Core Knowledge® MATHEMATICS

Introducing Multiplication

Teacher Guide

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Introducing Multiplication

Table of Contents

Introducing Multiplication

Teacher Guide

Core Knowledge Mathematics™

Introduction to the CKMath Program

Welcome to the Core Knowledge Math™ (CKMath) program, based on the carefully researched and designed Illustrative Math™ (IM) instructional materials. IM K-12 Math is a problem-based core curriculum that believes all students are able to understand and use mathematics. Students learn about math by doing math. They bring their current understanding of math and their world experiences to the classroom. In these lessons, students take an active role in the learning process by building on their previous knowledge, and by exploration to develop conceptual understanding instead of being told how to solve problems. Doing math includes: understanding problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning. Encouraging students to participate in mathematical practices with other students gives the opportunity for them to perceive themselves as mathematical thinkers and as part of a mathematical community. By observing students' understanding of concepts and their thought processes, teachers are able to direct student learning and guide them to recognize the connection between concepts and procedures.

Organization of Units and Lessons

Each unit is divided into sections. Each section revolves around specific goals.

- The **Section Overview** identifies the learning goals for each section of the unit and describes how students will work towards these goals. Sections are labeled by letters; e.g. Section A, Section B, and so on. Each section uses scaffolding to identify the Common Core Standards that apply to that section. In Third Grade, there are five areas covered by the Common Core Standards. They include Operations and Algebraic Thinking (3.OA), Number and Operations in Base Ten (3.NBT), Number and Operations – Fractions (3.NF), Measuring and Data (3.MD), and Geometry (3.G).

  The standards in each section are divided into three groups: **Building On, Addressing, and Building Towards**. A standard that reflects the work of prior grades and is being used to bridge to a grade-level standard is indicated as *Building On*. When the standard is focused on the grade-level work, the alignment is indicated as *Addressing*. A standard that is indicated as *Building Towards* means that the standard has not yet been achieved by the activities in that section.

- The **Center Overview** identifies the learning centers to be used in the unit. Each center has different stages, or levels. Students will progress through the stages as they master the objectives for each stage. Each center description includes the Common Core Standards that apply to that stage of the center, a stage narrative describing the activity with possible variations, and a list of materials needed for the center.

- The **Standards for Mathematical Practice (MP)** describe the types of thinking and behaviors students engage in as they are doing mathematics. Throughout the curriculum, the Teacher Guide identifies lessons and activities where different Mathematical Practices are likely to be observed.

  Standards for Mathematical Practice Student Facing Learning Targets

  MP1 I Can Make Sense of Problems and Persevere in Solving Them
  - I can ask questions to make sure I understand the problem.
  - I can say the problem in my own words.
  - I can keep working when things aren't going well and try again.
  - I can show at least one try to figure out or solve the problem.
  - I can check that my solution makes sense.

MP2 I Can Reason Abstractly and Quantitatively

- I can think about and show numbers in many ways.
- I can identify the things that can be counted in a problem.
- I can think about what the numbers in a problem mean and how to use them to solve the problem.
- I can make connections between real-world situations and objects, diagrams, numbers, expressions, or equations.

MP3 I Can Construct Viable Arguments and Critique the Reasoning of Others

- I can explain or show my reasoning in a way that makes sense to others.
- I can listen to and read the work of others and offer feedback to help clarify or improve the work.
- I can come up with an idea and explain whether that idea is true.

MP4 I Can Model with Mathematics

- I can wonder about what mathematics is involved in a situation.
- I can come up with mathematical questions that can be asked about a situation.
- I can identify what questions can be answered based on data I have.
- I can identify information I need to know and don't need to know to answer a question.
- I can collect data or explain how it could be collected.
- I can model a situation using a representation such as a drawing, equation, line plot, picture graph, bar graph, or a building made of blocks.
- I can think about the real-world implications of my model.

MP5 I Can Use Appropriate Tools Strategically

- I can choose a tool that will help me make sense of a problem. These tools might include counters, base-ten blocks, tiles, a protractor, ruler, patty paper, graph, table, or external resources.
- I can use tools to help explain my thinking.
- I know how to use a variety of math tools to solve a problem.

MP6 I Can Attend to Precision

- I can use units or labels appropriately.
- I can communicate my reasoning using mathematical vocabulary and symbols.
- I can explain carefully so that others understand my thinking.
- I can decide if an answer makes sense for a problem.

MP7 I Can Look for and Make Use of Structure

- I can identify connections between problems I have already solved and new problems.
- I can compose and decompose numbers, expressions, and figures to make sense of the parts and of the whole.
- I can make connections between multiple mathematical representations.
- I can make use of patterns to help me solve a problem.

MP8 I Can Look for and Express Regularity in Repeated Reasoning

- I can identify and describe patterns and things that repeat.
- I can notice what changes and what stays the same when working with shapes, diagrams, or finding the value of expressions.
- I can use patterns to come up with a general rule.

- Each unit contains between 8 - 25 **Lesson Plans**.

  Each lesson is designed to use 60 minutes. A typical lesson is divided into four phases; a warm-up activity, one or more instructional activities, the lesson synthesis, and a cool-down activity. Every activity within these phases is divided into three parts-the Launch, the Activity, and the Synthesis.

  - **Warm-up Activity**—The warm-up activity is designed to strengthen the idea of mathematical community. In these activities, students work with their peers. Students use their personal experiences and mathematical knowledge to develop ideas, ask questions, defend their responses, and evaluate the reasoning of others. A warm-up activity might review a context students have seen before, have them reflect on where the previous lesson left off, or preview a context or idea that will come up in that lesson.

    There are several **warm-up routines** that are used during the lessons.

    - Act It Out—This routine is for kindergarten and first grade students. It encourages young children to understand the relationship between words and numbers. It provides opportunities for students to make sense of story problems. In this routine, students listen to a story problem and act it out through movement, using their fingers, or objects to represent the action in the story.

    - Choral Count—This routine encourages students to make predictions and think about patterns. It also provides opportunities for students to justify their reasoning. In this routine, students count aloud starting from a given number. The count might be forwards or backwards. The teacher records the numbers on a chart as students say them. Students then stop and look at the written numbers to make predictions and look for patterns.

    - Estimation Exploration—Estimation Exploration encourages students to use what they know and what they can see to problem-solve for a rough evaluation of a quantity rather than giving a "wild guess." The estimates can be in the context of measurement, computation, or numerosity—estimating about a large group of objects (MP2). In this routine, students make estimates in response to a question about an image. They first think about estimates that would be sensible, but too high or too low. Then they make a reasonable estimate and discuss why their estimate makes sense.

    - How Many Do You See?—This routine encourages students to see groups when counting. Being able to see groups of objects in an organized way helps them visualize quantities and improves their ability to do mental computation. In this routine, students look at an image, which is typically an arrangement of dots or other shapes. Then students state how many dots or shapes they see. Also included in the discussion will be comments about the way they saw them or determined how many there were. This encourages students to see groups and patterns rather than count each item one by one.

    - Notice and Wonder—This routine provides an opportunity for students to bring their understandings and experiences to a problem. They share their ideas and ask questions without any pressure to answer or solve a problem. This routine reinforces the importance of making sense of situations before solving a problem. In this routine, students look at an image related to the topic of the lesson and are asked, "What do you notice?" The teacher writes all comments on a chart. They are then asked, "What do you wonder?", and their questions are also recorded on the chart.

- Number Talk—This routine provides an opportunity for students to practice mental math. It helps them solve problems and think about numbers in flexible ways. They not only justify their own reasoning, but critique the reasoning of others as they make sense of methods for solving problems. In this routine, a series of problems are presented one at a time. Students solve the problem in their head and signal when they have an answer. The teacher takes notes as they justify their answer and explain their method for solving.

- Questions About Us—This routine is used with kindergarten students. It provides them opportunities to learn more about their classmates and gives them practice asking questions, organizing quantities, counting, and analyzing data. In this routine, students ask their classmates a question with two choices. They keep track of the answers and count the responses. The teacher then asks follow up questions that students answer using the data that they collected.

- True or False?—This routine encourages students to make sense of equations, often without any computation. It provides another opportunity for students to justify their reasoning as they explain to others what they are thinking. In this routine, students are presented with a series of equations, one at a time. Some equations may be true, and some may be false. Students use what they know about place value, operations, and number relationships to decide if each is true or false. And then, students explain how they know.

- What Do You Know About _____?—This routine encourages students to share their experiences and understandings about a math topic. In this routine, students are presented with a number, expression, or are asked a general question about a math topic. They then list everything they know about that topic. The teacher writes what students say and then references the list later so that students can add more ideas.

- Which One Doesn't Belong?—This routine provides an opportunity for students to reason about characteristics of shapes, math tools, or other images to decide which one doesn't belong. Because any answer is correct, students are able to focus on communicating their reasoning and justifying their choice. In this routine, students are shown 4 different images, which may be numbers, equations, shapes, images, or diagrams. They decide which one doesn't belong and explain why.

- **Instructional Activities**—After the warm-up, lessons consist of one to three instructional activities.

  Instructional Activities include:

  - 5 Practices—Lessons that include this routine are designed to allow students to solve problems in ways that make sense to them. During the activity, students engage in a problem in meaningful ways and teachers monitor to uncover and nurture conceptual understandings. During the activity synthesis, students collectively reveal multiple approaches to a problem and make connections between these approaches (MP3).

  - Card Sort—A card sorting task gives students opportunities to analyze representations, statements, and structures closely, and make connections (MP2 and MP7). As students work, teachers monitor for the different ways groups choose their categories, and

encourage increasingly precise mathematical language (MP6).

- MLR1 Stronger and Clearer Each *(MLR stands for Mathematics Learning Routine.)*—Provides students with a structured and interactive opportunity to revise and refine both their ideas and their verbal and written output. *Embedded in grades 3–5.*

- MLR2 Collect and Display—Captures a variety of students' oral words and phrases into a stable, collective reference. Output can be organized, re-voiced, or explicitly connected to other languages in a display that all students can refer to, build on, or make connections with during future discussion or writing. *Embedded in grades K–5.*

- MLR3 Clarify, Critique, Correct—Gives students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. *Embedded in grades 3–5.*

- MLR4 Information Gap—Creates an authentic need for students to communicate. Partners or team members are given different pieces of necessary information that must be used together to solve a problem. *Embedded in grades 3–5.*

- MLR5 Co-craft Questions—Allows students to get inside a context before feeling pressure to produce answers, and creates opportunities for students to produce the language of mathematical questions. *Embedded in grades 2–5.*

- MLR6 Three Reads—Supports reading comprehension, sense-making, and meta-awareness of mathematical language. Students take time to understand mathematical situations and story problems, and plan their strategies before finding solutions. *Embedded in grades K–5.*

- MLR7 Compare and Connect—Fosters students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. *Embedded in grades K–5.*

- MLR8 Discussion Supports—Includes a large variety of teacher moves that support rich discussions about mathematical ideas, representations, contexts, and strategies. *Embedded in grades K–2.*

- **Lesson Synthesis—**After the instructional activities are completed, students take time to reflect on the knowledge they have gained during the instructional activities and incorporate his with their previous knowledge. The lesson synthesis activity should take 5–10 minutes. During this time, teachers help students with this process by asking questions verbally and having students respond orally or in a written journal, by asking students to add on to a graphic organizer or concept map, or some similar activity.

- **Cool-down Activity—**The cool-down activity is given to students at the end of the lesson. This activity should take about 5 minutes. Students work on the cool-down independently and turn it in. The teacher uses the cool-down as a formative assessment to determine if students understand the lesson and to adjust further instruction.
  Note: The Cool-down activity is identified in the introduction to the lesson plan and not at the end of the lesson.

- **Assessments**—There are several opportunities for assessment during each unit.

  - Pre-unit problems can be used as a pre-unit assessment.

  - Each instructional task includes expected student responses and suggestions to advance student thinking. Teachers will adjust their instruction depending on how the students respond to the task. Frequently there are suggested questions to help teachers better understand students' thinking.

- Practice problems are provided for each lesson that can be used for in-class practice, homework, or as a means to assess certain learning on a particular concept.
- Each section has a checklist to indicate that students are meeting the section goals.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit.

Unit Resources

Teacher Components

Teacher Guide: The Teacher Guide for each unit contains an overview of the sections in which the unit is divided, a description of the centers students will use with the unit, detailed lesson plans, and teacher resources. Within the overview of the unit sections can be found suggested activities from each unit section that can be used as a PLC activity for teachers. PLCs, or Professional Learning Communities provide teachers the ability to work collaboratively in recurring cycles of collective inquiry and action research to achieve better results for students. PLCs give teachers the opportunity to discuss and plan instruction with peers.

The first few pages of each detailed lesson plan are directed to the teacher. Support notes to the teacher are in gray boxes throughout the lesson plan. On these first pages can be found:

- Alignment to the Common Core Standards
- Learning Goals
  - Teacher-facing learning goals appear at the top of lesson plans. They are directed to the teacher and describe the mathematical and pedagogical goals of the lesson.
  - Student-facing learning goals are directed to the student and start with the word "Let's." These learning goals can be written on the board before class begins. They are used to invite students into the work of that day without giving away too much and spoiling the problem-based instruction.
- Lesson Purpose
- Suggestions for instruction for English Learners and Students with Disabilities
- Instructional Routines
- List of materials needed for the lesson
- Lesson Timeline
- Description of the Cool-Down Activity
- Teacher Reflection Question – The purpose of this question is to provide a direction to the teachers to think critically about their teaching during the lesson.
- Sample Student Responses

At the back of the Teacher Guide are Teacher Resources for the unit.

- Family Support Materials
- Assessments
- Cool Downs
- Instructional Masters

Student Component

Activity Book: The Activity Book is used by the students during the lessons. It coordinates with the lesson plans. It displays the student-facing learning goals for each lesson as well as activity sheets for some activities. Not all activities will use the Activity Book.

Introduction to Grade 3

The big ideas in grade 3 include: developing understanding of multiplication and division and strategies for multiplication and division within 100; developing understanding of fractions, especially unit fractions (fractions with numerator 1); developing understanding of the structure of rectangular arrays and of area; and describing and analyzing two-dimensional shapes.

Grade 3 is divided into eight units:

1. Introducing Multiplication
2. Area and Multiplication
3. Wrapping Up Addition and Subtraction within 1,000
4. Relating Multiplication to Division
5. Fractions as Numbers
6. Measuring Length, Time, Liquid Volume, and Weight
7. Two-dimensional Shapes and Perimeter
8. Putting It All Together

Unit 1: Introducing Multiplication

At a Glance

Unit 1 is estimated to be completed in 22-23 days including 2 days for assessment.

This unit is divided into three sections including 20 lessons and 1 optional lesson.

- Section A—Interpret and Represent Data on Scaled Graphs (Lessons 1-8)
- Section B—From Graphs to Multiplication (Lessons 9-15)
- Section C—Represent Multiplication with Arrays and the Commutative Property (Lessons 16-21)

On pages 8-9 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses four student centers.

- Sort and Display
- Capture Squares
- Five in a Row: Addition and Subtraction
- Five in a Row: Multiplication

Unit 1: Introducing Multiplication

Unit Learning Goals

- Students represent and solve multiplication problems through the context of picture and bar graphs that represent categorical data.

In this unit, students interpret and represent data on scaled picture graphs and scaled bar graphs. Then, they learn the concept of multiplication.

This is the first of four units that focus on multiplication. In this unit, students explore scaled picture graphs and bar graphs as an entry point for learning about equal-size groups and multiplication.

In grade 2, students analyzed picture graphs in which one picture represented one object and bar graphs that were scaled by single units. Here, students encounter picture graphs in which each picture represents more than one object and bar graphs that were scaled by 2 or 5 units. The idea that one picture can represent multiple objects helps to introduce the idea of equal-size groups.

Students learn that multiplication can mean finding the total number of objects in a groups of b objects each, and can be represented by $a \times b$. They then relate the idea of equal groups and the expression $a \times b$ to the rows and columns of an array. In working with arrays, students begin to notice the commutative property of multiplication.

In all cases, students make sense of the meaning of multiplication expressions before finding their value, and before writing equations that relate two factors and a product.

Later in the unit, students see situations in which the total number of objects is known but either the number of groups or the size of each group is not known. Problems with a missing factor offer students a preview to division.

Throughout the unit, provide access to connecting cubes or counters, as students may choose to use them to represent and solve problems.

Section A: Interpret and Represent Data on Scaled Graphs

Standards Alignments

Building On	2.MD.D, 2.MD.D.10, 2.NBT.B.5, 2.OA.C.3
Addressing	3.MD.B, 3.MD.B.3
Building Towards	3.MD.B.3

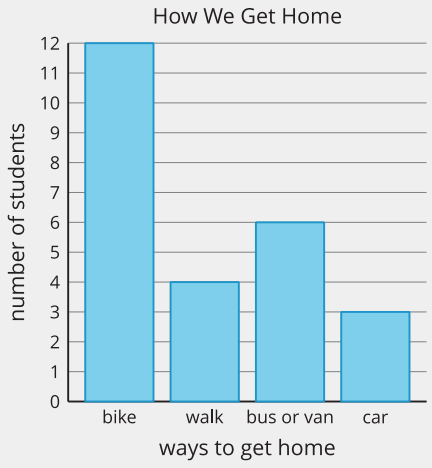
Section Learning Goals

- Interpret scaled picture and bar graphs.
- Represent data using scaled picture and bar graphs.
- Solve one- and two-step story problems using addition and subtraction.

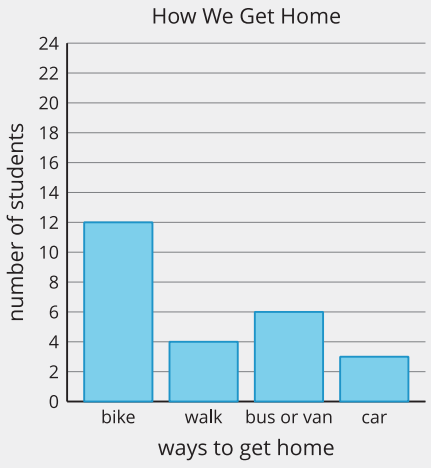
In this section, students interpret and draw picture graphs and bar graphs to represent data, building on their experience with data representation and with skip-counting by 2, 5, and 10 in grade 2.

Students see that each picture in a picture graph and each line or increment in a bar graph can represent more than one object. They work with familiar number scales of 2, 5, and 10.

bar graph



scaled bar graph



Students use the information in scaled bar graphs to solve one- and two-step "how many more" and "how many fewer" problems within 100. This work allows teachers to formatively assess students' fluency with addition and subtraction within 100, a grade 2 expectation.

PLC: Lesson 3, Activity 1, So Many Responses

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–3), Stage 3: Add within 20 (Supporting)
- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Section B: From Graphs to Multiplication

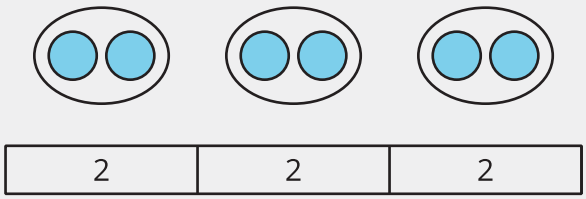
Standards Alignments

Building On	2.NBT.B.5
Addressing	3.OA.A, 3.OA.A.1, 3.OA.A.3, 3.OA.A.4, 3.OA.D.9
Building Towards	3.OA.A.1, 3.OA.C.7

Section Learning Goals

- Represent and solve multiplication problems involving equal groups.
- Understand multiplication in terms of equal groups.

In this section, students make sense of multiplication in terms of equal groups of objects. They use discrete drawings and tape diagrams that show equal groups to represent multiplication, and then relate these representations to expressions such as 3×2, interpreting them to mean "3 groups of 2."



Note that expressions of the form $a \times b$ could be interpreted to mean a groups of b or b groups of a. Because we tend to say "___ groups of ___" when referring to equal groups, however, in these materials we write multiplication expressions in that order:

$$\text{number of groups} \times \text{size of each group}$$

It is not necessary for students to use this convention as long as they can explain what each number in their expression represents.

Later, students write equations to represent multiplication situations and find unknown products or factors. In reasoning about the latter, they begin to make sense of the relationship between multiplication and division, without formally using the language of division.

👤↔👤 PLC: Lesson 10, Activity 2, Card Sort: Equal Groups

Suggested Centers

- Capture Squares (1–3), Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing

(Supporting)

- Five in a Row: Addition and Subtraction (1–2), Stage 7: Add within 1,000 without Composing (Supporting)
- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 8: Add within 1,000 with Composing (Supporting)

Section C: Represent Multiplication with Arrays and the Commutative Property

Standards Alignments

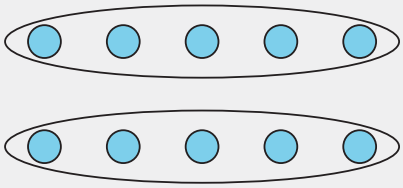
Building On	2.NBT.B.5, 2.OA.C.4
Addressing	3.MD.B.3, 3.OA.A, 3.OA.A.1, 3.OA.A.3, 3.OA.B.5, 3.OA.C.7, 3.OA.D.9
Building Towards	3.NBT.A.2

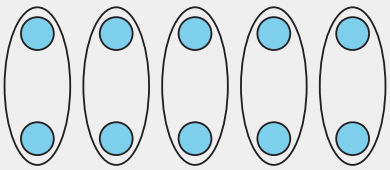
Section Learning Goals

- Represent and solve multiplication problems involving arrays.

In this section, students relate the idea of equal groups to the structure of an array, a representation introduced in grade 2.

Students see that the rows and columns of an array represent equal groups. The number of rows (or columns), the number of items in each row (or column), and the total number of objects in an array can therefore be represented with a multiplication equation. The equations may involve an unknown value, be it one of the factors or the product. As students reason about arrays, they also notice that multiplication is commutative.





PLC: Lesson 20, Activity 1, Learn More About Multiplication

Suggested Centers

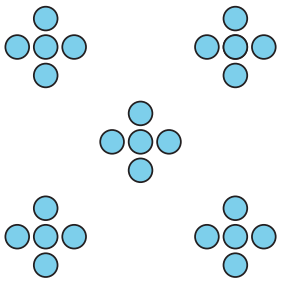
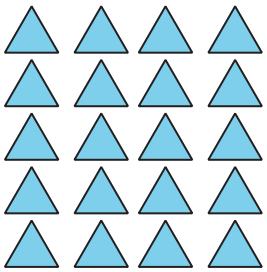
- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Throughout the Unit

Students work toward fluency in multiplying by 2, 5, and 10. The How Many Do You See routine is used

to encourage students to look for equal groups. It prompts students to subitize a group of dots as one unit, see the iterations of the groups, and skip-count to say the total number of dots they see in the image. This routine progresses from dots to drawings of equal groups to array formations.

Here is a sampling of the How Many Do You See warm-ups in the unit.

lesson 4	lesson 18
	

Number Talks are likewise designed to help students build fluency with equal groups and multiplication expressions. The sequence of expressions encourages students to relate multiplication to skip-counting. For example, in the sequence 1×10, 2×10, 3×10, 4×10, students can discover that the products increase in the same way as in skip-counting by 10. Some Number Talks elicit students' understanding of addition and subtraction within 100 in preparation for the work in an upcoming unit.

Here is a sampling of the Number Talk warm-ups in the unit.

lesson 5	lesson 15	lesson 19	lesson 20
$2 + 2 + 2 + 2$	1×10	10×2	$70 - 10$
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$	2×10	9×2	$68 - 10$
$5 + 5 + 5 + 5$	3×10	8×2	$70 - 12$
$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$	4×10	7×2	$68 - 12$

Materials Needed

LESSON	GATHER	COPY
A.1	• none	• none
A.2	• Sticky notes	• none
A.3	• none	• none
A.4	• none	• none
A.5	• Materials from a previous lesson	• none
A.6	• none	• none
A.7	• Materials from a previous lesson	• none
A.8	• none	• none
B.9	• Connecting cubes or counters	• none
B.10	• none	• Card Sort Equal Groups (groups of 2)
B.11	• Materials from a previous lesson	• none
B.12	• none	• none
B.13	• none	• none
B.14	• none	• Card Sort Unknown Numbers (groups of 2)
B.15	• none	• none
C.16	• Connecting cubes	• none
C.17	• Connecting cubes or counters	• Card Sort Arrays (groups of 2)
C.18	• Connecting cubes or counters	• none

C.19	• none	• none
C.20	• none	• none
C.21	• Connecting cubes or counters • Inch tiles • Tools for creating a visual display	• Centimeter Grid Paper - Standard (groups of 2)

Center: Sort and Display (1–3)

Stage 2: Picture or Bar Graphs

Lessons

- Grade3.1.A1 (supporting)
- Grade3.1.A2 (supporting)
- Grade3.1.A3 (supporting)
- Grade3.1.A4 (supporting)

Stage Narrative

Students sort 20–30 objects into three categories and make a picture or bar graph that shows how they sorted. Provide students with a group of items that will be interesting for them to work with such as:

- pattern blocks
- connecting cubes
- counters
- combination of the blocks, cubes, and counters
- sets of books

Students then ask their partner two questions that can be answered based on their graph.

Standards Alignments

Addressing 2.MD.D.10

Materials to Gather

Collections of objects

Materials to Copy

Sort and Display Stage 2 Recording Sheet (groups of 1)

Additional Information

Create collections of 20–30 objects with up to three attributes by which to sort.

Stage 3: Scaled Graphs

Lessons

- Grade3.1.A5 (addressing)
- Grade3.1.A6 (addressing)
- Grade3.1.A7 (addressing)
- Grade3.1.A8 (addressing)

Stage Narrative

Students sort 40–100 objects into 3–5 categories and make a scaled picture or bar graph that shows how they sorted. Provide students with a group of items that will be interesting for them to work with such as:

- pattern blocks
- connecting cubes
- counters
- combination of the blocks, cubes, and counters
- sets of books

Students then ask their partner two questions that can be answered based on their graph.

Standards Alignments

Addressing 3.MD.B.3

Materials to Gather

Collections of objects

Materials to Copy

Sort and Display Stage 3 Recording Sheet (groups of 1)

Additional Information

Create collections of 40–100 objects with up to five attributes by which to sort.

Stages used in Grade 2

Stage 1

Supporting

- Grade2.1.B

Stage 2

Addressing

- Grade2.1.B
- Grade2.1.C

Center: Capture Squares (1–3)

Stage 3: Add within 20

Lessons

- Grade3.1.A1 (supporting)
- Grade3.1.A2 (supporting)
- Grade3.1.A3 (supporting)
- Grade3.1.A4 (supporting)

Stage Narrative

Students spin to get a number (6–10) and flip a card (0–10) and find the sum. The spinner includes a wild space where students can choose their own number.

Standards Alignments

Addressing 1.OA.C.6, 2.OA.B.2

Materials to Gather

Colored pencils or crayons, Number cards 0–10, Paper clips

Materials to Copy

Capture Squares Stage 3 Gameboard (groups of 2), Capture Squares Stage 3 Spinner (groups of 2)

Stage 4: Subtract within 20

Lessons

- Grade3.1.B9 (supporting)
- Grade3.1.B10 (supporting)
- Grade3.1.B11 (supporting)

Stage Narrative

Students spin to get a number (16–20) and flip a card (0–10). They subtract the number on the card from the number on the spinner. The spinner includes a wild space where students can choose their own number.

Standards Alignments

Addressing 1.OA.C.6, 2.OA.B.2

Materials to Gather

Colored pencils or crayons, Number cards 0–10, Paper clips

Materials to Copy

Capture Squares Stage 4 Gameboard (groups of 2), Capture Squares Stage 4 Spinner (groups of 2)

Stage 5: Multiply with 2, 5, and 10

Lessons

- Grade3.1.B12 (addressing)
- Grade3.1.B13 (addressing)
- Grade3.1.B14 (addressing)
- Grade3.1.B15 (addressing)
- Grade3.1.C16 (addressing)
- Grade3.1.C17 (addressing)
- Grade3.1.C18 (addressing)
- Grade3.1.C19 (addressing)
- Grade3.1.C20 (addressing)
- Grade3.1.C21 (addressing)

Stage Narrative

Students roll a number cube and spin a spinner and find the product of the two numbers they generated. The spinner has numbers 2, 5, and 10 and a wild space where students can choose their own number.

Standards Alignments

Addressing 3.OA.C.7

Materials to Gather

Colored pencils or crayons, Number cubes, Paper clips

Materials to Copy

Capture Squares Stage 5 Gameboard (groups of 2), Capture Squares Stage 5 Spinner (groups of 2)

Additional Information

Each group of 2 needs one number cube.

Stages used in Grade 2

Stage 1

Supporting

- Grade2.2.A

Stage 2

Supporting

- Grade2.2.A

Stage 3

Addressing

- Grade2.2.A
- Grade2.2.B
- Grade2.2.C

Supporting

- Grade2.4.A
- Grade2.6.C

Stage 4

Addressing

- Grade2.2.B
- Grade2.2.C

Supporting

- Grade2.3.B
- Grade2.4.A
- Grade2.6.C

Center: Five in a Row: Addition and Subtraction (1–2)

Stage 6: Add within 100 with Composing

Lessons

- Grade3.1.A5 (supporting)
- Grade3.1.A6 (supporting)
- Grade3.1.A7 (supporting)
- Grade3.1.A8 (supporting)
- Grade3.1.B9 (supporting)

Stage Narrative

Partner A chooses two numbers and places a paper clip on each number. They add the numbers and place a counter on the sum. Partner B moves one of the paper clips to a different number, adds the numbers, and places a counter on the sum. Students take turns moving one paper clip, finding the sum, and covering it with a counter.

Standards Alignments

Addressing 1.NBT.C.4, 2.NBT.B.5

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Addition and Subtraction Stage 6 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 counters and 2 paper clips.

Stage 7: Add within 1,000 without Composing

Lessons

- Grade3.1.B10 (supporting)
- Grade3.1.B11 (supporting)
- Grade3.1.B12 (supporting)
- Grade3.1.B13 (supporting)

Stage Narrative

Partner A chooses two numbers and places a paper clip on each number. They add the numbers and place a counter on the sum. Partner B moves one of the paper clips to a different number, adds the numbers, and places a counter on the sum. Students take turns moving one paper clip, finding the sum, and covering it with a counter.

Standards Alignments

Addressing 2.NBT.B.7

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Addition and Subtraction Stage 7 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 counters and 2 paper clips.

Stage 8: Add within 1,000 with Composing

Lessons

- Grade3.1.B14 (supporting)
- Grade3.1.B15 (supporting)

Stage Narrative

Partner A chooses two numbers and places a paper clip on each number. They add the numbers and place a counter on the sum. Partner B moves one of the paper clips to a different number, adds the numbers, and places a counter on the sum. Students take turns moving one paper clip, finding the sum, and covering it with a counter.

Standards Alignments

Addressing 2.NBT.B.7

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Addition and Subtraction Stage 8 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 counters and 2 paper clips.

Stages used in Grade 2

Stage 5

Supporting

- Grade2.2.A

Stage 6

Addressing

- Grade2.2.A
- Grade2.2.B
- Grade2.9.B

Supporting

- Grade2.2.C
- Grade2.3.A
- Grade2.4.A
- Grade2.7.A

Stage 7

Addressing

- Grade2.7.A
- Grade2.7.B
- Grade2.7.C

Supporting

- Grade2.8.A
- Grade2.8.B

Stage 8

Addressing

- Grade2.7.B
- Grade2.7.C

Supporting

- Grade2.8.A
- Grade2.8.B

Center: Five in a Row: Multiplication (3–5)

Stage 1: Factors 1–5 and 10

Lessons

- Grade3.1.C16 (addressing)
- Grade3.1.C17 (addressing)
- Grade3.1.C18 (addressing)
- Grade3.1.C19 (addressing)
- Grade3.1.C20 (addressing)
- Grade3.1.C21 (addressing)

Stage Narrative

Students multiply using factors of 1–5 and 10. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments

Addressing 3.OA.C.7

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Multiplication and Division Stage 1 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 two-color counters and 2 paper clips.

Section A: Interpret and Represent Data on Scaled Graphs

Lesson 1: Make Sense of Data

Standards Alignments

Building On	2.MD.D, 2.MD.D.10
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Interpret picture graphs and bar graphs to generate questions (orally and in writing) about the data.

Student-facing Learning Goals

- Let's read and ask questions about data.

Lesson Purpose

The purpose of this lesson is to elicit students' prior understandings of single-unit scale picture graphs and bar graphs in preparation for upcoming work with scaled bar graphs.

In grade 2, students learned how to draw and label single-unit scale bar graphs and picture graphs and used categorical data presented in graphs to solve simple problems. In this lesson, students revisit the structure of picture graphs and bar graphs, the features of graphs that help communicate information clearly, and the information they can learn by analyzing a graph. Students learn that a **key** is the part of a picture graph that tells what each picture represents. Students contextualize and make sense of the data based on the title, the given values, and their own experiences (MP2).

Math Community

Prepare a space, such as a piece of poster paper, titled "Math Community" and a T-chart with the headers "Doing Math" and "Norms." Partition each of the columns into two sections: students and teacher. The two sections encourage the students and teachers to be mindful that both respective parties are responsible for the way math is being done in the classroom.

Mathematical Community	
Doing Math	*Norms*
Students	Students
Teacher	Teacher

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	15 min
Activity 1	10 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Today's lesson provided an opportunity to learn from your students. How were you able to incorporate your students' lived experience into the lesson?

Cool-down (to be completed at the end of the lesson)

5 min

Describe and Ask

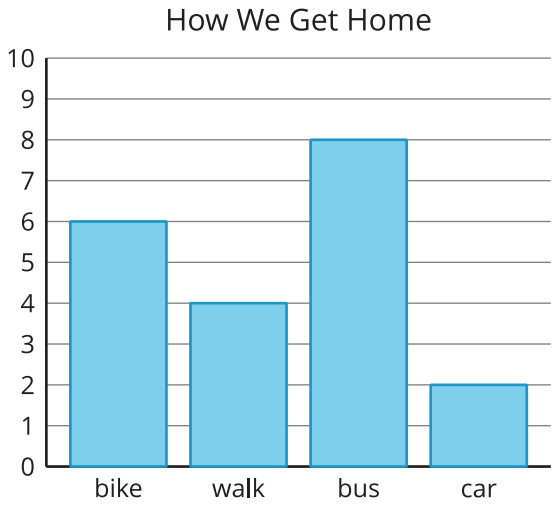
Standards Alignments

Building On	2.MD.D
Building Towards	3.MD.B.3

Student-facing Task Statement

A group of students were asked, "How do you get home from school each day?"

Their responses are shown in this bar graph:



Based on the data shown on the graph:

1. Write one fact you learned about how the students get home.
2. Write one question you could ask about how the students get home from school.

Student Responses

Sample responses:

1. More students take the bus home than ride bikes. Eight students take the bus home. Two students ride in a car.
2. How many more students take the bus than walk home? How many students ride their bikes home?

Begin Lesson

Warm-up

15 min

Notice and Wonder: Graphs

Standards Alignments

Building On 2.MD.D.10

The purpose of this warm-up is to elicit students' prior understandings about categorical data representations, which will be useful when students engage with single-unit scale picture and bar graphs in later activities. While students may notice and wonder many things about this graph, it is important to pay attention to the ways in which students make sense of a picture graph, the questions they have about the categorical data, and the contexts that make sense for the categorical data shown.

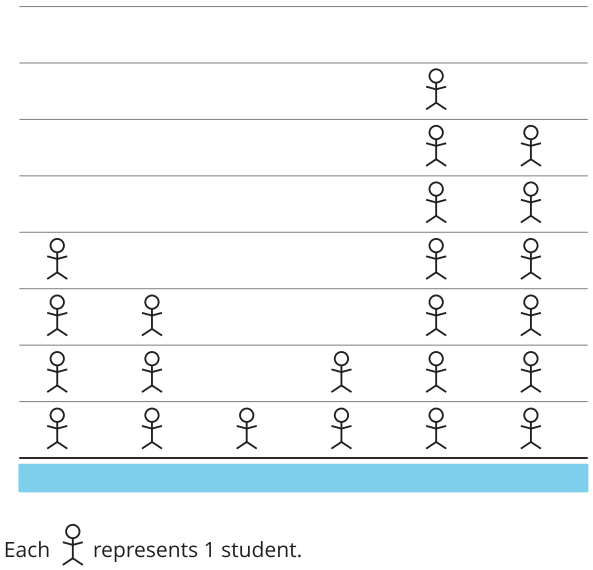
For all warm-up routines, consider establishing a small, discreet hand signal that students can display to indicate they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class. Since this is the first warm-up of the year, we allocated 15 minutes, instead of 10, to establish the structure of a routine.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Each represents 1 student.

Launch

- Groups of 2
- Display the graph.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What situations could the graph represent?" (Favorite day of the week. Favorite type of food. Types of animals that people saw in the park.)
- Math Community: Ask students to reflect on both individual and group actions while considering the question "What does it look

Student Responses

Students may notice:

- It looks like a picture graph.
- There is no title.
- The categories are hidden.
- There are stick people in each column.

Students may wonder:

- What is this graph about?
- What do the stick people represent?
- Why are there so many stick people in the last two columns?
- Where is the title?

and sound like to do math together as a math community? What am I doing? What are you doing?" (We talked to each other and to the teacher. We had quiet time to think. We shared our ideas. We thought about the math ideas and words we knew. You were writing down our answers. You were waiting until we gave the answers.)

- Record and display their responses under the "Doing Math" header.

Activity 1

10 min

Picture Time

Standards Alignments

Building On	2.MD.D.10
Building Towards	3.MD.B.3

The purpose of this activity is to elicit students' prior understandings about essential parts of a picture graph. The graph in this activity is the same as the one in the warm-up, but includes a title. Students are encouraged to consider what categories could be in the graph. Students contextualize and make sense of the data based on the title, the given values, and their own experiences (MP2). This is an opportunity for students to connect their lived experience to the mathematics, supporting the development of their math identities.

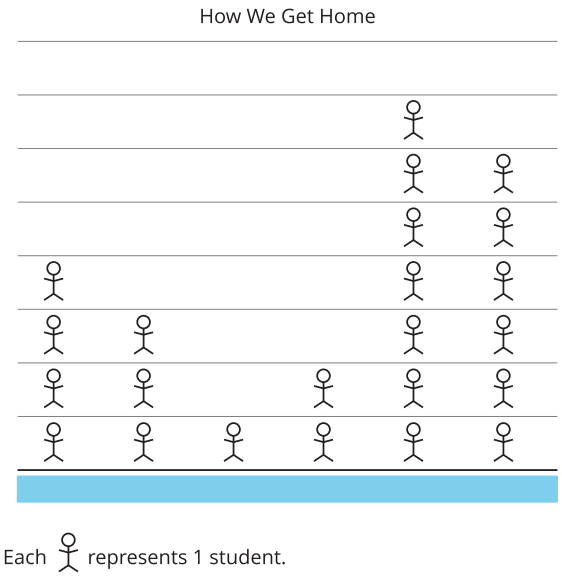
Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge to help students recall the terms "picture graph" and "key." Ask, "Why do we call this graph a picture graph?", "What kind of information does a key show?"
Supports accessibility for: Memory, Language

Student-facing Task Statement

What could the categories be for this picture graph?

Be prepared to explain your reasoning.



Student Responses

Sample responses:

- Car, because that's how I get home from school.
- Bus, because some students in our class take the bus home.

Launch

- Groups of 2
- Display the graph.
- "What is different about this graph from the first graph that we discussed?" (It has a title. We know what the graph is about.)
- 30 seconds: quiet think time
- Share responses.
- "The title of the graph helps us make sense of the data shown in the graph."
- As needed, remind students that data is information about the things or people in a group.
- "How do you and other students in our community get home from school?"
- Share responses.

Activity

- "This is a picture graph that represents how students get home from school. A **picture graph** shows how many in each group or category using pictures of the objects or symbols. Picture graphs have **keys** that tell what each picture represents."
- "What could the categories be in this picture graph? Be prepared to explain your reasoning."
- 2–3 minutes: partner work time

Synthesis

- Display the graph.
- Invite students to share possible categories for the graph.
- Consider asking: "How many categories will there be for this graph? How can you tell?"

Activity 2

20 min

Picture Graphs and Bar Graphs

Standards Alignments

Building On	2.MD.D.10
Building Towards	3.MD.B.3

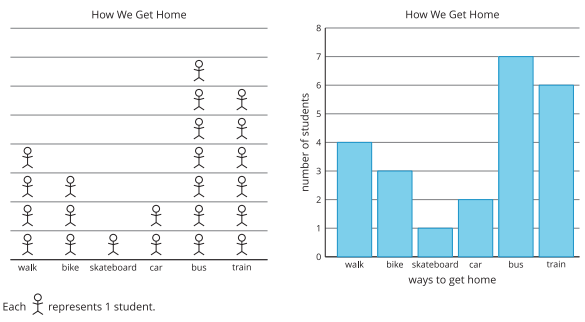
The purpose of this activity is to prepare students for work with scaled bar graphs in upcoming lessons. Now that students have reasoned about the parts of a picture graph, they look at how picture graphs and bar graphs are alike and how they are different. Students use the information presented on the axes of the bar graph to read the graph, interpret the categorical data presented in the graphs, and generate questions that can be answered using the graphs.

Access for English Learners

MLR8 Discussion Supports. Synthesis: For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Student-facing Task Statement

A group of students were asked, "How do you get home?" Their responses are shown in a picture graph and a bar graph.



1. How are the graphs the same? How are they different?
2. What can we learn about how students get

Launch

- Groups of 2
- Display the picture graph and the bar graph.
- "The second image is a bar graph. A **bar graph** shows how many in each group or category using the length of rectangles. How are the graphs alike? How are they different?"
- 2 minute: partner discussion
- Share and display responses.

Activity

- "What could you learn from the graphs about how students get home? Write two questions that the graphs could answer."

home based on the graphs?

3. Write two questions you could ask about how students get home based on the graphs.

Student Responses

1. Sample responses:
    - They both have a title at the top and categories at the bottom.
    - The picture graph has pictures to show the number in each category, but the bar graph uses bars.
    - The bar graph has labels on the bottom and the side to help you know what the bars mean.
    - The picture graph has a key, but the bar graph has the numbers on the side of the graph.
2. Sample responses:
    - 7 students take a bus home.
    - More students get home by bus or train than any other way.
3. Sample responses:
    - How many students walk home?
    - How many more students take a bus home than ride a bike?

- 7–10 minutes: partner work time

Synthesis

- Display the graphs.
- "What can we learn about how students get home from school based on the complete graphs?"
- "What questions could you ask about how students get home from school based on the graphs?"

Advancing Student Thinking

If students write questions that can't be answered with the graphs, consider asking:

- "How did you come up with your question?"
- "How could we come up with a question that could be answered with the graph?"

Lesson Synthesis

10 min

"What did we learn about picture graphs and bar graphs today?" (Bar graphs and picture graphs show us data. In a picture graph, a picture represents an object or a person. In a bar graph, the scale tells you how many objects or people. We can ask and answer questions about the data in graphs.)

If these ideas do not arise, consider asking the following questions:

- "What parts of graphs help us communicate the data in the graph with others?"
- "How are picture graphs and bar graphs the same? How are they different?"

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–3), Stage 3: Add within 20 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not read and interpret the bar graph accurately, or students write questions that can't be answered with the graph.

Next Day Support

- Use the launch of the next day's activity to have students interpret the picture graph and generate questions that could be answered with the picture graph.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 2: Represent Data and Solve Problems

Standards Alignments

Building On	2.MD.D.10, 2.OA.C.3
Addressing	3.MD.B, 3.MD.B.3
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Represent data using bar graphs and picture graphs.
- Solve one- and two-step problems using addition and subtraction within 20.

Student-facing Learning Goals

- Let's create graphs and answer questions.

Lesson Purpose

The purpose of this lesson is for students to solve one- and two-step problems about data represented in bar graphs.

Students solved one-step problems about data in grade 2. In this lesson, students first create a picture graph and bar graph that represent how they get home from school. Then, they solve one- and two-step "how many more" and "how many fewer" problems using data presented in a bar graph. Consider launching the lesson with a read-a-loud of *Last Stop on Market Street* by Matt de la Peña and Christian Robinson.

Math Community

Tell students they will have a chance to revise their math community ideas at the end of this lesson. As they work today they should think about actions that may be missing from the current list.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up)

Materials to Gather

- Sticky notes: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Think about who participated in math class today. What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Cool-down (to be completed at the end of the lesson)

5 min

Questions About a Bar Graph

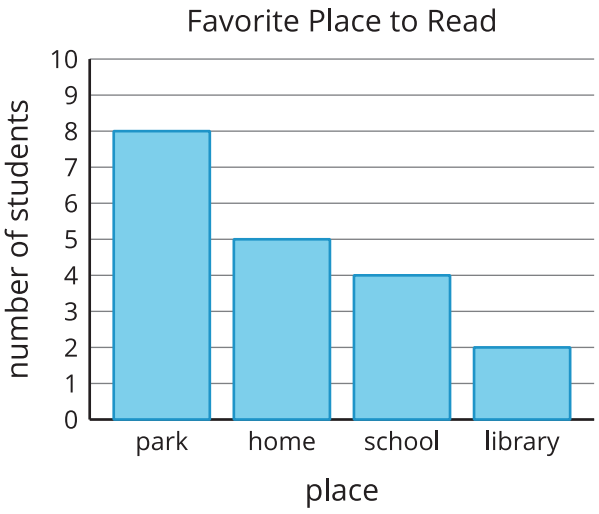
Standards Alignments

Addressing	3.MD.B.3
Building Towards	3.MD.B.3

Student-facing Task Statement

A group of students were asked, "Where is your favorite place to read?"

Their responses are shown in this bar graph:



1. How many more students chose the park than home as their favorite place to read?
2. True or false: More students like to read at the school or library than the park. Explain or show your reasoning.

Student Responses

1. 3 more students
2. False. Sample response: Six students $(4 + 2)$ like to read at the school or library, and 8 students like to read at the park.

Begin Lesson

Warm-up

10 min

How Many Do You See: Dots in Groups

Standards Alignments

Building On 2.OA.C.3

The purpose of this How Many Do You See is for students to subitize or use grouping strategies to describe the number of dots they see. They also make connections between the images to determine the number of dots. Grouping strategies and skip-counting by 2, 5, and 10 offer a review of grade 2 work and build toward multiplication in future lessons. In the synthesis, students revisit the language of "how many more" to prepare them to use data from a bar graph to solve "how many more" problems throughout this lesson.

Instructional Routines

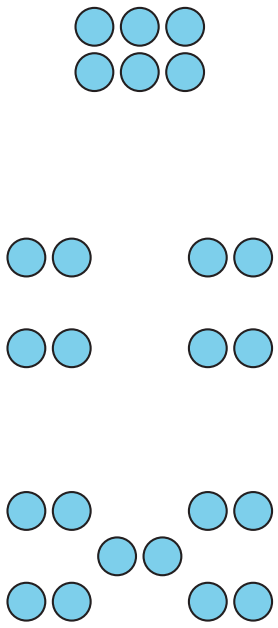
How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.



Student Responses

- 6: I see 2 rows of 3
- 8: I see 2 groups of 4
- 10: I see 5 groups of 2

- 30 seconds: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Synthesis

- "How many more dots were in the third image than in the second image? What equation matches your thinking?" (There were 2 more dots in the image. $8 + 2 = 10$.)

Activity 1

15 min

How We Get Home

Standards Alignments

Building On 2.MD.D.10

The purpose of this activity is for students to create a bar graph that includes features that help communicate the data clearly. A class picture graph is created, and students make a bar graph using that data. During the synthesis, focus attention on similarities and differences between picture and bar graphs. When you create the blank "ways to get home" picture graph for the

launch, feel free to adjust the categories based on how your students get home from school. When students label their graphs, including a title, a key, and numbers if they make a bar graph, they are communicating clearly and precisely (MP6).

Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Materials to Gather

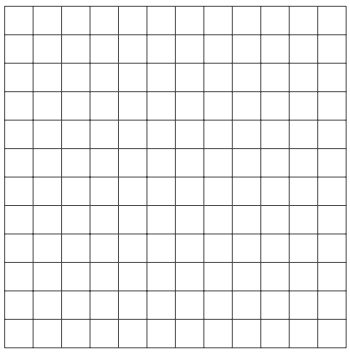
Sticky notes

Required Preparation

- Create a visual display with a blank bar graph that will be large enough to fit a column of sticky notes in each category.

Student-facing Task Statement

1. Follow your teacher's instructions to organize and represent the class data in a picture graph.
2. Represent the same data that shows how our class gets home on a bar graph.



Launch

- Groups of 2
- Display an empty picture graph with labels along the bottom axis for bike, walk, van, bus, car, and train.
- Give each student a small sticky note.
- Have each student draw a smiley face on their sticky note.
- "Put your smiley face on the graph based on how you get home from school."
- "What needs to be added to the class picture graph to communicate the data clearly?" (A title. A key so we know what each smiley face represents.)
- Facilitate addition of title and key to class picture graph.

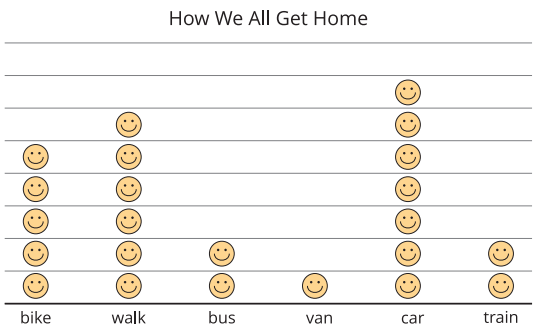
Activity

- "Represent the data shown in the class picture graph on a bar graph with your partner. Make sure to include the parts of

Student Responses

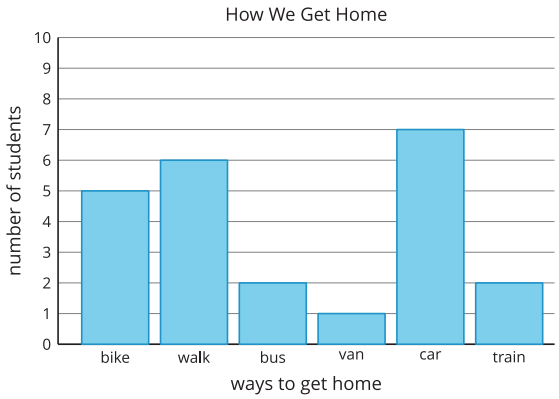
Sample responses:

1.



Each ☺ represents 1 student.

2.



the graph that will help someone else read it."

- 5–7 minutes: partner work time

Synthesis

- "How are our picture graph and bar graph alike?" (They both show the same data. They have the same categories.)
- "How are our picture graph and bar graph different?" (The picture graph has a key, but the bar graph has a scale. For the picture graph, you have to count each picture but in the bar graph, you can use the numbers on the side to tell how many.)

Advancing Student Thinking

If students create bar graphs that do not match the data in the class picture graph, consider asking:

- "Tell me about how you made your bar graph."
- "How could we use the data in the class picture graph to help make the bar graph?"

Activity 2

20 min

Questions About a Bar Graph

Standards Alignments

Addressing	3.MD.B
Building Towards	3.MD.B.3

The purpose of this activity is for students to answer one- and two-step "how many more" questions using data represented in a bar graph. Students decide if statements about the data in the bar graph from the previous activity are true or false and then answer questions about the data. When students use expression, equations, or describe adding or subtracting to find how many more or less, they show they can decontextualize and recontextualize the data to make sense of and solve the problems (MP2). You will generate the questions students answer in this task from the class graph.

Access for Students with Disabilities

Representation: Access for Perception. Read the directions and statements aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language

Student-facing Task Statement

1. Decide if each statement is true or false about how our class gets home. Explain your reasoning to your partner.
    a. More students walk than go home any other way.
    b. More students ride home on a bus than in a car.
    c. Fewer students walk home than ride their bikes.
    d. More students walk or ride their bikes than ride in a van.
2. Fill in the blanks as directed by your teacher, then answer each question.

Launch

- Groups of 2

Activity

- "Now you're going to use your bar graph to decide if statements are true or false."
- 1–2 minutes: independent work time
- 3–5 minutes: partner discussion
- As students work, decide which categories will go in the question stems for the next problem.
- Consider providing these sentence stems if students need support explaining their reasoning:

a. "How many more students _____ than _____?"
b. "How many more students _____ or _____ than _____?"

Student Responses

1. Sample response: I know that the first statement is false because more students ride their bike than walk home.
2. For example, if one question is completed as "How many more students bike than walk?" and 6 students bike and 4 students walk, students should explain how they know 2 more students bike than walk.

- "I knew the statement was false because . . ."
- "I knew the statement was true because . . ."

- "How did you know if each statement was true or false?"
- Share responses.
- Guide the whole class to fill in blanks in question stems using the previously identified categories.
- "Use the data in your bar graph to answer the questions."
- 3–5 minutes: partner work time

Synthesis

- Ask students to share their responses to each compare problem.
- "Do you have any lingering questions about how to answer these questions from the bar graph?"
- Consider asking: "What equation matches your thinking?"

Advancing Student Thinking

If students find differences that do not match the data in the graph, consider asking:

- "How did you answer the questions?"
- "How could you use the graphs to answer the questions?"

Lesson Synthesis

10 min

Display a student-created bar graph.

Generate a few questions for students to answer about how they get home using the bar graph. For example, you might ask:

- "How many more students _____ than _____?"

- "How many fewer students _____ than_____?"
- "How many more students _____ or _____ than _____?"

Have students answer the questions and share their reasoning.

Math Community

After the Cool-down, give students 2–3 minutes to discuss any revisions to the "Doing Math" actions in small groups. Share ideas as a whole group and record any revisions.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–3), Stage 3: Add within 20 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students draw lines into the bars of the bar graph to find the number represented by the bar by counting.

Next Day Support

- During the launch of the next day's activity, have students discuss how the scale on the bar graph can be used to determine the number of people or objects in each category.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 3: Scaled Picture Graphs

Standards Alignments

Building On	2.NBT.B.5
Addressing	3.MD.B
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Interpret scaled picture graphs to generate questions (orally and in writing) about the data.

Student-facing Learning Goals

- Let's explore scaled picture graphs.

Lesson Purpose

The purpose of this lesson is for students to read and answer questions about scaled picture graphs.

In previous lessons, students reviewed how to create and interpret single-unit scale picture graphs. In this lesson, students learn that a **scaled picture graph** is a picture graph where each picture represents an amount other than 1. They read, interpret, and answer questions about scaled picture graphs with a scale of 2 and 5, and generate questions that can be answered by these graphs.

Math Community

Tell students that, at the end of the lesson, they will be asked to identify specific actions from their "Doing Math" list (both teacher and student sections) they personally experienced.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

In this lesson, students make sense of scaling a picture graph by a number other than 1. How does this support the work that students will do

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

with multiplication later in this unit?

Cool-down (to be completed at the end of the lesson)

5 min

Birds in the Park

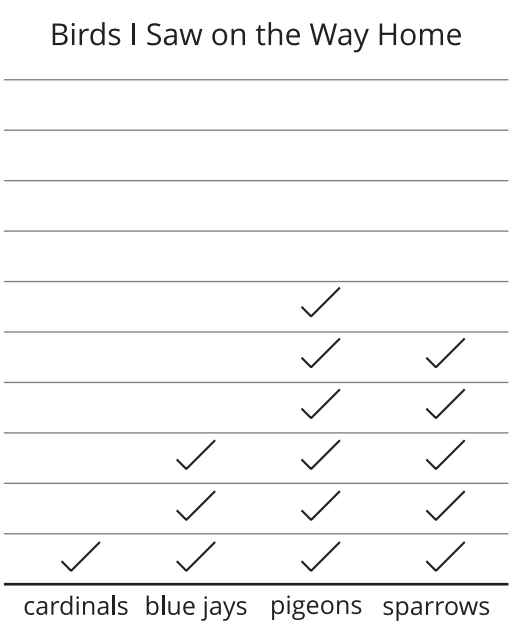
Standards Alignments

Addressing 3.MD.B

Student-facing Task Statement

Jada collected data to see how many of each type of bird she saw on her way home.

The data is shown in this picture graph:



Each ✓ represents 2 birds.

Based on the data on the graph:

1. How many sparrows did Jada see on the way home?

2. Write one question you could ask about the birds Jada saw on the way home.

Student Responses

1. 10 sparrows
2. Sample responses: How many birds did Jada see on the way home? How many blue jays and cardinals did Jada see on the way home?

Begin Lesson

Warm-up

10 min

Number Talk: Addition

Standards Alignments

Building On 2.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for adding within 100. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to add up the total number of students represented in a picture graph. When students use strategies based on place value to add they look for and make use of structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $50 + 10$
- $50 + 12$
- $60 + 13$
- $65 + 13$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.

Student Responses

- 60: Five tens and 1 ten make 6 tens, which is 60.
- 62: It's just like the first one, but there are 2 ones so it would be 62.
- 73: It's like the second problem, but there's 1 more ten and 1 more one. So each one goes up by 1 so it's 73.
- 78: There are 7 tens and 8 ones so it's 78.

- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How was place value helpful as you added these numbers?" (I was able to use tens and ones to help me find the sum.)
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Activity 1

15 min

So Many Responses

PLC Activity

Standards Alignments

Addressing	3.MD.B
Building Towards	3.MD.B.3

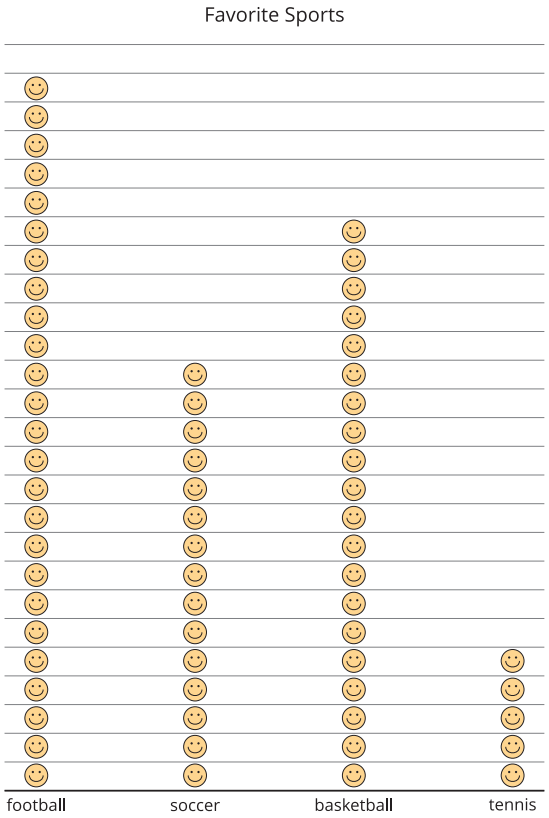
The purpose of this activity is for students to read a scaled picture graph. A scale of 5 is used to encourage skip-counting because students skip-counted by 5 in grade 2. The questions in the task focus on the structure of a scaled picture graph and strategies for reading them.

Student-facing Task Statement

1. A group of students were asked, "What is your favorite sport?" Their responses are shown in this picture graph:

Launch

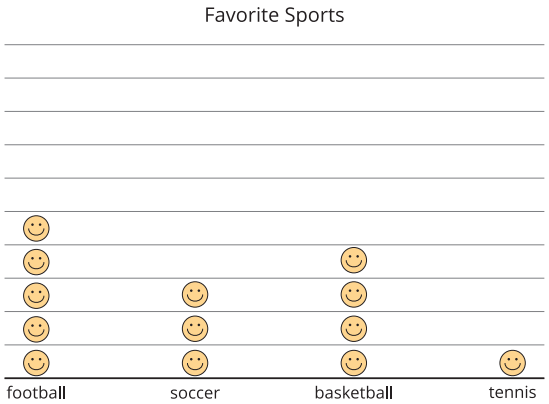
- Groups of 2
- "What is your favorite sport or activity outside of school?"
- Share responses.



Each ☺ represents 1 student.

How many students are represented in the graph?

2. Their responses are also shown in this picture graph:



Each ☺ represents 5 students.

How is counting the total number of

- Display the first image of the single-unit scale picture graph.
- "What do you notice? What do you wonder?" (Students may notice: The graph is about students' favorite sports. There are a lot of smiley faces. Each smiley face represents 1 student. It takes a lot of time to count up the students in each category. Students may wonder: How many student responses are shown in the whole graph? How could we make the graph take up less space?)
- 1 minute: quiet think time
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity

- "Work with your partner to find how many students are represented in the graph."
- 3–5 minutes: partner work time
- Monitor for students who group the smiley faces by 2, 5, or 10 to make them easier to count.
- Have students who grouped the smiley faces share their strategies for how they found the total number of students represented in the graph.
- If no students use this strategy ask, "How could grouping the pictures in the graph make them easier to count?" (We could circle tens so we could count by ten. It would be easier to keep track of your count than by counting by ones.)
- Display the second image of the scaled picture graph.
- "How could we count the total number of students in this graph?"
- 2 minutes: partner work time
- Math Community: As students work,

students in this graph different from counting the total number of students in the first graph?

Student Responses

1. 65 students.
2. We count by 5 to find the total instead of counting by 1. We are able to count to find the total a lot faster.

monitor for examples of the "Doing Math" actions.

Synthesis

- "In a graph where there's a lot of data we can adjust the scale so each picture represents more than 1 object. When each picture represents something other than 1, we say that it's a **scaled picture graph**. The key tells us that in this graph, each smiley face represents 5 students."

Advancing Student Thinking

If students count the students in the scaled picture graph and get a total other than 65, consider asking:

- "How did you find the total number of students represented in the graph?"
- "How could you use counting by 5 to find the total number of students represented in the graph?"

Activity 2

20 min

Questions about Scaled Picture Graphs

Standards Alignments

Addressing	3.MD.B
Building Towards	3.MD.B.3

The purpose of this activity is for students to interpret a scaled picture graph and write questions that can be asked based on the data represented in a scaled picture graph.

Access for English Learners

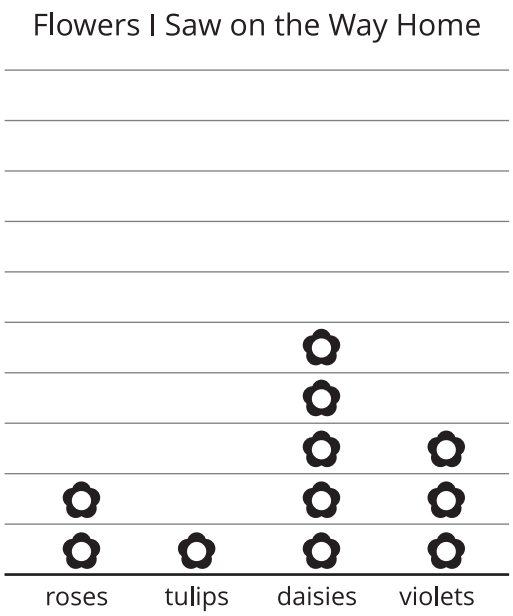
MLR8 Discussion Supports. Use multimodal examples to show the meaning of a symbol. Use verbal descriptions along with gestures, drawings, or concrete objects to show how each flower on the graph is a symbol that represents five flowers that were seen in the park.
Advances: Listening, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were needed to solve the problem. Display the sentence frame, "The next time I read a scaled picture graph I will pay attention to . . . ."
Supports accessibility for: Conceptual Processing

Student-facing Task Statement

1. Andre collected data to see how many of each type of flower he saw on the way home. The data is shown in this picture graph:



Each ✿ represents 5 flowers.

a. How many of each type of flower did

Launch

- Groups of 2
- Display the graphs for all to see.
- "What are some strategies you could use to read the graphs?" (In the Flowers I Saw on the Way Home graph I could count each category by 5 since each picture represents 5 flowers.)
- 1 minute: quiet think time
- Share and record responses.

Activity

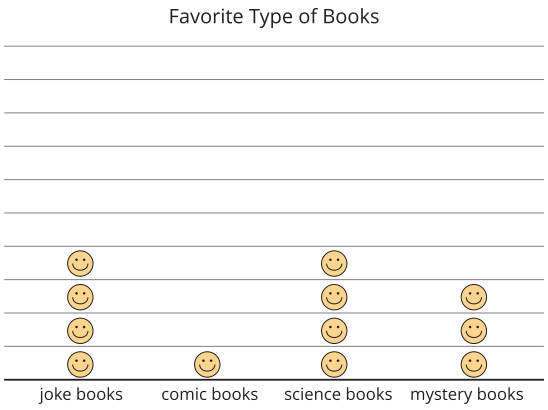
- "Now, we're going to answer some questions about the scaled picture graphs. You will also have a chance to write your own question that can be asked based on each graph."
- 8–10 minutes: partner work time
- If there is time, have groups trade books and answer each other's questions.
- Math Community: As students work, monitor for examples of the "Doing Math" actions.

Andre see on the way home?

roses	tulips	daisies	violets
_____	_____	_____	_____

b. Write 2 questions that you could ask about the flowers Andre saw on his way home.

2. A group of students were asked, "What is your favorite type of book?" Their responses are shown in this picture graph:



a. How many students liked each type of book? How do you know?

b. Write 2 questions that you could ask about students' favorite types of books based on the graph.

Student Responses

1. a. 10, 5, 25, 15

   b. Sample responses: How many more roses did Andre see than tulips? How many fewer violets did Andre see than daisies?

2. a. 8 liked joke books, 2 liked comic books, 8 liked science books, and 6 liked mystery books. Sample response: Each smiley face represents 2 students so to find the number in each category I counted by 2.

Synthesis

- Have students share responses to the questions they answered and explain their reasoning.
- Consider asking:
  - "How did you use the scale to answer the questions?" (I counted by the number that each picture represented, like by 5 for the flowers and by 2 for the students.)
- Share a variety of student written questions.
- Consider asking:
  - "How did you know your question could be answered with the graph?" (The data you needed to answer the question was in the graph.)
  - If time ask, "What questions cannot be answered by this graph?" (How many students' favorite type of book is graphic novels?)

b. Sample responses: How many more students like science books than like mystery books? How many students liked joke books or comic books?

Advancing Student Thinking

If students answer questions about the graph with numbers that don't match the graph, consider asking:

- "How did you answer the questions about the graph?"
- "What does the key tell us about each picture in the graph?"

Lesson Synthesis

10 min

Display the images of the two "Favorite Sports" graphs.

"Today we learned about scaled picture graphs. Why would we make a scaled picture graph?" (When there is a lot of data to represent, it is faster to use a scale.)

"How is reading scaled picture graphs different from reading graphs that have a scale of 1?" (Each picture doesn't represent 1 thing so you need to look at the scale. In a scaled picture graph you can count by the scale to find the total in each category instead of counting by 1.)

Math Community

After the cool-down, ask students to individually reflect on the question "Which 'Doing Math' action did you feel was most important in your work today, and why?" Have students write their responses on the bottom of their cool-down paper, on a separate sheet of paper, or in a math journal.

Collect and read their responses after class. These responses will offer insight into how students feel about their own mathematical work and help you make personal connections to the norms they will be creating during days 4–6.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–3), Stage 3: Add within 20 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students say 5 sparrows are represented in the graph.

Next Day Support

- Use the next day's warm-up to practice counting by 5, and discuss how this could be used to read a scaled picture graph.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 4: Create Scaled Picture Graphs

Standards Alignments

Addressing	3.MD.B, 3.MD.B.3
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Represent data using scaled picture graphs.

Student-facing Learning Goals

- Let's make a scaled picture graph.

Lesson Purpose

The purpose of this lesson is for students to create a scaled picture graph to represent categorical data.

In a previous lesson, students interpreted and answered questions about scaled picture graphs. In this lesson, they gather and organize data about ways that students would like to travel and represent the data in a scaled picture graph with a scale of 2. Students make sense of how to represent a single student on a scaled picture graph that has a scale of 2.

Math Community

Explain to students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Tell students that some of these norms may apply to both you and me, however there may be things you need me to do to support you in doing math each day. Offer an example, such as "It may help us share our ideas as a whole class if we have the norm 'Listen as others share their ideas.'" Tell students you will pause at two different points of the lesson to identify norms that help everyone do math.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Cool-down (to be completed at the end of the lesson)

5 min

Complete the Picture Graph

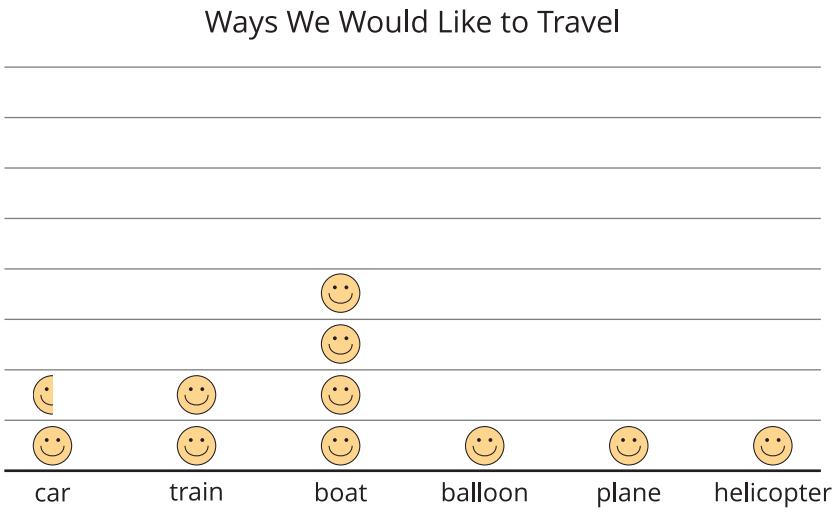
Standards Alignments

Addressing 3.MD.B.3

Student-facing Task Statement

A group of students were asked, "How would you like to travel?"

Their responses are shown in this picture graph:



Each ☺ represents 2 students.

Four students were absent when this data was collected. They would like to travel by plane.

Add their data to the graph.

Student Responses

Students draw two more smiley faces in the plane column.

Begin Lesson

Warm-up

10 min

How Many Do You See: More Groups of Dots

Standards Alignments

Building Towards 3.MD.B.3

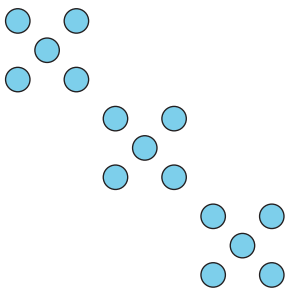
The purpose of this activity is for students to subitize or use grouping strategies to describe the number of dots they see. Although the dots have been deliberately grouped by 5 to elicit counting by 5 as a strategy, students may see 2 groups of 5 as 10. Grouping strategies and skip-counting by 2, 5, and 10 offer a review of grade 2 work and build toward multiplication in future lessons.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?

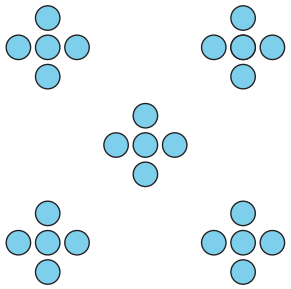


Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."



Student Responses

- 15: I saw 3 groups of 5.
- 25: I saw 5 groups of 5.

- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Synthesis

- "What pattern did you see first and how did this help you figure out the total?" (I saw that the dots were in groups of 5. This helped me because I know how to count by 5.)
- Consider asking:
  - "Did anyone see the dots the same way but would explain it differently?"
  - "Does anyone want to add an observation to the way _____ saw the dots?"

Activity 1

15 min

Ways to Travel

Standards Alignments

Addressing	3.MD.B
Building Towards	3.MD.B.3

The purpose of this activity is for students to gather and organize categorical data about their classmates. Students record their classmates' preferred way to travel and discuss advantages and disadvantages of displaying categorical data in a table.

To make the data collection process faster, students can collect their responses within their group first and then each group can share out how many students chose each way of travel.
Their names can be pre-printed in a table for them or they could only write the person's first name and way of travel abbreviation (given in the task statement) in the table. In the next activity, students create a scaled picture graph for this categorical data.

Student-facing Task Statement

How would you like to travel?

- car (C)
- train (T)
- boat (B)
- balloon (Bal)
- plane (P)
- helicopter (H)

student's name	way of traveling

Student Responses

Answers vary.

Launch

- Groups of 4

Activity

- "Today you will survey your classmates. You will start with your group and then we will collect the group data as a class."
- Give directions for how students should collect the categorical data (see suggestions in the narrative).
- 10 minutes: students record classmate responses

Synthesis

- "What's helpful about having this data in the form of a list? What's not helpful?" (We know the way each person would like to travel. It's hard to see how many people would like to travel each way.)
- Math Community: Ask students to reflect on both individual and group actions while considering the question "What norms, or expectations, were we mindful of as we did math together in our math community?"
- Record and display their responses under the "Norms" header.

Activity 2

20 min

Create a Scaled Picture Graph

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to apply understandings from previous lessons to create a picture graph with a scale of 2 from the categorical data they gathered. Students are guided to use a scale of 2 but can choose their own symbol. Depending on the data, students may need to use a half symbol in order to represent an odd number of students choosing a specific method of travel. This idea is discussed in the synthesis.

Students will use their scaled picture graphs again in the next lesson.

Access for English Learners

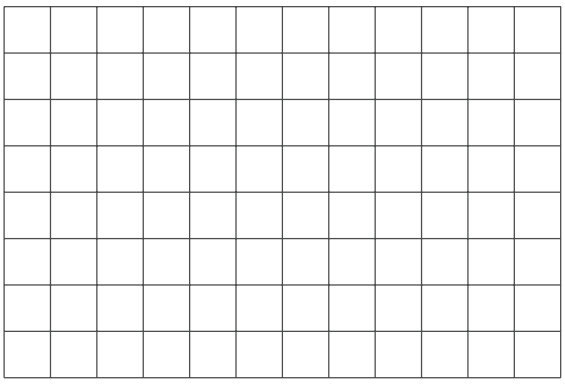
MLR8 Discussion Supports. Synthesis: When students compare graphs, display the following sentence frames: "The symbol I chose to represent _____ is _____, because . . . ", "One way our graphs are the same is . . .", and "One way our graphs are different is . . . ."
Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Invite students to begin by creating a physical model of a picture graph. Provide access to physical objects, such as connecting cubes, that students can use to represent each person, and then organize into groups of 2.
Supports accessibility for: Visual-spatial processing, Conceptual processing

Student-facing Task Statement

Represent our survey data in a scaled picture graph where each picture represents 2 students.



Student Responses

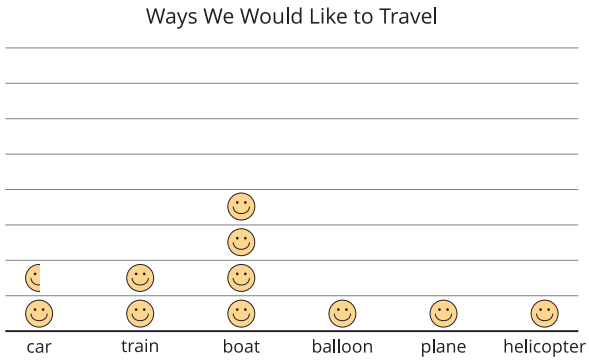
Student graphs should match class data. Sample response:

Launch

- Groups of 2
- "How can we represent our survey data in a picture graph without having to draw a picture for each student in our class?" (We can make each symbol represent more than one student so we don't have to draw as much.)

Activity

- "Represent the data that you collected in your own scaled picture graph where each picture represents 2 students."
- 10 minutes: independent work time
- Circulate as students work:
    - Encourage them to include a title, category labels, and key.



Each ☺ represents 2 students.

  - Pay attention to how students are grouping by 2.
  - Support students with questions they may have (especially around representing odd number amounts).
- "Compare your graph with your partner."
- 2 minutes: partner discussion
- Monitor for a graph that uses a half picture to show an odd number of students in one of the categories to share during the activity synthesis.

Synthesis

- Display selected student work.
- "How does this graph represent the survey data from our class?"
- "How did _____ represent the number of students who picked a way of travel when it was an odd number?"
- "What questions do you have about creating a scaled picture graph?" (Could a face represent 3 students or 5 students? Can you use whatever picture you want to represent 2 students?)

Advancing Student Thinking

If students choose symbols that are time-consuming to draw, consider asking:

- "How did you choose the symbol to use on your graph?"
- "How could you make your symbol easier to draw?"

Lesson Synthesis

10 min

Display a scaled picture graph from today's lesson. "What if 2 more students chose to travel by balloon? How could we represent that on this graph?" (Add 1 more picture in that category.)

"What if 1 more student chose to travel by car? How could we represent that on this graph?" (Add half of the picture in that category.)

Math Community

Revisit the "Norms" list. Ask students to discuss with a partner when a norm was helpful as they did math. Add any missing ideas or revise earlier ones.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–3), Stage 3: Add within 20 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students draw 4 smiley faces on the graph instead of 2.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 5: Represent Data in Scaled Bar Graphs

Standards Alignments

Addressing	3.MD.B, 3.MD.B.3
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Represent data using scaled bar graphs.

Student-facing Learning Goals

- Let's make a scaled bar graph.

Lesson Purpose

The purpose of this lesson is for students to create a scaled bar graph.

In a previous lesson, students collected categorical class data and learned how to create a scaled picture graph. Students now make connections between scaled picture graphs and scaled bar graphs, and expand the idea of a scale that is more than one to bar graphs. In this lesson students choose a scale of 2 or 5 for their bar graph.

Math Community

Tell students that, at the end of the lesson, they will be asked to identify specific examples of norms they experienced as they did math.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 2

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

Based on students' prior work with scaled picture graphs, what strategy did you anticipate

Activity 1	10 min
Activity 2	25 min
Lesson Synthesis	10 min
Cool-down	5 min

today? What strategy did you not anticipate?

Cool-down (to be completed at the end of the lesson)

5 min

Complete a Scaled Bar Graph

Standards Alignments

Addressing 3.MD.B.3

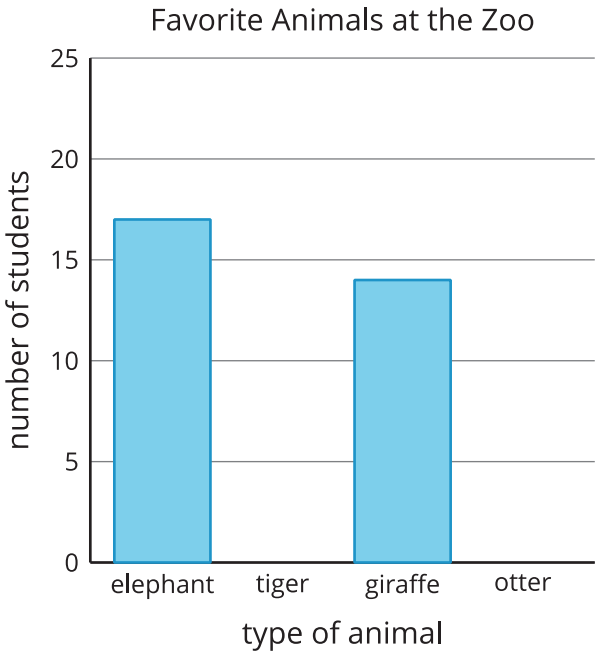
Student-facing Task Statement

Students visiting the zoo were asked, "What is your favorite animal at the zoo?"

Their responses are shown in this table:

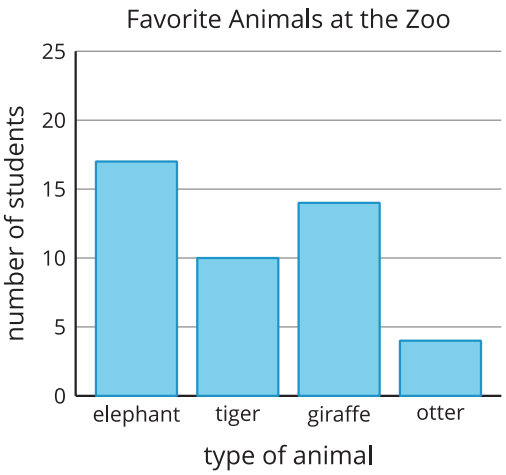
animal	number of students
elephant	17
tiger	10
giraffe	14
otter	4

Use the data in the table to complete the scaled bar graph.



Student Responses

Student bar graphs should show 10 students chose tigers and 4 students chose otters.



Begin Lesson

Warm-up

10 min

Number Talk: Twos and Fives

Standards Alignments

Addressing 3.MD.B.3

The purpose of this Number Talk is to elicit strategies students have for counting by 2 and 5. These understandings help students develop fluency and will be used later in this lesson when students will need to be able to scale bar graphs.

When students notice the number of equal addends are doubled in the second expression, they are looking for and making sense of structure (MP7). When they notice the pattern repeats in the second pair of expressions and use the pattern to find the value of the sum, they are also looking for and expressing regularity in repeated reasoning (MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$

Student Responses

- 8: There were four 2s so I counted by 2 four times like 2, 4, 6, 8.
- 16: I knew that 4 twos was 8 and now there are two groups of 8, so I doubled 8 to get 16.
- 20: I knew that 2 fives is 10 and there are 4 fives, so I doubled 10.
- 40: I knew that four fives was 20 and now there are 2 groups of 20, so I doubled 20 to get 40.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the first two expressions help you solve the third and fourth expressions?" (If you notice there are double the number of twos or fives you can just double your sum.)
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Activity 1

10 min

Compare Bar Graphs

Standards Alignments

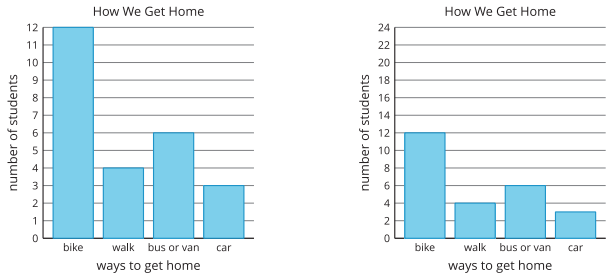
Addressing	3.MD.B
Building Towards	3.MD.B.3

The purpose of this activity is to introduce students to a scaled bar graph. Students consider a

single-unit scale bar graph next to a bar graph with a scale of 2, both representing the same set of categorical data. They discuss similarities and differences between a single-unit scale bar graph and a bar graph with a scale of 2.

Student-facing Task Statement

A class was asked, "How do you get home from school?" Their responses are shown in these two bar graphs:



Discuss with your partner: How are the two graphs alike? How are they different?

Student Responses

- They both have bars, the same categories, labels, etc.
- One counts by ones for the scale and the other counts by twos (call this out as "scaling").
- The 12 on the first graph goes higher than the 12 in the second graph.

Launch

- Groups of 2

Activity

- Display the images.
- "How are these bar graphs alike? How are they different?"
- 1 minute: quiet think time
- 4 minutes: partner discussion

Synthesis

- "What was the same? What was different?"
- Share and record responses.
- If it doesn't come up, elicit the idea that the scale on the second graph counts by 2.
- "When each jump on the scale is some number other than 1, we say that it's a **scaled bar graph**."
- "Why would it be helpful to make a scaled bar graph?" (If you didn't want to count one by one. If you wanted to show larger numbers on your graph.)

Activity 2

25 min

Create a Scaled Bar Graph

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to create a scaled bar graph. Students decide on a scale of 2 or 5, so it will be important to ask students why they chose their scale and how accurately they can tell the exact number the bar represents (MP6). In the activity synthesis, students discuss how they represented an odd number of students with a scale of 2 and a number of students that was not a multiple of 5 on a scale of 5. This question should be adjusted based on the data your class collects.

Access for English Learners

MLR7 Compare and Connect. Synthesis: Give students time to study the student work displayed with both scales. During the whole-class discussion, ask students, "What do the graphs have in common?", "How are they different?", "Why do the different graphs lead to the same outcome?"
Advances: Representing, Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after they have represented one method of travel on a graph.
Supports accessibility for: Organization, Attention

Materials to Gather

Materials from a previous lesson

Required Preparation

- Each student needs the picture graph they creaed in the previous lesson.

Student-facing Task Statement

Represent the data we collected earlier in a **scaled bar graph**.

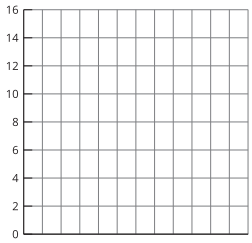
Use the graph with a scale of 2 or the graph with a scale of 5. If you have time, you can make 2 graphs. Be sure to label your title and categories.

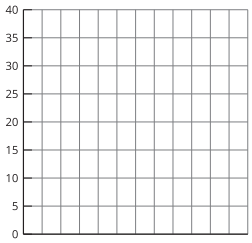
Launch

- Groups of 2
- Make sure each student has their scaled picture graph from the previous lesson.

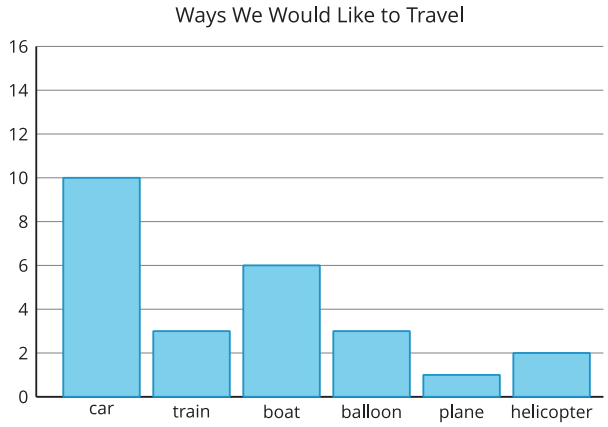
Activity

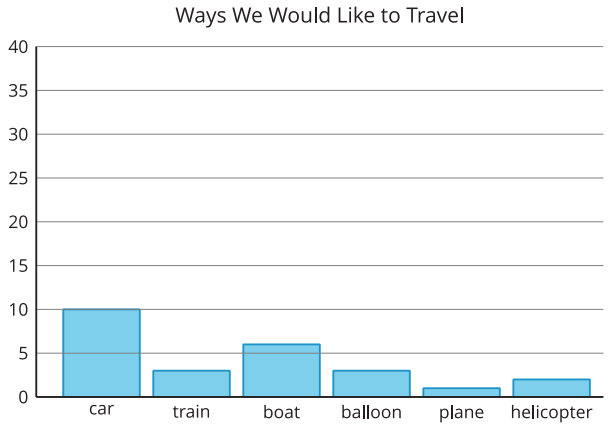
- "Today, we will represent the data we collected yesterday about ways we would





Student Responses





like to travel in a scaled bar graph."

- "Decide with your partner whether you want to use a scale of 2 or 5. Be prepared to explain your choice. If you have time, try making one graph with a scale of 2 and one with a scale of 5."
- 12 minutes: partner work time
- Consider asking, "How did your scaled picture graph help you to make your scaled bar graph?"

Synthesis

- "What scale did you and your partner choose? Why?"
- Display student work with both scales.
- "How did you represent a way of travel that didn't land right on the numbers in the scale?" (It was between the 2 numbers, so I had to make a guess about where the bar should stop.)
- "What differences do you notice when the graph is with a scale of 2 and when the graph is with a scale of 5?" (Sample responses: The bars look taller when the scale is 2. It's hard to tell what number some bars represent when the scale is 5. It was easier to count up to larger amounts when the scale was 5.)

Advancing Student Thinking

If students draw the top of the bar in a location that doesn't correspond with the ways we would like to travel data, consider asking:

- "How did you decide where the top of the bar would end?"
- "How could you use counting by 2 (or 5) to help you decide where the top of the bar should end?"

Lesson Synthesis

10 min

Display a scaled bar graph from the lesson.

"We've been learning about how to make scaled bar graphs. If you were going to help a friend create a scaled bar graph, what advice would you give them?" (I would tell them that means that the scale goes up by numbers other than 1. They should look at the number of people in each category and think about whether those numbers are easy to count by 2 or 5 or some other number.)

Be sure to highlight ideas about using scales of 2 or 5.

Math Community

After the cool-down, give students 2–3 minutes to discuss in small groups any revisions to the "Norms" section. Collect and record any revisions.

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students draw bars that use a scale of 1 instead of a scale of 5.

Next Day Support

- Use the launch of the next day's activity to discuss the difference between a bar graph that has a scale of 1 and a scaled bar graph.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 6: Choose a Scale

Standards Alignments

Addressing 3.MD.B.3

Teacher-facing Learning Goals

- Choose an appropriate scale for a bar graph that represents a given data set.

Student-facing Learning Goals

- Let's choose a scale for our bar graph.

Lesson Purpose

The purpose of this lesson is for students to consider the advantages and disadvantages of various bar graph scales.

In previous lessons, students created scaled picture and bar graphs with a given scale of 2 or 5. This lesson extends this work to allow students to choose the scale for their bar graph and reflect on the advantages or disadvantages of their choices. Through the work of the lesson, students notice that they can choose a scale based on the numbers in the data set and that the scale can make a graph easier or more difficult to read (MP6).

Math Community

Tell students they will reflect on their identified norms at the end of this lesson.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

In tomorrow's lesson, students solve one- and two-step "how many more" and "how many fewer" problems using data presented in scaled bar graphs. Based on the work you have seen students doing in previous lessons, what

Lesson Synthesis	10 min
Cool-down	5 min

strategies do you anticipate each student will use to solve these problems? How will you encourage each student to share their understandings and listen to one another's strategies?

Cool-down (to be completed at the end of the lesson)

5 min

Reflection on Bar Graphs and Scale

Standards Alignments

Addressing 3.MD.B.3

Student-facing Task Statement

1. How did you decide on the scale for your graph in the last activity?
2. What was the most important thing you learned today that will help when you make your next scaled bar graph?

Student Responses

1. Sample response: I chose a scale of 5 so I would have less numbers to write on my scale.
2. Sample response: I learned to think about the numbers in my graph to help me choose a scale.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Bar Graph Scales

Standards Alignments

Addressing 3.MD.B.3

The purpose of this warm-up is to elicit the idea that adjusting the scale changes the size of the bars in a bar graph and can make it easier or more difficult to interpret. While students may notice and

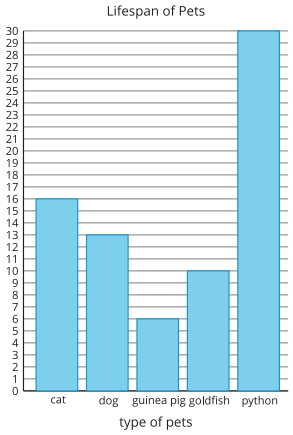
wonder many things about these graphs, the different scales in the bar graphs are the most important discussion points.

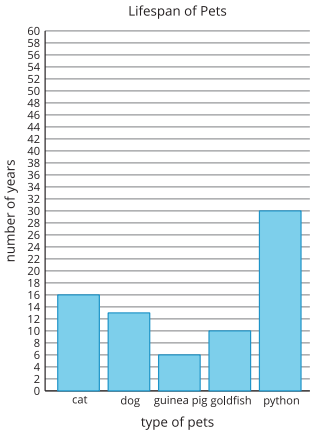
Instructional Routines

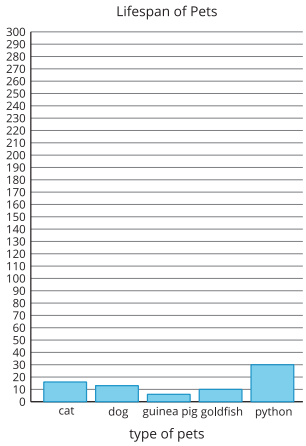
Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?







Student Responses

Students may notice:

- The data is the same.
- The jumps on the scales are different. The

Launch

- Groups of 2
- Display the graphs.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How are the three graphs different?" (They show the same data, but the bars are different heights. They have different scales.)

highest number on the graphs is different.

- It's harder to tell the number for guinea pigs on the last graph.

Students may wonder:

- Why are the jumps on the scales different?
- Why are the lines the same on each graph?

Activity 1

20 min

Represent Pattern Blocks

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to analyze a scale and create a scaled bar graph. Students consider a large collection of pattern blocks and decide which scale will work best to represent the categorical data. They consider three students' ideas, choose a scale of 2, 5, or 10, and create a scaled bar graph to represent the categorical data. Students must justify why they agree that a particular scale would be best. During the activity and whole-class discussion, students share their thinking and have opportunities to listen to and critique the reasoning of their peers (MP3). Providing a variety of scales for students to choose from allows for discussion about the benefits of using larger scales for larger groups of objects and the effect of a scale on how easy it may be to read and interpret data in a graph.

Access for Students with Disabilities

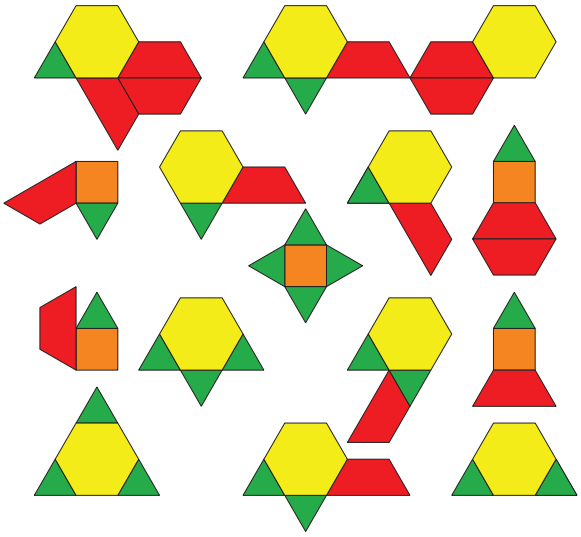
Representation: Access for Perception. Provide access to pattern blocks to model the collection of pattern blocks in the student-facing task statement.
Supports accessibility for: Organization, Visual-Spatial Processing

Student-facing Task Statement

Here is a collection of pattern blocks.

Launch

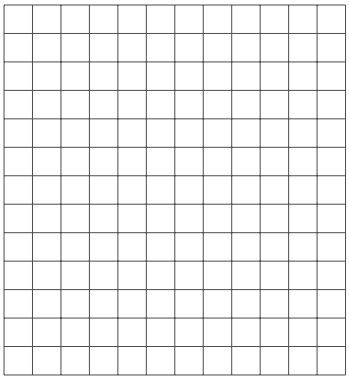
- Groups of 2
- Display the image.
- "Take a minute to consider these pattern blocks and think about how you could



Mai, Noah, and Priya want to make a bar graph to represent the number of triangles, squares, trapezoids, and hexagons in the collection.

- Mai says the scale of the bar graph should be 2.
- Noah says the scale of the bar graph should be 5.
- Priya says the scale of the bar graph should be 10.

1. Who do you agree with? Explain your reasoning.
2. Use the scale that you chose to create a scaled bar graph to represent the collection of pattern blocks.



represent them in a scaled bar graph."

- 30 seconds: quiet think time

Activity

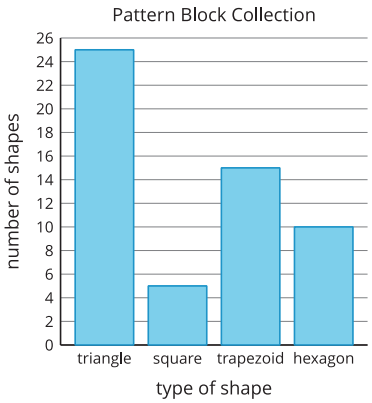
- "Now answer the questions about organizing and representing the pattern blocks in a bar graph with your partner. Be prepared to justify your choice of scale."
- 12 minutes: partner work
- Monitor for students who used each of the scales to create their bar graph.

Synthesis

- Display selected student work showing each of the scales.
- "What scale did you use for your bar graph? Why did you choose that scale?" (I used a scale of 5 because each amount can be counted by 5. I used a scale of 10 so I don't have to make as many marks on the scale.)

Student Responses

1. I agreed with Mai because there are a lot of blocks and counting by 2 is easy. My partner agreed with Priya because it would be really fast to count by 10.
2. Sample response:



Advancing Student Thinking

If students use a scale of 2, consider asking:

- "How did you decide on the scale to use in your graph?"
- "How would using a scale of 5 or 10 affect your graph?"

Activity 2

15 min

Represent More Data in a Scaled Bar Graph

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to represent data in a scaled bar graph. In this activity, the categorical data is presented in a table. Students choose a scale and make a scaled bar graph

of the categorical data. Students have prior experience with scales of 2, 5, and 10, and are not directed to a specific scale in this activity. However, due to the larger numbers, it is likely that students choose a scale of 5 or 10. If students struggle to get started, you could suggest a scale of 5 or 10. In the whole-class discussion, students share how their choice of scale affected their graph.

Students will use their scaled bar graphs again in the next lesson.

Access for English Learners

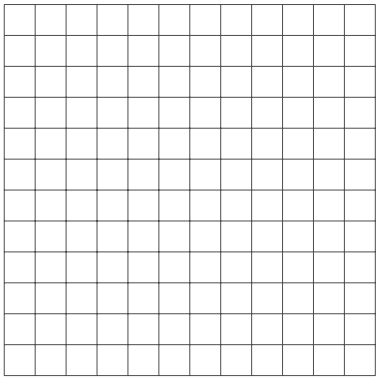
MLR8 Discussion Supports. During small-group discussion, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Student-facing Task Statement

All the third-grade students at school were asked, "What is your favorite time of the year?" Their responses are shown in this table:

favorite time of the year	number of students
winter	24
spring	13
summer	40
fall	22

Use the data from this table to create a scaled bar graph.



Launch

- Groups of 4
- "What is your favorite time of the year?"
- 30 seconds: quiet think time
- Share responses.
- "We are going to make a scaled bar graph to represent some 3rd grade students' favorite times of the year."

Activity

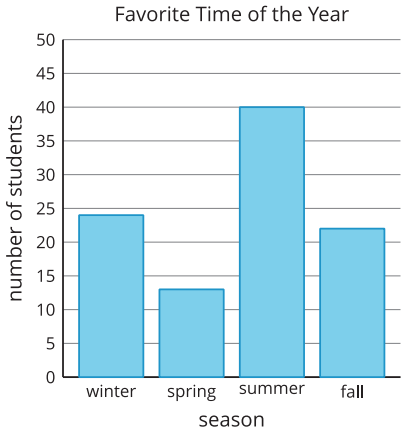
- "Represent the data shown in the table in a scaled bar graph. Think about a scale that makes sense with the number of students."
- 5–7 minutes: independent work time
- "Share your graphs with your small-group. Discuss the scales you chose to use."
- 2–3 minutes: small-group discussion

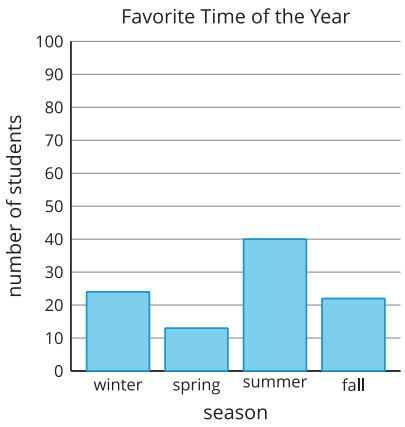
Synthesis

- "How did the scale you chose for your graph affect how your graph looked in the end?" (Certain scales make it easier or

Student Responses

Sample responses:





more difficult to read the data. For example, with a scale of 10, it might be more difficult to read the exact values from the graph.)

Lesson Synthesis

10 min

Display the bar graphs from today's lesson.

"What did you learn today that will help you make decisions about how to create scaled graphs in the future?" (You can pick scales that match the data. If there's mostly larger numbers, you might pick a scale like 5 or 10. The scale can help make the graph easier to read.)

Math Community

After the cool-down, ask students to individually reflect on the following question: "Which one of the norms did you feel was most important in your work today, and why?" Students can write their responses on the bottom of their cool-down paper, on a separate sheet of paper, or in a math journal.

Tell students that as their math community works together over the course of the year, the group will continually add to and revise its “Doing Math” and “Norms” actions and expectations.

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students have responses they'd like to share with a partner.

Next Day Support

- Before the first activity, pair students up to discuss their responses.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 1, Section B: Ways to Represent Data

Lesson 7: Answer Questions about Scaled Bar Graphs

Standards Alignments

Building On	2.OA.C.3
Addressing	3.MD.B.3
Building Towards	3.MD.B.3

Teacher-facing Learning Goals

- Solve one-step "how many more" and "how many fewer" problems within 100, based on the data presented in scaled bar graphs.

Student-facing Learning Goals

- Let's solve problems based on data represented in bar graphs.

Lesson Purpose

The purpose of this lesson is for students to solve one-step "how many more" and "how many fewer" problems based on data presented in a scaled bar graph.

In grade 2, students solved simple Put Together, Take Apart, and Compare problems using data represented in a single-unit scaled bar graph.

In this lesson, students solve one-step Compare problems using data represented in scaled bar graphs.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

Think about a time you recently made a mistake during math class. How did you leverage your mistake to show students that mistakes are just

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

learning in process?

Cool-down (to be completed at the end of the lesson)

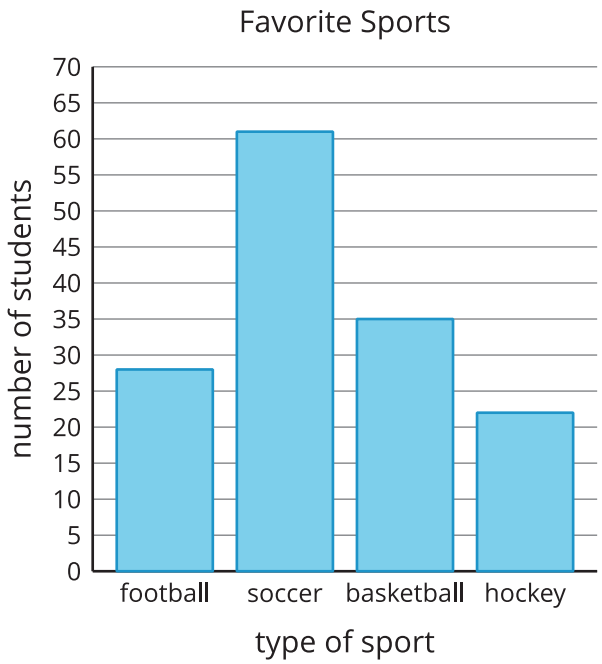
5 min

Favorite Sports

Student-facing Task Statement

A group of students were asked, "What is your favorite sport?"

Their responses are shown in this bar graph:



Use the graph to answer the questions.

1. How many more students chose soccer than football? Show your thinking using expressions or equations.
2. How many fewer students chose hockey than basketball? Show your thinking using expressions or equations.

Student Responses

1. 33 students. Sample response: $28 + 2 = 30$, $30 + 31 = 61$, and $2 + 31 = 33$.
2. 13 students. Sample response: $35 - 22 = 13$

Begin Lesson

Warm-up

10 min

How Many Do You See: Groups of Dots

Standards Alignments

Building On	2.OA.C.3
Building Towards	3.MD.B.3

The purpose of this How Many Do You See is for students to subitize or use grouping strategies to describe the images they see.

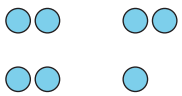
When students notice that some of the dots are in equal groups and skip-count to find the total number of dots they are looking for and making use of structure (MP7).

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?

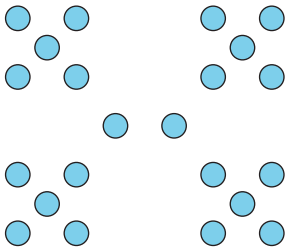


Launch

- Groups of 2
- "How many do you see? How you do see them?"
- Flash the image.
- 30 seconds: quiet think time

Activity

- Display the image.



Student Responses

Sample responses:

- 7: I see 3 groups of 2 and one more.
- 22: I counted by 5 for the 4 groups of 5 to get 20, then added 2 more for the dots in the middle.

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Synthesis

- "How did thinking about groups help you find the total number of dots?" (Some of the dots were in groups and some were not. I used skip-counting to count the groups that were the same size, then added on the rest of the dots.)
- Consider asking:
  - "Who can restate the way _____ saw the dots in different words?"
  - "Did anyone see the dots the same way but would explain it differently?"

Activity 1

15 min

Questions about Favorite Time of the Year

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to use data presented in scaled bar graphs to solve one-step "how many more" and "how many fewer" problems. Students use scaled bar graphs that they created in the previous lesson that contain data about the favorite time of the year. Answering questions about a graph with which they are familiar prepares them for the next task in which they answer questions about a new graph. This activity provides an opportunity for formative assessment of students addition and subtraction methods. In grade 2, students were expected to fluently add and subtract within 100.

Materials to Gather

Materials from a previous lesson

Required Preparation

Students will need their Favorite Time of the Year graphs from the previous lesson.

Student-facing Task Statement

Use your Favorite Time of the Year graph to answer the questions. Show your thinking using expressions or equations.

1. How many students are represented in the graph?
2. How many students chose spring or fall as their favorite season?
3. How many more students chose summer than winter?
4. How many fewer students chose spring than fall?

Student Responses

1. 99 students. Sample response:
   $24 + 13 = 37$
   $37 + 40 + 77$
   $77 + 22 = 99$
2. 35 students. Sample response:
   $13 + 22 = ?$
   $13 + 2 = 15$
   $15 + 20 = 35$
3. 16 more students. Sample response:
   $40 - 24$
   $40 - 20 = 20$
   $20 - 4 = 16$
4. 9 fewer students. Sample response:
   $13 + ? = 22$
   $13 + 7 = 20$
   $20 + 2 = 22$
   $7 + 2 = 9$

Launch

- Groups of 2
- Make sure that students have their Favorite Time of the Year graphs from a previous lesson.
- "Take a minute to look over the questions you'll answer using the Favorite Time of the Year graph from a previous lesson."
- 1 minute: quiet think time

Activity

- "Work with your partner to answer the questions."
- 7–10 minutes: partner work time
- Monitor for different strategies students use to add or subtract, particularly strategies that use tens and ones.

Synthesis

- Have students share responses to the last 2 questions.
- "How are these questions the same? How are they different?" (They both ask about the difference between 2 categories. The first question uses "more" and the second question uses "fewer.")
- As students share, use this as an opportunity to highlight addition and subtraction strategies in which students use tens and ones.

Advancing Student Thinking

If students find the sum of the quantities in the compare problems or do not find a value, consider asking:

- "What is this problem about?"
- "How would you describe two categories in the graph using the phrase, 'more than' or 'fewer than'?"

Activity 2

20 min

Questions About Bugs in the Garden

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to use data presented in scaled bar graphs to solve one-step "how many more" and "how many fewer" problems. The graph in the previous activity was familiar to students since they had created it in the previous lesson, but the graph used in this activity is new to students. Because the graph has a scale of 10, students need to estimate values that do not show an exact multiple of 10. As a result, answers may vary slightly. Accept all answers that align to reasonable estimates.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Involve both students in sharing their response with the whole class. While one student speaks, invite the other student to follow along and point to the corresponding parts of the bar graph on the display.
Advances: Speaking, Representing, Listening

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, "Does this situation remind anyone of something we have seen, read, or done before?"
Supports accessibility for: Social-Emotional Functioning

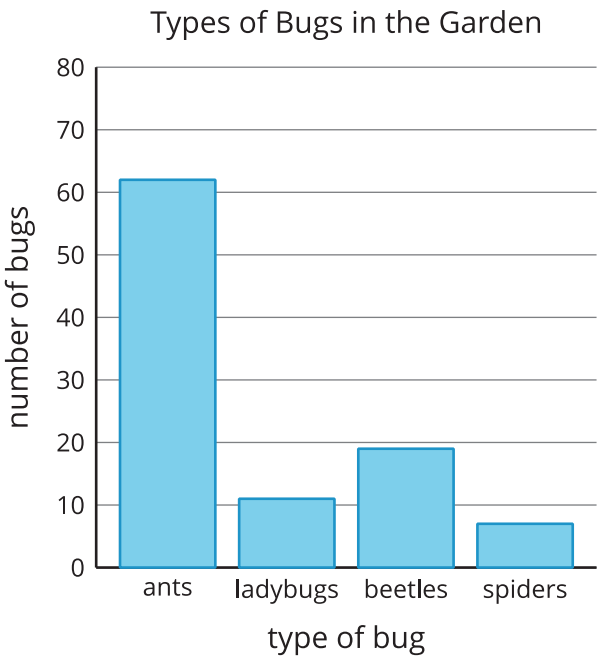
Student-facing Task Statement

Data was collected to see how many of each type of bug were in a garden. The data is shown

Launch

- Groups of 2
- "Look at the scaled bar graph and tell your

in this bar graph:



Use the bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many bugs were in the garden?
2. How many more ants were in the garden than spiders?
3. How many fewer ladybugs were there than ants?
4. Work with your partner to write 2 questions that could be asked about the bugs in the garden.
5. Trade with another group and answer each others' questions.

Student Responses

1. 99 bugs. Sample response:
   $62 + 11 + 19 + 7 = ?$
   $11 + 19 = 30$
   $30 + 62 = 92$

partner one thing you notice."

- 1 minute: partner discussion

Activity

- "Work with your partner to use the data in the bar graph to complete the first four problems."
- 7–10 minutes: partner work time
- Monitor for different strategies students use to add or subtract, particularly strategies that use tens and ones.
- "Now, trade the questions you wrote for the fourth question with another group and answer their questions."
- 2–3 minutes: partner work time

Synthesis

- Have students share responses for the first three problems. Be sure to share a variety of reasonable estimates for the values in each category to make sure students know it's okay if they don't know the exact value for sure.
- Have 2–3 groups share a question they wrote and have the whole class solve it. As students share, use this as an opportunity to highlight addition and subtraction strategies in which students use tens and ones.

$92 + 7 = 99$

2. 53 more. Sample response: $61 - 8$ is $61 - 1 - 7$, which is 53.
3. 51 fewer. Sample response: $62 - 11 = 51$
4. Sample response: How many of the bugs were spiders or beetles?
5. Sample response: 26 of the bugs were spiders or beetles. $7 + 19 = 6 + 1 + 19$, which is 26.

Advancing Student Thinking

If students find sums for compare problems or do not find a solution, consider asking:

- "Tell me about how you started this problem?"
- "How could you use the information from the graph to solve the problem?"

Lesson Synthesis

10 min

Display the bar graph from the last activity in today's lesson.

"What were some strategies that were helpful today as you answered questions about the data represented in bar graphs?" (I used the bar graph to get the numbers before I added or subtracted. I used the bar graph to find the answers by looking at one of the bars and counting up to the other bar in the problem.)

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students show they understand which categories to compare, but they use numbers that are not based on the scale of the graph.

Next Day Support

- Use the launch of the next day's activity to brainstorm tips for reading a scaled graph.

The work of this lesson builds from the addition and subtraction concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 2, Section C: Represent and Solve Story Problems

Lesson 8: More Questions about Scaled Bar Graphs

Standards Alignments

Addressing 3.MD.B.3

Teacher-facing Learning Goals

- Solve one- and two-step "how many more" and "how many fewer" problems within 100, based on the data presented in scaled bar graphs.

Student-facing Learning Goals

- Let's solve problems using data shown on bar graphs.

Lesson Purpose

The purpose of this lesson is for students to solve one- and two-step "how many more" and "how many fewer" problems, based on data presented in a scaled bar graph.

This lesson introduces Three Reads (MLR 6) to support students in making sense of and solving situations. In this lesson students continue to interpret graphs that represent quantities that are not exact multiples of the scale and require students to estimate values. As a result, answers may vary slightly. Accept all answers that align to reasonable estimates.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

MLR6 Three Reads (Activity 1), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

Who got to do math today in class and how do you know? Identify the norms or routines that allowed those students to engage in mathematics. How can you adjust these norms and routines so all students do math tomorrow?

Lesson Synthesis	10 min
Cool-down	5 min

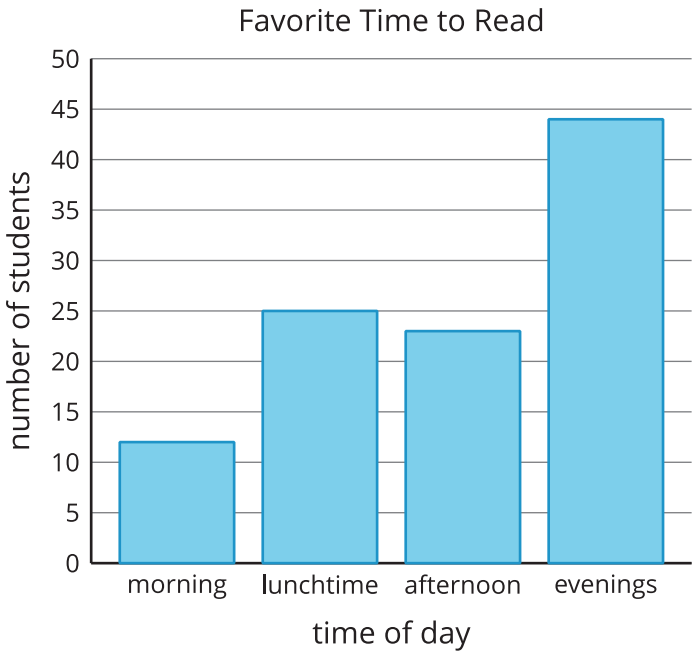
Cool-down (to be completed at the end of the lesson)

5 min

Reading Time

Student-facing Task Statement

A group of students were asked, "What is your favorite time to read?" Their responses are shown in this bar graph:



Use the graph to answer the questions.

1. How many fewer students like to read in the morning than in the afternoon? Show your thinking using expressions or equations.
2. How many more students like to read in the evening than in the morning or at lunchtime? Show your thinking using expressions or equations.

Student Responses

1. 12. Sample responses: $23 - 11 = 12$

2. 7. Sample responses:
   - $44 - (12 + 25)$
   - $44 - 10 - 20 = 14$
   - $14 - 2 - 5 = 7$

Begin Lesson

Warm-up

10 min

Number Talk: Repeated Addition

Standards Alignments

Addressing 3.MD.B.3

The purpose of this Number Talk is to elicit strategies and understandings students have for adding groups of 2 and groups of 5. These understandings help students develop fluency and will be helpful later in this lesson when students need to be able to use data in scaled bar graphs to solve one- and two-step "how many more" and "how many fewer" problems. Students use the structure of the expressions and repeated reasoning when they use methods based on skip-counting by 2 or 5 or counting on 2 or 5 from a previous known value (MP7, MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $2 + 2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.

Student Responses

- 10: I skip-counted by 2 five times: 2, 4, 6, 8, 10.
- 12: It is one more 2 than the first problem, $10 + 2 = 10$.
- 30: I counted by 5.
- 35: It is one more 5 than the problem before this.

- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What is a strategy that you hadn't used before that you might use in the future?" (Thinking about one more group if I already know part of the problem. Skip-counting.)
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Do you agree or disagree? Why?"

Activity 1

20 min

New School Year

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is to introduce MLR6, Three Reads, and solve a two-step "how many fewer" problem using data presented in a scaled bar graph. The routine prompts students to read a problem three times for different purposes to support them in making sense of the problem (MP1).

Instructional Routines

MLR6 Three Reads

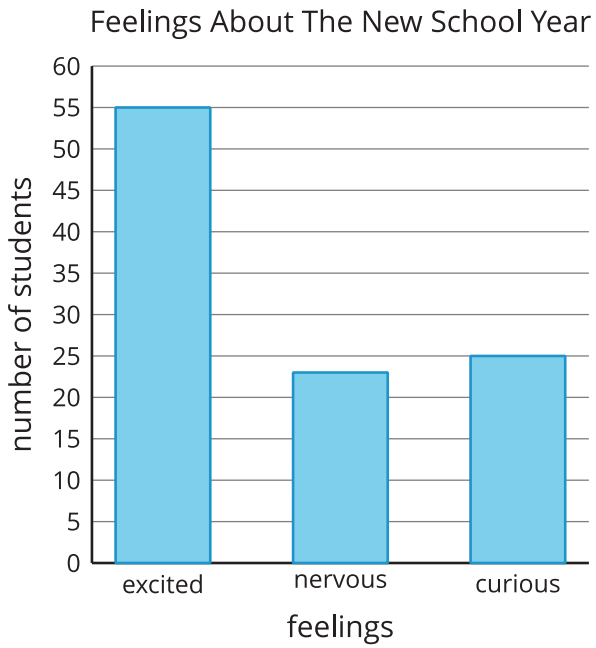
Student-facing Task Statement

A group of students were asked, "How are you

Launch

- Groups of 2

feeling about the new school year?" Their responses are shown in this bar graph:



How many more students are excited about the new school year than are nervous or curious?

Student Responses

7 students

MLR6 Three Reads

- "Keep your books closed."
- Display only the graph, without revealing the question.
- "We are going to read this graph 3 times."
- 1st read: "Take a moment to read the data displayed by this graph"
- "What is this graph about?" (It's about how students are feeling about the new year.)
- 1 minute: partner discussion
- Listen for and clarify any questions about the context.
- 2nd read: "Read and interpret the graph a second time. What quantities are represented? What can be counted or measured in this situation?" (We can count the number of students in each category.)
- 30 seconds: quiet think time
- 2 minutes: partner discussion
- Record quantities on a display for all to see.
- Reveal the question.
- 3rd read: Read the question aloud.
- "What are some strategies we can use to solve this problem?" (We could add the 24 plus 25 to see how many students are nervous or curious. We could subtract how many students are nervous or curious from the number of students that are excited.)
- 30 seconds: quiet think time
- 1–2 minutes: partner discussion

Activity

- 5-7 minutes: partner work time
- Monitor for students who:
  - add 25+23 and then subtract 55-48
  - subtract 55-23 and then subtract 32-25
  - subtract 55-25 and then subtract

30-23

Synthesis

- Have 1-2 selected students share their solution and strategy.
- Display the steps of Three Reads routine and keep displayed for the next activity.
- "How did reading the problem three times help you make sense of the problem?" (I was more comfortable with the problem each time I read it. It made more sense every time I read the problem. I noticed different details each time I read it.)

Activity 2

15 min

Use Bar Graphs to Solve Problems

Standards Alignments

Addressing 3.MD.B.3

The purpose of this activity is for students to practice the Three Reads math language routine on their own and use data presented in a scaled bar graph to solve a two-step "how many more" situation. The Three Reads routine has students read a problem three times for different purposes to support them to make sense of the problem and persevere in solving it (MP1).

Access for English Learners

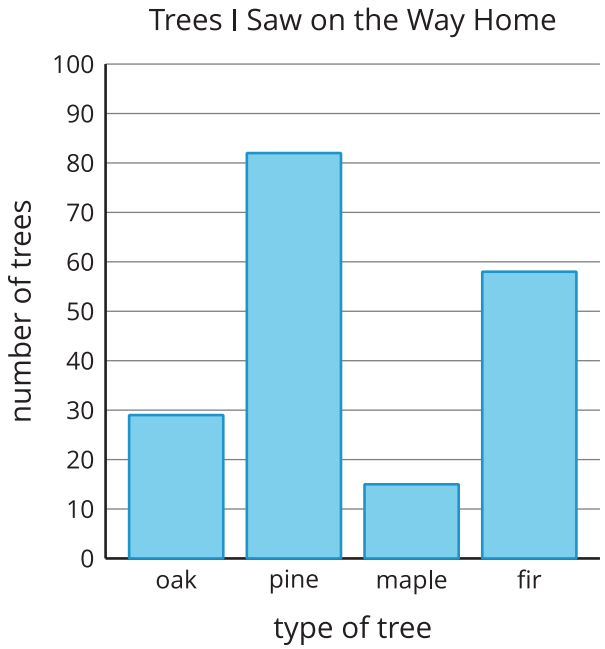
MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Listening, Speaking

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which problem to start with or decide the order to complete the task.
Supports accessibility for: Social-Emotional Functioning

Student-facing Task Statement

The bar graph shows how many of each type of tree Clare saw on the way home. Use the graph to answer the questions. Show your thinking using expressions or equations.



1. How many more pine trees did Clare see than fir trees?
2. How many more pine trees did Clare see than oak or maple trees?
3. How many fewer oak trees did Clare see than pine trees?
4. How many fewer maple or oak trees did Clare see than fir trees?

Student Responses

1. 24 more. Sample responses:
   - $82 - 20 = 62$
     $62 - 4 = 58$
   - $58 + 2$
     $60 + 20$

Launch

- Groups of 2
- "What types of plants do you see on your way home from school?" (Trees. Bushes. Flowers. Vines.)
- Share responses.

Activity

- Display the graph.
- "Work independently to use the graph to solve the problems. You can use the Three Reads routine if it's helpful to you."
- 7–10 minutes: independent work time
- Monitor for students who solve the one-step problems by:
  - using the bars on the graph to count up to or back from one number to another.
  - using equations to show adding or subtracting by place.
- Monitor for students who solve two-step problems by:
  - adding two numbers then subtracting the sum from another number.
  - subtracting a number, then subtracting again from the difference.

Synthesis

- Have 1-2 students share the method, including any expressions or equations they used to solve the first problem.
- Have 2-3 students share the expressions or equations they used to solve the last problem.
- "How are these methods the same? How

$80 + 2$
$2 + 20 + 2$

2. 38 more. Sample responses:
   - $20 + 10 = 30$
     $9 + 5 = 14$
     $30 + 14 = 44$
     $82 - 40 = 42$
     $42 - 4 = 38$
   - $82 - (29 + 15)$
3. 53 fewer. Sample responses:
   - $82 - 20 = 62$
     $62 - 2 = 60$
     $60 - 7 = 53$
   - $29 + 1$
     $30 + 50$
     $80 + 2$
     $1 + 50 + 2$
4. 14 fewer. Sample response:
   - $20 + 10 = 30$
     $9 + 5 = 14$
     $30 + 14 = 44$
     $58 - 44 = 14$
   - $58 - (15 + 29)$

are they different?" (Some people added the maple and oak trees together and then subtracted it from the number of fir trees. Some people subtracted the number of maple trees from fir trees and then subtracted the number of oak trees. Each way found the same number of trees.)

Advancing Student Thinking

If students find the sum of all categories or don't find a solution to the problems, consider asking:

- "Tell me about how you started this problem?"
- "How could you use the information from the graph to solve the problem?"

Lesson Synthesis

10 min

Display equations from the first problem in the last activity.

"How did you use what you know about tens and ones to solve the problems?" (Some questions I saw I could just count by 10 on the graph because the scale was 10. We didn't subtract all at once. We subtracted the tens, then the ones. We were thinking about how to get to the next ten to make adding the tens easier.)

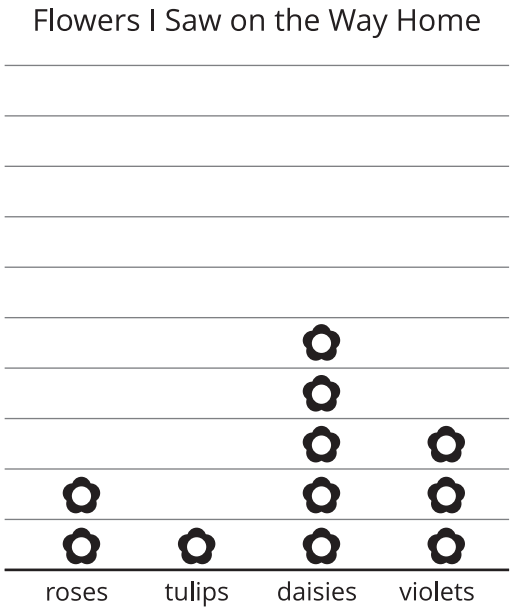
"In the future, how could you use the Three Reads strategy on your own, without a partner?" (We could use it any time by thinking what the problem is about. We can read the problem to look for what can be measured or counted in the problem. We can read it to think about strategies we could use to solve the problem.)

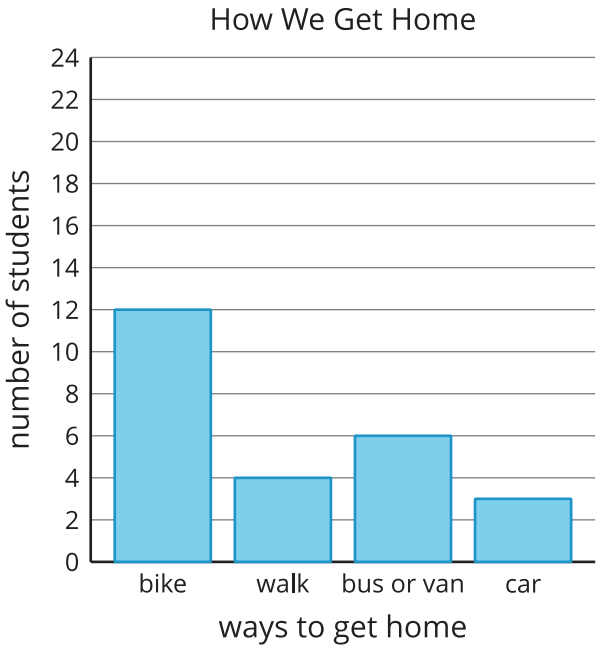
Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Student Section Summary

In this section, we created scaled picture graphs and scaled bar graphs.





We asked and answered questions that could be answered by the graphs.

- How many more daisies were seen than violets?
- How many fewer students walk home than bike home?
- How many more students bike home than walk or ride in a car?

Complete Cool-Down

Response to Student Thinking

Students find sums rather than the differences.

In the two-step compare problem, students only find the difference between evening and morning (or evening and lunchtime).

The work of this lesson builds from the addition and subtraction concepts developed in a prior unit.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Prior Unit Support

Grade 2, Unit 2, Section C: Represent and Solve Story Problems

Section B: From Graphs to Multiplication

Lesson 9: Multiplication as Equal Groups

Standards Alignments

Building On	2.NBT.B.5
Addressing	3.OA.A, 3.OA.A.1
Building Towards	3.OA.A.1

Teacher-facing Learning Goals

- Build an understanding of multiplication as equal groups.
- Represent a situation involving equal groups in a way that makes sense to students.

Student-facing Learning Goals

- Let's work with equal groups of things.

Lesson Purpose

The purpose of this lesson is for students to use scaled picture graphs as an introduction to **multiplication** as equal groups.

Scaled picture graphs provide an equal grouping context that naturally elicits **multiplication.** Multiplication expressions aren't introduced in this lesson so that students spend more time with concrete representations of multiplication before being introduced to the more abstract representation. The next few lessons focus on the meaning and representations of multiplication, not the product. While students may want to go right to finding the product, it is important to focus on the meaning of multiplication as equal groups and the ways in which it can be represented in the discussions.

Throughout this section, make connecting cubes or counters available to students who need them.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Connecting cubes or counters: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did students' work with scaled picture graphs and bar graphs set up the introduction of multiplication in today's lesson?

Cool-down (to be completed at the end of the lesson)

5 min

Represent Equal Groups

Standards Alignments

Addressing 3.OA.A

Student-facing Task Statement

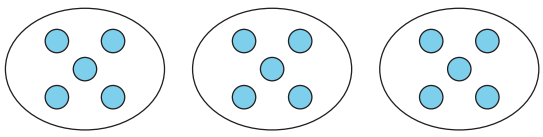
Jada has 3 bags. Each bag has 5 bracelets in it.

Represent the situation.

Student Responses

Sample responses:

- Students make 3 groups of 5 counters or 3 groups of 5 connecting cubes.
- Students create a drawing of 3 groups with 5 items in each.



Begin Lesson

Warm-up

10 min

Number Talk: More Addition

Standards Alignments

Building On 2.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for addition within 100. It also provides an opportunity to observe student strategies as they work toward becoming fluent in addition within 1,000.

When students use strategies based on place value to add they look for and make use of structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $40 + 35$
- $45 + 35$
- $45 + 36$
- $34 + 58$

Student Responses

- 75: There are 7 tens and 5 ones, which is 75.
- 80: There are 7 tens and 5 plus 5 makes another ten, which would be 80.
- 81: It's like the one right before it, but there's 1 more, which would make 81.
- 92: There are 8 tens and 12 ones. If I break apart the 12 into a ten and 2 ones, then I have 9 tens and 2 ones, which is 92.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did you compose new tens as you solved these problems?" (In the second problem I composed a ten from the 2 fives. In the third problem I composed a new ten from the 5 and the 6 and still had 1 leftover.)
- Consider asking:

- "Who can restate _____ 's reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone approach the problem in a different way?"
- "Does anyone want to add on to____'s strategy?"

Activity 1

15 min

From Scaled Graphs to Equal Groups

Standards Alignments

Addressing	3.OA.A
Building Towards	3.OA.A.1

The purpose of this activity is for students to connect scaled picture graphs to situations involving equal groups. The scale of the picture graph will be used to help students think about a category of the graph as a situation involving equal groups.

The launch of the activity is an opportunity for students to share their experiences and ask questions about the graph to ensure each student has access to the context. If it is helpful, display a few images of different types of signs students may see in their community.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were important or most useful to solve the problem. Display the sentence frame, "The next time I read a scaled picture graph, I will pay attention to . . . "
Supports accessibility for: Visual-Spatial Processing

Materials to Gather

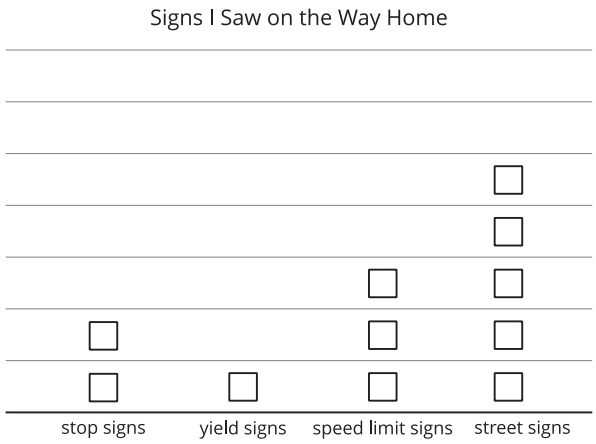
Connecting cubes or counters

Required Preparation

Each student needs 20 connecting cubes or counters.

Student-facing Task Statement

Elena collected data about the kind of signs she saw on the way home. The data is shown in this picture graph:



Each ☐ represents 2 signs.

1. Represent the number of speed limit signs Elena saw on the way home.
2. Which statement describes the number of speed limit signs Elena saw? Explain your reasoning.

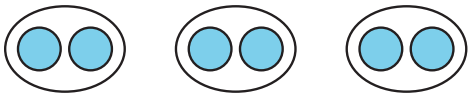
   A. There are 3 pictures and each picture represents 1 speed limit sign.

   B. There are 3 pictures and each picture represents 2 speed limit signs.

   C. There are 2 pictures and each picture represents 2 speed limit signs.
3. How could this drawing represent the street signs Elena saw on the way home?

Student Responses

1. Sample response:



Launch

- Groups of 2
- Give students access to connecting cubes or counters.
- "We're going to look at a scaled picture graph about signs that Elena saw on the way home. What types of signs do you see in the community?" (stop signs, speed limit signs, street signs, billboards)
- 30 seconds: quiet think time
- 1 minute: partner discussion
- Share responses.
- Display the graph.

Activity

- "Work independently to represent the number of speed limit signs that Elena saw on the way home."
- 1 minute: independent work time.
- Monitor for students who create drawings of equal groups similar to the one shown in the last problem to display during the synthesis.
- 1 minute: partner discussion
- "Work with your partner to complete the next problem."
- 1 minute: quiet think time
- 2 minutes: partner discussion
- "How did you know which statement described the speed limit signs that Elena saw on the way home?" (There were 3 pictures on the graph. Each picture represents 2 signs.)
- Share responses.
- "Take a few minutes to complete the last problem on your own."
- 3 minutes: independent work time
- "Share your responses with your partner."

2. B. Sample response: B describes 6 signs and that's how many speed limit signs Elena saw on the way home. Also, there were 3 pictures in the graph and each picture represents 2 signs.
3. Sample responses: There are 5 pictures in the graph and there are 5 groups. Each picture represents 2 signs and there are 2 dots in each group. There are 10 dots in the groups and 10 signs in the graph.

- 1 minute: partner discussion

Synthesis

- Display a student-created drawing of equal groups that represents the speed limit signs that Elena saw on the way home and the drawing in the last problem.
- "These drawings show equal groups. How did these drawings help you represent the data in the picture graph?" (The key told us that each picture represented 2 signs and the drawings helped us see the 2 signs. Each group showed 2 signs.)

Activity 2

20 min

Equal Group Situations

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to represent situations involving equal groups in a way that makes sense to them. Have connecting cubes available for students to use to represent the situation, if they would like. Students may also draw a picture. One partner could use the objects while one draws and then switch for each problem. The focus of the discussion is on the important quantities of each situation and how students used their representation to model each quantity (MP4).

In the launch of the activity, it may be helpful to ask students to tell their partner a quick story or ask any questions about the focus of each of the three contexts to ensure each student has access. It may also be helpful to display images for students to reference.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Involve both partners in sharing their response with the whole class. While one student speaks, invite the other student to follow along and point to where the numbers are in their representations.
Advances: Representing, Listening

Materials to Gather

Connecting cubes or counters

Required Preparation

Each student needs 20 connecting cubes or counters.

Student-facing Task Statement

Represent each situation.

1. There are 4 people wearing shoes. Each person is wearing 2 shoes.
2. There are 2 boxes of markers. Each box has 10 markers.
3. There are 3 basketball teams. Each team has 5 players.

Student Responses

Sample responses:

1.
   - 4 groups of 2 connecting cubes
   - A drawing of 4 groups with 2 dots in each group
2.
   - 2 groups of 10 counters
   - A drawing of 2 groups with 10 dots in each group
3.
   - 3 groups of 5 connecting cubes
   - A drawing of 3 groups with 5 tallies in each group

Launch

- Groups of 2
- Give students access to connecting cubes or counters.
- "What are some places you see groups of 2 in the community? Groups of 5? Groups of 10?" (Shoes. Socks. Wings. Hands have 5 fingers. Flowers can have 5 petals. Markers come in packs of 10. Ten people on a bus.)
- 30 seconds: quiet think time
- 1 minute: partner discussion
- Share and record responses.
- Choose a student-generated example with small numbers or display this situation: "There are 3 flowers. Each flower has 5 petals."
- "How could you represent this situation?"
- 30 seconds: quiet think time
- 1-2 minutes: partner work time
- Share and record responses. Focus on how the representation connects to the problem.
- Consider asking:
  - "How did you represent the 3 flowers?"
  - "How did you represent the 5 petals on each flower?"
  - "Did someone represent this differently?"

Activity

- "Now you are going to represent some more situations involving equal groups with your partner."
- 5–7 minutes: partner work time
- If some students finish earlier than others, encourage them to write their own situation and trade with their partner.

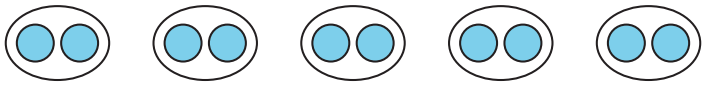
Synthesis

- Ask 2-3 students to share their work for each problem. Be sure to share a variety of different representations.
- In each, ask how the numbers in the situation are represented in their work.
- "How do the representations help you picture the situation?" (I can pretend the objects are the things in the story like the shoes. The drawing is like a picture of what's happening in the story.)

Lesson Synthesis

10 min

Display a representation of equal groups from the lesson.



"The situations we looked at today were all **multiplication**. Multiplication is how we represent the total number of objects when you have a certain number of equal groups. For example, in this picture, we would say we have 5 groups of 2."

"Describe a situation with equal groups that you could represent as multiplication." (Packs of pencils, bins or baskets with the same number of things in each one, pairs of shoes, rows of seats on the bus.)

Suggested Centers

- Capture Squares (1–3), Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–2), Stage 6: Add within 100 with Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students indicate that they mixed up the number of groups and the number of objects in each group by making 5 groups of 3.

Next Day Support

- Use the next day's warm-up to discuss how to represent a situation involving equal groups, differentiating between the number of groups and the number of objects in each group.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 8, Section A: Odd and Even

Lesson 10: Drawings, Situations, and Diagrams, Oh My!

Standards Alignments

Addressing	3.OA.A, 3.OA.A.1
Building Towards	3.OA.A.1

Teacher-facing Learning Goals

- Interpret a situation involving equal groups and represent it with a diagram.
- Make sense of tape diagrams that represent multiplication.

Student-facing Learning Goals

- Let's represent equal groups.

Lesson Purpose

The purpose of this lesson is for students to connect situations involving equal groups to tape diagrams.

This lesson introduces tape diagrams as a way to represent equal groups and multiplication, building on students' work with scaled picture graphs and discrete drawings of equal groups. Students deepen their understanding of multiplication as they connect tape diagrams to situations that involve equal groups. They are then introduced to multiplication **expressions** as a way to represent the quantities and situations encountered in the lesson. This happens at the end to allow students to work with other representations of multiplication before they learn about abstract symbols that represent multiplication.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 2), Notice and Wonder (Warm-up)

Materials to Copy

- Card Sort Equal Groups (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did connecting different representations of multiplication during the card sort support students in developing their understanding of multiplication?

Cool-down (to be completed at the end of the lesson)

5 min

Boxes of Shirts

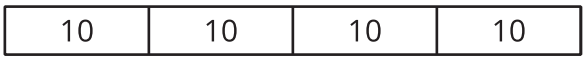
Standards Alignments

Addressing 3.OA.A

Student-facing Task Statement

The store has 4 boxes. Each box has 10 shirts in it.

Does this diagram match the situation? Explain your reasoning.



Student Responses

Yes, because the 4 parts represent the 4 boxes and the 10 in each part represents the 10 shirts in each box.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Socks

Standards Alignments

Addressing 3.OA.A.1

The purpose of this warm-up is to elicit different strategies for counting objects arranged in groups of 2, which will be useful when students multiply by 2 in a later activity. While students may notice and wonder many things about these images, flexible ways of seeing the groups and strategies for finding the total number of objects are the important discussion points.

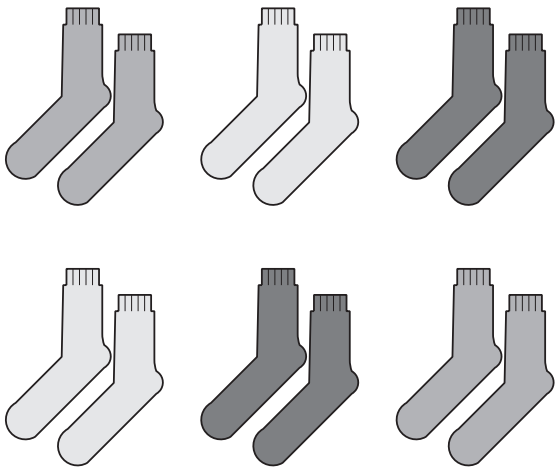
When students see the socks are grouped by 2 and use that to find the total, they are looking for and making use of structure (MP7).

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- The socks are in pairs.
- There are 12 socks.
- You could count by 2 to find the total number of socks.

Students may wonder:

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How does this problem relate to what we know about multiplication?" (There are equal groups of socks, we can say there are 6 groups of 2.)

- How many socks are there?
- How many pairs of socks are there?
- Why are there 6 pairs of socks?

Activity 1

15 min

Scaled Picture Graph to Diagram

Standards Alignments

Addressing	3.OA.A
Building Towards	3.OA.A.1

The purpose of this activity is for students to build on the work they have done with scaled picture graphs to use the tape diagram as a new representation of multiplication. The scale of the picture graph will be used to help students think about a category of the graph as a situation involving equal groups.

To add movement to this activity, students could find someone in the class who represented a different category than they did or represented the same category in a different way. When they find a person, they can describe what is the same and what is different about their representations.

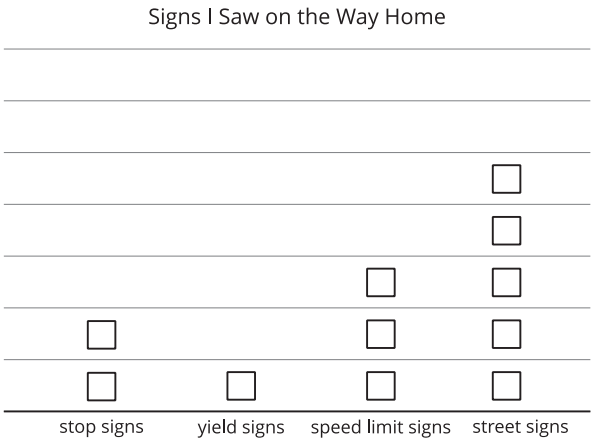
Access for English Learners

MLR8 Discussion Supports. Synthesis: When students compare the diagram and the scaled picture graph, display sentence frames to support whole-class discussion: "_____ and _____ are the same because . . .", and "_____ and _____ are different because . . . ."
Advances: Speaking, Representing

Student-facing Task Statement

Launch

- Groups of 2
- Display the picture graph and tape diagram.
- "What do you notice? What do you wonder?" (The key says each square represents 2 signs. The diagram shows 3



2	2	2

1. How does the diagram show the speed limit signs that Elena saw on the way home?
2. Represent the data from another category in the graph with your own drawing or diagram.

Student Responses

1. Sample response: There are 3 pictures (squares) in the graph and 3 sections of the diagram. The graph tells us that each picture (square) represents 2 signs. In the diagram, each section is labeled with a 2.
2. Sample responses for street signs:

2	2	2	2	2

groups of 2. I wonder whether they represent the same thing.)

- 1 minute: quiet think time
- 1 minute: partner discussion time
- "How does the diagram show the speed limit signs that Elena saw on the way home?"
- Share responses.

Activity

- "Now independently represent the data from another category in the graph with your own drawing or diagram."
- 1-2 minutes: independent work time
- "Share how you represented the data in your drawing or diagram with your partner."
- 2–3 minutes: partner discussion
- Monitor for students who create a tape diagram to represent one of the other categories to use during the synthesis.

Synthesis

- Have students share different ways they represented a category in the graph.
- Display a student created tape diagram or make a quick sketch of one to represent the street signs Elena saw on the way home.
- "Which category does this tape diagram represent? How do you know?"
- "How is the diagram the same as the scaled picture graph?" (Each picture and each part of the tape diagram represents 2 signs.)
- "How is the diagram different than the scaled picture graph?" (In the graph, you have to read the key to know that each picture shows two signs. In the diagram, each part is labeled with a 2.)

Activity 2

20 min

Card Sort: Equal Groups

PLC Activity

Standards Alignments

Addressing	3.OA.A
Building Towards	3.OA.A.1

The purpose of this activity is for students to connect situations involving equal groups to drawings and tape diagrams. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7). Students explain why two cards match and have opportunities to critique and question their peers' reasoning (MP3). When explaining, students have opportunities to revise their language to make their explanations more precise and clear (MP6). After sorting and describing their sort, students notice that all of the representations reinforce the meaning of multiplication as a way to express equal groups.

Students will spend all of the next lesson working with expressions. Keep the equal groups cards for the next lesson.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Attention, Organization

Instructional Routines

Card Sort

Materials to Copy

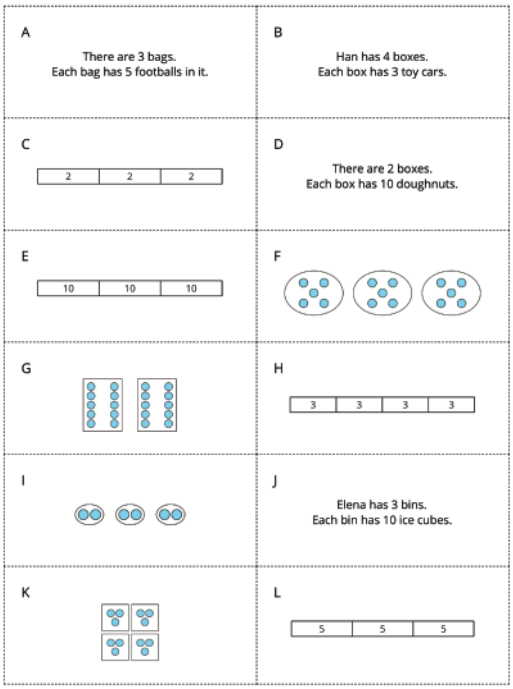
Card Sort Equal Groups (groups of 2)

Required Preparation

Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

1. Your teacher will give you a set of cards that show drawings, situations, and diagrams. Find the cards that match. Be ready to explain your reasoning.



2. Create a drawing or diagram for each situation.

   a. There are 4 bags. Each bag has 2 strawberries.

   b. There are 4 hands. Each hand has 5 fingers.

Student Responses

1.
   - A, F, and L
   - B, H, and K
   - C and I
   - D and G
   - E and J
     Sample response: E and J go together because the bins are like the sections in the diagram and the numbers are

Launch

- Groups of 2
- Give each group of 2 students a set of cards.
- "This set of cards includes drawings, situations, and diagrams. Take a little time to think about how some of the cards could match."
- 1 minute: quiet think time
- "Find the cards that match. Work with your partner to justify your choices. Be ready to explain your reasoning."
- 5 minutes: partner work time
- Have students share the matches they made and how they know those cards go together.
- Listen for the language students use to describe their match. If students only reference the numbers that match, consider asking:
  - "What do you mean when you say _____?"
  - "How could you use the words 'equal groups' to explain?"

Activity

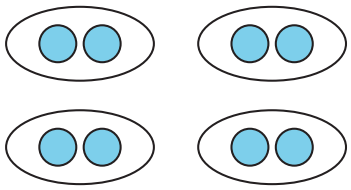
- "Now you're going to create a drawing or diagram to represent two different situations."
- 3–5 minutes: independent work time
- Monitor for the students who draw equal groups and students who draw a tape diagram.

Synthesis

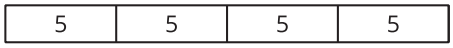
- For each situation in the last 2 problems, display 2–3 student representations, at least one drawing of equal groups and one tape diagram. Leave them displayed.

like the ice cubes.

2. a. Sample response:



b. Sample response:



- "What do all these representations have in common?" (They all show equal groups. They all have groups. Each group has the same amount.)
- "Where are the 4 bags and 2 strawberries in each drawing or diagram?"
- "Where are the 4 hands and 5 fingers in each drawing or diagram?"

Advancing Student Thinking

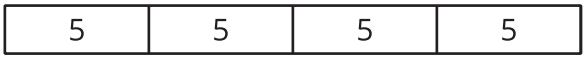
If students create representations that do not match the number of groups or size of the groups in the situations, consider asking:

- "How did you represent the situation?"
- "How could you show the groups in the situation? How could you show the objects in each group?"

Lesson Synthesis

10 min

Display the tape diagram.



"Today's lesson was all about multiplication. How can a diagram show multiplication?" (A diagram can show multiplication because you can draw the number of groups and write how many are in each group. You can see that there are 4 parts in this diagram, so there are 4 groups and the 5 tells you there are 5 things in each group.

Display:

4 groups and 5 in each group

4×5

"You may remember that an **expression** has at least 2 numbers and at least one math operation. A multiplication expression is how we represent the number of groups and number in each group in a

situation. For example, the multiplication expression 4×5 would represent this diagram because we have 4 groups and 5 in each group." Point to the 4 and 5 in the diagram and the expression as you explain.

"The symbol in the middle of the expression is the multiplication symbol. 4×5 can be read as '4 groups of 5.'"

Suggested Centers

- Capture Squares (1–3), Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–2), Stage 7: Add within 1,000 without Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students answer no and explain that the diagram should have 10 parts with 4 in each part.

Next Day Support

- During the launch of the next day's activity, have students discuss a matching situation and diagram, such as cards E and J.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 8, Section A: Odd and Even

Lesson 11: Multiplication Expressions

Standards Alignments

Addressing 3.OA.A.1

Teacher-facing Learning Goals

- Write multiplication expressions to represent situations involving equal groups and diagrams.

Student-facing Learning Goals

- Let's write multiplication expressions.

Lesson Purpose

The purpose of this lesson is for students to use multiplication expressions to represent equal groups.

In previous lessons, students represented situations involving equal groups with drawings and tape diagrams. Students were also shown how to represent equal groups as an expression. In this lesson, students connect the structure of drawings, tape diagrams, and multiplication situations to the structure of multiplication expressions (MP7). Students create diagrams and drawings to represent multiplication expressions and ultimately write their own expressions to represent drawings, diagrams, and situations (MP2).

When generating multiplication expressions, consider using the convention of the number of groups as the first factor and the size of the groups as the second factor. However, it is not necessary for students to write the factors in this order. It is important that students connect their expressions to the corresponding situations and representations. They should be able to correctly explain what each factor represents in their expressions. If students ask questions about the idea of commutativity, consider recording the questions publicly for future investigation.

To allow time for students to focus on the meaning of multiplication, it is not an expectation that students find the product of each expression in this lesson. In subsequent lessons, students will work on strategies for finding the product. If students mention the product in today's lesson, it is okay to note that, but try to maintain focus on the connections between the expression and the diagrams.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR2 (Activity 2)

Instructional Routines

Choral Count (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	15 min
Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

Cool-down (to be completed at the end of the lesson)

5 min

Write an Expression

Standards Alignments

Addressing 3.OA.A.1

Student-facing Task Statement

There were 6 envelopes. Each envelope had 2 notes in it.

Write a multiplication expression to represent the situation. Explain or show your reasoning. Create a drawing or diagram if it's helpful.

Student Responses

6×2 or 2×6. The 6 represents the 6 envelopes. The 2 represents the 2 notes in each envelope.

Begin Lesson

Warm-up

10 min

Choral Count: Twos and Fives

Standards Alignments

Addressing 3.OA.A.1

The purpose of this Choral Count is for students to practice counting by 5 and 2 and notice patterns in the count. These understandings help students begin to develop fluency and will be helpful later in this lesson when students write multiplication expressions.

When students notice patterns in the count, such as in the count by 5 that the ones place alternates between 0 and 5, they look for and express regularity in repeated reasoning (MP8).

Instructional Routines

Choral Count

Student Responses

0	0
5	2
10	4
15	6
20	8
25	10
30	12
35	14
40	16
45	18
50	20

Sample response:

In the fives column:

- The ones place alternates 0 and 5.
- The tens place changes after every two counts.

In the twos column:

- The ones place counts 0, 2, 4, 6, 8, then repeats.

Launch

- "Count by 5, starting at 0."
- Record as students count. See Student Responses for recording structure.
- Stop counting and recording at 50.

Activity

- "What patterns do you see?"
- 1-2 minutes: quiet think time
- Record responses.
- Repeat activity. Count by 2, start at 0 and stop at 20.

Synthesis

- "How could some of the patterns help you with counting by these numbers?" (I know that the next count by 5 should end in 5. I know that the next count by 2 should have a 2 in the ones place.)
- Consider asking:
  - "Who can restate the pattern in

- The tens column changes after 5 counts.

different words?"

- "Does anyone want to add an observation on why that pattern is happening here?"
- "Do you agree or disagree? Why?"

Activity 1

10 min

Multiplication Expression Match

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to match drawings, tape diagrams, and situations to multiplication expressions (MP2). Students build on their understanding of how the structure of drawings, tape diagrams, and multiplication situations show equal groups and connect this to the structure of a multiplication expression (MP7). This will be helpful later in the lesson when students create drawings or diagrams to match expressions and write expressions that represent drawings, diagrams, and situations.

Materials to Gather

Materials from a previous lesson

Required Preparation

- Each group of 2 needs 1 card from the card sort in the previous lesson.
- Post these expressions around the room:
  - 3×5
  - 4×3
  - 3×2
  - 2×10
  - 3×10

Student-facing Task Statement

Your teacher will give you a card showing a drawing, a diagram, or a situation.

Match it to one of the expressions posted around the room. Be prepared to explain your reasoning.

Student Responses

- A, F, and L: 3×5
- B, H, and K: 4×3
- C and I: 3×2
- D and G: 2×10
- E and J: 3×10

Launch

- Groups of 2
- Give each group of 2 students 1 card from the Instructional master.

Activity

- "Work with your partner to find the expression that matches your card. Then discuss how you know the expression matches your card."
- 2 minutes: partner work time

Synthesis

- Have students standing near each expression share how they know their card matches the expression.
- Consider asking:
  - "Where do you see each number in the expression on your card?"

Activity 2

15 min

Expressions to Drawings and Diagrams

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to demonstrate a conceptual understanding of multiplication expressions by creating drawings of equal groups or tape diagrams that match expressions. Drawings of equal groups and tape diagrams are familiar representations to students from previous lessons and support students as they make sense of multiplication expressions.

Three expressions are given, but the focus of the synthesis is the second expression, 3×4. This provides an opportunity to support students on the first problem as you monitor and then let

them try the second and third expressions on their own. To keep things simple and allow ideas about commutativity to develop over time, in this activity we suggest you display student responses that follow the convention of groups as the first factor and the size of the groups as the second factor.

If there is time, and you want to include more movement, this activity could be done as a gallery walk.

Access for English Learners

MLR2 Collect and Display. Collect the language students use to describe the diagrams for each of the expressions. Display words and phrases such as: "5 groups of 2", "there are 5 groups, and 2 in each group", and "there are 5 equal groups". During the synthesis, invite students to suggest ways to update the display and to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Disabilities

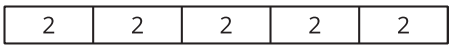
Action and Expression: Develop Expression and Communication. Provide access to a variety of tools: mini-whiteboards and counters.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

1. Create a drawing or diagram for each expression. Explain your reasoning.
    a. 5×2
    b. 3×4
    c. 3×10
2. Write your own expression and matching diagram. Explain your reasoning.

Student Responses

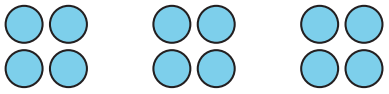
1. Sample responses:
    a. I showed 5 groups of 2.



    b. There are 3 groups and 4 in each group.

Launch

- Groups of 2
- Review key understandings of multiplication:
    - "Multiplication is how we express equal groups."
    - "We use the multiplication symbol to create an expression like 5×10 which represents the total number of objects in '5 groups of 10' or '5 tens.'"
- Display expressions.
- "Think about the drawings or diagrams you could make for these expressions."
- 30 seconds: quiet think time



c. It is 3 groups of 10.

10	10	10

2. Answers vary.

Activity

- "Work with your partner to create a drawing or diagram for each expression. Then, write your own expression and matching diagram. Explain your reasoning."
- 5–7 minutes: partner work time
- Monitor for student-created drawings and tape diagrams to share during synthesis.

Synthesis

- For the expression 3×4, display 2 different representations side by side (one drawing of equal groups and one tape diagram).
- "How are they the same? How are they different?"
- If time, consider asking:
  - "How would the diagram change if the expression was 5×4?" (There would be 5 sections instead of 3.)
  - "How would the diagram change if the expression was 3×5?" (Each section would be 5 instead of 4.)

Activity 3

10 min

Write Multiplication Expressions

Standards Alignments

Addressing 3.OA.A.1

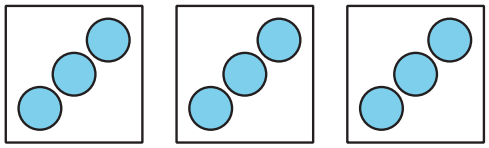
The purpose of this activity is for students to write expressions to represent drawings of equal groups, tape diagrams, and multiplication situations. As students work, continually ask where each number in the expression is in the drawing, diagram, or situation.

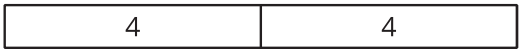
If students finish early, ask them to find something in the room they can represent with a

multiplication expression. Have them record what they represented and their expression.

Student-facing Task Statement

Write a multiplication expression to match each representation. Explain your reasoning.

1. 

2. 

3. There were 2 packs of water. Each pack had 6 bottles of water.

Student Responses

1. 3×3: It's 3 groups of 3.
2. 2×4 or 4×2: There were 2 parts and each part had 4 in it.
3. 2×6 or 6×2: There are 2 groups with 6 in each group.

Launch

- Groups of 2
- "Now you are going to write multiplication expressions to represent a drawing, a diagram, and a situation. Take a minute to look them over before you begin working."
- 1 minute: quiet think time

Activity

- "Work with your partner to write a multiplication expression to match each representation. Explain your reasoning."
- 3–5 minutes: partner work

Synthesis

- Share responses.
- "Why does each of the representations show multiplication?" (They all show groups where there is the same number of things in each group.)

Advancing Student Thinking

If a student writes a multiplication expression that doesn't match the given representation, consider asking:

- "How could you describe the equal groups in this drawing/diagram/situation?"
- "How could you turn your statement into a multiplication expression?"

Lesson Synthesis

10 min

Display the drawing of equal groups, tape diagram, situation, and expression from Activity 2.

"We've learned about different ways to represent multiplication. Share something that you learned today about multiplication with your partner." (Multiplication can be shown with drawings of equal groups or with diagrams. Multiplication can be real-world situations that involve equal groups. Multiplication can be expressed using the multiplication symbol ($\times$). An expression like 4×5 means the total number of objects in 4 groups of 5.)

Suggested Centers

- Capture Squares (1–3), Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–2), Stage 7: Add within 1,000 without Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students draw 2 groups of 6.

Next Day Support

- Use the next day's warm-up to have students practice differentiating the groups in the image from the number of dots in each group.

Lesson 12: Represent and Solve Multiplication Problems

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

Teacher-facing Learning Goals

- Represent and solve multiplication problems.

Student-facing Learning Goals

- Let's represent and solve problems involving equal groups.

Lesson Purpose

The purpose of this lesson is for students to represent and solve multiplication problems.

In previous lessons, students learned different ways to represent equal group situations with drawings, tape diagrams, and expressions. The purpose of this lesson is for students to solve problems involving equal groups with a representation of their choice.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up), MLR5 Co-craft Questions (Activity 1)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What strategy did each student seem most comfortable using to find products today?

Cool-down (to be completed at the end of the lesson)

5 min

Ducks in a Pond

Standards Alignments

Addressing 3.OA.A.3

Student-facing Task Statement

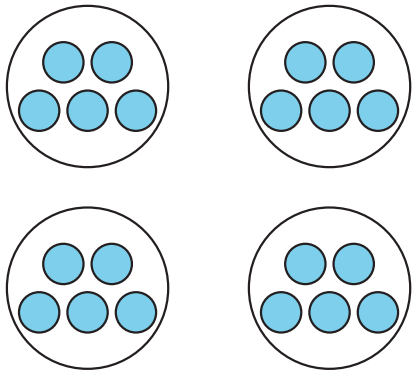
There are 4 ponds. Each pond has 5 ducks. How many ducks are there altogether?

Use diagrams, symbols, or other ways to show your thinking.

Student Responses

20 ducks. Sample responses:

- 4×5
- 5, 10, 15, 20



Begin Lesson

Warm-up

10 min

How Many Do You See: Lots of Dots

Standards Alignments

Addressing 3.OA.A.1

The purpose of this warm-up is for students to subitize or use grouping strategies to describe the images they see. When students decompose the images into groups of 10 to count efficiently, they are

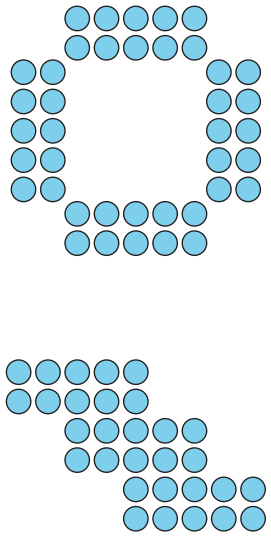
looking for and making use of structure (MP7). For these images, students may need them displayed for a longer amount of time in order to see the structure.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?



Student Responses

Sample responses:

- 40: I counted by 5 (or 2 or 10) to 40.
- 30: I saw that each step had 10 so I counted by 10 to 30.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash image.
- 30 seconds: quiet think time

Activity

- Display image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses. Use multiplication expressions when students share explanations involving equal groups.
- Repeat for each image.

Synthesis

- "What pattern was helpful in finding the total number of dots?"
- Consider asking:
  - "Who can restate the way _____ saw the dots in different words?"
  - "Did anyone see the dots the same way but would explain it differently?"
  - "Does anyone want to add an observation to the way _____ saw the dots?"

Activity 1

15 min

Tyler's Boxes

Standards Alignments

Addressing 3.OA.A.3

The purpose of this activity is for students to use the Co-craft Questions math language routine to make sense of a multiplication situation before solving. Students are first asked to generate questions they could ask about part of a problem. Then, students are given the full problem and asked to solve it. The activity concludes with students reflecting on the representations they used. In this activity, students will need to see the full problem to solve. Before the lesson, record the problem and have it hidden until the appropriate time in the lesson or write it for all to see at that point during the activity.

This activity uses MLR5 Co-craft Questions. Advances: writing, reading, representing

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

Tyler has 3 boxes. He has 5 baseballs in each box. How many baseballs does he have altogether? Show your thinking using diagrams, symbols, or other representations.

Student Responses

15. Sample responses:

- 

Launch

- Groups of 2

MLR5 Co-Craft Questions

- Display only the problem stem, "Tyler has 3 boxes." without revealing the question.
- "Write a list of mathematical questions that could be asked about this situation." (What's in the boxes? How many things are in the boxes? How many things does he have altogether?)
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.

- Tyler has 15 baseballs because I counted by 5.
- Tyler has 15 baseballs. 3×5 or 5×3

- "What do these questions have in common? How are they different?"
- Reveal the task (students open books), and invite additional connections.

Activity

- "Think about how you'll solve the problem."
- 30 seconds: quiet think time
- 2–3 minutes: partner work time

Synthesis

- Display student work with different representations of the problem one at a time (drawings of equal groups, tape diagrams, and expressions). If no student writes an expression, write one for students to analyze.
- "How does each representation help us see what's happening in the problem?"

Activity 2

20 min

Solve Equal Groups Problems

Standards Alignments

Addressing 3.OA.A.3

The purpose of this activity is for students to use what they've learned about multiplication to solve and represent situations that involve equal groups. Students now have experience with multiple representations and have the opportunity to choose which representation is most helpful to represent multiplication situations.

The launch of the activity is an opportunity for students to share their experiences and ask questions about the objects to ensure each student has access to the context. If it is helpful, display images of the items for students to reference.

Access for English Learners

MLR8 Discussion Supports. Synthesis: As students describe their representations of the problems, use gestures to emphasize the number of groups and how many are in each group. For example, trace with your finger around each group, and point to each object in each group to show how many there are.
Advances: Listening, Representing

Access for Students with Disabilities

Representation: Access for Perception. Provide access to connecting cubes. Ask students to identify correspondences between the concrete and visual representation used.
Supports accessibility for: Conceptual Processing

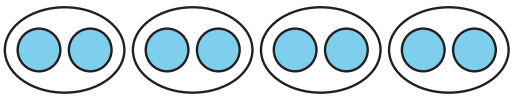
Student-facing Task Statement

Solve each problem. Show your thinking using diagrams, symbols, or other representations.

1. There are 4 soccer fields. Two teams are on each field. How many teams are there altogether?
2. There are 7 windows. Each window has 2 pieces of glass. How many pieces of glass are there in the windows?
3. Jada has 5 bags. Each bag has 10 earrings. How many earrings does Jada have?
4. Kiran has 4 boxes. Each box has 5 pencils in it. How many pencils does Kiran have?
5. Andre has 3 bags of carrots. Each bag has 10 carrots. How many carrots does Andre have?

Student Responses

1. 8 teams. Sample response:



I used the groups to count the dots. Each dot represents a team.

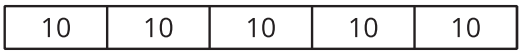
Launch

- Groups of 2
- Write the list of objects (“teams, earrings, pencils, pieces of glass, carrots”) on a display for all students to see.
- “Take a minute to read this list. When you are done, discuss the objects you know or have questions about.”
- 3 minutes: partner discussion
- Share a few responses.
- “Now we’re going to represent and solve more problems with equal groups. Take a couple of minutes to begin working on the problems before you work in partners.”
- 2–3 minutes: independent work time

Activity

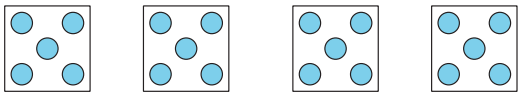
- “Work with your partner to solve each problem.”
- 5–7 minutes: partner work
- Identify students who use different representations like drawings of equal groups or tape diagrams as they solve the problems.

2. 14 pieces of glass. Sample response: 7×2. I counted by 2 seven times because each window had 2 pieces of glass.
3. 50 earrings. Sample response:



I used a diagram to show 5 groups of 10, then counted by 10 to find the total.

4. 20 pencils. Sample response:



I used a drawing to show 4 boxes with 5 in each box.

5. 30 carrots. Sample response: I counted the 3 bags of 10 like 10, 20, 30.

Synthesis

- For each problem, display different representations, one at a time.
- "How does this representation help us see what's happening in the problem?" (You can see there are 4 groups for the fields and 2 dots in each group for the teams. The diagram is split into 5 parts for the 5 bags, and each section has a 10 in it for the number of earrings.)
- "How could each representation help us solve the problem?" (Counting the dots. Counting by 10.)
- If needed, "What expression could we write to represent this situation?"

Advancing Student Thinking

If students add or subtract instead of multiply to solve the problems, consider asking:

- "Tell me about how you solved this problem."
- "How does the problem involve equal groups?"

Lesson Synthesis

10 min

Display samples of student work with different representations (drawings of equal groups, tape diagrams, and equations).

"Which representation did you find most helpful today and why?" (Drawings of equal groups because I could see what was happening in the problem. Diagrams because it helped me understand the problem, but I didn't have to draw all the dots.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)

- Five in a Row: Addition and Subtraction (1–2), Stage 7: Add within 1,000 without Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students add or subtract the numbers in the problem instead of multiplying.

Next Day Support

- During the launch of the next day's activity, have students discuss what the situations have in common that make them multiplication problems.
- Launch the lesson by asking students to recap the important points of the previous lessons.

Lesson 13: Multiplication Equations

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

Teacher-facing Learning Goals

- Relate equations to multiplication situations and diagrams.
- Write equations for multiplication situations and diagrams using a symbol for the unknown number.

Student-facing Learning Goals

- Let's learn about multiplication equations.

Lesson Purpose

The purpose of this lesson is for students to relate multiplication equations to situations and diagrams and write equations.

In previous lessons students represented situations and diagrams with multiplication expressions. In this lesson, students learn the meaning of **factor** and **product**. Students do not have to use the vocabulary in this lesson as they will continue to have opportunities to do so throughout the year. In future lessons, students will represent situations and diagrams with equations that use a symbol for the unknown number, which may be either a factor or the product.

Consider continuing to use the convention of groups as the first factor and the size of the groups as the second factor when writing equations. However, it is not necessary for students to write the factors in this order. It is important that students connect their equations to the corresponding situations and representations (MP2). They should be able to correctly explain what each factor represents in their equations. If students ask questions about the idea of commutativity, consider recording the questions publicly for future investigation.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How were the terms factor and product helpful as students began working with multiplication equations?

Cool-down (to be completed at the end of the lesson)

5 min

Match the Equation

Standards Alignments

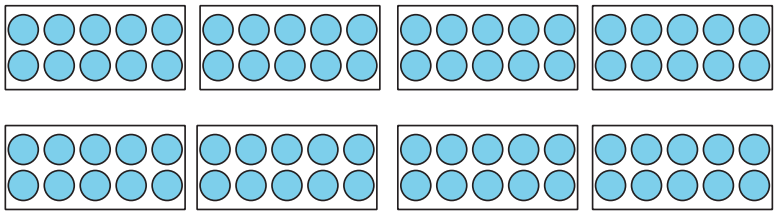
Addressing 3.OA.A.1

Student-facing Task Statement

Select **all** the drawings, diagrams, and situations that could represent the equation.

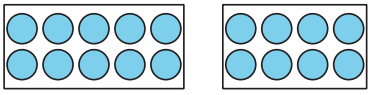
$$80 = 8 \times 10$$

A.

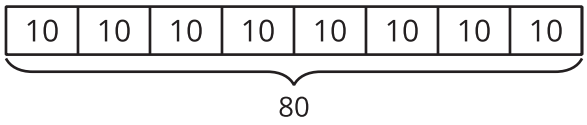


B. Andre has 8 boxes. Each box has 10 cars in it. He has 80 cars altogether.

C.

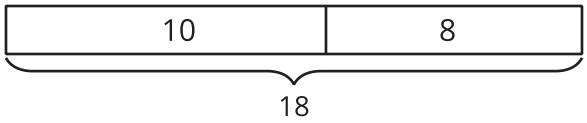


D.



E. Andre had 8 boxes. Then, he found 10 more boxes. How many boxes does Andre have?

F.



Student Responses

A, B, and D

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Representations

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

This warm-up prompts students to carefully analyze and compare features of expressions and equations. When students compare the drawing, expression, and equations, they must use language precisely to describe how each is the same or different (MP6). Listen to the language students use to describe the different characteristics of each expression and equation and connect students' language to the new terms, factor and product, that are introduced in the synthesis.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A

B

3×5

C

$2 \times 5 = 10$

D

$7 + 8 = 15$

Student Responses

Sample responses:

- A is the only one that doesn't show any numbers.
- B is the only one that doesn't show the total.
- C is the only one that doesn't show 15.
- D is the only one that is not about multiplication and is not about groups of 5.

Launch

- Groups of 2
- Display the image, expression, and equations.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2-3 minutes: partner discussion
- Share and record responses.

Synthesis

- "How is C different from the other ways we've represented equal groups before?" (It has an equal sign. It's an equation.)
- "C is a multiplication **equation** because it contains a multiplication symbol and the equal sign."
- "There are words that help us talk about different parts of the multiplication equation. The **factors** are the numbers being multiplied. The **product** is the result of multiplying some numbers. In the equation in C, the numbers 2 and 5 are the factors. The product is 10. Keep these words in mind today as we work with other multiplication equations."

Activity 1

20 min

Multiplication Equation Match

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

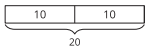
The purpose of this activity is for students to match multiplication equations to situations and representations. Students make explicit connections between the factors and the number of groups or the number of objects in each group and between the product and the total number of objects. These connections are brought out explicitly during the synthesis. When students make explicit connections between multiplication situations and equations, they are reasoning abstractly and quantitatively (MP2).

Student-facing Task Statement

Find an equation from the list that can represent each situation, diagram, or drawing. Record the equation. Be prepared to explain your reasoning.

- $3 \times 5 = 15$
- $10 = 5 \times 2$
- $16 = 8 \times 2$
- $4 \times 10 = 40$
- $30 = 6 \times 5$
- $4 \times 5 = 20$
- $2 \times 10 = 20$
- $4 \times 2 = 8$
- $50 = 5 \times 10$

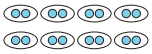
1.



2.

Andre had 5 pairs of socks.

3.

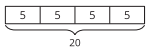


4.

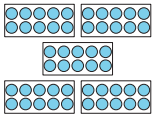
6 hands were on the table. Each hand had 5 fingers.

5.

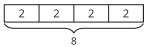
6.



7.



8.



9.

There were 4 boxes of markers. Each box had 10 markers.

Launch

- Groups of 2 and 4
- "Think about how you might match these equations to a situation or diagram."
- 1 minute: quiet think time

Activity

- "Take turns finding a situation or diagram that matches each equation. Explain your reasoning to your partner."
- 5–7 minutes: partner discussion
- Monitor for students who make direct connections between each factor representing the number in each group or the number of groups and the product representing the total number of objects to share during the synthesis.
- "Get together with another group to discuss the matches you made."
- 3-5 minutes: small-group discussion

Synthesis

- "Were there any matches you disagreed on? How did you come to an agreement?" (We went back and recounted the dots together.)
- For the first, fourth, and seventh pairs that match ask, "How does the equation represent the situation (or diagram)?" (The 2 represents the 2 parts in the diagram.

Student Responses

1. $2 \times 10 = 20$
2. $10 = 5 \times 2$
3. $16 = 8 \times 2$
4. $30 = 6 \times 5$
5. $3 \times 5 = 15$
6. $4 \times 5 = 20$
7. $50 = 5 \times 10$
8. $4 \times 2 = 8$
9. $4 \times 10 = 40$

The 5 represents the 5 fingers on each hand. The 50 represents how many dots were in the groups altogether.)

Activity 2

15 min

Write Multiplication Equations

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

The purpose of this activity is for students to write equations that match situations and diagrams. Students use what they learned in the last activity to use multiplication equations to represent situations and diagrams. In the lesson synthesis, use the words factor and product to help students connect the vocabulary to the concepts.

Access for English Learners

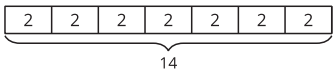
MLR7 Compare and Connect. Synthesis: Invite groups to prepare a visual display that shows their reasoning for one of the equations using details such as different colors, arrows, labels, diagrams or drawings. Give students time to investigate each others' work. Ask, "Which details or language helped you understand the displays?", "Did anyone create the same equation, but would explain it differently?"
Advances: Representing, Conversing

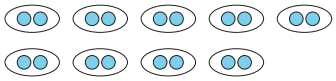
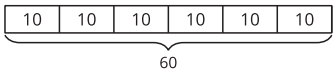
Access for Students with Disabilities

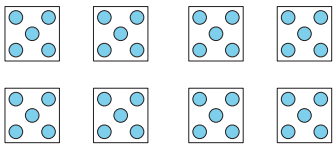
Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which problem to start with or decide the order to complete the task.
Supports accessibility for: Social-Emotional Functioning

Student-facing Task Statement

Write an equation that represents each situation, drawing, or diagram. Be prepared to explain your reasoning.

1. A package has 6 pairs of socks.
2. 

3. Diego has 7 sections in his notebook. Each section has 10 pages.
4. 
5. 

6. Elena has 4 bags of oranges. Each bag has 5 oranges in it.
7. 

Student Responses

1. $6 \times 2 = 12$ or $2 \times 6 = 12$. Sample response:

Launch

- Groups of 2

Activity

- "Work with your partner to write an equation that represents each situation and diagram."
- 5-7 minutes: partner work time
- Monitor for students who can justify the equations they wrote by explaining the meaning of the factors and products in their equations.

Synthesis

- For each problem have a student share their equation. Consider asking:
    - "How does this equation make sense for this situation, drawing, or diagram?"
    - "What parts of the situation, drawing, or diagram were especially helpful as you wrote the equation?"

There are 6 groups of socks with 2 socks in each group. This makes 12 socks.

2. $7 \times 2 = 14$ or $2 \times 7 = 14$. Sample response: There are 7 parts and each part has 2 in it. If you count by 2 seven times, you get 14.
3. $7 \times 10 = 70$ or $10 \times 7 = 70$. Sample response: There are 7 sections and each section has 10 pages, so there are 70 pages altogether.
4. $9 \times 2 = 18$ or $2 \times 9 = 18$. Sample response: There are 9 circles and each circle has 2 dots in it. This is a total of 18 circles.
5. $6 \times 10 = 60$ or $10 \times 6 = 60$. Sample response: There are 6 parts with 10 in each part. This makes 60.
6. $4 \times 5 = 20$ or $5 \times 4 = 20$. Sample response: There are 4 bags and each bag has 5 oranges. This is 20 oranges.
7. $8 \times 5 = 40$ or $5 \times 8 = 40$. Sample response: There are 40 circles because there are 8 squares and each has 5 dots.

Lesson Synthesis

10 min

Display:

Expression: 3×5
Equation: $3 \times 5 = 15$

"Today we learned about equations and how we can use them to represent multiplication. In this equation, 3 and 5 are the factors and 15 is the product."

"How are multiplication expressions and equations alike?" (They both use the multiplication symbol. They both have factors.)

"How are multiplication expressions and equations different?" (Equations have an equal sign. Multiplication equations have numbers on both sides of the equal sign.)

"When would each be helpful?" (Expressions are helpful when you want to describe a situation. Equations are helpful if you are trying to find the product.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 7: Add within 1,000 without Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students choose drawings, diagrams, or situations that represent addition instead of multiplication.

Next Day Support

- Use the launch of the next day's activity to discuss how problems that involve multiplication are different than problems that involve addition.

Lesson 14: Write and Solve Equations with Unknowns

Standards Alignments

Addressing	3.OA.A.1, 3.OA.A.3, 3.OA.A.4, 3.OA.D.9
Building Towards	3.OA.C.7

Teacher-facing Learning Goals

- Relate equations to multiplication situations and diagrams using a symbol for the unknown number.
- Write equations for multiplication situations and diagrams using a symbol for the unknown number.

Student-facing Learning Goals

- Let's work with equations with unknown numbers.

Lesson Purpose

The purpose of this lesson is for students to relate equations to and write equations for multiplication situations and diagrams using a symbol for the unknown number.

Students have worked with addition and subtraction equations with a symbol to represent the unknown number in grades 1 and 2. Students build on that work and the work with multiplication equations in the previous lesson as they encounter multiplication equations that have a symbol for the unknown number for the first time.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Number Talk (Warm-up)

Materials to Copy

- Card Sort Unknown Numbers (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How do tape diagrams help students make sense of equations in which the unknown number is in different positions?

Cool-down (to be completed at the end of the lesson)

5 min

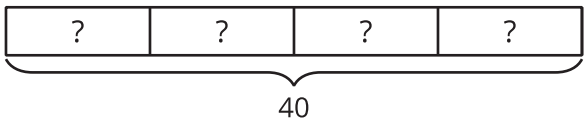
Unknown and a Number

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.4

Student-facing Task Statement

1. Write an equation to match the diagram. Use a symbol for the unknown.



2. Find the number that makes the equation true. Rewrite the equation with that number. Explain your reasoning.

Student Responses

1. $4 \times ? = 40$ or $? \times 4 = 40$
2. $4 \times 10 = 40$ or $10 \times 4 = 40$. Sample response: If I count by ten 4 times I get 40, so I know the missing number is 10.

Begin Lesson

Warm-up

10 min

Number Talk: Fives

Standards Alignments

Addressing	3.OA.D.9
Building Towards	3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying by 5. These understandings help students develop fluency and will be helpful later in this lesson when students represent and solve a problem involving groups of 5.

When students reason why as one factor increases by 1, the product increases by 5, they are looking for and expressing the regularity they notice in the expressions (MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 1×5
- 2×5
- 3×5
- 4×5

Student Responses

- 5: It's only 1 group of 5.
- 10: I added 5 more on to the first answer.
- 15: I added $5 + 5 + 5$.
- 20: I counted by 5.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What pattern do you see as you look at all of the problems? Why is that happening?" (The factor that isn't 5 goes up by 1 each time. The products increase by 5. We added another group of 5.)

Activity 1

15 min

Card Sort: Unknown Numbers

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.4

The purpose of this activity is for students to relate equations to multiplication situations and diagrams using a symbol for the unknown number. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7). Students explain their matches to their peers and revise their language for precision and clarity when they describe how the numbers and symbols in the equations match the representations (MP3, MP6). In the synthesis, students explain the meaning of the factors and products, and what a symbol in an equation represents.

Access for English Learners

MLR8 Discussion Supports. Invite students to take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "I noticed ___ , so I matched . . ." Encourage students to challenge each other when they disagree.
Advances: Conversing, Representing

Instructional Routines

Card Sort

Materials to Copy

Card Sort Unknown Numbers (groups of 2)

Required Preparation

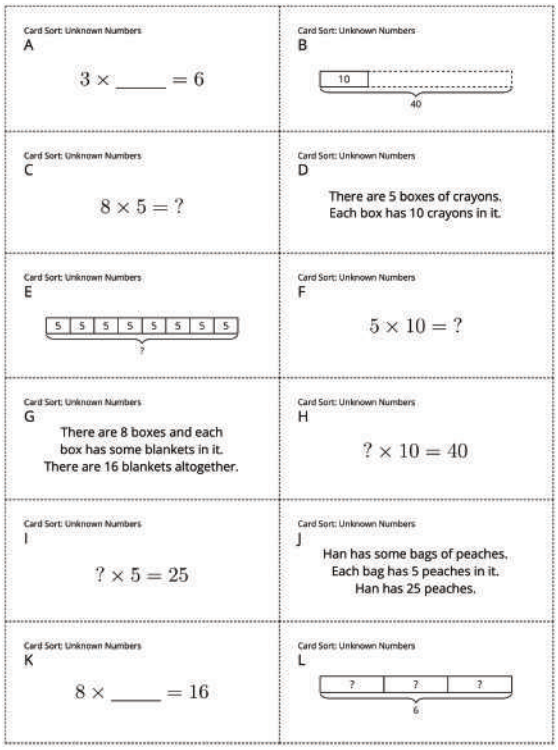
Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards. Match each equation with a situation or diagram.

Launch

- Groups of 2
- Display: $4 \times 5 = ?$
- "What might this equation mean?" (There are 4 groups of 5. There's a number in the



Student Responses

- A and L
- B and H
- C and E
- D and F
- G and K
- I and J

Sample response: I know that A and L go together because there are 3 groups that together make 6, but we don't know the size of each group.

equation that we don't know. We don't know the total.)

- 2 minutes: partner discussion
- Share responses.
- "Different symbols can be used to represent the unknown number in an equation. Some that are common are question marks, blank spaces, and boxes."
- "For example, in the equation $80 = 8 \times 10$, if we didn't know the product we could write ? = 8×10." Display these equations as you explain.
- "If we didn't know one of the factors, what is an equation you could write using a symbol for the unknown number?" ($80 = \_\_\_\_\_ \times 10, 80 = ? \times 10$)
- Distribute one set of pre-cut cards to each group of students.

Activity

- "This set of cards includes equations, situations, and diagrams. Match each equation to a situation or diagram. Work with your partner to justify your choices."
- 5-7 minutes: partner work time
- Monitor for students who explain the meaning of the factors and the product, specifically that the symbol is for a missing number that represents a missing amount in the diagram or situation.

Synthesis

- Have students share their matches and how they know those cards go together.
- Choose 1-2 equations (at least one with a missing factor) and ask:
    - "What does each number in the equation represent?"
    - "What does each question mark (or blank or box) represent?"

    - "How can we figure out which number goes in the blank to make the equation true?"
- Listen for the language students use to describe their matches and the equations, diagrams, and situations clearly and precisely. As needed, ask:
    - "What do you mean when you say _____?"
    - "Was the unknown the product or one of the factors? Explain."
    - "What did the unknown factor represent? Explain."
- Highlight terms such as "factor" and "product."

Activity 2

20 min

Write Equations with an Unknown Number

Standards Alignments

Addressing 3.OA.A.3, 3.OA.A.4

The purpose of this activity is for students to write equations for multiplication situations and diagrams using a symbol for the unknown number. When students write an equation to represent a situation, including a symbol for the unknown number, they model a situation with mathematics (MP4).

Students find an unknown factor or unknown product in multiplication problems. In this task, the unknown factor diagrams and situations only include the "how many groups" problem type and the factors 2, 5, and 10. This sets students up to skip-count to find the unknown number.

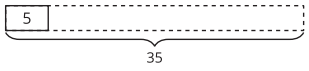
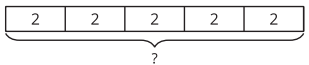
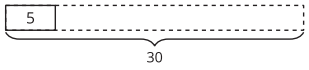
This problem type will be revisited extensively in future lessons and will be related to division. It is not necessary to make the connection to division now. In the synthesis students explain how the equations they wrote represent the diagram or situation.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were important or most useful to pay attention to. Display the sentence frame, "The next time I write an equation with an unknown number, I will . . . ."
Supports accessibility for: Visual-Spatial Processing

Student-facing Task Statement

- Write an equation to represent each diagram or situation. Use a symbol for the unknown. Be prepared to share your reasoning.
- Find the number that makes each equation true. Rewrite the equation with the solution.

diagram or situation	equation with symbol	equation with solution
		
Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?		
		
		
The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does		

Launch

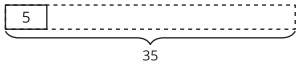
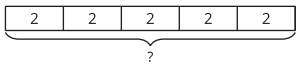
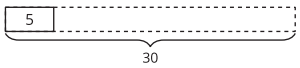
- Groups of 2

Activity

- "Now you will practice writing your own equations with a symbol for the unknown."
- 2–3 minutes: independent work time
- "Share your equations with your partner. Discuss how you know each equation matches the diagram or situation."
- 2–3 minutes: partner discussion
- Have a whole-class discussion focused on how the equations match the different representations.
- Consider asking:
  - "How did you use the representations to write an equation with a symbol for the unknown?" (I looked for what was missing in the diagram. I thought about the situation to figure out if it was the number in each group, the number of groups, or the total that was missing.)
- "Now find the missing number in each equation and write a new equation that includes the solution."
- 3–5 minutes: partner work time

diagram or situation	equation with symbol	equation with solution
the school have?		

Student Responses

diagram or situation	equation with symbol	equation with solution
	$? \times 5 = 35$ $5 \times ? = 35$	$7 \times 5 = 35$ $5 \times 7 = 35$
Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?	$? \times 5 = 45$ $5 \times ? = 45$	$9 \times 5 = 45$ $5 \times 9 = 45$
	$5 \times 2 = ?$ $2 \times 5 = ?$	$5 \times 2 = 10$ $2 \times 5 = 10$
	$? \times 5 = 30$ $5 \times ? = 30$	$6 \times 5 = 30$ $5 \times 6 = 30$
The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does the school have?	$6 \times 10 = ?$ $10 \times 6 = ?$	$6 \times 10 = 60$ $10 \times 6 = 60$

Synthesis

- "What strategies did you use to find the unknown numbers?" (I counted by 2 to get the total. I counted by different numbers until I found a number that gave me the product.)
- "How did each equation change as you found the unknown number?" (The symbol was replaced with the number.)

Advancing Student Thinking

If students write an equation without a symbol for the unknown number, consider asking:

- "How does your equation represent the diagram (or situation)?"
- "How could you show the number that was unknown with a symbol?"

Lesson Synthesis

10 min

Display:

$$6 \times 5 = ?$$
$$6 \times ? = 30$$
$$? \times 5 = 30$$

"Today we found the unknown number in multiplication equations."

"How was finding an unknown factor different from finding an unknown product?" (If we didn't know the product, we could skip-count by the number the right number of times, like 5, 10, 15, 20, 25, 30. If we didn't know one of the factors, we might have to skip-count by the number enough times to get to the product. If we didn't know one of the factors, we might not know what to skip-count by, just the number of counts.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 8: Add within 1,000 with Composing (Supporting)

Complete Cool-Down

Response to Student Thinking

Students write a number for the unknown that doesn't make the equation true.

Next Day Support

- Use the launch of the next day's activity to brainstorm ways to find the unknown number in a multiplication equation.

Lesson 15: More Factors, More Problems

Standards Alignments

Addressing	3.OA.A.3, 3.OA.A.4, 3.OA.D.9
Building Towards	3.OA.C.7

Teacher-facing Learning Goals

- Solve multiplication problems.

Student-facing Learning Goals

- Let's solve more multiplication problems.

Lesson Purpose

The purpose of this lesson is for students to solve multiplication problems.

Students write equations with a symbol for the unknown to represent multiplication problems and then solve the problems. As in the previous lesson, some problems are unknown factor problems which students do not relate to division until a future unit. Students put together what they have learned about drawings, diagrams, expressions, and equations to solve multiplication problems.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As students worked in their small-groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Cool-down (to be completed at the end of the lesson)

5 min

Solve the Problem

Standards Alignments

Addressing 3.OA.A.3

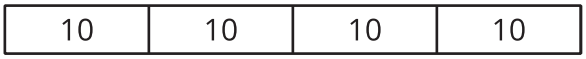
Student-facing Task Statement

Solve each problem. Explain or show your reasoning.

1. There are 4 boxes. Each box has 10 toys. How many toys are there?
2. Elena has 10 socks. She puts them in piles of 2. How many piles does she make?

Student Responses

1. 40 toys. Sample response:



2. 5 piles. Sample response: $10 = ? \times 2$ I know that 5×2 is 10.

Begin Lesson

Warm-up

10 min

Number Talk: Tens

Standards Alignments

Addressing 3.OA.D.9
Building Towards 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplication by 10. These understandings help students develop fluency and will be helpful later in this lesson when students need to be able to represent and solve a problem involving groups of 10.

When students reason why as one factor increases by 1, the product increases by 10, they look for and

express the regularity they notice in the expressions (MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 1×10
- 2×10
- 3×10
- 4×10

Student Responses

- 10: it's only 1 group of 10.
- 20: I added 10 more on to the first answer.
- 30: I added $10 + 10 + 10$.
- 40: I counted by 10.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression

Synthesis

- "What pattern do we see as we look at all of the problems? Why is that happening?" (The factor that isn't 10 goes up by 1 each time. The products increase by 10 because I have one more group of 10.)
- Consider asking:
  - "Did anyone use a different strategy"?
  - "Did anyone have the same strategy but would explain it differently?"

Activity 1

15 min

Represent Situations with Equations

Standards Alignments

Addressing 3.OA.A.3, 3.OA.A.4

The purpose of this activity is for students to represent a situation with a multiplication equation including a symbol for the unknown, and find the number that makes the equation true. Students are able to use an earlier representation to help them solve the problem, however some students may just write the equation and skip-count to find the product. Either is okay. In the synthesis, share different ways students represented the problem beyond the equation. If students used repeated addition, avoid saying 'multiplication is repeated addition' because while repeated addition is one way to find the product, it is not the meaning of multiplication.

To add movement to this activity, students can work in groups of 4 to make a poster for one of the problems. After each group is done, they can do a gallery walk to look for things that are the same or different in the posters.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Memory

Student-facing Task Statement

For each problem:

- Write an equation with a symbol for the unknown to represent the situation.
- Find the number that makes the equation true. Show your reasoning.

1. There are 15 plates. Han placed 5 plates on each table. How many tables have plates on them?
    a. equation:
    b. solution:
2. Lin made 6 sandwiches. She used 2 slices of bread for each sandwich. How many pieces of bread did she use?
    a. equation:
    b. solution:
3. Han has 60 ice cubes. The ice cubes are in trays of 10. How many trays of ice cubes does Han have?

Launch

- Groups of 2
- "We are going to solve some problems about equal groups that you may see when you are making or eating a meal."
- "What are some equal groups that you might see when making or eating a meal?"
- Share responses.
- "Think about how you could represent these problems in a way that could help you write an equation with an unknown number for each problem."
- 1 minute: quiet think time
- 1 minute: partner discussion

Activity

- "Now, independently work on these problems."
- 5–7 minutes: independent work time
- As you circulate, consider asking:

a. equation:

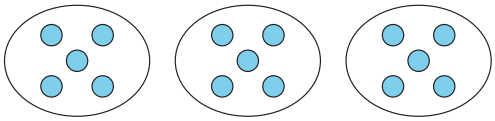
b. solution:

Student Responses

1. a. $? \times 5 = 15$ or $5 \times ? = 15$

   b. $3 \times 5 = 15$ or $5 \times 3 = 15$

   Sample response:



2. a. $6 \times 2 =$ _____ or $2 \times 6 =$ _____

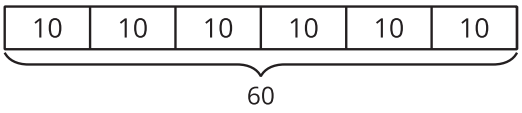
   b. $6 \times 2 = 12$ or $2 \times 6 = 12$

   Sample response: I counted 2, 4, 6, 8, 10, 12.

3. a. $? \times 10 = 60$ or $10 \times ? = 60$

   b. $6 \times 10 = 60$ or $10 \times 6 = 60$

   Sample response:



- "How could you represent this situation?"
- "What information is missing from the situation?"

Synthesis

- Display samples of student work for each problem next to each other, including a sample of a drawing of equal groups and a sample of a tape diagram.
- "Where do we see the parts of the problem in the drawing and the diagram?" (The number of objects in each group are the dots in the drawing, but the number is written in each part of the diagram.)
- "How did you use the factors in each equation to find the product?" (The factors told me how many groups there were and how many were in each group.)
- "How are drawings and diagrams useful for finding the solution to the problem?" (You can count the dots in the drawing. The diagram can be used to count by 10.)

Activity 2

20 min

Multiplication Mashup

Standards Alignments

Addressing 3.OA.A.3

The purpose of this activity is for students to practice solving multiplication problems in which the unknown amount can be the number of groups, the number in each group, or the total. The first three problems have the unknown in each of those locations. The sequence of these problems, the context, and the use of the same factors and product encourages students to use a known fact to find the unknown factor in the "how many in each group" problem. Students will make the

connection between this problem type and division in a future unit. Students are able to choose the representation they use to represent and solve the problems.

Access for English Learners

MLR8 Discussion Supports. Monitor and clarify any questions about the context. As students look over the problems, ask, "Are there any words that are unfamiliar or that you have questions about?"
Advances: Reading, Representing

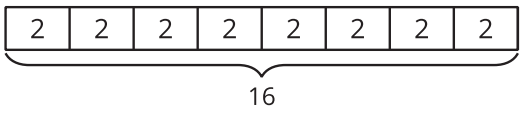
Student-facing Task Statement

Solve each problem. Explain or show your reasoning.

1. Clare has 16 socks. She puts them in piles of 2. How many piles can she make?
2. Diego has 8 piles of socks. Each pile of socks has 2 socks. How many socks does Diego have?
3. Andre has 16 socks. He puts them in 8 groups that are the same size. How many socks are in each group?
4. The store has 9 boxes. Each box has 5 shirts. How many shirts are there?
5. A store has 80 sweaters. There are 8 sweaters in each pile on a shelf. How many piles of sweaters are on the shelf?

Student Responses

1. 8 piles. Sample response:



2. 16 socks. Sample response: It's 8 groups of 2. I counted the piles of two like 2, 4, 6, 8, 10, 12, 14, 16.
3. 2 socks. Sample response:

Launch

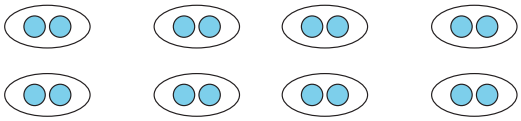
- Groups of 2
- "Take a minute to look over these problems. What representations or strategies might be helpful to you as you solve these problems?"
- 1 minute: quiet think time
- Share and record responses.

Activity

- "Work with your partner to solve each problem."
- 8-10 minutes: partner work time
- Circulate and consider the following questions to focus students on the structure of the situations:
    - "What information is missing in the situation?"
    - "How could you represent this situation?"

Synthesis

- Share student work for each problem and ask students to explain their reasoning. Be sure to share a variety of strategies and representations.
- As students share, consider asking the



4. 45 shirts. Sample response: $5 + 5 = 10$, $10 + 10 = 20$, $20 + 20 = 40$. That's 8 boxes. One more is $40 + 5 = 45$.
5. 10 piles. Sample response: $80 = ? \times 8$. I know 10×8 is 80. So 10 groups of 8 must be 80.

class:

- "Why does this strategy make sense?"
- "Why does this representation make sense?"
- "Did anyone solve this problem in a different way?"
- "What do you notice is the same about the representations and strategies that we are using to solve these problems?"

Lesson Synthesis

10 min

"Today we solved multiplication problems using any strategy or representation that we wanted."

"What strategy or representation do you find most helpful when you are solving these types of problems? Why?" (I like to draw equal groups so I can see how many groups there are and how many are in each group. I think a diagram is nice to draw because I don't have to draw all the things, but I can still see the groups. I like to use an equation so I can see where the unknown number is.)

"What are the most important things to remember when you are solving multiplication problems?" (There are always groups that are the same size. You could be looking for the number of groups, how many things are in each group, or the total number of things in all the groups.)

Suggested Centers

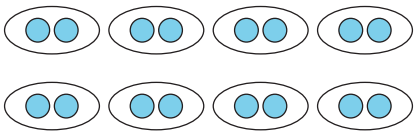
- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–2), Stage 8: Add within 1,000 with Composing (Supporting)

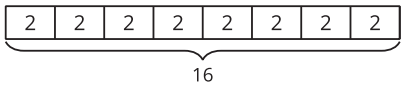
Student Section Summary

In this section, we learned about equal groups. We created drawings and diagrams to represent situations that involve equal groups.

situation drawing diagram

Diego has 8 piles of socks. Each pile of socks has 2 socks.





We wrote multiplication expressions and equations to represent equal groups.

expression	equation
8×2	$8 \times 2 = 16$

We learned that the numbers that are multiplied are called **factors** and the number that is the result of multiplying is called a **product**. In the equation $8 \times 2 = 16$, the numbers 8 and 2 are the factors and 16 is the product.

Complete Cool-Down

Response to Student Thinking

Students find solutions other than 40 toys and 5 piles.

Students do not clearly show how they found the solution or do not show a solution.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Section C: Represent Multiplication with Arrays and the Commutative Property

Lesson 16: Arrange Objects Into Arrays

Standards Alignments

Building On	2.OA.C.4
Addressing	3.OA.A.1

Teacher-facing Learning Goals

- Build arrays with physical objects and describe them in terms of multiplication.
- Describe an array as an arrangement of objects into rows with an equal number of objects in each row and into columns with an equal number in each column.

Student-facing Learning Goals

- Let's make some arrays.

Lesson Purpose

The purpose of this lesson is for students to describe arrays and arrange objects into arrays.

In grade 2, students were introduced to arrays and related them to addition expressions and equations. In this lesson, students deepen their understanding of arrays as they arrange physical objects and relate arrays to multiplication and equal groups. Students use connecting cubes to represent arrays in Activity 2 and in the cool-down.

When working with array situations, students may see the equal groups in an array in either the rows or the columns. For example, when representing 3 rows of 5 chairs, they may create a 3 by 5 array or a 5 by 3 array. This is fine as long as students can correctly describe where the "3 rows of 5 chairs" are in their array. Students will learn about commutativity in the last lesson in this section, so if questions about commutativity arise, record them publicly for discussion in that lesson.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Connecting cubes: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In previous grades, students saw examples of arrays, such as 10-frames, and counted objects arranged in arrays. How does that previous experience support their work with equal groups in arrays in this lesson?

Cool-down (to be completed at the end of the lesson)

5 min

Array Arrangement

Standards Alignments

Addressing 3.OA.A.1

Student-facing Task Statement

Arrange 12 cubes into an array.

Explain or show how the array is related to multiplication.

Student Responses

Sample response: It's related to multiplication because there are 4 cubes in each row, which is like 3 equal groups.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Eggs

Standards Alignments

Building On	2.OA.C.4
Addressing	3.OA.A.1

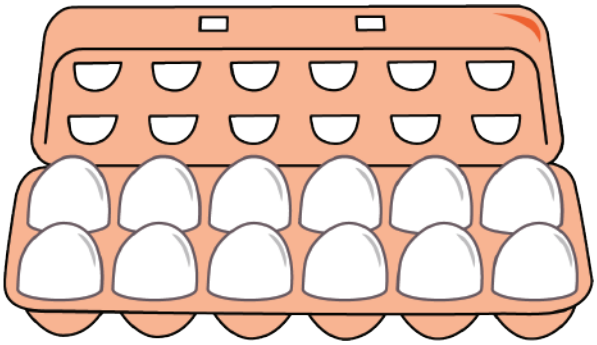
The purpose of this warm-up is to elicit ideas students have about objects arranged in an array, which will be useful when students arrange equal groups into arrays in a later activity. While students may notice and wonder many things about this image, ideas around arrangement and equal groups are the important discussion points. When students notice the arrangement of the eggs they look for and make use of structure (MP7).

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 12 eggs.
- There are 2 groups of 6.
- I see 6 groups of 2.

Students may wonder:

- How many eggs would be in two cartons?
- Are there other equal groups that I don't see?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How does having the eggs in a carton help you see equal groups?" (I can see how they could be split into equal groups. I can see 6 eggs in each row. I can see 6 groups of 2.)
- "The eggs are arranged in an array. An **array** is an arrangement of objects in rows and columns. Each column must contain the same number of objects as the other columns, and each row must have the same number of objects as the other rows."

Activity 1

15 min

Compare Equal Groups and Arrays

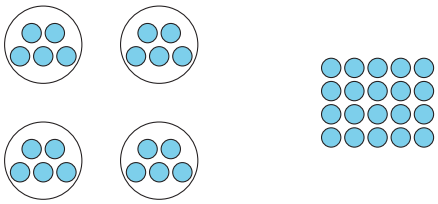
Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to describe an array as an arrangement of objects into rows with an equal number of objects in each row and into columns with an equal number in each column. This will be helpful in the next activity when students arrange objects into arrays and describe arrays in terms of multiplication.

When students decide whether or not they agree with Noah about seeing equal groups in the array and explain their reasoning, they construct a viable argument and critique the reasoning of others (MP3).

Student-facing Task Statement



1. How does arranging the dots into an array affect how you see the number?
2. Noah says he sees equal groups in the drawing with 4 circles and 5 dots in each circle, but says there are no equal groups in the array. Do you agree with Noah? Explain your reasoning.

Student Responses

1. Sample response: The array allows you to see different numbers inside the 20, like 2

Launch

- Groups of 2
- Display the images.
- "Consider these drawings. On the left is a drawing showing equal groups. On the right is an array. How are the drawings alike? How are they different?" (Alike: They both have 20 dots. They both have groups of 5. Different: The dots are arranged differently. The second group has the dots in rows. The first drawing has the dots in circles.)
- 1–2 minutes: quiet think time
- Share and record responses.

Activity

- "Work with your partner to describe how arranging the dots into an array affects how you see the number."
- 2–3 minutes: partner work time

groups of 10 or 5 groups of 4.

2. Sample response: No, there are 5 dots in each row, so each row is an equal group.

- Share responses.
- "Read what Noah says about the dots and decide whether you agree or disagree. Be ready to share your reasoning."
- 1 minute: independent work time
- "Now discuss whether you agree or disagree with Noah with your partner."
- 2–3 minutes: partner discussion

Synthesis

- "How are arrays related to equal groups?" (There are the same number of dots in each row, so the rows are equal groups. There are the same number of dots in each column, so the columns are equal groups.)

Activity 2

20 min

Arrange Into Arrays

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to build arrays with physical objects and describe the arrays in terms of multiplication. Students focus on where equal groups can be seen in arrays. Students will write expressions and equations to represent arrays in future lessons. In the activity, students are asked to create different arrays with 24 cubes. It is not an expectation of grade 3 for students to find all the arrays for a given number.

When students notice that the rows or columns in an array have the same number of objects and relate this to equal groups, they look for and make use of structure (MP7).

Keep connecting cubes out for the cool-down.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Create a visual display of the various arrays created by the students. As students discuss each array, annotate the display with the language used, such as "array", "rows", "columns", and "equal groups".
Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify what is important or most useful to pay attention to. Display the sentence frame, "The next time I arrange objects in an array, I will remember to . . . ."
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Memory

Materials to Gather

Connecting cubes

Required Preparation

- Each group of 2 needs 60 cubes.

Student-facing Task Statement

1. Use cubes to make 6 groups of 5.
    - Arrange them into an array.
    - Explain or show how the array is related to equal groups.
2. Count out 20 cubes.
    - Arrange them into as many arrays as you can.
    - Explain or show how each array is related to equal groups.
3. Count out 24 cubes.
    - Arrange them into as many arrays as you can.
    - Explain or show how each array is related to equal groups.

Launch

- Groups of 2
- Give each group of 2 students at least 60 connecting cubes.

Activity

- 7–10 minutes: partner work time
- Monitor for students who relate the same number of objects in each row and column in an array to how multiplication expresses equal groups of objects.

Synthesis

- Have students share different arrays they made for the last problem.
- "Why were you able to create different arrays with 24 objects?" (There are many ways to put 24 things into equal groups.)

Student Responses

1. Students may arrange the cubes into a 5 by 6, 6 by 5, 3 by 10, 10 by 3, 15 by 2, or 2 by 15 array.

   Sample response: The array is related to multiplication because there is 5 in each row, which is like having equal groups, and that is what multiplication represents.
2. Students may arrange the cubes into a 2 by 10, 10 by 2, 5 by 4, or 4×5 array.

   Sample response: The array is related to multiplication because I see 2 cubes in each row. The rows are like equal groups.
3. Students may arrange the cubes into a 4 by 6, 6 by 4, 3 by 8, 8 by 3, 12 by 2, or 2 by 12 array.

   Sample response: The array is related to multiplication because I see 4 cubes in each column. The columns are like equal groups.

Lesson Synthesis

10 min

"Today we arranged objects into arrays and described how arrays are related to equal groups."

"What did you think about when you arranged cubes into arrays?" (I dealt the cubes into equal groups and then arranged them into an array. I thought about equal groups I could use to make the total and made each group a row with the same number of things in each row. I arranged them until there were the same number of cubes in each row and the same number of cubes in each column.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students aren't sure how to arrange cubes into an array.

Next Day Support

- Use the next day's warm-up to brainstorm tips for arranging objects into an array.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 8, Section B: Rectangular Arrays

Lesson 17: Match and Draw Arrays

Standards Alignments

Addressing 3.OA.A.1

Teacher-facing Learning Goals

- Relate arrays to drawings of equal groups and describe them in terms of multiplication.

Student-facing Learning Goals

- Let's match arrays to equal groups and draw arrays.

Lesson Purpose

The purpose of this lesson is for students to relate arrays to drawings of equal groups and describe arrays in terms of multiplication.

Students first match arrays to drawings of equal groups. Then, they redraw drawings of equal groups as arrays. The work of this lesson connects to upcoming lessons when students represent arrays with expressions and equations. Make connecting cubes or counters available to students who need them.

Keep collecting ideas that arise about commutativity.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Card Sort (Activity 1), MLR1 Stronger and Clearer Each Time (Activity 1), Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Connecting cubes or counters: Activity 2

Materials to Copy

- Card Sort Arrays (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

What question do you wish you had asked today? When and why should you have asked it?

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

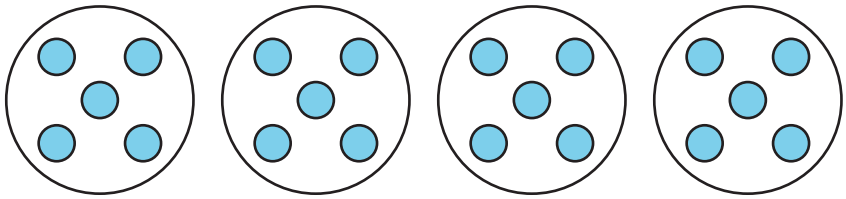
5 min

Draw and Describe

Standards Alignments

Addressing 3.OA.A.1

Student-facing Task Statement



1. Redraw the equal groups as an array.
2. Describe how the diagram and the array are related.

Student Responses

1. Sample response:Students create a 4×5, 5×4, 2×10, or 10×2 array.
2. Sample response: There are 5 dots in each group and there are 5 dots in each row. There are 4 groups and there are 4 rows. They both have 20 dots.

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Arrangements

Standards Alignments

Addressing 3.OA.A.1

The purpose of this warm-up is for students to compare four arrangements of dots to elicit the attributes, or structure, of an array. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use, such as rows, corners, groups, and array.

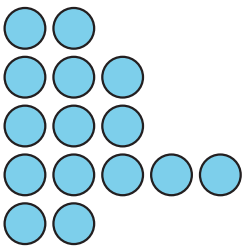
Instructional Routines

Which One Doesn't Belong?

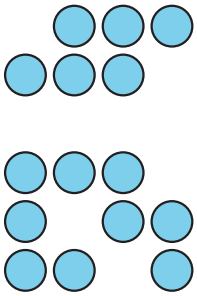
Student-facing Task Statement

Which one doesn't belong?

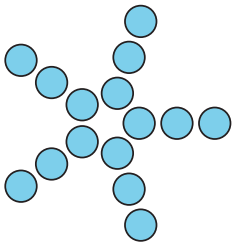
A



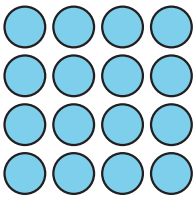
B



C



D



Student Responses

- A is the only where the equal groups aren't in a straight line.
- B is the only where the equal groups and the dots in the equal groups aren't always right next to one another.

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "Why is B not an array?" (It has the same number of dots in each row, but not the same in each column. One of the columns only has 3 dots. The 3rd row is missing.)
- Consider asking:
  - "Let's find at least one reason why each one doesn't belong."

- C is the only one where the equal groups are not organized in rows.
- D is the only one that doesn't have 15 dots.

Activity 1

20 min

Card Sort: Arrays

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to relate drawings of equal groups to arrays. Specifically, students look for arrays that have the same number of objects in each row or column as each drawing has in each group. In some arrays, the equal groups in the drawing are represented as rows, and in some, they are represented in columns. Students have the opportunity to explain the connections they see between the drawings and arrays, receive feedback from their peers, and revise their explanation for precision and clarity (MP3, MP6). This will be useful in future lessons when students record multiplication expressions and equations to represent arrays.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Attention, Organization

Instructional Routines

Card Sort, MLR1 Stronger and Clearer Each Time

Materials to Copy

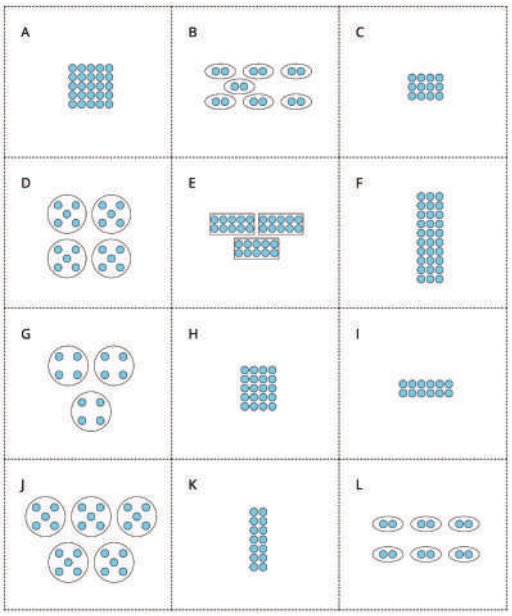
Card Sort Arrays (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2 or 4 students.

Student-facing Task Statement

1. Match the drawings of equal groups and arrays that are alike. Be prepared to explain your reasoning.



2. Choose a match you and your partner made. Write down how you know the drawing matches the array.

Student Responses

1.
   - A and J
   - B and K
   - C and G
   - D and H
   - E and F
   - I and L
2. Sample response: I and L match because the array has 2 dots in each column and the drawing has 2 dots in each group. Also, the drawing has 6 groups and the array has 6 columns. They both have 12 dots.

Launch

- Groups of 2 or 4
- Distribute one set of pre-cut cards to each group of students.

Activity

- "This set of cards includes drawings of equal groups and arrays. Match each drawing to an array. Work with your partner to justify your choices."
- 8 minutes: partner work time
- "Independently choose a match you and your partner made. Write down how you know that the drawing matches the array."
- 2 minutes: independent work time

MLR1 Stronger and Clearer Each Time

- "Share your response to why your cards match with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time

Synthesis

- Have 2-3 students share the matches they made and describe how they know those cards go together.
- "Did your group agree on the matches? What did you look for to decide two cards were matches?" (Yes, we looked for equal groups that had the same number of dots in a group as one of the rows in the array.)

- Listen for language students use to describe their matches and the structure of the arrays. As needed, ask:
  - "What do you mean by _____?"
  - "What else could we call _____?"
  - "How could you use 'equal groups' to explain your match?"
- Highlight the use of terms like row, column, and equal groups.

Activity 2

15 min

Draw Arrays

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to draw arrays from a given arrangements of dots. Students draw an array from dots in equal groups to reinforce the definition of an array and then draw as many arrays as they can from 16 randomly placed dots. Having cubes or counters for students to physically rearrange would be helpful in this activity.

Access for English Learners

MLR8 Discussion Supports. To support partner discussion, display the following sentence frames: "This array matches the diagram because . . .", and "This array shows multiplication because . . . ." *Advances: Conversing, Representing*

Materials to Gather

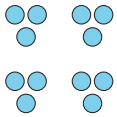
Connecting cubes or counters

Student-facing Task Statement

1. a. Draw 1 way the dots could be rearranged into an array.

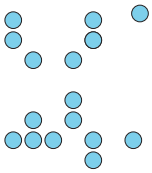
Launch

- Groups of 2
- Give students access to connecting cubes



b. Explain or show how the array is related to multiplication.

2. a. Draw ways that the dots could be arranged into arrays. Draw as many ways as you can.



b. Explain or show how each array is related to multiplication.

Student Responses

1. Sample responses:
   a. 4 by 3 array, 3 by 4 array, 6 by 2 array, 2 by 6 array
   b. The array shows multiplication because each row has 3 dots, which is an equal group.
2. Sample responses:
   a. 2 by 8 array, 8 by 2 array, 4 by 4 array
   b. The array shows multiplication because each column has 2 dots, which is an equal group.

or counters.

Activity

- "Work independently to draw a way that the first group of dots in problem 1 could be arranged into an array."
- 2 minutes: independent work time
- "Discuss how you arranged your dots and how the array is related to multiplication with your partner."
- 1 minute: partner discussion
- "How did you rearrange the dots to make an array?" (Since there were 3 in each group, I put 3 dots in each row. I saw 2 groups of 6, so I made 2 rows of 6.)
- Consider asking:
  - "Did anyone create a different array?"
- "Now you are going to make as many arrays as you can from 16 dots."
- 2–3 minutes: independent work time
- "Share how you rearranged the dots into arrays with your partner. See if together you can come up with any other arrays."
- 3–5 minutes: partner work time

Synthesis

- "What kinds of equal groups did you make from 16 dots? How can you see the equal groups in the arrays you made?" (I can make 2 groups of 8. I drew it as 2 rows of 8 dots.)

Lesson Synthesis

10 min

"Today we made drawings that showed how groups of dots could be rearranged into arrays."

"What do you need to think about when you draw an array?" (Make sure the rows and columns all have the same number of dots. Make the number of groups the number of columns or row in the array and then draw how many are in each group in each column or row. All the dots have to be used.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students do not describe how the rows and columns are connected to equal groups.

Next Day Support

- Use the next day's warm-up to have students discuss how they see equal groups in the array.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 8, Section B: Rectangular Arrays

Lesson 18: Represent Arrays with Expressions

Standards Alignments

Addressing 3.OA.A.1

Teacher-facing Learning Goals

- Represent multiplication situations with arrays and multiplication expressions.

Student-facing Learning Goals

- Let's represent situations with arrays and expressions.

Lesson Purpose

The purpose of this lesson is for students to represent multiplication situations with arrays and multiplication expressions.

In a previous lesson, students arranged objects into arrays and described the arrays in terms of equal groups. In this lesson, students write expressions to represent arrays to further connect arrays and multiplication (MP2).

As students connect arrays to expressions, they may write 3×5 or 5×3 to represent 3 rows of 5 chairs. This is fine as long as students can correctly describe where the "3 rows of 5 chairs" are in their array or expression. Keep collecting ideas that arise about commutativity.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up)

Materials to Gather

- Connecting cubes or counters: Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

In an upcoming lesson, students will learn about the commutative property of multiplication. What do you notice in their work from today's

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

lesson that you might leverage in that future lesson?

Cool-down (to be completed at the end of the lesson)

5 min

Array Situation

Standards Alignments

Addressing 3.OA.A.1

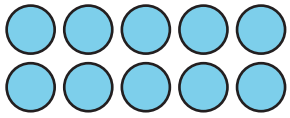
Student-facing Task Statement

There are 2 rows of plants. Each row has 5 plants.

1. Draw an array to represent the situation.
2. Write an expression to represent the situation.

Student Responses

1. Sample response:



2. 2×5 or 5×2

Begin Lesson

Warm-up

10 min

How Many Do You See: An Array of Shapes

Standards Alignments

Addressing 3.OA.A.1

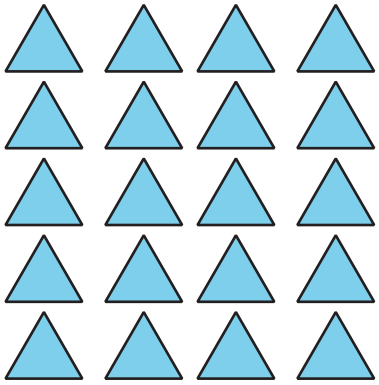
The purpose of this How Many Do You See is for students to subitize or use grouping strategies to describe the images they see. When students use the structure of the array to figure out how many objects are shown, they look for and make use of structure (MP7).

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?



Student Responses

Sample responses:

- 20: I saw 4 groups of 5 or 5 groups of 4.
- 20: I see 2 groups of 10 or 10 groups of 2.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How did seeing equal groups help you know how many triangles there were in the array?"
- Consider asking:
  - "Who can restate the way _____ saw the triangles in different words?"
  - "Did anyone see the triangles the same way but would explain it differently?"

Activity 1

20 min

Represent Array Situations

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to represent multiplication situations with arrays and multiplication expressions. Students should have the option to use math tools, such as counters or connecting cubes, to create the arrays before they draw them. Connecting situations, arrays, and expressions reinforces the idea that multiplication can be used to express the total number of objects in equal groups (MP2).

Access for English Learners

MLR2 Collect and Display. Amplify language used to describe arrays. On a visible display, record words, phrases and expressions such as: row, column, each, for every, 3 by 5, 5 by 3, 5×3, and 3×5. Include diagrams and annotations. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Representing, Reading, Speaking

Access for Students with Disabilities

Representation: Internalize Comprehension. Make connections between representations visible. Invite students to use gestures or drawings as they verbally describe correspondences between their arrays and expressions. For example, "The 4 in my expression 4×5, shows the number of rows, and the 5 shows that there are 5 cars in each row."
Supports accessibility for: Conceptual Processing, Memory

Materials to Gather

Connecting cubes or counters

Required Preparation

- Each group of 2 will need 20 connecting cubes or counters.

Student-facing Task Statement

Use objects or drawings to represent each

Launch

- Groups of 2

situation with an array.

1. There are 3 rows of chairs. Each row has 5 chairs.
2. There are 4 rows of cars. Each row has 5 cars in it.
3. There are 2 rows of eggs. Each row has 6 eggs.
4. There are 2 teams of students lined up. Each team has 10 students.

Write a multiplication expression to represent each situation.

Student Responses

1. 3 by 5 or 5 by 3 array drawn or built, 3×5 or 5×3
2. 4 by 5 or 5 by 4 array drawn or built, 4×5 or 5×4
3. 6 by 2 or 2 by 6 array drawn or built, 2×6 or 6×2
4. 10 by 2 or 2 by 10 array drawn or built, 2×10 or 10×2

- Give students access to connecting cubes or counters.
- Display the first situation.
- "Take a minute to represent this situation with an array. You can use drawings or objects."
- 2 minutes: independent work time
- "Discuss your ideas with your partner."
- 2 minutes: partner discussion
- Share responses. Emphasize ways students used equal groups to create their arrays.

Activity

- "Work with your partner to represent the next three situations with an array. Be prepared to share how you see equal groups in your array."
- 5 minutes: partner work time
- Have students share an array for problems 2–4. Try to show both drawings and arrays made of objects.
- "How do you see equal groups in your arrays?" (The rows can show the number of groups, and then you put however many are in each group across the row.)
- Display the first situation again.
- "Let's revisit the first situation. What multiplication expression would represent this situation?"
- 30 seconds: quiet think time
- Share responses. Emphasize how students use equal groups to write the expression.
- "Work with your partner to write multiplication expressions for the other three situations."
- 2 minutes: partner work time

Synthesis

- "How did you use equal groups to write

your multiplication expression?" (I thought about how many groups were in the situation and then how many things were in each group. For example, I know that 2 teams of 10 students is 2 groups of 10, so I can write 2×10.)

Activity 2

15 min

Connect Arrays to Expressions

Standards Alignments

Addressing 3.OA.A.1

The purpose of this activity is for students to apply their knowledge from previous activities to draw arrays to match multiplication expressions. Have connecting cubes or counters available for students who need them. In the launch, students use their bodies to make an array for the expression 4×6. Feel free to adjust this expression to better fit the number of students in your class.

Student-facing Task Statement

Draw an array for each multiplication expression. Be prepared to share your reasoning.

1. 2×3
2. 5×2
3. 4×4

Student Responses

1. 3 by 2 or 2 by 3 array
2. 2 by 5 or 5 by 2 array
3. 4 by 4 array

Launch

- Groups of 2
- Display 4×6
- "Work together as a class to use your bodies to create an array for this multiplication expression."
- If the number of students does not exactly match the product, ask extra students to monitor the array and be prepared to explain where they see parts of the expression in the array.
- 3–5 minutes: whole-class work time

Activity

- "Work with your partner to draw an array for each multiplication expression. Be ready to share connections you notice between the multiplication expressions and arrays."
- 2–3 minutes: partner work time

Synthesis

- Share an array or two for each expression.
- "What connections did you and your partner see between the multiplication expressions and arrays?" (The factors tell us how many things are in each row and column. For 2×3, we drew an array with 2 columns that have 3 things in each column. For 2×3, we drew an array with 2 rows that have 3 things in each row.)

Advancing Student Thinking

If students draw equal groups that are not arrays, consider asking:

- "How did you use the expression to create your drawing?"
- "How could we rearrange your drawing into an array?"

Lesson Synthesis

10 min

Display a situation from the first activity, and an array and an expression that represents the situation.

"We learned that multiplication is how we express the total number of objects in equal groups."

"How did your knowledge of equal groups help you create arrays and write expressions for multiplication situations?" (I thought about how many groups there were and drew the groups as each row [or column]. Then the number of groups tells me how many rows [or columns] there are. The array and the expression represent the total number of objects in the problem.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students don't write a multiplication expression that matches the array they created.

Next Day Support

- Use the launch of the next day's activity to have students discuss how to write an expression that represents an array.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support

Grade 2, Unit 8, Section B: Rectangular Arrays

Lesson 19: Solve Problems Involving Arrays

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3, 3.OA.C.7, 3.OA.D.9

Teacher-facing Learning Goals

- Represent an array situation with an equation with a symbol for the unknown number.
- Solve multiplication problems involving arrays.

Student-facing Learning Goals

- Let's solve problems involving arrays.

Lesson Purpose

The purpose of this lesson is for students to represent an array situation using an equation with a symbol for the unknown number and solve.

In previous lessons, students represented multiplication situations using arrays and multiplication expressions with an emphasis on equal groups. Equal groups continue to be emphasized in this lesson as students learn that finding the product in a multiplication equation gives the total number of objects in the related array.

As students connect arrays to equations, they may write $3 \times 5 = 15$ or $5 \times 3 = 15$ to represent 3 rows of 5 chairs. This is fine as long as students can correctly describe where the "3 rows of 5 chairs" are in their array or equation. Keep collecting ideas that arise about commutativity.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

MLR5 Co-craft Questions (Activity 1), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

Who has been sharing their ideas in class lately? Make a note of students whose ideas have not

Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.

Cool-down (to be completed at the end of the lesson)

5 min

Clare's Cards

Standards Alignments

Addressing 3.OA.A.3

Student-facing Task Statement

Clare has 3 rows of baseball cards. Each row has 10 cards. How many cards does she have?

1. Write an equation with a symbol for the unknown number to represent the situation.
2. Find the number that makes the equation true. Explain or show your reasoning.

Student Responses

1. $3 \times 10 = ?$ or $10 \times 3 = ?$
2. 30. Sample response: 30 because 3 rows of 10 is 10, 20, 30.

Begin Lesson

Warm-up

10 min

Number Talk: One Less Group

Standards Alignments

Addressing 3.OA.C.7, 3.OA.D.9

The purpose of this Number Talk is to elicit strategies and understandings students have about equal groups in multiplication expressions and see a pattern as one factor is decreased. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to use multiplication to answer questions about array situations.

When students notice that as the number being multiplied by 2 decreases the product decreases by a group of 2, they look for and express regularity in repeated reasoning (MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 10×2
- 9×2
- 8×2
- 7×2

Student Responses

- 20: 2 groups of 10 is 20.
- 18: I counted by 2 (2, 4, 6, 8, 10, 12, 14, 16, 18).
- 16: I counted by 2 or it's one group of 2 less than the last equation.
- 14: I counted by 2 or it's one group of 2 less than the last equation.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What do you notice about the products as we worked through this series of problems?" (They decrease by 2.)
- "Why does the product decrease by 2 each time?" (The number of groups decreased by 1.)

Activity 1

25 min

Array of Colors

Standards Alignments

Addressing 3.OA.A.1, 3.OA.A.3

The purpose of this activity is for students to use the Co-craft math language routine to write questions that can be asked about array situations and to relate array situations to equations. Students should be encouraged to use whatever strategy or representation feels appropriate to them. Given their prior experiences, they may represent the situation with an array and skip-count or consider equal groups in other ways to find the total.

This activity uses *MLR5 Co-craft Questions. Advances*: writing, reading, representing.

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

There are 7 rows. Each row has 5 crayons. How many crayons are there?

1. Solve this problem. Explain or show your reasoning.
2. Represent the situation with an array and an equation with a symbol for the unknown.

Student Responses

1. Sample response: There are 35 crayons because I skip-counted by 5.
2. A 7 by 5 array or a 5 by 7 array. $7 \times 5 = ?$ or $5 \times 7 = ?$

Launch

- Groups of 2

MLR5 Co-craft Questions

- "Keep your books closed."
- Display: "There are 7 rows."
- "Write a list of mathematical questions that could be asked about this situation." (What's in the rows? How many things are in each row? How many things are there altogether?)
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "What do these questions have in common? How are they different?" (They all have to do with rows. They have to do with more detail about the rows, like what's in the rows and how many are in each row.)
- Reveal the task (students open books), and invite additional connections.

Activity

- "Complete the first problem by solving this situation in any way that makes sense to you."
- 3–4 minutes: independent work time
- 1–2 minutes: partner discussion
- Share a variety of student representations and solution strategies.
- "Think about the situation we have been considering. How could you represent the situation using an equation with a symbol for the unknown?"
- 2 minutes: quiet think time
- "Share your equation with your partner. Together, rewrite the equation with the solution you found in place of the symbol."
- 2 minutes: partner work

Synthesis

- "What equation(s) did you write?"
- Display an equation with a symbol and one with the solution.
- "How does each part of the equation connect to the situation?" (The 7 is the number of rows. The 5 is the number of crayons in each row. The 35 represents the total number of crayons, but it was a question mark when we didn't know how many there were.)

Activity 2

10 min

Tyler's Trees

Standards Alignments

Addressing 3.OA.A.3

The purpose of this activity is for students to write an equation with a symbol for the unknown to represent an array situation. Then, they answer the question in the multiplication situation. Encourage students to use whatever strategy or representation feels appropriate to them. Given their prior experiences, they might represent the situation with an array and skip-count or consider equal groups in other ways to find the total.

In the launch of the activity, it may be helpful to ask students to tell their partner a quick story or ask any questions about the context of the first problem. To ensure all students have access, it may also be helpful to display images for students to reference about coconut trees or Mexico.

Access for English Learners

MLR8 Discussion Supports. Synthesis: If necessary, invite students to repeat their question using mathematical language. For example, "Can you say that again, using the word array?"
Advances: Listening, Speaking

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Activity: Invite students to show thinking using connecting cubes or counters.
Supports accessibility for: Conceptual Processing

Student-facing Task Statement

For each problem:

- Write an equation with a symbol for the unknown to represent the situation.
- Solve the problem. Show your reasoning.

1. A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?
2. After learning about growing coconuts in Mexico, Tyler wants to plant coconut trees in his backyard in Florida. His mom will only let him plant 2 rows of 4 trees in his backyard.

   How many trees will Tyler plant?

Launch

- Groups of 2
- Display the image.
- "Coconut trees are grown as a crop in warm climates and have lots of uses. Sometimes they are grown in rows. What are some other crops that are grown in rows?" (Corn. Strawberries. Carrots.)

Activity

- "Now you are going to practice what we just learned about solving array situations and writing an equation with a symbol for the unknown."
- 5–7 minutes: independent work time
- As you circulate, consider asking:
  - "How does each number or symbol

Student Responses

1. Equation with unknown: $5 \times 9 =$ _____ or $9 \times 5 =$ _____. Solution: 45 trees. Sample response: I counted by five 9 times. 5, 10, 15, 20, 25, 30, 35, 40, 45
2. Equation with unknown: $2 \times 4 = ?$ or $4 \times 2 = ?$ Solution: 8 trees. Sample response: I know that 2 groups of 4 is 8.

in your equation connect to the situation?"

- "How are you using equal groups to find the solution to the problem?"

- "Share your strategy with your partner. Ask any questions you have about your partner's ideas."
- 2 minutes: partner discussion

Synthesis

- "What questions do you still have about solving array problems or writing an equation with a symbol for the unknown?"

Lesson Synthesis

10 min

Display the information from the first problem in Activity 2.

A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?

$5 \times 9 = ?$ $5 \times 9 = 45$

"How do each of these equations represent the array situation?" (The equation with the symbol represents the 5 rows of trees and the 9 trees in each row in the situation before we knew there were 45 trees. The equation with the 45 includes the solution to the problem because there were 45 trees.)

Consider asking: "What does the question mark in the first equation represent in the situation? What does the 45 in the second equation represent in the situation?"

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students write an equation that doesn't use a symbol for the unknown number.

Next Day Support

- Before the warm-up have students discuss how they can represent the problem with an equation before they know the solution, and how they can represent the problem with an equation once they know the solution.

Lesson 20: The Commutative Property

Standards Alignments

Building On	2.NBT.B.5
Addressing	3.OA.B.5
Building Towards	3.NBT.A.2

Teacher-facing Learning Goals

- Describe the commutative property of multiplication using arrays.

Student-facing Learning Goals

- Let's learn about the commutative property.

Lesson Purpose

The purpose of this lesson is for students to describe the commutative property of multiplication using arrays.

In previous lessons, students used drawings of equal groups and arrays to represent multiplication situations. They also connected multiplication expressions and equations to these representations. In this lesson, students are introduced to the commutative property. Students will notice that the same product can be represented by different situations, arrays, or equations. Re-organizing the arrays or reversing the order of the factors in a multiplication expression does not change the total number of objects. It is important that students connect their equations to the corresponding situations and representations. They should be able to correctly explain what each factor and the product represents in their equations.

Note that students are not expected to use the name of the property. They should, however, be able to rely on their conceptual understanding of multiplication to explain why the product does not change when the order of the factors changes.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What part of the lesson went really well today in terms of students learning? What did you do that made that part go well?

Cool-down (to be completed at the end of the lesson)

5 min

Multiplication Reflection

Standards Alignments

Addressing 3.OA.B.5

Student-facing Task Statement

Summarize what you learned about multiplication today.

Student Responses

Sample response: If we switch the order of the numbers we're multiplying, we get the same product.

Begin Lesson

Warm-up

10 min

Number Talk: Subtraction

Standards Alignments

Building On	2.NBT.B.5
Building Towards	3.NBT.A.2

The purpose of this Number Talk is to elicit strategies and understandings students have for subtracting within 100. It also provides an opportunity to observe student strategies as they work toward becoming fluent in addition within 1,000.

When students use strategies based on place value to subtract, they look for and make use of structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $70 - 10$
- $68 - 10$
- $70 - 12$
- $68 - 12$

Student Responses

- 60: Seven tens minus 1 ten would be 6 tens, which is 60.
- 58: It's just like the first one, but the number you're subtracting from is 2 less, so the difference would be 2 less.
- 58: This is like the first problem, but you're subtracting 2 more from 70, so instead of 60, it would be 58.
- 56: It's like the first problem, but you're starting with 2 less and taking 2 more, so it would be 4 less than the first answer. I subtracted tens from tens and ones from ones and ended up with 5 tens and 6 ones.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How was place value helpful as you found the difference in these problems?" (I subtracted the tens first then adjusted my answer each time. I subtracted tens from tens and ones from ones.)
- Consider asking:
    - "Who can restate _____'s reasoning in a different way?"
    - "Did anyone have the same strategy but would explain it differently?"
    - "Did anyone approach the problem in a different way?"
    - "Does anyone want to add on to _____'s strategy?"

Activity 1

20 min

Learn More About Multiplication

PLC Activity

Standards Alignments

Addressing 3.OA.B.5

The purpose of this activity is to introduce the commutative property. Students write array situations for a pair of arrays and discuss similarities and differences. While the situations will have the same total number of objects, how the objects are grouped should be different. Then, students write equations to go with the arrays and situations, and make connections between the representations (MP2). Students notice that, while the order of the factors in the multiplication equation changes, the product does not change (MP7).

Access for English Learners

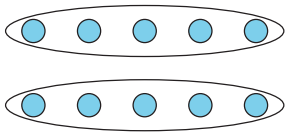
MLR8 Discussion Supports. Synthesis: Create a visual display of the equations and corresponding arrays. As students describe their connections between the equations and the situations, annotate the display to illustrate the connections. For example, below each number, write either rows, columns, or total.
Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Access for Perception. Students may benefit from the opportunity to observe a demonstration that shows the grouping of dots in the arrays. For example, prepare a display of Image A and Image B showing only the dots. Then, invite students to watch as you circle the groups accordingly.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

What do you notice? What do you wonder?



Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?" (Students may notice: Both groups of dots are arranged as arrays. They both have 10 dots. One array has groups of

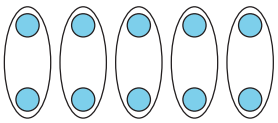


Image A

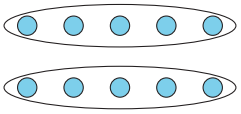
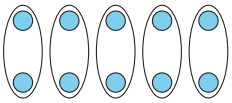


Image B



1. a. Write an array situation for each array.

   Image A

   Image B

   b. How are the situations the same? How are the situations different?

2. a. Write an equation for each situation.

   Image A Image B

   b. How does your equation connect to the situation and array?

   Image A

   Image B

Student Responses

Sample responses:

1. a. There are 2 rows of students. Each row has 5 students. There are 5 groups of students. Each group has 2 students. or There are 2 bags with 5 apples in each bag. There are 5 bags with 2 apples in each bag.

   b. The first situation is 2 rows of 5, but the second situation is 5 columns of 2. The situations both describe 10 objects.

2. One array has groups of 5. Students may wonder: Why are the dots grouped differently? Are the arrays the same?)

- 1 minute: quiet think time
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity

- "Let's consider these two arrays in more detail. Write an array situation for each array."
- 3–5 minutes: independent work time
- "Share your situations with your partner. Together, consider how the situations you wrote for the first array are the same and different from the situations you wrote for the second array."
- 3 minutes: partner work time
- "What was the same and what was different about the situations you wrote?"
- Share responses.
- "Now, write an equation for each situation you just came up with to match the arrays."
- 1 minute: independent work time
- "Share your equations with your partner and discuss how each number in your equations connects to your situations and the array."
- 2 minutes: partner discussion

Synthesis

- "Let's write down the equations we came up with."
- Display $2 \times 5 = 10$ and $5 \times 2 = 10$
- "How do each of the numbers in the equations connect to the situation you wrote?" (5 is the number of columns in one situation, but it's how many dots are in

2. a. $2 \times 5 = 10$ and $5 \times 2 = 10$
   b. The 5 and the 2 are switched in the second equation just like the rows and columns are switched. The product is the same.

each row in the other situation. 2 is the number of groups in one situation, but it's how many are in each group in the other situation. 10 is the total number of objects in both situations.)

Activity 2

15 min

Revisit Arrays

Standards Alignments

Addressing 3.OA.B.5

The purpose of this activity is to reinforce the idea of the commutative property. In this activity, students write two equations to match an array to show again that reversing the order of the factors does not change the product. If students do not immediately see how they might write different equations for the array, encourage them to consider different ways of grouping the dots in the array, similar to the previous activity. Students use the vocabulary they have learned for describing arrays and multiplication to explain why both equations match an array with their partner. The Stronger and Clearer Each Time routine allows students to receive feedback and revise their explanation for clarity (MP3, MP6).

If students finish early, consider drawing another array. Have students write two equations for the array and consider how they can think of the rows or columns as equal groups.

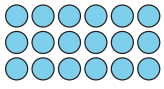
This activity uses MLR1 Stronger and Clearer Each Time. *Advances: reading, writing*

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

1. Write 2 multiplication equations that represent the array.



Launch

- Groups of 2
- "Talk to your partner about equations that could represent this array."

2. Explain why both equations can represent the array.

Student Responses

1. $3 \times 6 = 18$ and $6 \times 3 = 18$
2. Sample response: Both equations can represent the same array because we could think about the array showing 6 groups of 3 or 3 groups of 6. If we think of the columns as groups of 3, there are 6 of the groups. If we think about the rows as groups of 6, there are 3 of the groups. In either case, the array has 18 dots, so both equations have a total of 18.

- 1 minute: partner discussion

Activity

- "Write two equations for this array. If it helps you, you can imagine grouping the dots as in the previous activity. Then write down why both equations can represent the array."
- 5 minutes: independent work time

Synthesis

- "Let's write down the equations we came up with."
- Display the equations $3 \times 6 = 18$ and $6 \times 3 = 18$.

MLR1 Stronger and Clearer Each Time

- "Share why both equations can represent the array with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time
- Have students share the revisions they made to their initial draft.

Lesson Synthesis

10 min

Display a 3 by 4 array and the equations $3 \times 4 = 12$ and $4 \times 3 = 12$.

"What did we learn from thinking about arrays and seeing pairs of equations like this today?" (The

order of the factors does not change the product or the total number of objects in the array or situation. Connecting the numbers in your equations to arrays and situations helps clarify what each number means.)

Display $3 \times 4 = 4 \times 3$.

"Since $3 \times 4 = 12$ and $4 \times 3 = 12$, we can write $3 \times 4 = 4 \times 3$."

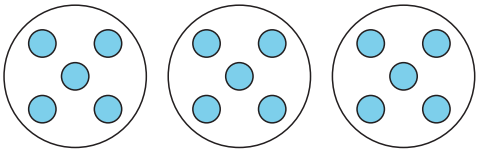
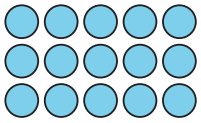
"The idea that we can multiply two numbers in any order and get the same product is called the commutative property."

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Student Section Summary

In this section, we learned how equal groups are related to arrays and how to represent arrays with expressions and equations.

drawing of equal groups	array	expression
		3×5 equation $3 \times 5 = 15$

We also learned that we can multiply numbers in any order and get the same product.

$3 \times 5 = 15$ $\quad$ $5 \times 3 = 15$ $\quad$ $3 \times 5 = 5 \times 3$

Lesson 21: Game Night Seating Plan (Optional)

Standards Alignments

Addressing 3.MD.B.3, 3.OA.A, 3.OA.A.3

Teacher-facing Learning Goals

- Make choices and assumptions.
- Represent data using scaled bar graphs to communicate results.
- Solve real-world problems involving equal groups.

Student-facing Learning Goals

- Let's plan a game night.

Lesson Purpose

The purpose of this lesson is for students to use their understanding of equal groups to solve a design problem.

This lesson is optional because it does not address any new mathematical content standards. It does provide students with an opportunity to apply precursor skills of mathematical modeling.

In previous lessons, students created scaled bar graphs and solved problems involving equal groups. In this lesson, they use these ideas as they make a seating arrangement.

Students first examine a diagram showing equal groups and consider the situations it could represent. Upon learning that the diagram represents a seating chart, they consider the information needed to set up the seating arrangement for a game night. Students then plan a seating arrangement given some constraints—the total number of game tables and a combination of games that each involve a certain number of players—and create a display to present their solution. Finally, students create a scaled bar graph to represent the number of players that can play each game in their seating solution. Throughout the lesson, students make sense of problems and persevere in solving them (MP1).

Students model with mathematics as they define quantities and variables that are relevant in the situation, communicate their solution, and translate a mathematical solution back into context (MP4).

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Connecting cubes or counters: Activity 1
- Inch tiles: Activity 1
- Tools for creating a visual display: Activity 1

Materials to Copy

- Centimeter Grid Paper - Standard (groups of 2): Activity 1
- Centimeter Grid Paper - Standard (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min

Teacher Reflection Question

How did the modeling task support collaboration between students?

Begin Lesson

Warm-up

10 min

Notice and Wonder: Squares and Circles

Standards Alignments

Addressing 3.OA.A

This warm-up serves two goals: to elicit observations about equal groups in seating arrangements, and to identify variables that might be important when solving a real-world problem in which limited information is given. These conversations prepare students to design seating arrangements given some constraints later in the lesson.

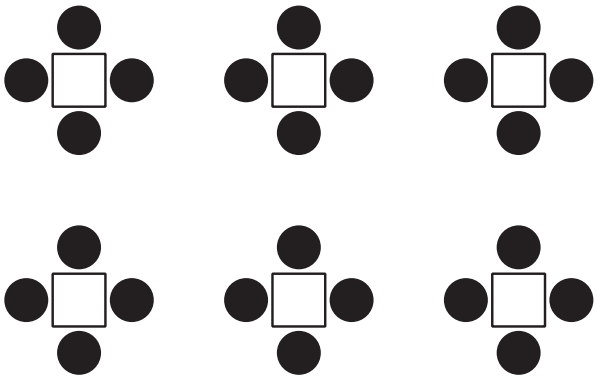
During the synthesis, highlight observations about equal groups and reveal that the image shows a seating chart. Ask students to identify the information they may need if they were to be in charge of planning the seating arrangement for a game night. As students brainstorm questions to help them gather necessary information and clarify the problem, they engage in aspects of mathematical modeling (MP4).

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 6 squares.
- There are 4 circles around each square.
- There are 24 circles.
- The squares are arranged like an array with 2 rows and 3 squares in each row.

Students may wonder:

- Why are the circles around the squares?
- Does something go inside the squares?
- What situation could this represent?
- Do the squares represent tables? Do the circles represent chairs?
- Is this a game board?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "Somebody drew the diagram to illustrate a real-world situation. What could that situation be?" (Table arrangement. Game board. Sports arrangement.)
- "The image shows table arrangements for a game night."
- "If you were in charge of planning the seating arrangement for a game night, what questions would you need to ask to have enough information to plan?" (How many tables are there? How many people can play each game? How many people can sit at each table? How many chairs are there? How many guests are going to the game night? How big are the tables?)
- Share and record responses. Keep this visible for the next activity.
- "In the next activity you will create a seating chart for a game night. You will get answers to some of these questions. For other questions, you will need to make some decisions."

Activity 1

25 min

Game Night

Standards Alignments

Addressing 3.MD.B.3, 3.OA.A.3

The purpose of this activity is for students to plan a seating arrangement. Students are only given the information of the number of players required for each game and the total number of tables. The numbers 2, 4, 5, and 10 have been chosen to reflect the multiplication work students have done in previous lessons. Students make their own decisions about other aspects of the scenario before planning their seating arrangement and also choose how to represent their seating arrangement (MP4).

Students may want answers from the teacher before making the arrangement. Encourage them to make their own assumptions as long as it does not contradict the given information.

Access for English Learners

MLR8 Discussion Supports. Clarify any questions about the context. Give students 1–2 minutes to read and make sense of the task. Ask, "Are there any words that are unfamiliar or that you have questions about?"
Advances: Reading, Representing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Differentiate the degree of difficulty or complexity. Some students may benefit from the opportunity to complete the task with fewer game types.
Supports accessibility for: Organization, Attention

Materials to Gather

Connecting cubes or counters, Inch tiles, Tools for creating a visual display

Materials to Copy

Centimeter Grid Paper - Standard (groups of 2)

Student-facing Task Statement

Your club is planning a game night.

Guests can play one of four different games that require a different number of players:

Launch

- Groups of 2 or 4
- Give each group tools for creating a visual display and access to inch tiles, graph

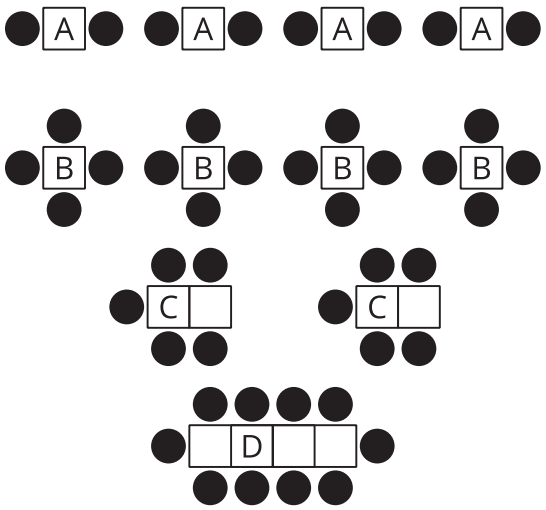
- Game A - 2 players
- Game B - 4 players
- Game C - 5 players
- Game D - 10 players

The game room has 16 identical square tables, where one person can sit on each side.

1. Make a seating plan that shows the table arrangement so that each guest can play one of the games.
2. Make a poster that includes:
    a. a seating chart
    b. an explanation about how you decided on your seating plan
    c. how many people can play games in the room with your seating plan

Student Responses

Sample response:



We made sure that there were tables for each game. We used 4 tables for game D, which left us 12 tables. We decided to use 4 tables for each of the other games. So 8 people could play game A, 16 people could play game B, 10 people could play game C, and 10 people could play paper, and connecting cubes or counters.

- "The first part of the task answers some of our questions such as the number of people needed for each game and the total number of tables. You can decide the information that is not given. In the poster you make, include the information you assumed and explain what new information you got as a result. Also, include how many people can play games in the room with your seating plan."

Activity

- 20 minutes: small-group work time
- Monitor for groups that:
    - Describe their assumptions and explain how their assumption impacted the arrangement. For example, if they wanted to set up 6 games of Game A, then they need space for 12 people.
    - Make assumptions about the total number of people.

Synthesis

- Invite previously selected students to display their posters for all to see.
    - "What does this arrangement tell us about the situation?" (It shows us how many of each game are played. It shows us how many people can play each game. It shows us how many people can play games in the room if it's set up like this.)
    - "What multiplication expression represents the number of people that can play Game A? B? C? D?"

game D. We added $8 + 16 + 10 + 10$ to find out that 44 people could play games in the room if we used this seating chart.

Advancing Student Thinking

If students find it challenging to make decisions about unknown information, consider asking:

- "Tell me about how you've designed your seating chart so far?"
- "Is there information given in the problem for what you're choosing? What are some choices you have about _____? How would it affect your seating chart if you _____?"

Activity 2

10 min

Game Night on a Graph

The purpose of this activity is for students to represent their game night plans on a scaled bar graph. In the synthesis, students consider how their graph communicates information about their game night plan.

Materials to Copy

Centimeter Grid Paper - Standard (groups of 2)

Required Preparation

- Each student needs a sheet of grid paper.

Student-facing Task Statement

Make a scaled bar graph that shows the number of guests that can play each of the games A, B, C, and D.

Be sure to include:

- a title and other labels
- a scale that counts by a number other than

Launch

- Groups of 2 or 4
- Give each student grid paper.
- "Now you're going to make a scaled bar graph that shows how many people can play each game with your room arrangement."
- "Discuss what scale your groups will use for

1

Student Responses

Sample response:

your graph."

- 1 minute: small-group discussion
- "You can work with your group, but everyone in your group will make their own graph. You can also choose a different scale than the rest of your group."

Activity

- 5-7 minutes: small-group work time

Synthesis

- Display graphs that used different scales.
- "How did choosing different scales affect the graphs?" (Some of the graphs have shorter bars because each jump on the graph is worth more. Some of the graphs are easier to read than others.)
- "What information does this bar graph give us about the situation?" (The number of people at each game, the types of games. We can find the total number of people if we add them all up.)

Lesson Synthesis

10 min

"Today, we made seating arrangements based on some given information and other things we decided."

"Which decisions affected your arrangement? Were there any decisions that did not affect your arrangement?" (We decided that there would be 2 of games A, B, and C played at the same time. This affected the number of people who could play game D.)

Suggested Centers

- Capture Squares (1–3), Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

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Core Knowledge MATHEMATICS™

Family Support Materials

Family Support Materials

Introducing Multiplication

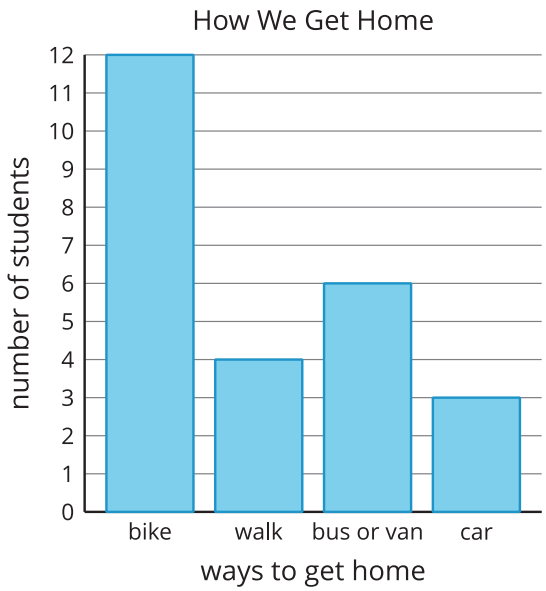
In this unit, students represent and interpret data on scaled bar graphs and picture graphs. Then, they are introduced to the concept of multiplication.

Section A: Interpret and Represent Data on Scaled Graphs

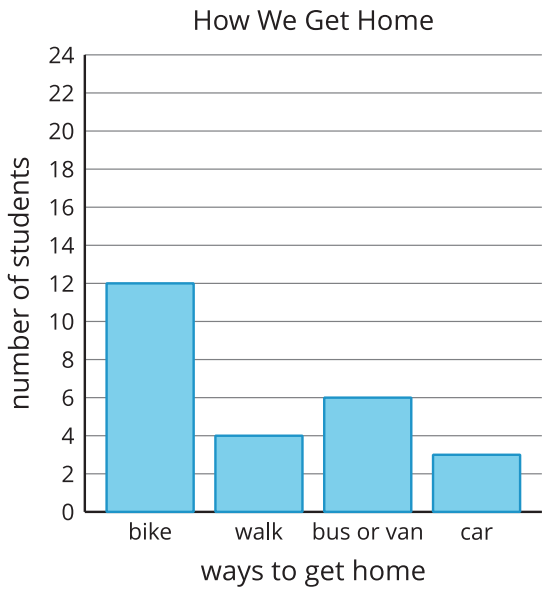
In this section, students make sense of and draw picture graphs and bar graphs. They see that each picture in a picture graph, or each step on a bar graph, can represent more than one object. They work with scales of 2, 5, and 10 (where each picture or step represents 2 objects, 5 objects, or 10 objects).

Students use the scaled bar graphs to solve “how many more” and “how many fewer” problems where the numbers are within 100.

bar graph



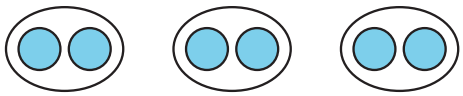
scaled bar graph



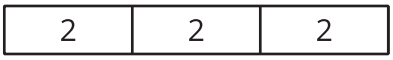
Section B: From Graphs to Multiplication

In this section, students use the idea of "each picture representing multiple objects" to think about equal-size groups and learn about multiplication. They create drawings and tape diagrams to represent situations that involve equal-size groups.

drawing of equal groups



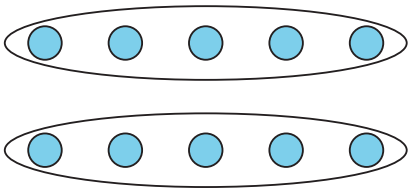
tape diagram

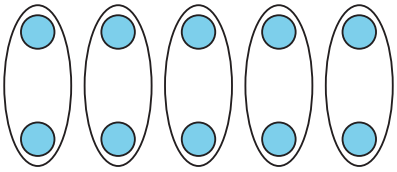


Students learn that we can write 3×2 to represent these drawings and interpret the expression to mean "3 groups of 2." Later, they write equations to represent multiplication situations. They also find unknown factors and products in equations (for example, $4 \times ? = 12$ and $5 \times 4 = ?$).

Section C: Represent Multiplication with Arrays and the Commutative Property

In this section, students connect the equal-group representations to arrays. An array is a set of objects organized in rows and columns. Students look for equal-size groups in arrays like in these diagrams:





Students write expressions to represent arrays. For example, in the shown arrays, we can write 2×5 (or 2 groups of 5) and 5×2 (or 5 groups of 2).

Try it at home!

Near the end of the unit, ask your student to find examples of equal-size groups or arrays at home, or use household objects to make such groups or arrays.

Questions that may be helpful as they work:

- How many groups are there?
- How many are in each group?
- Represent the objects with a drawing, a diagram, and an expression. How does your drawing and diagram match the expression?

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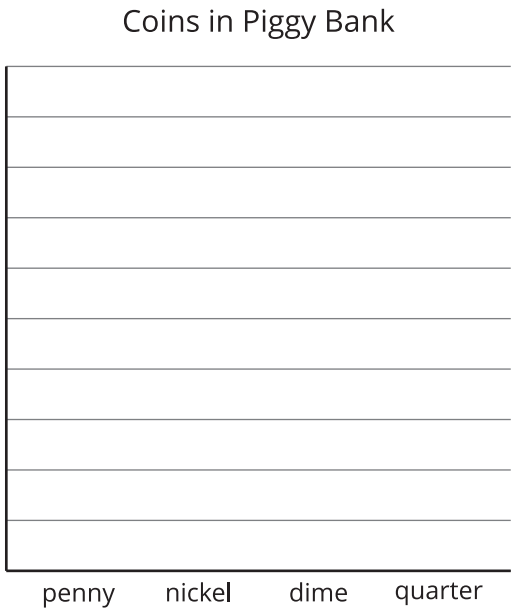
Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment

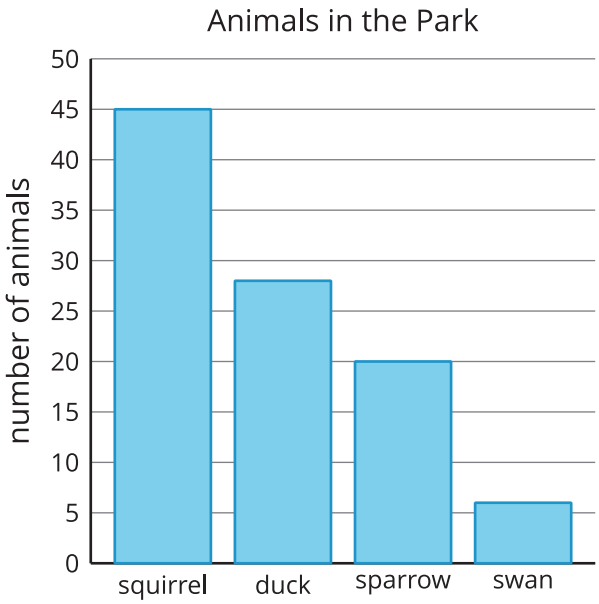
Introducing Multiplication: Section A Checkpoint

1. The table shows the number of different coins in a piggy bank. Create a scaled bar graph to represent the data. Consider a scale of 5 or 10.

coin	number
penny	49
nickel	15
dime	27
quarter	36



2. The bar graph shows the number of different animals at a park.



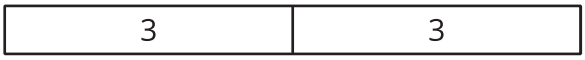
a. How many squirrels and sparrows are there altogether? Explain or show your reasoning.

b. How many more squirrels are there at the park than sparrows and swans combined? Explain or show your reasoning.

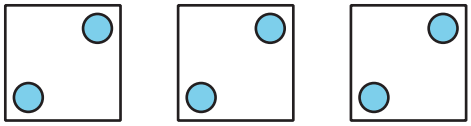
Introducing Multiplication: Section B Checkpoint

1. Jada has 2 bowls. Each bowl has 3 apples. Select **all** representations of Jada's apples.

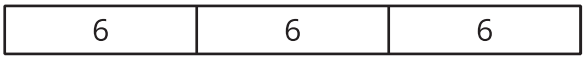
A.



B.



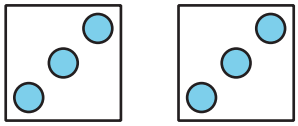
C.



D. 2×3

E. $2 + 3$

F.



2. a. There are 4 bunches of grapes. Each bunch has 10 grapes. How many grapes are there? Explain or show your reasoning.

b. There are 30 people in some cars. Each car has 5 people in it. How many cars are there? Explain or show your reasoning.

Introducing Multiplication: Section C Checkpoint

1. Draw an array that represents the expression 3×4. Explain or show your reasoning.

2. There are 3 rows of chairs in the room. Each row has 7 chairs. How many chairs are there?

   a. Draw an array to represent the situation.

   b. Write an equation to represent the situation. Use a symbol for the unknown.

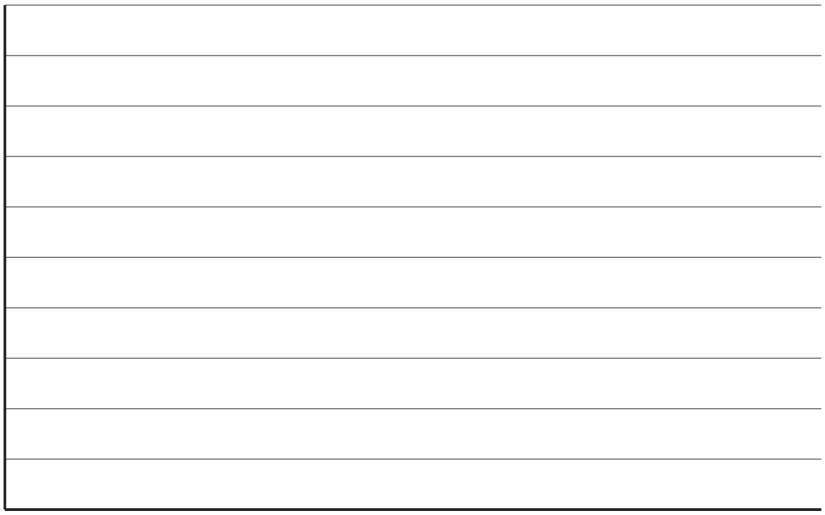
   c. Solve the problem. Explain or show your reasoning.

Introducing Multiplication: End-of-Unit Assessment

1. The table shows the favorite seasons of some students.

favorite season	number of students
fall	36
winter	23
spring	29
summer	48

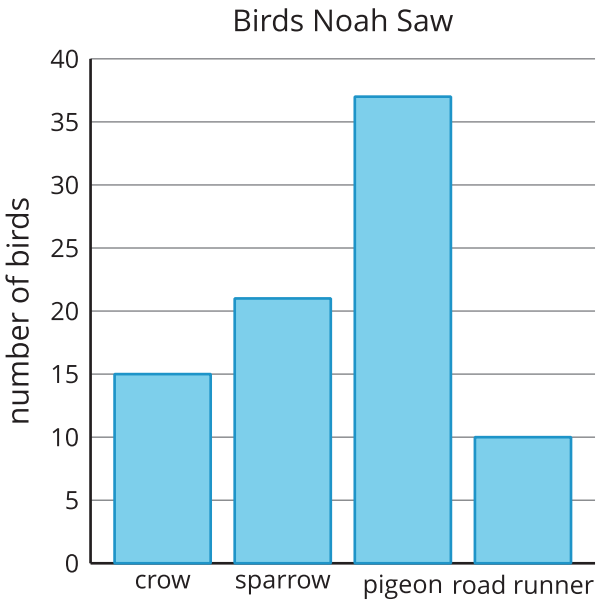
a. Create a scaled bar graph to represent the data. Use 2, 5, or 10 for your scale.



b. Which of the scales, 2, 5, or 10, will work to make a scaled bar graph on the grid?

c. Explain how you chose your scale.

2. The bar graph shows the kinds of birds that Noah saw one day.



Select **all** true statements.

A. Noah saw 3 crows.

B. Noah saw 37 pigeons.

C. Noah saw 22 more pigeons than crows.

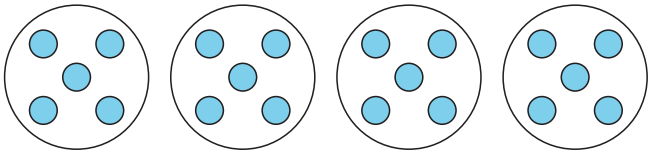
D. Noah saw fewer sparrows than road runners.

E. Noah saw fewer crows than pigeons.

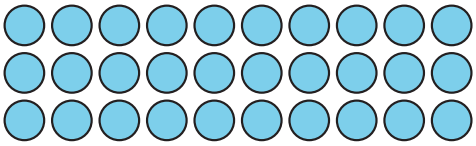
F. Noah saw 5 fewer crows than road runners.

3. Write a multiplication expression that could represent the number of dots in each drawing.

A



B



4. Elena has 5 bags. Each bag has 8 rubber bands.

   How many rubber bands does Elena have? Explain or show your reasoning.

5. There are 3 soccer teams on the field. Each team has 10 players. How many soccer players are on the field altogether?

   A. 7

   B. 13

   C. 20

   D. 30

6. Kiran has 18 cards. He arranges the cards in 3 rows. Each row has the same number of cards.

   a. Explain how the equation $3 \times ? = 18$ relates to Kiran's cards.

   b. How many cards are in each row? Explain how you know.

7. Find the number that makes each equation true.

   a. $4 \times 5 =$ _____

   b. $2 \times 6 =$ _____

   c. $3 \times 4 =$ _____

   d. $5 \times$ _____ $= 35$

   e. _____ $\times 10 = 40$

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Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment

Assessment Answer Keys
Assessment: Section A Checkpoint

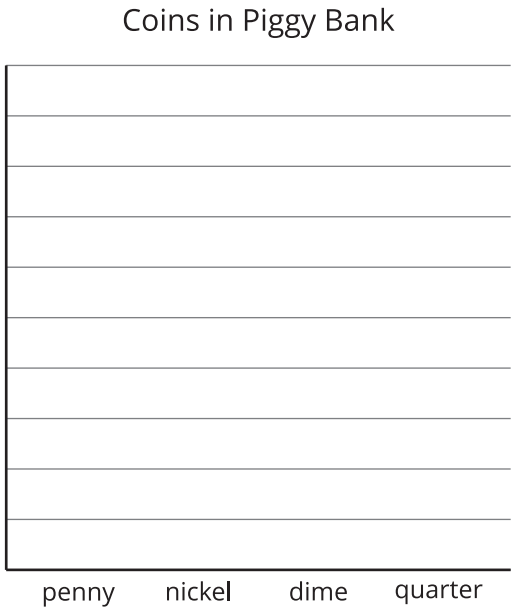
Problem 1

Goals Assessed

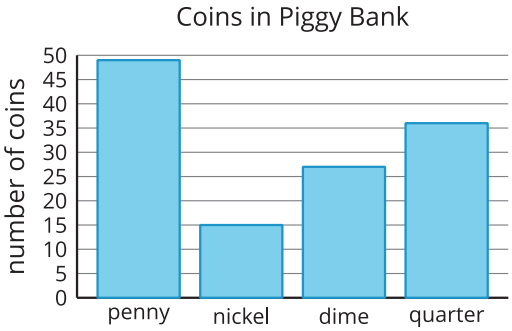
- Represent data using a picture graph and a bar graph.

The table shows the number of different coins in a piggy bank. Create a scaled bar graph to represent the data. Consider a scale of 5 or 10.

coin	number
penny	49
nickel	15
dime	27
quarter	36



Solution

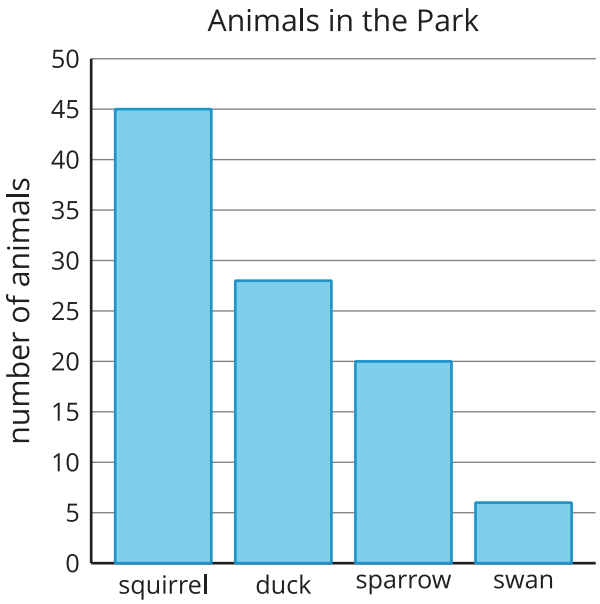


Problem 2

Goals Assessed

- Interpret scaled picture and bar graphs.
- Solve one- and two-step story problems using addition and subtraction.

The bar graph shows the number of different animals at a park.



a. How many squirrels and sparrows are there altogether? Explain or show your reasoning.

b. How many more squirrels are there at the park than sparrows and swans combined? Explain or show your reasoning.

Solution

a. 65. $45 + 20$

b. 19. $45 - 26$

Assessment: Section B Checkpoint

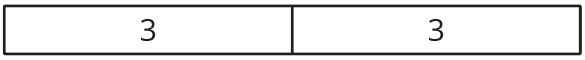
Problem 1

Goals Assessed

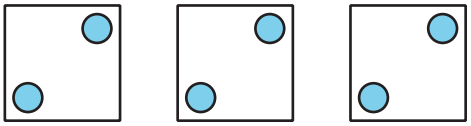
- Understand multiplication in terms of equal groups.

Jada has 2 bowls. Each bowl has 3 apples. Select **all** representations of Jada's apples.

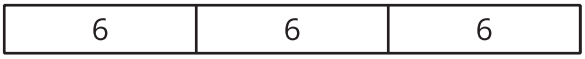
A.



B.



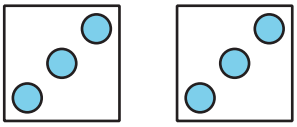
C.



D. 2×3

E. $2 + 3$

F.



Solution

["A", "D", "F"]

Problem 2

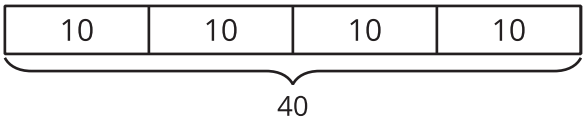
Goals Assessed

- Represent and solve multiplication problems involving equal groups.

a. There are 4 bunches of grapes. Each bunch has 10 grapes. How many grapes are there? Explain or show your reasoning.
b. There are 30 people in some cars. Each car has 5 people in it. How many cars are there? Explain or show your reasoning.

Solution

a. There are 40 grapes. Sample responses: $4 \times 10 = 40$ or $10 \times 4 = 40$



b. There are 6 cars. I counted 5, 10, 15, 20, 25, 30 and that's 6 times to get to 30. So there are 6 cars.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Represent and solve multiplication problems involving arrays.

Draw an array that represents the expression 3×4. Explain or show your reasoning.

Solution

Sample response: Student draws a 3 by 4 array or 4 by 3 array. There are 3 rows of 4 dots so that's 3×4 dots.

Problem 2

Goals Assessed

- Represent and solve multiplication problems involving arrays.

There are 3 rows of chairs in the room. Each row has 7 chairs. How many chairs are there?

a. Draw an array to represent the situation.
b. Write an equation to represent the situation. Use a symbol for the unknown.
c. Solve the problem. Explain or show your reasoning.

Solution

a. Student draws a 3 by 7 array or 7 by 3 array.
b. $3 \times 7 = ?$ or $7 \times 3 = ?$
c. Sample response: There are 21 chairs total.
   $7 + 7 = 14$
   $14 + 7 = 21$

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments

Addressing 3.MD.B.3

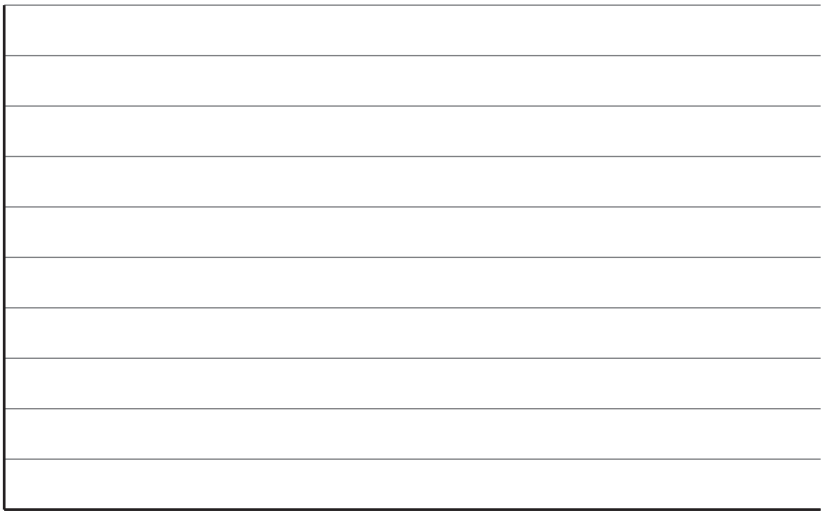
Narrative

Students choose a scale for a scaled bar graph and make the graph. Given the provided graphing space, they have 2 choices for a scale. They can use a scale of 5 and then the largest bar goes almost to the top or they can use a scale of 10, decreasing the height of the bars. Students could choose a scale of 2 if they extend the given grid by hand. This will influence their answer for the second question but should be considered as fully correct.

The table shows the favorite seasons of some students.

favorite season	number of students
fall	36
winter	23
spring	29
summer	48

a. Create a scaled bar graph to represent the data. Use 2, 5, or 10 for your scale.

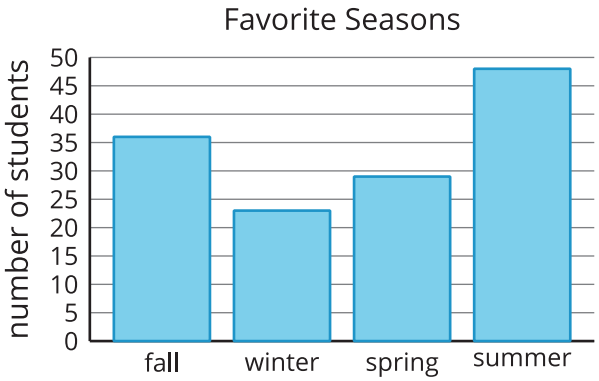


b. Which of the scales, 2, 5, or 10, will work to make a scaled bar graph on the grid?

c. Explain how you chose your scale.

Solution

a. Sample response:



b. Sample response: There are 10 lines. If I make each one represent 2 students, that only goes up to 20 students and I can't show the data. With a scale of 5, it goes up to 50 and I can show all of the numbers. I can also show all of the numbers with a scale of 10 because then it would go up to 100 students.

c. Sample response: I decided to choose 5 for a scale because all of the data fit and I was able to draw the bars pretty accurately.

Problem 2

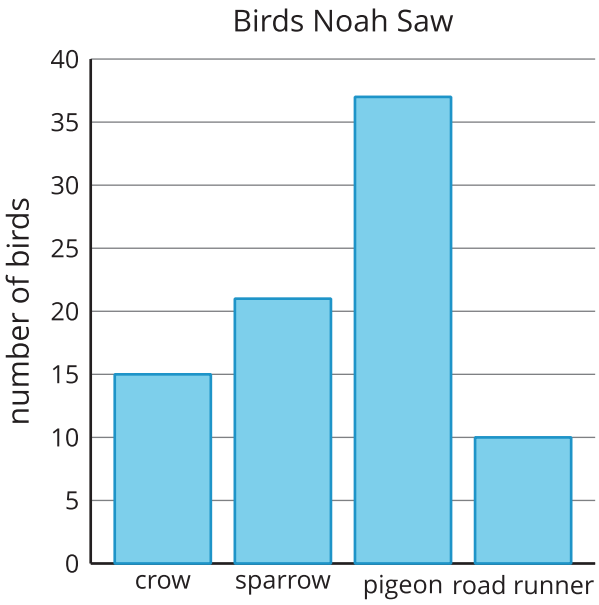
Standards Alignments

Addressing 3.MD.B.3

Narrative

Students read a scaled bar graph and answer questions about the data. Students may select A (and not select B or C) if they do not read the scale on the graph. Students may select D or F if they confuse fewer and more.

The bar graph shows the kinds of birds that Noah saw one day.



Select **all** true statements.

A. Noah saw 3 crows.

B. Noah saw 37 pigeons.

C. Noah saw 22 more pigeons than crows.

D. Noah saw fewer sparrows than road runners.

E. Noah saw fewer crows than pigeons.

F. Noah saw 5 fewer crows than road runners.

Solution

["B", "C", "E"]

Problem 3

Standards Alignments

Addressing 3.OA.A.1

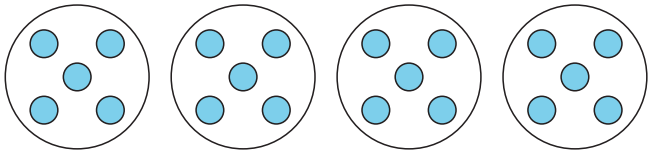
Narrative

Students write multiplication expressions to represent the number of dots in different images. These include an array and an equal groups image. In each case, students may write the order of

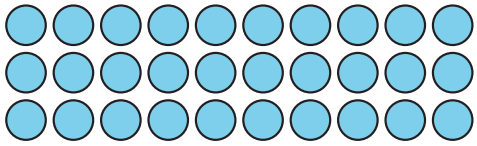
the factors in two different ways. Students could possibly see the diagrams differently, that is they could write 2×10 for the first if they group pairs of 5 dots. This is not likely but if they write a multiplication expression whose value is 20 for the first diagram or 30 for the second diagram they may understand the meaning of multiplication but may view the diagram differently.

Write a multiplication expression that could represent the number of dots in each drawing.

A



B



Solution

A: 4×5 or 5×4

B: 3×10 or 10×3

Problem 4

Standards Alignments

Addressing 3.OA.A.3

Narrative

Students solve a problem about an equal groups situation. Students may solve the problem using an expression or equation or they may make a drawing. Listed in increasing order of abstraction, possible drawings include:

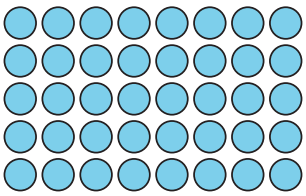
- a drawing of bags and rubber bands
- an array
- a tape diagram

Elena has 5 bags. Each bag has 8 rubber bands.

How many rubber bands does Elena have? Explain or show your reasoning.

Solution

40. Sample response: There are 5 rows of 8 circles and 40 circles total.



Problem 5

Standards Alignments

Addressing 3.OA.A.3

Narrative

This item assesses an understanding of equal groups situations. No explanation is required and the numbers are deliberately chosen to foster mental calculation. Other items assess student ability to represent these situations with equations and diagrams. Students who do not answer this question correctly may need further review of multiplication. Students who select A or B are likely performing the wrong operation with the numbers 3 and 10. Response C probably indicates incomplete work.

There are 3 soccer teams on the field. Each team has 10 players. How many soccer players are on the field altogether?

A. 7

B. 13

C. 20

D. 30

Solution

D

Problem 6

Standards Alignments

Addressing 3.OA.A.3

Narrative

Students interpret an array situation with an unknown number of columns as a multiplication equation and then solve the equation. The numbers are friendly so that they can solve the equation by inspection or by calculating. Students may also draw an array of dots and stop when they reach 18 total dots.

Kiran has 18 cards. He arranges the cards in 3 rows. Each row has the same number of cards.

a. Explain how the equation $3 \times ? = 18$ relates to Kiran's cards.
b. How many cards are in each row? Explain how you know.

Solution

a. If ? is the number of cards in each row, then $3 \times ? = 18$ because 18 is how many he has altogether and 3 is the number of rows.
b. 6. I put dots in 3 rows and there are 3, 6, 9, 12, 15, 18 dots. It takes 6 dots in each row to get 18 total.

Problem 7

Standards Alignments

Addressing 3.OA.A.4

Narrative

Students find an unknown in a multiplication equation using a way of reasoning that makes sense to them. Fluency with these facts is a yearlong progression, so any strategy for solving a multiplication equation at this point of the year is okay.

Find the number that makes each equation true.

a. $4 \times 5 =$ _____
b. $2 \times 6 =$ _____
c. $3 \times 4 =$ _____

d. $5 \times \_\_\_\_\_ = 35$

e. $\_\_\_\_\_ \times 10 = 40$

Solution

a. 20

b. 12

c. 12

d. 7

e. 4

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Core Knowledge MATHEMATICS™

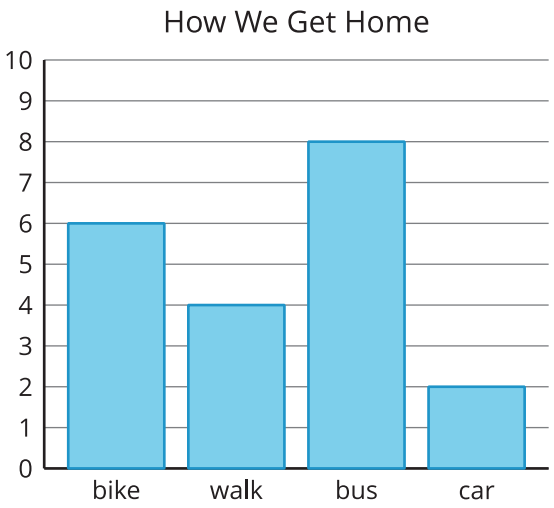
Lesson Cool Downs

Lesson 1: Make Sense of Data

Cool Down: Describe and Ask

A group of students were asked, "How do you get home from school each day?"

Their responses are shown in this bar graph:



Based on the data shown on the graph:

1. Write one fact you learned about how the students get home.

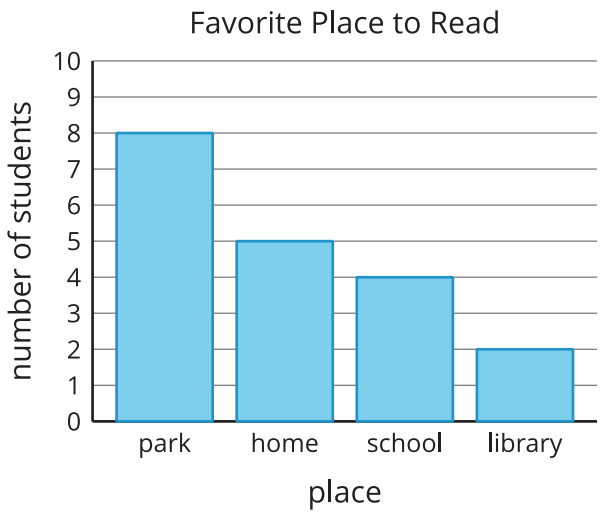
2. Write one question you could ask about how the students get home from school.

Lesson 2: Represent Data and Solve Problems

Cool Down: Questions About a Bar Graph

A group of students were asked, "Where is your favorite place to read?"

Their responses are shown in this bar graph:



1. How many more students chose the park than home as their favorite place to read?

2. True or false: More students like to read at the school or library than the park. Explain or show your reasoning.

Lesson 3: Scaled Picture Graphs

Cool Down: Birds in the Park

Jada collected data to see how many of each type of bird she saw on her way home.

The data is shown in this picture graph:

Each ✓ represents 2 birds.

Based on the data on the graph:

1. How many sparrows did Jada see on the way home?

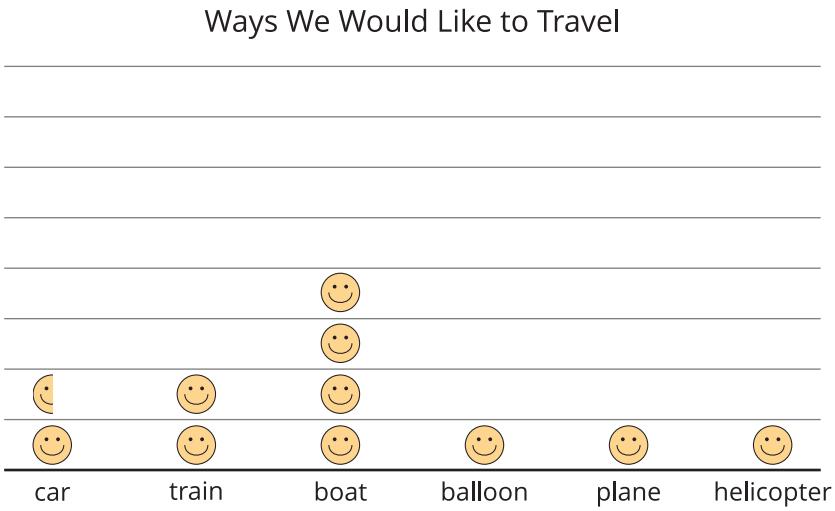
2. Write one question you could ask about the birds Jada saw on the way home.

Lesson 4: Create Scaled Picture Graphs

Cool Down: Complete the Picture Graph

A group of students were asked, “How would you like to travel?”

Their responses are shown in this picture graph:



Each ☺ represents 2 students.

Four students were absent when this data was collected. They would like to travel by plane.

Add their data to the graph.

Lesson 5: Represent Data in Scaled Bar Graphs

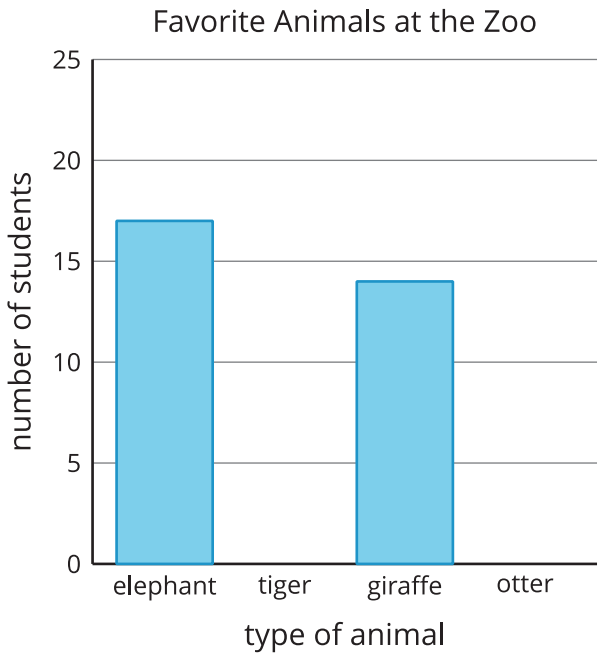
Cool Down: Complete a Scaled Bar Graph

Students visiting the zoo were asked, "What is your favorite animal at the zoo?"

Their responses are shown in this table:

animal	number of students
elephant	17
tiger	10
giraffe	14
otter	4

Use the data in the table to complete the scaled bar graph.



Lesson 6: Choose a Scale

Cool Down: Reflection on Bar Graphs and Scale

1. How did you decide on the scale for your graph in the last activity?

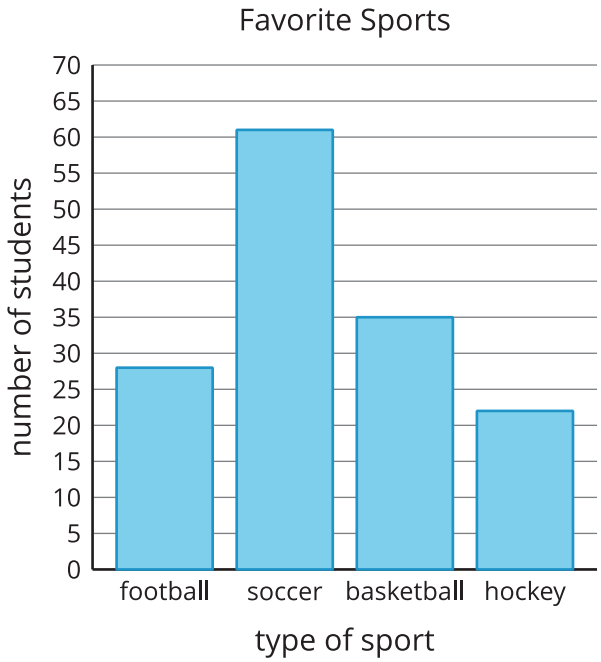
2. What was the most important thing you learned today that will help when you make your next scaled bar graph?

Lesson 7: Answer Questions about Scaled Bar Graphs

Cool Down: Favorite Sports

A group of students were asked, “What is your favorite sport?”

Their responses are shown in this bar graph:



Use the graph to answer the questions.

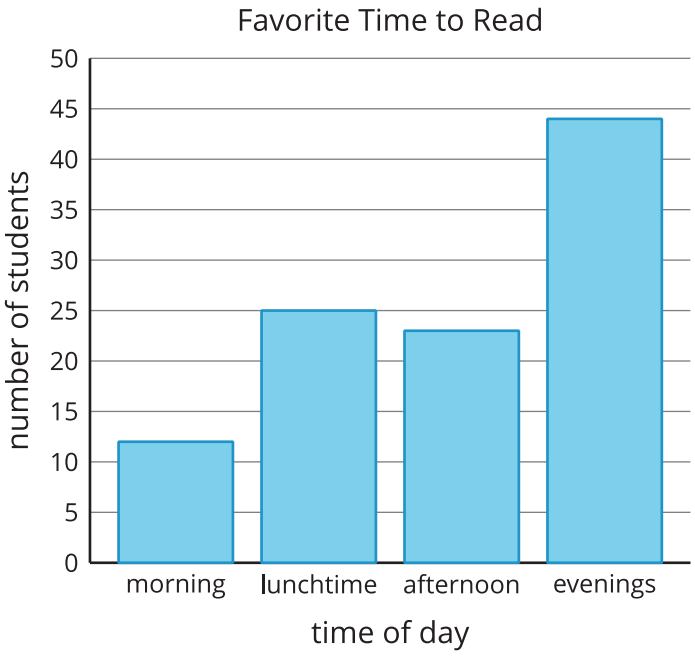
1. How many more students chose soccer than football? Show your thinking using expressions or equations.

2. How many fewer students chose hockey than basketball? Show your thinking using expressions or equations.

Lesson 8: More Questions about Scaled Bar Graphs

Cool Down: Reading Time

A group of students were asked, “What is your favorite time to read?” Their responses are shown in this bar graph:



Use the graph to answer the questions.

1. How many fewer students like to read in the morning than in the afternoon? Show your thinking using expressions or equations.

2. How many more students like to read in the evening than in the morning or at lunchtime? Show your thinking using expressions or equations.

Lesson 9: Multiplication as Equal Groups

Cool Down: Represent Equal Groups

Jada has 3 bags. Each bag has 5 bracelets in it.

Represent the situation.

Lesson 10: Drawings, Situations, and Diagrams, Oh My!

Cool Down: Boxes of Shirts

The store has 4 boxes. Each box has 10 shirts in it.

Does this diagram match the situation? Explain your reasoning.

Lesson 11: Multiplication Expressions

Cool Down: Write an Expression

There were 6 envelopes. Each envelope had 2 notes in it.

Write a multiplication expression to represent the situation. Explain or show your reasoning. Create a drawing or diagram if it's helpful.

Lesson 12: Represent and Solve Multiplication Problems

Cool Down: Ducks in a Pond

There are 4 ponds. Each pond has 5 ducks. How many ducks are there altogether?

Use diagrams, symbols, or other ways to show your thinking.

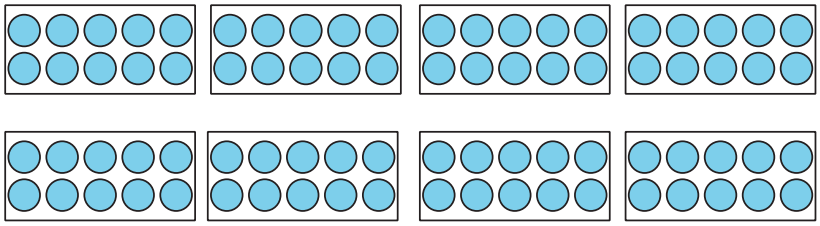
Lesson 13: Multiplication Equations

Cool Down: Match the Equation

Select **all** the drawings, diagrams, and situations that could represent the equation.

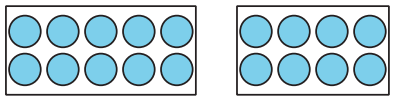
$$80 = 8 \times 10$$

A.

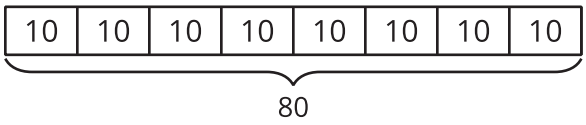


B. Andre has 8 boxes. Each box has 10 cars in it. He has 80 cars altogether.

C.

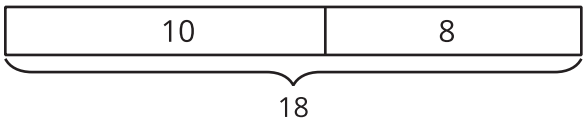


D.



E. Andre had 8 boxes. Then, he found 10 more boxes. How many boxes does Andre have?

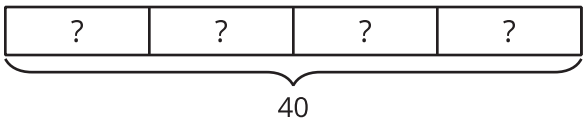
F.



Lesson 14: Write and Solve Equations with Unknowns

Cool Down: Unknown and a Number

1. Write an equation to match the diagram. Use a symbol for the unknown.



2. Find the number that makes the equation true. Rewrite the equation with that number. Explain your reasoning.

Lesson 15: More Factors, More Problems

Cool Down: Solve the Problem

Solve each problem. Explain or show your reasoning.

1. There are 4 boxes. Each box has 10 toys. How many toys are there?

2. Elena has 10 socks. She puts them in piles of 2. How many piles does she make?

Lesson 16: Arrange Objects Into Arrays

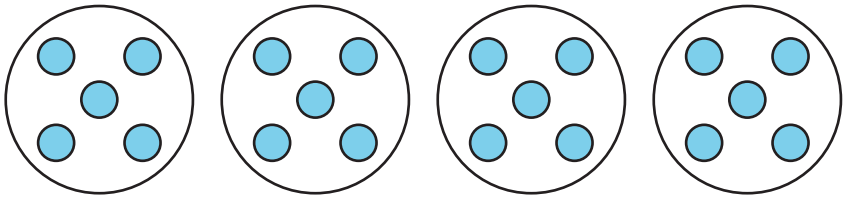
Cool Down: Array Arrangement

Arrange 12 cubes into an array.

Explain or show how the array is related to multiplication.

Lesson 17: Match and Draw Arrays

Cool Down: Draw and Describe



1. Redraw the equal groups as an array.

2. Describe how the diagram and the array are related.

Lesson 18: Represent Arrays with Expressions

Cool Down: Array Situation

There are 2 rows of plants. Each row has 5 plants.

1. Draw an array to represent the situation.

2. Write an expression to represent the situation.

Lesson 19: Solve Problems Involving Arrays

Cool Down: Clare's Cards

Clare has 3 rows of baseball cards. Each row has 10 cards. How many cards does she have?

1. Write an equation with a symbol for the unknown number to represent the situation.

2. Find the number that makes the equation true. Explain or show your reasoning.

Lesson 20: The Commutative Property

Cool Down: Multiplication Reflection

Summarize what you learned about multiplication today.

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Instructional Masters

Instructional Masters for Introducing Multiplication

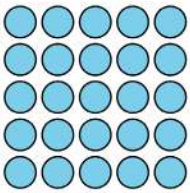
address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade3.1.17.1	Card Sort Arrays	2	no	yes	no	no
Activity Grade3.1.14.1	Card Sort Unknown Numbers	2	no	yes	no	no
Activity Grade3.1.21.1	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.1.21.2	Centimeter Grid Paper - Standard	2	yes	no	no	no
Activity Grade3.1.10.2	Card Sort Equal Groups	2	no	yes	no	no
Center	Sort and Display Stage 2 Recording Sheet	1	yes	no	no	no
Center	Capture Squares Stage 3 Gameboard	2	yes	no	no	no
Center	Capture Squares Stage 3 Spinner	2	no	no	no	no
Center	Sort and Display Stage 3 Recording Sheet	1	yes	no	no	no
Center	Five in a Row Addition and Subtraction Stage 6 Gameboard	2	no	no	no	no
Center	Capture Squares Stage 4 Gameboard	2	yes	no	no	no
Center	Capture Squares Stage 4 Spinner	2	no	no	no	no
Center	Five in a Row Addition and Subtraction Stage 7 Gameboard	2	no	no	no	no

Center	Capture Squares Stage 5 Spinner	2	no	no	no	no
Center	Capture Squares Stage 5 Gameboard	2	yes	no	no	no
Center	Five in a Row Addition and Subtraction Stage 8 Gameboard	2	no	no	no	no
Center	Five in a Row Multiplication and Division Stage 1 Gameboard	2	no	no	no	no

Card Sort Arrays

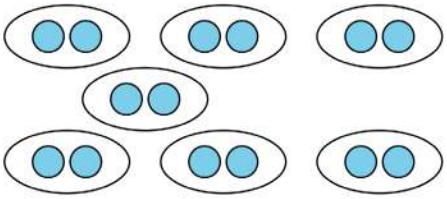
Card Sort: Arrays

A



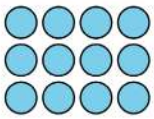
Card Sort: Arrays

B



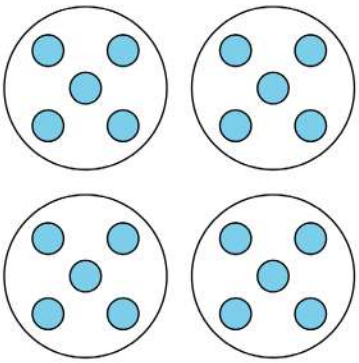
Card Sort: Arrays

C



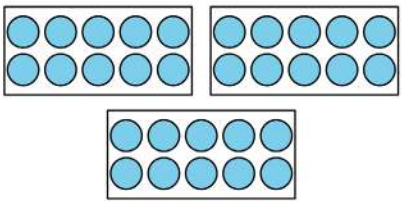
Card Sort: Arrays

D



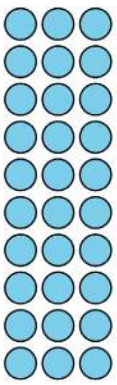
Card Sort: Arrays

E



Card Sort: Arrays

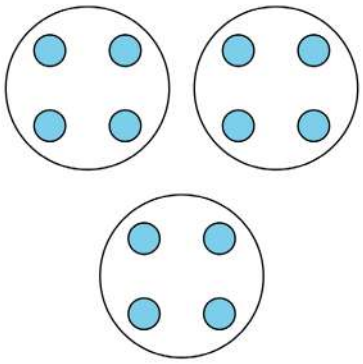
F



Card Sort Arrays

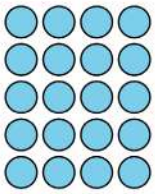
Card Sort: Arrays

G



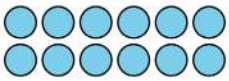
Card Sort: Arrays

H



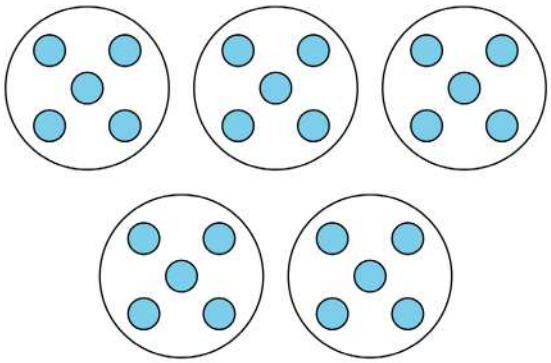
Card Sort: Arrays

I



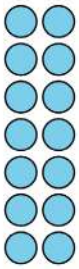
Card Sort: Arrays

J



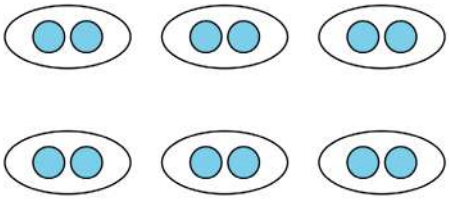
Card Sort: Arrays

K



Card Sort: Arrays

L

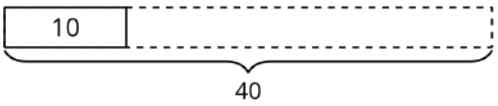


Card Sort Unknown Numbers

Card Sort: Unknown Numbers
A

$$3 \times \_\_\_\_\_ = 6$$

Card Sort: Unknown Numbers
B



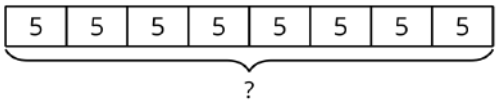
Card Sort: Unknown Numbers
C

$$8 \times 5 = ?$$

Card Sort: Unknown Numbers
D

There are 5 boxes of crayons.
Each box has 10 crayons in it.

Card Sort: Unknown Numbers
E



Card Sort: Unknown Numbers
F

$$5 \times 10 = ?$$

Card Sort: Unknown Numbers
G

There are 8 boxes and each box has some blankets in it.
There are 16 blankets altogether.

Card Sort: Unknown Numbers
H

$$? \times 10 = 40$$

Card Sort: Unknown Numbers
I

$$? \times 5 = 25$$

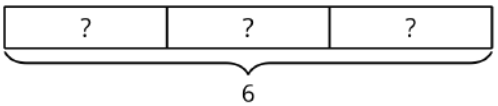
Card Sort: Unknown Numbers
J

Han has some bags of peaches.
Each bag has 5 peaches in it.
Han has 25 peaches.

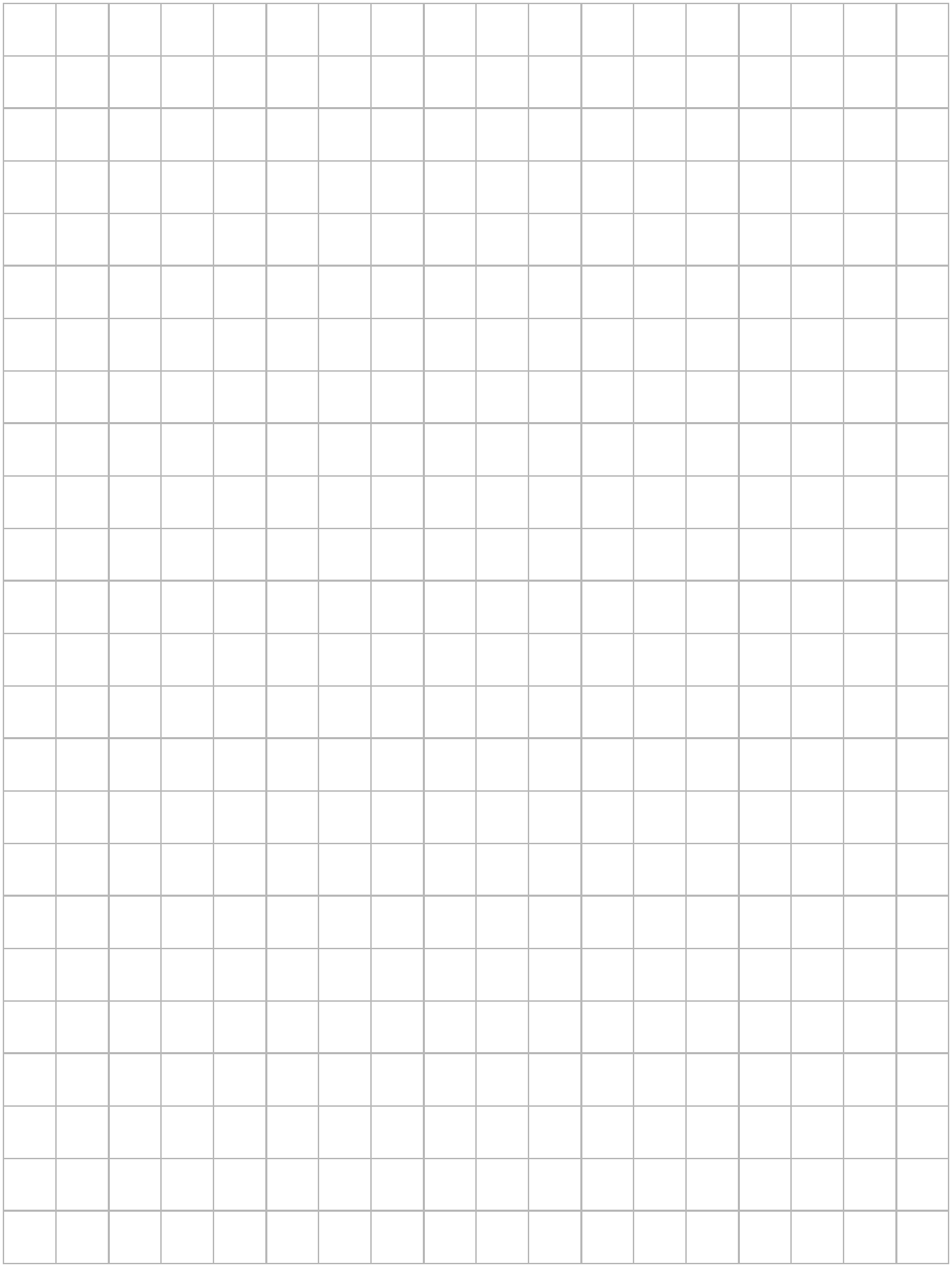
Card Sort: Unknown Numbers
K

$$8 \times \_\_\_\_\_ = 16$$

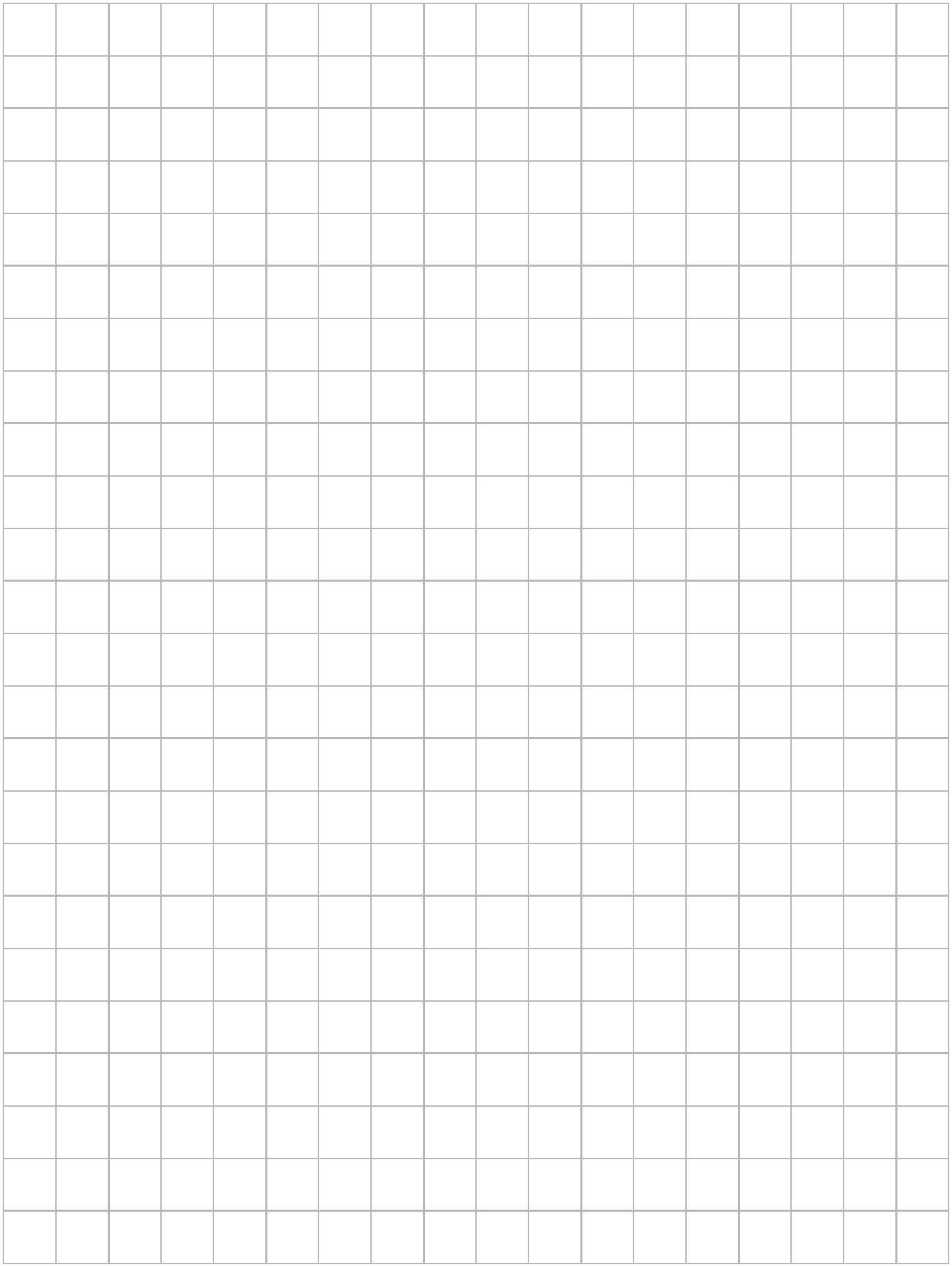
Card Sort: Unknown Numbers
L



Centimeter Grid Paper - Standard



Centimeter Grid Paper - Standard



Card Sort Equal Groups

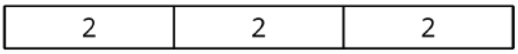
Card Sort: Equal Groups
A

There are 3 bags.
Each bag has 5 footballs in it.

Card Sort: Equal Groups
B

Han has 4 boxes.
Each box has 3 toy cars.

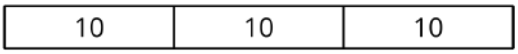
Card Sort: Equal Groups
C



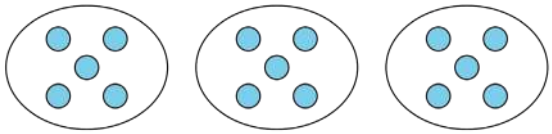
Card Sort: Equal Groups
D

There are 2 boxes.
Each box has 10 doughnuts.

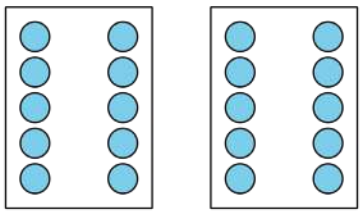
Card Sort: Equal Groups
E



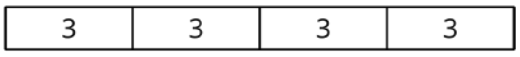
Card Sort: Equal Groups
F



Card Sort: Equal Groups
G



Card Sort: Equal Groups
H



Card Sort Equal Groups

Card Sort: Equal Groups

I

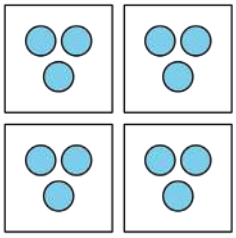
Card Sort: Equal Groups

J

Elena has 3 bins.
Each bin has 10 ice cubes.

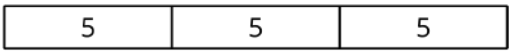
Card Sort: Equal Groups

K



Card Sort: Equal Groups

L



Sort and Display Stage 2 Recording Sheet

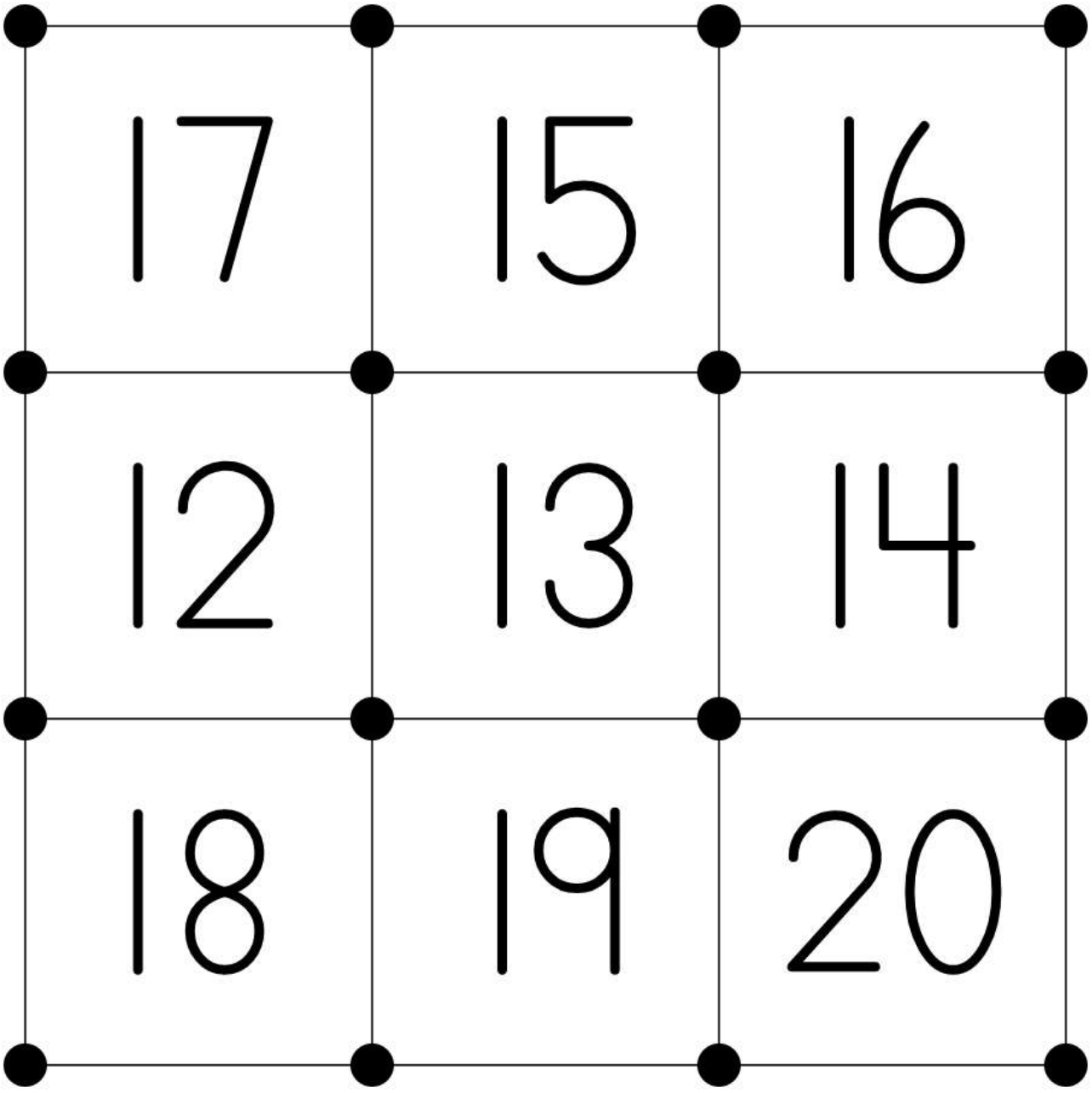
Directions:

- Choose 3 categories to sort your objects into.
- Make a picture graph or bar graph to show how you sorted.
- Ask your partner 2 questions that can be answered based on your graph.

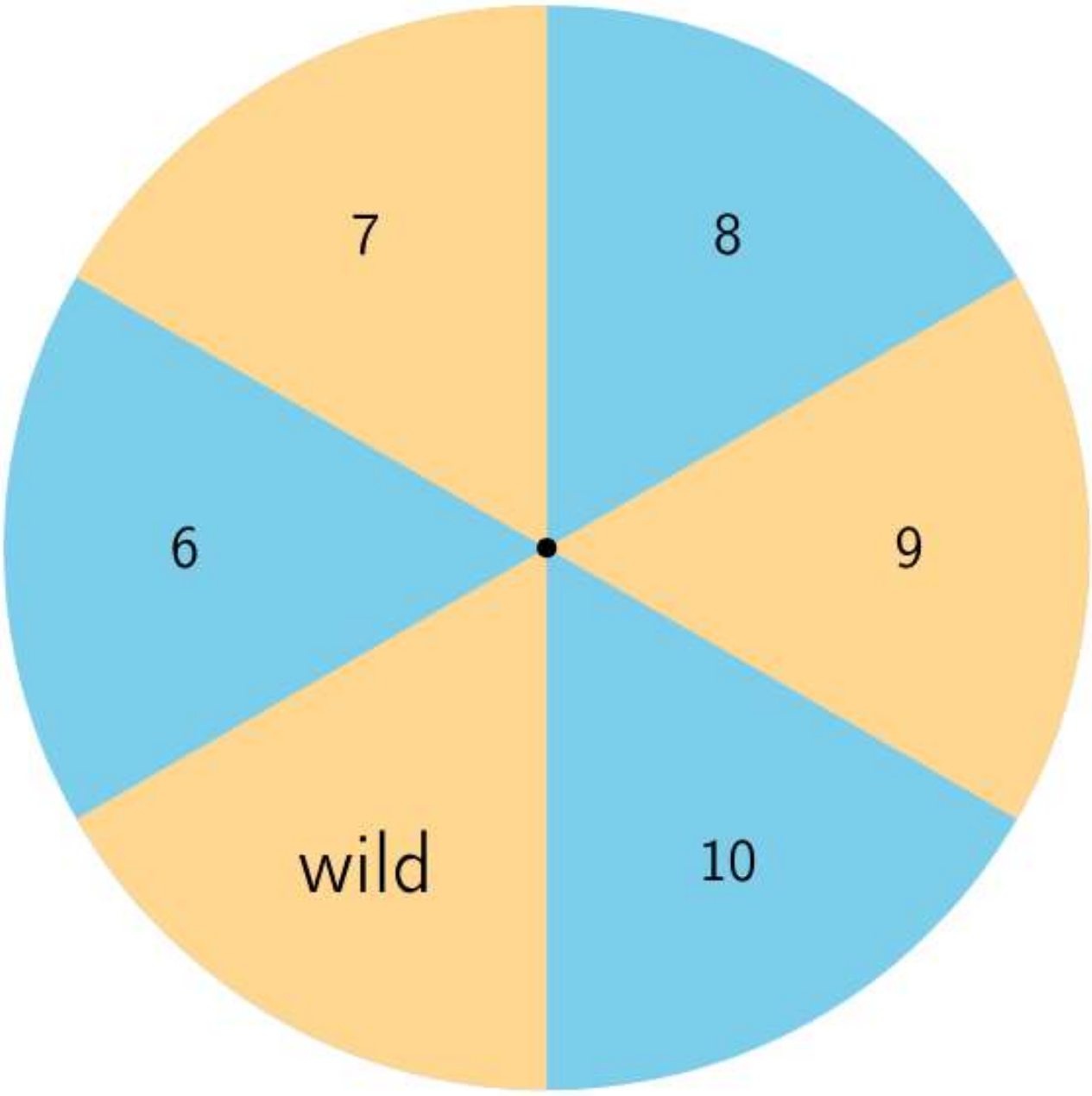
Capture Squares Stage 3 Gameboard

Directions:

- On your turn:
  - Spin the spinner and take 1 number card. Find the sum.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - If you can't draw a line, spin again and take a new card.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.



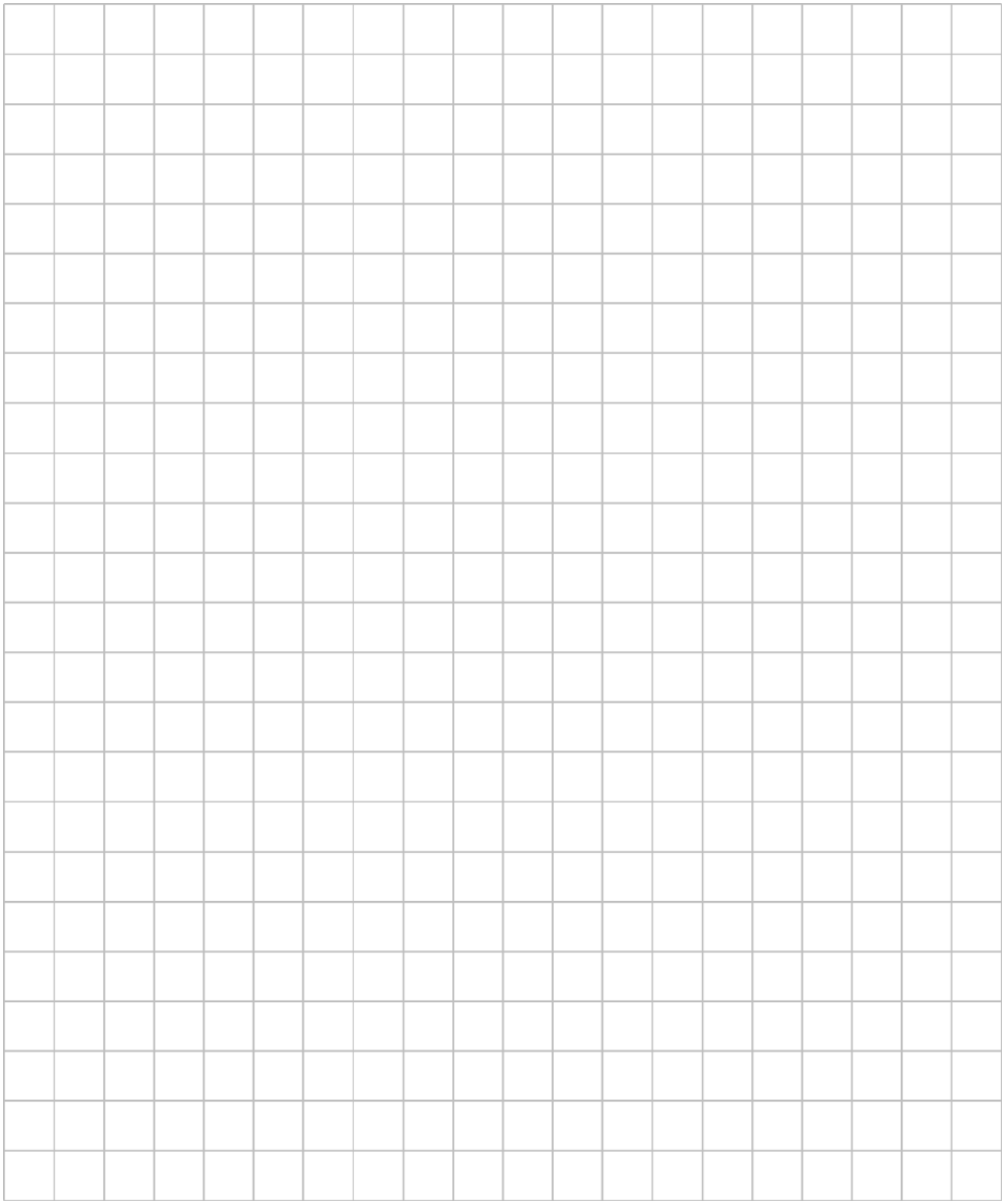
Capture Squares Stage 3 Spinner



Sort and Display Stage 3 Recording Sheet

Directions:

- Choose 3–5 categories to sort your objects into.
- Make a scaled picture or bar graph to show how you sorted.
- Ask your partner 2 questions that can be answered based on your graph.



Five in a Row Addition and Subtraction Stage 6 Gameboard

Directions: (two-digit plus two-digit)

- Partner A: Put a paper clip on 2 numbers in the grey rows. Cover the sum of the 2 numbers with a counter.
- Partner B: Move 1 of the paper clips, add the numbers, and cover the sum with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

81	91	54	46	90
84	83	35	82	53
60	92	99	73	51
73	42	44	53	92
100	75	82	61	64

16	27	25	34	35
65	19	57	26	48

Five in a Row Addition and Subtraction Stage 6 Gameboard

Directions: (one-digit plus two-digit)

- Partner A: Put a paper clip on 2 numbers in the grey rows. Cover the sum of the 2 numbers with a counter.
- Partner B: Move 1 of the paper clips, add the numbers, and cover the sum with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

75	64	24	26	63
65	25	22	31	55
58	30	67	32	66
72	56	54	34	71
74	23	33	73	57

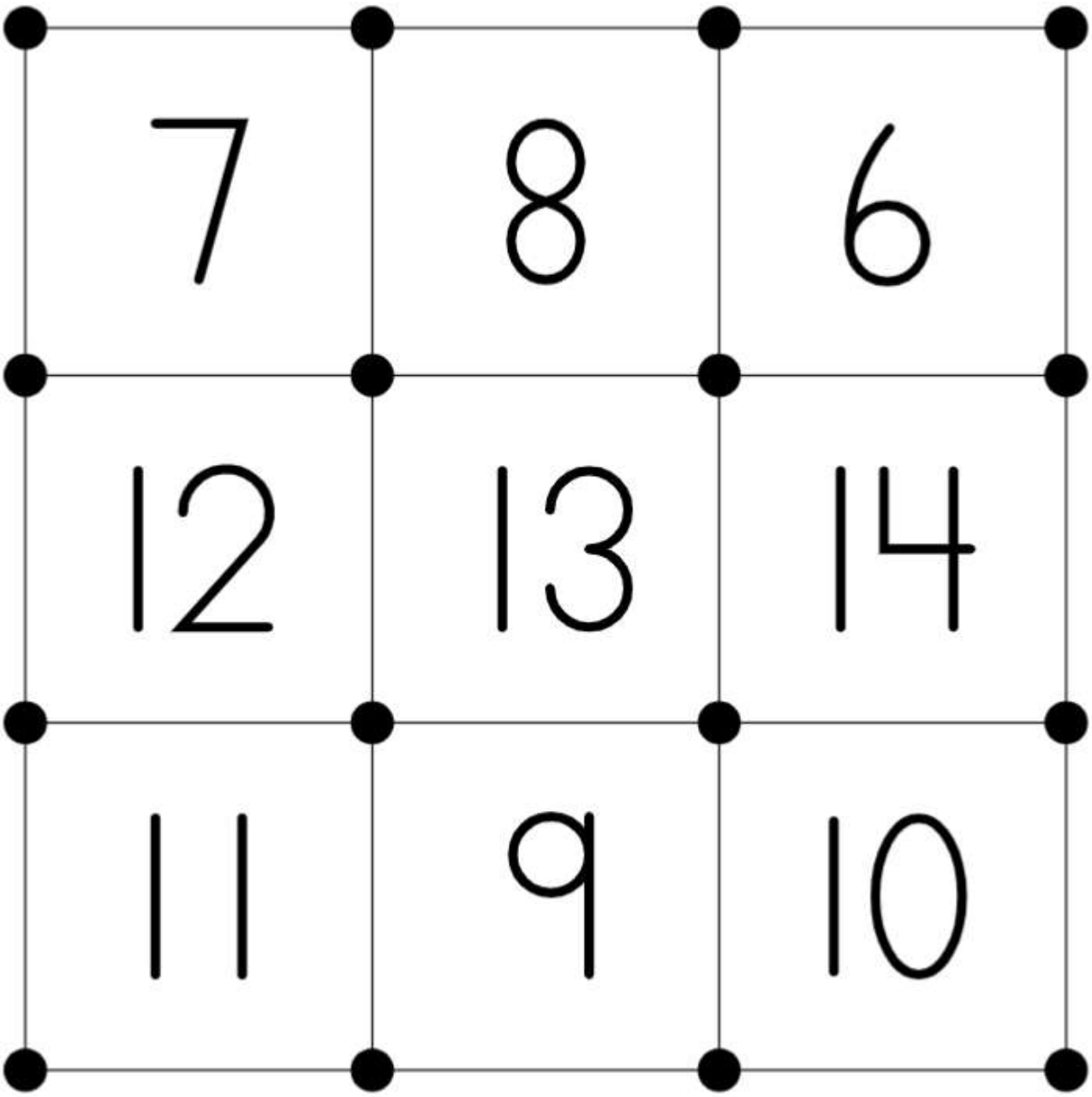
5	6	7	8	9

17	25	49	58	66

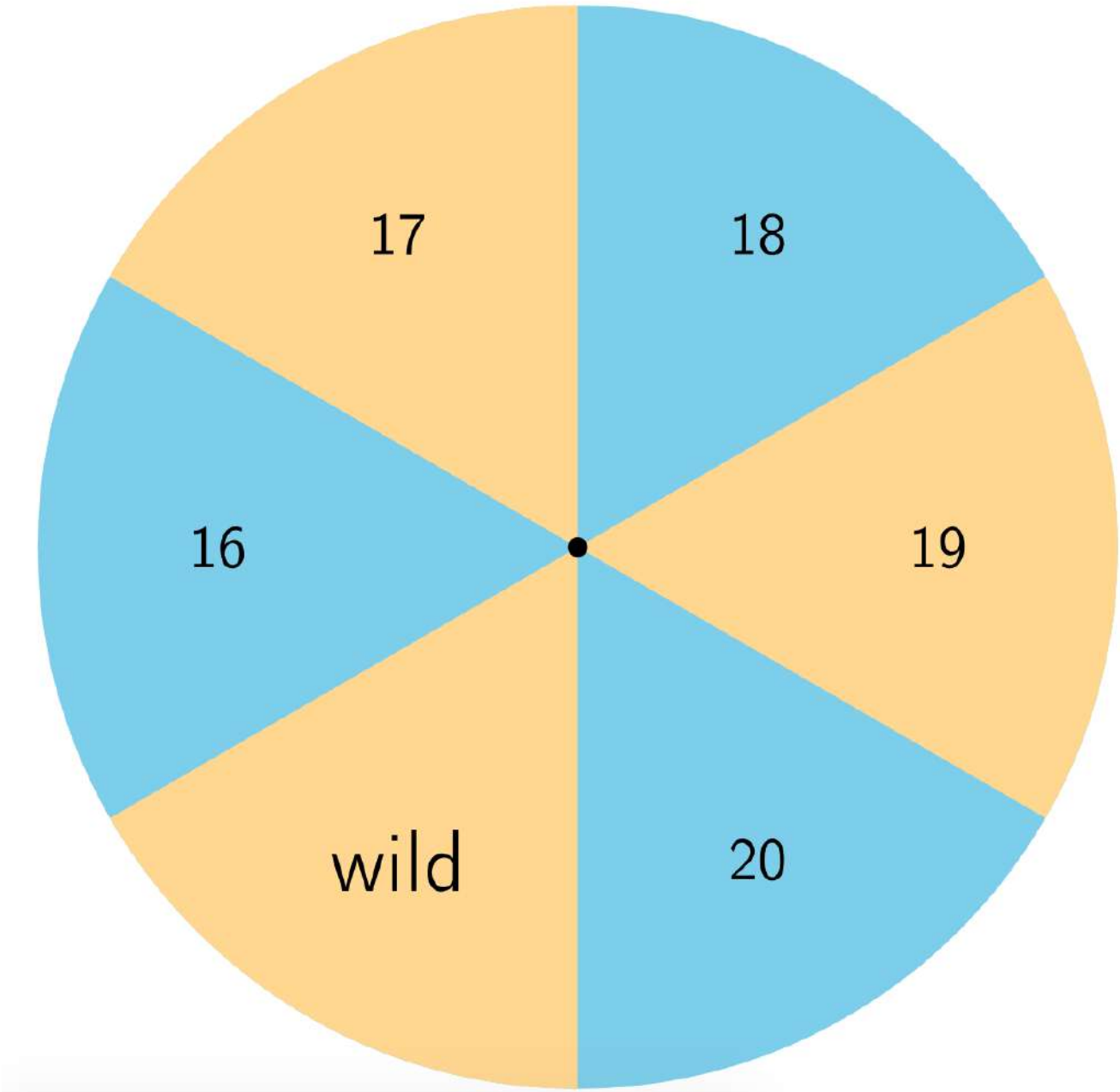
Capture Squares Stage 4 Gameboard

Directions:

- On your turn:
  - Spin the spinner and take 1 number card. Subtract the number on the card from the number on the spinner.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - If you can't draw a line, spin again and take a new card.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.



Capture Squares Stage 4 Spinner



Five in a Row Addition and Subtraction Stage 7 Gameboard

Directions:

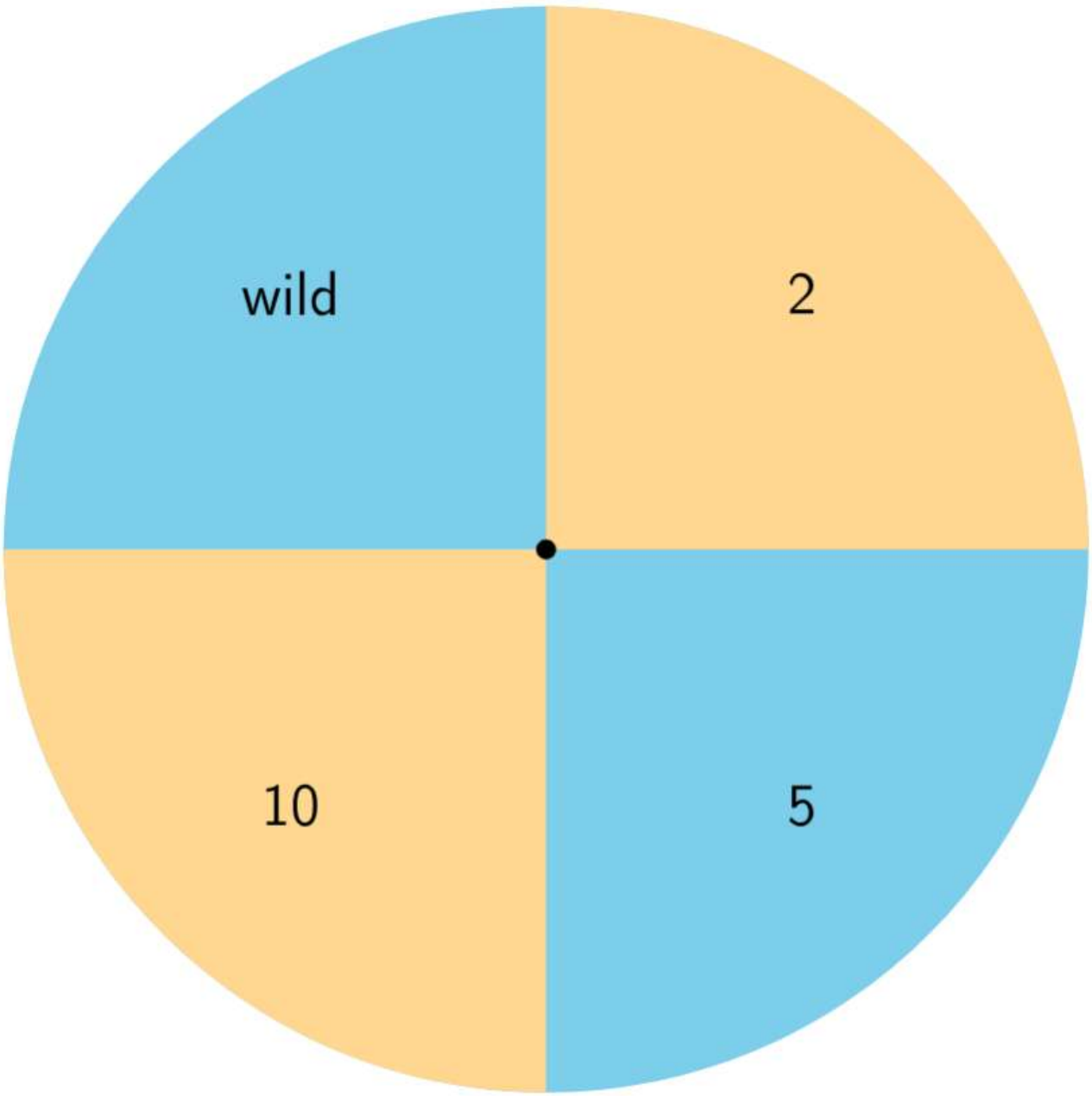
- Partner A: Put a paper clip on 2 numbers in the grey rows. Cover the sum of the 2 numbers with a counter.
- Partner B: Move 1 of the paper clips, add the numbers, and cover the sum with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

704	669	621	442	784
497	695	323	956	44
586	413	784	576	614
297	386	378	867	532
873	99	134	531	665

263	100	352	65	10

34	432	604	313	521

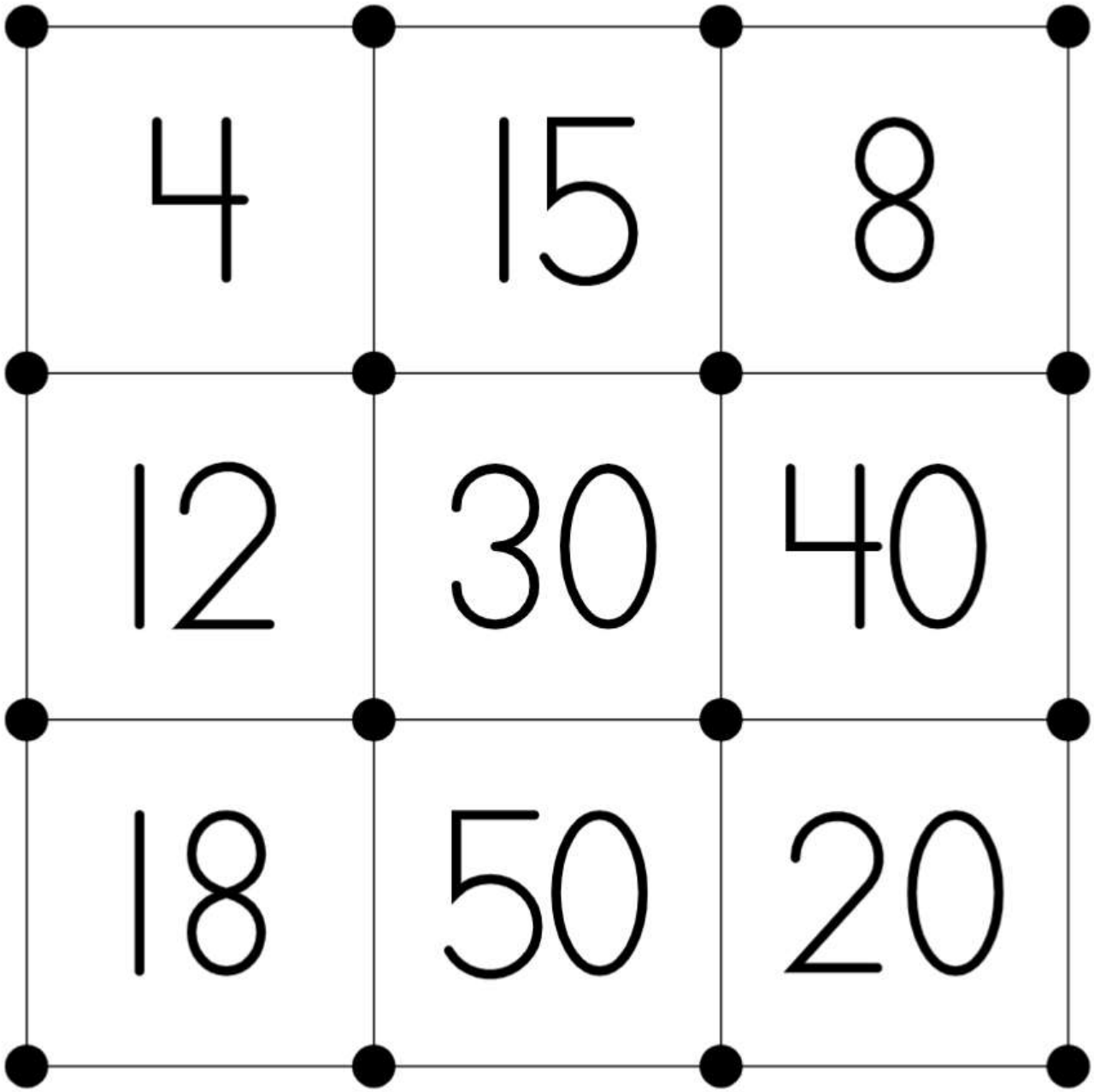
Capture Squares Stage 5 Spinner



Capture Squares Stage 5 Gameboard

Directions:

- On your turn:
  - Roll the number cube and spin the spinner. Find the product.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - If you can't draw a line, roll and spin again.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.



Five in a Row Addition and Subtraction Stage 8 Gameboard

Directions:

- Partner A: Put a paper clip on 2 numbers in the grey rows. Cover the sum of the 2 numbers with a counter.
- Partner B: Move 1 of the paper clips, add the numbers, and cover the sum with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

918	935	335	401	313
446	407	585	929	709
352	613	440	591	754
715	748	630	896	429
346	890	737	307	624

45	67	78	84	39

670	362	851	546	268

Five in a Row Multiplication and Division Stage 1 Gameboard

Directions:

- Partner A:
  - Put a paper clip on 2 numbers in the grey row.
  - Multiply the numbers.
  - Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

40	2	3	30	5
6	20	8	15	10
100	15	2	16	50
12	9	16	20	25
4	1	50	4	100

1	2	3	4	5	10

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