1 on the number line. Explain your reasoning.



- b. Locate and label $\frac{5}{6}$ on the number line. Explain your reasoning.



3. What fraction of the large rectangle is shaded? Select **all** that apply.



A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{2}{6}$

D. $\frac{1}{3}$

E. $\frac{4}{6}$

4. Which fraction is equivalent to $\frac{9}{3}$?

A. $\frac{1}{3}$

B. $\frac{10}{4}$

C. $\frac{6}{2}$

D. $\frac{8}{2}$

5. Write two fractions that are equivalent to $\frac{1}{2}$.

6. Diego says that there are no fractions equivalent to 2 because 2 is a whole number, not a fraction. Do you agree with Diego? Explain or show your reasoning.

7. Select **all** the true statements.

A. $\frac{1}{3} > \frac{1}{4}$

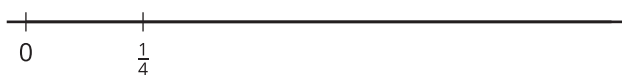
B. $\frac{9}{4} < \frac{11}{4}$

C. $\frac{2}{6} > \frac{2}{4}$

D. $\frac{1}{4} < \frac{1}{5}$

E. $\frac{3}{8} < \frac{4}{8}$

8. a. Locate and label $\frac{1}{2}$ on the number line.



b. Noah locates and labels $\frac{1}{2}$ here and says "It's halfway between the tick marks for 0 and $\frac{1}{4}$." Explain why Noah is not correct.



c. What is the number Noah labeled $\frac{1}{2}$? Explain your reasoning.



Assessment Answer Keys

Check Your Readiness A, B, C and D
End-of-Unit Assessment

Assessment Answer Keys

Assessment: Section A Checkpoint

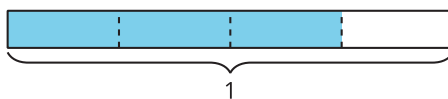
Problem 1

Goals Assessed

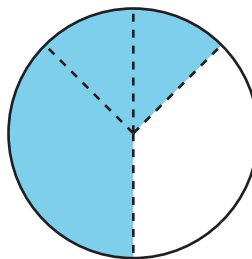
- Understand that fractions are built from unit fractions such that a fraction $\frac{a}{b}$ is the quantity formed by a parts of size $\frac{1}{b}$.

Select **all** diagrams where $\frac{3}{4}$ of the whole rectangle or circle is shaded.

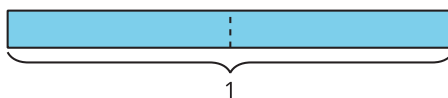
A.



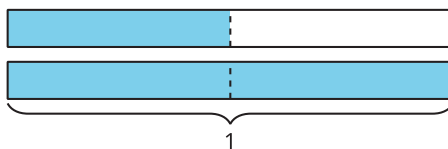
B.



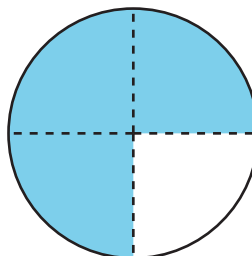
C.



D.



E.



Solution

["A", "E"]

Problem 2

Goals Assessed

- Understand that fractions are built from unit fractions such that a fraction $\frac{a}{b}$ is the quantity formed by a parts of size $\frac{1}{b}$.
- Understand that unit fractions are formed by partitioning shapes into equal parts.

Shade $\frac{5}{6}$ of the rectangle.

Solution

Sample response:



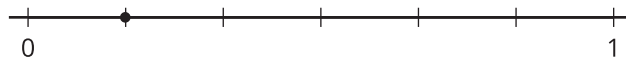
Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Understand a fraction as a number and represent fractions on the number line.

What fraction is marked on the number line?



Solution

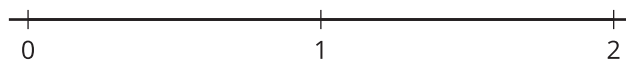
$$\frac{1}{6}$$

Problem 2

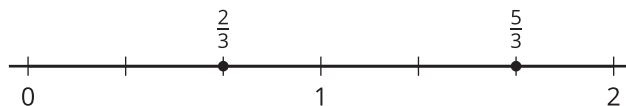
Goals Assessed

- Understand a fraction as a number and represent fractions on the number line.

Locate and label $\frac{2}{3}$ and $\frac{5}{3}$ on the number line:



Solution

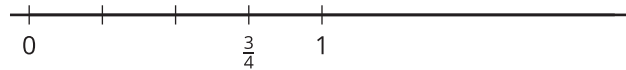


Problem 3

Goals Assessed

- Understand a fraction as a number and represent fractions on the number line.

Locate and label the number 1 on the number line. Explain or show your reasoning.

**Solution**

First I found $\frac{1}{4}$, knowing that $\frac{3}{4}$ is three $\frac{1}{4}$ s, and then I kept going to get $\frac{4}{4}$ or 1.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Explain equivalence of fractions in special cases and express whole numbers as fractions and fractions as whole numbers.

Select **all** the true equations.

A. $\frac{7}{8} = \frac{3}{4}$

B. $\frac{2}{6} = \frac{1}{3}$

C. $\frac{4}{8} = \frac{3}{6}$

D. $\frac{1}{6} = \frac{2}{8}$

E. $\frac{1}{2} = \frac{2}{3}$

Solution

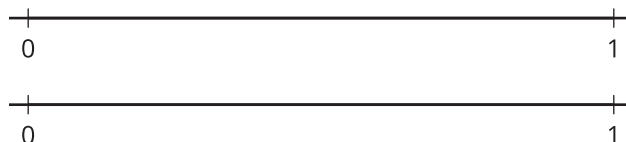
["B", "C"]

Problem 2

Goals Assessed

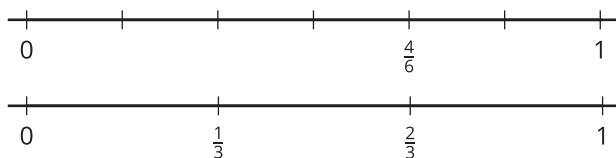
- Explain equivalence of fractions in special cases and express whole numbers as fractions and fractions as whole numbers.

Find a fraction that is equivalent to $\frac{4}{6}$. Use the number lines if they are helpful.



Solution

$$\frac{2}{3}$$



Problem 3

Goals Assessed

- Explain equivalence of fractions in special cases and express whole numbers as fractions and fractions as whole numbers.

- a. Circle the fraction that is equal to a whole number: $\frac{1}{4}$ $\frac{3}{4}$ $\frac{11}{4}$ $\frac{12}{4}$
- b. Write three different fractions that are equal to 2.

Solution

- a. Students circle $\frac{12}{4}$.
- b. Sample response: $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$

Assessment: Section D Checkpoint

Problem 1

Goals Assessed

- Compare two fractions with the same numerator or denominator, record the results with the symbols $>$, $=$, or $<$, and justify the conclusions.

Use $<$, $>$, or $=$ to make each statement true.

- a. $\frac{1}{3}$ _____ $\frac{1}{2}$
b. $\frac{5}{8}$ _____ $\frac{4}{8}$
c. $\frac{2}{6}$ _____ $\frac{1}{3}$
d. $\frac{3}{2}$ _____ $\frac{3}{4}$

Solution

Sample responses:

- a. $\frac{1}{3} < \frac{1}{2}$
b. $\frac{5}{8} > \frac{4}{8}$
c. $\frac{2}{6} = \frac{1}{3}$
d. $\frac{3}{2} > \frac{3}{4}$

Problem 2

Goals Assessed

- Compare two fractions with the same numerator or denominator, record the results with the symbols $>$, $=$, or $<$, and justify the conclusions.

Elena ate $\frac{1}{3}$ of a loaf of bread while Clare ate $\frac{1}{4}$ of the same loaf of bread. Clare says that she ate more of the bread because 4 is greater than 3.

Do you agree with Clare? Explain or show your reasoning.

Solution

Sample response: Clare is not correct, Elena ate more of the bread. Thirds are bigger than fourths since the whole is split into fewer parts.

Assessment: End-of-Unit Assessment

Problem 1

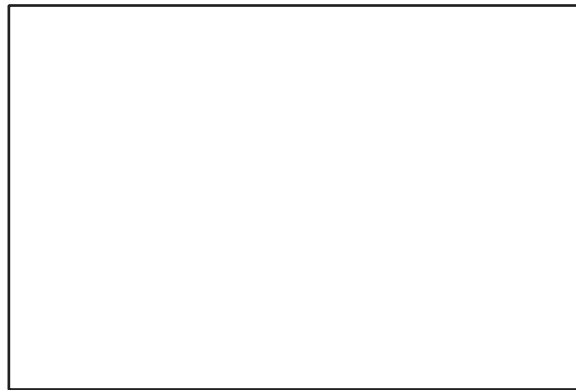
Standards Alignments

Addressing 3.G.A.2, 3.NF.A.1

Narrative

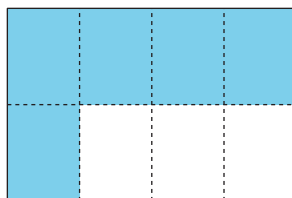
Students shade a fraction of a rectangle. They can divide the rectangle into 8 equal parts in several different ways. The most likely choices are all vertical cuts or one horizontal cut and 3 vertical cuts as shown in the sample response. Since students are making the divisions freehand, the 8 parts do not need to be exactly equal. They also may choose to shade any 5 of the pieces.

Shade $\frac{5}{8}$ of the rectangle.



Solution

Sample response:



Problem 2

Standards Alignments

Addressing 3.NF.A.2.a, 3.NF.A.2.b

Narrative

Students locate numbers on the number line. First they locate and label 1 on a number line where a single unit fraction has been labeled. They need to understand that 1 is $\frac{4}{4}$ to do this problem.

Then students locate a fraction given the location of 1 so they need to understand the meaning of the denominator as the number of equal parts in the whole and the numerator as the number of those parts.

- a. Locate and label 1 on the number line. Explain your reasoning.

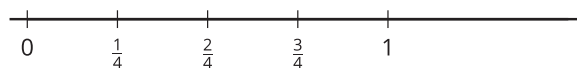


- b. Locate and label $\frac{5}{6}$ on the number line. Explain your reasoning.

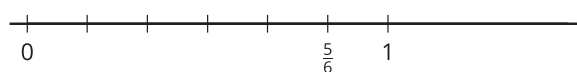


Solution

- a. Sample response: I marked off fourths until I reached $\frac{4}{4}$ which is 1.



- b. Since $\frac{5}{6}$ is 5 equal parts and each part is $\frac{1}{6}$, I started by making 6 equal parts between 0 and 1 and then the 5th one is $\frac{5}{6}$.



Problem 3

Standards Alignments

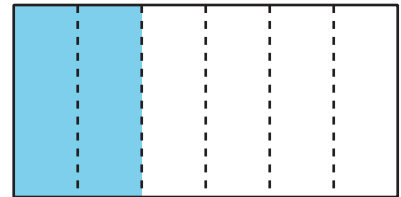
Addressing 3.NF.A.1, 3.NF.A.3.b

Narrative

Students identify equivalent ways of expressing a fraction. Students may select A if they see that

the rectangle is divided into 6 equal pieces but do not notice that 2 pieces are shaded. They may select B if they count the shaded region as one piece and each of the unshaded rectangles as one piece. Students may not select D if they fail to see the 3 equal groups of 2 small rectangles in the whole. Students may select E if they focus on the unshaded area instead of the shaded area.

What fraction of the large rectangle is shaded? Select **all** that apply.



- A. $\frac{1}{6}$
- B. $\frac{1}{5}$
- C. $\frac{2}{6}$
- D. $\frac{1}{3}$
- E. $\frac{4}{6}$

Solution

["C", "D"]

Problem 4

Standards Alignments

Addressing 3.NF.A.3.a, 3.NF.A.3.c

Narrative

Students identify a fraction that is equivalent to a whole number expressed as a fraction. While they are not directly asked to write $\frac{9}{3}$ as a whole number, the most likely reasoning is to identify that this is 3 wholes and so is $\frac{6}{2}$. Students may select A if they confuse the meaning of the numerator and denominator and think of the fraction $\frac{1}{3}$. They may select B if they add 1 to the numerator and the denominator. They may select D if they subtract 1 from the numerator and the denominator.

Which fraction is equivalent to $\frac{9}{3}$?

- A. $\frac{1}{3}$
- B. $\frac{10}{4}$
- C. $\frac{6}{2}$
- D. $\frac{8}{2}$

Solution

C

Problem 5

Standards Alignments

Addressing 3.NF.A.3.b

Narrative

Students write two fractions that are equivalent to $\frac{1}{2}$. No representation is suggested so students may draw a diagram to solve the problem or may use their understanding of the meaning of fractions in terms of equal parts of a whole.

Write two fractions that are equivalent to $\frac{1}{2}$.

Solution

Sample response: $\frac{2}{4}$ and $\frac{4}{8}$

Problem 6

Standards Alignments

Addressing 3.NF.A.3.c

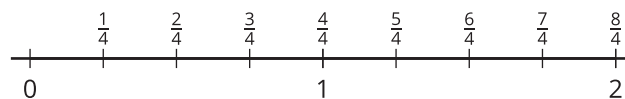
Narrative

Students examine a false claim about writing a whole number as a fraction. To write 2 as a whole number, they can explain that each whole is equivalent to 4 fourths (for example) so 2 wholes is equivalent to 8 fourths. Students may draw a diagram, such as a number line, to support their reasoning and they may pick any denominator for their fraction.

Diego says that there are no fractions equivalent to 2 because 2 is a whole number, not a fraction. Do you agree with Diego? Explain or show your reasoning.

Solution

Diego is right that 2 is a whole number but it can also be written as a fraction. The number line shows that 2 and $\frac{8}{4}$ are at the same location so they represent the same number.



Problem 7

Standards Alignments

Addressing 3.NF.A.3.d

Narrative

Students decide if fraction comparison statements are correct. Students who do not select A, B, or E have likely either misread the numbers or need further practice with the meaning of the numerator and denominator in a fraction. Students who select C or D are probably comparing the denominators and not thinking about what they say about the size of the fraction.

Select **all** the true statements.

- A. $\frac{1}{3} > \frac{1}{4}$
- B. $\frac{9}{4} < \frac{11}{4}$
- C. $\frac{2}{6} > \frac{2}{4}$
- D. $\frac{1}{4} < \frac{1}{5}$

E. $\frac{3}{8} < \frac{4}{8}$

Solution

["A", "B", "E"]

Problem 8

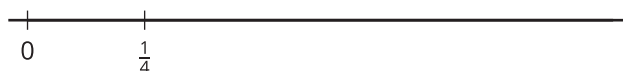
Standards Alignments

Addressing 3.NF.A.2.a, 3.NF.A.2.b, 3.NF.A.3.b

Narrative

Students locate different numbers on a number line. In order to find $\frac{1}{2}$ given the location of $\frac{1}{4}$ they need to recall that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$. Then they analyze a common error where a point halfway from 0 to $\frac{1}{4}$ has been labeled $\frac{1}{2}$. Because there are four $\frac{1}{4}$ s in a whole, that means there are 8 of this labeled number in a whole and so it is $\frac{1}{8}$.

- a. Locate and label $\frac{1}{2}$ on the number line.



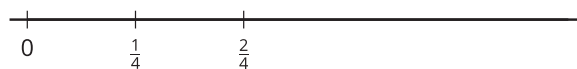
- b. Noah locates and labels $\frac{1}{2}$ here and says "It's halfway between the tick marks for 0 and $\frac{1}{4}$." Explain why Noah is not correct.



- c. What is the number Noah labeled $\frac{1}{2}$? Explain your reasoning.

Solution

- a. Sample response: $\frac{2}{4}$ is the same as $\frac{1}{2}$ because if I double it I get $\frac{4}{4}$ or 1.



- b. No, Noah is not correct. He has located half of a fourth, not half of a whole.
 c. Fraction strips show that half of $\frac{1}{4}$ is $\frac{1}{8}$. There are 8 of those parts in 1 so each one is $\frac{1}{8}$.

Lesson Cool Downs

Lesson 1: Name the Parts

Cool Down: Partition a Rectangle

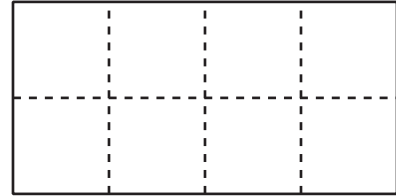
Partition the rectangle into eighths.



Lesson 2: Name Parts as Fractions

Cool Down: Label the Parts

1. Label each part with the correct fraction.



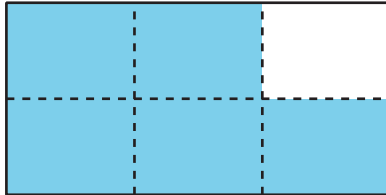
2. Partition and shade the rectangle to show $\frac{1}{4}$.



Lesson 3: Non-unit Fractions

Cool Down: Shaded Fraction

The rectangle represents 1 whole. What fraction is shaded? Explain your reasoning.



Lesson 4: Build Fractions from Unit Fractions

Cool Down: Represent a Fraction

This strip represents 1 whole. Partition the diagram and shade it to represent $\frac{6}{8}$.



Lesson 5: To the Number Line

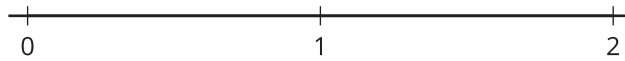
Cool Down: Reflection

Describe something you really understand well after today’s lesson or describe something that was confusing or challenging.

Lesson 6: Locate Unit Fractions on the Number Line

Cool Down: Locate and Label

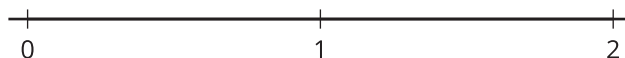
Locate and label $\frac{1}{8}$ on the number line. Explain your reasoning.



Lesson 7: Non-unit Fractions on the Number Line

Cool Down: Where is $\frac{5}{3}$?

Locate and label $\frac{2}{3}$ and $\frac{5}{3}$ on the number line. Explain your reasoning.



Lesson 8: Fractions and Whole Numbers

Cool Down: Where is 1?

Locate and label 1 on the number line.
Explain your reasoning.



Lesson 9: All Kinds of Numbers on the Number Line

Cool Down: Where is 1 Now?

Locate and label 1 on the number line. Explain your reasoning.



Lesson 10: Equivalent Fractions

Cool Down: Find the Equivalent Fractions

Each diagram represents 1.

Select **all** the diagrams whose shaded parts represent equivalent fractions.
Explain your reasoning.

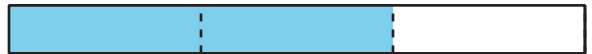
A



B



C



D



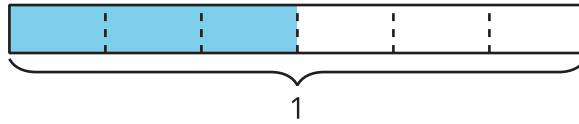
E



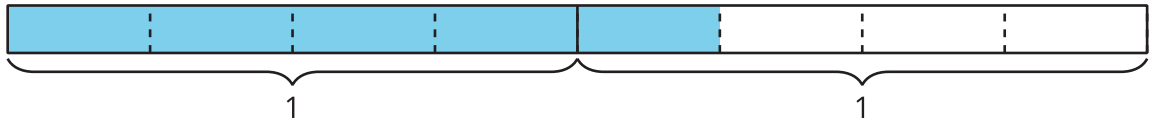
Lesson 11: Generate Equivalent Fractions

Cool Down: Two Fraction Names for Each Diagram

1. Write two fractions that the shaded part of this diagram represents.



2. Show that the shaded part of this diagram represents both $\frac{5}{4}$ and $\frac{10}{8}$.



Lesson 12: Equivalent Fractions on a Number Line

Cool Down: Equivalence on the Number Line

Use the number line(s) to decide whether $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent. Explain your reasoning.



Lesson 13: Whole Numbers and Fractions

Cool Down: Fraction to Whole Number and Whole Number to Fraction

1. Is $\frac{18}{4}$ a whole number? Explain or show your reasoning.

2. Write 2 as a fraction. Explain or show your reasoning.

Lesson 14: How Do You Compare Fractions?

Cool Down: How Would You Decide?

How would you decide if $\frac{6}{4}$ is equivalent to $\frac{3}{4}$? Explain or show your reasoning.

Lesson 15: Compare Fractions with the Same Denominator

Cool Down: Same Denominator

1. Which is the greater fraction: $\frac{7}{8}$ or $\frac{6}{8}$? Explain or show your reasoning.

2. Use the symbols $>$ or $<$ to make the statement true.

$$\frac{7}{8} \quad \underline{\hspace{1cm}} \quad \frac{6}{8}$$

Lesson 16: Compare Fractions with the Same Numerator

Cool Down: Same Numerator

Use the symbols $>$ and $<$ to make the statement true. Explain or show your reasoning.

$$\frac{4}{3} \quad \underline{\hspace{1cm}} \quad \frac{4}{6}$$

Lesson 17: Compare Fractions

Cool Down: All Kinds of Comparisons

1. Use the symbols $>$, $<$, or $=$ to make each statement true.

a. $\frac{4}{6}$ _____ $\frac{2}{6}$

b. $\frac{8}{8}$ _____ $\frac{4}{4}$

2. An ant crawled $\frac{3}{6}$ of the length of a bench. A spider crawled $\frac{3}{4}$ of the length of the same bench.

a. Which animal crawled farther? Explain or show your reasoning.

b. Write a statement using the symbols $>$, $<$, or $=$ to represent your answer.

Instructional Masters

Instructional Masters for Fractions as Numbers

| address | title | students per copy | written on? | requires cutting? | card stock recommended? | color paper recommended? |
|-----------------------|--------------------------------------|-------------------|-------------|-------------------|-------------------------|--------------------------|
| Activity Grade3.5.5.1 | Card Sort: Number Lines | 2 | no | yes | no | no |
| Activity Grade3.5.5.2 | Fold and Label Number Lines | 1 | yes | yes | no | no |
| Activity Grade3.5.7.1 | Number Line Scoot Stage 2 Gameboard | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 2 Gameboard | 2 | no | no | no | no |
| Activity Grade3.5.7.1 | Number Line Scoot Stage 2 Directions | 2 | no | yes | no | no |
| Center | Number Line Scoot Stage 2 Directions | 2 | no | yes | no | no |
| Activity Grade3.5.1.2 | Fold and Name | 4 | yes | yes | no | no |
| Activity Grade3.5.4.1 | Secret Fractions Stage 1 Gameboard | 2 | yes | no | no | no |
| Center | Secret Fractions Stage 1 Gameboard | 2 | yes | no | no | no |
| Activity Grade3.5.2.1 | Partition the Strips | 2 | yes | yes | no | no |
| Activity Grade3.5.4.1 | Secret Fractions Stage 1 Cards | 2 | no | yes | no | no |
| Center | Secret Fractions Stage 1 Cards | 2 | no | yes | no | no |
| Activity Grade3.5.3.2 | Fraction Match Part 2 | 4 | no | yes | no | no |

| | | | | | | |
|----------------------------|----------------------------------------------------------------|---|-----|-----|----|----|
| Activity Grade3.5.1.1.1 | Card Sort: Partitions | 2 | no | yes | no | no |
| Activity Grade3.5.15.2 | Spin to Win Recording Sheet | 2 | yes | no | no | no |
| Activity Grade3.5.3.2 | Fraction Match Part 1 | 2 | no | yes | no | no |
| Activity Grade3.5.15.2 | Spin to Win Spinner | 2 | no | no | no | no |
| Center | Mystery Number Stage 2 Directions | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 1 Gameboard | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 1 Spinner | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 1 Directions | 2 | no | yes | no | no |
| Center | Mystery Number Stage 3 Gameboard | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 3 Gameboard | 2 | no | no | no | no |
| Center | Number Line Scoot Stage 3 Directions | 2 | no | yes | no | no |
| Center | Rolling for Fractions Stage 1 Recording Sheet | 1 | yes | no | no | no |
| Center | Five in a Row Multiplication and Division Stage 2 Gameboard | 2 | no | no | no | no |

Card Sort: Number Lines

Card Sort: Number Lines
A



Card Sort: Number Lines
B



Card Sort: Number Lines
C



Card Sort: Number Lines
D



Card Sort: Number Lines
E



Card Sort: Number Lines
F



Card Sort: Number Lines
G



Card Sort: Number Lines
H



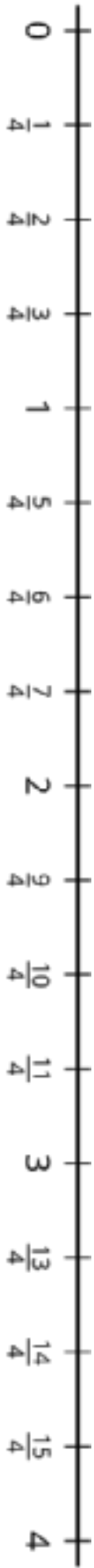
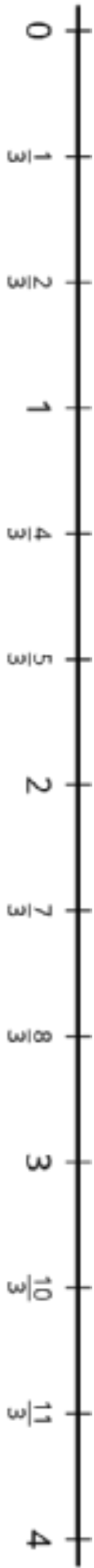
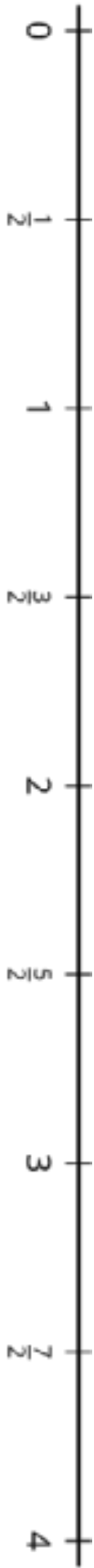
Card Sort: Number Lines
I



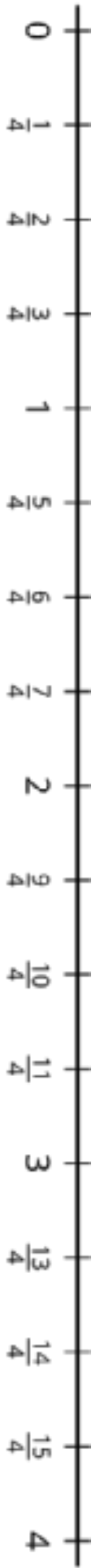
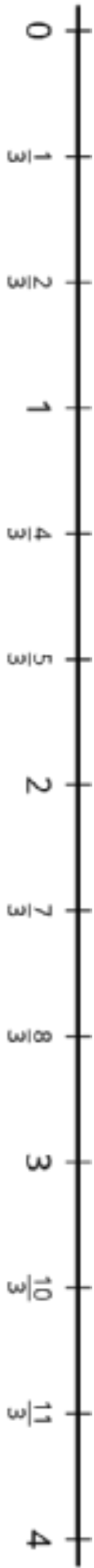
Fold and Label Number Lines



Number Line Scoot Stage 2 Gameboard



Number Line Scoot Stage 2 Gameboard



Number Line Scoot Stage 2 Directions

Directions:

- Place a small cube on zero on each number line.
 - On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, or 4 as the denominator for your fraction.
 - Count aloud as you move a cube that distance on the appropriate number line.
 - Take turns rolling and moving one cube.
 - If a cube lands *exactly* on the last tick mark of a number line, that partner keeps the cube and puts a new one at 0.
 - The first player to collect 5 cubes wins.
-

Directions:

- Place a small cube on zero on each number line.
- On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, or 4 as the denominator for your fraction.
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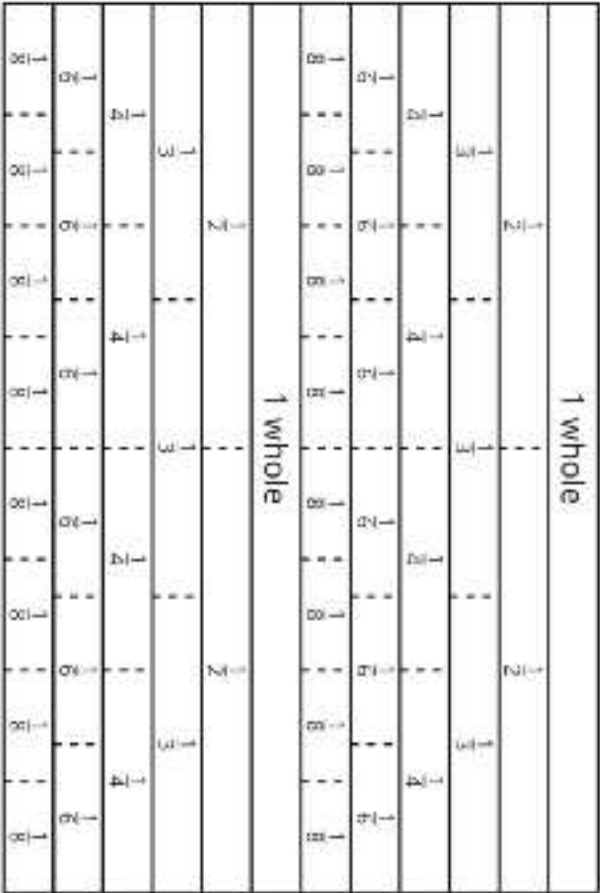
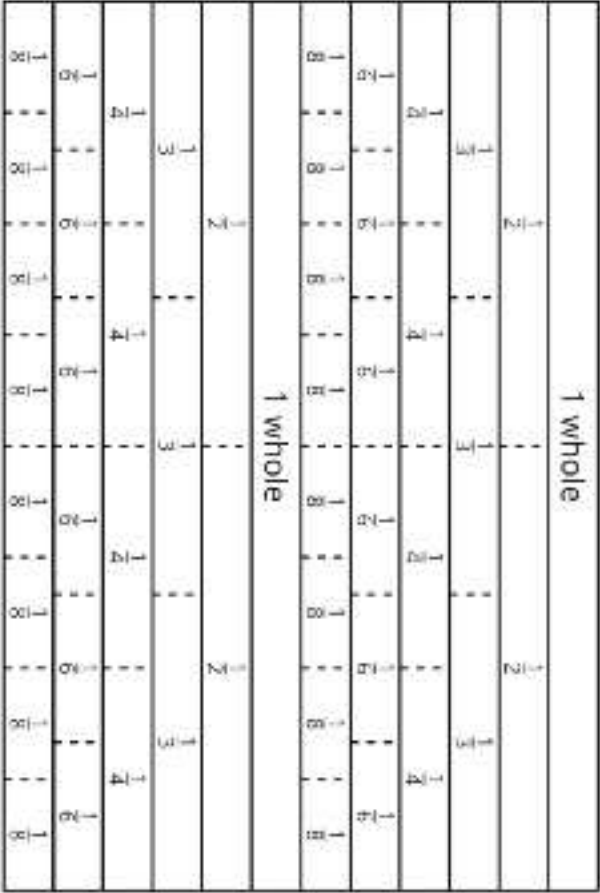
Fold and Name

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Secret Fractions Stage 1 Gameboard

Directions:

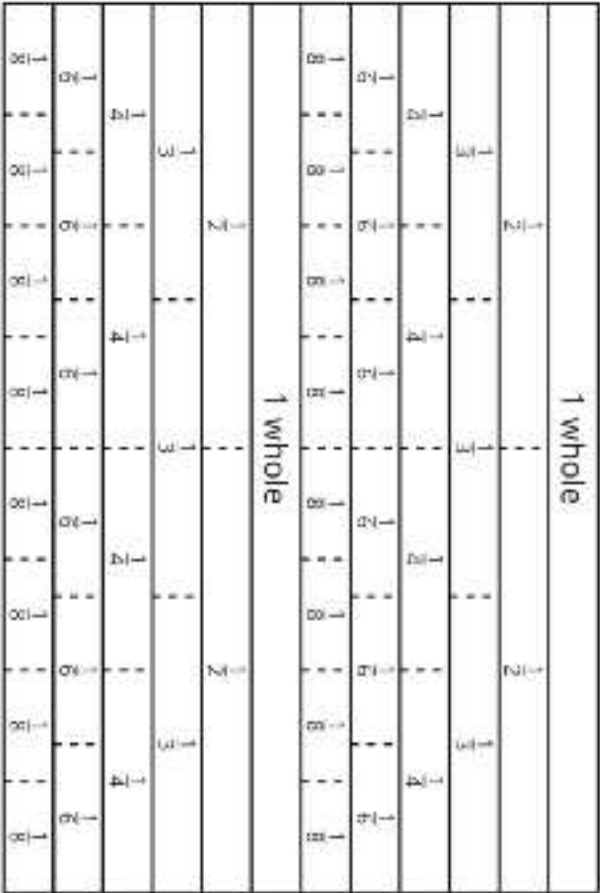
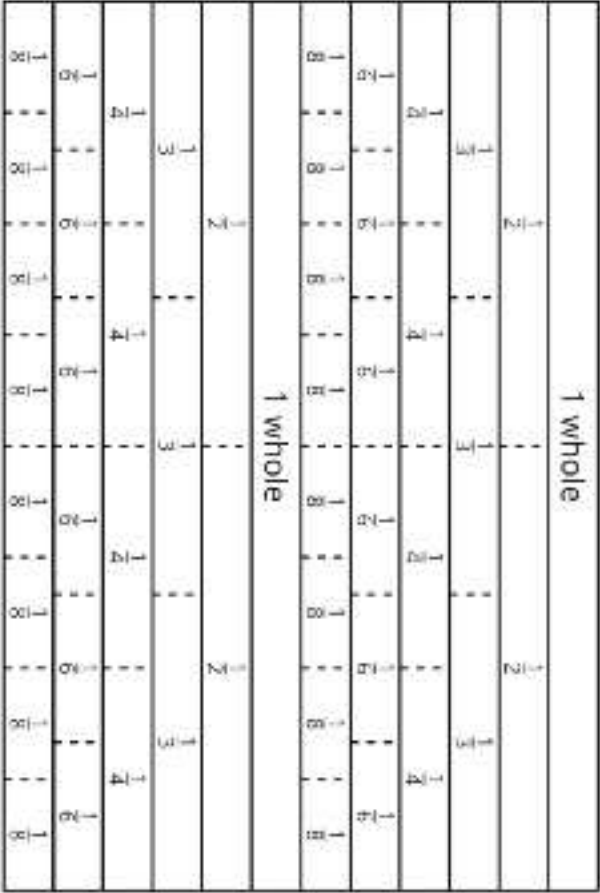
- Place the secret fraction cards in a stack face down.
- Place the unit fraction cards in a stack face down.
- Each partner:
 - Pick 2 secret fraction cards. These are the fractions you are trying to make with your unit fractions.
- Partner A you can:
- Pick up one unit fraction card.
- Trade both of your secret fractions for 2 new secret fractions from the pile.
- When you have enough unit fractions to make your secret fraction, fill in your secret fraction on the gameboard and pick a new secret fraction.
- First partner to make 3 secret fractions wins.



Secret Fractions Stage 1 Gameboard

Directions:

- Place the secret fraction cards in a stack face down.
- Place the unit fraction cards in a stack face down.
- Each partner:
 - Pick 2 secret fraction cards. These are the fractions you are trying to make with your unit fractions.
- Partner A you can:
- Pick up one unit fraction card.
- Trade both of your secret fractions for 2 new secret fractions from the pile.
- When you have enough unit fractions to make your secret fraction, fill in your secret fraction on the gameboard and pick a new secret fraction.
- First partner to make 3 secret fractions wins.



[illegible]

1 whole

Secret Fractions Stage 1 Cards

Unit Fraction

$$\frac{1}{2}$$

Unit Fraction

$$\frac{1}{2}$$

Unit Fraction

$$\frac{1}{3}$$

Unit Fraction

$$\frac{1}{3}$$

Unit Fraction

$$\frac{1}{3}$$

Unit Fraction

$$\frac{1}{4}$$

Unit Fraction

$$\frac{1}{4}$$

Unit Fraction

$$\frac{1}{4}$$

Unit Fraction

$$\frac{1}{4}$$

Unit Fraction

$$\frac{1}{6}$$

Unit Fraction

$$\frac{1}{6}$$

Unit Fraction

$$\frac{1}{6}$$

Unit Fraction

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Unit Fraction

$$\frac{1}{8}$$

Unit Fraction

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Unit Fraction

$$\frac{1}{8}$$

Unit Fraction

$$\frac{1}{8}$$

Unit Fraction

$$\frac{1}{8}$$

Secret Fractions Stage 1 Cards

| | | | |
|----------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> |
| <div>Secret Fraction</div> <div>$\frac{2}{2}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{3}$</div> | <div>Secret Fraction</div> <div>$\frac{3}{3}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{4}$</div> |
| <div>Secret Fraction</div> <div>$\frac{3}{4}$</div> | <div>Secret Fraction</div> <div>$\frac{4}{4}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{6}$</div> | <div>Secret Fraction</div> <div>$\frac{4}{6}$</div> |
| <div>Secret Fraction</div> <div>$\frac{5}{6}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{8}$</div> | <div>Secret Fraction</div> <div>$\frac{3}{8}$</div> | <div>Secret Fraction</div> <div>$\frac{5}{8}$</div> |
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Secret Fractions Stage 1 Cards

| | | | |
|--------------------------------------------------------------|--------------------------------------------------------------|--------------------------------------------------------------|--------------------------------------------------------------|
| <div>Unit Fraction</div> <div>$\frac{1}{2}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{2}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{3}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{3}$</div> |
| <div>Unit Fraction</div> <div>$\frac{1}{3}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{4}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{4}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{4}$</div> |
| <div>Unit Fraction</div> <div>$\frac{1}{4}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> |
| <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{6}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> |
| <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> |

Secret Fractions Stage 1 Cards

| | | | |
|----------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> | <div>Unit Fraction</div> <div>$\frac{1}{8}$</div> |
| <div>Secret Fraction</div> <div>$\frac{2}{2}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{3}$</div> | <div>Secret Fraction</div> <div>$\frac{3}{3}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{4}$</div> |
| <div>Secret Fraction</div> <div>$\frac{3}{4}$</div> | <div>Secret Fraction</div> <div>$\frac{4}{4}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{6}$</div> | <div>Secret Fraction</div> <div>$\frac{4}{6}$</div> |
| <div>Secret Fraction</div> <div>$\frac{5}{6}$</div> | <div>Secret Fraction</div> <div>$\frac{2}{8}$</div> | <div>Secret Fraction</div> <div>$\frac{3}{8}$</div> | <div>Secret Fraction</div> <div>$\frac{5}{8}$</div> |
| | | | |

Fraction Match Part 2

Fraction Match Part 2
Q

$$\frac{3}{6}$$

Fraction Match Part 2
S

Fraction Match Part 2
U

$$\frac{3}{3}$$

Fraction Match Part 2
W

Fraction Match Part 2
R

$$\frac{5}{6}$$

Fraction Match Part 2
T

Fraction Match Part 2
V

$$\frac{3}{4}$$

Fraction Match Part 2
X

Fraction Match Part 2
Q

$$\frac{3}{6}$$

Fraction Match Part 2
S

Fraction Match Part 2
U

$$\frac{3}{3}$$

Fraction Match Part 2
W

Fraction Match Part 2
R

$$\frac{5}{6}$$

Fraction Match Part 2
T

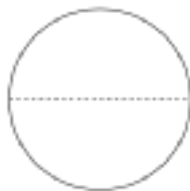
Fraction Match Part 2
V

$$\frac{3}{4}$$

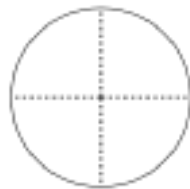
Fraction Match Part 2
X

Card Sort: Partitions

Card Sort: Partitions
A



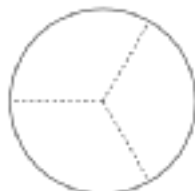
Card Sort: Partitions
D



Card Sort: Partitions
G



Card Sort: Partitions
J



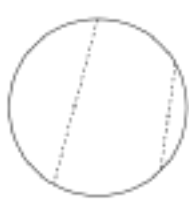
Card Sort: Partitions
B



Card Sort: Partitions
E



Card Sort: Partitions
H



Card Sort: Partitions
K



Card Sort: Partitions
C



Card Sort: Partitions
F



Card Sort: Partitions
I

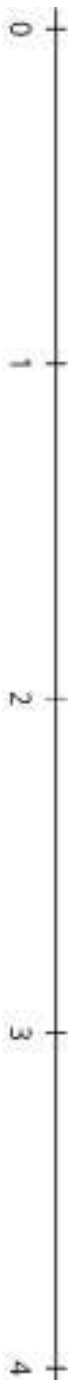


Card Sort: Partitions
L



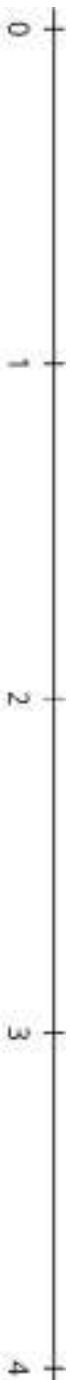
Spin to Win Recording Sheet

Round 1:



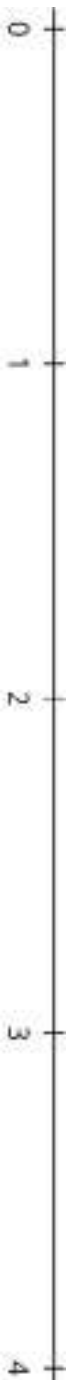
Use $>$, $<$ or $=$ to compare:

Round 2:



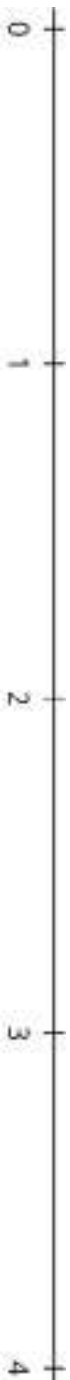
Use $>$, $<$ or $=$ to compare:

Round 3:



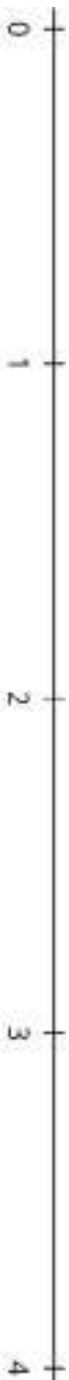
Use $>$, $<$ or $=$ to compare:

Round 4:



Use $>$, $<$ or $=$ to compare:

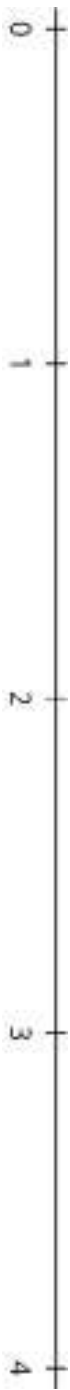
Round 5:



Use $>$, $<$ or $=$ to compare:

Spin to Win Recording Sheet

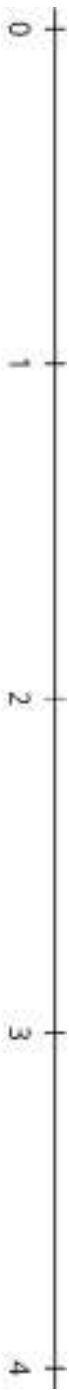
Round 6:



Use $>$, $<$ or $=$ to compare:



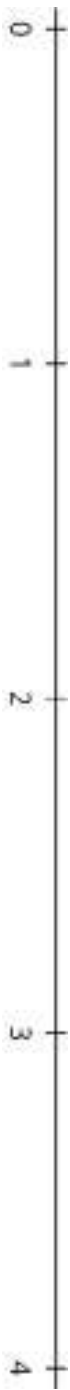
Round 7:



Use $>$, $<$ or $=$ to compare:



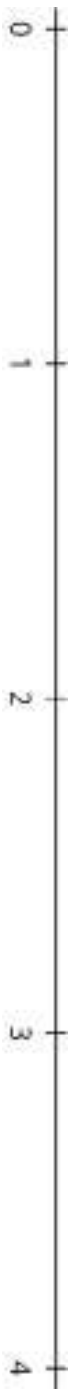
Round 8:



Use $>$, $<$ or $=$ to compare:



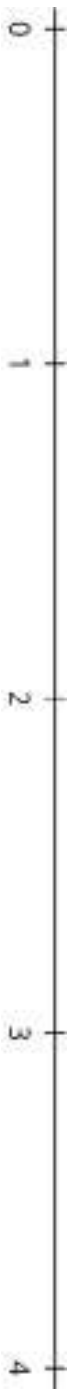
Round 9:



Use $>$, $<$ or $=$ to compare:



Round 10:



Use $>$, $<$ or $=$ to compare:



Fraction Match Part 1

Fraction Match Part 1
A

$$2 \frac{2}{2}$$

Fraction Match Part 1
E

$$4 \frac{4}{6}$$

Fraction Match Part 1
I



Fraction Match Part 1
M



Fraction Match Part 1
B

$$2 \frac{2}{3}$$

Fraction Match Part 1
F

$$5 \frac{5}{8}$$

Fraction Match Part 1
J



Fraction Match Part 1
N



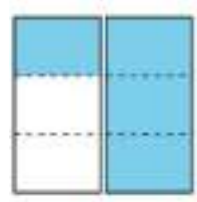
Fraction Match Part 1
C

$$2 \frac{2}{4}$$

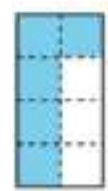
Fraction Match Part 1
G

$$7 \frac{7}{8}$$

Fraction Match Part 1
K



Fraction Match Part 1
O



Fraction Match Part 1
D

$$2 \frac{2}{6}$$

Fraction Match Part 1
H

$$4 \frac{4}{3}$$

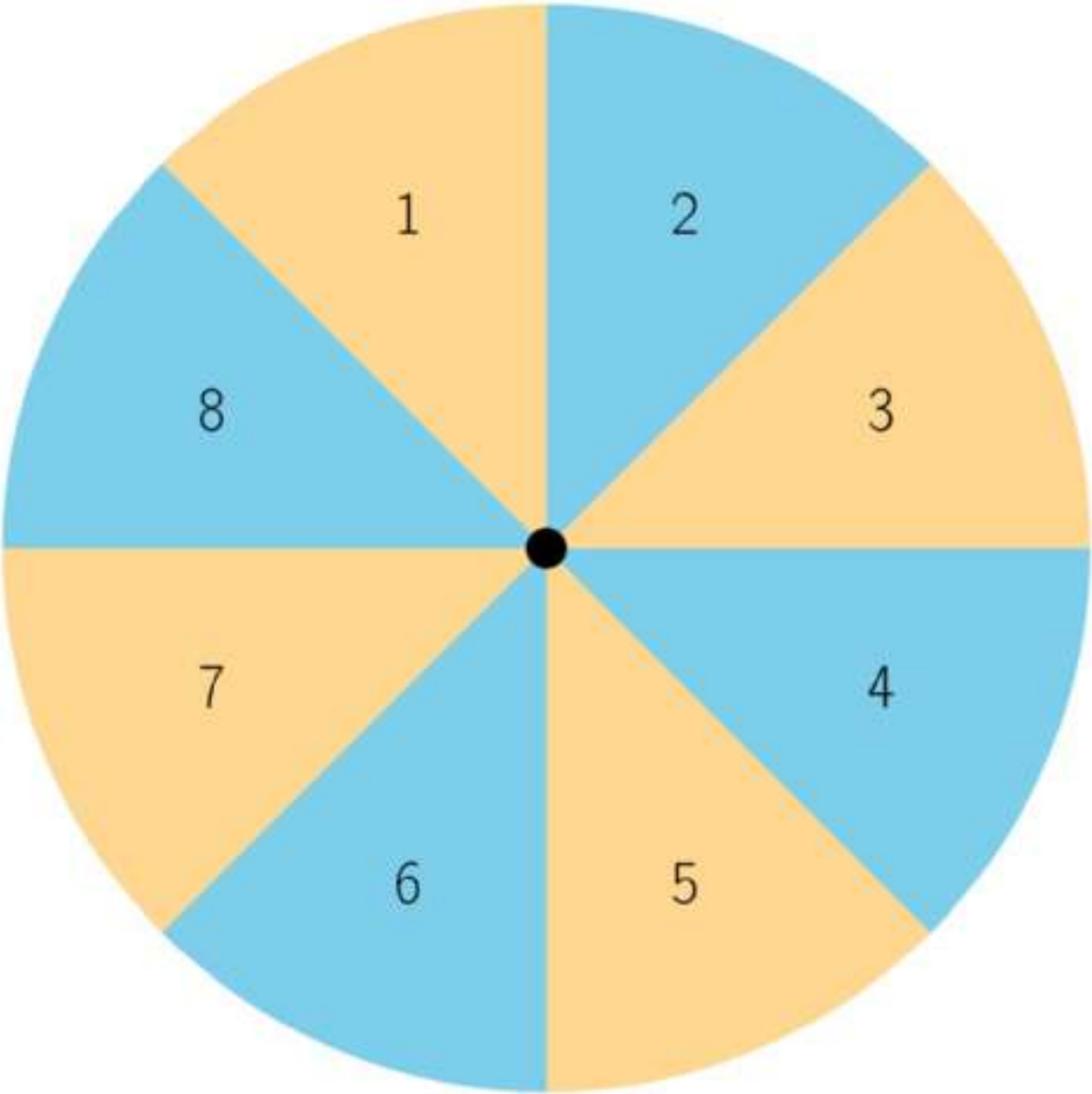
Fraction Match Part 1
L



Fraction Match Part 1
P



Spin to Win Spinner



Mystery Number Stage 2 Directions

Directions:

- Partner A:
 - Pick 3 cards and make a mystery three-digit number. Don't show your partner!
 - Give your partner a clue about your mystery number. You can use the sentences below to help you give clues, or make up your own.
- Partner B:
 - Guess your partner's mystery number.
- If Partner B guesses the mystery number, switch roles.
- If Partner B does not guess the mystery number, Partner A gives another clue. Go back and forth guessing the number and giving clues until Partner B guesses the mystery number.

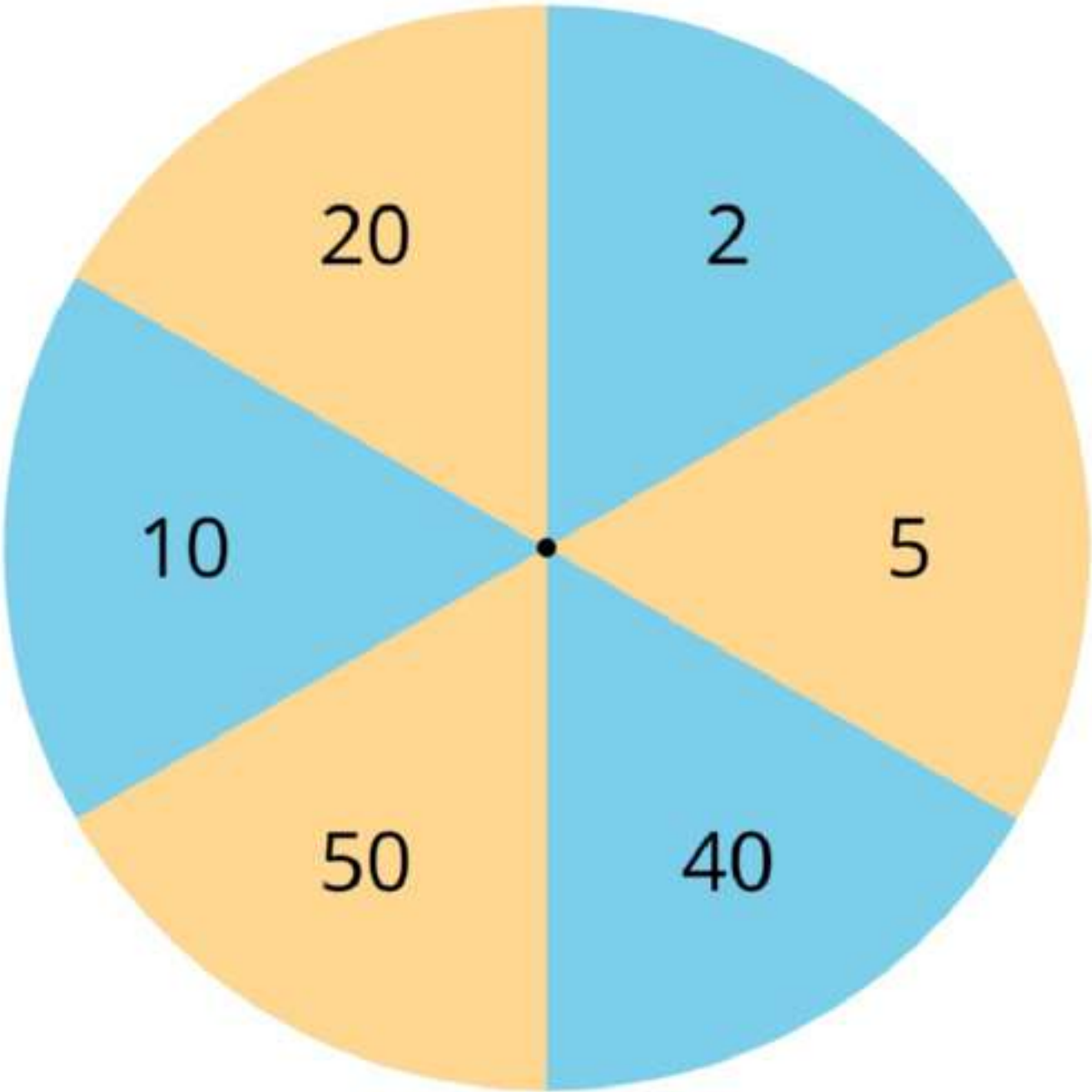


Example clues:

- The mystery number has more than ____ hundreds.
- The mystery number has less than ____ ones.
- The mystery number is greater than ____.
- The mystery number is less than ____.
- The mystery number has more hundreds than ones.
- The mystery number has more ones than tens.

Number Line Scoot Stage 1 Gameboard





Number Line Scoot Stage 1 Directions

Directions:

- Place a small cube on zero on each number line.
 - On your turn:
 - Spin the spinner.
 - Count aloud as you move that distance on one or more number lines.
 - You can use your whole spin on one number line or split it between multiple number lines.
 - Take turns spinning and moving.
 - If a cube lands *exactly* on the last tick mark of a number line, that player keeps the cube and puts a new one at 0.
 - The first player to collect 5 cubes wins.
-

Directions:

- Place a small cube on zero on each number line.
- On your turn:
 - Spin the spinner.
 - Count aloud as you move that distance on one or more number lines.
 - You can use your whole spin on one number line or split it between multiple number lines.
- Take turns spinning and moving.
- If a cube lands *exactly* on the last tick mark of a number line, that player keeps the cube and puts a new one at 0.
- The first player to collect 5 cubes wins.

Mystery Number Stage 3 Gameboard

Directions:

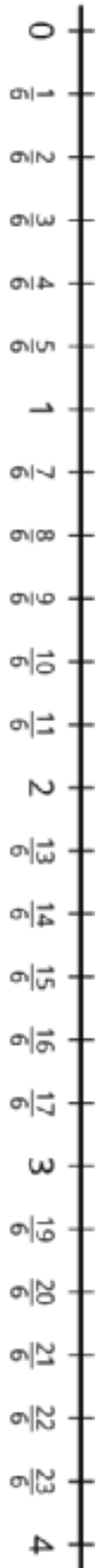
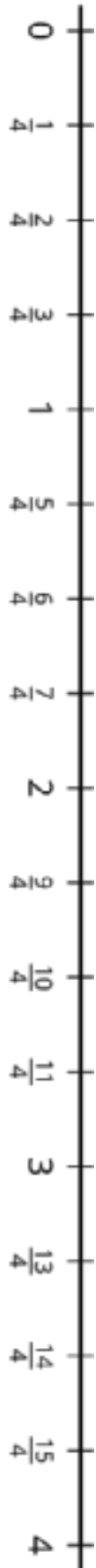
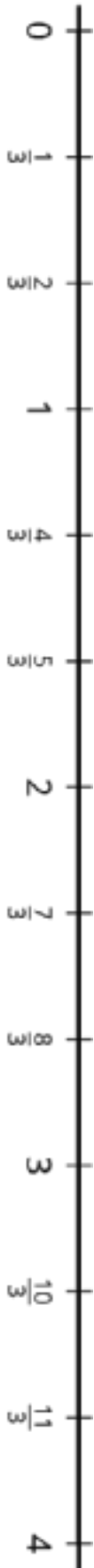
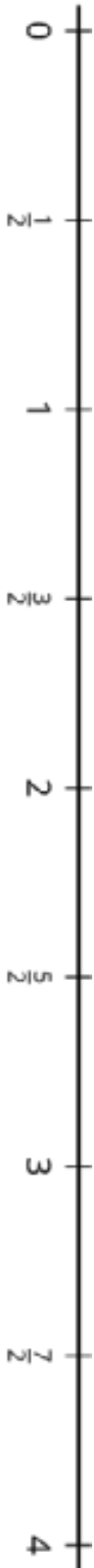
- Partner A:
 - Pick a number on the game board. Don't tell your partner!
 - Give your partner a clue about your mystery number. You can use the vocabulary below to help you give clues, or make up your own.
- Partner B:
 - Guess your partner's mystery number.
- If Partner B guesses the mystery number, switch roles.
- If Partner B does not guess the mystery number, Partner A gives another clue. Go back and forth guessing the number and giving clues until Partner B guesses the mystery number.

Vocabulary:

numerator, denominator, greater than, less than, equivalent, whole, odd, even

| | | | |
|----------------|----------------|---------------|----------------|
| $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{4}{6}$ | $\frac{5}{3}$ |
| $\frac{2}{6}$ | $\frac{7}{4}$ | $\frac{3}{2}$ | $\frac{5}{6}$ |
| $\frac{4}{6}$ | $\frac{5}{2}$ | $\frac{6}{6}$ | $\frac{18}{6}$ |
| $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{2}{2}$ | $\frac{1}{4}$ |
| $\frac{13}{4}$ | $\frac{15}{6}$ | $\frac{2}{3}$ | $\frac{7}{4}$ |

Number Line Scoot Stage 3 Gameboard



Number Line Scoot Stage 3 Directions

Directions:

- Place a small cube on zero on each number line.
 - On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, 4, 6, or 8 as the denominator for your fraction.
 - Count aloud as you move a counter that distance on the appropriate number line.
 - Take turns rolling and moving one cube.
 - If a cube lands *exactly* on the last tick mark of a number line, that partner keeps the cube and puts a new one at 0.
 - The first player to collect 5 cubes wins.
-

Directions:

- Place a small cube on zero on each number line.
- On your turn:
 - Roll a number cube.
 - The number you rolled is the numerator of your fraction. Choose whether you want to use 2, 3, 4, 6, or 8 as the denominator for your fraction.
 - Count aloud as you move a counter that distance on the appropriate number line.
- Take turns rolling and moving one cube.
- If a cube lands *exactly* on the last tick mark of a number line, that partner keeps the cube and puts a new one at 0.
- The first player to collect 5 cubes wins.

Rolling for Fractions Stage 1 Recording Sheet

- Each partner:
 - Roll 6 number cubes. If you roll any fives, they count as a wild and can be any number you'd like.
 - See if you can fill in a statement to show equivalent fractions.
 - If you cannot make equivalent fractions, re-roll as many cubes as you'd like.
 - If you can make equivalent fractions, record your statement and show or explain how you know the fractions are equivalent. You get 1 point for each pair of equivalent fractions you write.
- Take turns. The partner who has the most points once the recording sheet is full wins the game.

| | |
|-----------------------------------------------------|-----------------------------------------------------|
| $\frac{\square}{\square} = \frac{\square}{\square}$ | $\frac{\square}{\square} = \frac{\square}{\square}$ |
| $\frac{\square}{\square} = \frac{\square}{\square}$ | $\frac{\square}{\square} = \frac{\square}{\square}$ |

Five in a Row Multiplication and Division Stage 2 Gameboard

Directions:

- Partner A:
 - Put a paper clip on 2 numbers in the grey rows.
 - Multiply the numbers.
 - Cover the product of the 2 numbers with a counter.
- Partner B:
 - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.



| | | | | | |
|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 12 | 14 |
| 15 | 16 | 18 | 20 | 21 | 24 |
| 25 | 27 | 28 | 30 | 32 | 35 |
| 36 | 40 | 42 | 45 | 48 | 49 |
| 54 | 56 | 63 | 64 | 72 | 81 |

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|

| | | | |
|---|---|---|---|
| 6 | 7 | 8 | 9 |
|---|---|---|---|

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