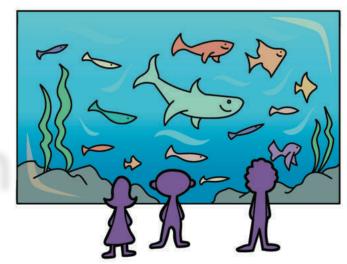


# Relating Multiplication to Division



**Teacher Guide** 



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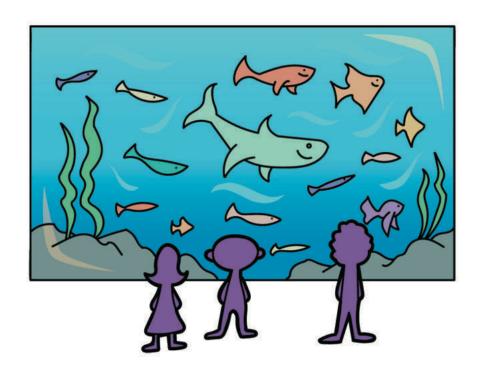
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# Relating Multiplication to Division

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# Relating Multiplication to Division Teacher Guide

Core Knowledge Mathematics  $^{\scriptscriptstyle\mathsf{TM}}$ 

# **Unit 4: Relating Multiplication to Division**

#### At a Glance

Unit 4 is estimated to be completed in 23-24 days including 2 days for assessment.

This unit is divided into four sections including 21 lessons and 1 optional lesson.

- Section A—What is Division? (Lessons 1-5)
- Section B—Relate Multiplication and Division (Lessons 6-11)
- Section C—Multiplying Larger Numbers (Lessons 12-17)
- Section D—Dividing Larger Numbers (Lessons 18-22)

On pages 7-8 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses six student centers.

- Rectangle Rumble
- Five in a Row: Multiplication
- Capture Squares
- Compare
- How Close?
- Can You Draw It?

# **Unit 4: Relating Multiplication to Division**

#### **Unit Learning Goals**

 Students learn about and use the relationship between multiplication and division, place value understanding, and the properties of operations to multiply and divide whole numbers within 100.
 They also represent and solve two-step word problems using the four operations.

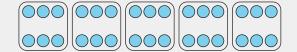
This unit introduces students to the concept of division and its relationship to multiplication.

Previously, students learned that multiplication can be understood in terms of equal-size groups. The expression  $5 \times 2$  can represent the total number of objects when there are 5 groups of 2 objects, or when there are 2 groups of 5 objects.

Here, students make sense of division also in terms of equal-size groups. For instance, the expression  $30 \div 5$  can represent putting 30 objects into 5 equal groups, or putting 30 objects into groups of 5. They see that, in general, dividing can mean finding the size of each group, or finding the number of equal groups.

30 objects put into 5 equal groups

30 objects put into groups of 5

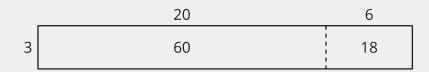




Students use the relationship between multiplication and division to develop fluency with single-digit multiplication and division facts. They continue to reason about products of two numbers in terms of the area of rectangles whose side lengths represent the factors, decomposing side lengths and applying properties of operations along the way.



As they multiply numbers greater than 10, students see that it is helpful to decompose the two-digit factor into tens and ones and distribute the multiplication. For instance, to find the value of  $26 \times 3$ , they can decompose the 26 into 20 and 6, and then multiply each by 3.



Toward the end of the unit, students solve two-step problems that involve all four operations. In some situations, they work with expressions that use parentheses to indicate which operation is completed first (for example:  $276 + (45 \div 5) = ?$ ).

Unit 4 1

#### **Section A: What is Division?**

#### **Standards Alignments**

Addressing 3.NBT.A.2, 3.OA.A.2, 3.OA.A.3

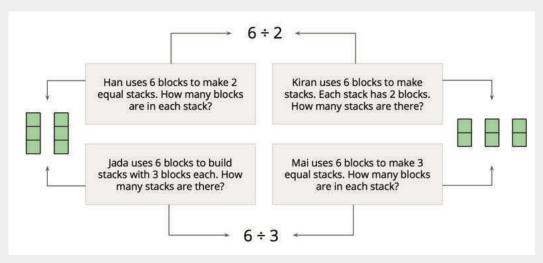
Building Towards 3.OA.A.3

#### **Section Learning Goals**

Represent and solve "how many groups?" and "how many in each group?" problems.

In this section, students encounter situations involving the questions "how many in each group?" and "how many groups?" They make sense of division in terms of finding the answers to these questions.

The focus here is on interpreting descriptions, diagrams, and expressions that represent division situations. Students see that the same diagram or expression can represent different questions. For example, the expression  $6 \div 2$  can represent two different questions about 6 blocks being put into stacks of 2 or into 2 equal stacks.



Later, students generalize their observations about division situations and interpret division expressions without a context.

♣ ↔ ♣ PLC: Lesson 4, Activity 3, Stacks of Blocks

#### **Suggested Centers**

- Rectangle Rumble (3–5), Stage 2: Factors 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)
- Capture Squares (1–3), Stage 6: Multiply with 1–5 (Supporting)

# **Section B: Relate Multiplication and Division**

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.NBT.A.3, 3.OA.A.2, 3.OA.A.3, 3.OA.B.6, 3.OA.C.7, 3.OA.D.9

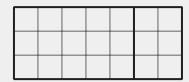
#### **Section Learning Goals**

- Understand division as a missing-factor problem.
- Use properties of operations to develop fluency with single-digit multiplication facts, and their related division facts.

In this section, students explicitly relate division to the missing factor in a multiplication equation. For example, the quotient in  $30 \div 6 =$  \_\_\_\_\_ is the missing factor in \_\_\_\_\_  $\times$  6 = 30. They use this insight and their knowledge of multiplication facts to identify division facts.

To develop fluency, students reason about patterns in a multiplication table and notice that multiplication is commutative. For instance, if they know the value of  $4 \times 7$ , they also know that of  $7 \times 4$ .

Students also reason about the product of two factors by decomposing one of the factors. For instance, to find the value of  $7 \times 3$ , they can decompose the 7 into 5 and 2 and find the value of  $(5 \times 3) + (2 \times 3)$ . Visually, the product can be represented by the area of a 7-by-3 rectangle that has been decomposed into two rectangles that are 5 by 3 and 2 by 3.



This line of reasoning develops students' intuition for the distributive property of multiplication. (Note that students are not expected to know the names of the properties of operations.)

🏝 ↔ 🚨 PLC: Lesson 7, Activity 1, Division Round Table

#### **Suggested Centers**

- Capture Squares (1–3), Stage 6: Multiply with 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)
- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

Unit 4 Section B 3

# **Section C: Multiplying Larger Numbers**

#### **Standards Alignments**

Building On 3.OA.B.5

Addressing 3.MD.C.7.c, 3.NBT.A.3, 3.OA.A.3, 3.OA.B.5, 3.OA.D.8

Building Towards 3.OA.C.7

#### **Section Learning Goals**

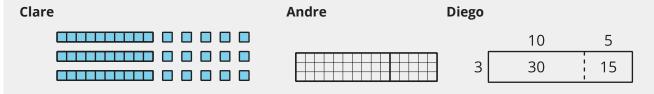
• Use properties of operations and place value understanding to develop strategies to multiply within 100 and to multiply one-digit numbers by a multiple of 10.

In this section, students use various strategies based on place value and properties of operations to multiply larger numbers.

Students first multiply one-digit numbers and multiples of 10 and observe the associative property of multiplication. They interpret  $3 \times 20$  to mean 3 groups of 2 tens, which is 6 tens. This means  $3 \times 20$  can be evaluated by finding  $3 \times 2 \times 10$  or  $6 \times 10$ .

These insights enable students to then multiply other one- and two-digit factors (not limited to multiples of 10) and find products within 100.

The representations used here (base-ten blocks, gridded rectangles, and ungridded diagrams) encourage students to also use their understanding of place value and to decompose two-digit factors into tens and ones as they multiply.



♣ ↔ ♣ PLC: Lesson 14, Activity 1, A Factor Greater than Ten

#### **Suggested Centers**

- Compare (1–5), Stage 3: Multiply within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)
- Compare (1–5), Stage 2: Add and Subtract within 20 (Supporting)
- How Close? (1–5), Stage 4: Add to 1,000 (Supporting)

# **Section D: Dividing Larger Numbers**

#### **Standards Alignments**

3.MD.C.7, 3.NBT.A.3, 3.OA.A.2, 3.OA.A.3, 3.OA.A.4, 3.OA.B.5, 3.OA.C.7,

3.OA.D.8

Building

Towards 3.OA.A.3

#### **Section Learning Goals**

• Use properties of operations, place value understanding, and the relationship between multiplication and division to divide within 100.

In this section, students perform division in which the quotient or divisor is larger than 10. They apply what they know about place value, the two interpretations of division, and the relationship between multiplication and division to divide larger numbers.

The numbers in the division expressions encourage students to see the divisor as either the number of groups or the number in each group. For example, they may interpret  $57 \div 3$  to mean dividing 57 into 3 equal groups. However, given  $90 \div 15$ , students may make groups of 15 and see how many are needed to make 90. This flexibility helps students choose methods that are most efficient for them for any given problem.

Students also use the relationship between multiplication and division and place value understanding to find quotients. For instance, to find the value of  $78 \div 3$ , students may reason as follows:

$$3 \times 10 = 30$$
  $3 \times 20 = 60$   
 $3 \times 10 = 30$   $3 \times 6 = 18$   
 $3 \times 6 = 18$   $20 + 6 = 26$ 

In both cases, students see that there are 3 groups of 26 in 78.

# ♣ ↔ ♣ PLC: Lesson 21, Activity 1, Apple Adventure

#### **Suggested Centers**

- Compare (1–5), Stage 4: Divide within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)
- Can You Draw It? (1–5), Stage 2: Grade 2 Shapes (Supporting)

Unit 4 Section D 5

#### **Throughout the Unit**

Some of the warm-ups early in the unit continue to develop fluency in addition and subtraction within 1,000. The rest of the warm-ups are designed to develop fluency with multiplying and dividing within 100. Students initially reason visually to find the product of a multiple of 10 by a single-digit whole number, then transition to working with expressions and equations. Warm-ups are also used to introduce and reinforce important ideas such as the relationship between multiplication and division and properties of operations.

Here is a sampling of the warm-ups in the unit.

lesson 3	lesson 7	lesson 8	lesson 10
Number Talk	How Many Do You See?	Number Talk	How Many Do You See?
120 + 120		$4 \times 10$	
121 + 119		40 ÷ 4	
125 + 115		40 ÷ 10	
129 + 111		60 ÷ 6	

# **Materials Needed**

LESSON	GATHER	COPY
A.1	<ul><li>Connecting cubes or counters</li><li>Tools for creating a visual display</li></ul>	● none
A.2	<ul><li>Connecting cubes or counters</li><li>Tools for creating a visual display</li></ul>	• none
A.3	• none	• none
A.4	• none	• none
A.5	<ul> <li>Tools for creating a visual display</li> </ul>	• Card Sort: All About Bugs (groups of 2)
B.6	• none	• none
B.7	• none	Division Round Table (groups of 1)
B.8	<ul> <li>Materials from a previous activity</li> </ul>	<ul> <li>Card Sort: Multiplication (groups of 2)</li> <li>Card Sort: Multiplication Recording Sheet (groups of 2)</li> </ul>
B.9	• none	• none
B.10	<ul> <li>Colored pencils, crayons, or markers</li> </ul>	• none
B.11	• none	<ul> <li>Card Sort: Different Expressions, Same Rectangle (groups of 2)</li> <li>Centimeter Grid Paper - Standard (groups of 2)</li> </ul>
C.12	Base-ten blocks	Centimeter Grid Paper - Standard (groups of 2)

Unit 4 Materials Needed 7

C.13	<ul> <li>Base-ten blocks</li> <li>Connecting cubes or counters</li> <li>Tools for creating a visual display</li> </ul>	Centimeter Grid Paper - Standard (groups of 2)
C.14	Base-ten blocks	• none
C.15	<ul><li>Base-ten blocks</li><li>Sticky notes</li><li>Tools for creating a visual display</li></ul>	Centimeter Grid Paper - Standard (groups of 2)
C.16	Base-ten blocks	<ul> <li>Centimeter Grid Paper - Standard (groups of 2)</li> <li>Number Cards (0-10) (groups of 2)</li> </ul>
C.17	Base-ten blocks	Centimeter Grid Paper - Standard (groups of 2)
D.18	<ul><li>Base-ten blocks</li><li>Connecting cubes or counters</li></ul>	Centimeter Grid Paper - Standard (groups of 2)
D.19	Base-ten blocks	• none
D.20	Base-ten blocks	<ul> <li>Centimeter Grid Paper - Standard (groups of 2)</li> <li>Compare Stage 4 Division Cards (groups of 2)</li> </ul>
D.21	• none	• none
D.22	<ul><li>none</li></ul>	Centimeter Grid Paper - Standard (groups of 2)

# **Center: Rectangle Rumble (3-5)**

# Stage 2: Factors 1–5

#### Lessons

- Grade3.4.A1 (supporting)
- Grade3.4.A2 (supporting)
- Grade3.4.A3 (supporting)

#### **Stage Narrative**

Students generate factors with a number cube and a spinner with the numbers 1–5. Students use a  $15 \times 15$  grid.

#### **Standards Alignments**

Addressing 3.MD.C.7

#### **Materials to Gather**

# Colored pencils, crayons, or markers, Number cubes, Paper clips

#### **Materials to Copy**

Rectangle Rumble Stage 2 Grid (groups of 2), Rectangle Rumble Stage 2 Spinner (groups of 2)

#### **Additional Information**

Each group of students need a paper clip, a number cube, and two different color writing utensils.

# Stage 3: Factors 1-10

#### Lessons

- Grade3.4.B7 (addressing)
- Grade3.4.B8 (addressing)
- Grade3.4.B9 (addressing)
- Grade3.4.B10 (addressing)
- Grade3.4.B11 (addressing)

#### **Stage Narrative**

Students generate factors with two spinners, one that shows 1–5 and one that shows 6–10. Students use a  $20 \times 20$  grid.

#### **Standards Alignments**

Addressing 3.MD.C.7

#### **Materials to Gather**

# **Materials to Copy**

Colored pencils, crayons, or markers, Paper clips

Rectangle Rumble Stage 3 Grid (groups of 2), Rectangle Rumble Stage 3 Spinners (groups of 2)

#### **Additional Information**

Each group of students need two paper clips and two different color writing utensils.

# Center: Five in a Row: Multiplication (3–5)

# Stage 2: Factors 1-9

#### Lessons

- Grade3.4.A1 (supporting)
- Grade3.4.A2 (supporting)
- Grade3.4.A3 (supporting)
- Grade3.4.A4 (supporting)
- Grade3.4.A5 (supporting)
- Grade3.4.B6 (supporting)

#### **Stage Narrative**

Students multiply using factors of 1–9. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Materials to Gather**

#### **Materials to Copy**

Paper clips, Two-color counters

Five in a Row Multiplication and Division Stage 2 Gameboard (groups of 2)

#### **Additional Information**

Each group of 2 needs 25 two-color counters and 2 paper clips.

# **Center: Capture Squares (1–3)**

# Stage 6: Multiply with 1-5

#### Lessons

- Grade3.4.A4 (supporting)
- Grade3.4.A5 (supporting)
- Grade3.4.B6 (supporting)

#### **Stage Narrative**

Students roll a number cube and spin a spinner and find the product of the two numbers they generated. The spinner has numbers 2–5.

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Materials to Gather**

Colored pencils or crayons, Number cubes, Paper clips

#### **Materials to Copy**

Capture Squares Stage 6 Gameboard (groups of 2), Capture Squares Stage 6 Spinner (groups of 2)

#### **Additional Information**

Each group of 2 needs one number cube.

# Stage 7: Multiply with 6-9

#### Lessons

- Grade3.4.B7 (addressing)
- Grade3.4.B8 (addressing)
- Grade3.4.B9 (addressing)
- Grade3.4.B10 (addressing)
- Grade3.4.B11 (addressing)

#### Stage Narrative

Students roll a number cube and spin a spinner and find the product of the two numbers they generated. The spinner has the numbers 6–9.

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Materials to Gather**

Colored pencils or crayons, Number cubes, Paper clips

#### Additional Information

Each group of 2 needs one number cube.

# **Stages used in Grade 2**

#### Stage 1

#### **Supporting**

• Grade2.2.A

#### Stage 2

#### Supporting

• Grade2.2.A

#### Stage 3

#### **Addressing**

- Grade2.2.A
- Grade2.2.B
- Grade2.2.C

#### Supporting

- Grade2.4.A
- Grade2.6.C

#### Stage 4

#### Addressing

- Grade2.2.B
- Grade2.2.C

#### **Supporting**

- Grade2.3.B
- Grade2.4.A
- Grade2.6.C

#### **Materials to Copy**

Capture Squares Stage 7 Gameboard (groups of 2), Capture Squares Stage 7 Spinner (groups of 2)

# **Center: Compare (1-5)**

# Stage 2: Add and Subtract within 20

#### Lessons

- Grade3.4.C13 (supporting)
- Grade3.4.C14 (supporting)

#### **Stage Narrative**

Students use cards with addition and subtraction expressions within 20.

#### **Standards Alignments**

Addressing 1.OA.C.6

#### **Materials to Copy**

Compare Stage 2 Addition Cards to 20 (groups of 2), Compare Stage 2 Subtraction Cards to 20 (groups of 2)

# **Stage 3: Multiply within 100**

#### Lessons

- Grade3.4.C12 (addressing)
- Grade3.4.C15 (addressing)
- Grade3.4.C16 (addressing)
- Grade3.4.C17 (addressing)

#### **Stage Narrative**

Students use cards with multiplication expressions within 100.

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Materials to Copy**

Compare Stage 3-8 Directions (groups of 2), Compare Stage 3 Multiplication Cards (groups of 2)

# **Stage 4: Divide within 100**

#### Lessons

- Grade3.4.D18 (addressing)
- Grade3.4.D19 (addressing)
- Grade3.4.D20 (addressing)

#### **Stage Narrative**

Students use cards with division expressions within 100.

This stage of the Compare center is used in grades 3, 4, and 5. When used in grade 3 or 4, remove the cards with two-digit divisors.

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Materials to Copy**

Compare Stage 3-8 Directions (groups of 2), Compare Stage 4 Division Cards (groups of 2)

Center: How Close? (1-5)

**Stage 4: Add to 1,000** 

#### Lessons

Grade3.4.C13 (supporting)

Grade3.4.C14 (supporting)

#### **Stage Narrative**

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 8 cards and chooses 6 of them to create 2 three-digit numbers. Each student adds the numbers. The score for the round is the difference between each student's sum and 1,000. Students pick new cards so that they have 8 cards in their hand and then start the next round. The player with the lowest score wins.

This center stage is the first time Number Cards 0–10 are used in Grade 3, so they are provided as a Instructional master. Students will continue to use these throughout the year. Consider copying them on cardstock or laminating them and keeping them organized to be used repeatedly.

#### **Standards Alignments**

Addressing 2.NBT.B.7, 3.NBT.A.2

#### **Materials to Copy**

How Close? Stage 4 Recording Sheet (groups of 1), Number Cards (0-10) (groups of 2)

# **Stage 5: Multiply to 100**

#### Lessons

- Grade3.4.C12 (addressing)
- Grade3.4.C15 (addressing)
- Grade3.4.C16 (addressing)
- Grade3.4.C17 (addressing)
- Grade3.4.D18 (addressing)
- Grade3.4.D19 (addressing)
- Grade3.4.D20 (addressing)

#### **Stage Narrative**

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 4 cards and chooses 2–3 of them to use to create a multiplication expression. Each student multiplies the numbers. The score for the round is the difference between each student's product and 100. Students pick new cards so that they have 4 cards in their hand and then start the next round. The player with the lowest score wins.

#### Variation:

Students can write the related division equation for each multiplication equation they record.

#### **Standards Alignments**

Addressing 3.OA.B.5

#### **Materials to Gather**

#### **Materials to Copy**

Number cards 0-10

How Close? Stage 5 Recording Sheet (groups of 1)

# **Stages used in Grade 2**

#### Stage 1

#### Addressing

- Grade2.1.A
- Grade2.1.B
- Grade2.1.C

#### Stage 2

#### **Addressing**

- Grade2.1.A
- Grade2.1.B
- Grade2.1.C

# Stage 3

#### Addressing

- Grade2.1.A
- Grade2.1.B
- Grade2.1.C

#### Supporting

- Grade2.4.A
- Grade2.4.B
- Grade2.7.A
- Grade2.7.B

#### Stage 4

#### Addressing

- Grade2.7.B
- Grade2.7.C

#### Supporting

- Grade2.8.A
- Grade2.8.B

# Center: Can You Draw It? (1-5)

# **Stage 2: Grade 2 Shapes**

#### Lessons

- Grade3.4.D19 (supporting)
- Grade3.4.D20 (supporting)

#### **Stage Narrative**

Partner A chooses a shape card and describes it to their partner. If Partner B draws the shape correctly, they keep the card. Shape cards include triangles, quadrilaterals, and hexagons.

#### **Standards Alignments**

Addressing 2.G.A.1

#### **Materials to Copy**

Centimeter Dot Paper - Standard (groups of 1), Shape Cards Grade 2 (groups of 2)

#### **Additional Information**

Before playing, remove any cards that do not show a triangle, quadrilateral, or hexagon.

# **Stages used in Grade 2**

#### Stage 1

#### **Supporting**

Grade2.6.A

#### Stage 2

#### Addressing

- Grade2.6.A
- Grade2.6.B
- Grade2.6.C

## **Section A: What is Division?**

# **Lesson 1: How Many Groups?**

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

Building Towards 3.OA.A.3

#### **Teacher-facing Learning Goals**

 Solve "how many groups?" problems in a way that makes sense to them.

#### **Student-facing Learning Goals**

• Let's represent and solve problems.

#### **Lesson Purpose**

The purpose of this lesson is for students to solve "how many groups?" problems in a way that makes sense to them.

In a previous unit, students were introduced to multiplication. They interpreted products as the total number of objects in a given number of groups of equal size. Students represented groups of equal size using drawings, tape diagrams, and arrays.

The purpose of this lesson is to introduce problems that involve putting objects into groups of equal size, starting with "how many groups?" problems. Even though the structure of the problems suggests division, students may use their understanding of multiplication or any strategy that makes sense to them to solve the problems. If students use connecting cubes, encourage them to draw a picture to match their work. In the lesson synthesis, students have a chance to think about how they would define division. The definition and symbol for division will be introduced in subsequent lessons.

#### Access for:

Students with Disabilities

**3** English Learners

Representation (Activity 1)

MLR8 (Activity 2)

#### **Instructional Routines**

How Many Do You See? (Warm-up)

#### **Materials to Gather**

- Connecting cubes or counters: Activity 1
- Tools for creating a visual display: Activity 1

#### **Lesson Timeline**

Warm-up 10 mi	
Activity 1 20 mi	n
Activity 2 15 mi	n
Lesson Synthesis 10 mi	n
Cool-down 5 mi	n

#### **Teacher Reflection Question**

In this lesson, students are introduced to division for the first time. How is their understanding of multiplication influencing and supporting how they solve division problems?

# **Cool-down** (to be completed at the end of the lesson)

3 5 min

How Many Bags?

# **Standards Alignments**

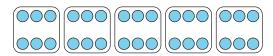
Addressing 3.OA.A.3

# **Student-facing Task Statement**

Lin has 30 apples to share with her friends. She is putting them in bags, with 6 apples in each bag. How many bags does she need? Explain or show your reasoning.

#### **Student Responses**

She will need 5 bags. If I put the 30 apples into groups of 6 there will be 5 groups.



-----Begin Lesson ------

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Warm-up © 10 min

How Many Do You See: Apples

#### **Standards Alignments**

Building Towards 3.OA.A.3

The purpose of this How Many Do You See is for students to subitize or use grouping strategies to describe the images they see.

#### **Instructional Routines**

How Many Do You See?

#### **Student-facing Task Statement**

How many do you see? How do you see them?



#### **Student Responses**

Sample responses:

- 16. 4 rows (or columns) of 4.
- 16. I multiplied  $8 \times 2$ .
- 23. I see 4 groups of 4, and I see 7 more apples in the other crate, which makes 23.

#### Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

#### **Activity**

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

#### **Synthesis**

- "How did the organization of the apples help you see how many there were?" (I saw 2 rows of 4 which I knew was 8, then I doubled that to find how many apples were in the crate. I saw groups of 4 so I multiplied 4 x 4 then added the other apples.)
- Consider asking:
  - "Who can restate the way \_\_\_ saw the apples in different words?"
  - o "Did anyone see the apples the same

way but would explain it differently?"

 "Does anyone want to add an observation to the way \_\_\_\_ saw the apples?"

Activity 1 © 20 min

How Many Apples?

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

The purpose of this activity is for students to represent and solve "how many groups?" problems. Encourage students to use whatever strategy and visual representation that make sense to them. Students create a poster of their solution to the first problem with a partner. In the next activity, students participate in a gallery walk of the posters.

Monitor for students who represent the situation with:

- concrete objects: putting 24 cubes into groups of 8
- drawings of objects: drawing 24 apples and then splitting them into groups of 8 or circling groups of 8
- arrays: drawing 3 rows of 8 apples in each to reach 24

When students represent the situation with objects, concrete drawings, or abstract drawings they are reasoning abstractly and quantitatively (MP2).

#### Access for Students with Disabilities

Representation: Access for Perception. Begin by showing a physical demonstration of what the poster might look like, using a different problem, to support understanding of the context. Supports accessibility for: Social-Emotional Functioning

#### **Materials to Gather**

Connecting cubes or counters, Tools for creating a visual display

Unit 4 Lesson 1

#### **Student-facing Task Statement**

Solve each problem. Show your thinking using objects, a drawing, or a diagram.

- 1. If 24 apples are put into boxes with 8 apples in each box, how many boxes are there?
- 2. If 42 apples are put into boxes with 6 apples in each box, how many boxes are there?
- 3. If 32 apples are put into boxes with 4 apples in each box, how many boxes are there?

#### **Student Responses**

- 3 boxes. Sample reasoning: The student draws 8 apples in each row and notices there are 3 rows.
- 2. 7 boxes. Sample reasoning: The student draws 42 apples and circles groups of 6 until no apples are left.
- 3. 8 boxes. Sample reasoning: The student gets 32 cubes and puts them in groups of 4 for each box.

#### Launch

- Groups of 2
- Give students access to connecting cubes or counters.

#### **Activity**

- "Solve these problems and show your thinking using objects, a drawing, or a diagram."
- 6–8 minutes: independent work time
- As student work, consider asking:
  - "How can you represent what you are thinking?"
  - "Where can we see the apples in your work?"
  - "Where can we see how many boxes there are in your work?"
- Monitor for students who solve the first problem in the same way. Arrange them into groups of 2 to create a poster together.
- "Now you are going to create a poster to show your thinking on the first problem."
- "You are going to work with a partner who solved the problem in the same way you did."
- Give each group tools for making a visual display.
- 6–8 minutes: partner work time

#### **Synthesis**

• Display posters around the room.

#### **Advancing Student Thinking**

If students don't find a solution to the problems, consider asking: "What is this problem about?" and "How could you represent the problem?"

Activity 2 © 15 min

Gallery Walk: Apples in Boxes

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

The purpose of this activity is for students to consider what is the same and what is different about the ways that they solved a "how many groups?" problem in the previous activity.

As students visit the posters, identify 2–3 students who show particularly well that this problem is about finding how many groups are made. Select them to share their explanations in the next lesson.

#### Access for English Learners

MLR8 Discussion Supports: Display sentence frames to support student writing: "One thing I noticed was the same . . .", "One thing I noticed was different . . .", "One thing that was the same . . .", "One difference was . . . ."

Advances: Writing, Speaking

#### **Student-facing Task Statement**

- 1. Visit the posters around the room with your partner. Discuss what is the same and what is different about the thinking shown on each poster.
- Reflect on what you saw. Write down one thing that was the same and one thing that was different about the thinking shown on each poster.

#### **Student Responses**

- 1. Answers vary.
- 2. Sample responses:
  - On each poster, the idea is to put 24 apples into boxes of 8 apples.

#### Launch

• Groups of 2

#### **Activity**

- "As you visit the posters with your partner, discuss what is the same and what is different about the thinking shown on each poster."
- 8–10 minutes: gallery walk

#### **Synthesis**

- Give students a chance to ask questions they have about any posters.
- "What is the same about the thinking shown on the posters?"

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- There are always 3 boxes in the end.
- On some posters, we can see how the apples got into each group. On other posters, we can only see the end result.
- "What is different about the thinking shown on the posters?"

# **Lesson Synthesis**

① 10 min

"Today we solved problems about putting apples into boxes. How were these problems the same as multiplication? How were they different?" (These problems had groups of equal size. In multiplication we counted how many things were altogether. In these problems we knew that already. We were trying to find how many groups we could make.)

"The problems we solved today are division problems. How would you define division based on the problems we saw today?" (Division is about putting into groups of equal size. I would say it's about finding how many groups you can make.)

#### **Suggested Centers**

- Rectangle Rumble (3–5), Stage 2: Factors 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

# ----- Complete Cool-Down ----

#### **Response to Student Thinking**

Students represent 6 bags of apples instead of 6 apples in each bag.

#### **Next Day Support**

 During the launch of the next day's activity, pass back the cool-down and have students work in pairs to represent the problem with counters and discuss the solution to the problem.

# **Lesson 2: How Many in Each Group?**

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

Building Towards 3.OA.A.3

#### **Teacher-facing Learning Goals**

• Solve "how many in each group?" problems in a way that makes sense to them.

#### **Student-facing Learning Goals**

• Let's represent and solve more problems.

#### **Lesson Purpose**

The purpose of this lesson is for students to solve "how many in each group?" problems in a way that makes sense to them.

Previously, students solved "how many groups?" problems in a way that made sense to them. In this lesson students extend problems involving sharing into groups of equal size to include "how many in each group?" problems. Students again have the flexibility to represent and solve problems using any strategy that makes sense to them in this lesson. If students use connecting cubes, encourage them to draw a picture to match their work. At the end of this lesson, **division** is defined as finding the number of groups or finding the size of each group when we share into groups of equal size.

#### Access for:

Students with Disabilities

Engagement (Activity 1)

## **3** English Learners

• MLR7 (Activity 3)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

#### Materials to Gather

- Connecting cubes or counters: Activity 1
- Tools for creating a visual display: Activity 1

#### **Lesson Timeline**

#### Warm-up 10 min

#### **Teacher Reflection Question**

What did you say, do, or ask during the lesson

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Activity 1	15 min
Activity 2	10 min
Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

synthesis that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

**Cool-down** (to be completed at the end of the lesson)

© 5 min

Apples in Bags

#### **Standards Alignments**

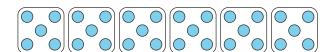
Addressing 3.OA.A.3

#### **Student-facing Task Statement**

Lin has 30 apples. She makes 6 bags with the same number of apples in each bag to give to her friends. How many apples are in each bag? Explain or show your reasoning.

#### **Student Responses**

Each bag has 5 apples. If I put the 30 apples into 6 groups one by one there will be 5 apples in each group.



----- Begin Lesson -----

Warm-up © 10 min

Notice and Wonder: More Apples

### **Standards Alignments**

Building Towards 3.OA.A.3

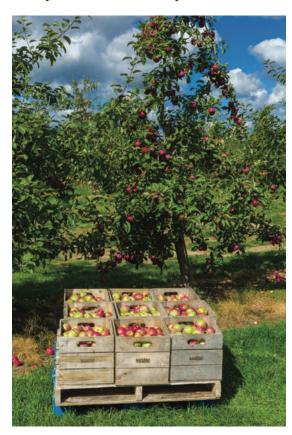
The purpose of this warm-up is to elicit the idea that many different questions could be asked about a situation, which will be useful when students solve problems in a later activity.

#### **Instructional Routines**

Notice and Wonder

### **Student-facing Task Statement**

What do you notice? What do you wonder?



### **Student Responses**

Students may notice:

- Some apples are in boxes.
- Some apples are still on the tree.
- There are 9 boxes of apples

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

If not mentioned in students' responses, ask:
 "What mathematical questions could we ask
 about this picture?" (How many apples are in
 each box? Are there more apples in the boxes
 than on the trees? How many apples are in
 the boxes altogether?)

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#### Students may wonder:

- How did the apples get into the boxes?
- How many apples are in boxes?
- Does each box have the same number of apples?

Activity 1 © 15 min

**How Many Apples?** 

### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

The purpose of this activity is for students to represent and solve "how many in each group?" problems using whatever strategy and visual representation make sense to them. Students create a poster of their solution to the first problem with a partner. In the next activity, they participate in a gallery walk of the posters.

Monitor for students who represent the situation with:

- concrete objects: putting 20 cubes into 4 groups one by one
- drawings of objects: drawing 20 apples and then splitting them into 4 groups
- arrays: drawing 4 rows with one apple in each row to reach 20

#### Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk (problem). Supports accessibility for: Attention

#### **Materials to Gather**

Connecting cubes or counters, Tools for creating a visual display

### **Student-facing Task Statement**

Solve each problem. Show your thinking using objects, a drawing, or a diagram.

#### Launch

Groups of 2

- 1. If 20 apples are packed into 4 boxes with each box having the same number of apples, how many apples are in each box?
- 2. If 36 apples are packed into 6 boxes with each box having the same number of apples, how many apples are in each box?
- 3. If 45 apples are packed into 9 boxes with each box having the same number of apples, how many apples are in each box?

#### **Student Responses**

- 1. 5 apples. Sample reasoning: The student draws the apples 1 by 1 into 4 rows so that in the end there are 5 apples in each row.
- 2. 6 apples. Sample reasoning: The student draws 36 apples and thinks about how to split them into 6 groups that are the same size. Then they circle 6 groups of 6.
- 3. 5 apples. Sample reasoning: The student gets 45 cubes and puts them 1 by 1 into 9 groups for each box.

- Give students access to connecting cubes and counters.
- "Turn and talk to your partner about how you might solve these problems."
- 1–2 minutes: partner discussion

#### **Activity**

- "Solve these problems and show your thinking using objects, a drawing, or a diagram."
- 5–7 minutes: independent work time
- As student work, consider asking:
  - "How can you represent what you are thinking?"
  - "Where can you see the boxes in your work?"
  - "Where can you see how many apples are in each box in your work?"
- Monitor for students who solve the first problem in the same way. Arrange them into groups of 2 to create a poster together.
- "Now you are going to create a poster to show your thinking on the first problem."
- "You are going to work with a partner who solved the problem in the same way you did."
- Give each group tools for creating a visual display.
- 5–7 minutes: partner work time

#### **Synthesis**

Display posters around the room.

### **Advancing Student Thinking**

If students don't find a solution to the problems, consider asking: "What is this problem about?" and "How could you represent the problem?"

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Activity 2 © 10 min

Gallery Walk

### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

The purpose of this activity is for students to consider what is the same and what is different about the ways that they solved a "how many in each group?" problem in the previous activity. As students visit the posters, identify 2–3 students who show particularly well that this problem is about finding how many there are in each group. Select them to share in the next activity.

### **Student-facing Task Statement**

Visit the posters around the room with your partner. Discuss what is the same and what is different about the thinking shown on each poster.

#### **Student Responses**

Sample responses:

- On each poster, the idea is to put 20 apples into 4 groups.
- There are always 5 apples in each box.
- On some posters, we can see how the apples got into each group. On other posters, we can only see the end result.

#### Launch

Groups of 2

### **Activity**

- "As you visit the posters with your partner, discuss what is the same and what is different about the thinking shown on each poster."
- 5–7 minutes: gallery walk

#### **Synthesis**

- Give students a chance to ask questions they have about any posters.
- "What is the same about the thinking shown on the posters?"
- "What is different about the thinking shown on the posters?"

Activity 3 © 10 min

All the Apples

### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

The purpose of this activity is for students to consider what is the same and what is different about the "how many groups?" and "how many in each group?" problems they solved in a previous lesson and in this lesson. The discussion should highlight that in "how many groups?" problems we know the size of each group and in "how many in each group?" problems we know how many groups there are. In order to describe how the problems are the same and how they are different, students attend to the structure of the problems, that is what is given in each situation and what is unknown (MP7).

### Access for English Learners

MLR7 Compare and Connect. Synthesis: Lead a discussion comparing, contrasting, and connecting the different representations. Ask, "What specific words or language helped you understand how to solve the problems? Are there any additional details or language that you have questions about?" To amplify student language, and illustrate connections, follow along and point to the relevant parts of the displays as students speak.

Advances: Representing, Conversing

# **Required Preparation**

• Gather the 2–3 posters from the previous lesson and this lesson that highlight counting the groups in a "how many groups?" problem and finding how many in each group in a "how many in each group?" problem.

# **Student-facing Task Statement**

If 24 apples are put into If 20 apples are packed boxes with 8 apples in into 4 boxes with each each box, how many box having the same boxes are there? number of apples, how

many apples are in each box?

#### Launch

- Groups of 2
- Display the problems.
- Display the 2-3 previously selected posters for each problem.

Unit 4 Lesson 2

Discuss with your partner:

- How are these problems alike?
- How are they different?
- What is alike and what is different about how these problems are represented and solved?

#### **Student Responses**

Sample responses:

- Both problems were about apples.
- In both problems there were the same numbers of apples in each box.
- In both problems we knew the total number of apples.
- In the first problem we were trying to find out how many boxes of apples we could make, but we knew how many apples were in each box.
- In the problem from today we knew how many boxes there were, but we didn't know how many apples were in each box.

#### **Activity**

- "Here are two problems we've worked on. Yesterday we made posters for the first one and today we made posters for the second one."
- "Here are a few posters for each problem."
- "Talk to your partner about how these problems are alike and how they are different. Also talk about what is alike and what is different about how the problems are represented and solved."
- 3-5 minutes: partner discussion

#### **Synthesis**

- "What did you and your partner notice was alike?"
- "What did you and your partner notice was different?"
- Share and record responses.
- As students share, encourage them to use the posters to show examples of what they notice.

# **Lesson Synthesis**

① 10 min

"Yesterday, we solved problems that asked about how many groups we could make. Today we solved problems that asked about how many things are in each group. Both of these ideas are division."

"**Division** is finding the number of groups or finding the size of each group when we share into groups of equal size."

### **Suggested Centers**

- Rectangle Rumble (3–5), Stage 2: Factors 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

# 

# **Response to Student Thinking**

Students represent 6 apples in each bag instead of 6 bags with the same number of apples in each.

### **Next Day Support**

 During the launch of the next day's activity pass back the cool-down and have students work in partners to represent the problem with counters and discuss the solution to the problem.

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# **Lesson 3: Division Situation Drawings**

### **Standards Alignments**

Addressing 3.NBT.A.2, 3.OA.A.2

#### **Teacher-facing Learning Goals**

- Interpret and relate drawings and descriptions of division situations.
- Understand that a division situation may involve finding an unknown number of groups or finding an unknown number of objects in each group.

#### **Student-facing Learning Goals**

 Let's represent division situations with drawings.

#### **Lesson Purpose**

The purpose of this lesson is for students to interpret descriptions or drawings of division situations and recognize whether they involve finding an unknown number of groups or finding an unknown number of objects in each group.

Students see the two types of division situations side-by-side in this lesson. They understand that division is finding the number in each group or the size of each group and can match division situations to drawings. Students learn that the same drawing can match either type of division situation. This is because the drawings represent the end result after division has occurred. From the drawing, we cannot tell whether the number of groups or the number of objects in each group was known. The division symbol,  $\div$ , is introduced in the lesson synthesis.

#### Access for:

- **③** Students with Disabilities
- Engagement (Activity 3)

# **S** English Learners

MLR8 (Activity 3)

#### Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Number Talk (Warm-up)

#### **Lesson Timeline**

Warm-up 10 min

#### **Teacher Reflection Question**

How are students leveraging the drawings they used for multiplication to solve division

# $\textbf{Cool-down} \hspace{0.2cm} \text{(to be completed at the end of the lesson)}$

© 5 min

**Party Favors** 

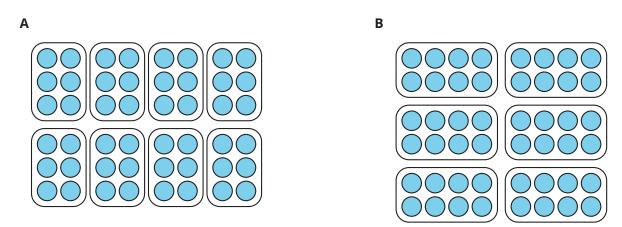
## **Standards Alignments**

Addressing 3.OA.A.2

## **Student-facing Task Statement**

Clare has 48 markers. She puts 8 markers into each goodie bag for her birthday party. How many bags will she use?

Which drawing matches the situation? Explain your reasoning.



# **Student Responses**

Sample response: Drawing B matches the situation because it shows 8 markers in each bag. After the 48 markers are put into groups of 8, there will be 6 bags.

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----- Begin Lesson -----

Warm-up © 10 min

Number Talk: The More Things Change...

### **Standards Alignments**

Addressing 3.NBT.A.2

The purpose of this Number Talk is to elicit strategies and understandings students have for adding within 1,000, particularly around adjusting numbers in a sum to make them easier to add. These understandings help students develop fluency for adding within 1,000.

When students notice that the same value is being removed from one addend and added to the other and the value of the sum does not change, they look for and make use of structure (MP7).

#### **Instructional Routines**

Number Talk

#### **Student-facing Task Statement**

Find the value of each expression mentally.

- 120 + 120
- 121 + 119
- 125 + 115
- 129 + 111

#### **Student Responses**

- 240. I just doubled 120.
- 240. I noticed that 121 is 1 more than 120 and 119 is one less than 120, so the value is the same as for 120 + 120.
- 240. I took 5 from 125 and added it to 115. Then it's 120 + 120 again.
- 240. I took 9 from 129 and added it to 111. Then it's 120 + 120 again.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

#### **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### **Synthesis**

- "Why do you think all of these expressions have the same value?" (Even though each number is changing, the same amount is being added to one number and subtracted from the other number, so the total is the same.)
- Consider asking:

- "Who can restate \_\_\_\_'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone approach the problem in a different way?"
- "Does anyone want to add on to \_\_\_\_\_'s strategy?"

Activity 1 © 10 min

**Groups of Students** 

# **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to physically represent the difference between making 2 groups and making groups of 2. Ten students will put themselves into 2 groups and then groups of 2. The rest of the students observe how the groups were made to highlight the difference between "how many groups?" problems and "how many in each group?" problems.

### **Student-facing Task Statement**

- 1. What did you notice about how the students put themselves into groups of 2?
- 2. What did you notice about how the students put themselves into 2 groups?

### **Student Responses**

- 1. Sample responses:
  - They could just pair up.
  - They didn't need to know how many groups they were making and just made sure 2 students were in each group.

#### Launch

- Groups of 2
- Invite 10 students to come to the front of the class.
- "These students are going to put themselves into groups in different ways. If you are observing, take notes on what you notice about how they make the groups."

### **Activity**

- Ask the 10 students to put themselves into groups of 2.
- Give observers a chance to take notes.

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• There were 5 groups of 2 students in the end.

#### 2. Sample responses:

- They had to figure out how many students would be in each group.
- They put people into groups one by one.
- There were 2 groups of 5 students in the end.

- Ask the 10 students to put themselves into 2 groups.
- Give observers a chance to take notes.
- Ask the students to return to their seats.
- "Talk with a partner about what you noticed about how the students put themselves into groups of 2 and 2 groups."
- 2–3 minutes: partner discussion

#### **Synthesis**

- Ask students who observed to share what they noticed.
- Highlight ideas that help clarify differences between "how many groups?" and "how many in each group?"

Activity 2 © 10 min

Elena's Colored Pencils

# **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to match a division situation to a drawing of equal groups. Students should be able to explain why the situation matches drawing A, which shows 2 groups of 6, and why it does not match drawing B, which shows 6 groups of 2.

This activity uses *MLR1 Stronger* and *Clearer Each Time*. *Advances: reading, writing* 

#### **Instructional Routines**

MLR1 Stronger and Clearer Each Time

### **Student-facing Task Statement**

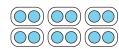
Elena has 12 colored pencils. She has 2 boxes and wants to put the same number of colored pencils in each box. How many colored pencils will go in each box?

Which drawing matches the situation? Explain your reasoning.

Α









### **Student Responses**

Drawing A matches the situation because it shows 2 boxes. After the 12 colored pencils are put into 2 boxes, there will be 6 in each box.

#### Launch

- Groups of 2
- "Today we are going to look at drawings to represent division situations. Take a minute to read this situation."
- 1 minute: independent work time

### **Activity**

- "Work independently to decide which drawing matches this situation and explain your reasoning."
- 2–3 minutes: independent work time

### **Synthesis**

#### **MLR1 Stronger and Clearer Each Time**

- "Share your response with your partner.
   Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 2–3 minutes: structured partner discussion
- Repeat with 2 different partners.
- "Which drawing did you decide matches? How do you know?"
- "How do you know the other drawing does not match this situation?" (Drawing B is 6 groups of 2 colored pencils. That would be like if she had 6 boxes, not 2 boxes.)

**Activity 3** 

① 15 min

Which Drawing Matches?

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### **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to relate division situations and drawings of equal groups (MP2). Each given drawing matches two different situations. Students learn that the same drawing can represent both a "how many groups?" problem and a "how many in each group?" problem because the drawing shows the end result, not how the groups were made. When students interpret one diagram as representing two different story types they state clearly how each part of the diagram corresponds to the story, including what corresponds to the unknown in the story (MP6).

# **Access for English Learners**

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed \_\_\_\_, so I matched . . . . " Encourage students to challenge each other when they disagree.

Advances: Listening, Speaking, Representing

### Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select at least 3 of the 6 problems to complete.

Supports accessibility for: Organization, Attention, Social-emotional skills

# **Student-facing Task Statement**

Match each situation to a drawing. Be prepared to explain your reasoning.

- Mai has 8 markers. She puts 4 markers in each box. How many boxes of markers are there?
- 2. Kiran has 20 pens. He puts 2 pens at each table. How many tables can he put pens on?
- 3. Lin has 8 colored pencils. She puts them



В



C

#### Launch

- Groups of 2
- "We're going to look at some situations that involve writing or drawing tools. What are some things we use to write or draw?"
- 30 seconds: quiet think time
- Share and record responses.

### **Activity**

- "You are going to match six situations and drawings that could represent them.
   Take a few minutes to decide which drawing matches each situation."
- 3-5 minutes: independent work time

into 2 bags. Each bag has the same number of colored pencils. How many colored pencils will be in each bag?



- 4. Priya has 15 crayons. She puts 5 crayons on each desk. How many desks will have crayons?
- 5. Noah has 20 pencils. He puts the same number of pencils into 10 boxes. How many pencils will be in each box?
- 6. Jada has 15 markers.
  She puts the same
  number of markers on 3
  tables. How many
  markers will be on each
  table?

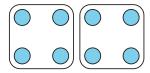
### **Student Responses**

- 1. A
- 2. B
- 3. A
- 4. C
- 5. B
- 6. C

- "Share your ideas with your partner."
- 2-3 minutes: partner discussion

#### **Synthesis**

- Invite students to share which drawing matches each situation.
- Focus on one drawing and the two situations it can represent, such as:



Mai has 8 markers. She puts 4 markers in each box. How many boxes of markers are there?

Lin has 8 colored pencils. She puts them into 2 bags. Each bag has the same number of colored pencils. How many colored pencils will be in each bag?

"How can the same drawing represent both situations?" (We didn't see how the groups were made, but in the end, the same number and size of groups were made in both situations. The drawing can represent putting 8 markers into boxes with 4 markers in each box and finding that they fit into 2 boxes. It can also represent putting 8 pencils into 2 bags with the same number of pencils in each bag and finding that you can put 4 pencils in each bag.)

# **Advancing Student Thinking**

If students say that the drawing can't match both situations, consider asking:

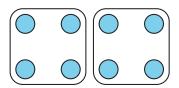
- "How could we make a drawing for each situation?"
- "What might we draw first to represent the first situation with 8 objects? What about with the second situation with 8 objects?"

Unit 4 Lesson 3

# **Lesson Synthesis**

① 10 min

Continue to display the drawing and situations from the last activity, such as:



Mai has 8 markers. She puts 4 markers in each box. How many boxes of markers are there?

Lin has 8 colored pencils. She puts them into 2 bags. Each bag has the same number of colored pencils. How many colored pencils will be in each bag?

"Today we matched drawings to division situations. There are two types of division situations and we saw today that the same drawing can represent both types of situations."

"What is the same and what is different about these division situations?" (Both situations have the numbers 8, 2, and 4 in them. Both involve putting objects into equal groups. The objects are different, one is about markers and the other is about colored pencils. One situation tells us how many items go into each container and the other tells us how many containers there are.)

"In the first situation, we need to figure out how many groups there are. We know there are 4 markers in each box, but we don't know how many boxes there will be. In the second situation, we need to figure out how many in each group. We know there are 2 bags, but we don't know how many colored pencils will be in each bag."

"Now that we are dividing, we need a new symbol to write division expressions. If we wanted to represent '8 divided into groups of 4' we would write:  $8 \div 4$ ."

"What expression could we write to represent '8 divided into 2 groups'?"  $(8 \div 2)$ 

### **Suggested Centers**

- Rectangle Rumble (3–5), Stage 2: Factors 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

-- Complete Cool-Down ----

### **Response to Student Thinking**

Students choose drawing A, which shows 8 bags instead of 8 markers in each bag.

#### **Next Day Support**

 During the launch of the next day's activity, have students discuss why drawing B matches the situation.

# **Lesson 4: Interpret Division Expressions**

### **Standards Alignments**

Addressing 3.NBT.A.2, 3.OA.A.2

#### **Teacher-facing Learning Goals**

- Interpret division expressions.
- Understand that the same division expression can be used to represent both types of division situations.

### **Student-facing Learning Goals**

• Let's make sense of division expressions.

#### **Lesson Purpose**

The purpose of this lesson is for students to interpret division expressions and understand that the same division expression can be used to represent both types of division situations.

Students first match a division expression to a situation that it could represent. Then, students learn that the same division expression can match both "how many groups?" and "how many in each group?" problems depending on how the **divisor**, the number we are dividing by, is interpreted. Students then have a chance to match drawings and expressions to situations before they write their own division expressions in a subsequent lesson.

#### Access for:

- Students with Disabilities
- Representation (Activity 1)

- English Learners
- MLR2 (Activity 2)

#### Instructional Routines

Number Talk (Warm-up)

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	10 min
Activity 2	10 min
Activity 3	15 min

#### **Teacher Reflection Question**

What aspects of today's lesson allowed each of your students to see themselves as productive mathematical reasoners?

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Lesson Synthesis	10 min
Cool-down	5 min

# $\textbf{Cool-down} \hspace{0.2cm} \text{(to be completed at the end of the lesson)}$

© 5 min

Han's Tops

### **Standards Alignments**

Addressing 3.OA.A.2

## **Student-facing Task Statement**

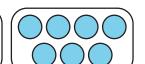
Han has 14 tops. He shares the tops equally between 2 boxes. How many tops will be in each box?

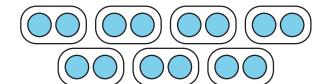
В.

D.

Select **all** the ways that we could represent the situation.

A.





C.

14 ÷ 2

 $14 \div 7$ 

# **Student Responses**

A, C

---- Begin Lesson ------

Warm-up © 10 min

Number Talk: More or Less?

### **Standards Alignments**

Addressing 3.NBT.A.2

The purpose of this Number Talk is to elicit strategies and understandings students have for subtracting within 1,000, particularly around adding up to find differences. These understandings help students develop fluency for subtracting within 1,000.

#### **Instructional Routines**

Number Talk

### **Student-facing Task Statement**

Find the value of each expression mentally.

- 500 **–** 475
- 504 **–** 475
- 512 475
- 512 449

### **Student Responses**

- 25: I added 25 to 475 to get to 500.
- 29: I thought about the first problem and I added 4 to 25 to get to 504.
- 37: I started at 475 and added 25 to get to 500, then I added 10 to get 510, and 2 more to get to 512. 25 + 10 + 2 = 37.
- 63: I know that 49 is 51 away from 500 and then I added 12 more.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

### **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### **Synthesis**

- "Why is the value of 512 475 greater than the value of 504 475?" (Since the 475 doesn't change, but 512 is larger than 504, the difference between the numbers is greater.)
- Consider asking:
  - "Who can restate \_\_\_\_'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to \_\_\_\_\_'s strategy?"

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Activity 1 © 10 min

**Spinning Tops** 

### **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to match division expressions to division situations. Students should justify their matches by articulating how the numbers in the expression connect to what is happening in the situation (MP2).

### **1** Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were needed to solve the problem. Display the sentence frame, "The next time I match division expressions to division situations, I will look for . . . ."

Supports accessibility for: Attention, Memory

### **Student-facing Task Statement**

Spinning tops are popular around the world. Here are spinning tops from a few different cultures.











Match each situation about spinning tops with an expression that can represent it.

- 1. Clare has a collection of 24 A.  $24 \div 2$  spinning tops in four colors. She has the same number of tops in black, white, red, and green. How many tops of each color does she have?
- 2. Priya and her friend are decorating 24 wooden tops with

B.  $12 \div 2$ 

#### Launch

- Groups of 2
- Display the image.
- "These are toys called spinning tops, or sometimes just called tops. They are played with in many cultures. What are some other toys that you know about?"
- 30 seconds: quiet think time
- Share responses.
- "Now we're going to work with some situations that involve spinning tops. We'll see situations about other toys in upcoming activities."

### **Activity**

 "Work with your partner to match each situation to a division expression."

paint. If each person is painting the same number of tops, how many tops is each person painting?

3. A store has 24 tops from around the world displayed in 6 boxes. Each box contains the same number of tops. How many tops are in each box?

each friend 2 trompos, how many friends can get them as

4. Diego has 12 trompos that he wants to give as gifts. If he gives each friend 2 trompos how

 $C. 24 \div 4$ 

E.  $24 \div 6$ 

5. Six friends are playing with 12 dreidels. If everyone is playing with the same number of dreidels, how many dreidels does each person have?

- 3–5 minutes: partner work time
- Monitor for students who can justify their matches by explaining how the numbers in the expression represent the situation.

#### **Synthesis**

- Select previously identified students to share.
- Consider asking: "How do the numbers in the expression represent what is in the situation?"

# **Student Responses**

1. C

gifts?

- 2. A
- 3. E
- 4. B
- 5. D

Activity 2 © 10 min

Cars in Boxes

# **Standards Alignments**

Addressing 3.OA.A.2

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The purpose of this activity is for students to understand that the same division expression can be used to represent both types of division situations. Students are given two situations and asked to match a division expression to one of the situations, but the expression matches both situations given. It is okay if students do not recognize that the expression matches both situations in the activity, because it will be discussed in the activity synthesis. Students learn that the number we are dividing by is called the **divisor** and understand that the divisor can represent the size of the groups or the number of groups. When students explain that a divisor can be interpreted differently based on the situation it represents, they reason abstractly and quantitatively (MP2).

# Access for English Learners

MLR2 Collect and Display. Circulate and collect the language students use as they consider the two situations. Listen for and clarify any questions about the context. On a visible display, record words and phrases such as: put in groups, split, divide, number of groups, etc. During the synthesis, add "divisor" to the display and highlight connections to any related language. Advances: Conversing, Reading

### **Student-facing Task Statement**

Consider these two situations.

A. Han has 21 toy cars. He puts the same number of cars in each of 3 boxes. How many cars will be in each box? B. Han has 21 toy cars. He wants to put 3 cars in each box. How many boxes will he need?

Which situation does the expression  $21 \div 3$  represent? Explain your reasoning.



#### **Student Responses**

The expression matches both situations. Sample response: Situation A is 21 things put into 3 boxes, that is 21 divided by 3. Situation B is 21 put into groups of 3. That is also 21 divided by 3.

#### Launch

- Groups of 2
- "Let's look a bit closer at division expressions. Take a minute to read these two situations."
- 1 minute: quiet think time

### **Activity**

- "Work with your partner to decide which situation the expression represents."
- 2–3 minutes: partner work time

#### **Synthesis**

- Invite students to share their responses and reasoning.
- "How can the same expression represent two different situations?" (Both situations involve the same numbers, 21 and 3. Both situations involve putting 21 objects into equal groups. In one case, the 3 is the number of objects in the group, but

- in the other, it is the number of groups. Both situations are talking about 21 divided by 3, just in different ways.)
- "We noticed that the number that we are dividing by, 3, can have two different meanings. It can mean 3 groups or 3 objects in each group."
- "When we divide, the number we divide by is called the **divisor**. In the expression  $27 \div 3$ , the divisor is 3."

# **Activity 3**

Stacks of Blocks

(15 min



♣ ↔ ♣ PLC Activity

### **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to apply what they have learned about representations of division to match drawings and expressions to division situations (MP2). In doing so, they solidify their understanding that the same division expression can represent both types of division situations. The given drawings enable students to see the number of groups and how many objects are in each group. The work here helps students make connections across the three representations before they write their own division expressions and solve division problems in a subsequent lesson. When students describe how one equation can represent different stories they attend to precision in the language they use and the correspondence that they establish between the equation and the stories (MP6).

# **Student-facing Task Statement**

Match each situation to a drawing and an expression that represent the situation. Be prepared to explain your reasoning.

1. Kiran uses 6 blocks to make stacks. Fach stack has 2 blocks. How many stacks are

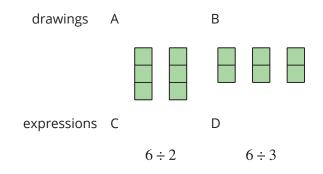
#### Launch

- Groups of 2
- "Now that we've represented division situations with both drawings and expressions, we're going to match some situations with both representations."

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there?

- 2. Han uses 6 blocks to make two equal stacks. How many blocks are in each stack?
- 3. Jada uses 6 blocks to build stacks with 3 blocks each. How many stacks are there?
- 4. Mai uses 6 blocks to make 3 equal stacks. How many blocks are in each stack?



#### **Student Responses**

- 1. B and C
- 2. A and C
- 3. A and D
- 4. B and D

- "Read these situations."
- 1-2 minutes: quiet think time
- "Talk to your partner about what is the same and what is different about these situations."
- 2–3 minutes: partner discussion
- Share responses.

### **Activity**

- "Now work with your partner to match each situation to a drawing and an expression that represents the situation."
- 3–5 minutes: partner work time

### **Synthesis**

- "Let's consider the first two situations about Kiran and Han. Why can we use the same expression to represent these situations but different drawings?" (Both situations are represented by 6 divided by 2, but in the first situation the 2, or the divisor, is how many blocks are in each stack. In the second situation the 2 is the number of stacks.)
- "Now let's look at Han and Jada's situations.
   Why can we use the same drawing to
   represent these situations but not the
   same expression?" (Both of these situations
   describe 2 groups of 3, so they match the
   same drawing. The first situation is 6
   blocks divided between 2 stacks and the
   second situation is 6 blocks divided into
   groups of 3.)

# **Lesson Synthesis**

① 10 min

Display some expressions from the lesson, such as  $6 \div 2$  and  $6 \div 3$ .

"Is there any way to tell the expressions that represent a 'how many groups?' problem from the expressions that represent a 'how many in each group?' problem?" (No, not by just looking at the expression. We would have to look back at the situation or the drawing.)

"Division expressions can be interpreted two ways and we can't really tell what type of division situation is being represented unless we have a situation or a drawing that goes with the expression."

#### **Suggested Centers**

- Capture Squares (1–3), Stage 6: Multiply with 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

### ---- Complete Cool-Down ----

### **Response to Student Thinking**

Students select responses that correspond to 7 groups of 2 instead of 2 groups of 7.

#### **Next Day Support**

 During the launch of the next day's activity, have students work in partners to discuss a correct response to this cool-down.

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# **Lesson 5: Write Division Expressions**

#### **Standards Alignments**

Addressing 3.NBT.A.2, 3.OA.A.2, 3.OA.A.3

#### **Teacher-facing Learning Goals**

- Solve "how many groups?" and "how many in each group?" problems.
- Write division expressions to represent division situations.

### **Student-facing Learning Goals**

 Let's write division expressions and solve "how many groups?" and "how many in each group?" problems.

#### **Lesson Purpose**

The purpose of this lesson is for students to write division expressions to represent division situations and solve "how many groups?" and "how many in each group?" problems.

Students sort division situations for whether the number of groups is unknown or the number of objects in each group is unknown and write division expressions to represent each situation (MP2). Students then have a chance to use the representations they have learned in this section to solve division problems.

This lesson has a Student Section Summary.

#### Access for:

- Students with Disabilities
- Engagement (Activity 1)

- **3** English Learners
  - MLR8 (Activity 1)

#### **Instructional Routines**

Card Sort (Activity 1), MLR7 Compare and Connect (Activity 2), Number Talk (Warm-up)

#### **Materials to Gather**

• Tools for creating a visual display: Activity 2

#### **Materials to Copy**

 Card Sort: All About Bugs (groups of 2): Activity 1

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

### **Teacher Reflection Question**

How have students' strategies for solving division problems evolved from the first lesson in this unit?

# Cool-down (to be completed at the end of the lesson)

© 5 min

Ant Legs

### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

# **Student-facing Task Statement**

Twenty-four legs belong to 4 ants. All ants have the same number of legs.

- 1. Write a division expression to represent this situation.
- 2. How many legs does each ant have? Explain or show your reasoning.

# **Student Responses**

- 1.  $24 \div 4$
- 2. 6 legs. Sample response: A drawing with 4 groups of 6.

Warm-up © 10 min

Number Talk: What's the Same?

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#### **Standards Alignments**

Addressing 3.NBT.A.2

The purpose of this Number Talk is to elicit strategies and understandings students have for subtracting within 1,000, particularly with expressions with a constant difference. These understandings help to develop fluency for subtracting within 1,000. Consider drawing number lines as students share their strategies to emphasize that the difference of the two numbers in each expression is not changing.

#### **Instructional Routines**

Number Talk

#### **Student-facing Task Statement**

Find the value of each expression mentally.

- 225 **–** 100
- 227 **–** 102
- 230 105
- 220 95

## **Student Responses**

- 125: The difference between 100 and 200 is 100 and then it's 25 more to 225.
- 125: I noticed that 2 was added to both numbers in the first problem. So now it's 98 to 200, but it's 27 more to get to 227.
   98 + 27 = 125
- 125: Five is added to each number from the first problem, so the difference between the numbers is still 125.
- 125: This time 5 was subtracted from both numbers. I added 5 to 95 to get to 100 and then it's 120 more to 220, so the value is still 125.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

### **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### **Synthesis**

- "What do you notice about these expressions?" (They all have the same value.)
- "Why do they all have the same value?" (Since the same number is added or subtracted to both numbers from the original expression, the difference does not change.)
- Consider asking:
  - "Who can restate \_\_\_\_\_'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"

 "Does anyone want to add on to \_\_\_\_\_'s strategy?"

Activity 1 © 15 min

Card Sort: All about Bugs

#### **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this activity is for students to determine whether a situation is about an unknown number of groups or an unknown number of objects in each group. After sorting the situations, students write a division expression to represent each situation. The fact that the structure of the expressions is the same for representing an unknown number of groups or an unknown number of objects in each group further emphasizes that division expressions can be interpreted two ways. As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).

As students explain their reasoning around the unknown in the situation, encourage students to describe how they would start to solve the problem to make it clear what is unknown in the situation.

# Access for English Learners

MLR8 Discussion Supports. Synthesis. Display a sentence frame to support whole-class discussion: "We noticed \_\_\_\_\_ so we . . . ."

Advances: Speaking, Representing

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Organization, Attention

#### **Instructional Routines**

Card Sort

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#### **Materials to Copy**

Card Sort: All About Bugs (groups of 2)

#### **Required Preparation**

• Create a set of cards from the Instructional master for each group of 2.

#### **Student-facing Task Statement**



- 1. Your teacher will give you a set of cards that show situations. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.
  - A. Mole crickets have special legs for digging. Ten special legs belong to 5 mole crickets. How many special legs does each mole cricket have?
- B. A beetle has a pair of antennae for sensing heat, touch, smell, and more. If there are 8 antennae, how many beetles are there?
- C. Fourteen antennae belong to a group of bees. If each bee has 2 antennae, how many bees are there?
- D. There are 12 wings. If each dragonfly has 4 wings, how many dragonflies are there?
- E. Thirty legs
- F. There are 50 spots

#### Launch

- Groups of 2
- Display the image.
- "We are going to work with some situations involving insects. Insects are a type of bug. These are all insects."
- "What are some parts of the insects we could count?" (legs, eyes, wings, antennae, body segments)
- If needed, clarify what antennae are.
- Distribute one set of pre-cut cards to each group of students.

#### **Activity**

- "In this activity, you will sort some cards into categories of your choosing. When you sort the situations, you should work with your partner to come up with categories."
- 5 minutes: partner work time
- Select groups to share their categories and how they sorted their cards.
- Choose as many different types of categories as time allows, but ensure that one set of categories distinguishes between "how many groups?" and "how many in each group?" problems.
- If no students sorted the cards by type of division situation, give them a minute to do so, and then discuss how they know which type of division each situation represents.
- "Now work with your partner to sort your cards by 'how many groups?' and 'how

belong to 5 ants. If all the ants have the same number of legs, how many legs does each ant have? on 5 butterflies. If each butterfly has the same number of spots, how many spots does each butterfly have?

2. Write a division expression to represent each situation. Be ready to explain your reasoning.

### **Student Responses**

- 1. Sample response:
  - The situations are about finding the number of groups: B, C, D
  - The situations are about finding the number of objects in each group: A, E, F
- 2. A:  $10 \div 5$ 
  - B:  $8 \div 2$
  - C:  $14 \div 2$
  - D:  $12 \div 4$
  - E:  $30 \div 5$
  - $F: 50 \div 5$

many in each group?' problems."

- "Once you have sorted your cards, write a division expression to represent each situation."
- 5 minutes: partner work time

### **Synthesis**

- Invite students to share the expression for each situation.
- Consider asking:
  - "What does each number represent in the expression?"
  - "Where do you see the number of groups in the expression?"
  - "Where do you see the number of objects in each group in the expression?"

Activity 2 © 20 min

Solve a Buggy Problem

# **Standards Alignments**

Addressing 3.OA.A.3

In this activity, students consolidate their understanding of the types of division situations and their representations to solve division problems.

During the synthesis, arrange and display students' posters by type, as sorted in the previous

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activity.

This activity uses MLR7 Compare and Connect. Advances: representing, conversing

#### **Instructional Routines**

MLR7 Compare and Connect

#### **Materials to Gather**

Tools for creating a visual display

#### **Student-facing Task Statement**

Your teacher will assign a problem to your group.

Create a visual display that shows your thinking and your solution to the problem.

#### **Student Responses**

- 1. 2 special legs
- 2. 4 beetles
- 3. 7 bees
- 4. 3 dragonflies
- 5. 6 legs
- 6. 10 spots

#### Launch

- Groups of 2
- Assign each group one of the problems from the previous activity to solve.
- Give each group tools for creating a visual display.

### Activity

#### **MLR7 Compare and Connect**

- "Create a visual display that shows your thinking about the problem you were assigned. You may want to include details such as notes, diagrams, drawings, etc. to help others understand your thinking."
- 5 minutes: partner work time
- 8–10 minutes: gallery walk

#### **Synthesis**

- "What is the same about the two types of division problems?" (They both involve putting things into equal groups.)
- "What is different about them?"
   (Sometimes we know how many things are in each group and we need to find how many groups we can make. Sometimes we know how many groups there are, but we

need to find how many things are in each group.)

# **Lesson Synthesis**

(10 min

"Over the last few lessons we have been learning about division. We represented and solved two kinds of division problems. Let's summarize what we know about division together."

"What are some of the big ideas we have learned about division?" (Division is about equal groups. We can find how many groups or how many there are in each group. We can represent division with drawings. We can write division expressions to represent division situations.)

Organize the class ideas on a chart with two columns, with representations of "how many groups?" in one column and those of "how many in each group?" in the other (as in the student lesson summary).

#### **Suggested Centers**

- Capture Squares (1–3), Stage 6: Multiply with 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

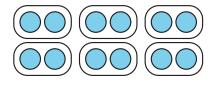
# **Student Section Summary**

In this section, we learned that division is finding the number of groups or finding the size of each group when we put objects into groups of equal size. We represented division situations with drawings and expressions, and solved division problems.

"How many groups?"

"How many in each group?"

Han has 12 colored pencils. He wants to put 2 colored pencils in each box until he's out of colored pencils. How many boxes does Han need? Elena has 12 colored pencils. She has 2 boxes and wants to put the same number of colored pencils in each box. How many pencils will be in each box?



 $12 \div 2$ 

 $12 \div 2$ 

Unit 4 Lesson 5

# ------ Complete Cool-Down

## **Response to Student Thinking**

Students write an expression other than  $24 \div 4$  to match the situation or don't find a solution to the problem.

### **Next Day Support**

• Before the warm-up, have students to work in partners to discuss a correct response to this cool-down.

# **Section B: Relate Multiplication and Division**

### Lesson 6: Division as an Unknown Factor

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.B.6

#### **Teacher-facing Learning Goals**

- Explain the relationship between multiplication and division equations.
- Interpret division equations and multiplication equations with a missing factor.

### **Student-facing Learning Goals**

 Let's connect division equations to multiplication equations.

### **Lesson Purpose**

The purpose of this lesson is for students to relate multiplication and division and recognize division as an unknown factor problem.

Previously, students learned to interpret and write division expressions. They connected division to multiplication informally, recognizing that both operations involved equal groups. In this lesson, students analyze related multiplication and division equations to formalize the relationship between multiplication and division. In the lesson synthesis, students learn that the result in a division equation is called a **quotient**.

#### Access for:

- Students with Disabilities
- Engagement (Activity 2)

# English Learners

MLR7 (Activity 1)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

#### **Lesson Timeline**

Warm-up 10 min

#### **Teacher Reflection Question**

In this lesson, students formally relate

Unit 4 Lesson 6 63

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

multiplication and division for the first time. How is their previous knowledge of multiplication and division supporting them in understanding this relationship?

Cool-down	(to be completed at the end of the l	lesson)
-----------	--------------------------------------	---------

© 5 min

**Boxed Muffins** 

### **Standards Alignments**

Addressing 3.OA.B.6

### **Student-facing Task Statement**

There are 30 muffins for the bake sale. Each box has 6 muffins. How many boxes are there?

Tyler wrote two equations for this problem.

$$\_\_ \times 6 = 30$$

$$30 \div 6 =$$
\_\_\_\_\_

He says the same number goes in each blank even though one equation is a multiplication equation and the other equation is a division equation. Is he correct? Explain or show your reasoning.

# **Student Responses**

Tyler is right. Sample response: Five goes in both blanks because it is the number of boxes, we just write it in different places for multiplication equations and division equations.

----- Begin Lesson

Warm-up © 10 min

Notice and Wonder: Missing Numbers

## **Standards Alignments**

Addressing 3.OA.B.6

The purpose of this warm-up is to elicit the idea that multiplication and division are related, which will be useful as students learn to understand division as an unknown factor problem. While students may notice and wonder many things about these equations, ideas about how multiplication and division are alike and different are the important discussion points.

Students have seen division expressions, but this will be their first time seeing division equations.

#### Instructional Routines

Notice and Wonder

## **Student-facing Task Statement**

What do you notice? What do you wonder?

$$3 \times ? = 12$$

$$12 \div 3 = ?$$

## **Student Responses**

Students may notice:

- The 12 is being broken into 3 groups or groups of 3.
- Both equations have a 3, a 12, and a question mark, but they are not in the same places.
- Putting 4 for the question mark would make sense for both equations.

Students may wonder:

- Is the missing number the same in both equations?
- What is the missing number?
- Are the two equations related?

#### Launch

- Groups of 2
- Display the equations.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

## **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

 "Today, we are going to work more with multiplication and division equations like these."

Unit 4 Lesson 6 65

Activity 1 © 15 min

**Equations about Onions** 

#### **Standards Alignments**

Addressing 3.OA.B.6

The purpose of this activity is for students to formalize the relationship between multiplication and division equations. They see that the unknown quantity in a division situation can be represented as a missing factor in a multiplication equation or as a quotient in a division equation. The synthesis should emphasize that both equations are appropriate ways to represent a situation that involves equal groups.

This activity gives students an opportunity to make sense of each quantity and how it relates to the situation (MP2). As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).

# **Access for English Learners**

*MLR7 Compare and Connect.* Synthesis: Create a visual display of the problem. "What did Lin and Mai's approaches have in common? How were they different?" As students share their reasoning, annotate the display to illustrate connections. For example, below each equation, write the words total, number of groups, and number in each group depending on the equation and student input.

Advances: Listening, Representing

# **Student-facing Task Statement**

A farmer puts 14 onions into 2 bags, with the same number of onions in each bag.

Lin says the situation should be represented by the equation:



Mai says the



#### Launch

- Groups of 2
- "Where are some places that we get food?"
- 30 seconds: quiet think time
- Share responses.
- "This situation is about getting food from a farmer's market. Farmers markets are places where people in the community come together and sell food they've grown or food they've made."

situation should be represented by the equation:

$$14 \div 2 =$$

Whose equation do you agree with? Be ready to explain your reasoning.

## **Student Responses**

Sample responses:

- Lin, because we know the total and how many groups there are, but we don't know how many are in each group so one of the factors should be missing.
- Mai, because we know the total is being split into 2 groups, so we're dividing to find the number in each group.

#### **Activity**

- "Read how Lin and Mai are thinking about this situation and decide who you agree with and why."
- 3 minutes: independent work time
- Monitor for students who agree with the different equations to pair for discussion.
- 3 minutes: partner discussion
- Monitor for students who can articulate why either student is correct.

#### **Synthesis**

- "After discussing your ideas with your partner, who do you agree with? Explain your reasoning." (They are both correct because the situation can be represented with a multiplication equation with a missing factor or a division equation. Both equations show the number of onions in each box as the missing number.)
- If students don't see that both Lin and Mai are correct, consider asking, "How could both equations represent this situation?"

Activity 2 © 20 min

At the Farmers' Market

# **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.B.6

The purpose of this activity is for students to understand how multiplication equations correspond to diagrams and equations they have used to represent division situations. The focus should be on relating the unknown factor to the unknown number of groups or the unknown number of objects in each group. In their explanations, students should make direct

Unit 4 Lesson 6 67

connections between the situations, representations, and equations (MP2).

## Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Some students may benefit from feedback that emphasizes effort and time on task. For example, give feedback after each row or encourage students to work on the next row if they have difficulty with a specific row on the chart. Supports accessibility for: Attention, Social-Emotional Functioning

#### **Student-facing Task Statement**

Complete each row. Be prepared to explain your reasoning.

situation	drawing or diagram	multiplication equation	division equation
Elena's family buys 18 avocados at the farmers market. The avocados are in bags of 3 each.	00000		18 ÷ 3 =
Andre sees 25 tomatoes. They are in 5 bunches. Each bunch has the same number of tomatoes.		5 × ? = 25	25÷5=?
Lin orders 6 banana fritters. The fritters are served on 2 plates and each plate has the same number of fritters.	0000	2×?=6	
	00000 00000 00000 00000 00000	×10 = 30	30 ÷ 10 =

# **Student Responses**

#### Launch

- Groups of 2
- "We're going to complete this table. Take a minute to look at what might be missing from the table."
- 1 minute: quiet think time
- Clarify any questions students have about the situations in the table.

## **Activity**

- "Complete each row. Be prepared to explain your reasoning."
- 5–7 minutes: independent work time
- "Share your reasoning with your partner."
- 2–3 minutes: partner discussion
- Monitor for different ways that students explain their responses. Listen for strategies that make clear connections between the quantities in the situation and their representations. For example, students may say the 2 plates match with the 2 groups in the drawing or the 2 in the equations represents the 2 groups in the drawing and the 2 plates in the situation.
- If students finish early, or to add movement to the activity, consider asking groups of 4 to create a poster that show their own situation and the corresponding drawing, multiplication

situation	drawing or diagram	multiplication equation	division equation
Elena's family buys 18 avocados at the farmers market. The avocados are in bags of 3 each.		×3 = 18	18 ÷ 3 =
Andre sees 25 tomatoes. They are in 5 bunches. Each bunch has the same number of tomatoes.		5 × ? = 25	25 ÷ 5 = ?
Lin orders 6 banana fritters. The fritters are served on 2 plates and each plate has the same number of fritters.	0000	2 × ? = 6	6 ÷ 2 = ?
Tyler sees 30 oranges. They are in boxes and each box has 10 oranges.	00000 00000 00000 00000	×10 = 30	30 ÷ 10 =

equation, and division equation. They can then do a gallery walk.

#### **Synthesis**

- Share student responses for each row.
- Consider asking "How does the equation (or drawing) show the numbers or amounts in the situation?"
- "What relationship did you see between the multiplication equations and the division equations?" (The equations used the same numbers to represent the situation. The answer for the division equation was always one of the missing factors for the multiplication equation.)

## **Advancing Student Thinking**

If students don't record a multiplication equation and division equation for each row, consider asking:

- "How did you decide what type of equation to write?"
- "How could we represent this diagram (or situation) with a multiplication (or division) equation? Where do we see each part of the equation in the diagram (or situation)?"

# **Lesson Synthesis**

(10 min

"Today we focused on connecting multiplication and division equations that represent the same situation."

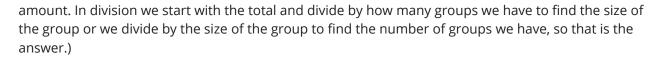
Display: A farmer puts 14 onions into 2 bags, with the same number of onions in each bag.

$$2 \times \square = 14$$

$$14 \div 2 = \Box$$

"The two equations here have the same parts: 2, 14, and an unknown amount. Why are they arranged differently if they represent the same situation?" (In multiplication, the factors are the number of groups and the size of each group. The number on the other side of the equation is the total

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"We call the result in a division equation the **quotient**. For example, in  $14 \div 2 =$  \_\_\_\_, the result is unknown, so we are finding the value of the quotient. In the completed equation  $14 \div 2 = 7$ , we see that the value of the quotient is 7."

## **Suggested Centers**

- Capture Squares (1–3), Stage 6: Multiply with 1–5 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

## ----- Complete Cool-Down ----

# **Response to Student Thinking**

Students say that different numbers should be used to complete each equation.

#### **Next Day Support**

 During the launch of the next day's activity, pass back the cool-down and have students discuss how the missing number in both equations relates to the situation.

# **Lesson 7: Relate Multiplication and Division**

#### **Standards Alignments**

Addressing 3.NBT.A.3, 3.OA.A.2, 3.OA.A.3, 3.OA.B.6

#### **Teacher-facing Learning Goals**

- Represent situations involving equal groups using multiplication and division equations with a symbol for the unknown quantity.
- Use multiplication and division within 100 to solve problems involving equal groups.

## **Student-facing Learning Goals**

 Let's make more connections between multiplication and division.

## **Lesson Purpose**

The purpose of this lesson is for students to use the relationship between multiplication and division to write equations and solve problems.

In previous lessons, students built a foundation of division understanding and connected multiplication and division equations. Here, they use a variety of representations to show how multiplication and division are related and write multiplication or division equations to represent division situations.

#### Access for:

#### Students with Disabilities

Engagement (Activity 1)

# **S** English Learners

MLR8 (Activity 2)

#### **Instructional Routines**

How Many Do You See? (Warm-up)

## Materials to Copy

• Division Round Table (groups of 1): Activity 1

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min

## **Teacher Reflection Question**

Which question did you ask today that best supported students' understanding of how multiplication and division equations are

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Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

related? What did students say or do that showed the question was effective?

**Cool-down** (to be completed at the end of the lesson)

© 5 min

**Sharing Roses** 

## **Standards Alignments**

Addressing 3.OA.B.6

## **Student-facing Task Statement**

Clare has 14 roses. She wants to give each of her teachers 2 roses. How many teachers can she give roses to?

Write a multiplication equation and a division equation to represent the situation. Use symbols for the unknown and explain your reasoning.

# **Student Responses**

 $? \times 2 = 14$  and  $14 \div 2 = ?$ 

Sample response: I know she has 14 roses and wants to put them in groups of 2. The question is asking me how many groups there will be, which is how many teachers will get roses and is represented by the "?".

Note: It is most important that students accurately explain that the size of the groups is known, but not the number of groups. They may reverse the factors in the multiplication equation.

Warm-up © 10 min

How Many Do You See: Tens

# **Standards Alignments**

Addressing 3.NBT.A.3

The purpose of this How Many Do You See is for students to use grouping strategies to describe the images they see.

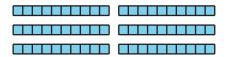
When students use grouping to find the total in a multiple of tens, they look for and make use of structure (MP7).

#### **Instructional Routines**

How Many Do You See?

## **Student-facing Task Statement**

How many do you see? How do you see them?



## **Student Responses**

60. Sample responses:

- I see 6 groups of 10.
- I see 2 tens in each row and there's 3 rows, so there are 6 tens.
- I see 2 tens in each group and there are 3 groups, so there are 6 tens.

#### Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

#### **Activity**

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

## **Synthesis**

- "What expressions could we record for the different ways that students saw the tens?"  $(6 \times 10$ , because some students saw 6 groups of 10.  $3 \times (10 \times 2)$ , because some students saw 2 rows of 10, then multiplied by 3.  $(3 \times 10) \times 2$ , because some students multiplied 3 times 10 for each column, then multiplied by 2.)
- Consider asking:
  - "Who can restate in different words the way \_\_\_\_\_ saw the tens?"

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- "Did anyone see the tens the same way but would explain it differently?"
- "Does anyone want to add an observation to the way \_\_\_\_\_ saw the tens?"

# **Activity 1**

( 20 min

Division Round Table

♣ ↔ ♣ PLC Activity

## **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.B.6

The purpose of this activity is for students to solidify what they have learned about the relationship between multiplication and division. Students start by creating a drawing of equal groups. They then get a drawing created by another student in their group and write a division situation to match it. Then, they pass their paper and use the drawing of equal groups and the situation to write a multiplication equation. In the final round of this "carousel" structure, students write a division equation to match the other representations.

When students relate drawings, situations, and equations they reason abstractly and quantitatively (MP2). As students look through each other's work, they add to the representations and can defend different points of view. Students are able to critique the work of others and construct viable arguments (MP3).

Students work on the same box on a graphic organizer as the other students in their group, so if they struggle, encourage them to talk to their group. Remind students that what they are creating should match what has already been filled in.

# Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Check in and provide each group with feedback that encourages collaboration and community. For example, supporting students in participating, passing the paper to the right, and writing the symbol.

Supports accessibility for: Social-Emotional Functioning, Language

# **Materials to Copy**

Division Round Table (groups of 1)

#### **Student-facing Task Statement**

Your teacher will give you a sheet of paper with 4 boxes on it and instruct you to draw or write something in each box.

After working on each box, pause and wait for your teacher's instructions for the next box.

- 1. Draw equal groups in Box 1 on your recording sheet.
- 2. In Box 2, write a description of a division situation that matches the drawing you just received.
- 3. In Box 3, write a multiplication equation that matches the drawing and division situation you just received. Use a symbol for the unknown quantity.
- 4. In Box 4, write a division equation that matches the drawing, division situation, and multiplication equation you just received.

  Use a symbol for the unknown quantity.



# **Student Responses**

Answers vary.

#### Launch

- Groups of 4
- Give each student a recording sheet.
- "In the first box on your sheet, create a drawing that shows equal groups of objects. This drawing will be used by other students in your group to fill in the other boxes."
- 3 minutes: independent work time

#### **Activity**

- "Pass your paper to your right. In Box 2, write a description of a division situation that matches the drawing you were just passed."
- 3 minutes: independent work time
- "Pass your paper to your right. In Box 3, write a multiplication equation that matches the drawing and division situation you just received. Use a symbol for the unknown quantity."
- 2 minutes: independent work time
- "Pass your paper to your right. In Box 4, write a division equation that matches the drawing, division situation, and multiplication equation you just received. Use a symbol for the unknown quantity."
- 2 minutes: independent work time
- "Pass your paper one more time. You should have your original drawing back."
- "Talk to your group about which box was the most difficult for you to fill in. Share ideas about what helped you most during this activity."
- 5 minutes: small-group discussion

#### **Synthesis**

 "What strategies were shared in your group?" (When I wasn't sure about writing a

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- situation, I looked back at the drawing and tried to imagine something I could be dividing that looks like the drawing. When I was writing an equation it helped me to imagine the situation happening.)
- "As you look at your paper, what are some connections you notice between multiplication and division?" (I can use both multiplication and division to represent the same drawing or situation. The multiplication equations are all missing a factor, but the division equations are all missing the quotient.)

Activity 2 © 15 min

Sets of School Supplies

# **Standards Alignments**

Addressing 3.OA.A.3

The purpose of this activity is for students to represent and solve problems involving equal groups. Students can solve the problem first or write the equation first, depending on the order that makes the most sense to them. Students write equations with a symbol standing for the unknown quantity to represent each problem, but can write either a multiplication equation or a division equation. A multiplication equation and a division equation that represent the same problem are highlighted in the synthesis.

# **3** Access for English Learners

*MLR8 Discussion Supports:* Prior to writing the equations, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

## **Student-facing Task Statement**

For each situation:

- a. Write an equation with a symbol for the unknown quantity to represent the situation.
- b. Solve the problem and find the unknown number in the equation. Be prepared to explain your reasoning.
- 1. Kiran had 32 paper clips. He gave each student 4 paper clips. How many students received paper clips?

a.	Equation:	
b.		

2. There are 28 books in 4 stacks. If each stack has the same amount of books, how many books are in each stack?

a.	Equation:	
h		

3. There are 6 boxes. Each box has 8 erasers. How many erasers are there?

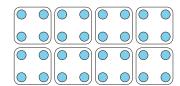
a.	Equation:	
b.		

4. Lin had 36 sticky notes. She placed 6 sticky notes on each notebook. How many notebooks received sticky notes?

a.	Equation:	
b.		

# **Student Responses**

- 1. a.  $4 \times ? = 32$ ,  $? \times 4 = 32$ , or  $32 \div 4 = ?$ 
  - b. 8 students. Sample reasoning:



#### Launch

- Groups of 2
- "These situations are all about things that you could find on a desk or around a desk.
   What are some things that you could find on a desk or around a desk?"
- 30 seconds: quiet think time
- Share responses.

#### **Activity**

- "Read through each situation and write an equation with a symbol that represents the unknown quantity for each situation. Then, solve and determine the unknown number in each equation. You can solve the problem first or write an equation first depending on what order makes the most sense to you. Be prepared to explain your reasoning."
- 7–10 minutes: independent work time
- Monitor for students who write a division equation and a multiplication equation for the same situation to share during the synthesis.
- "Now, share your equations and your solutions with your partner. Take turns sharing your equations and solutions."
- 3–5 minutes: partner discussion

# **Synthesis**

- Have students share a division equation and a multiplication equation that were written to represent the same division situation and display for all to see.
- Discuss differences in equations students wrote.
- Consider asking:
  - "How does each number in the equations represent the situation?"

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I drew groups of 4 until I got to 32, then counted the groups. There were 8 groups so I knew 8 students received paper clips.

2. a. 
$$4 \times \underline{\hspace{1cm}} = 28$$
,  $\times 4 = 28$ , or  $28 \div 4 = \underline{\hspace{1cm}}$ 

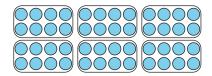
b. 7 books. Sample reasoning:



I started with 4 rows since the books were in 4 stacks. I knew  $4 \times 7$  is 28 so I put 7 in each row.

3. a. 
$$6 \times 8 = ?$$
,  $8 \times 6 = ?$ ,  $? \div 8 = 6$ ,  $? \div 6 = 8$ 

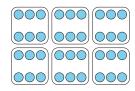
b. 48 erasers. Sample reasoning:



I drew 6 circles to show the boxes and put 8 dots in each circle. Then, I counted the total number of dots to get 48. I knew  $5 \times 8$  is 40 and one more group of 8 would make 48.

4. a. 
$$\frac{\times 6 = 36, 6 \times \_\_\_}{36 \div 6} = 36, \text{ or}$$

b. 6 notebooks. Sample reasoning:



I drew groups of 6 until I got to 36. I was able to make 6 groups so 6 notebooks get sticky notes.

- "\_\_\_\_ wrote \_\_\_\_ and \_\_\_\_ wrote \_\_\_\_ to represent the same problem. How are those equations the same and different?" (One of the equations is a division equation, but the other equation is a multiplication equation with an unknown factor. They used different symbols for the unknown amount. Both symbols in the equation represent the missing \_\_\_\_ in the situation.)
- Have students share strategies they used to solve the problem.

## **Advancing Student Thinking**

If students don't find a solution to the problems, consider asking: "What is this problem about?" and "How could you represent the problem?"

# **Lesson Synthesis**

① 10 min

Display:  $24 \div 4 = ?$ 

"What would be the related multiplication equation?" ( $4 \times ? = 24$  or  $? \times 4 = 24$ )

"How are they related?" (The missing number in the division equation is the number of groups or the number in each group and that's what the missing number in the multiplication equation represents.)

Display:  $4 \times ? = 28$ 

"What would be the related division equation?" ( $28 \div 4 = ?$ )

"How are they related?" (The multiplication equation is missing the number in each group and that is what the quotient represents in the division equation.)

## **Suggested Centers**

- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

---- Complete Cool-Down ----

## **Response to Student Thinking**

Students write a multiplication equation and division equation to represent the situation, but don't explain their reasoning.

#### **Next Day Support**

 During the launch of the next day's activity, have students discuss the parts of each equation and how they represent the situation.

Unit 4 Lesson 7

# **Lesson 8: Relate Quotients to Familiar Products**

## **Standards Alignments**

Addressing 3.OA.B.6, 3.OA.C.7

#### **Teacher-facing Learning Goals**

• Identify known single-digit multiplication facts and their related division facts.

#### **Student-facing Learning Goals**

 Let's consider the products and quotients we know right away or can find quickly.

#### **Lesson Purpose**

The purpose of this lesson is for students to practice identifying multiplication facts within 100 and to use products they know to determine unknown quotients.

In this lesson, students check in on their progress towards fluent multiplication within 100 and sort their facts into categories. Then, students use the multiplication facts they know to generate related division facts. Knowing related facts will help students multiply and divide in future lessons.

#### Access for:

## Students with Disabilities

Representation (Activity 1)

# **3** English Learners

MLR8 (Activity 1)

#### **Instructional Routines**

Card Sort (Activity 1), Number Talk (Warm-up)

#### **Materials to Gather**

Materials from a previous activity: Activity 2

#### Materials to Copy

- Card Sort: Multiplication (groups of 2): Activity 1
- Card Sort: Multiplication Recording Sheet (groups of 2): Activity 1

## **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min

## **Teacher Reflection Question**

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure

Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

tomorrow to ensure each student's ideas are a part of the collective learning?

**Cool-down** (to be completed at the end of the lesson)

5 min

Multiplication and Division Facts

## **Standards Alignments**

Addressing 3.OA.B.6

## **Student-facing Task Statement**

Think about the multiplication facts that you know. How have they changed since the beginning of the year?

## **Student Responses**

Sample responses: I used to only know the 5s and 10s, but now I've used those facts to learn more. I only knew a few at the beginning of the year, but now I know a lot more.

-----Begin Lesson ------

Warm-up © 10 min

Number Talk: Multiplication and Division

# **Standards Alignments**

Addressing 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying and dividing within 100. These understandings help students develop fluency and identify division facts that are related to known products.

When students use the relationship between multiplication and division to find division facts they don't

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know, they are looking for and making use of structure (MP7).

#### **Instructional Routines**

Number Talk

#### **Student-facing Task Statement**

Find the value of each expression mentally.

- 4×10
- 40 ÷ 4
- 40 ÷ 10
- 60 ÷ 6

## **Student Responses**

- 40: I counted by 10, like 10, 20, 30, 40. I just knew it.
- 10: Since I know that  $4 \times 10 = 40$ , I know that 40 divided by 4 is 10, because 10 is the missing factor.
- 4: This is just like  $40 \div 4$ , but the other factor is missing, so the quotient would be 4.
- 10: Since I know that 6 times 10 is 60, I know that 60 divided by 6 would be 10, because 10 is the missing factor.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

## **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

## **Synthesis**

 "How do the first 3 expressions show that multiplication and division are related?" (The two division expressions both have one of the factors missing. In the first expression, the 10 is missing. In the second, the 4 is missing.)

Activity 1 © 20 min

Card Sort: Multiplication

# **Standards Alignments**

Addressing 3.OA.C.7

The purpose of this activity is for students to check-in on their progress towards fluent

multiplication within 100. Students work in groups of 2 to sort products into groups they know right away, can find quickly, or don't know yet. The launch provides time for a class discussion about what it means to know a fact quickly. Students identify five products with which they'd like to be more proficient, share their strategies, and practice finding the products they choose. The cards from this activity will be used in the next activity.

# Access for English Learners

MLR8 Discussion Supports. Synthesis: Display a sentence frame to support whole-class discussion: "The next time I multiply \_\_\_\_\_ and \_\_\_\_\_, I will . . . ."

Advances: Listening, Speaking

#### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

#### **Instructional Routines**

Card Sort

## **Materials to Copy**

Card Sort: Multiplication (groups of 2), Card Sort: Multiplication Recording Sheet (groups of 2)

## **Required Preparation**

- Create a set of cards from the Instructional master for each group of 2.
- The Multiplication Fact sort cards from this activity will be used again in the next activity.

# **Student-facing Task Statement**

Quiz your partner on their multiplication facts. Sort your partner's facts into one of these columns:

- 1. know it right away
- 2. can find it quickly
- 3. don't know it yet

Multiplication expressions I'm going to practice:

#### Launch

- Groups of 2
- "Today we're going to revisit the multiplication facts to see how many you've learned so far. Remember, though, you have the rest of the year to learn them."
- "We all know what it means to know a product right away, but what does it mean to know a product quickly?" (We can figure it out in a couple of seconds with a

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- 1.
- 2.
- 3.
- 4.
- 5.

## **Student Responses**

Answers vary.

- strategy. We can figure it out in less than 5 seconds.)
- Discuss as a class and come to an agreement about what it means to find a product quickly.
- Consider asking:
  - "Does anyone want to add on to what \_\_\_\_\_ says it means to find a product quickly?"
  - "Does anyone have different ideas about what it means to find a product quickly?"
  - "Based on this discussion, does anyone want to revise their ideas about what it means to find a product quickly?"
- Give each group one set of pre-cut cards and a sort table.

## **Activity**

- "Take some time to quiz each other on multiplication facts. As you quiz your partner, use the table to sort the expressions into three groups that show if they know it right away, they can find it quickly, or they don't know it yet."
- 7–10 minutes: partner work time
- "Choose 5 multiplication facts that you don't know yet and write down the expressions. These are the products you will practice finding."
- 1 minute: independent work time
- "Now, share the products you want to practice with your partner and have them help you think of some strategies you could use to find the products quickly."
- "After you have some strategies, take some time to practice finding the products you chose."
- 5–7 minutes partner work time

## **Synthesis**

 "What were some useful strategies for finding products you didn't know yet?" (Thinking of a product I already know and using that product to find the one I didn't know yet. Using products of 2, 5, and 10 to figure out other products.)

## **Advancing Student Thinking**

If students don't yet have a strategy for one of the facts they've chosen to practice, consider asking:

- "What have you tried so far to find this product?"
- "Could you check in with another group to see if they could suggest a strategy for finding this product?"

Activity 2 © 15 min

If I Know, Then I Know

# **Standards Alignments**

Addressing 3.OA.B.6, 3.OA.C.7

The purpose of this activity is for students to identify division facts that are related to multiplication facts that they know. Students complete "If I know, then I know" statements using their multiplication fact cards from the previous activity. Give students time, if needed, to determine the product before generating the related division equation. Some students may generate 4 related division equations for each product by moving the quotient to the left side of the equal sign. If this comes up, recognize that this is possible, but keep the emphasis on generating two related division facts, one for each of the factors as the unknown number.

When students use the relationship between multiplication and division identify two division facts from a multiplication fact, they look for and make use of structure (MP7).

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#### **Materials to Gather**

Materials from a previous activity

#### **Required Preparation**

Each group of 2 needs a set of cards from the previous activity.

#### **Student-facing Task Statement**

If I know  $4 \times 5 = 20$ , then I know \_\_\_\_\_.

- 1. Set the multiplication fact cards in a stack face down.
- 2. Take turns drawing a multiplication fact card.
- 3. Use the multiplication fact on the card to record a multiplication equation in the "If I know . . ." column.
- 4. Then, record related division equations in the "Then I know . . ." column.

If I know ,	then I know

# **Student Responses**

Sample responses:

#### Launch

- Groups of 2
- "Read the first statement of the activity.
   Talk with your partner about how you could finish the statement."
- 1 minute: partner discussion
- Share responses.

#### **Activity**

- "Now, you're going to take turns drawing a card and using the fact you chose to complete an 'If I know, then I know' statement with the multiplication fact that you drew and the related division facts. Take some time to figure out the multiplication fact together if you need to. After every turn, record the multiplication equation and related division equations in the table."
- 7–10 minutes: partner work time

#### **Synthesis**

"How many division equations were you able to come up with for each multiplication equation? Explain your reasoning." (2, because I could come up with 1 equation where the quotient was 1 factor and 1 equation where it was the other factor. 4, because I could come up with 1 for each of the factors being the quotient, and I could have the quotient on the right or the left of the equal sign.)

If I know ,	then I know
$3 \times 4 = 12$	$12 \div 4 = 3, \ 12 \div 3 = 4$
$2 \times 8 = 16$	$16 \div 2 = 8, 16 \div 8 = 2,$ $8 = 16 \div 2, 2 = 16 \div 8$

# **Lesson Synthesis**

(10 min

"Today we thought about multiplication facts that we know and worked on some that we don't know yet. How did this help you with finding division facts?" (We could use a multiplication fact to find related division facts. We realized that if we know a multiplication fact, then there are some division facts that we know too.)

## **Suggested Centers**

- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

## ---- Complete Cool-Down -----

# **Response to Student Thinking**

The work in this lesson builds from the multiplication concepts developed in a prior unit.

#### **Prior Unit Support**

Grade 3, Unit 1, Section B: From Graphs to Multiplication

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# **Lesson 9: Patterns in the Multiplication Table**

## **Standards Alignments**

Addressing 3.OA.C.7, 3.OA.D.9

#### **Teacher-facing Learning Goals**

- Identify arithmetic patterns in the multiplication table and use them to find unknown multiplication facts.
- Recognize that multiplication is commutative.

## **Student-facing Learning Goals**

• Let's find patterns in the multiplication table and use them to multiply.

#### **Lesson Purpose**

The purpose of this lesson is for students to identify and explain patterns in the multiplication table.

Students may have worked with the multiplication table in an optional lesson in a previous unit. In this lesson, they observe patterns and structures in the multiplication table that highlight properties of multiplication and are helpful for multiplying numbers. Although there is an opportunity to highlight multiple properties, the focus of this lesson is the commutative property (though students are not expected to name the property). Students notice that multiplying two numbers in any order gives the same product and make use of this observation to find unknown products (MP8).

#### Access for:

#### Students with Disabilities

Action and Expression (Activity 2)

# **3** English Learners

MLR2 (Activity 1)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

#### **Teacher Reflection Question**

What surprised you about student thinking in the first activity?

Lesson Synthesis	10 min
Cool-down	5 min

# **Cool-down** (to be completed at the end of the lesson)

© 5 min

Find the Missing Product

# **Standards Alignments**

Addressing 3.OA.D.9

## **Student-facing Task Statement**

What number should replace the question mark? Explain or show your reasoning.

×	1	2	3	4	5	6	7	8	9	10
1	1									
2	2	4								
3	3	6	9							
4	4	8	12	16				?		
5	5	10	15	20	25					
6	6	12	18	24	30	36				
7	7	14	21	28	35	42	49			
8	8	16	24	32	40	48	56	64		
9	9	18	27	36	45	54	63	72	81	
10	10	20	30	40	50	60	70	80	90	100

# **Student Responses**

32. Sample responses: The table shows that  $4 \times 8$  is 32, and I know that  $8 \times 4$  has the same value as  $4 \times 8$ , so it is also 32. I know that  $4 \times 4$  or 4 groups of 4 is 16. I added another 16 to get  $8 \times 4$ , and 16 + 16 = 32.

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Warm-up 
© 10 min

Notice and Wonder: Multiplication Table

#### **Standards Alignments**

Addressing 3.OA.D.9

The purpose of this warm-up is to elicit the idea that the product of two factors on the multiplication table is found where the row and column of each factor intersect. While students may notice and wonder many things about these products, the patterns in the multiplication table and how the table is structured are the important discussion points.

#### **Instructional Routines**

Notice and Wonder

#### **Student-facing Task Statement**

What do you notice? What do you wonder?

×	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

## Student Responses

Students may notice:

- The numbers in the top row and the leftmost column are factors.
- The product is lined up with a factor on the top and a factor to the left.
- The row that starts with 5 counts by 5 as you move to the right, like 5, 10, 15, etc.
- The column that starts with 3 counts by 3 as you move down the column, like 3, 6, 9, etc.
- There are many patterns in the table.

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

- If not mentioned in students' responses, explain: "A multiplication table uses rows and columns to show products of two numbers. The numbers in the leftmost column and the top row are factors."
- "Each number in the (non-shaded part of the) table is the result of multiplying the two factors in the same row and column as that number."
- "What are some patterns that you see in the multiplication table and why do they work?"

 The table is like some tables we saw before, but those tables had to do with addition.

#### Students may wonder:

- What do the numbers on the table mean?
- How does the table work?
- Why are the numbers smaller in the top left part of the table, then larger in the bottom right part of the table?
- (As we move right on the 3s row or down in the 3s column, the products increase by 3, because we are adding groups of 3. The number 15 appears in two places because we can find  $3 \times 5$  or  $5 \times 3$  to get 15. We see 12 in two places in the table because we can get 12 by counting by 3 like 3, 6, 9 12 or counting by 4 like 4, 8, 12.)
- "Find all the places where 20 appears. Which pairs of factors multiply to 20?" (4 and 5)

Activity 1 © 20 min

Products in the Table

## **Standards Alignments**

Addressing 3.OA.D.9

The purpose of this activity is for students to apply multiplication strategies based on properties of operations to find products in a multiplication table. While students may use various strategies based on properties of operations, look for opportunities to highlight strategies based on the commutative property. Students consider how known products that are already in the table can help find an unknown product in the multiplication table.

When students use a multiplication fact that they know to determine a multiplication fact that they don't know, they look for and make use of structure (MP7).

# **3** Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they find the missing products on the table and describe the strategies they used. On a visible display, record words and phrases such as: add one more group, the same factors, switch the order, take one group away, double. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

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## **Student-facing Task Statement**

Here is a partially completed multiplication table.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5					
2	2	4	6	8	10		Α			
3	3	6	9	12	15				В	
4	4	8	12	16	20	С				
5	5	10	15	20	25			D		
6	6	12	18	24	30					Ш
7	7	14	21	28	35		F			
8	8	16	24	32	40	48			G	
9	9	18	27	36	45	54	63			
10	10	20	30	40	50	60	70	80		

- Use the products in the table to help you find the numbers that should replace letters A-G. Be prepared to explain your reasoning.
- 2. Find the number that should go in three other empty cells in the table. Use:
  - a. 7 as a factor
  - b. 9 as a factor
  - c. 10 as a factor

Be prepared to explain your reasoning.

# **Student Responses**

- 1. A: 14 D: 40 F: 49 B: 27 E: 60 G: 72 C: 24
- 2. Sample responses:
  - a.  $7 \times 4 = 28$
  - b.  $9 \times 4 = 36$
  - c.  $10 \times 10 = 100$

#### Launch

- Groups of 2
- "We'll work with another multiplication table in this activity. How is this table different from the first table we saw?" (It has more products than the first table. It doesn't have all of the products in it. Some of the boxes have letters in them.)
- 1 minute: quiet think time
- Share responses.

## **Activity**

- "Use the numbers in the table to help you find the numbers that should replace the letters A-G. Think about how the numbers that are already in the table might help."
- "Afterwards, find numbers that should go in three other empty cells in the table. Be prepared to explain your reasoning."
- 5–7 minutes: independent work time
- "Share with your partner how you found the missing numbers in the table."
- 3–5 minutes: partner discussion
- Monitor for students who:
  - use  $7 \times 2$ , which is in the table, to find  $2 \times 7$  or A
  - add one more group of 4 to 20 to find C
  - use a product from the 9s row to find a product in the 9s column

## **Synthesis**

 Select previously identified students to share how they used the numbers that were in the table to find unknown products. If possible, display and annotate the table to illustrate students' reasoning. Activity 2 © 15 min

If I Know, Then I Know: Multiplication

## **Standards Alignments**

Addressing 3.OA.C.7

The purpose of this activity is for students to articulate how they use known products to find unknown products, using a structure similar to that used in an earlier lesson. Students may describe strategies that are based on any property of operations. The focus should be on the description of the strategy (such as "multiplying two numbers in any order gives the same product") rather than remembering the property on which the strategy is based (such as "commutative property").

#### Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Synthesis: Identify connections between strategies that result in the same outcomes but use differing approaches. Supports accessibility for: Memory, Conceptual Processing

# **Student-facing Task Statement**

In each row, write down at least two
multiplication facts you can figure out
because you know the given multiplication
fact in the left column. Be prepared to share
your reasoning.

If I know ,	then I also know
$2 \times 4$	$4 \times 2$ , $4 \times 4$ , $2 \times 8$
$3 \times 5$	
$4 \times 10$	
$7 \times 2$	
$5 \times 8$	

2. If time permits, complete the rest of the multiplication table. Use the multiplication facts you know to find those you don't know.

#### Launch

• Groups of 2

#### **Activity**

- "In the right column, work independently to write down at least two multiplication facts you can figure out because you know the given multiplication fact in the left column."
- 3–5 minutes: independent work time
- "Now, share the facts that you found with your partner. Record any facts that your partner found that you didn't find. Be sure to explain your reasoning."
- 3–5 minutes: partner work time

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## **Student Responses**

1. Sample responses:

If I know ,	then I also know
2 × 4	$4 \times 2, 4 \times 4, 2 \times 8$
$3 \times 5$	$6 \times 5$ , $5 \times 3$ , $4 \times 5$
$4 \times 10$	$10 \times 4, 9 \times 4, 8 \times 10$
$7 \times 2$	$7 \times 4$ , $8 \times 2$ , $14 \times 2$
5 × 8	$6 \times 8, 4 \times 8, 8 \times 8$

2. Students complete the multiplication table from the first activity.

#### **Synthesis**

 For each given product, invite 1–2 students to share the products they found and how they were related to the given product.

# **Lesson Synthesis**

① 10 min

"Today we used products that we knew to find products that we didn't know."

"What patterns did you find helpful?" (We can write the factors in any order, the result is still the same, like  $3 \times 6$  has the same value as  $6 \times 3$ . If we know  $3 \times 5$  is 15 and 6 is  $2 \times 3$ , then  $6 \times 5$  is twice  $3 \times 5$  or  $2 \times (3 \times 5)$ , or twice 15, which is 30. We can find the value of  $8 \times 2$  by thinking of 8 as 3 + 5 and then finding  $3 \times 2$  and  $5 \times 2$ . When 2, 4, 6, 8, and 10 is a factor, the product is even. When 5 is a factor, the product alternates between 5 and 10. When 10 is a factor, the product ends in 0.)

Record the patterns students noticed.

## **Suggested Centers**

- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

# **Response to Student Thinking**

Students find the product of 4 and 8 by drawing a discrete diagram or counting one-by-one.

## **Next Day Support**

• Before the next day's warm-up, have students discuss which facts on the table could be used to find  $4 \times 8$ .

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# Lesson 10: Explore Multiplication Strategies with Rectangles

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

#### **Teacher-facing Learning Goals**

• Use area diagrams to explore strategies based on properties of multiplication.

#### **Student-facing Learning Goals**

 Let's use rectangles to explore multiplication strategies.

#### **Lesson Purpose**

The purpose of this lesson is for students to use area diagrams to explore multiplication strategies based on properties of operations.

Previously, students examined patterns in the multiplication table and used them to find products within 100 and to notice properties of multiplication—the commutative property, in particular. In this lesson, they analyze strategies for finding the area of rectangles to explore distributive and associative properties. They study gridded rectangles that have been decomposed into smaller parts and expressions that represent how the decomposition could help us find the area. Students see how the strategies, along with the diagrams and the expressions that represent them—can help us find the product of two numbers.

As students make sense of expressions and interpret them in terms of parts of area diagrams (MP1), they practice reasoning quantitatively and abstractly (MP2).

#### Access for:

Students with Disabilities

• Engagement (Activity 2)

# **3** English Learners

MLR8 (Activity 2)

#### **Instructional Routines**

How Many Do You See? (Warm-up), MLR2 Collect and Display (Activity 1)

#### Materials to Gather

Colored pencils, crayons, or markers:

#### Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

## **Teacher Reflection Question**

Reflect on times you observed students listening to one another's ideas today in class. What norms would help each student better attend to their classmates' ideas in future lessons?

# **Cool-down** (to be completed at the end of the lesson)

© 5 min

Mark or Shade Parts to Find Area

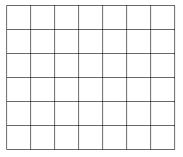
## **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

# **Student-facing Task Statement**

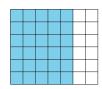
Here is a rectangle whose area can be found by finding  $6 \times 7$ .

- 1. Mark or shade the rectangle to show that we can write  $2 \times (3 \times 7)$  or  $(6 \times 5) + (6 \times 2)$  to find its area.
- 2. What is the value of  $6 \times 7$ ? Explain or show your reasoning.



# **Student Responses**

1. Sample responses:



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2. 42. I know that  $6 \times 5$  is 30 and  $6 \times 2$  is 12, and 30 + 12 = 42.

----- Begin Lesson

Warm-up © 10 min

How Many Do You See: Squares

## **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

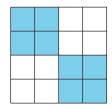
The purpose of this How Many Do You See is for students to use grouping strategies to describe the quantities they see.

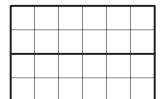
#### **Instructional Routines**

How Many Do You See?

# **Student-facing Task Statement**

How many do you see? How do you see them?



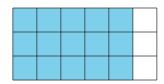


#### Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

## Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.
- Keep the images displayed for the launch of the next activity.



#### **Student Responses**

- 16 squares: I see 4 groups of 4. I see 4 columns and 2 groups of 2 in each column. I see 2 x 4 or 8 blue squares and 2 x 4 or 8 white squares.
- 24 squares: I see 2 groups of 12. I see 4 rows with 6 in each row.
- 18 squares: I see 5 columns of 3 and then 1 more column of 3. I see 6 groups of 3.

## **Synthesis**

- "How can we use amounts that we can see quickly to find the total number of squares?" (We can look for repetition of the number of squares that we can easily see. We can add to or multiply the number of squares we can easily see.)
- Consider asking:
  - "Who can restate the way \_\_\_\_\_ saw the squares in different words?"
  - "Did anyone see the squares the same way but would explain it differently?"
  - "Does anyone want to add an observation to the way \_\_\_\_\_ saw the squares?"

Activity 1 © 20 min

From Diagrams to Expressions

# **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

The purpose of this activity is for students to analyze different ways of decomposing a gridded rectangle to find the total number of squares in a rectangle. For example, they see that the area of a rectangle that is 3 units by 6 units can be found adding  $3 \times 5$  and  $3 \times 1$  and relate that strategy to the expression  $(3 \times 5) + (3 \times 1)$ . The area can also be found by decomposing the rectangle into two halves or finding  $3 \times 3$  twice, which is represented by  $2 \times (3 \times 3)$ .

The reasoning here allows students to visually make sense of strategies for multiplication that are based on the associative and distributive properties of multiplication. The focus is not on naming the properties, but rather on interpreting the expressions and relating them to the quantities in the diagrams (MP7).

This activity uses MLR2 Collect and Display. Advances: conversing, reading, writing

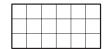
Unit 4 Lesson 10

#### **Instructional Routines**

MLR2 Collect and Display

## **Student-facing Task Statement**

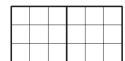
Andre and Elena are finding the area of this rectangle.



Andre writes  $6 \times 3$ .

He marks the rectangle like this:

He then writes:

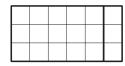


 $2 \times (3 \times 3)$  $2 \times 9 = 18$ 

Elena writes  $3 \times 6$ .

She marks the rectangle like this:

She then writes:

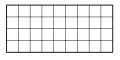


 $3 \times (5+1)$   $(3 \times 5) + (3 \times 1)$ 15+3

18

## 1. Discuss with a partner:

- a. How are Andre and Elena's strategies alike? How are they different?
- b. How are the numbers in Andre's expressions related to his diagram?
- c. How are the numbers in Elena's expressions related to her diagram?
- 2. Here is another rectangle.



Its area can be found by finding  $4 \times 9$ .

- a. Mark or shade the rectangle in a way that would help you find its area.
- b. Write one or more expressions that can represent your work on the

#### Launch

- Groups of 2
- Display the first problem.
- "Take a minute to make sense of how Andre and Elena found the area of a rectangle."
- 1 minute: quiet think time

## **Activity**

- "Work with your partner to discuss how their strategies are alike and different, and how the numbers in each of their expressions relate to their diagrams."
- 5–7 minutes: partner work time
- Share responses.
- "Work with your partner to complete the second problem."
- 3–5 minutes: partner work time

#### **MLR2 Collect and Display**

- Circulate, listen for and collect ways that students decompose the rectangle and the language students use to describe the strategies they used. Listen for: decomposed, smaller parts, smaller rectangles,  $2 \times 2 \times 9$ ,  $2 \times 18$ ,  $4 \times 5 + 4 \times 4$ , and  $5 \times 4 + 4 \times 4$ .
- Record students' diagrams, words, and phrases on a visual display and update it throughout the lesson.

#### **Synthesis**

 "Are there any other words or phrases that are important to include on our display?"

diagram and show how you find the area.

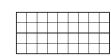
#### **Student Responses**

- 1. Sample responses:
  - a. Alike: They both decomposed their diagrams and wrote new expressions. Different: They wrote different expressions to start with. Andre decomposed his diagram into two equal squares. He wrote new expressions with only multiplication. Elena decomposed her rectangle into two different-size rectangles. She wrote new expressions with addition and multiplication.
  - b. Andre: The numbers in  $2 \times 3 \times 3$  refer to the 2 large squares that are 3 by 3 each. The numbers in  $2 \times 9$  refer to the 2 large squares with 9 unit squares in each.
  - c. Elena: The 3 in  $3 \times (5+1)$  refers to the shorter side of 3 units and the 5+1 refers to the longer side decomposed into 5 units and 1 unit. The  $3 \times 5$  and  $3 \times 1$  refer to the area of the smaller rectangles she created.
- 2. Sample responses:

a.



a.



b.  $4 \times (5 + 4)$  or  $(4 \times 5) + (4 \times 4)$ , which is 20 + 16 or 36

b.  $2 \times (2 \times 9)$  or  $2 \times 18$ , which is 18 + 18 or 36.

- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.

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#### **Advancing Student Thinking**

If students count one-by-one to find the area if the rectangle in the second problem, consider asking:

- "How did you find the area of the rectangle?"
- "How could you use a product you already know to find the area of the rectangle? How could you show your strategy on the rectangle?"

Activity 2 © 15 min

From Expressions to Diagrams

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

In this activity, students are given expressions that represent strategies for finding the area of rectangles. The strategies are based on the distributive property and the associative property of multiplication. Students interpret the expressions by marking or shading area diagrams and connect each expression to the product of two factors (MP2). For instance, they see that to find the value of  $2 \times (2 \times 6)$  is to find the value of  $4 \times 6$  or  $6 \times 4$ .

## Access for English Learners

MLR8 Discussion Supports: Synthesis: Create a visual display of the diagrams. As students share their strategies, annotate the display to illustrate connections. For example, trace the area showing 5 columns of 3, and write  $5 \times 3$ .

Advances: Speaking, Representing

## Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select at least 2 of the 3 problems to complete.

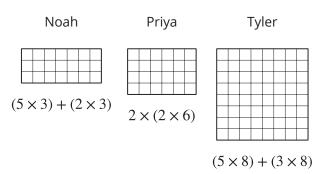
Supports accessibility for: Organization, Attention, Social-emotional skills

#### **Materials to Gather**

Colored pencils, crayons, or markers

## **Student-facing Task Statement**

Here are some rectangles and expressions that show how three students saw the area of the rectangles.



For each rectangle:

- 1. Name the two factors that can be multiplied to find its area.
- 2. Mark or shade each rectangle to show how each student saw the area. Be prepared to explain your reasoning.

## **Student Responses**

Noah	Priya	Tyler
1. $3 \times 7$ or $7 \times 3$	1. $4 \times 6$ or $6 \times 4$	1. 8 × 8 2.
2.	2.	

#### Launch

- Groups of 2
- "Take a minute to read the directions of the activity. Then, talk to your partner about what you are asked to do."
- 1 minute: quiet think time
- 1 minute: partner discussion
- Answer any clarifying questions from students.
- Give students access to colored pencils, crayons, or markers.

## **Activity**

- "Mark or shade each diagram to represent how each student found the area."
- 3–5 minutes: independent work time
- "Share with your partner how you used the rectangles to show each expression."
- 3–5 minutes: partner discussion

## **Synthesis**

- "What are the two factors you can multiply to find the area of Noah's rectangle?" (7 and 3)
- "How are those numbers related to the expression that he wrote:  $(5 \times 3) + (2 \times 3)$ ?"  $(7 \times 3)$  is 21. Finding  $5 \times 3$ , which is 15, then adding  $2 \times 3$ , which is 6, also gives 21.)
- "Where do you see the two factors in his expression?" (The 7 is 5 and 2 combined. The 3 is in the  $5 \times 3$  and  $2 \times 3$ .)
- Repeat the line of questioning with Priya's rectangle and expression.

# **Lesson Synthesis**

① 10 min

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"Today, we used diagrams to find the area of rectangles with certain side lengths. We decomposed the rectangles in different ways and wrote different expressions."

"What were some strategies for decomposing the rectangles to find their areas?" (Partition one side into smaller parts and find the area of smaller rectangles within the original one. Partitioning the rectangle into two halves and finding the area of each half and then doubling it.)

"How might these strategies help us multiply two numbers?" (They show that we can break apart or decompose one of the numbers and multiply smaller numbers and then combine the results. Using diagrams and writing expressions can help us see and record the parts.)

## **Suggested Centers**

- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

## **Response to Student Thinking**

Students find the total number of squares in the rectangles, but don't mark or shade the rectangle to represent one of the given expressions.

#### **Next Day Support**

 Before the warm-up, pass back the cooldown and have students discuss how they could represent each of the given expressions by marking or shading parts of the rectangular area.

# Lesson 11: Multiplication Strategies on Ungridded Rectangles

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

## **Teacher-facing Learning Goals**

- Apply associative and distributive properties of multiplication to find products within 100.
- Recognize that multiplication is associative and can be distributed over addition.

#### **Student-facing Learning Goals**

 Let's use different strategies to find the area of ungridded rectangles.

#### **Lesson Purpose**

The purpose of this lesson is for students to represent multiplication strategies on an ungridded rectangle.

Previously, students used gridded rectangles to represent strategies based on the distributive and associative properties. Here, they use the same strategies, but represent them on an area diagram without a grid. Then, students match expressions that could represent the area of the same rectangle, without using diagrams. The reasoning helps students work toward fluent multiplication within 100.

This lesson has a Student Section Summary.

#### Access for:

#### Students with Disabilities

• Engagement (Activity 2)

## English Learners

MLR2 (Activity 1)

#### Instructional Routines

Card Sort (Activity 2), Which One Doesn't Belong? (Warm-up)

#### **Materials to Copy**

 Card Sort: Different Expressions, Same Rectangle (groups of 2): Activity 2

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• Centimeter Grid Paper - Standard (groups of 2): Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

## **Teacher Reflection Question**

Which students came up with an unexpected strategy in today's lesson? What are some ways you can be more open to the ideas of each and every student?

# **Cool-down** (to be completed at the end of the lesson)

© 5 min

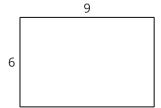
Expressions for a Rectangle

## **Standards Alignments**

Addressing 3.OA.C.7

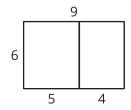
## **Student-facing Task Statement**

- 1. Mark or shade this rectangle to show a strategy for finding its area.
- 2. Write one or more expressions that represent how you find the area.



# **Student Responses**

- 1. Sample response:
- 2.  $(6 \times 5) + (6 \times 4)$



#### ---- Begin Lesson -----

Warm-up © 10 min

Which One Doesn't Belong: Multiplication in Many Forms

#### **Standards Alignments**

Addressing 3.OA.C.7

This warm-up prompts students to compare four representations of multiplication. It gives students a reason to use language precisely as they talk about characteristics of the items being compared. During the synthesis, ask students to explain the meaning of any terminology they use, such as strategies, area, and parts.

#### **Instructional Routines**

Which One Doesn't Belong?

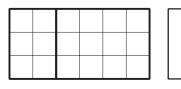
## **Student-facing Task Statement**

Which one doesn't belong?

Α

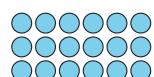
В

D



2 4

**c** (3 × 2) + (3 × 4)



## **Student Responses**

- A is the only one that shows square units.
- B is the only one that doesn't have 3 represented as a number or countable objects.

#### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

## **Activity**

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

## **Synthesis**

 "What number do the diagrams and the expression in C represent?" (18) "How do you know?" (There are 18 dots in the array. There are 18 squares in the rectangle. If I add up the parts of the expression or the parts of the rectangle, I get 18.)

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- C is the only one that isn't a diagram.
- D is the only one that doesn't show  $3 \times 2$  or  $3 \times 4$  separately.
- "What might be the length of the unlabeled side of the rectangle in B? How do you know?"
  (3, because the rectangle is the same one as in A, just not showing a grid. 3, because 3 × 2 = 6 and 3 × 4 = 12.
- Consider asking:
  - "Let's find at least one reason why each one doesn't belong."

Activity 1 © 15 min

Mark, then Express

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

The purpose of this activity is for students to find the area of ungridded rectangles using strategies based on the distributive and associative properties. Students represent these strategies on rectangles with no grid. This will be helpful in future lessons as students use area diagrams to represent the multiplication of larger numbers.

## **3** Access for English Learners

*MLR2 Collect and Display.* Direct attention to words collected and displayed from the previous lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Reading, Representing, Conversing

#### **Student-facing Task Statement**

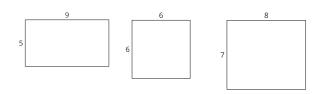
For each rectangle:

- Mark or shade each rectangle to show a strategy for finding its area.
- Write one or more expressions that can represent how you find the area.

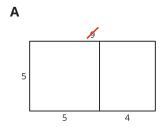
A B C

#### Launch

- Groups of 2
- "We are going to find the area of more rectangles. How are these rectangles different from the rectangles we worked with in the last lesson?" (They don't have a grid in them. We can't see the squares.)
- 30 seconds: quiet think time



#### **Student Responses**



 $5 \times (5 + 4)$  or  $(5 \times 5) + (5 \times 4)$ , which is 25 + 20 or 45.

6 6 3  $2 \times (3 \times 6)$  or  $2 \times 18$ , which is 36.

7

 $4 \times (7 \times 2)$ , which is  $4 \times 14$  or 56.

Share responses.

## **Activity**

- "Mark or shade each rectangle to help you find its area. Then write one or more expressions that represent your work and show how you found the area."
- 5–7 minutes: independent work time
- "Share how you found the area of each rectangle with your partner. Be sure to ask and answer any questions you have about your partner's strategy."
- 3–5 minutes: partner discussion

#### **Synthesis**

 "How was showing your strategy on a rectangle with no grid different than showing your strategy on a rectangle with a grid?" (I just estimated where I thought I should split the rectangle. I was thinking more about the numbers than counting all the squares.)

## **Advancing Student Thinking**

If students say they aren't sure where to mark or shade the rectangle because they can't see the squares, consider asking:

- "What numbers are you multiplying to find the area?"
- "How could you decompose one of the factors to help you find the product? How would you show that on the diagram?"

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Activity 2 © 20 min

Card Sort: Different Expressions, Same Rectangle

#### **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.C.7

In this sorting activity, students identify expressions that could represent the area of the same rectangle and explain their reasoning. To do so, they apply their understanding of properties of multiplication and draw rectangles as needed as they interpret parts of the expressions. Some students may sort expressions based only on the value of the expressions. Encourage them to explain or show how they know, for instance, that  $8 \times 6$  and  $3 \times 6 + 5 \times 6$  can represent the area of the same rectangle (MP2, MP7). Some of the expressions from this activity are used in the synthesis to highlight the commutative, distributive, and associative properties of multiplication.

#### **1** Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Attention, Focus

#### **Instructional Routines**

Card Sort

## **Materials to Copy**

Card Sort: Different Expressions, Same Rectangle (groups of 2), Centimeter Grid Paper -Standard (groups of 2)

# **Required Preparation**

• Create a set of cards from the Instructional master for each group of 2 or 4.

## **Student-facing Task Statement**

Your teacher will give you a set of cards with expressions that represent areas of rectangles.

#### Launch

- Groups of 2 or 4
- Give each group a set of pre-cut cards from

Sort the expressions into groups so that the expressions in each group can represent the area of the same rectangle. Be prepared to explain your reasoning.

You can draw rectangles if you find them helpful.

A	В	c	D
$(7 \times 2) \times 2$	6×7	7×4	8×3
E	F	G	н
$3\times 6 + 5\times 6$	$(4 \times 3) \times 2$	4 × (2 × 3)	4×9
E	J	K	L
$(5 \times 6) + (2 \times 6)$	$2 \times (2 \times 9)$	8×6	$(5 \times 4) + (2 \times 4)$

#### **Student Responses**

Sample responses:

- A, C, and L (28 square units, 7 units by 4 units)
- B and I (42 square units, 6 units by 7 units)
- D, F, and G (24 square units, 8 units by 3 units)
- E and K (48 square units, 6 units by 8 units)
- J and H (36 square units, 9 units by 4 units)

the Instructional master.

Give students access to grid paper.

#### **Activity**

- "This set of cards includes expressions that represent areas of rectangles. Group together expressions that can represent the area of the same rectangle."
- "Work with your partner to explain your sorting decisions. You can draw rectangles if you find them helpful."
- 8 minutes: partner work time

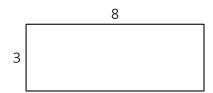
#### **Synthesis**

- Invite students to share their sorting results, drawings (if any), and explanations on how they know those expressions go together.
- Record each group of expressions. Discuss the connections between the expressions, illustrating them on a drawing of a rectangle. For instance, for

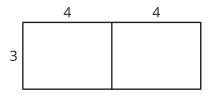
$$8 \times 3$$
  
 $(4 \times 3) \times 2)$   
 $4 \times (2 \times 3)$ 

ask questions such as:

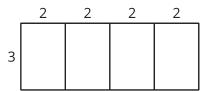
- "Where do we see the 8 in  $4 \times (2 \times 3)$ ?"
- "What's the area of this rectangle? What could its side lengths be?"
- o (Draw and label a rectangle.)



• "Where do we see the  $(4 \times 3) \times 2$  in the rectangle?"



• "Where do we see the  $4 \times (2 \times 3)$  in the rectangle?"



# **Lesson Synthesis**

① 10 min

"Today we matched expressions that could represent the same rectangle. Let's think about what some of the matching expressions show us about multiplication."

"What expressions show us that we can decompose one of the factors, then multiply them separately?" (E and K, C and L, B and I)

Display the expressions on cards F and G.

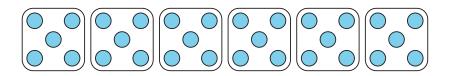
"What do these expressions show us about multiplication?" (When there are more than 2 factors, we can decide which two factors to multiply first without changing the result).

#### **Suggested Centers**

- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Addressing)

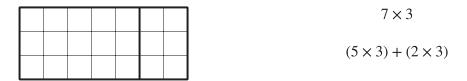
# **Student Section Summary**

In this section, we learned how multiplication and division are related.



$$6 \times 5 = ?$$
  $30 \div 5 = ?$   $30 \div 6 = ?$ 

We used strategies to multiply and divide and worked towards fluent multiplication and division within 100.



## 

## **Response to Student Thinking**

Students find the area of the rectangle, but don't record an expression that represents their strategy.

#### **Next Day Support**

 Before the warm-up, highlight a strategy for finding the area on the rectangle, and discuss how to write expressions that would represent the strategy.

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# **Section C: Multiplying Larger Numbers**

# **Lesson 12: Multiply Multiples of Ten**

#### **Standards Alignments**

Addressing 3.NBT.A.3

#### **Teacher-facing Learning Goals**

 Multiply one-digit whole numbers by multiples of 10 using strategies based on place value and the properties of operations.

#### **Student-facing Learning Goals**

 Let's multiply one-digit numbers times multiples of 10.

## **Lesson Purpose**

The purpose of this lesson is for students to multiply one-digit numbers by multiples of 10.

The work of this lesson connects to previous work because students have used strategies based on properties of operations to multiply within 100. Now, students extend this work and consider place value to multiply one-digit numbers by multiples of 10. Students complete a problem in context in which they explore how 180 can be grouped into multiples of ten in different ways. Students analyze two strategies for multiplying a single-digit number by a multiple of ten, then complete similar problems using the strategy of their choice. Throughout the lesson the associative property is used as a strategy to think of problems like  $3 \times 60$  as 18 tens or  $18 \times 10$ .

When students decompose multiples of ten in different ways as a strategy to multiply, they are looking for and making use of structure (MP7).

#### Access for:

**③** Students with Disabilities

**3** English Learners

Engagement (Activity 1)

MLR8 (Activity 1)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

#### **Materials to Gather**

• Base-ten blocks: Activity 1, Activity 2

#### **Materials to Copy**

- Centimeter Grid Paper Standard (groups of 2): Activity 1
- Centimeter Grid Paper Standard (groups of 2): Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

#### **Teacher Reflection Question**

Students have used place value in past lessons to add and subtract. In this lesson, how do they begin using place value as a strategy to multiply multiples of 10?

# **Cool-down** (to be completed at the end of the lesson)

3 5 min

What's the Value?

## **Standards Alignments**

Addressing 3.NBT.A.3

## **Student-facing Task Statement**

Find the value of  $6 \times 40$ . Explain or show your reasoning.

## **Student Responses**

240. Sample response: I decomposed 40 into  $4 \times 10$ , then multiplied  $6 \times 4$  to get 24. Twenty-four tens is 240.

## -----Begin Lesson ------

Warm-up © 10 min

Notice and Wonder: Tens

#### **Standards Alignments**

Addressing 3.NBT.A.3

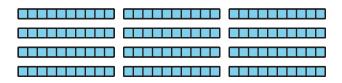
The purpose of this warm-up is to elicit the idea that 3 groups of 40 can also be seen as 12 groups of 10, which will be useful when students multiply one-digit whole numbers by multiples of 10 in a later activity. While students may notice and wonder many things, seeing that the total can be decomposed into rows of 30 and further decomposed into units of 10 are the important discussion points.

#### **Instructional Routines**

Notice and Wonder

#### **Student-facing Task Statement**

What do you notice? What do you wonder?



## **Student Responses**

Students may notice:

- These are diagrams of base-ten blocks.
- There are 10 squares (or ones) in each rectangle or block.
- There are 4 rows and 3 columns of tens.
- There are 3 tens in each row.
- There are 30 in each row.
- I could skip count by 30 to find the total.
- There are 12 groups of ten.

Students may wonder:

- How many squares (or ones) are in each section?
- How many squares are in each row?

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

- "What is the value the diagram represents?" (120)
- "How could noticing groups of ten help us find the total number of squares?" (There are 3 groups of 4 tens, which is 12 tens. There are 4 groups of 30, which is 12 tens. We could count by tens to find the total. We know 12 tens would be 120.)
- Record equations that reflect student thinking such as  $3 \times 4 \times 10 = 12 \times 10$  and  $4 \times 30 = 12 \times 10$ .

How many squares are there in all?

Activity 1 © 15 min

A Whole Lot of Dollars

#### **Standards Alignments**

Addressing 3.NBT.A.3

The purpose of this activity is for students to work with products of whole numbers and multiples of 10 in a concrete and familiar context before reasoning more abstractly about them. Given some numbers of dollar bills (for instance, four \$20 bills), students write expressions to represent the amount (4  $\times$  20) and then find its value using strategies that make sense to them. For example, they may count by 20 four times, think of \$20 in terms of two \$10 bills and find  $4 \times 2 \times 10$  (or  $8 \times 10$ ). Consider giving students access to play money, if available, to help them visualize the quantities and support their reasoning.

The reasoning here prompts students to use strategies based on place value and properties of operations (especially the associative property). It prepares students to work more flexibly with products involving factors and multiples of 10 in which the product is greater than 100.

## Access for English Learners

*MLR8 Discussion Supports*. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

#### **@** Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Check in and provide each group with feedback that encourages collaboration and community. For example, ensuring each member of the group has a chance to share their solution and thinking.

Supports accessibility for: Social-Emotional Functioning

#### **Materials to Gather**

## **Materials to Copy**

Base-ten blocks

Centimeter Grid Paper - Standard (groups of 2)

#### **Student-facing Task Statement**

Six friends are playing a board game that uses play money. The paper bills come in \$5, \$10, \$20, \$50, and \$100.

1. Every player received \$100 to start. Which of the following could be the bills that a player received?

Write an expression to represent the play bills and the amount in dollars.

bills	expression	dollar amount
one \$100 bill		
four \$20 bills		
ten \$10 bills		
ten \$5 bills		
five \$20 bills		
twenty \$10 bills		
twenty \$5 bills		
two \$50 bills		

- 2. At one point in the game, Noah had to pay Lin \$150. He gave her that amount using the same type of bill.
  - a. Which bill and how many of it could Noah have used to make \$150? Name all the possibilities.
  - b. Write an expression for each way that Noah could have paid Lin.
- 3. The table shows what the players had at the end of the game. The person with the most money wins. Who won the game?

Write an expression to represent the bills each person has and the amount in dollars.

player	bills	expression	dollar amount
	nine \$10 bills		
Andre	and ten \$5		
	bills		

#### Launch

- Groups of 2
- "We're going to solve a problem about a game that involves play money. What do you know about games that involve play money?"
- 1 minute: quiet think time
- Share responses.
- Give students access to base-ten blocks, grid paper, and play money, if available.

#### **Activity**

- "Work with your partner to complete the problems."
- 7-10 minutes: partner work time
- In the last problem monitor for students who use the following strategies to highlight in the synthesis:
  - Count by multiples of 10 to find a total, such as 50, 100, 150, 200, 250, 300.
  - Use place value to find a total, such as knowing that 14 tens is 10 tens or 100, and 4 more tens or 40, which makes 140.

#### **Synthesis**

- Invite students to share different combinations of the same bill that could be used to make \$150. Record and display expressions for each combination.
- Select previously identified students to share their strategies for how they found one of the totals in the last problem.

player	bills	expression	dollar amount
Clare	fourteen \$10 bills		
Jada	ten \$10 bills and three \$50 bills		
Lin	eight \$20 bills		
Noah	six \$50 bills		
Tyler	twenty-one \$10 bills		

# **Student Responses**

1.	bills	expression	dollar amount
	one \$100 bill	$1 \times 100$	100
	four \$20 bills	$4 \times 20$	80
	ten \$10 bills	$10 \times 10$	100
	ten \$5 bills	$10 \times 5$	50
	five \$20 bills	$5 \times 20$	100
	twenty \$10 bills	$20 \times 10$	200
	twenty \$5 bills	$20 \times 5$	100
	two \$50 bills	$2 \times 50$	100

- 2. a. Three \$50 bills, fifteen \$10 bills, and thirty \$5 bills
  - b.  $3 \times 50$ ,  $15 \times 10$ , and  $30 \times 5$
- 3. Noah won the game.

player	bills	expression	dollar amount
Andre	nine \$10 bills and ten \$5 bills	$(9 \times 10) + (10 \times 5)$	140
Clare	fourteen \$10 bills	$14 \times 10$	140
Jada	ten \$10 bills and three \$50 bills	$(10 \times 10) + (3 \times 50)$	250
Lin	eight \$20	$8 \times 20$	160

player	bills	expression	dollar amount
	bills		
Noah	six \$50 bills	6 × 50	300
Tyler	twenty- one \$10 bills	21 × 10	210

## **Advancing Student Thinking**

If students don't find the product of one-digit whole numbers and multiples of 10 in the last problem, consider asking:

- "What have you tried so far to find the product?"
- "How could you represent the product with base-ten blocks?"

Activity 2 © 20 min

**Two Strategies** 

## **Standards Alignments**

Addressing 3.NBT.A.3

The purpose of this activity is for students to continue to reason about products of a whole number and a multiple of 10, this time using base-ten blocks to support their thinking. They analyze two strategies for multiplying. Both strategies are based on place value, but the second strategy also uses the associative property to think about  $8\times30$  as  $8\times3\times10$  or  $24\times10$ .

#### **Materials to Gather**

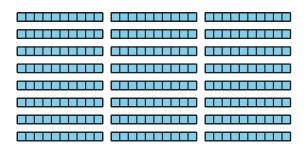
## **Materials to Copy**

Base-ten blocks

Centimeter Grid Paper - Standard (groups of 2)

#### **Student-facing Task Statement**

1. Two students used base-ten blocks to find the value of  $8 \times 30$ .



- Jada counted: 30, 60, 90, 120, 150, 180, 210, 240, and said the answer is 240.
- Kiran said he knew  $8 \times 3$  is 24, then found  $24 \times 10$  to get 240.

How are Jada and Kiran's strategies alike? How are they different?

- 2. Find the value of each expression. Explain or show your reasoning.
  - a.  $5 \times 60$
  - b.  $8 \times 50$
  - c.  $4 \times 30$
  - d.  $7 \times 40$
  - e.  $9 \times 20$

## **Student Responses**

- 1. Sample responses: They are alike because they both get 240. Kiran's strategy decomposes the 30 into  $3 \times 10$ , then multiplies the  $3 \times 8$  before multiplying  $24 \times 10$ . Jada just counts by 30 without decomposing anything.
- 2. a. 300. Sample response:  $5 \times 6 = 30$ .  $30 \times 10 = 300$ .
  - b. 400. Sample response: The student creates 8 groups of 50 with base-ten blocks and counts by 50 to get to 400.
  - c. 120. Sample response: The student

#### Launch

- Groups of 2
- "Take some time to look at Jada and Kiran's strategies for multiplying 8 x 30."
- 30 seconds: quiet think time
- "Talk to your partner about how we can see Jada and Kiran's strategies in the diagram." (We can see Jada's skip counting by 30 in the rows. The 8 in Kiran's strategy is the 8 rows and the 3 is the 3 tens in each row, so there are 24 tens.)
- 2–3 minutes: partner discussion
- Share responses.
- Give students access to grid paper and base-ten blocks.

#### **Activity**

- "Work with your partner on the first problem."
- 2–3 minutes: partner discussion
- Invite students to share how the strategies are alike and how they're different.
- "How was Kiran able to turn  $8 \times 30$  into  $24 \times 10$ ?" (Eight times 30 is like 8 groups of 3 tens, so that's like 24 tens. You can see  $8 \times 30$  and  $24 \times 10$  in the same diagram, so they are the same amount.)
- "Now, work with your partner to find the value of other products."
- 5–7 minutes: partner work time
- Monitor for students who use the associative property as a strategy to highlight during the synthesis.

#### **Synthesis**

• Select 2–3 students who used a strategy based on the associative property (for example, thinking of  $7 \times 40$  as 28 tens) to share their responses.

- creates a 4 by 30 rectangle and sees 12 tens.  $12 \times 10 = 120$ .
- d. 280. Sample response:  $7 \times 40 = 28 \times 10.28 \times 10 = 280.$
- e. 180. Sample response: The student counts by 20 nine times, like 20, 40, 60, 80, 100, 120, 140, 160, 180.
- Consider asking:
  - "Where do we see the original expression in \_\_\_\_\_'s work?"
  - "How did \_\_\_\_\_ change the original expression to make it easier to find the total?"
  - "How does \_\_\_\_\_'s strategy for multiplying work?"

# **Lesson Synthesis**

() 10 min

"Today we multiplied one-digit whole numbers by multiples of 10."

"How did thinking about tens help us find the value of products that were larger than we had found before?" (Using tens helped us count or multiply a lot faster. If we know  $5 \times 6$ , we can think of that many tens to find  $5 \times 60$ . We can use what we already know to find other products.)

"What were some strategies that were helpful as you multiplied one-digit whole numbers by multiples of 10?" (Decomposing one of the factors and finding smaller products. Using place value to multiply by 10 since we know 10 tens is 100.)

## **Suggested Centers**

- Compare (1–5), Stage 3: Multiply within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)

---- Complete Cool-Down ----

#### **Response to Student Thinking**

Students give the correct product of  $6\times40$ , but don't provide any reasoning around how they found the product.

#### **Next Day Support**

• During the launch of the next day's activity, have students discuss their reasoning about how they know  $6 \times 40 = 240$ .

# **Lesson 13: Solve Problems With Equal Groups**

#### **Standards Alignments**

Addressing 3.OA.A.3, 3.OA.B.5

Building Towards 3.OA.C.7

#### **Teacher-facing Learning Goals**

 Multiply within 100, where one factor is a teen number, in a way that makes sense to them.

#### **Student-facing Learning Goals**

• Let's multiply some teen numbers.

#### **Lesson Purpose**

The purpose of this lesson is for students to solve problems involving multiplication within 100, where one factor is a teen number, in a way that makes sense to them.

The work of this lesson connects to previous work because students have used strategies to multiply one-digit factors. Now, they have the opportunity to extend these strategies to the multiplication of teen numbers. Students may use area diagrams and expressions to represent multiplication strategies, which they used in the previous section. Students solve problems involving the multiplication of teen numbers, then make a poster of their work with a student who solved in a similar way. During the gallery walk, students see a variety of ways to represent and solve the problem (MP2). This will be helpful in the next lesson where students make sense of specific representations of multiplying within 100 when one factor is a teen number.

#### Access for:

#### Students with Disabilities

Engagement (Activity 2)

## **3** English Learners

MLR7 (Activity 2)

#### **Instructional Routines**

Estimation Exploration (Warm-up)

#### **Materials to Gather**

- Base-ten blocks: Activity 1
- Connecting cubes or counters: Activity 1
- Tools for creating a visual display: Activity 1

#### Materials to Copy

 Centimeter Grid Paper - Standard (groups of 2): Activity 1

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

#### **Teacher Reflection Question**

Which students had opportunities to share their diagrams and thinking during whole-class discussion? How did you select these students?

**Cool-down** (to be completed at the end of the lesson)

© 5 min

Bags of Oranges

#### **Standards Alignments**

Addressing 3.OA.A.3

## **Student-facing Task Statement**

There are 6 bags of oranges and each bag has 11 oranges. How many oranges are in the bags? Show your thinking using objects, a drawing, or a diagram.

## **Student Responses**

66. Sample response: Students use base-ten blocks to make 6 groups of 11.

----- Begin Lesson -----

Warm-up © 10 min

Estimation Exploration: Multiply Teens

## Standards Alignments

Addressing 3.OA.B.5

The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer

based on experience and known information.

#### **Instructional Routines**

**Estimation Exploration** 

#### **Student-facing Task Statement**

 $4 \times 18$ 

Record an estimate that is:

too low	about right	too high

#### **Student Responses**

Sample responses

• Too low: 40

About right: 40–79

Too high: 80

#### Launch

- Groups of 2
- Display the expression.
- "What is an estimate that's too high? Too low? About right?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

## **Synthesis**

- Consider asking:
  - "Is anyone's estimate less than \_\_\_? Is anyone's estimate greater than \_\_\_?"
  - "Based on this discussion does anyone want to revise their estimate?"

Activity 1 © 20 min

Problems with Teen Numbers

# **Standards Alignments**

Addressing 3.OA.A.3, 3.OA.B.5

Building Towards 3.OA.C.7

The purpose of this activity is for students to work with problems that involve multiplication within 100 where one factor is a teen number. This is the first time students have worked with problems with numbers in this range, so they should be encouraged to use the tools provided to them during the lesson if they choose (MP5). Students should also be encouraged to use strategies and representations from the previous section. As students are paired to create posters for the next activity, try to include a variety of approaches for students to see during the gallery walk in the next activity such as:

- Counting by the teen number.
- Counting by the single digit number.
- Use the distributive property to decompose the teen number to multiply in parts.
- Use the distributive property and place value understanding to decompose the teen number into tens and ones to multiply in parts.

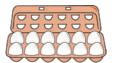
#### **Materials to Gather**

Base-ten blocks, Connecting cubes or counters, Tools for creating a visual display

#### **Student-facing Task Statement**

Solve each problem. Show your thinking using objects, a drawing, or a diagram.

- 1. A seller at a farmers market has 7 dozen eggs when they close for the day. How many eggs does the seller have?
- 2. At the farmers market there's a space for performers to play music with some chairs for people to sit and listen. There are 5 rows of chairs and each row has 15 chairs. How many chairs are there?
- 3. A booth at a farmers market has a table top that has lengths of 4 feet and 16 feet. What is the area of the table top?



#### **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- "Turn and talk to your partner about how the first problem is the same and different than problems you've seen before." (It involves equal groups. It's about a farmers market. It's asking for the total number of something. It uses the word dozen instead of saying the number.)
- 1–2 minutes: partner discussion
- If needed clarify that a dozen is 12 and refer students to the illustration of a dozen eggs.
- Give students access to connecting cubes or counters, grid paper, and base-ten blocks.

#### **Activity**

 "Solve these problems and show your thinking using objects, a drawing, or a

#### **Student Responses**

- 84 eggs. Sample responses: The student draws 7 groups with 12 dots in each group. The student creates groups of 12 using base-ten blocks.
- 2. 75 chairs. Sample responses: The student draws 5 groups with 15 dots in each group. The student creates 5 groups of 15 using base-ten blocks. The student uses the grid paper to make a rectangle with 5 rows with 15 squares in each row.
- 3. 64 square feet. Sample responses: The student uses the grid paper to make a 4 by 16 rectangle.

diagram."

- 6–8 minutes: independent work time
- As students work, consider asking:
  - "What strategies have you used before that you could try here?"
  - "How can you represent your thinking about the problem?"
  - "Where can you see \_\_\_\_\_ in your work?"
- Monitor for students who solve the second problem in the same way to pair to create a poster together. Try to include a variety of approaches.
- "Now you are going to create a poster to show your thinking on the second problem with a partner who solved the problem in the same way you did."
- Give each group tools for creating a visual display.
- 6–8 minutes: partner work time

## **Synthesis**

• Display posters around the room.

## **Advancing Student Thinking**

If students say they aren't sure how to start the problem, consider asking:

- "What is the problem about?"
- "How could you use base-ten blocks or grid paper to help you solve the problem?"

**Activity 2** 

(15 min

Gallery Walk: Problems with Teen Numbers

#### **Standards Alignments**

Addressing 3.OA.B.5

The purpose of this activity is for students to consider what is the same and what is different about the ways that students solved problems involving multiplication of a teen number. Students may notice representations that were used, as well as different strategies that were used to find the total in the problem. The important thing is that students see a variety of ways to represent and solve the problem.

## Access for English Learners

*MLR7 Compare and Connect.* Synthesis: After the Gallery Walk, lead a discussion comparing, contrasting, and connecting the different representations. "How did the number of chairs show up in each method? Why did the different approaches lead to the same outcome?" To amplify student language, and illustrate connections, follow along and point to the relevant parts of the displays as students speak.

Advances: Representing, Conversing

#### Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity. Supports accessibility for: Social-Emotional Functioning

## **Student-facing Task Statement**

As you visit the posters with your partner, discuss what is the same and what is different about the thinking shown on each poster.

## **Student Responses**

Sample responses: They all showed  $5 \times 15$ , even if it was in different ways. They found a total of 75 chairs. Some students showed the problem with base-ten blocks, some drew groups, and some made arrays. The numbers were grouped in different ways to make the total easier to find. Some groups used 5 and some groups used 10.

#### Launch

- Groups of 2
- "Before you begin the gallery walk, what are some things you could look for as you look at other students' work?" (Ways they showed their thinking. How they found the solution to the problem.)
- Share responses.

#### **Activity**

 "Visit the posters. Discuss with your partner what is the same and what is different about the thinking on each poster."

• 8–10 minutes: gallery walk

#### **Synthesis**

- Give students a chance to ask questions they have about any posters.
- "What is the same about the thinking shown on the posters?"
- "What is different about the thinking shown on the posters?"

# **Lesson Synthesis**

() 10 min

"Today we solved some problems that involved multiplying teen numbers. What were some strategies or representations you saw today that you'd like to try in the future?" (One of the posters that I saw used groups of 5 to find the total. One of the groups broke the teen number into tens and ones. One group used a grid to represent the problem. One of the groups used base-ten blocks to represent the problem.)

"How did your work multiplying smaller numbers help you multiply teen numbers?" (I broke the teen numbers apart like I did with smaller numbers that are challenging to multiply.)

#### **Suggested Centers**

- Compare (1–5), Stage 2: Add and Subtract within 20 (Supporting)
- How Close? (1–5), Stage 4: Add to 1,000 (Supporting)

## ----- Complete Cool-Down

## **Response to Student Thinking**

Students don't find a solution to the problem.

#### **Next Day Support**

 During warm-up, have students discuss how diagrams could be used to represent the problem in the cool-down.

# Lesson 14: Ways to Represent Multiplication of Teen Numbers

#### **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

#### **Teacher-facing Learning Goals**

 Make sense of representations of multiplication (base-ten blocks and area diagrams) where one factor is a teen number.

#### **Student-facing Learning Goals**

 Let's make sense of some ways to represent the multiplication of teen numbers.

#### **Lesson Purpose**

The purpose of this lesson is for students to make sense of representations of the multiplication of teen numbers.

The work of this lesson connects to previous work because students have solved problems involving multiplication of teen numbers in ways that make sense to them. In the previous section they also used the distributive property to find products of single-digit factors using facts they know. In this lesson, students consider and connect different representations of a strategy used in the previous section that can also be used to multiply a teen number. This will be helpful in the next lesson when students solve these types of problems and choose how to represent the problem.

#### Access for:

Students with Disabilities

**3** English Learners

Representation (Activity 2)

• MLR8 (Activity 2)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

#### **Materials to Gather**

Base-ten blocks: Activity 1

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

## **Teacher Reflection Question**

How did their previous work with area diagrams support students in their work today with multiplying teen numbers?

Cool-down (to be completed at the end of the lesson)

⑤ 5 min

Multiply and Explain

## **Standards Alignments**

Addressing 3.OA.B.5

## **Student-facing Task Statement**

Find the value of  $5\times15$  and explain how it's represented in the diagram.

	10	5
5	50	25

## **Student Responses**

75. Sample response: I can see  $5 \times 10$  in the large part of the rectangle and  $5 \times 5$  in the smaller part. If we add those parts of the rectangle, we get the product of  $5 \times 15$ , which is 75.

----- Begin Lesson

Warm-up © 10 min

Notice and Wonder: Seeing Groups

#### **Standards Alignments**

Addressing 3.OA.B.5

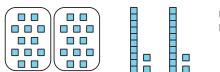
The purpose of this warm-up is to elicit the idea that while there are multiple ways to represent 2 groups of 12, some ways are more useful than others. While students may notice and wonder many things about the images, how 2 images show the groups of 12 have been organized using place value and how this type of decomposition can be helpful in finding the total are the important discussion points.

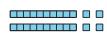
#### **Instructional Routines**

Notice and Wonder

#### **Student-facing Task Statement**

What do you notice? What do you wonder?





#### **Student Responses**

Students may notice:

- All the diagrams show 2 groups.
- Each group has 12 squares.
- Two of the diagrams show tens and ones.
- It's easier to count the 12 in the diagrams with groups of ten because I can think 10, 11, 12.
- Each diagram has the same number of squares.
- You can think of all the diagrams as  $2 \times 12$ .
- In the diagrams on the right I see 20 and 4.

Students may wonder:

- How could we multiply  $2 \times 12$ ?
- Why are the squares scattered in one diagram?
- Why are there tens and ones in some of the diagrams?

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

"The image on the left is a drawing of equal groups. The other images are base-ten diagrams. What is the same and different about these representations?" (They all show 12. They all show 2 groups of the same size. In the base-ten diagrams you can see the tens easier. It's harder to see the tens in the first drawing.)

Why are the tens and ones turned sideways in the last diagram?

# **Activity 1**

① 20 min

A Factor Greater than Ten

♣ ↔ ♣ PLC Activity

#### **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

The purpose of this activity is for students to see how, when multiplying a number larger than ten, the distributive property can be used to decompose the factor into tens and ones, creating two smaller products. Base-ten blocks are used to help students visualize what is happening when a factor is decomposed to make two more easily known products. Factors slightly larger than ten can be naturally decomposed into a ten and some ones using place value. This will be useful in subsequent lessons as students progress towards fluent multiplication and division within 100.

When students see that you can decompose a teen number into tens and ones and use this to multiply teen numbers, they look for and make use of structure (MP7).

#### Materials to Gather

Base-ten blocks

## **Student-facing Task Statement**

1. Tyler says he can use base-ten blocks to find the value of  $7 \times 13$  because he knows  $7 \times 10$  and  $7 \times 3$ . He says this diagram proves his thinking.

Do you agree or disagree? Explain your reasoning.



2. Use Tyler's method to find the value of

#### Launch

- Groups of 2
- Give students base-ten blocks.

#### **Activity**

- "Take a few minutes to look at Tyler's strategy and decide if you agree or disagree with it."
- 2–3 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor for students who connect the

 $3 \times 14$ . Explain or show your reasoning.

#### **Student Responses**

- 1. Sample response: I agree because the whole diagram represents  $7 \times 13$ , but I see the  $7 \times 10$  in the tens and the  $7 \times 3$  in the ones. So, to find the value of  $7 \times 13$  you could add the  $7 \times 10$  and the  $7 \times 3$ .
- 2. 42. Sample response:  $3 \times 10 = 30$ ,  $3 \times 4 = 12$ , 30 plus 12 is 42.

- expressions  $7 \times 10$  and  $7 \times 3$  to the tens and ones portion of the place value diagram and  $7 \times 13$  to the entire diagram.
- Have students share why they agreed or disagreed with Tyler's strategy with a focus on using the place value diagram as a justification.
- Consider asking:
  - "Where do we see the  $7 \times 13$  in the place value diagram?"
  - "Where do the 10 and the 3 come from?"
  - $\circ$  "How could you use the place value diagram to figure out how to find the value of  $7 \times 13$ ?"
- "Now use Tyler's method on your own to find the value of  $3 \times 14$ ."
- 2–3 minutes: independent work time
- "Share your solution and your reasoning with your partner."
- 2–3 minutes: partner discussion

#### **Synthesis**

- Display base-ten blocks or place value diagrams that students used to solve. As students explain their work, write multiplication expressions to represent them.
- "How does this diagram (or the base-ten blocks) show how Tyler's method could be used to multiply 3 × 14?" (We can see there are 3 tens which is 30. We can see that there are 3 groups of 4 ones which is 12. 30 + 12 is 42. The whole diagram represents 3 × 14.)
- If there is time, ask students to find the value of  $4 \times 12$  and  $5 \times 16$  using the baseten blocks and Tyler's strategy.

#### **Advancing Student Thinking**

If students say they don't see  $7 \times 10$  and  $7 \times 3$  in Tyler's diagram, consider asking:

- "Where do you see  $7 \times 13$  in the diagram?"
- "If we separate the tens and ones, what expression could we use to describe the tens? The ones?"

Activity 2 © 15 min

Ways to Represent

#### **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

The purpose of this activity is for students to make sense of different ways of representing multiplication of a teen number. Students analyze a gridded area diagram, base-ten blocks, and an area diagram labeled with side lengths. When they discuss how the different diagrams represent the same product, students reason abstractly and quantitatively (MP2).

## **3** Access for English Learners

*MLR8 Discussion Supports*. Synthesis: Show a visual display of the diagrams. As students share their observations, annotate the display to illustrate connections. For example, on each diagram, annotate the decomposition of 15 into 10 and 5 by circling the groups of 10 and the groups of 5. *Advances: Listening, Representing* 

## Access for Students with Disabilities

Representation: Access for Perception. Begin by showing a demonstration explaining how you see the product in each of the 3 different models using a different problem to support understanding of the context.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

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#### **Student-facing Task Statement**

Andre, Clare, and Diego represented the same expression. Their representations are shown below.

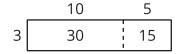
#### **Andre**



#### Clare



#### Diego



- 1. Where do you see the factors in each diagram?
- 2. Where do you see the product in each diagram?

## **Student Responses**

- 1. I see the factors in the side lengths of Andre's and Diego's diagrams. In Clare's I see 3 rows and 15 in each row.
- 2. In Andre's and Clare's diagrams I see the product in the total number of squares. In Diego's diagram I see 30 and 15. To get the product you would add these numbers together.

#### Launch

- Groups of 2
- "We're going to look at three different ways students showed the same expression.
   What do you notice? What do you wonder?" (Students may notice: You can see all the squares in the first 2 diagrams, but not in the last one. The middle diagram looks like base-ten blocks. Students may wonder: Why would you choose to use one of these diagrams? What numbers were they multiplying?)
- 1 minute: quiet think time
- Share responses.

#### **Activity**

- "Work with your partner to tell how you see the factors in each diagram and how you see the product in each diagram."
- 5–7 minutes: partner work time

## **Synthesis**

- "How are these ways of representing 3 x 15 the same?" (They all represent 3 times 15. They all show the 15 being decomposed into 10 and 5. They are all shaped like a rectangle.)
- "How are these ways of representing 3 x 15 different?" (Clare used base-ten blocks, but Andre and Diego used rectangles. Diego didn't show the squares in his rectangle, but Clare and Andre did.)
- "How could we represent the strategy shown in all the diagrams with expressions?" ( $3 \times 10$  and  $3 \times 5$  or  $10 \times 3$  and  $5 \times 3$ .)

# **Lesson Synthesis**

① 10 min

Display:

$$7 \times 6$$

$$(5 \times 6) + (2 \times 6)$$

$$3 \times 15$$

$$(3 \times 10) + (3 \times 5)$$

"Today we saw some different ways to represent strategies we can use to multiply teen numbers. How are the strategies we use to multiply teen numbers like the strategies we used to multiply smaller numbers in past lessons?" (We can use facts that we know to find facts that we don't know. We can break down one of the factors into smaller parts to make it easier to multiply.)

#### **Suggested Centers**

- Compare (1–5), Stage 2: Add and Subtract within 20 (Supporting)
- How Close? (1–5), Stage 4: Add to 1,000 (Supporting)

----- Complete Cool-Down

# **Response to Student Thinking**

The work in this lesson builds from the multiplication concepts developed in a prior unit.

#### **Prior Unit Support**

Grade 3, Unit 1, Section B: From Graphs to Multiplication

# **Lesson 15: Equal Groups, Larger Numbers**

## **Standards Alignments**

Addressing 3.MD.C.7.c, 3.OA.A.3, 3.OA.B.5

Building Towards 3.OA.C.7

#### **Teacher-facing Learning Goals**

• Multiply within 100, where one factor is a teen number.

#### **Student-facing Learning Goals**

 Let's solve some problems with equal groups that have larger numbers.

#### **Lesson Purpose**

The purpose of this lesson is for students to multiply within 100, where one factor is a teen number.

The work of this lesson connects to previous work because students have seen a variety of ways to represent and solve problems in which one of the factors is a teen number. In this lesson students use their choice of strategy and representation to solve these types of problems. Students participate in a gallery walk to highlight different ways that they solved problems with a synthesis that highlights the area diagram to represent one of the problems. Students' attention is drawn to scaling of area diagrams during the warm-up. While it's not important that student representations are exact, it is important that any area diagrams presented to students are to scale.

#### Access for:

#### Students with Disabilities

Engagement (Activity 1)

# English Learners

MLR8 (Activity 2)

#### **Instructional Routines**

Which One Doesn't Belong? (Warm-up)

#### **Materials to Gather**

Base-ten blocks: Activity 1

Sticky notes: Activity 2

Tools for creating a visual display: Activity 1

#### **Materials to Copy**

 Centimeter Grid Paper - Standard (groups of 2): Activity 1

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

## **Teacher Reflection Question**

What question do you wish you had asked today? When and why should you have asked it?

**Cool-down** (to be completed at the end of the lesson)

© 5 min

Find the Area

## **Standards Alignments**

Addressing 3.OA.A.3

## **Student-facing Task Statement**

A rectangle is 6 feet by 15 feet. What is the area of the rectangle? Explain or show your reasoning.

# **Student Responses**

90 square feet. Sample response:  $6 \times 10 = 60$ ,  $6 \times 5 = 30$ , 60 + 30 = 90

----- Begin Lesson

Warm-up © 10 min

Which One Doesn't Belong: Rectangles

# **Standards Alignments**

Addressing 3.MD.C.7.c

This warm-up prompts students to compare four area diagrams that have been decomposed into two areas, each representing a product. It gives students a reason to use language precisely (MP6). It gives

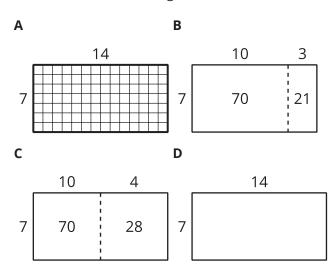
the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use, such as side lengths, area, parts, and decompose.

#### **Instructional Routines**

Which One Doesn't Belong?

#### **Student-facing Task Statement**

Which one doesn't belong?



#### **Student Responses**

Sample responses:

- A is the only one that doesn't have any blank rectangles inside.
- B is the only one that doesn't represent  $7 \times 14$ .
- C is the only one where the lengths of the lines don't fit the size of the numbers.
- D is the only one that doesn't have any lines inside the rectangle.

#### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

#### **Synthesis**

- "Why didn't it make sense for the rectangle in C to be split in half?" (Because one part of the rectangle should have been larger than the other because 70 is greater than 28. The part with 28 should have been smaller than the part with 70.)
- Consider asking: "Let's find at least one reason why each one doesn't belong."

Activity 1 © 20 min

**Equal Groups, Larger Numbers** 

## **Standards Alignments**

Addressing 3.OA.A.3, 3.OA.B.5

Building Towards 3.OA.C.7

The purpose of this activity is for students to solve problems that involve multiplication where one factor is a teen number. Students may solve and represent the problem any way they choose. In problem 3, look for different ways in which students are using area diagrams to highlight in the posters for the gallery walk in the next activity. Students reason abstractly and quantitatively when they interpret the stories and represent them with diagrams, expressions, or equations (MP2).

## Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select at least 2 of the 4 problems to complete.

Supports accessibility for: Organization, Attention, Social-emotional Skills

#### Materials to Gather

Base-ten blocks, Tools for creating a visual display

## **Student-facing Task Statement**

Solve each problem. Explain or show your reasoning.

- 1. Noah sees a large painted mural that has side lengths of 15 feet and 4 feet. What is the area of the mural?
- 2. Noah's family buys a mosaic that has 12 rows and 8 columns of 1 inch tiles. What is the area of the mosaic?
- 3. At the art festival, Noah uses sidewalk chalk to help decorate a rectangular piece of

## **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

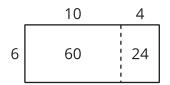
- Groups of 2
- "Where are some places that you see art in your community?"
- 30 seconds: quiet think time
- Share responses.
- "These problems are about a student visiting a local art festival. This is where local artists come to display and sell their work."
- Give students access to grid paper and

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- sidewalk that is 6 feet by 14 feet. What is the area of the piece of sidewalk that Noah helped decorate?
- 4. At the art festival, Noah buys a pack of stickers. There are 5 sheets and each sheet has 16 stickers. How many stickers are in the pack?

## **Student Responses**

- 60 square feet. Sample response: The student draws an area diagram and decomposes it into a 10 by 4 rectangle and a 5 by 4 rectangle, adding 40 and 20 to find the product.
- 2. 96 square inches. Sample response:  $10 \times 8 = 80, 2 \times 8 = 16, 80 + 16 = 96$
- 3. 84 square feet. Sample response:  $6 \times 10 = 60, 6 \times 4 = 24, 60 + 24 = 84$



4. 80 stickers. Sample response: The student makes 5 groups of 16 using base-ten blocks, multiplies  $5 \times 10$  and  $5 \times 6$ , then adds 50 and 30 to find the product.

- base-ten blocks.
- "Turn and talk to your partner about how you might solve these problems."
- 1–2 minutes: partner discussion

#### Activity

- "Solve these problems. Explain or show your reasoning."
- 6–8 minutes: independent work time
- As students work, consider asking:
  - "How can you represent your thinking about the problem?"
  - "Where can you see \_\_\_\_\_ in your work?"
- Monitor for:
  - different ways that students use the area diagram in the third problem
  - students who represent the third problem in the same way to pair to create a poster together
- "Now you are going to create a poster to show your thinking on the third problem.
   You are going to work with a partner who solved the problem in the same way you did."
- Give each group tools for creating a visual display.
- 6–8 minutes: partner work time

## **Synthesis**

• Display posters around the room.

## **Advancing Student Thinking**

If students don't find a solution to the problems, consider asking: "What is this problem about?" and "How could you represent the problem?"

Activity 2 © 15 min

Gallery Walk: Equal Groups, Larger Numbers

## **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

The purpose of this activity is for students to see how other students solved one of the problems that involves a factor of a teen number. While students look at each other's work, they will leave sticky notes describing why they think the answer does or does not make sense (MP3). The synthesis will look specifically at examples of how students used the area diagram to represent the problem.

## Access for English Learners

*MLR8 Discussion Supports.* Synthesis: As students share their observations about the selected posters, annotate the poster to illustrate connections. For example, circle the factors and the product and write "factors" and "product" respectively.

Advances: Listening, Representing

#### Materials to Gather

Sticky notes

## **Student-facing Task Statement**

As you visit the posters with your partner, discuss what is the same and what is different about the thinking shown on each poster.

## **Student Responses**

Sample responses: They are the same because they show the multiplication of the same numbers. They are different because they are labeled differently. The diagram doesn't have a grid. The side lengths are labeled and the rectangle is broken into parts.

#### Launch

- Groups of 2
- "Before you begin the gallery walk, what are some representations you expect to see as you look at other students' work?" (I expect to see rectangles because some of the problems were about the area of rectangles. Base-ten blocks because we were multiplying teen numbers.)
- Share responses.
- Give students sticky notes.

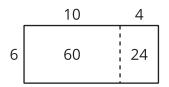
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#### **Activity**

- "As you visit the posters with your partner, discuss what is the same and what is different about the thinking shown on each poster. Also, leave a sticky note describing why you think the solution does or does not make sense."
- 8–10 minutes: gallery walk
- Monitor for different uses of the area diagram to highlight, specifically:
  - a fully gridded area diagram with no labels and no decomposition
  - a gridded area diagram that was gridded, but also decomposed into parts or labeled along the sides or in the parts of the rectangle
  - a partitioned rectangle that was drawn with no grid, but labeled with side lengths or the area of the parts of the rectangle

## **Synthesis**

- Select 2–3 posters that used a diagram to represent the problem. Discuss them in the order shown above.
- For each poster, ask:
  - "How are the factors shown in the diagram?"
  - "How is the product shown in the diagram?"
  - "How could we represent this strategy with expressions?"
- If students don't use an ungridded diagram, display one that represents the problems and ask the same questions.



# **Lesson Synthesis**

① 10 min

Display image from the warm-up.

"Today we solved problems that involved the multiplication of a teen number. What would be the advantages and disadvantages of using each of these diagrams?" (A is nice because you can see all the squares, but it would take a long time to draw and you can't see easier multiplication facts to find the total. D is fast to draw, but it just shows the numbers you're multiplying which doesn't help you find the product. B is fast to draw and it helps you because you just add 70 and 21 to find the product of  $7 \times 13$ .)

#### **Suggested Centers**

- Compare (1–5), Stage 3: Multiply within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)

---- Complete Cool-Down

## **Response to Student Thinking**

Students use an inefficient method that results in an error, such as drawing 6 groups of 15 and miscounting or skip counting by 6 and making a mistake in the count.

#### **Next Day Support**

 Before the warm-up, pass back the cooldown and have students discuss strategies they could use to find the product.

# **Lesson 16: Multiply Numbers Larger than 20**

## **Standards Alignments**

Building On 3.OA.B.5 Addressing 3.OA.B.5 Building Towards 3.OA.C.7

#### **Teacher-facing Learning Goals**

- Multiply within 100, where one factor is greater than 20.
- Use properties based on place value and properties of operations to multiply.

## **Student-facing Learning Goals**

 Let's multiply numbers that are larger than 20.

#### **Lesson Purpose**

The purpose of this lesson is for students to multiply within 100, where one factor is greater than 20.

Previously, students have used strategies based on place value and properties of operations to find products of a whole-number and a teen number. Here, students extend this work to larger two-digit numbers. Students first analyze different strategies and then apply one or more of them to find the value of other products. The lesson also includes an optional game students can play to apply what they've learned to multiply larger numbers.

#### **Access for:**

## Students with Disabilities

• Engagement (Activity 3)

# English Learners

MLR8 (Activity 1)

#### **Instructional Routines**

Number Talk (Warm-up)

#### **Materials to Gather**

• Base-ten blocks: Activity 1, Activity 2

#### **Materials to Copy**

- Centimeter Grid Paper Standard (groups of 2): Activity 1
- Centimeter Grid Paper Standard (groups of 2): Activity 2

Number Cards (0-10) (groups of 2): Activity
 3

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Activity 3	10 min
Lesson Synthesis	5 min
Cool-down	5 min

#### **Teacher Reflection Question**

When do your students feel successful in math? How do you know?

**Cool-down** (to be completed at the end of the lesson)

© 5 min

Multiply Numbers Greater than 20

# **Standards Alignments**

Addressing 3.OA.B.5

# **Student-facing Task Statement**

What's the value of  $4 \times 24$ ? Explain or show your reasoning.

## **Student Responses**

96. Sample response:  $4 \times 20 = 80$ ,  $4 \times 4 = 16$ , 80 + 16 = 96

----- Begin Lesson ----

Warm-up © 10 min

Number Talk: Three Times Some Numbers

#### **Standards Alignments**

Addressing 3.OA.B.5

This Number Talk encourages students to think about the multiplication of one-digit numbers and multiples of 10 and to rely on place value to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students multiply numbers larger than 20.

#### **Instructional Routines**

Number Talk

#### **Student-facing Task Statement**

Find the value of each expression mentally.

- 3 × 10
- 3 × 20
- $\bullet$  3 × 50
- 3 × 25

## **Student Responses**

- 30: Three groups of 10 is 30. I just knew it.
- 60: Twenty is twice 10, so the product is twice 30, which is 60. Twenty is  $2 \times 10$ , and  $3 \times 2 \times 10$  is  $6 \times 10$  or 6 groups of 10, which is 60.
- 150: Fifty is 10 + 20 + 20, so  $3 \times 50$  is  $(3 \times 10) + (3 \times 20) + (3 \times 20)$  or 30 + 60 + 60, which is 150. Fifty is  $5 \times 10$ , so  $3 \times 50$  is  $3 \times 5 \times 10$  or  $15 \times 10$ , which is 150.
- 75: Twenty-five is half of 50, so  $3 \times 25$  is half of  $3 \times 50$  or half of 150, which is 75. It's  $3 \times 5$  more than  $3 \times 20$ , so it's 15 more than 60 or 60 + 15, which is 75.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

## **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### **Synthesis**

• "How did the first three problems help you solve the last problem?" (Since I knew  $3 \times 20$  was 60, I just added  $3 \times 5$  to that by adding 15. I broke  $3 \times 25$  into  $3 \times 10$ ,  $3 \times 10$ , and  $3 \times 5$  to make it easier to multiply. Since 25 is half of 50, I took half of  $3 \times 50$  to find  $3 \times 25$ , and half of 150 is 75.)

Activity 1 © 15 min

 $4 \times 23$ , Represented

## **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

Previously, students multiplied two factors where one factor was a whole number and the other a teen number. The purpose of this activity is for students to make sense of multiplication of a one-digit number and a two-digit number greater than 20. Students analyze representations used to find  $4 \times 23$  and articulate reasons for using strategies that are based on the distributive property and place value to find products (MP7). Along the way, they see that decomposing the two-digit factors into tens and ones is particularly helpful for multiplying. They also reinforce what they know about the connections between area diagrams and multiplication expressions.

#### Access for Students with Disabilities

*MLR8 Discussion Supports.* Before partner work, remind students to use words such as decompose, tens, and ones.

Advances: Speaking, Representing

#### **Materials to Gather**

Base-ten blocks

## **Student-facing Task Statement**

1. Here is how Clare and Andre represented  $4 \times 23$ .

Clare Andre

- a. How does each diagram show  $4 \times 23$ ?
- b. How could we use Clare's diagram to find the value of  $4 \times 23$ ?

## **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- "Take a minute to make sense of how Clare and Andre represented 4 x 23."
- 1 minute: quiet think time
- Give students access to grid paper and base-ten blocks.

## **Activity**

"Now, work with your partner to complete

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- c. How could we use Andre's diagram to find the value of  $4 \times 23$ ?
- 2. Diego tried different ways to partition or split a diagram to help him find the value of  $4 \times 23$ .

A 12 11 4

15 8

В

10 13 4

**D**20 3
4

- a. What do you notice about the numbers in his diagrams?
- b. Which diagram would you use to find the value of  $4 \times 23$ ? Explain your reasoning.
- 3. Find the value of  $3 \times 28$ . Show your thinking using diagrams, symbols, or other representations.

## **Student Responses**

- 1. Sample response:
  - a. Clare's blocks show 4 groups of 2 tens and 3 ones. Andre's diagram shows a grid or an array with 4 rows of 23 squares.
  - b. The arrangement shows 8 tens and 12 ones, so its value is 80 + 12, which is 92.
  - c. The first two large parts of the rectangle show 4 rows of 10 each, so that's 40 + 40 or 80. The last little part

- the first two problems about  $4 \times 23$ ."
- 8–10 minutes: partner work time
- Monitor for the diagrams that students choose in Diego's problem and for their explanations.
- Pause for a discussion. Invite students to share their responses.
- If not mentioned by students, clarify that all diagrams can be used to multiply 4 and 23, but not all are equally practical.
- Select a student to explain why diagram D might be more productive than the other diagrams.
- "Now, work independently to complete the last problem."
- 2–3 minutes: independent work time

#### **Synthesis**

• "What representations did you use in the last question to show  $3 \times 28$ ? How did they help you find the product?" (Base-ten blocks helped me break the 28 into tens and ones. A diagram with a grid helped me break 28 into smaller numbers. A diagram with no grid helped me think about the numbers as I labeled it without worrying about all the little squares.)

of the rectangle shows 4 rows of 3, which is 12. The total number of squares is 92.

#### 2. Sample response:

- a. The numbers labeling the long side of the rectangle always add up to 23. The label for the shorter side is always 4.
- b. Diagram D, because I can find  $4 \times 20$  and  $4 \times 3$  more easily than I can multiply the other numbers.

#### 3. 84. Sample response:

	20	8
3	60	24

$$3 \times 20 = 60, 3 \times 8 = 24, 60 + 24 = 84$$

## **Advancing Student Thinking**

If students choose a diagram in the second problem that doesn't help them find the product of  $4 \times 23$ , consider asking:

- "Tell me about how you chose which diagram you would use to find the value of  $4 \times 23$ ."
- "Is there another diagram that would make finding the value of  $4 \times 23$  easier? How would it make it easier?"

# Activity 2 © 15 min

Some Fine Products

# **Standards Alignments**

Building On 3.OA.B.5

The purpose of this activity is for students to continue to multiply single-digit whole numbers and numbers greater than 20. The opening problem encourages students to apply place value

reasoning (decomposing two-digit numbers into tens and ones) and properties of operations to reason numerically about products. Because one of the factors is small, however, students may use repeated addition (such as 43+43) to find subsequent products. In the synthesis, emphasize strategies that are based on place value, connecting the numerical expressions with diagrams as needed.

#### **Materials to Gather**

Base-ten blocks

## **Student-facing Task Statement**

1. To find the value of  $2 \times 37$ , Mai started by writing this equation:

$$2 \times 30 = 60$$

Describe or show what Mai would do to finish finding the value of  $2 \times 37$ .

- 2. Find the value of each product. Show your reasoning.
  - a.  $3 \times 32$
  - b.  $2 \times 43$
  - c.  $4 \times 22$
  - d.  $3 \times 29$

## **Student Responses**

- 1. Sample responses: She would find  $2 \times 7$ , which is 14, and add 14 to 60, which gives  $74.2 \times 7 = 14$  and 60 + 14 = 74.
- 2. a. 96. Sample response:

$$3 \times 30 = 90$$

$$3 \times 2 = 6$$

$$90 + 6 = 96$$

b. 86. Sample response:

$$(2 \times 40) + (2 \times 3) = 80 + 6 = 86$$

c. 88. Sample response:

$$(4 \times 20) + (4 \times 2) = 80 + 8 = 88$$

d. 87. Sample response:

#### Materials to Copy

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- "Take a look at how Mai started to multiply 2 × 37. Then, talk to your partner about why you think Mai decided to start multiplying this way." (Thirty-seven is a large number to multiply, so she broke it into tens and ones. 2 × 30 shows the multiplication of the tens.)
- 1 minute: partner discussion
- Share responses.
- Give students access to grid paper and base-ten blocks.

## **Activity**

- "Take a few minutes to work independently on the activity. Afterwards, share your responses with your partner."
- 5 minutes: independent work time
- 5 minutes: partner work time

#### **Synthesis**

- Invite 1–2 students to share their responses and reasoning. Display or record their work for all to see.
- Discuss responses that highlight use of place value and properties of operations.

$$3 \times 20 = 60$$
  
 $3 \times 9 = 27$   
 $60 + 27 = 87$ 

 $4 \times 22$ , for instance, can be found using the associative property ( $2 \times (2 \times 22)$ ) or distributive property ( $4 \times 20 + 4 \times 2$ ).

 Consider using diagrams or base-ten blocks to reinforce the meaning of expressions as needed.

## **Advancing Student Thinking**

If a student says they don't know how to start the problem, consider asking:

- "What would you do if the factors were 3 and 10? What about 3 and 20?"
- "How could you use base-ten blocks or diagrams to help you find these products?"

# **Activity 3 (optional)**

(1) 10 min

Play Close to 100, Multiplication

# **Standards Alignments**

Addressing 3.OA.B.5 Building Towards 3.OA.C.7

The purpose of this activity is for students to play a game in which they are able to apply the strategies they've learned for multiplying teen numbers and numbers over 20. Students use digits to create an expression that has a value as close to 100 as possible. The first game includes teen numbers and the second game includes numbers over 20.

This activity is optional because it provides extra practice for multiplying by factors that are teen numbers and factors greater than 20. Depending on the time available, students can play 1 or 2 games.

# Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community

Supports accessibility for: Social-Emotional Functioning

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## **Materials to Copy**

Number Cards (0-10) (groups of 2)

#### **Required Preparation**

• Create a set of cards from the Instructional master for each group of 2.

## **Student-facing Task Statement**

Play Close to 100, Multiplication with a partner.

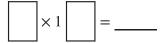
- 1. Place the cards face down.
- 2. Each player draws 4 cards.
- 3. Each player chooses 2 cards to complete the expression to make a value as close to 100 as possible. Write the 2 digits and the product.
- 4. Player closest to 100 wins.
- 5. Play 5 rounds. Player who wins the most rounds wins.

Game	1
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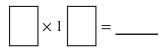
Round 1



Round 2



Round 3



Round 4



Round 5

#### Launch

- Groups of 2
- Distribute one set of pre-cut cards to each group of students.
- "We're going to play a game called Close to 100. Let's read through the directions together and play 1 round together."
- Play a round against the class, displaying the numbers from the cards and thinking through decisions aloud.

#### **Activity**

- "Now, you will play a game of Close to 100 with your partner. The game will have 5 rounds."
- 5–7 minutes: partner game time
- If time allows, have students play the second game of Close to 100. Inform students that the numbers are different in the second game.

## **Synthesis**

 "What were some strategies that were helpful as you played Close to 100?" (I used rounding to think about how large the product would be. I multiplied the tens and ones, then combined them to find the product.)

× 1 =			
Game 2			
Round 1			
× 2 =			
Round 2			
× 2 =			
Round 3			
× 2 =			
Round 4			
× 2 =			
Round 5			
× 2 =			
Student Responses			

# **Lesson Synthesis**

Answers vary.

© 5 min

"In the past few lessons, we saw and used different strategies to multiply larger numbers."

"What were some of those strategies?" (Using base-ten blocks, drawing gridded and ungridded diagrams, decomposing the tens and ones and multiplying them separately or using easier multiplication facts.)

"What strategies for multiplying do you prefer for finding the value of a product such as  $3 \times 24$ ?" (I like breaking the larger factor into smaller parts to make it easier to multiply.)

#### **Suggested Centers**

- Compare (1–5), Stage 3: Multiply within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)

## **Response to Student Thinking**

Students find the product of  $4 \times 24$  by adding 24 repeatedly.

#### **Next Day Support**

 Launch the lesson by asking students to recap the important points of the previous lessons.

# **Lesson 17: Use the Four Operations to Solve Problems**

## **Standards Alignments**

Addressing 3.NBT.A.3, 3.OA.B.5, 3.OA.D.8

#### **Teacher-facing Learning Goals**

- Represent two-step word problems using equations with a letter standing for the unknown quantity.
- Solve two-step word problems using the four operations.

## **Student-facing Learning Goals**

 Let's use the four operations to solve problems.

#### **Lesson Purpose**

The purpose of this lesson is for students to solve two-step problems using all four operations.

Previously, students have solved two-step problems involving addition, subtraction, and multiplication. Here they consider what mathematical questions could be asked about a situation and then solve two-step problems that include division where the factors are limited to single-digit numbers. Parentheses are revisited as a tool students can use to specify which operation happens first in the equation so that it matches the situation they are representing.

This lesson has a Student Section Summary.

#### Access for:

## Students with Disabilities

• Engagement (Activity 2)

# English Learners

MLR5 (Activity 2)

#### **Instructional Routines**

True or False (Warm-up)

#### **Materials to Gather**

Base-ten blocks: Activity 2

## **Materials to Copy**

 Centimeter Grid Paper - Standard (groups of 2): Activity 2

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#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

#### **Teacher Reflection Question**

How has your students' understanding of twostep word problems evolved from previous lessons? How have their experiences with multiplication and division in this unit influenced their problem solving strategies?

# **Cool-down** (to be completed at the end of the lesson)

O 5 min

Andre's Balloons

## **Standards Alignments**

Addressing 3.OA.D.8

# Student-facing Task Statement

Andre has 125 balloons. He and 4 friends hung up some balloons for a party at school and now there are 80 balloons left. If each person hung up the same number of balloons, how many balloons did each person hang up?

- 1. Write an equation with a letter for the unknown quantity to represent the situation.
- 2. Solve the problem. Explain or show your reasoning.

# **Student Responses**

- 1.  $(125 80) \div 5 = b$
- 2. 9 balloons. Sample response: I subtracted 125-80 to see how many balloons Andre and his friends hung up and got 45. Then, I divided 45 by 5 to see how many balloons each person hung up and got 9.

Warm-up 
© 10 min

True or False: Multiply by 10

## **Standards Alignments**

Addressing 3.NBT.A.3, 3.OA.B.5

The purpose of this True or False is to elicit strategies and understandings students have for multiplying one-digit whole numbers by multiples of 10. The reasoning students do here helps to deepen their understanding of the associative property as they decompose multiples of ten to make multiplying easier.

#### **Instructional Routines**

True or False

## **Student-facing Task Statement**

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\bullet$  2 × 40 = 2 × 4 × 10
- $2 \times 40 = 8 \times 10$
- $3 \times 50 = 15 \times 10$
- $3 \times 40 = 7 \times 10$

## **Student Responses**

- True:  $4 \times 10$  is 40, so the sides are the same
- True:  $2 \times 4$  is 8 in the first equation, so  $8 \times 10$  is also equal to  $2 \times 40$ .
- True: 15 is  $3 \times 5$ , so I could write  $3 \times 5 \times 10$  and  $5 \times 10$  is 50.
- False:  $3 \times 40$  is the same as  $3 \times 4 \times 10$  or  $12 \times 10$ , not  $7 \times 10$ .

#### Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

#### **Activity**

- Share and record answers and strategy.
- Repeat with each statement.

#### **Synthesis**

- "How can you justify your answer without finding the value of both sides?"
- Consider asking:
  - "Who can restate \_\_\_\_\_'s reasoning in a different way?"
  - "Does anyone want to add on to \_\_\_\_\_'s reasoning?"
  - "Can we make any generalizations based on the statements?"

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Activity 1 © 15 min

Questions about a Situation

#### **Standards Alignments**

Addressing 3.OA.D.8

The purpose of this activity is for students to consider a situation and think about all the mathematical questions they could ask about it. This gives students a chance to make sense of the situation before they are asked to solve problems. Students might choose to write a multiplication equation like  $(g \times 6) + 94 = 142$ . Acknowledge that this represents this situation, but focus the discussion in the synthesis on division to connect to the work in the next section.

## **Student-facing Task Statement**

What questions could you ask about this situation?

There are 142 guests at a party. All the guests are in 2 rooms. Room A has 94 guests. Room B has 6 tables that each have the same number of guests. There are 4 pieces of silverware and 1 plate for each guest.

## **Student Responses**

Sample responses:

- How many people will be in Room B?
- How many pieces of silverware are at each table?
- How many guests fit at each table in Room B?
- How many pieces of silverware are in each room?

#### Launch

- Groups of 2
- "This situation is about planning for a party. What are some things that you have to think of when you plan for a party?" (Making enough food. Places for people to sit or hang out. Activities for people to do.)
- 1 minute: quiet think time
- Share responses.

#### **Activity**

- "Now, work with your partner to come up with as many questions as you can about this situation."
- 3–5 minutes: partner work time
- Share and record responses.
- Display: "How many guests fit at each table in Room B?" or circle the question if mentioned by a student.
- "Now work with your partner to answer this question." (I found 142 – 94 to find out how many guests were in Room B. There

- were 48 guests and 6 tables, I put the same amount of guests at each table and there were 8 guests at each table.)
- 3–5 minutes: partner work time

## **Synthesis**

- "What information did we use in the problem?" (The number of guests at the party. The number of guests in Room A. The number of tables in Room B.)
- "How could we record an equation with a letter for the unknown quantity that would represent the problem? Explain your reasoning." ( $(142-94) \div 6 = g$ . We had to find 142-94 to find out how many people were in Room B. We had to divide the number of people in room B by 6 to find out how many guests were at each table. The g represents how many guests fit at each table in Room B.)
- Display:  $(142 94) \div 6 = g$
- If students don't use parentheses, say: "In this equation, we can use parentheses to show that we subtracted first."
- "The parentheses show us that the subtraction is done first in the equation to represent the problem. Keep this in mind as you work on the next activity."

Activity 2 © 20 min

Party Problems

# Standards Alignments

Addressing 3.OA.D.8

The purpose of this activity is for students to solve two-step word problems using all four operations. Students should be encouraged to solve the problem first or write the equation first, depending on their preference. Encourage students to use parentheses if needed to show what is being done first in their equations.

When students make sense of situations to solve two-step problems they reason abstractly and quantitatively (MP2).

## **Access for English Learners**

*MLR5 Co-Craft Questions.* Keep books or devices closed. Display only the problem stem, without revealing the question. Give students 2–3 minutes to write a list of mathematical questions that could be asked about this situation, before comparing their questions with a partner. Invite each group to contribute one written question to a whole-class display. Ask the class to make comparisons among the shared questions and their own. Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

## **@** Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Some students may benefit from feedback that emphasizes effort and time on task. For example, check in and give feedback after each party planning problem.

Supports accessibility for: Attention

#### **Materials to Gather**

Base-ten blocks

## **Student-facing Task Statement**

For each problem:

- a. Write an equation to represent the situation. Use a letter for the unknown quantity.
- b. Solve the problem. Explain or show your reasoning.
- 1. Kiran is making paper rings each day to decorate for a party. From Monday to Thursday he was able to complete 156 rings.

## **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- Give students access to grid paper and base-ten blocks.

#### **Activity**

 "Work independently to solve these problems and write an equation with a letter for the unknown quantity to represent each situation. You can choose to solve the problem first or write the

Friday, Kiran and 2 friends worked on making more rings. Each of them made 9 more rings. How many rings did they make over the week?

- 2. Mai has 168 muffins. She put 104 of the muffins in a basket. She packed the rest of the muffins into 8 boxes with the same number of muffins. How many muffins were in each box?
- 3. There are 184 cups on a table. Three tables with 8 people at each table come up to get drinks and each use a cup. How many cups are on the table now?

## **Student Responses**

- 1. a.  $156 + 3 \times 9 = r$ 
  - b. 183 rings. Sample response:  $3 \times 9 = 27.156 + 27 = 183.$
- 2. a.  $(168 104) \div 8 = m$ 
  - b. 8 muffins. Sample response: I subtracted 104 from 168 to find out how many muffins were going into the boxes, which was 64. Then, I divided  $64 \div 8$  to see how many muffins would be in each box, which was 8.
- 3. a.  $184 (3 \times 8) = c$ 
  - b. 160 cups. Sample response:  $3 \times 8 = 24.184 24 = 160.$

equation first."

- 5–7 minutes: independent work time
- "Share your solutions and your equations with your partner. Also, tell your partner if you think their solutions and equations make sense or why not."
- 5–7 minutes: partner discussion

## **Synthesis**

- For each problem have a student share their equation and discuss how it represents the problem.
- Consider asking:
  - "Where do we see \_\_\_\_\_ from the problem in the equation?"
  - "What information from the situation did we need to solve and write our equation?"
  - "How are parentheses used in the equation?"

# **Advancing Student Thinking**

If students don't write a single equation to represent both steps of the problem, consider asking:

- "What equations did you write for each part of the problem?"
- "How could you combine your equations into one equation that would represent the problem?"

# **Lesson Synthesis**

① 10 min

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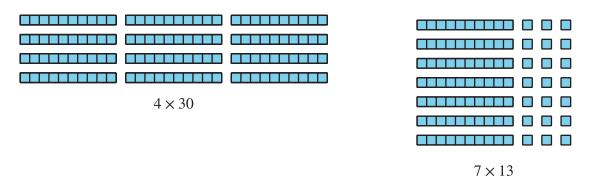
"Today we used multiplication, division, addition, and subtraction to solve two-step problems. What were some strategies that were helpful as you solved these types of problems?" (It was helpful to represent the situation with a drawing to help me think about what was happening in the situation. It helped to think about the information that I needed. It helped to think about how to represent each part of the problem before I put it all together into an equation.)

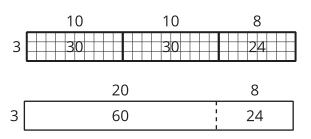
#### **Suggested Centers**

- Compare (1–5), Stage 3: Multiply within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)

# **Student Section Summary**

In this section, we learned how to multiply single-digit numbers by multiples of ten. We used strategies to multiply teen numbers and numbers greater than 20.





 $3 \times 28$ 

# **Response to Student Thinking**

Students solve the problem, but don't write an equation with a symbol for the unknown to represent the situation.

## **Next Day Support**

 Before the warm-up, pass back the cooldown and have students discuss how the problem could be represented with an equation with a symbol for the unknown number.

# **Section D: Dividing Larger Numbers**

# **Lesson 18: Larger Numbers in Equal Groups**

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.A.3

#### **Teacher-facing Learning Goals**

 Solve problems involving division within 100, with quotients over 10, in a way that makes sense to them.

#### **Student-facing Learning Goals**

Let's divide with larger numbers.

#### **Lesson Purpose**

The purpose of this lesson is for students to solve problems involving division within 100, with quotients over 10, in a way that makes sense to them.

In previous lessons, students learned how multiplication and division are related. They also used strategies based on properties of operations to multiply larger numbers.

In this lesson, students use a strategy of their choice to solve division problems with larger quotients than in previous lessons. Students should be encouraged to use whatever strategy and representation makes sense to them. The problem allows teachers an opportunity to see how students apply their learning from the unit to a new problem.

#### Access for:

Students with Disabilities

**3** English Learners

Representation (Activity 1)

MLR7 (Activity 1)

#### **Instructional Routines**

5 Practices (Activity 1), What Do You Know About \_\_\_\_\_? (Warm-up)

#### **Materials to Gather**

Base-ten blocks: Activity 1, Activity 2

## Materials to Copy

 Centimeter Grid Paper - Standard (groups of 2): Activity 1

 Connecting cubes or counters: Activity 1, Activity 2

## Centimeter Grid Paper - Standard (groups of 2): Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

## **Teacher Reflection Question**

What division strategies did your students use when working with larger numbers today?

# Cool-down (to be completed at the end of the lesson)

© 5 min

**Recess Teams** 

## **Standards Alignments**

Addressing 3.OA.A.3

# **Student-facing Task Statement**

At recess, 42 students played a game. There were 3 teams with the same number of students on each team. How many students were on each team?

Show your thinking using diagrams, symbols, or other representations.

# Student Responses

14. Any strategy is acceptable at this point in the unit. Sample responses:

- Students draw 42 things and circle 3 equal groups of size 14.
- Students draw base-ten diagrams and put 1 ten and 4 ones into each group.
- Students write a series of multiplication equations with 3 as a factor and build up to 42.

Warm-up © 10 min

What Do You Know About Division?

## **Standards Alignments**

Addressing 3.OA.A.2

The purpose of this What Do You Know About \_\_\_\_ is to invite students to share what they know and how they can represent division.

#### Instructional Routines

What Do You Know About \_\_\_\_?

#### **Student-facing Task Statement**

What do you know about division?

## **Student Responses**

Sample responses:

- Division is finding the number in each group in a situation with equal groups.
- Division is finding the number of groups in a situation with equal groups.
- I can write division equations or expressions with the symbol ÷.
- Division is related to multiplication.
- I can multiply to find the answer to a division problem.

#### Launch

- Display the word division.
- "What do you know about division?"
- 1 minute: quiet think time

#### **Activity**

- Record responses.
- "How could we represent division?" (a drawing, with connecting cubes, an equation, an expression)

#### **Synthesis**

- "We have already learned a lot about division.
   We are going to continue to learn about division by working with larger numbers than we have before."
- Consider asking: "What connections do you see between different answers?"

Activity 1 © 20 min

Groups on a Field Trip

#### **Standards Alignments**

Addressing 3.OA.A.3

The purpose of this 5 Practices activity is to elicit students' existing strategies for finding the value of quotients with larger numbers. Students should be encouraged to use whatever strategy or representation makes sense to them.

Monitor and select students with the following strategies to share in the synthesis:

- make groups of 4 and see how many groups there are (with objects, drawings, arrays, or base-ten blocks)
- multiply up starting with  $4 \times 10$
- break the dividend up into tens and ones and divide each part

If appropriate, discuss connections between the strategies as they are shared, rather than after all strategies are shared. It is not essential that all the listed strategies are discussed, as students will consider these ideas in upcoming lessons. The main goal here is to elicit what students currently understand.

When students make sense of the contextual division problem they reason abstractly and quantitatively (MP2). Students who use the relationship between multiplication and division make use of structure (MP7).

# **Access for English Learners**

MLR7 Compare and Connect. Synthesis: Invite students to prepare a visual display that shows the strategy they used to figure out the number of groups. Encourage students to include details that will help others interpret their thinking. For example, specific language, using different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each other's work. During the whole-class discussion, ask students, "Did anyone solve the problem the same way, but would explain it differently? How did the groups of 4 show up in each method? Why did the different approaches lead to the same outcome?"

Advances: Representing, Conversing

## Access for Students with Disabilities

*Representation: Access for Perception.* Synthesis: As students identify correspondences between strategies, follow along and point to the relevant parts of each strategy to amplify student thinking and illustrate connections.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

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#### Instructional Routines

5 Practices

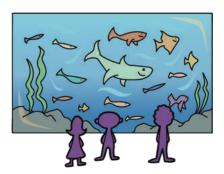
#### **Materials to Gather**

Base-ten blocks, Connecting cubes or counters

#### **Student-facing Task Statement**

There are 48 students going on a field trip to the aquarium. They visit the exhibits in groups of 4 students. How many groups will there be?

Show your thinking using diagrams, symbols, or other representations.



## **Student Responses**

12. Sample responses:

- I know that  $4 \times 10$  is 40 and then  $4 \times 2$  is 8, so there are 10 + 2 = 12 groups of 4 in 48.
- I know that 5 groups of 4 is 20, so 10 groups of 4 is 40. There are 8 students left, which make 2 groups of 4. Adding 10 and 2 gives 12.

#### **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Display the statement: "On a field trip to the aquarium, a class of students is splitting into groups."
- "What do you notice? What do you wonder?" (Students may notice: They are going to the aquarium in groups. They can't go in one big group. Students may wonder: How many kids are in the class? How many kids are in each group? Are the groups the same size?)
- 1 minute: quiet think time
- Share and record responses.
- Give students access to connecting cubes or counters, base-ten blocks, and grid paper.

#### **Activity**

- "Solve this problem. Use whatever strategy or representation that works best for you."
- 5 minutes: independent work time
- As students work, consider asking:
  - "Where are the groups of 4 in your work?"
  - "Where is the number of groups in your work?"
- "Share your response and strategy with your partner. Talk about how they are alike and how they are different."
- 3 minutes: partner discussion

## **Synthesis**

- Select previously identified students to share in the order listed in the activity narrative.
- As each strategy is presented, invite the class to ask questions.
- Keep all the strategies displayed.
- "How are the strategies alike?" (All the strategies used  $4 \times 10 = 40$  to help break the problem into smaller parts. The strategies with division and multiplication equations both had 10 + 2 = 12.)
- "How are the strategies different?" (In some strategies, students drew a representation.
   In others, students wrote multiplication or division expressions or equations.)

## **Advancing Student Thinking**

If students say they aren't sure how to start the problem, consider asking: "What is the problem about?" and "How could you represent the problem?"

Activity 2 © 15 min

Bus Ride and Lunch Groups

# **Standards Alignments**

Addressing 3.OA.A.3

The purpose of this activity is for students to consider their strategies as they solve two other division problems involving equal groups with larger numbers. The divisor in the first problem is a low one-digit number. Students can see from the given situation that it is the number of groups. In the second problem, the divisor is a teen number, and the context suggests that it is the size of one group. Students are likely to adjust their strategy based on these observations. Focus the discussion on how students may have reasoned differently given a larger divisor or given what they understand about the situation.

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#### **Materials to Gather**

Base-ten blocks, Connecting cubes or counters

#### **Student-facing Task Statement**

For each question, show your thinking using diagrams, symbols, or other representations.

- On another field trip, 72 students and teachers rode in 3 buses to a science museum, with the same number of people in each bus. How many people rode in each bus?
- 2. During lunch, the 72 people sat at long tables, with 12 people at each table. How many tables did they use?

#### **Student Responses**

- 1. 24 people. Sample responses:
  - I split 72 connecting cubes into 3 groups and saw that there are 24 cubes in each group.
  - o If we put 10 people in each bus, that's 30 people. If we put 10 more people in each bus, that's 60 people, and there are 12 more people who are not yet on a bus. Putting 4 more on each bus takes care of the 12 people. 10 + 10 + 4 = 24.
  - I know that 3 groups of 20 is 60 and 3 groups of 4 is 12, so 3 groups of 24 is 72.
- 2. 6 tables. Sample responses:
  - I put 72 cubes into groups of 12 and made 6 groups.
  - I know there are 2 groups of 12 in 24, so the 24 people in each bus take up 2 tables. This means 6 tables for the 72 people in 3 buses.

#### **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- "Read the two problems about another field trip. Think about what strategies you might use to solve them."
- 1 minute: quiet think time
- "Share your thinking with your partner."
- 1 minute: partner discussion
- Give students access to connecting cubes or counters, base-ten blocks, and grid paper.

#### **Activity**

- "Work independently on the problems for a few minutes. Then, share your responses with your partner."
- 5 minutes: independent work time
- 3 minutes: partner discussion
- Monitor for students who use different representations (drew different diagrams or wrote different types of expressions or equations) for the two problems.

#### **Synthesis**

- Invite students to share their responses. Display or record their reasoning.
- Poll the class on whether they used a different strategy for solving the second problem than used for the first.
- Ask those who used a different strategy: "Why did you change your strategy?" (Sample responses:
  - In the first, the 3 represents 3 groups. In the second, the 12 is how

 I subtracted 12 from 72 six times until there was no more to subtract.

- many in each group.
- In the first, the number used to divide is smaller. In the second, the number is larger.)

#### **Advancing Student Thinking**

If students say they aren't sure how to get started on the problem, consider asking: "What is the problem about?" and "How could you represent the problem?"

# **Lesson Synthesis**

(10 min

"Today we found the value of quotients with larger numbers than we have worked with before."

"We saw problems that involved dividing 48 by 4, 72 by 3, and 72 by 12."

"Reflect on the strategies you used. Did the size of the number being divided—48 and 72—affect how you solved the problem? If so, how?" (When the number was larger, I broke it up into more parts. When the number was smaller, I used a drawing, but when it was larger, I used another way.)

# **Suggested Centers**

- Compare (1–5), Stage 4: Divide within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)

# ----- Complete Cool-Down

# **Response to Student Thinking**

Students do not find a solution to the problem.

#### **Next Day Support**

 Before the warm-up, pass back the cooldown and have students work in small groups to make corrections.

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# **Lesson 19: Ways to Divide Larger Numbers**

#### **Standards Alignments**

Addressing 3.NBT.A.3, 3.OA.A.2, 3.OA.B.5, 3.OA.C.7

#### **Teacher-facing Learning Goals**

- Recognize that division of larger numbers can still mean finding the number of groups or finding the size of each group.
- Use base-ten blocks to represent division where the quotient is more than 10.

#### **Student-facing Learning Goals**

Let's make sense of representations of division.

#### **Lesson Purpose**

The purpose of this lesson is for students to recognize that the two interpretations of division still apply when dividing larger numbers and to use base-ten diagrams to interpret and represent division within 100.

Prior to this lesson, students have interpreted and represented division in terms of making equal-size groups. In this lesson, they revisit the two interpretations of division and recall that the divisor can be seen as either the number of groups or the size of each group.

Students use base-ten blocks and diagrams to analyze and represent division expressions such as  $55 \div 5$  and  $84 \div 7$ . They see that, depending on the numbers involved, one interpretation of division may be more useful or productive than the other.

Students also recognize that it is helpful to use tens and ones to make equal groups (for example, to think of 84 as 8 tens and 4 ones, rather than 84 ones), and to decompose tens into ones as needed.

#### Access for:

#### **③** Students with Disabilities

• Representation (Activity 1)

# **3** English Learners

MLR8 (Activity 1)

#### **Instructional Routines**

True or False (Warm-up)

#### **Materials to Gather**

• Base-ten blocks: Activity 1, Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

#### **Teacher Reflection Question**

How is students' knowledge of place value and the properties of operations helping them make sense of division strategies?

# **Cool-down** (to be completed at the end of the lesson)

3 5 min

Find the Value

# **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

# **Student-facing Task Statement**

Find the value of  $51 \div 3$ . Use base-ten blocks if they are helpful. Explain or show your reasoning.

# **Student Responses**

17. Sample responses: Students make 51 with base-ten blocks and make 3 equal groups, decomposing tens as needed. Students make a drawing that shows 51 placed into 3 groups or a drawing that shows 51 placed into groups of 3.

# ----- Begin Lesson

Warm-up © 10 min

True or False: Ones, Tens, Twenties

#### **Standards Alignments**

Addressing 3.NBT.A.3

The purpose of this True or False is to reinforce the relationship between tens and ones (that 1 ten is equal to 10 ones, or 1 group of 10 is 10 groups of 1). This will be helpful when students use base-ten blocks to represent division and decompose tens into ones to facilitate the process of dividing. It also allows students to practice finding the product of a one-digit whole number and a multiple of 10.

#### **Instructional Routines**

True or False

#### **Student-facing Task Statement**

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\bullet \quad 4 \times 10 = 40 \times 1$
- $\bullet \quad 4 \times 20 = 4 \times 2 \times 10$
- $8 \times 20 = 8 \times 2 \times 1$
- $8 \times 20 = 16 \times 10$

## **Student Responses**

- True: 4 tens is 40 ones. Both sides are equal to 40.
- True: The  $2 \times 10$  is 20, so both sides show 4 groups of 20.
- False: On the left is 8 groups of 20. On the right is 8 groups of 2, since  $2 \times 1$  is 2.
- True: 20 is  $2 \times 10$ , so  $8 \times 20$  is the same as  $8 \times 2 \times 10$ , which is  $16 \times 10$ .

#### Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

## **Activity**

- Share and record answers and strategy.
- Repeat with each statement.

## **Synthesis**

- "How can you justify your answer without finding the value of both sides?"
- Consider asking:
  - "Who can restate \_\_\_\_\_'s reasoning in a different way?"
  - "Does anyone want to add on to \_\_\_\_\_'s reasoning?"

**Activity 1** 

① 20 min

Divide with Base-Ten Blocks

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

The purpose of this activity is for students to use strategies based on place value to find quotients greater than 10. Students use base-ten blocks to represent quotients with single-digit divisors, for which it is intuitive to think of the divisor as the number of groups. In a later activity, students will be reminded that the divisor can also be interpreted as the size of each group.

Working with base-ten blocks encourages students to divide out the tens and then the ones, and to see that sometimes it is necessary to decompose one or more tens to finish putting the dividend into equal groups. When students represent a quotient using base-ten blocks they reason abstractly and quantitatively (MP2).

# **3** Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

#### **(iii)** Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most important when deciding how to divide up the blocks. Display the sentence frame: "The next time I use base-ten blocks to divide, I will look for/pay attention to . . . . "
Supports accessibility for: Visual-Spatial Processing, Attention

#### **Materials to Gather**

Base-ten blocks

## **Student-facing Task Statement**

- 1. Use base-ten blocks to represent each expression. Then, find its value.
  - a.  $55 \div 5$
  - b.  $45 \div 3$
- 2. Find the value of each expression. Use baseten blocks if you find them helpful.
  - a.  $63 \div 3$
  - b.  $84 \div 7$

#### Launch

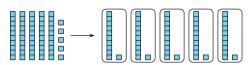
- Groups of 2
- Give each group base-ten blocks.
- "Use base-ten blocks to represent  $39 \div 3$ ."
- 1-2 minutes: independent work time
- Select a student who divided the blocks into 3 groups of 13 to share their final representation, such as:

c.  $100 \div 5$ 



#### **Student Responses**

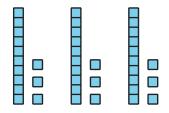
1. a. 11



b. 15



- 2. a. 21 (3 groups of 21 ones, or 3 groups of 2 tens and 1 one)
  - b. 12 (7 groups of 12, or 7 groups of 1 ten and 2 ones)
  - c. 20 (5 groups of 20 ones, or 5 groups of 2 tens)



- "Why are there 3 groups?" (We are dividing by 3.)
- "How could the blocks have been divided to end up like this?" (The tens were put into 3 groups and then ones placed one by one into 3 groups until none were left.)
- Highlight that the blocks could have been divided up by the tens and then the ones.

#### **Activity**

- "Work with your partner on the first problem."
- 5 minutes: partner work time
- Pause for a discussion.
- "What was different about using the blocks to find  $45 \div 3$  and using them to find  $55 \div 5$ ?" (For  $45 \div 3$ , it was necessary to decompose 1 ten to finish putting 45 into 3 equal groups. That's not necessary for  $55 \div 5$  because there was already the right number of tens and ones to make the 5 groups.)
- "Now, work independently to find the value of quotients in the second problem."
- 6–8 minutes: independent work time

### **Synthesis**

- Invite students to share their responses and reasoning for the last set of quotients.
- Ask students who used base-ten blocks or drew diagrams: "Was it necessary to decompose any of the tens into ones to divide?" (It wasn't necessary for 63 ÷ 3 because there was already the right number of tens and ones to put into 3

- groups. It wasn't necessary for  $100 \div 5$  because I started with 10 tens and there was already the right number of tens to put into 5 groups.)
- "Why was it necessary or helpful to decompose the tens in 84?" (After putting 7 tens in 7 groups, there's still 1 ten and 4 ones. The 1 ten couldn't be split into 7 groups.)

Activity 2 © 15 min

Different Ways to Show Division

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.C.7

The purpose of this activity is to show that the two meanings of division still apply when dividing larger numbers and that, in some cases, one interpretation may be more helpful than the other.

Students first analyze two ways of using base-ten blocks to represent  $65 \div 5$  and see that the divisor, 5, can be interpreted to mean the number of groups or the size of one group. They then consider how they might interpret and represent the divisor in other quotients. The reasoning here prepares students to reason more strategically as they divide larger numbers.

#### **Materials to Gather**

Base-ten blocks

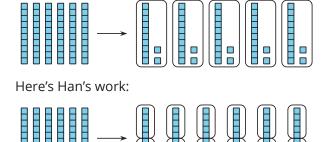
#### **Student-facing Task Statement**

Jada and Han used base-ten blocks to represent  $60 \div 5$ .

Here is Jada's work:

#### Launch

- Groups or 2–4
- Give base-ten blocks to each group.
- Ask students to keep their materials closed.
- "Use base-ten blocks to find the value of  $60 \div 5$ ."



- 1. Make sense of Jada's and Han's work.
  - a. What did they do differently?
  - b. Where do we see the value of  $60 \div 5$  in each person's work?
- 2. How would you use base-ten blocks so you could represent these expressions and find their value? Be prepared to explain your reasoning.
  - a.  $64 \div 4$ : Would you make 4 groups or groups of 4?
  - b.  $72 \div 6$ : Would you make 6 groups or groups of 6?
  - c.  $75 \div 15$ : Would you make 15 groups or groups of 15?

# **Student Responses**

- 1. Sample response:
  - a. Jada made 5 groups and put 1 ten in each. She decomposed the last ten into 10 ones and put 2 ones in each group. Han split each ten into 2 groups with 5 ones in each.
  - b. In Jada's work, it's the number in each group, which is 12. In Han's work, it's the number of groups, which is 12.
- 2. Sample response:
  - a. 4 groups, because it's quicker than making groups of 4 and counting the groups.
  - b. 6 groups, because it doesn't require

• 1–2 minutes: independent work time

#### **Activity**

- "Now take a look at Jada and Han's work in the activity. Which of them represented the division the same way you did?"
- "Work with your partner to make sense of Jada's and Han's work and complete the first problem."
- Pause for a brief discussion.
- "How was Jada's and Han's representation different? How did each of them interpret 60 ÷ 5?" (Jada saw the 5 as the number of groups. Han saw the 5 as the number in each group.)
- Poll the class on how they interpreted  $60 \div 5$  when they represented it during the launch.
- "Now, work independently on the second set of problems."
- 5 minutes: independent work time

#### **Synthesis**

- Invite students to share their responses and reasoning for the last set of problems.
- "How did you decide whether the divisor, the number we're dividing by, is the number of groups or the amount in each group?" (It depends on the number. In the first two problems, the divisor was 4 and 6, so it was easier to think about 4 groups and 6 groups. In the last problem, the divisor was 15. It was easier to think about how many groups of 15 are in 75 than to think about making 15 groups from 75.)

decomposing the 7 tens into ones.

c. Groups of 15, because it's quicker to make groups of 1 ten and 5 ones, than to make 15 groups and figuring out how many to put in each group.

# **Lesson Synthesis**

(10 min

"Today, we recalled that the divisor in a division expression can be seen as the number of groups or the size of each group."

Display:  $96 \div 8$ 

"If you are representing this quotient with base-ten blocks, would you put 9 tens and 6 ones in 8 groups, or would you put them into groups of 8?" (I would put them into 8 groups. Eight of the tens can go into 8 groups easily. The 1 remaining ten and 6 ones make 16 ones, so 2 ones go in each group. I would put them into groups of 8. I know 10 groups of 8 is 80, so that takes care of the 8 tens. The 1 remaining ten and 6 ones make 16, which is 2 groups of 8.)

### **Suggested Centers**

- Compare (1–5), Stage 4: Divide within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)
- Can You Draw It? (1–5), Stage 2: Grade 2 Shapes (Supporting)

----- Complete Cool-Down ----

#### **Response to Student Thinking**

Students do not determine the quotient of  $51 \div 3$ .

#### **Next Day Support**

 During the launch of the next day's activity, highlight important ideas from the previous lesson.

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# **Lesson 20: Strategies for Dividing**

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

#### **Teacher-facing Learning Goals**

- Analyze strategies for representing and reasoning about division.
- Divide within 100 using strategies based on place value and properties of operations.

#### **Student-facing Learning Goals**

Let's use different strategies to divide.

#### **Lesson Purpose**

The purpose of this lesson is for students to analyze representations and strategies for finding quotients with larger numbers and to divide within 100.

Previously, students use base-ten blocks, diagrams, and other representations or strategies to reason about division within 100. In this lesson, they extend and formalize this work to include writing a series of equations to find the value of a quotient.

In analyzing various strategies to represent division, students reinforce their understanding of place value, properties of operations, and the relationship of multiplication and division.

#### Access for:

### Students with Disabilities

Representation (Activity 2)

# **3** English Learners

MLR8 (Activity 1)

#### **Instructional Routines**

Number Talk (Warm-up)

#### **Materials to Gather**

Base-ten blocks: Activity 2

#### **Materials to Copy**

- Centimeter Grid Paper Standard (groups of 2): Activity 2
- Compare Stage 4 Division Cards (groups of 2): Activity 3

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Activity 3	10 min
Lesson Synthesis	5 min
Cool-down	5 min

### **Teacher Reflection Question**

Who has been sharing their ideas in class lately? Make a note of students whose ideas have not been featured in class and look for an opportunity for them to share their thinking in tomorrow's lesson.

# **Cool-down** (to be completed at the end of the lesson)

① 5 min

One More Division

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

# **Student-facing Task Statement**

Find the value of  $96 \div 6$ . Explain or show your reasoning.

# **Student Responses**

16. Sample responses:

- A drawing showing 6 groups with 1 ten and 6 ones in each group.
- I know that  $10 \times 6$  is 60 and  $6 \times 6$  is 36, and 60 + 36 = 96. 10 + 6 = 16.

----- Begin Lesson

Warm-up © 10 min

Number Talk: Multiplication and Division

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for using multiplication to help them divide. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to find the value of quotients.

#### **Instructional Routines**

Number Talk

#### **Student-facing Task Statement**

Find the value of each expression mentally.

- 3×5
- 6×5
- $\bullet$  10  $\times$  5
- 65 ÷ 5

## **Student Responses**

- 15: I just know it.
- 30: It's double 15 since 6 is double 3.
- 50: I just know it.
- 13: I know that 65 is 50 + 15. There are 10 groups of 5 in 50 since  $5 \times 10 = 50$  and 3 groups of 5 in 15 since  $5 \times 3 = 15$ . That's 13 groups of 5 in 65.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

## **Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### **Synthesis**

- "How does thinking about multiplication help you divide?" (I can think about what number multiplied by 5 will be 65. I can break that into smaller products I know.)
- Consider asking:
  - "Who can restate \_\_\_\_\_'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to \_\_\_\_\_'s strategy?"

Activity 1 © 15 min

Ways to Divide

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

The purpose of this activity is for students to transition from reasoning about division concretely or visually (using base-ten diagrams) to doing so more abstractly (by writing equations). It also reinforces the connections between multiplication and division.

Students make sense of three different strategies of dividing 78 by 3 and attend to the connections between the visual and numerical representations of the same quotient. As they do so, they practice reasoning quantitatively and abstractly (MP2).

During the synthesis, discuss how place value units play a role in all three strategies.

# **Access for English Learners**

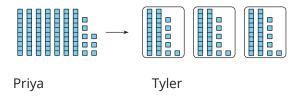
*MLR8 Discussion Supports.* Synthesis: Display Lin, Priya, and Tyler's strategies. As students share their observations, annotate the display to illustrate connections. For example, annotate where students see the divisor, dividend, and quotient on each diagram.

Advances: Listening, Representing

# **Student-facing Task Statement**

1. Lin, Priya, and Tyler found the value of 78 ÷ 3. Their work is shown. Make sense of each student's work.

Lin



#### Launch

- Groups of 2
- "Take a couple minutes to make sense of Lin, Priya, and Tyler's work."
- 2 minutes: quiet think time

## Activity

- "Work with your partner to make sense of Lin, Priya, and Tyler's work and to complete the activity."
- 7–8 minutes: partner work time
- Monitor for students who:

$$3 \times 10 = 30$$
  $3 \times 20 = 60$   
 $3 \times 10 = 30$   $3 \times 6 = 18$   
 $3 \times 6 = 18$   
 $3 \times 26 = 78$   $20 + 6 = 26$ 

- 2. How are the three students' work alike?
- 3. How are they different?

#### **Student Responses**

- 1. Answers vary.
- 2. Sample responses:
  - They all saw the 3 as the number of groups and found the size of each group. They all got 26 for the result.
  - We can see the tens and ones, and 3,
     20, and 6, in everyone's work.
- 3. Sample responses:
  - Lin uses base-ten diagrams. Priya and Tyler both wrote equations.
  - Lin started with 7 tens and 8 ones and put them into 3 equal groups. To do that, she decomposed 1 of the tens into ones.
  - Priya knew that  $3 \times 10 = 30$  and another  $3 \times 10$  is another 30, which makes 60. To get to 78, 18 is needed, and 18 is  $3 \times 6$ . Each group has 10 + 10 + 6.
  - Tyler knew that 3 groups of 20 make 60, and that adding another 3 groups of 6 gives 18 more to get to 78. He added 20 and 6.

- make connections between the equations and the base-ten diagram
- recognize how Tyler's equations and reasoning are different from Priya's equations
- Identify students who can explain these connections or distinctions to share during the synthesis.

#### **Synthesis**

- Invite students to share how Lin, Priya, and Tyler's work are alike and how they are different.
- Select previously identified students to share additional connections that they noticed.
- "Why does it make sense that Priya and Tyler wrote multiplication equations to find the value of a quotient?" (Multiplication and division can represent the same equalgroups situation. To divide 78 by 3 is to find how many are in each of 3 groups or how many groups of 3 there are. We can multiply up to 78 to find the answer.)
- Consider asking: "What new ideas about dividing numbers did you learn and would like to try? Talk to a new partner about why you'd like to try them."

**Activity 2** 

① 15 min

How Would You Divide?

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

The purpose of this activity is for students to practice finding the value of division expressions using any strategy that makes sense to them. They may divide the dividend into equal groups or use the divisor to multiply up to the given dividend. They may choose to represent the division or multiplication with base-ten blocks or by drawing diagrams.

During the synthesis, highlight strategies that rely on place value, properties of operations, and the relationship between multiplication and division (MP7).

#### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Synthesis: Make connections between representations visible. Elicit from students the connections between the different strategies and representations shared by students.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

#### **Materials to Gather**

Base-ten blocks

## **Student-facing Task Statement**

Find the value of each quotient. Explain or show your reasoning. Organize it so it can be followed by others.

- 1.  $80 \div 5$
- 2.  $68 \div 4$
- 3.  $91 \div 7$

If you have time: Eighty-four students on a field trip are put into groups. Each group has 14 students. How many groups are there?

#### **Student Responses**

- 1. 16. Sample response: 5 groups of 10 is 50. 5 groups of 6 is 30. 10 + 6 = 16
- 2. 17. Sample response:

#### **Materials to Copy**

Centimeter Grid Paper - Standard (groups of 2)

#### Launch

- Groups of 2
- Give students access to base-ten blocks and grid paper.

#### **Activity**

- "Find the value of each quotient. You can use whatever strategy or representation you prefer."
- 8–10 minutes: independent work time
- Monitor for a variety of strategies and representations. Identify students to share in the synthesis.
- "Share your favorite way to divide with your partner."
- 2–3 minutes: partner discussion

$$4 \times 10 = 40$$

$$4 \times 5 = 20$$

$$4 \times 2 = 8$$

$$4 \times 17 = 68$$

3. 13. Sample response: If I put 70 in 7 groups, that's 10 in each group. If I put the remaining 21 into the 7 groups, that's 3 more in each group. 10 + 3 = 13

If you have time: 6 groups. Sample response:

- A base-ten drawing showing 6 tens and 24 ones arranged into 6 groups of 1 ten and 4 ones each.
- I know  $2 \times 14 = 28$  and  $4 \times 14$  is twice 28, which is 56. Adding another 28 makes 84, so  $6 \times 14 = 84$ .

$$2 \times 14 = 28$$

$$2 \times 14 = 28$$

$$2 \times 14 = 28$$

$$6 \times 14 = 84$$

#### **Synthesis**

- Have previously selected students share their responses. Display or record the strategies and representations students use.
- For each problem, consider polling the class on the strategy they used.

# **Activity 3 (optional)**

(10 min

Compare, Divide within 100

# **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

The purpose of this optional activity is for students to practice evaluating division expressions in order to make comparisons. Compare is a center that focuses on the procedural skills needed to solve single- and multi-step word problems. In this stage, students will use division to evaluate and compare quotients within 100.

This stage of the Compare center is used in grades 3, 4, and 5. When used in grade 3, remove the cards with two-digit divisors.

#### **Materials to Copy**

Compare Stage 4 Division Cards (groups of 2)

#### **Required Preparation**

• Create a set of cards from the Instructional master for each group of 2. Remove the cards with two-digit divisors.

#### **Student-facing Task Statement**

Play Compare with 2 players.

- 1. Shuffle the cards and split the deck between the players.
- 2. Each player turns over a card.
- 3. Compare the values. The player with the greater value keeps both cards.
- 4. Play until you run out of cards. The player with the most cards at the end of the game wins.

#### **Student Responses**

No response required.

#### Launch

- Groups of 2
- Give each group of students a set of cards from the Instructional master.
- "Now let's play Compare to practice what you learned about dividing numbers."
- "Take a minute to read the directions with your partner."
- 1 minute: quiet think time
- Answer any questions about the directions.
   If needed, play a turn with the class.

#### **Activity**

- 5–7 minutes: partner work time
- If time permits, provide extra time to play Compare.

## **Synthesis**

- Display:  $92 \div 4$  and  $72 \div 3$ .
- "Suppose these are the two cards you draw. How would you decide which expression has the greatest value?"

# **Lesson Synthesis**

⑤ 5 min

"Today, we used a variety of strategies and representations to divide larger numbers. How do you like to divide larger numbers? Why?" (I like to use multiplication because I can use the multiplication facts I know to divide. I like to divide in parts because I can think about smaller division facts I know. I like to

use drawings of base-ten blocks and think about putting them into equal groups because I can use tens and ones to divide.)

#### **Suggested Centers**

- Compare (1–5), Stage 4: Divide within 100 (Addressing)
- How Close? (1–5), Stage 5: Multiply to 100 (Addressing)
- Can You Draw It? (1–5), Stage 2: Grade 2 Shapes (Supporting)

### **Response to Student Thinking**

Students do not find the value of  $96 \div 6$ .

#### **Next Day Support**

 Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

# **Lesson 21: Solve Problems Using the Four Operations**

#### **Standards Alignments**

Addressing 3.OA.D.8

#### **Teacher-facing Learning Goals**

- Represent two-step word problems using equations with a letter standing for the unknown quantity.
- Solve two-step word problems using the four operations.

#### **Student-facing Learning Goals**

 Let's represent and solve problems using all four operations.

#### **Lesson Purpose**

The purpose of this lesson is for students to represent and solve two-step word problems using the four operations.

Previously, students have represented and solved two-step word problems using addition, subtraction, multiplication, and division with smaller numbers. In this lesson, students continue to deepen their understanding of two-step word problems as they consider what they need to know to solve problems and think about the relationship between numbers in a problem. Students write equations with a letter standing for the unknown quantity to represent these problems.

This lesson has a Student Section Summary.

#### **Access for:**

#### **1** Students with Disabilities

Action and Expression (Activity 1)

# **3** English Learners

MLR8 (Activity 2)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

### **Lesson Timeline**

Warm-up	10 min
Activity 1	20 min

## **Teacher Reflection Question**

Check in with your norms and routines. Are they promoting engagement from all your students? Are there any adjustments you might make so

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Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

that all students do math tomorrow?

# **Cool-down** (to be completed at the end of the lesson)

© 5 min

Apples at the Farm Stand

#### **Standards Alignments**

Addressing 3.OA.D.8

#### **Student-facing Task Statement**

A booth at the apple orchard has 225 apples. 165 apples are not in baskets. The rest of the apples are in 6 baskets with the same number of apples in each basket. How many apples are in each basket?

- 1. Write an equation to represent this situation. Use a letter for the unknown quantity.
- 2. Solve the problem. Explain or show your reasoning.

# Student Responses

- 1.  $165 + (6 \times n) = 225$  or  $225 165 = (6 \times n)$  or  $(225 165) \div 6 = n$
- 2. 10 apples. Sample response: I subtracted 165 from 225 to find out how many apples were in baskets. It was 60 apples. I know that  $6\times10$  is 60, so there would be 10 apples in each basket.

----- Begin Lesson -----

Warm-up © 10 min

Notice and Wonder: Apples Again

# **Standards Alignments**

Addressing 3.OA.D.8

The purpose of this warm-up is to elicit the idea that many different questions could be asked about this situation, which will be useful when students solve problems in a later activity. While students may notice and wonder many things about this situation, the various questions that could be asked about the situation are the important discussion points.

#### **Instructional Routines**

Notice and Wonder

### **Student-facing Task Statement**

What do you notice? What do you wonder?

A farmer picked some apples.

Some of the apples are packed into boxes and some are not.

#### **Student Responses**

Students may notice:

- We don't have enough information to solve for anything.
- We don't know how many apples there are.
- We don't know how many boxes were packed.
- We don't know how many apples are not in boxes.

Students may wonder:

- How many apples were there to start with?
- How many boxes are there?
- How many apples are in each box?
- How many apples are not in a box?
- Are all the boxes the same size?

#### Launch

- Groups of 2
- Display the situation.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

- "What does it mean that some apples are packed into boxes and some are not?" (Some apples are in groups and some apples are just loose in one big group.)
- "What questions could we ask about this situation?" (How many apples did the farmer pick? How many boxes had apples in them? How many apples were in each box?)

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# **Activity 1**

① 20 min

Apple Adventure

♣ ↔ ♣ PLC Activity

#### **Standards Alignments**

Addressing 3.OA.D.8

The purpose of this activity is for students to think about what they need to know to solve twostep word problems. Students choose numbers that make sense together to complete the problem from the warm-up. They articulate relationships between the quantities in the problem to justify their number choices. If students quickly find a combination of numbers that work, encourage them to see if there are other possibilities or to write a completed situation with the numbers they have chosen.

Students who do not choose a matching set of numbers quickly make sense of and persevere in solving the problem as they consider the relationship between the different quantities and the restrictions that puts on which numbers can describe the situation (MP1).

#### Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to plan a strategy, including the tools they will use, for completing the chart. If time allows, invite students to share their plan with their partner before they begin.

Supports accessibility for: Conceptual Processing, Organization

# **Student-facing Task Statement**

A farmer picked some apples. Some of the apples are packed into boxes and some are not.

From the list, choose 4 numbers that would make sense together in this situation. Write your choices in the table. Be ready to explain how your numbers make sense together.

(	400	300	240	12
	350	290	230	10
	340	280	170	5

#### Launch

- Groups of 2 and 4
- Keep the situation from the warm-up displayed.
- "Suppose the boxes the farmer packs are all the same size."

#### **Activity**

"A list of numbers is shown in the activity.
 Work with your partner to choose 4
 numbers that would make sense together
 in this situation. If you find one

total number	number of apples not in		apples in
of apples	boxes	boxes	each box



#### **Student Responses**

Sample responses:

total number of apples	number of apples not in boxes	number of boxes	number of apples in each box
400	350	5	10
350	230	10	12
290	240	10	5
350	290	12	5

combination of numbers that works, you can look for other combinations."

- 8–10 minutes: partner work time
- Groups of 4
- "Share with another group of students how your number choices make sense."
- 2–3 minutes: small-group discussion

# **Synthesis**

 Display this partially completed row in the table, such as:

total number of apples	number of apples not in boxes		number of apples in each box
200	152	8	

- "If you were given this information, how would you find the number of apples in each box?" (I could subtract 152 from the total of 200 and divide the answer by 8.)
- "What equation can we write to represent the situation in this example? Let's use a letter for the quantity that we don't know."
- 1 minute: quiet think time
- Record equations that students wrote, for instance:

$$\circ$$
 (200 – 152) ÷ 8 =  $n$ 

$$\circ$$
 8 ×  $n$  + 152 = 200

$$\circ$$
 152 + 8 ×  $n = 200$ 

Activity 2 © 15 min

**Apple Days** 

#### **Standards Alignments**

Addressing 3.OA.D.8

The purpose of this activity is for students to represent a problem with an equation using a letter for the unknown quantity and solve the problem. Students should be encouraged to use whatever strategy or representation makes sense to them.

The synthesis focuses on student thinking for the first problem. Students might represent the situation with:

- a tape diagram or an area diagram
- an equation that uses multiplication
- an equation that uses division

If students struggle to get started on a problem, encourage them to create a drawing or diagram. Students may also represent the situation or solve the problem before writing an equation if that makes more sense to them. While this activity is focused on independent practice, students can discuss with a partner if needed.

# **Access for English Learners**

*MLR8 Discussion Supports.* Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.

Advances: Reading, Representing

# **Student-facing Task Statement**

Tyler and Clare are helping with a festival at an apple orchard.



- 1. Tyler is stacking apples to sell at the event. There are 85 apples for his display. He has already made 5 rows of 10 apples. How many apples are left?
  - a. Write an equation with a letter for the unknown quantity to represent this situation.
  - b. Solve the problem. Explain or

#### Launch

- "Now we're going to solve some problems about an event at the apple orchard."
- Survey the class on their familiarity with events or activities at farms or orchards.
- If students are familiar, ask: "What are some things you might see or do at a farm or an orchard?"
- If students are unfamiliar, share some activities that might take place at an orchard. Consider showing some images of a market or an event at an orchard.
- 1–2 minutes: partner discussion

show your reasoning.

- 2. Clare is helping sell baked goods at the event. A customer buys 8 brownies that cost \$3 each. Clare adds that money to the cash box and now there is \$125 in the cash box. How much money was in the cash box before that purchase?
  - a. Write an equation with a letter for the unknown quantity to represent this situation.
  - b. Solve the problem. Explain or show your reasoning.
- 3. The market at the orchard had 200 jars of applesauce for sale. At the end of the event, 184 jars had been sold. The rest of the jars were shared equally among 4 people who work there. How many jars of applesauce did each person get?
  - a. Write an equation with a letter for the unknown quantity to represent this situation.
  - b. Solve the problem. Explain or show your reasoning.

# **Student Responses**

1. a. 
$$85 - (5 \times 10) = a$$
 or  $(5 \times 10) + a = 85$ 

b. 35 apples

2. a. 
$$m + (8 \times 3) = 125$$
 or  $125 - (8 \times 3) = m$ 

b. \$101

3. a. 
$$(200 - 184) \div 4 = t$$
 or  $200 = 184 + (4 \times t)$ 

b. 4 jars of applesauce

Share responses.

#### **Activity**

- 8–10 minutes: independent work time
- Monitor for:
  - a variety of strategies students use to solve or represent each problem
  - lingering questions or common misconceptions and how students overcome them
- Consider asking:
  - "How could you represent that?"
  - "What information do you know that might help you?"

#### **Synthesis**

 Select students who used different strategies to share their responses and reasoning for each problem.

# **Advancing Student Thinking**

If students say they aren't sure how to get started on the problem, consider asking: "What is the problem about?" and "How could you represent the problem?"

# **Lesson Synthesis**

① 10 min

"What did you find most challenging about solving these problems?" (There's a lot of information to keep track of. I have a hard time understanding how all the numbers are related to each other.)

"What ideas do you have for overcoming those challenges?" (drawing a diagram and labeling it with the numbers that we know, reading the problem carefully and acting it out, organizing what we know and don't know in a table)

"How did you know if your answer made sense?" (I put the number back into the problem and did the math to check if it makes sense. I made an estimate first so that I had an idea of about how large the answer should be.)

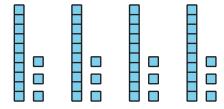
# **Student Section Summary**

In this section, we divided larger numbers and solved problems that involve division.

We used base-ten blocks, diagrams, and equations to represent the numbers we divided. To help us divide, we used what we know about place value, equal groups, and the relationship between multiplication and division.

For example, here are some ways we could find the value of  $52 \div 4$ :

• Put 5 tens and 2 ones into 4 equal groups.



- Think about how many groups of 4 are in 52.
- Use multiplication facts and write equations.

10 groups of 4 make 40. 3 groups of 4 make 12. 13 groups of 4 make 52.  $4 \times 10 = 40$  $4 \times 3 = 12$ 

$$10 + 3 = 13$$
  
 $4 \times 13 = 52$ 

At the end of the section, we used all four operations to solve problems.

----- Complete Cool-Down

# **Response to Student Thinking**

Students don't find a solution to the problem.

### **Next Day Support**

 Before the next day's warm-up, pass back the cool-down and work in small groups to make corrections.

# **Lesson 22: School Community Garden (Optional)**

#### **Standards Alignments**

Addressing 3.MD.C.7, 3.OA.A.2, 3.OA.A.3, 3.OA.A.4, 3.OA.D.8

Building Towards 3.OA.A.3

#### **Teacher-facing Learning Goals**

- Represent and solve "How many groups?" and "How many in each group?" problems in a real-world context.
- Solve two-step problems in a real-world context.

#### **Student-facing Learning Goals**

Let's plan a school garden.

#### **Lesson Purpose**

The purpose of this lesson is to use multiplication and division to model a real-world design problem.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling.

In previous lessons, students multiplied and divided numbers within 100. They related division to multiplication by understanding division as an unknown factor problem. They used properties of operations and place value understanding to develop strategies to multiply and divide within 100.

In this lesson, they use their understanding of multiplication and division to plan a school garden. In the first activity, students solve problems that involve the spacing between plants in a row and between the rows. In the second activity, they plan a garden. They choose the types of vegetables and fruit to grow, how many plants to grow, and the arrangement of the plants. They also consider the yield of the garden. Students represent their plans with diagrams and expressions or equations.

Students model with mathematics (MP4) as they consider constraints, make assumptions and decisions about quantities, think about how to represent the relationships among quantities, and check their solutions in terms of the situation.

#### Access for:

#### **③** Students with Disabilities

• Action and Expression (Activity 2)

# **S** English Learners

• MLR7 (Activity 2)

#### **Instructional Routines**

Notice and Wonder (Warm-up)

## **Materials to Copy**

 Centimeter Grid Paper - Standard (groups of 2): Activity 2

#### **Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	25 min
Lesson Synthesis	10 min

### **Teacher Reflection Question**

In what ways has the math community you are fostering improved? What actions can you take to improve areas that need work?

----- Begin Lesson

Warm-up © 10 min

Notice and Wonder: Garden

# **Standards Alignments**

Building Towards 3.OA.A.3

The purpose of this warm-up is to introduce the context of gardening, which will be useful when students plan a garden in a later activity. While students may notice and wonder many things about this image, the spacing of the plants is the important discussion point.

#### **Instructional Routines**

Notice and Wonder

Unit 4 Lesson 22 201

## **Student-facing Task Statement**

What do you notice? What do you wonder?



#### **Student Responses**

Students may notice:

- There are plants in rows.
- Each plant is spaced out in the row.
- There is an orange watering can.
- The different plants are different heights.

Students may wonder:

- What is being planted?
- Why are the plants spaced or arranged that way?
- How big will the plants get?

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### **Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### **Synthesis**

- "Many families, schools, or communities have gardens to grow their own fruits and vegetables."
- "If we were to start a school garden, what are some things we would have to think about?" (What plants do we want to grow? How many? How would we arrange the plants? How much space would the garden take up? Do we have enough space? How many fruits and vegetables would we harvest?)
- "This photograph shows a strawberry patch at the beginning of the season. Plants need to be spaced out so they have enough room to grow."
- "Today we'll learn about some plants and think about the arrangements that help them grow the best and plan part of a community garden."

**Activity 1** 

① 15 min

Produce

# Standards Alignments

Addressing 3.OA.A.2, 3.OA.A.3, 3.OA.A.4

The purpose of this activity is for students to use multiplication and division to solve problems about growing strawberries. They draw diagrams and write expressions or equations to represent each situation, and solve the problem. The given context encourages students to think of equal groups, arrays, or rectangular areas.

In previous lessons, students have been asked to write either an equation or expression. This activity gives them an opportunity to make a choice. Alternatively, teachers may opt to instruct students to write either an equation or expression.

#### **Student-facing Task Statement**

For each situation, draw a diagram and write an equation or expression.

- 1. A strawberry patch has 7 rows with 8 strawberry plants in each row.
  - a. How many strawberry plants are in the patch?



- b. To grow strawberries in the best way, the rows should be 4 feet apart. Each plant in the row should be 2 feet apart. How long and wide is the strawberry patch?
- c. You can harvest 12 strawberries per plant. How many strawberries will grow in each row?
- With your partner, take turns explaining where you see the numbers in the expression or equation you wrote in your diagram.

## **Student Responses**

- 1. Sample responses:
  - a. 56 plants. Students draw a 7 by 8 array.

 $7 \times 8, 8 \times 7, \text{ or } 8 \times 7 = 56.$ 

b. 24 feet and 14 feet or 28 feet and 16

#### Launch

- Groups of 2
- "Strawberries are grown in patches arranged in rows."
- "In this activity, you'll see how strawberries are arranged and how many fruits a strawberry plant will produce. In the next activity you will plan your own garden."

#### **Activity**

- 2 minutes: independent work time
- 10 minutes: partner work time
- Monitor for students who:
  - write multiplication or division expressions or equations
  - are able to represent the same situation with both multiplication and division

### **Synthesis**

- Invite previously selected students to share.
- "How did the diagrams help you think about the problem?"
- "Where do you see the numbers from the equation in the diagram?"

feet. Students draw a diagram of a rectangular area. (Students may account for various amounts of space on the ends or sides of the strawberry patch in addition to the space in between the plants or rows resulting in expressions or equations with values greater than 24 or 14.)

$$6 \times 4 = 24, 7 \times 2 = 14,$$
  
 $(6 \times 4) + 2 + 2, 7 \times 2 + 1 + 1 = 16.$ 

c. 96 strawberries. Students draw 8 groups with 1 tens and 2 ones in each group.

$$(8 \times 10) + (8 \times 2) = 80 + 16,$$
  
 $12 \times 8 = 96.$ 

2. No response required.

**Activity 2** ① 25 min

Plan the Garden

# **Standards Alignments**

Addressing 3.MD.C.7, 3.OA.A.3, 3.OA.A.4, 3.OA.D.8

The purpose of this activity is to use students' experience with multiplication and division within 100 to plan a school garden. In this activity, students make choices about which produce to grow. The choices are guided by some constraints, such as a desired yield. Students draw diagrams to represent how the plants are arranged in the garden and how they bear fruit. If needed, students can be provided with images to see how the different plants grow.

# Access for English Learners

MLR7 Compare and Connect: Invite groups to prepare a visual display that shows the strategies they used to plan part of the community garden. Encourage students to include details that will help others interpret their thinking. For example, specific language, using different colors, shading, arrows, labels, notes, diagrams or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students, "What kinds of additional details or language helped you understand the displays? Were there any additional details or language that you have questions about?"

Advances: Representing, Conversing

#### Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to plan a strategy, including the tools they will use for designing their garden. If time allows, invite students to share their plan with another group.

Supports accessibility for: Organization, Social-Emotional Functioning

# Materials to Copy

Centimeter Grid Paper - Standard (groups of 2)

# **Student-facing Task Statement**

- 1. Read the information about some plants you could grow in a garden. Then, circle 2 plants to grow in your part of the school garden.
  - a. strawberries
  - b. cantaloupe
  - c. zucchini
  - d. tomatoes
  - e. pinto beans
  - f. potatoes
- 2. Plan your garden. Both of your plants should harvest between 50–100 fruits or vegetables.
  - a. How many of each plant will you grow?

#### Launch

- Groups of 2 or 4
- "In this activity, you will plan part of the school garden."
- Give students access to grid paper.

#### **Activity**

- "Work with your partner to complete the activity. Pay attention to the amounts of fruits or vegetables your garden should produce."
- 15 minutes: partner work time
- As students are working, consider asking:
  - "What choices did you make?"
  - "What were your reasons?"
  - "How many fruits and vegetables will you be able to harvest?"

Unit 4 Lesson 22 205

- b. Predict how many fruits or vegetables you will harvest. Show or explain your reasoning.
- 3. Make a diagram that shows how the plants are arranged and how much space is needed.

#### **Growing Requirements**





strawberries

cantaloupes

- Grow in patches
- Space rows: 4 feet
   apart
- Space plants: 2 feet apart
- Each plant produces 12 strawberries.
- Grow on vines
- Space rows: 4 feet apart
- Space plants: 1 foot apart
- Each plant produces about 8 cantaloupes.





zucchini

- tomatoes
- Grow on vines
- Space rows: 5 feet
   apart
- Space plants: 1 foot apart
- Each plant produces about 8 zucchini.
- Grow on vines
- Space rows: 4 feet apart
- Space plants: 2 feet apart
- Each plant produces about 20 tomatoes.

- Monitor for students who:
  - write expressions or equations

### **Synthesis**

- Invite students to share their plans.
- "How did the growing information help you plan your garden?" (I was able to see how to arrange the rows and columns in my garden. The growing information told me how much fruit I will get.)





#### pinto beans

#### potatoes

- Grow on bushes in pods
- Space rows: 2 feet apart
- Space plants: 1 foot apart
- Each plant produces 20–25 pods and each pod produces about 5 beans.

- Grow in rows
- Space rows: 2–3 feet apart
- Space plants: 1 foot apart
- Each plant produces 5–10 potatoes.

## **Student Responses**

Sample responses:

- 1. Strawberries and cantaloupes.
- 2. a. 6 strawberry plants and 8 cantaloupe plants.
  - b.  $6 \times 12 = 72$ ,  $8 \times 8 = 64$ . There would be 72 strawberries and 64 cantaloupes.
- 3. Students draw an appropriate diagram such as an array or area diagram that shows the arrangement and space required for the field based on the growing information.

# **Lesson Synthesis**

(10 min

"Today, many of you used multiplication and division to arrange your plants and find out how much you could harvest from your garden."

"Besides arrangements of plants and the harvest, what else should we consider to maintain a garden?" (How much water, sun, and soil each plant needs. What to do if a plant is unhealthy.)

Unit 4 Lesson 22 207

# **CKMath**™ Core Knowledge MATHEMATICS™

# Family Support Materials

## **Family Support Materials**

### **Relating Multiplication to Division**

In this unit, students make sense of division and learn to multiply and divide whole numbers within 100. They also use the four operations to represent and solve two-step word problems. Students work toward these end-of-year goals:

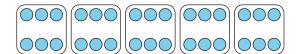
- fluently multiply and divide within 100
- know from memory all products of two one-digit numbers

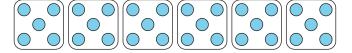
#### **Section A: What is Division?**

In this section, students think about division in terms of equal-size groups, just as they have done with multiplication. For instance, the expression  $30 \div 5$  can represent putting 30 objects into 5 equal groups, or putting 30 objects into groups of 5. Students see that, in general, dividing can mean answering the question "how much is in one group?" or "how many equal groups can be made?"

30 objects put into 5 equal groups

30 objects put into groups of 5





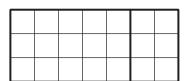
### **Section B: Relating Multiplication and Division**

In this section, students make connections between the result of division and the missing factor in a multiplication equation.

For example, the value of  $30 \div 6$  is the missing factor in  $\underline{\phantom{a}} \times 6 = 30$ . This understanding helps students recognize division facts based on the multiplication facts they know.

Students also learn to use properties of operations to multiply. For example, if they know  $3 \times 7$ , they also know  $7 \times 3$ .

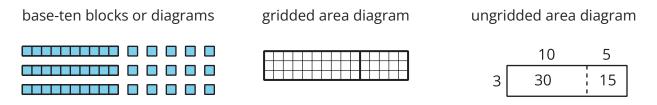
They can also decompose (or break apart) the 7 in  $7 \times 3$  into 5 and 2, and then find  $(5 \times 3) + (2 \times 3)$ . An area diagram can show this strategy for multiplying.



### **Section C: Multiplying Larger Numbers**

In this section, students use different strategies to multiply larger numbers. First, they multiply a single-digit number by a multiple of 10, relying on what they know about place value. For instance,  $2\times40$  means 2 groups of 4 tens, or  $2\times4\times10$ . Then, they multiply a single-digit number by other two-digit numbers.

Students see that it is helpful to break apart the two-digit numbers by place value, into tens and ones. For example,  $3 \times 15$  can be calculated by finding  $3 \times 10$  and  $3 \times 5$ . They use base-ten blocks or diagrams and area diagrams (with and without a grid) to help them find such products.



### **Section D: Dividing Larger Numbers**

In this section, students divide larger numbers. They continue to use the relationship between multiplication and division and their understanding of place value to find quotients. For example, to find the value of  $78 \div 3$ , they may think about putting 78 in 3 equal groups and use multiplication to find what is in each group.

$$3 \times 10 = 30$$
  $3 \times 20 = 60$   
 $3 \times 10 = 30$   $3 \times 6 = 18$   
 $3 \times 6 = 18$   $20 + 6 = 26$   
 $10 + 10 + 6 = 26$ 

#### Try it at home!

Near the end of the unit, ask your student to find answers to the following problems:

- 6 × 16
- 98 ÷ 7

Questions that may be helpful as they work:

- How did you break up the problem to make it easier for you to solve?
- Can you rewrite the division problem as a multiplication problem?

## **CKMath**™ Core Knowledge MATHEMATICS™

## **Unit Assessments**

Check Your Readiness A, B, C and D End-of-Unit Assessment

## Relating Multiplication to Division: Section A Checkpoint

1	l. Noah has 36 books.	There are	4 shelves	on his	bookshelf.	He puts	the same	number	of
	books on each shelf	f.							

- a. Write a division expression to represent the situation.
- b. How many books are on each shelf? Explain or show your reasoning.
- 2. There are 35 flowers in the garden. There are 7 flowers in each row.
  - a. Make a drawing to represent the situation.

b. How many rows of flowers are there?

## Relating Multiplication to Division: Section B Checkpoint

1. There are 24 students in the choir. They are standing in 3 equal rows. How many students are in each row? Select **all** equations that represent this situation.

A. 
$$3 \times 12 = ?$$

B. 
$$3 \times ? = 24$$

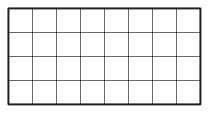
C. 
$$24 \times 3 = ?$$

D. 
$$? \div 24 = 3$$

E. 
$$24 \div 3 = ?$$

2. Han knows  $4 \times 5 = 20$  and  $4 \times 3 = 12$ .

Mark or shade the diagram to show how Han can use these facts to find the value of  $4 \times 8$ .



3. Find the value of each product.

a. 
$$5 \times 3$$

b. 
$$6 \times 3$$

$$c.6 \times 6$$

## Relating Multiplication to Division: Section C Checkpoint

1. Find the value of  $7 \times 60$ . Explain or show your reasoning.

2. Explain how each diagram represents  $4 \times 13$ .

Α



В



Use one of the diagrams to find the value of  $4 \times 13$ .

3. Andre began reading a book that has 192 pages. If he reads 15 pages for each of the next 5 days, how many pages will he still have to read to finish the book?

a. Write an equation to represent the situation. Use a letter for the unknown quantity.

b. Solve the problem. Explain or show your reasoning.

## Relating Multiplication to Division: Section D Checkpoint

1. Find the value of each quotient. Explain or show your reasoning.

a. 
$$60 \div 4$$

b. 
$$96 \div 6$$

2. Noah has 117 building blocks. He builds a shape with 25 blocks. Then, he puts the rest of the blocks in 4 bins, with the same number of blocks in each bin.

How many blocks are in each bin? Explain or show your reasoning.

## Relating Multiplication to Division: End-of-Unit Assessment

- 1. Select all expressions that have a value of 24.
  - A.  $2 \times 12$
  - B.  $3 \times 8$
  - C.  $4 \times 5$
  - D.  $6 \times 4$
  - E.  $20 \times 4$
- 2. Select **all** situations that match the equation  $48 \div 6 = ?$ .
  - A. There are 48 volleyball players on 6 equal teams. How many players are on each team?
  - B. There are 48 basketball teams at the tournament. There are 6 players on each team. How many basketball players are at the tournament?
  - C. There are 48 kids swimming in the pool. Then 6 kids leave the pool. How many kids are swimming in the pool now?
  - D. There are 6 buses. Each bus has 48 students on it. How many students are there altogether?
  - E. There are 48 oranges in the box. Jada puts 6 oranges in each bag. How many bags does Jada need for all the oranges?

3. There are 5 rows of flowers in the garden. Each row has the same number of flowers. There are 30 flowers altogether.

How many flowers are in each row? Select **all** equations that represent the situation.

- A.  $5 \times ? = 30$
- B.  $5 \times 30 = ?$
- C.  $30 \times ? = 5$
- D.  $5 \div ? = 30$
- E.  $30 \div 5 = ?$
- F.  $? \div 5 = 30$
- 4. Lin covers her desk with 77 sticky notes. The sticky notes are in 7 equal rows. How many sticky notes are in each row?
  - a. Write a division equation to represent the situation. Use a symbol for the unknown quantity.
  - b. Write a multiplication equation to represent the situation. Use a symbol for the unknown quantity.
  - c. Solve the problem. Explain or show your reasoning.

5. Find the value of each expression.

a. 
$$50 \div 5$$

b. 
$$36 \div 2$$

c. 
$$75 \div 5$$

6. Find the value of each expression. Explain or show your reasoning.

a. 
$$11 \times 8$$

b. 
$$7 \times 40$$

$$c. 5 \times 13$$

7. There are 278 students at the school. 197 go home by foot or by car. The rest take a school bus.	
a. How many students take the bus home? Explain or show your reasoning.	
b. Each bus holds 35 students. Explain why the students who take the bus cannot all fi in 2 buses.	t
c. There are 3 buses and each bus carries the same number of students. How many students are in each bus? Explain or show your reasoning.	

## **CKMath**™ Core Knowledge MATHEMATICS™

## Assessment Answer Keys

Check Your Readiness A, B, C and D End-of-Unit Assessment

## Assessment Answer Keys Assessment: Section A Checkpoint

#### Problem 1

#### **Goals Assessed**

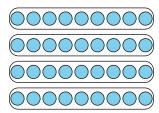
• Represent and solve "How many groups?" and "How many in each group?" problems.

Noah has 36 books. There are 4 shelves on his bookshelf. He puts the same number of books on each shelf.

- a. Write a division expression to represent the situation.
- b. How many books are on each shelf? Explain or show your reasoning.

#### Solution

- a.  $36 \div 4$
- b. 9 books. Sample response:



#### Problem 2

#### **Goals Assessed**

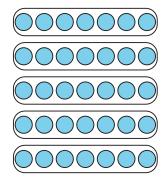
Represent and solve "How many groups?" and "How many in each group?" problems.

There are 35 flowers in the garden. There are 7 flowers in each row.

- a. Make a drawing to represent the situation.
- b. How many rows of flowers are there?

### Solution

a. Sample response:



b. 5

2 Grade 3

## **Assessment: Section B Checkpoint**

#### Problem 1

#### **Goals Assessed**

• Understand division as a missing factor problem.

There are 24 students in the choir. They are standing in 3 equal rows. How many students are in each row? Select **all** equations that represent this situation.

- A.  $3 \times 12 = ?$
- B.  $3 \times ? = 24$
- C.  $24 \times 3 = ?$
- D.  $? \div 24 = 3$
- E.  $24 \div 3 = ?$

#### Solution

["B", "E"]

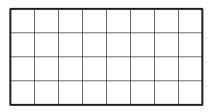
#### Problem 2

#### **Goals Assessed**

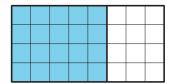
• Use properties of operations to develop fluency with single-digit multiplication facts, and their related division facts.

Han knows  $4 \times 5 = 20$  and  $4 \times 3 = 12$ .

Mark or shade the diagram to show how Han can use these facts to find the value of  $4 \times 8$ .



#### Solution



The shaded part is  $4 \times 5$  or 20 and the unshaded part is  $4 \times 3$ , or 12, so the total number of small squares is 20 + 12 or 32. It's also  $4 \times 8$ .

#### Problem 3

#### **Goals Assessed**

• Use properties of operations to develop fluency with single-digit multiplication facts, and their related division facts.

Find the value of each product.

- a.  $5 \times 3$
- b.  $6 \times 3$
- c.  $6 \times 6$

#### Solution

- a. 15
- b. 18
- c. 36

4 Grade 3

### **Assessment: Section C Checkpoint**

#### Problem 1

#### **Goals Assessed**

• Use properties of operations and place value understanding to develop strategies to multiply within 100 and to multiply one-digit numbers by a multiple of 10.

Find the value of  $7 \times 60$ . Explain or show your reasoning.

#### Solution

420. Sample Response: 60 is 6 tens and  $7 \times 6$  is 42 so  $7 \times 60$  is 42 tens and that's 420.

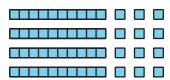
#### Problem 2

#### **Goals Assessed**

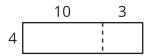
• Use properties of operations and place value understanding to develop strategies to multiply within 100 and to multiply one-digit numbers by a multiple of 10.

Explain how each diagram represents  $4 \times 13$ .





В



Use one of the diagrams to find the value of  $4 \times 13$ .

#### Solution

- a. Sample response:
  - i. There are 4 rows or groups of squares and there are 13 in each row so altogether there are  $4 \times 13$  squares.
  - ii. The diagram shows that I can add the products  $4 \times 10$  and  $4 \times 3$  to get  $4 \times 13$ .
- b. 52. Sample response: I see that there are 40 squares and then 12 more so that's 52 altogether.

#### Problem 3

#### **Goals Assessed**

• Use properties of operations and place value understanding to develop strategies to multiply within 100 and to multiply one-digit numbers by a multiple of 10.

Andre began reading a book that has 192 pages. If he reads 15 pages for each of the next 5 days, how many pages will he still have to read to finish the book?

- a. Write an equation to represent the situation. Use a letter for the unknown quantity.
- b. Solve the problem. Explain or show your reasoning.

#### Solution

- a.  $192 5 \times 15 = p$ . p is the number of pages Andre still needs to read.
- b. 117 pages. Sample response:  $5 \times 10 = 50$ ,  $5 \times 5 = 25$  so  $5 \times 15 = 75$ , 192 70 = 122 and 122 5 = 117, so there are 117 pages left to read.

6 Grade 3

### **Assessment: Section D Checkpoint**

#### Problem 1

#### **Goals Assessed**

• Use properties of operations, place value understanding and the relationship between multiplication and division to develop strategies to divide within 100.

Find the value of each quotient. Explain or show your reasoning.

- a.  $60 \div 4$
- b.  $96 \div 6$

#### Solution

- a. 15. Sample reasoning:  $10 \times 4 = 40$ ,  $5 \times 4 = 20$ , 40 + 20 = 60, and 10 + 5 = 15.
- b. 16. Sample reasoning:  $10 \times 6 = 60$ ,  $6 \times 6 = 36$ , 60 + 36 = 96, and 10 + 6 = 16.

#### Problem 2

#### **Goals Assessed**

• Use properties of operations, place value understanding and the relationship between multiplication and division to develop strategies to divide within 100.

Noah has 117 building blocks. He builds a shape with 25 blocks. Then, he puts the rest of the blocks in 4 bins, with the same number of blocks in each bin.

How many blocks are in each bin? Explain or show your reasoning.

#### Solution

23. Sample reasoning: 117 - 25 = 92,  $92 \div 4 = 23$ .

#### **Assessment: End-of-Unit Assessment**

#### Problem 1

#### **Standards Alignments**

Addressing 3.OA.C.7

#### **Narrative**

Students are building toward fluency with multiplication and division facts by the end of the year. This item gives students an opportunity to demonstrate fluency for multiplication facts within 100. If a student incorrectly answers several questions in this item then they may need to spend some extra time practicing multiplication. Students who select E are probably using addition instead of multiplication and students who select C are probably confusing  $4 \times 5$  with  $4 \times 6$ . Students who select both C and D need more practice with single digit multiplication.

Select all expressions that have a value of 24.

- A.  $2 \times 12$
- B.  $3 \times 8$
- C.  $4 \times 5$
- D.  $6 \times 4$
- E.  $20 \times 4$

#### Solution

["A", "B", "D"]

#### Problem 2

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.B.6

#### **Narrative**

Students match a division equation with situations. While students can do the matching by solving

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each problem and trying to check if it has the same solution as the given equation, this is likely to be time consuming and responses B and D go beyond grade level standards. Response A is a "how many in each group" version of division and response E is a "how many groups" division situation. The distractors all use the same numbers, namely the numbers given in the situation, but they all have the wrong operation. The operation for B and D is multiplication and the operation for C is subtraction.

Select **all** situations that match the equation  $48 \div 6 = ?$ .

- A. There are 48 volleyball players on 6 equal teams. How many players are on each team?
- B. There are 48 basketball teams at the tournament. There are 6 players on each team. How many basketball players are at the tournament?
- C. There are 48 kids swimming in the pool. Then 6 kids leave the pool. How many kids are swimming in the pool now?
- D. There are 6 buses. Each bus has 48 students on it. How many students are there altogether?
- E. There are 48 oranges in the box. Jada puts 6 oranges in each bag. How many bags does Jada need for all the oranges?

#### Solution

["A", "E"]

#### Problem 3

#### **Standards Alignments**

Addressing 3.OA.A.2, 3.OA.B.6

#### **Narrative**

Students match a situation with multiplication and division equations. While students can do the matching by solving the problem and trying to solve each equation, this is likely to be more time consuming than interpreting the equations in terms of the situation. One multiplication equation, response A, matches the situation and it matches regardless of whether students think of the first number in a product as the number of groups or the number of things in each group. The distractors all use the same numbers, namely the numbers given in the situation, but none of them makes sense in terms of the situation.

This item complements the second item on the assessment where students are given an equation and find matching situations.

There are 5 rows of flowers in the garden. Each row has the same number of flowers. There are 30 flowers altogether.

How many flowers are in each row? Select **all** equations that represent the situation.

- A.  $5 \times ? = 30$
- B.  $5 \times 30 = ?$
- C.  $30 \times ? = 5$
- D.  $5 \div ? = 30$
- E.  $30 \div 5 = ?$
- F.  $? \div 5 = 30$

#### Solution

["A", "E"]

#### Problem 4

#### **Standards Alignments**

Addressing 3.OA.A.3, 3.OA.A.4, 3.OA.B.6

#### **Narrative**

Students solve a division problem. No solution method is specified so students might draw a picture or they might use what they know about the relationship between multiplication and division to solve the problem.

Lin covers her desk with 77 sticky notes. The sticky notes are in 7 equal rows. How many sticky notes are in each row?

- a. Write a division equation to represent the situation. Use a symbol for the unknown quantity.
- b. Write a multiplication equation to represent the situation. Use a symbol for the unknown quantity.
- c. Solve the problem. Explain or show your reasoning.

10 Grade 3

#### Solution

- a.  $77 \div 7 = ? \text{ or } 77 \div ? = 7$
- b.  $7 \times ? = 77 \text{ or } ? \times 7 = 77$
- c. There are 11 sticky notes in each row. I know that  $10 \times 7 = 70$  so  $11 \times 7 = 77$ .

#### Problem 5

#### **Standards Alignments**

Addressing 3.OA.B.5, 3.OA.C.7

#### **Narrative**

This item gives students an opportunity to show fluency with division within 100. Except for the first problem which students may "just know" students will likely have to use what they know about multiplication and the distributive property or they may draw a diagram. No line of reasoning is requested, however, as they will have an opportunity to show their reasoning on other items.

Find the value of each expression.

- a.  $50 \div 5$
- b.  $36 \div 2$
- c.  $75 \div 5$

#### Solution

- a. 10
- b. 18
- c. 15

#### Problem 6

#### **Standards Alignments**

Addressing 3.NBT.A.3, 3.OA.B.5

#### **Narrative**

Students evaluate multiplication expressions. For the first and third problems they may use the

distributive property. For the second problem they may think of the product as a number of tens and use their understanding of place value. They may also draw diagrams or write equations to support their reasoning.

Find the value of each expression. Explain or show your reasoning.

- a.  $11 \times 8$
- b.  $7 \times 40$
- c.  $5 \times 13$

#### Solution

- a. 88, because it's 10 eights or 80 and then 1 more 8.
- b. 280, because 40 is 4 tens so 7 times as much is  $7 \times 4$  or 28 tens and that's 280.
- c. 65, because 5 tens is 50 and  $5 \times 3$  is 15, and 50 + 15 = 65.

#### Problem 7

#### **Standards Alignments**

Addressing 3.OA.D.8

#### **Narrative**

Students find multiple solutions to a two-step problem using subtraction and division. If students subtract incorrectly this may influence their answer to the second and third parts. So they could find that it is possible that all of the students riding the buses can fit in 2 buses. Their answer to the last question will also need to be evaluated based on how they answered the first question.

To find the number of people who take the bus, students are likely to show their work with an algorithm or using equations. To find how many are on each bus, students may use words, a diagram, or write equations.

There are 278 students at the school. 197 go home by foot or by car. The rest take a school bus.

- a. How many students take the bus home? Explain or show your reasoning.
- b. Each bus holds 35 students. Explain why the students who take the bus cannot all fit in 2 buses.
- c. There are 3 buses and each bus carries the same number of students. How many students are in each bus? Explain or show your reasoning.

12 Grade 3

#### Solution

- a. 81. I took away 200 but that was 3 too many so I added 3 back.
- b. Sample response: Since each bus holds 35 students, 2 buses hold  $2 \times 35$  students and that's 70. That's less than 81 so there is not enough space.
- c. 27. Sample response: I first found  $3 \times 10 = 30$  so that's 10 students in each bus and 30 students total. Then I can put another 10 students in each bus and that's 60 students total. That leaves 21 students (81 60 = 21) so that's 7 more in each bus.

## **CKMath**<sup>™</sup> Core Knowledge MATHEMATICS<sup>™</sup>

# Lesson Cool Downs

## **Lesson 1: How Many Groups?**

**Cool Down: How Many Bags?** 

Lin has 30 apples to share with her friends. She is putting them in bags, with 6 apples in each bag. How many bags does she need? Explain or show your reasoning.

## **Lesson 2: How Many in Each Group?**

**Cool Down: Apples in Bags** 

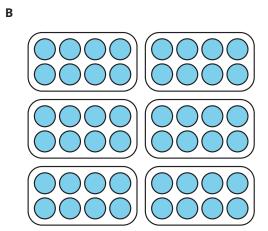
Lin has 30 apples. She makes 6 bags with the same number of apples in each bag to give to her friends. How many apples are in each bag? Explain or show your reasoning.

## **Lesson 3: Division Situation Drawings**

### **Cool Down: Party Favors**

Clare has 48 markers. She puts 8 markers into each goodie bag for her birthday party. How many bags will she use?

Which drawing matches the situation? Explain your reasoning.



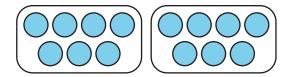
## **Lesson 4: Interpret Division Expressions**

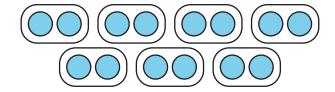
## **Cool Down: Han's Tops**

Han has 14 tops. He shares the tops equally between 2 boxes. How many tops will be in each box?

Select **all** the ways that we could represent the situation.

A. B.





C. D.

 $14 \div 2$   $14 \div 7$ 

## **Lesson 5: Write Division Expressions**

### **Cool Down: Ant Legs**

Twenty-four legs belong to 4 ants. All ants have the same number of legs.

- 1. Write a division expression to represent this situation.
- 2. How many legs does each ant have? Explain or show your reasoning.

### Lesson 6: Division as an Unknown Factor

**Cool Down: Boxed Muffins** 

There are 30 muffins for the bake sale. Each box has 6 muffins. How many boxes are there?

Tyler wrote two equations for this problem.

$$\times 6 = 30$$

$$30 \div 6 =$$
\_\_\_\_\_

He says the same number goes in each blank even though one equation is a multiplication equation and the other equation is a division equation. Is he correct? Explain or show your reasoning.

## **Lesson 7: Relate Multiplication and Division**

### **Cool Down: Sharing Roses**

Clare has 14 roses. She wants to give each of her teachers 2 roses. How many teachers can she give roses to?

Write a multiplication equation and a division equation to represent the situation. Use symbols for the unknown and explain your reasoning.	5

## **Lesson 8: Relate Quotients to Familiar Products**

## **Cool Down: Multiplication and Division Facts**

Think about the multiplication facts that you know. How have they changed since the beginn of the year?								

## **Lesson 9: Patterns in the Multiplication Table**

## **Cool Down: Find the Missing Product**

What number should replace the question mark? Explain or show your reasoning.

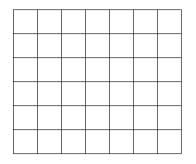
×	1	2	3	4	5	6	7	8	9	10
1	1									
2	2	4								
3	3	6	9							
4	4	8	12	16				?		
5	5	10	15	20	25					
6	6	12	18	24	30	36				
7	7	14	21	28	35	42	49			
8	8	16	24	32	40	48	56	64		
9	9	18	27	36	45	54	63	72	81	
10	10	20	30	40	50	60	70	80	90	100

## **Lesson 10: Explore Multiplication Strategies with Rectangles**

#### Cool Down: Mark or Shade Parts to Find Area

Here is a rectangle whose area can be found by finding  $6 \times 7$ .

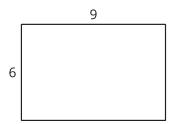
- 1. Mark or shade the rectangle to show that we can write  $2 \times (3 \times 7)$  or  $(6 \times 5) + (6 \times 2)$  to find its area.
- 2. What is the value of  $6 \times 7$ ? Explain or show your reasoning.



# **Lesson 11: Multiplication Strategies on Ungridded Rectangles**

# **Cool Down: Expressions for a Rectangle**

- 1. Mark or shade this rectangle to show a strategy for finding its area.
- 2. Write one or more expressions that represent how you find the area.



# **Lesson 12: Multiply Multiples of Ten**

**Cool Down: What's the Value?** 

Find the value of  $6 \times 40$ . Explain or show your reasoning.

# **Lesson 13: Solve Problems With Equal Groups**

# **Cool Down: Bags of Oranges**

There are 6 bags of oranges and each bag has 11 oranges. How many oranges are in the bags? Show your thinking using objects, a drawing, or a diagram.

# **Lesson 14: Ways to Represent Multiplication of Teen Numbers**

# **Cool Down: Multiply and Explain**

Find the value of  $5\times15$  and explain how it's represented in the diagram.

	10	5
5	50	25

# **Lesson 15: Equal Groups, Larger Numbers**

**Cool Down: Find the Area** 

A rectangle is 6 feet by 15 feet. What is the area of the rectangle? Explain or show your reasoning.

# **Lesson 16: Multiply Numbers Larger than 20**

**Cool Down: Multiply Numbers Greater than 20** 

What's the value of  $4 \times 24$ ? Explain or show your reasoning.

# Lesson 17: Use the Four Operations to Solve Problems

### Cool Down: Andre's Balloons

Andre has 125 balloons. He and 4 friends hung up some balloons for a party at school and now there are 80 balloons left. If each person hung up the same number of balloons, how many balloons did each person hang up?

- 1. Write an equation with a letter for the unknown quantity to represent the situation.
- 2. Solve the problem. Explain or show your reasoning.

# **Lesson 18: Larger Numbers in Equal Groups**

# **Cool Down: Recess Teams**

At recess, 42 students played a game. There were 3 teams with the same number of students on each team. How many students were on each team?

Show your thinking using diagrams, symbols, or other representations.

# **Lesson 19: Ways to Divide Larger Numbers**

**Cool Down: Find the Value** 

Find the value of  $51 \div 3$ . Use base-ten blocks if they are helpful. Explain or show your reasoning.

# **Lesson 20: Strategies for Dividing**

**Cool Down: One More Division** 

Find the value of  $96 \div 6$ . Explain or show your reasoning.

# **Lesson 21: Solve Problems Using the Four Operations**

# **Cool Down: Apples at the Farm Stand**

A booth at the apple orchard has 225 apples. 165 apples are not in baskets. The rest of the apples are in 6 baskets with the same number of apples in each basket. How many apples are in each basket?

- 1. Write an equation to represent this situation. Use a letter for the unknown quantity.
- 2. Solve the problem. Explain or show your reasoning.

# **CKMath**<sup>™</sup> Core Knowledge MATHEMATICS<sup>™</sup>

# Instructional Masters

# Instructional Masters for Relating Multiplication to Division

address	title	students written requires	written on?	requires	card stock color paper	color paper
Activity Grade3.4.5.1	Card Sort: All About Bugs	2	no	yes	ou	no
Activity Grade3.4.7.1	Division Round Table	1	yes	ou	ou	no
Activity Grade3.4.8.1	Card Sort: Multiplication Recording Sheet	2	yes	no	no	no
Activity Grade3.4.20.3	Compare Stage 4 Division Cards	2	no	yes	no	no
Center	Compare Stage 4 Division Cards	2	no	yes	no	no
Activity Grade3.4.11.2	Card Sort: Different Expressions, Same Rectangle	2	no	yes	no	no
Activity Grade3.4.8.1	Card Sort: Multiplication	2	no	yes	yes	no
Activity Grade3.4.11.2	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.4.12.1	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.4.12.2	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.4.13.1	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.4.15.1	Centimeter Grid Paper - Standard	2	yes	ou	no	no

OU	no	no	no	no	no	no	no	no	no	no	no	no	no	no	no	no
ou	ou	no	no	no	no	no	yes	yes	no	no	no	no	no	no	no	ou
Ou	no	no	no	no	no	no	yes	yes	no	no	no	no	no	no	no	no
yes	ou	no	yes	ou	ou	yes	ou	yes	ou	yes						
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Centimeter Grid Paper - Standard	Number Cards (0-10)	Number Cards (0-10)	Rectangle Rumble Stage 2 Grid	Rectangle Rumble Stage 2 Spinner	Five in a Row Multiplication and Division Stage 2 Gameboard	Capture Squares Stage 6 Gameboard	Capture Squares Stage 6 Spinner	Rectangle Rumble Stage 3 Grid	Rectangle Rumble Stage 3 Spinners	Capture Squares Stage 7 Gameboard						
Activity Grade3.4.16.1	Activity Grade3.4.16.2	Activity Grade3.4.17.2	Activity Grade3.4.18.1	Activity Grade3.4.18.2	Activity Grade3.4.20.2	Activity Grade3.4.22.2	Activity Grade3.4.16.3	Center	Center	Center	Center	Center	Center	Center	Center	Center

Center	Capture Squares Stage 7 Spinner	2	no	no	ou	no
Center	Compare Stage 3 Multiplication Cards	2	no	yes	ou	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	How Close? Stage 5 Recording Sheet	1	yes	no	no	no
Center	Compare Stage 2 Addition Cards to 20	2	no	yes	ou	no
Center	Compare Stage 2 Subtraction Cards to 20	2	no	yes	ou	no
Center	How Close? Stage 4 Recording Sheet	1	yes	no	ou	no
Center	Centimeter Dot Paper - Standard	1	yes	no	no	no
Center	Shape Cards Grade 2	2	no	yes	no	no

Ca	Card Sort: All About Bugs	
	Card Sort: All About Bugs A	Card Sort: All About Bugs D
	Mole crickets have special legs for digging. Ten special legs belong to 5 mole crickets. How many special legs does each mole cricket have?	There are 12 wings. If each dragonfly has 4 wings, how many dragonflies are there?
	Card Sort: All About Bugs B	Card Sort: All About Bugs E
	A beetle has a pair of antennae for sensing heat, touch, smell, and more. If there are 8 antennae, how many beetles are there?	Thirty legs belong to 5 ants. If all the ants have the same number of legs, how many legs does each ant have?
	Card Sort: All About Bugs C	Card Sort: All About Bugs F
	Fourteen antennae belong to a group of bees. If each bee has 2 antennae, how many bees are there?	There are 50 spots on 5 butterflies. If each butterfly has the same number of spots, how many spots does each butterfly have?

# **Division Round Table**

Box 3: Multiplication Equation	Box 1: Equal Groups Drawing
Box 4: Division Equation	Box 2: Division Situation

# Card Sort: Multiplication Recording Sheet

1. Sort your products into one of these columns:

a. know it right awayb. can find it quickly

c. don't know it yet

2. Review your strategies and practice the products that are in the "don't know it yet" column.

know it right away can find it quickly
can fin
can fin
can fin

 $57 \div 19$ 

Compare Stage 4 Division Cards	
Compare Stage 4	Compare Stage 4
$78 \div 6$	$84 \div 7$
Compare Stage 4	Compare Stage 4
$68 \div 4$	$65 \div 5$
Compare Stage 4	Compare Stage 4
90 ÷ 6	$45 \div 15$
Compare Stage 4	Compare Stage 4

Compare Stage 4 Division Cards	
Compare Stage 4	Compare Stage 4
$52 \div 13$	$84 \div 12$
Compare Stage 4	Compare Stage 4
$42 \div 7$	$56 \div 8$
Compare Stage 4	Compare Stage 4
$72 \div 9$	$64 \div 8$
Compare Stage 4	Compare Stage 4

 $81 \div 9$ 

Compare Stage 4

Compare Stage 4

 $92 \div 4$ 

 $69 \div 3$ 

Compare Stage 4

Compare Stage 4

$$84 \div 4$$

 $57 \div 19$ 

Compare Stage 4 Division Cards	
Compare Stage 4	Compare Stage 4
$78 \div 6$	$84 \div 7$
Compare Stage 4	Compare Stage 4
$68 \div 4$	$65 \div 5$
Compare Stage 4	Compare Stage 4
90 ÷ 6	$45 \div 15$
Compare Stage 4	Compare Stage 4

Compare Stage 4 Division Cards	
Compare Stage 4	Compare Stage 4
$52 \div 13$	$84 \div 12$
Compare Stage 4	Compare Stage 4
$42 \div 7$	$56 \div 8$
Compare Stage 4	Compare Stage 4
$72 \div 9$	$64 \div 8$
Compare Stage 4	Compare Stage 4

 $81 \div 9$ 

Compare Stage 4

Compare Stage 4

 $92 \div 4$ 

 $69 \div 3$ 

Compare Stage 4

Compare Stage 4

$$84 \div 4$$

# Card Sort: Different Expressions, Same Rectangle

Caid Solt. Dillefellt Explessions, Saille Nectangle	Salle nectaligie		
Card Sort: Different Expressions, Same Rectangle A	Card Sort: Different Expressions, Same Rectangle B	Card Sort: Different Expressions, Same Rectangle	Card Sort: Different Expressions, Same Rectangle D
$(7 \times 2) \times 2$	$6 \times 7$	$7 \times 4$	8  imes 3
Card Sort: Different Expressions, Same Rectangle E	Card Sort: Different Expressions, Same Rectangle F	Card Sort: Different Expressions, Same Rectangle <b>G</b>	Card Sort: Different Expressions, Same Rectangle H
$3 \times 6 + 5 \times 6$	$(4 \times 3) \times 2$	$4\times(2\times3)$	$4 \times 9$
Card Sort: Different Expressions, Same Rectangle	Card Sort: Different Expressions, Same Rectangle	Card Sort: Different Expressions, Same Rectangle K	Card Sort: Different Expressions, Same Rectangle L
$(5\times6)+(2\times6)$	$2 \times (2 \times 9)$	$8 \times 6$	$(5\times4)+(2\times4)$

Card Sort: Multiplication	Card Sort: Multiplication	: Multiplica	Card Sort: Multiplication
$1 \times 10$	$2 \times 10$	$3 \times 10$	! ! ! #
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	
$5 \times 10$	$6 \times 10$	7 × 10	

Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$9 \times 10$	$10 \times 10$		2 ×
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$3 \times 1$	$4 \times 1$	ა × 1	6 ×

4 × 5	ယ × ပ	2 × 5	⊢ × 5
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$10 \times 1$	$9 \times 1$	8 × 1	7 × 1
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication

$5\times5$ $6\times5$ $7\times5$ $8\times5$ $5\times5$ $9\times5$ $10\times5$ $10\times5$ $10\times5$ $10\times5$ $10\times5$					
Card Sort: Multiplication Card Sort: Multip	2 × 2	$1 \times 2$	$10 \times 5$	9 × 5	
Card Sort: Multiplication Card Sort: Multiplication Card Sort: Multiplication $7  imes 5$	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	
Card Sort: Multiplication Card Sort: Multiplication	∞ × ٽ	~7 × 5	6 × 5	თ × თ	
	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	

$7 \times 2$ $8 \times 2$ $9 \times$	Card Sort: Multiplication Card Sort: Multiplication Card Sort: Multiplication	$3 \times 2$ $4 \times 2$ $5 \times$	Card Sort: Multiplication Card Sort: Multiplication Card Sort: Multiplication	
$9 \times 2$	Card Sort: Multiplication		Card Sort: Multiplication	
$10 \times 2$	Card Sort: Multiplication	$6 \times 2$	Card Sort: Multiplication	

Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$1 \times 3$	2 × 3	ယ × မ	4 × 3
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
ပ × ယ	6 × 3	7 × 3	∞ × 3

Card Sort: Multiplication $9 imes3$	Card Sort: Multiplication $10 imes3$	Card Sort: Multiplication $1  imes 4$	Card Sort: Multiplication $2  imes 2$
ယ	$10 \times 3$	$1 \times 4$	
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$3 \times 4$	4 × 4	5 × 4	

Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$7 \times 4$	$8 \times 4$	$9 \times 4$	$10 \times 4$
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$1 \times 6$	$2 \times 6$	$3 \times 6$	$4 \times 6$

$9 \times 6$ $10 \times 6$ $1 \times 8$	Card Sort: Multiplication Card Sort: Multiplication Card Sort: Multiplication	$5 \times 6$ $6 \times 6$ $7 \times 6$	Card Sort: Multiplication Card Sort: Multiplication Card Sort: Multiplication
1 × 8	Card Sort: Multiplication		Card Sort: Multiplication
2 × 8	Card Sort: Multiplication	8 × 6	Card Sort: Multiplication

Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
ა × ∞	4 × 8	∵ × ∞	6 × 8
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
7 × 8	∞ × ∞	9 × 8	$10 \times 8$

Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$1 \times 7$	$2 \times 7$	$3 \times 7$	4 × 7
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
5 × 7	6 × 7	7 × 7	8 × 7

( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( (			
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$9 \times 7$	10 × 7	$1 \times 9$	$2 \times 9$
Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication	Card Sort: Multiplication
$3 \times 9$	$4 \times 9$	57 × 9	$6 \times 9$

$7 \times 9$	Card Sort: Multiplication
$8 \times 9$	Card Sort: Multiplication
$9 \times 9$	Card Sort: Multiplication
$10 \times 9$	Card Sort: Multiplication

	 I				1	1			

	 I				1	1			

	 I				1	1			

	 I				1	1	1		

	 I				1	1	1		

	 I				1	1	1		

	 I				1	1	1		

	 I				1	1	1	1	

	 I	1			1	1	1	1	1	

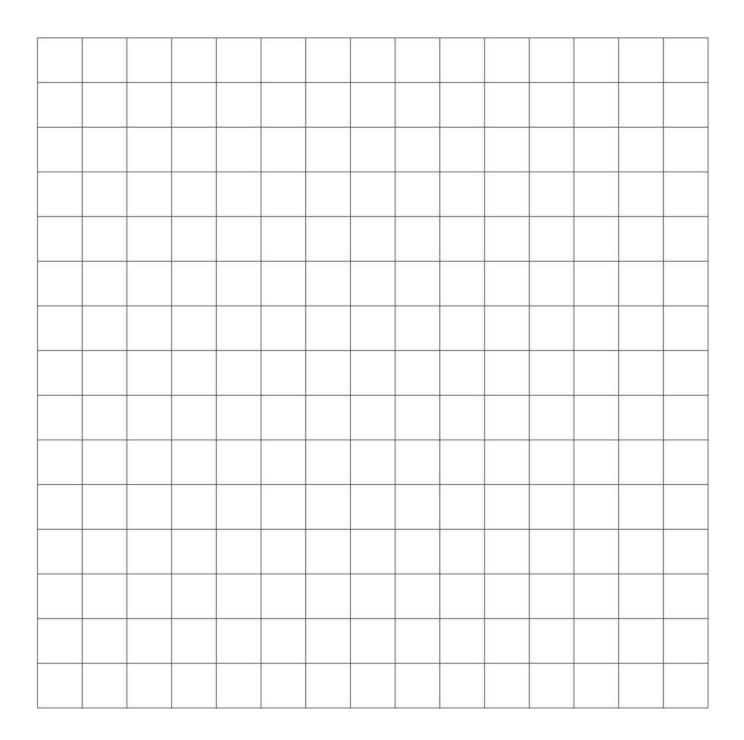
	 I	1			1	1	1	1	1	

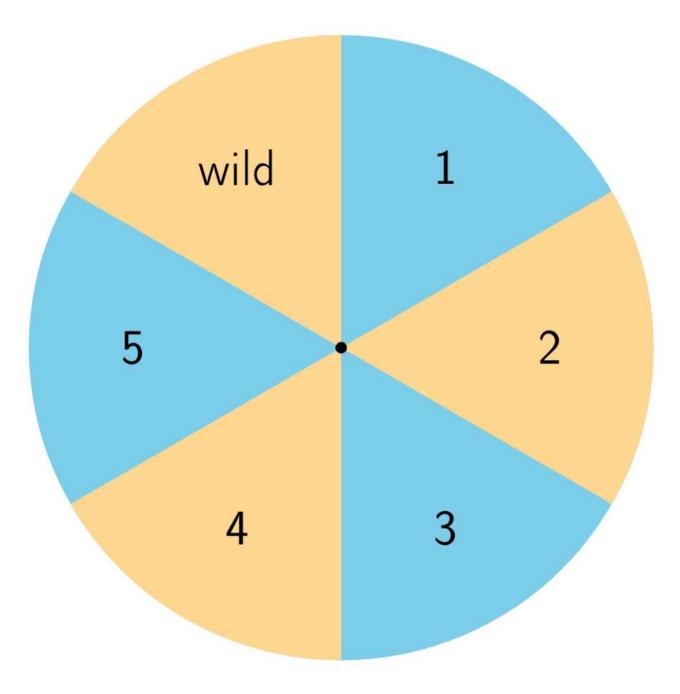
	 I	1			1	1	1	1	1	

	 I	1			1	1	1	1	1	

### Rectangle Rumble Stage 2 Grid

- Choose a color for your rectangles different from your partner.
- On your turn:
  - o Spin the spinner and roll the number cube.
  - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.





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### Five in a Row Multiplication and Division Stage 2 Gameboard

- Partner A:
  - Put a paper clip on 2 numbers in the grey rows.
  - Multiply the numbers.
  - o Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

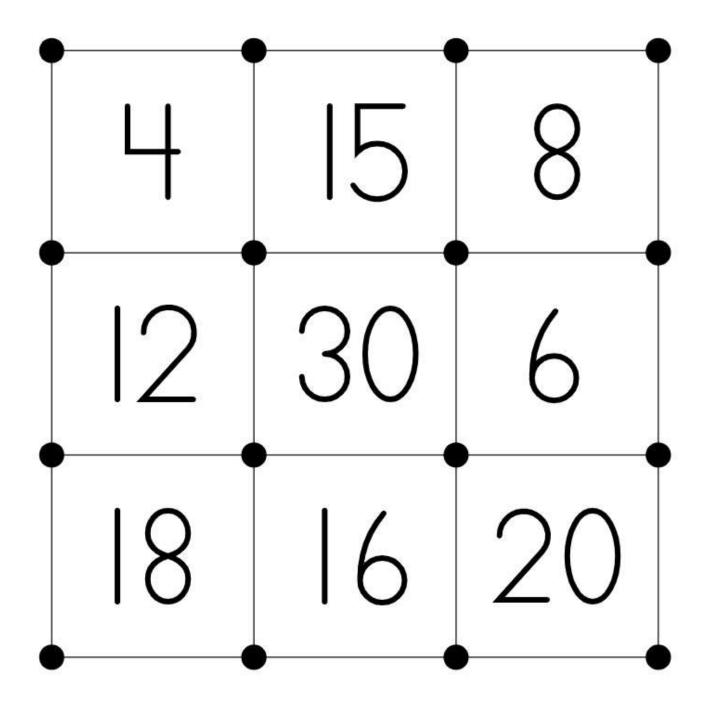
1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

|--|

6 7	8	9
-----	---	---



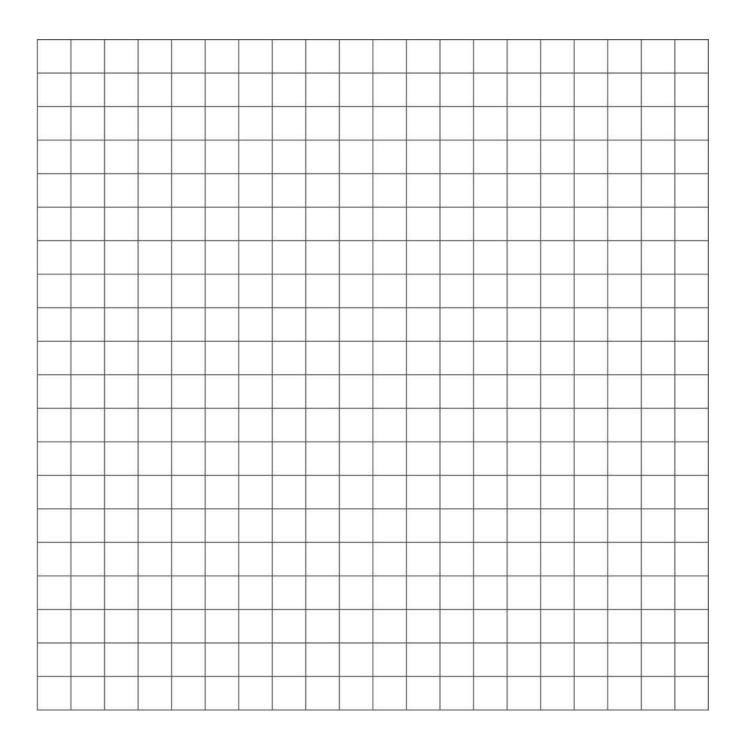
- On your turn:
  - o Roll the number cube and spin the spinner. Find the product.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - o If you can't draw a line, roll and spin again.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.

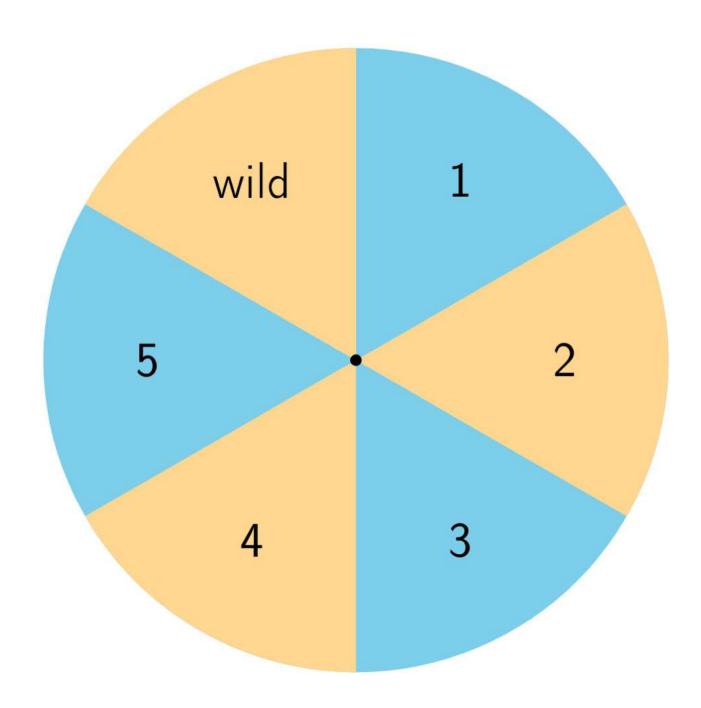


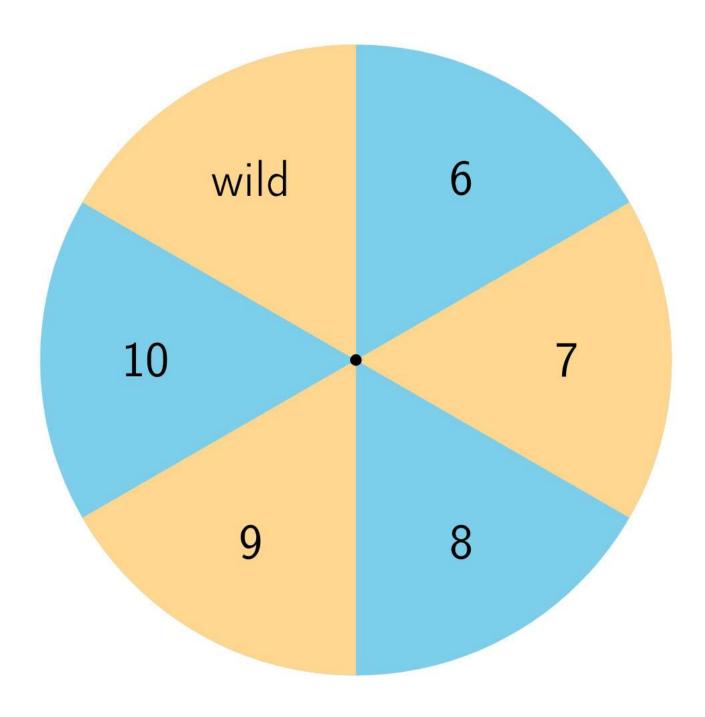


### Rectangle Rumble Stage 3 Grid

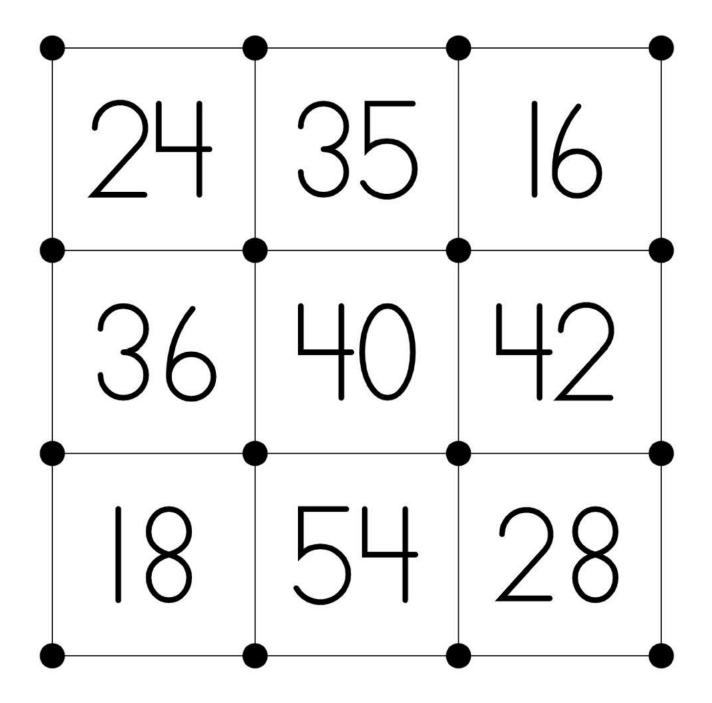
- Choose a color for your rectangles different from your partner.
- On your turn:
  - Spin each spinner.
  - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.







- On your turn:
  - o Roll the number cube and spin the spinner. Find the product.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - o If you can't draw a line, roll and spin again.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.





# Compare Stage 3 Multiplication Cards

Compare Stage 3	Compare Stage 3
$12 \times 9$	$12 \times 7$
Compare Stage 3	Compare Stage 3
$13 \times 7$	$14 \times 6$
Compare Stage 3	Compare Stage 3
$15 \times 6$	$10 \times 20$
Compare Stage 3	Compare Stage 3
$21 \times 4$	$19 \times 5$

# Compare Stage 3 Multiplication Cards

Compare Stage 3	Compare Stage 3
$18 \times 5$	$17 \times 4$
Compare Stage 3	Compare Stage 3
$16 \times 6$	14  imes 7
Compare Stage 3	Compare Stage 3
$31 \times 3$	$20 \times 4$
Compare Stage 3	Compare Stage 3
$8 \times 9$	$9 \times 7$

### Compare Stage 3 Multiplication Cards

Compare Stage 3 12 imes 5 13 imes 4 Compare Stage 3 13 imes 4 Compare Stage 3 9 imes 5

#### Compare Stage 3-8 Directions

#### Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

#### Compare Stage 3-8 Directions

#### Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

#### How Close? Stage 5 Recording Sheet

#### Directions:

- Each partner:
  - o Take 4 cards.
  - Choose 2–3 cards to multiply.
  - Write an equation to show the product of the numbers you chose.
  - Your score for each round is the difference between your product and 100.
- Take new cards so that you have 4 cards to start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.

round	multiplication equation	points for the round
1		
2		
3		
4		
5		
6		
7		
8		

Compare Stage 2

$$1 + 10 \\ 10 + 1$$

Compare Stage 2

$$\frac{2+9}{9+2}$$

Compare Stage 2

$$3 + 8 \\ 8 + 3$$

Compare Stage 2

$$\begin{array}{c} 4+7\\7+4\end{array}$$

Compare Stage 2

$$5+6 \\ 6+5$$

Compare Stage 2

$$2 + 10$$
  
 $10 + 2$ 

Compare Stage 2

$$3+9$$
$$9+3$$

Compare Stage 2

$$4+8\\8+4$$

Compare Stage 2 Addition Cards to 20	
Compare Stage 2 $5+7$	Compare Stage 2 $6+6$
7 + 5	0+0
Compare Stage 2	Compare Stage 2
3 + 10 $10 + 3$	$     \begin{array}{r}       4 + 9 \\       9 + 4     \end{array} $
Compare Stage 2	Compare Stage 2
$5+8 \\ 8+5$	$6+7 \\ 7+6$
	• • •
Compare Stage 2	Compare Stage 2
4 + 10	5 + 9
10 + 4	9 + 5

Compare Stage 2	Compare Stage 2
$6+8 \\ 8+6$	7+7
Compare Stage 2	Compare Stage 2
$5+10 \\ 10+5$	$6+9 \\ 9+6$
Compare Stage 2	Compare Stage 2
$7 + 8 \\ 8 + 7$	6 + 10 $10 + 6$
Compare Stage 2	Compare Stage 2
7+9 $9+7$	8+8

Compare Stage 2

8 + 99 + 8 Compare Stage 2

9 + 9

Compare Stage 2

$$9 + 10$$
 $10 + 9$ 

Compare Stage 2

$$10 + 10$$

Compare Stage 2

$$7+10\\10+7$$

Compare Stage 2

$$8 + 10 \\ 10 + 8$$

Compare Stage 2	Compare Stage 2
20 - 4	20 - 13
Compare Stage 2	Compare Stage 2
20 - 18	20 - 12
Compare Stage 2	Compare Stage 2
20 - 15	20 - 9
Compare Stage 2	Compare Stage 2
19 - 7	19 - 11

Compare Stage 2	Compare Stage 2
19 — 16	19 - 3
Compare Stage 2	Compare Stage 2
18 - 13	18 - 9
Compare Stage 2	Compare Stage 2
18 - 6	18 - 10
Compare Stage 2	Compare Stage 2
17 - 2	17 - 8

Compare Stage 2	Compare Stage 2
17 - 14	17 - 9
Compare Stage 2	Compare Stage 2
16 - 12	16 - 3
Compare Stage 2	Compare Stage 2
16 - 7	16 - 8
Compare Stage 2	Compare Stage 2
15 - 11	15 - 7

Compare Stage 2	Compare Stage 2
15 - 6	15 - 3
Compare Stage 2	Compare Stage 2
14-2	14 - 6
Compare Stage 2	Compare Stage 2
14 - 8	14 - 9
Compare Stage 2	Compare Stage 2
13 - 4	13 - 11

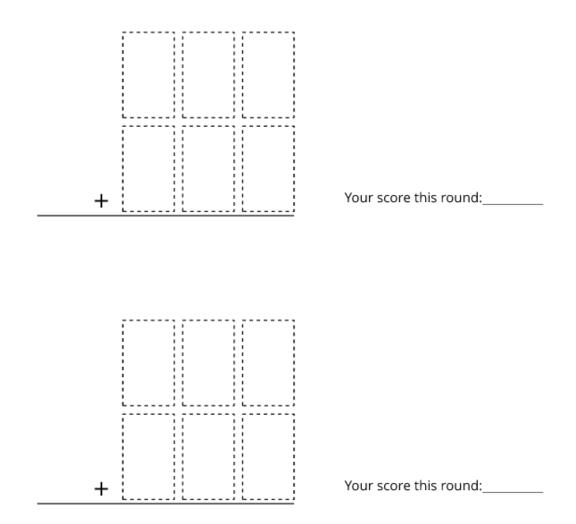
Compare Stage 2	Compare Stage 2
13 - 8	13 - 7
Compare Stage 2	Compare Stage 2
12 - 9	12 - 3
Compare Stage 2	Compare Stage 2
12 - 6	12 - 8
Compare Stage 2	Compare Stage 2
11 - 8	11 - 9

Compare Stage 2  $11-4 \qquad \qquad 11-2$ 

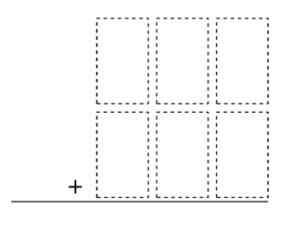
#### How Close? Stage 4 Recording Sheet

#### Directions:

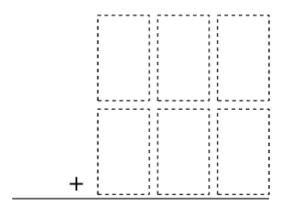
- Each partner:
  - o Take 8 cards.
  - o Choose 6 cards to make 2 three-digit numbers.
  - Write an equation to show the sum of the numbers you made.
  - Your score for each round is the difference between your sum and 1,000.
- Take 6 new cards and start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.



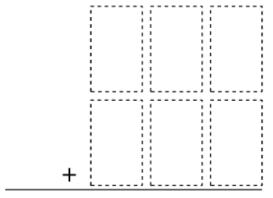
## How Close? Stage 4 Recording Sheet



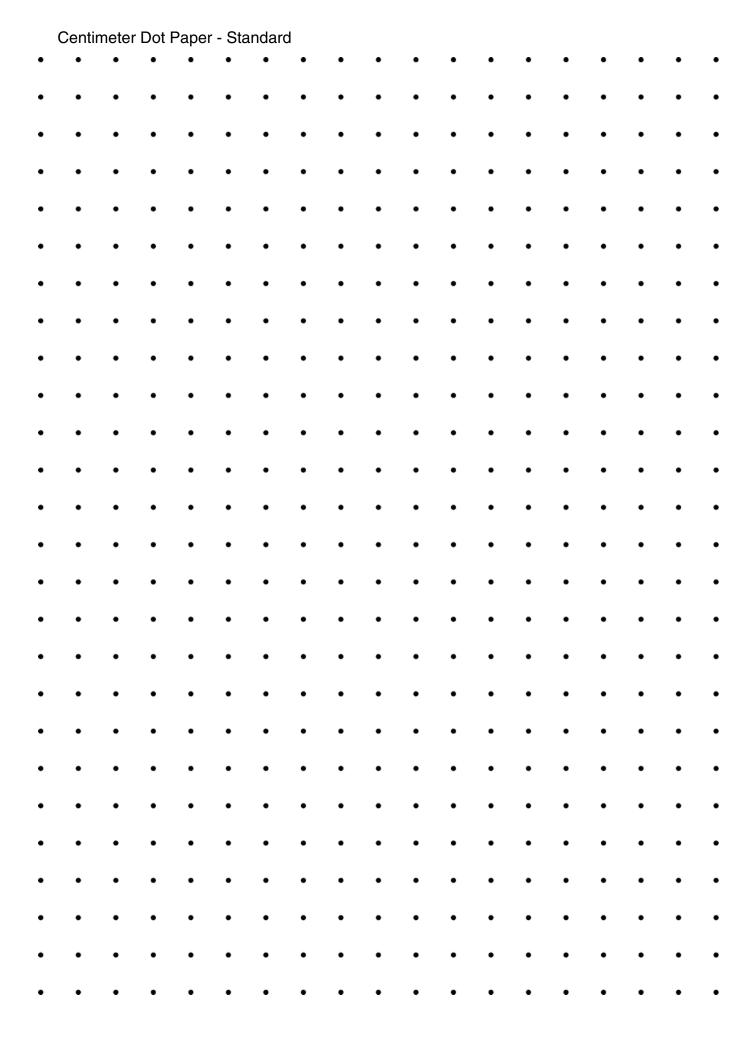
Your score this round:\_\_\_\_\_



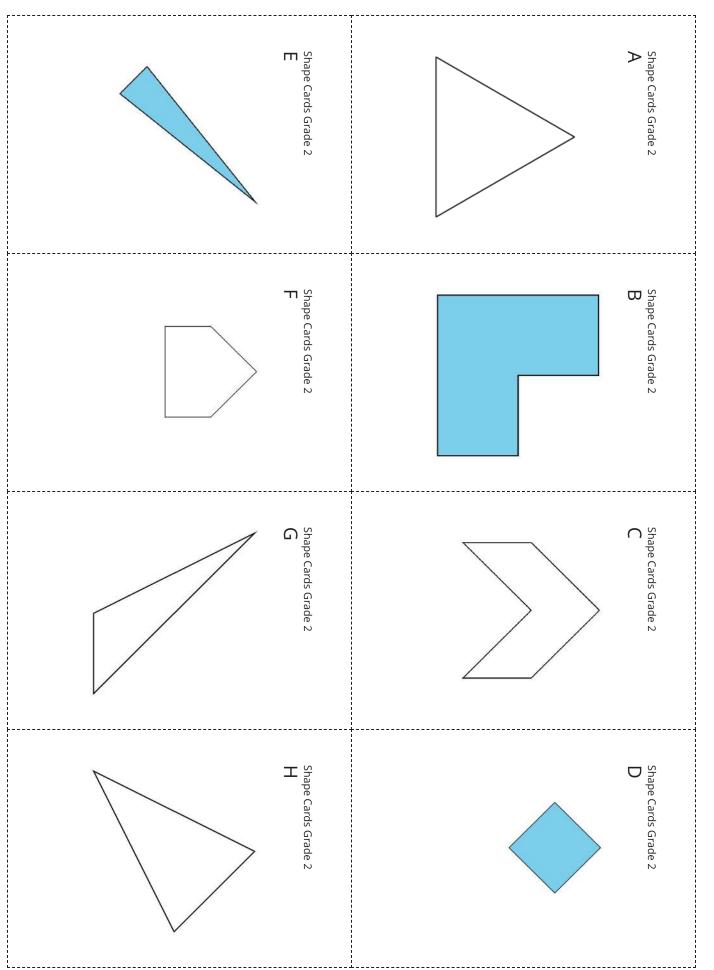
Your score this round:\_\_\_\_\_

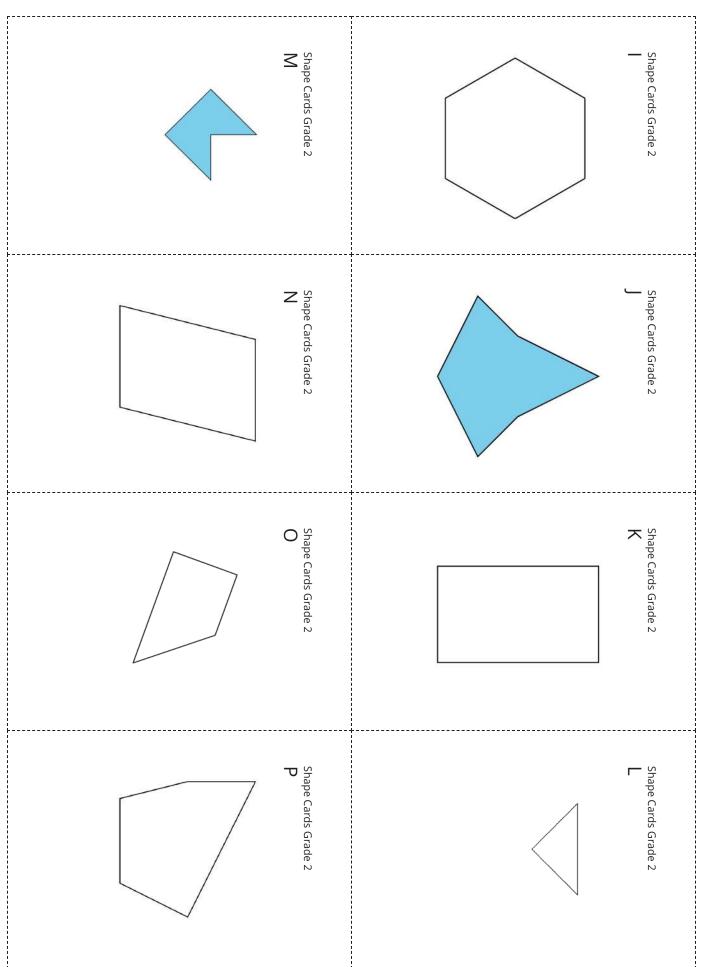


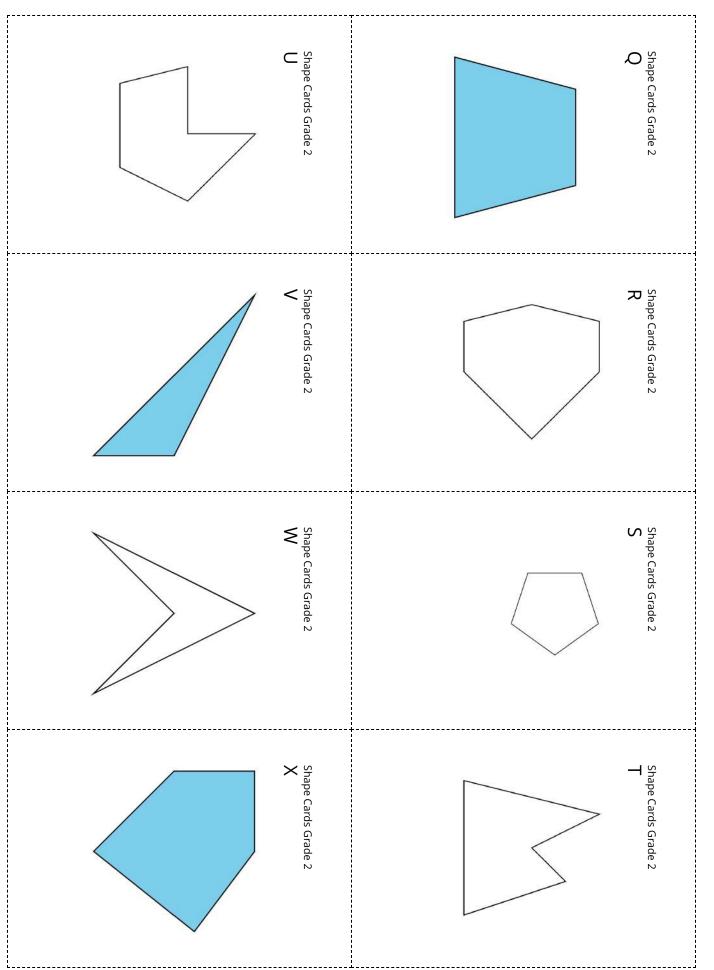
Your score this round:\_\_\_\_\_



pentagons	Shape Cards Grade 2	triangles	
hexagons	Shape Cards Grade 2	shape Cards Grade 2  quadrilaterals	







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