Proportional Relationships and Percentages

A train is traveling at...

Tax and Tip

Date: Sep. 12th
Time: 6:55 PM
Server: #27

1  Bread Stix  $9.50
2  Chicken Parm  $15.50
3  Chef Salad  $12.00
4  Lemon Soda  $2.00
5  Tea  $3.00

Subtotal  $42.00
Sales Tax  $3.99
Total  $45.99

THANK YOU FOR YOUR SHOPPING!

Percent increase and decrease

SALE 25% OFF

Measuring a soccer field
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# Proportional Relationships and Percentages

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Proportional Relationships and Percentages
Teacher Guide
Core Knowledge Mathematics™
Proportional Relationships and Percentage

Unit Narrative

Students began their work with ratios, rates, and unit rates in grade 6, representing them with expressions, tape diagrams, double number line diagrams, and tables. They used these to reason about situations involving color mixtures, recipes, unit price, discounts, constant speed, and measurement conversions. They extended their understanding of rates to include percentages as rates per 100, reasoning about situations involving whole-number percentages. They did not use the terms “proportion” and “proportional relationship” in grade 6.

A proportional relationship is a collection of equivalent ratios, and such collections are objects of study in grade 7. In previous grade 7 units, students worked with scale factors and scale drawings, and with proportional relationships and constants of proportionality. Although students have learned how to compute quotients of fractions in grade 6, these first units on scaling and proportional relationships do not require such calculations, allowing the new concept (scaling or proportional relationship) to be the main focus.

In this unit, students deepen their understanding of ratios, scale factors, unit rates (also called constants of proportionality), and proportional relationships, using them to solve multi-step problems that are set in a wide variety of contexts that involve fractions and percentages.

In the first section of the unit, students extend their use of ratios and rates to problems that involve computing quotients of fractions, and interpreting these quotients in contexts such as scaling a picture or running at constant speed (MP2). They use long division to write fractions presented in the form $\frac{a}{b}$ as decimals, e.g., $\frac{11}{30} = 0.3\overline{6}$.

The section begins by revisiting scale factors and proportional relationships, each of which has been the focus of a previous unit. Both of these concepts can be used to solve problems that involve equivalent ratios. However, it is often more efficient to view equivalent ratios as pairs that are in the same proportional relationship rather than seeing one pair as obtained by multiplying both entries of the other by a scale factor. From the scaling perspective, to see that one ratio is equivalent to another or to generate a ratio equivalent to a given ratio, a scale factor is needed—which may be different for each pair of ratios in the proportional relationship. From the proportional relationship perspective, all that is needed is the constant of proportionality—which is the same for every ratio in the proportional relationship.

The second section of the unit is about percent increase and decrease. Students consider situations for which percentages can be used to describe a change relative to an initial amount, e.g., prices before and after a 25% increase. They begin by considering situations with unspecified amounts, e.g., matching tape diagrams with statements such as “Compared with last year's strawberry harvest, this year's strawberry harvest increased by 25%”. They next consider situations with a specified amount and percent change, or with initial and final amounts, using double number line diagrams to find the unknown amount or percent change. Next, they use equations to represent such situations, using the distributive property to show that different expressions for the same amount are equivalent, e.g., $x - 0.25x = 0.75x$. So far, percent change in this section has focused
on whole-number rates per 100, e.g., 75%. The last lesson asks students to compute fractional percentages of given amounts.

In the third section of the unit, students begin by using their abilities to find percentages and percent rates to solve problems that involve sales tax, tip, discount, markup, markdown, and commission (MP2). The remaining lessons of the section continue the focus on situations that can be described in terms of percentages, but the situations involve error rather than change—describing an incorrect value as a percentage of the correct value rather than describing an initial amount as a percentage of a final amount (or vice versa).

The last section of the unit consists of a lesson in which students analyze news items that involve percent increase or decrease. In small groups, students identify important quantities in a situation described in a news item, use diagrams to map the relationship of the quantities, and reason mathematically to draw conclusions (MP4). This is an opportunity to choose an appropriate type of diagram (MP5), to state the meanings of symbols used in the diagram, to specify units of measurement, and to label the diagram accurately (MP6). Each group creates a display to communicate its reasoning and critiques the reasoning shown in displays from other groups (MP3).

These materials follow specific conventions for the use of language around ratios, rates, and proportional relationships. Please see the unit narrative for the second unit to read about those conventions.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Interpret**

- situations involving constant speed (Lesson 2)
- concrete problems involving percent increase and decrease (Lesson 7)
- problems involving sales tax and tip (Lesson 10)
- concrete situations involving percent error (Lesson 14)

**Explain**

- how to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 4)
- how to solve percent change problems (Lesson 6)
- strategies for solving percent problems with fractional percentages (Lesson 9)
- how to measure lengths and interpret measurement error (Lesson 13)
- strategies for solving percent error problems (Lesson 14)

**Unit 4**
Represent

- situations involving percent increase and decrease (Lesson 8)
- situations with percent error (Lesson 15)
- situations from the news involving percent change (Lesson 16)

In addition, students are expected to compare measurements, scale factors, and decimal and fraction representations, compare representations of an increase (or decrease) of an amount by a fraction or decimal, generalize about using constants of proportionality to solve problems efficiently and about relationships with percent increase and decrease, and justify why specific information is needed to solve percent change problems.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.
Learning Targets

Proportional Relationships and Percentages

Lesson 1: Lots of Flags
- I can find dimensions on scaled copies of a rectangle.
- I remember how to compute percentages.

Lesson 2: Ratios and Rates With Fractions
- I can solve problems about ratios of fractions and decimals.

Lesson 3: Revisiting Proportional Relationships
- I can use a table with 2 rows and 2 columns to find an unknown value in a proportional relationship.
- When there is a constant rate, I can identify the two quantities that are in a proportional relationship.

Lesson 4: Half as Much Again
- I can use the distributive property to rewrite an expression like $x + \frac{1}{2}x$ as $(1 + \frac{1}{2})x$.
- I understand that “half as much again” and “multiply by $\frac{3}{2}$” mean the same thing.

Lesson 5: Say It with Decimals
- I can use the distributive property to rewrite an equation like $x + 0.5x = 1.5x$.
- I can write fractions as decimals.
- I understand that “half as much again” and “multiply by 1.5” mean the same thing.

Lesson 6: Increasing and Decreasing
- I can draw a tape diagram that represents a percent increase or decrease.
- When I know a starting amount and the percent increase or decrease, I can find the new amount.
Lesson 7: One Hundred Percent
• I can use a double number line diagram to help me solve percent increase and decrease problems.

• I understand that if I know how much a quantity has grown, then the original amount represents 100%.

• When I know the new amount and the percentage of increase or decrease, I can find the original amount.

Lesson 8: Percent Increase and Decrease with Equations
• I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it.

Lesson 9: More and Less than 1%
• I can find percentages of quantities like 12.5% and 0.4%.

• I understand that to find 0.1% of an amount I have to multiply by 0.001.

Lesson 10: Tax and Tip
• I understand and can solve problems about sales tax and tip.

Lesson 11: Percentage Contexts
• I understand and can solve problems about commission, interest, markups, and discounts.

Lesson 12: Finding the Percentage
• I can find the percentage increase or decrease when I know the original amount and the new amount.

Lesson 13: Measurement Error
• I can represent measurement error as a percentage of the correct measurement.

• I understand that all measurements include some error.

Lesson 14: Percent Error
• I can solve problems that involve percent error.

Lesson 15: Error Intervals
• I can find a range of possible values for a quantity if I know the maximum percent error and the correct value.
Lesson 16: Posing Percentage Problems

- I can write and solve problems about real-world situations that involve percent increase and decrease.
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Required Materials

**Four-function calculators**

**Grocery store circulars**
Grocery store advertisements from the newspaper or that are picked up at the store. If students have Internet access, you could substitute an online version of this.

**Pre-printed slips, cut from copies of the Instructional master**

**Sticky notes**

**Tools for creating a visual display**
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.
Section: Proportional Relationships with Fractions

Lesson 1: Lots of Flags

Goals

- Compare (orally and in writing) the dimensions and scale factors of multiple scaled copies of the same figure.
- Explain (orally) how to estimate or calculate the percentage of a rectangular area that is covered by another region.
- Generate the dimensions for a scaled copy of an original figure that has fractional side lengths.

Learning Targets

- I can find dimensions on scaled copies of a rectangle.
- I remember how to compute percentages.

Lesson Narrative

In this unit students will be applying proportional relationships to solve problems with fractional ratios, rates, percents, and constants of proportionality. The purpose of this lesson is to start the unit with an engaging activity where these arise naturally, in the scaling of flags and in questions about what percentage of the flag is taken up by a particular part of the design.

Alignments

Building On

- 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.
- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.
• 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Instructional Routines
• MLR5: Co-Craft Questions
• MLR8: Discussion Supports
• Think Pair Share

Student Learning Goals
Let’s explore the U.S. flag.

1.1 Scaled or Not?

Warm Up: 5 minutes
This warm-up prompts students to reason about proportional relationships in geometric objects as a review of work done earlier in grade 7. As students discuss their answers with their partner, select students to share their answers to the second question during the whole-class discussion. Select students so that different sets of objects and their scale factors are represented in the discussion.

Building On
• 7.G.A.1

Building Towards
• 7.RP.A.2.a

Instructional Routines
• Think Pair Share

Launch
Arrange students in groups of 2. Give students 1 minute of quiet work time and another 1–2 minutes to share their solutions with their partner. Ask students to make sure they have the same objects identified in the first question. If one partner is missing a set of scaled objects, they should add them to their list during their partner discussion.

Anticipated Misconceptions
Students might think H is a scaled version of A or B. Suggest that they consider possible scale factors to get from, for example, A to H.
Student Task Statement

1. Which of the geometric objects are scaled versions of each other?

2. Pick two of the objects that are scaled copies and find the scale factor.

Student Response

1.
   - A and B
   - C, E, and K
   - D and I
   - G, J, and L

2. Answers vary. Sample response: A is 2 times the size of B. The height of A is 4, and the length of its base is 6. The height of B is 2, and the length of its base is 3.

Activity Synthesis

Select a couple of students, with a variety of answers to the second question, to share their answers to the second question. Ask students, “Did you encounter any objects that you initially believed were scaled versions of one another? How did you decide that they weren’t?”

1.2 Flags Are Many Sizes

15 minutes

In this introductory lesson students get a chance to recall what they have previously learned about ratios and proportional relationships. They will build on these ideas in the next few lessons where they will work with ratios and rates involving fractions. In this activity, students can leverage their
recent work on creating scale drawings to make connections between the dimensions of a different sized flag and the ratio of the side lengths.

A note about flag dimensions: The official government flag has sides with ratio $1 : 1.9$, i.e., the width of the flag is 1.9 times its height. However, many commercially sold flags use different ratios. This activity is working with the official ratio. If there is a flag displayed in the classroom, it would be interesting to check if it uses the official ratio or one of the other common commercial ratios, such as $2 : 3$ or $5 : 8$ or $6 : 10$.

Another interesting note about the flag: The current 50 star version of the flag was designed by a 17 year old HS student as a class project.

**Building On**

- 7.G.A.1

**Building Towards**

- 7.RP.A

**Instructional Routines**

- MLR5: Co-Craft Questions

**Launch**

Arrange students in groups of 3–4. If the classroom has a display of the flag which can be reached, ask a student to measure the dimensions of the flag.

Tell students, “The United States flag is displayed in many different sizes and for different purposes. One standard size is 19 feet by 10 feet. What would be a possible use for a flag of this size?”

Ask student where else they have seen flags displayed.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge of the definition of scale factor. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*
Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. Display the task statement without the questions and students to write possible mathematical questions about the situation. Invite students to share their questions with a partner before selecting 2–3 students to share with the class. Highlight mathematical language students use related to ratios and proportions. This helps students produce the language of mathematical questions and talk about the relationship between side lengths of a rectangle and scale factors.

Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

One standard size for the United States flag is 19 feet by 10 feet. On a flag of this size, the union (the blue rectangle in the top-left corner) is $7\frac{5}{8}$ feet by $5\frac{1}{8}$ feet.

There are many places that display flags of different sizes.

- Many classrooms display a U.S. flag.
- Flags are often displayed on stamps.
- There was a flag on the space shuttle.
- Astronauts on the Apollo missions had a flag on a shoulder patch.

1. Choose one of the four options and decide on a size that would be appropriate for this flag. Find the size of the union.

2. Share your answer with another group that used a different option. What do your dimensions have in common?

Student Response

Answers vary. Sample scale factors and dimensions:

- Classroom scale factor $\frac{1}{5}$ and union dimensions 1.525 feet by 1.075 feet
- Stamp scale factor $\frac{1}{240}$ and union dimensions 0.032 feet by 0.022 feet
- Shuttle scale factor $\frac{1}{2}$ and union dimensions 3.813 feet by 2.688 feet
- Patch scale factor $\frac{1}{60}$ and union dimensions 0.127 feet by 0.090 feet

Activity Synthesis

Record the groups' measurements in a table to show that there is a constant of proportionality.
1.3 What Percentage Is the Union?

15 minutes
This activity continues to look at the U.S. flag by asking questions about percentages, which students studied in grade 6. Later in this unit, students will continue working with percentages, including percent increase and decrease.

Knowing the side lengths of the flag and of the union allows you to compute the area of the flag and of the union. Students can then compute what percentage of the flag is taken up by the union. Finding out what percentage of the flag is red requires additional reasoning. Students can either compute the area of the red stripes or they can see what fraction of the non-union part of the flag is red.

This is a good opportunity for students to estimate their answers and get a visual idea of the size of different percentages.

Building On
- 7.G.A.1

Building Towards
- 7.RP.A

Instructional Routines
- MLR8: Discussion Supports

Launch
Arrange students in groups of 3–4. Tell students that they will continue to examine the United States flag, this time looking at area.

Student Task Statement
On a U.S. flag that is 19 feet by 10 feet, the union is \(7 \frac{5}{8}\) feet by \(5 \frac{3}{8}\) feet. For each question, first estimate the answer and then compute the actual percentage.

1. What percentage of the flag is taken up by the union?
2. What percentage of the flag is red? Be prepared to share your reasoning.

Student Response
Answers vary. Sample response:

1. Approximately 20%

2. 21.6%. Since the area of the union is about 41 square feet and the area of the flag is 190 square feet, the percentage is \(\frac{41}{190}\) or about 0.216.
2.
   ○ Approximately 40%
   ○ 41.5% Since the total red area is 78.875 square feet and the area of the flag is 190 square feet, the percentage is \( \frac{78.875}{190} \), or about 0.415.

**Are You Ready for More?**

The largest U.S. flag in the world is 225 feet by 505 feet.

1. Is the ratio of the length to the width equivalent to \( 1 : 1.9 \), the ratio for official government flags?

2. If a square yard of the flag weighs about 3.8 ounces, how much does the entire flag weigh in pounds?

**Student Response**

1. No.

2. About 3,000 pounds.

**Activity Synthesis**

The purpose of this discussion is to emphasize the use of proportion when working with areas of rectangles. Ask students to compare their estimated percentage with their calculated percentage. Ask students such as:

- “Did you use the image of the flag—a rectangle—to guide your estimate?”
- “Did you round the given dimensions to find the estimated percentage?”

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**Access for Students with Disabilities**

_Engagement: Develop Effort and Persistence._ Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I _____ because...”, “I noticed ____ so I...”, “Why did you...?”, “I agree/disagree because....”

_Supports accessibility for: Language; Social-emotional skills_

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**Access for English Language Learners**

_Speaking: MLR8 Discussion Supports._ Provide sentence frames to help students produce statements that compare their estimated percentage with their calculated percentage. For example, “The percentages are (similar/different) because ______.”

_Design Principle(s): Support sense-making, Optimize output (for comparison)
Lesson Synthesis
In this lesson, we used what we know about scale factors and dimensions of rectangles to answer questions related to the United States flag. Consider asking some of the following questions:

- “How can you tell whether two objects are scaled versions of one another?”
- “What properties stay the same when an object is scaled up or down?”
- “What strategies did you use to find properties of a scaled object, e.g. the dimensions of the union on a small flag?”
- “How did you go about finding the estimated percentage of total area on the United States flag (taken up by the union or the red region)?”

1.4 Colorado State Flag

Cool Down: 5 minutes
Building On
- 6.RP.A
- 7.G.A.1

Building Towards
- 7.RP.A.1

Student Task Statement
The side lengths of the state flag of Colorado are in the ratio 2 : 3. If a flag is 12 feet long, what is its height?

Student Response
8 feet
Student Lesson Summary

Imagine you have a painting that is 15 feet wide and 5 feet high. To sketch a scaled copy of the painting, the ratio of the width and height of a scaled copy must be equivalent to 15 : 5. What is the height of a scaled copy that is 2 feet across?

We know that the height is \( \frac{1}{3} \) the width, so \( h = \frac{1}{3} \cdot 2 = \frac{2}{3} \).

Sometimes ratios include fractions and decimals. We will be working with these kinds of ratios in the next few lessons.

Glossary

- percentage
Lesson 1 Practice Problems

Problem 1

Statement

A rectangle has a height to width ratio of 3 : 4.5. Give two examples of dimensions for rectangles that could be scaled versions of this rectangle.

Solution

Answers vary. Sample response: A rectangle measuring 6 units by 9 units and a rectangle measuring 9 units by 13.5 units.

Problem 2

Statement

One rectangle measures 2 units by 7 units. A second rectangle measures 11 units by 37 units. Are these two figures scaled versions of each other? If so, find the scale factor. If not, briefly explain why.

Solution

No, these two figures are not scaled versions of each other. The 2 unit side is scaled by a factor of 5.5 to correspond to the 11 unit side, but 7 multiplied by 5.5 is 38.5, not 37.

Problem 3

Statement

Ants have 6 legs. Elena and Andre write equations showing the proportional relationship between the number of ants, $a$, to the number of ant legs $l$. Elena writes $a = 6 \cdot l$ and Andre writes $l = \frac{1}{6} \cdot a$. Do you agree with either of the equations? Explain your reasoning.

Solution

Neither of them are correct. Although 6 and $\frac{1}{6}$ are the correct constants of proportionality, they are being multiplied by the wrong variables. For example, using Elena’s equation, 1 leg is equal to 6 ants.

(From Unit 2, Lesson 5.)

Problem 4

Statement

On the grid, draw a scaled copy of quadrilateral ABCD with a scale factor $\frac{2}{3}$.
Solution

Answers vary. Sample response on the right.

(From Unit 1, Lesson 4.)

Problem 5

Statement

Solve each equation mentally.

a. \( \frac{5}{2} \cdot x = 1 \)

b. \( x \cdot \frac{7}{3} = 1 \)

c. \( 1 \div \frac{11}{2} = x \)

Solution

a. \( x = \frac{2}{5} \)

b. \( x = \frac{3}{7} \)

c. \( x = \frac{2}{11} \)

(From Unit 1, Lesson 5.)
Problem 6

Statement
Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.

a. The height of the model train is 102 millimeters. What is the actual height of the train in meters? Explain your reasoning.

b. On the scale model, the distance between the wheels on the left and the wheels on the right is $1 \frac{1}{4}$ inches. The state of Wyoming has old railroad tracks that are 4.5 feet apart. Can the modern train travel on those tracks? Explain your reasoning.

Solution
a. 4.896 meters. Sample reasoning:
- The actual height is 48 times the scaled height. $102 \cdot 48 = 4,896$. 4,896 mm is 4.896 m.
- 102 mm is 0.102 m. The actual train is 48 times 0.102 m. $0.102 \cdot 48 = 4.896$.

b. No. Sample explanation: The modern train needs tracks that are 60 inches apart, because $1 \frac{1}{4} \cdot 48 = 60$. The old tracks are only 54 inches, so they are not wide enough.

(From Unit 1, Lesson 11.)
Lesson 2: Ratios and Rates With Fractions

Goals

- Compare and contrast (orally and in writing) different strategies for solving a problem involving equivalent ratios with fractional quantities.
- Explain (orally and in writing) how to find and use a unit rate to solve a problem involving fractional quantities.

Learning Targets

- I can solve problems about ratios of fractions and decimals.

Lesson Narrative

Before this unit, students worked with ratios of whole numbers and with whole number percentages. Now they start to work with ratios of fractions and fractional percentages. In this lesson they encounter situations where a ratio of fractions arises naturally. They compute scale factors and unit rates associated with ratios of fractions. They consider a situation involving a ratio where the second number is 100, in order to prepare for thinking about a percentage as a particular type of rate, and they compare rates associated with different ratios. The representations they use—tape diagrams and double number lines—are the same as they have used previously, but in the context of more complicated ratios.

The Mona Lisa task has more than one reasonable answer, and students must make sense of the situation in order to choose one (MP1).

Alignments

Building On

- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because \(3/4\) of \(8/9\) is \(2/3\). (In general, \((a/b) \div (c/d) = ad/bc\).) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
Addressing

- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Preparation

For the activity Scaling the Mona Lisa, consider showing a picture of the Mona Lisa painting.

Student Learning Goals

Let's calculate some rates with fractions.

2.1 Number Talk: Division

Warm Up: 5 minutes
The purpose of this number talk is to elicit strategies and understandings students have for dividing a fraction by a fraction. Later in this lesson, students will need to be able to divide a fraction by a fraction to solve problems in contexts.

Four problems are given. It may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Building On

- 6.NS.A.1
Building Towards

- 7.RP.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

Anticipated Misconceptions

Students may get stuck trying to remember a procedure to divide fractions or may also think the answer to each problem is just the whole number divided by 3. Help students reason about dividing a whole number by a unit fraction by asking what number if divided into thirds gives us an answer of ____ (5, 2, 1/2, 2 1/2)?

If students get stuck on the last problem, help them see that previous problems can be used to figure out an answer to this last one. Since $2\frac{1}{2}$ is half of 5 the answer is going to be half of the answer to $5 \div \frac{1}{3}$. They may also apply the distributive property to use the answer to $2 \div \frac{1}{3}$ and $\frac{1}{2} \div \frac{1}{3}$ to figure out the answer.

Student Task Statement

Find each quotient mentally.

- $5 \div \frac{1}{3}$
- $2 \div \frac{1}{3}$
- $\frac{1}{2} \div \frac{1}{3}$
- $2 \frac{1}{2} \div \frac{1}{3}$
Student Response

- 15
- 6
- $1 \frac{1}{2}$
- $7 \frac{1}{2}$

(Equivalent answers are also acceptable.)

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

2.2 A Train is Traveling at . . .

10 minutes

The purpose of this activity is to review different strategies for working with ratios and to prepare students to use these strategies with ratios involving fractions. The activity also foreshadows percentages by asking about the distance traveled in 100 minutes.

Monitor for different strategies like these:

- divide $\frac{15}{2} \div 6$ to find the distance traveled in 1 minute, and then multiply it by 100.
- draw a double number line.
• create a table of equivalent ratios.

Depending on their prior learning, students might lean towards the first strategy.

Building On
• 6.RP.A.3

Building Towards
• 7.RP.A

Instructional Routines
• Anticipate, Monitor, Select, Sequence, Connect
• MLR7: Compare and Connect

Launch
Give students 3 minutes of quiet work time. Encourage them to find more than one strategy if they have time. Follow with whole-class discussion around the various strategies they used.

Anticipated Misconceptions
Students might calculate the unit rate as $6 \div \frac{15}{2}$. Ask students what this number would mean in this problem? (This number means that it takes $\frac{4}{5}$ of a minute to travel 1 kilometer.) In this case, students should be encouraged to create a table or a double number line, since it will help them make sense of the meaning of the numbers.

Student Task Statement
A train is traveling at a constant speed and goes 7.5 kilometers in 6 minutes. At that rate:

1. How far does the train go in 1 minute?
2. How far does the train go in 100 minutes?

Student Response
1. $\frac{5}{4}$ kilometers or equivalent. Possible strategies:
   ◦ Double Number Line:
2. The train goes 125 kilometers (or equivalent) in 100 minutes. Possible strategies:

- Table:

<table>
<thead>
<tr>
<th>distance (km)</th>
<th>time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>6</td>
</tr>
<tr>
<td>$\frac{5}{4}$</td>
<td>1</td>
</tr>
</tbody>
</table>

- Unit Rate: $\frac{\frac{15}{2}}{6} = \frac{5}{4}$ or $\frac{\frac{15}{2}}{6} = 1 \frac{1}{4}$

**Activity Synthesis**

Select students to share the strategies they used. To the extent possible, there should be one student per strategy listed. If no students come up with one or more representations, create them so that students can compare and contrast.

- Divide ($\frac{15}{2} \div 6$) to find the number of kilometers traveled in 1 minute, and multiply by 100
- Double Number Line
- Table

Display strategies for all to see throughout the discussion.

Help students connect the strategies by asking:
Was there a place in your solution where you calculated $\frac{15}{2} \div 6$?

How can we see this value being used in the double number line? Table?

Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: “First, I _____ because...”, “I noticed _____ so I...”, “Why did you...?”, “I agree/disagree because....”

Supports accessibility for: Language; Social-emotional skills

Access for English Language Learners

*Speaking: MLR7 Compare and Connect.* Use this routine when students present the strategies they used to determine the distance traveled. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the different ways the unit rate and total distance can be seen in each representation (i.e., tape diagram, double number line and table). Listen for and amplify students' correct use of the term “unit rate.” These exchanges can strengthen students' mathematical language use as they reason to make sense of strategies used to calculate unit rates and distance traveled.

*Design Principle(s): Maximize meta-awareness*

2.3 Comparing Running Speeds

10 minutes

The purpose of this activity is to provide another context that leads students to calculate a unit rate from a ratio of fractions. This work is based on students' work in grade 6 on dividing fractions.

Students notice and wonder about two statements and use what they wonder to create questions that are collected for all to see. Each student picks a question secretly and calculates the answer, then shares the answer with their partner. The partner tries to guess the question. Most of the time in this activity should be spent on students engaging in partner discussion.

**Addressing**
- 7.RP.A.1

**Instructional Routines**
- MLR8: Discussion Supports
Launch

Arrange students into groups of 2. Display the two statements for all to see. Ask students to write down what they notice and wonder, and then use what they wonder to come up with questions that can be answered using the given information. Create a list of questions and display for all to see. Here are suggested questions to listen for:

- Who ran faster, Noah or Lin?
- How far would Lin run in 1 hour?
- How far did Noah run in 1 hour?
- How long would it take Lin to run 1 mile at that rate?
- How long would it take Noah to run 1 mile at that rate?

Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for: Organization; Attention*

Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Provide sentence frames to students to support their explanations of their process for finding the answer to their selected question. For example, “First, I ____. Then, I ____.”

*Design Principle(s): Support sense-making, Optimize output (for explanation)*

Anticipated Misconceptions

The warm-up was intended to remind students of some strategies for dividing fractions by fractions, but students may need additional support working with the numbers in this task.

Students might have a hard time guessing their partner’s question given only the answer. Ask their partners to share the process they used to calculate the solution, they might leave out numbers and describe in general the steps they took to find the answer first. If their partner is still unable to guess the question, have them share the specific number they used. If they need additional support to guess the question, have their partner show them their work on paper (without sharing the question they answered) and see if this helps them figure out the question.

*Unit 4 Lesson 2*
Student Task Statement

Lin ran 2 \(\frac{3}{4}\) miles in \(\frac{2}{5}\) of an hour. Noah ran 8 \(\frac{2}{3}\) miles in \(\frac{4}{5}\) of an hour.

1. Pick one of the questions that was displayed, but don’t tell anyone which question you picked. Find the answer to the question.

2. When you and your partner are both done, share the answer you got (do not share the question) and ask your partner to guess which question you answered. If your partner can’t guess, explain the process you used to answer the question.

3. Switch with your partner and take a turn guessing the question that your partner answered.

Student Response

Answers vary. Sample responses: Lin ran faster than Noah, because \(2 \frac{3}{4} \div \frac{2}{5} = 6 \frac{7}{8}\) and \(8 \frac{2}{3} \div \frac{4}{3} = 6 \frac{1}{2}\).

Lin ran 6 \(\frac{7}{8}\) miles in one hour since \(2 \frac{3}{4} \div \frac{2}{5} = 6 \frac{7}{8}\). Noah ran 6 \(\frac{1}{2}\) miles in one hour since \(8 \frac{2}{3} \div \frac{4}{3} = 6 \frac{1}{2}\).

It took Lin \(\frac{8}{55}\) of an hour to run a mile since \(\frac{2}{5} \div 2 \frac{3}{4} = \frac{8}{55}\). That is \(8 \frac{8}{11}\) minutes since \(\frac{8}{55} \cdot 60 = 8 \frac{8}{11}\). It took Noah \(\frac{2}{13}\) of an hour to run a mile since \(\frac{4}{3} \div 8 \frac{2}{3} = \frac{2}{13}\). That is \(9 \frac{3}{13}\) minutes since \(\frac{2}{13} \cdot 60 = 9 \frac{3}{13}\).

Are You Ready for More?

Nothing can go faster than the speed of light, which is 299,792,458 meters per second. Which of these are possible?

1. Traveling a billion meters in 5 seconds.

2. Traveling a meter in 2.5 nanoseconds. (A nanosecond is a billionth of a second.)

3. Traveling a parsec in a year. (A parsec is about 3.26 light years and a light year is the distance light can travel in a year.)

Student Response

1. Yes (200,000,000 meters per second)

2. No (400,000,000 meters per second)

3. No, because traveling 1 parsec in 1 year means traveling 3.26 times faster than the speed of light
Activity Synthesis
After both partners have a chance to guess each other’s question, ask a few different students to share their strategies for guessing which question their partner answered.

2.4 Scaling the Mona Lisa

Optional: 10 minutes (there is a digital version of this activity)
The purpose of this activity is to provide a context where a ratio of fractions arises naturally, and students need to find an equivalent ratio to solve the problem. The ratio $2\frac{1}{3} : 1\frac{3}{4}$ is equivalent to $10 : 7$, so a scaled copy of the Mona Lisa that is 10 inches by 7 inches would fit on the cover of the notebook. Other answers are possible. Some students might try to find the biggest possible copy that will fit on the cover, which would result in a different scale factor.

The digital version of the student materials includes an applet so that students can experiment with the context, because there are many related measurements within the context that can be hard to visualize. For example, the applet makes it clear that you can't simply scale down the Mona Lisa and make it perfectly fit on the notebook, since the notebook and the Mona Lisa are not scaled copies of each other. It also serves to remind students that the length and width of the Mona Lisa have to be scaled by the same factor, or the image becomes distorted.

Students discuss how they found their solution with a partner and determine if the scale factors they came up with are reasonable. Students must think about whether it makes sense to try to scale the picture so that it fills as much of the page as possible, or whether it makes more sense to leave room for a title. As students discuss with their partner, identify pairs of students who have a good argument that a certain scale factor makes more sense to use than another. Ask these students to share during the whole class discussion.

Building On
- 7.G.A.1

Addressing
- 7.RP.A.1

Instructional Routines
- MLR2: Collect and Display
- Think Pair Share

Launch
If desired, show students an image of the Mona Lisa.

If using the print version, and appropriate technology is available, consider displaying the applet for everyone to see to help students better understand the situation https://ggbm.at/j8B9vZKV.
The purpose of the applet is for experimenting and understanding the situation. If using it, demonstrate how it works, and ask students to think about:

- How to use the applet to create scale copies of the Mona Lisa. (Both dimensions have to be adjusted by the same factor.)
- Is it possible to scale down the Mona Lisa so that it perfectly covers the notebook? (No, choices have to be made about what the final product will look like.)

Arrange students in groups of 2. Give 3–5 minutes of quiet work time to do the problem. Then, ask them to take turns sharing with their partner the method used to calculate scale factor and reasonableness of their answers.

If using the digital activity, still have students work in groups of 2 and have them work individually on the problem with the applet, before sharing their method(s) to calculate scale factor with their partner.

**Access for English Language Learners**

*Conversing, Representing, Writing: MLR2 Collect and Display.* As students discuss the scale factor they used, listen for and collect mathematical language students use to describe the strategies they used to find scale factors. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language during paired and group discussions.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

**Anticipated Misconceptions**

Students might get stuck thinking the scaled copy needs to measure 11 inches by 9 inches. Ask students:

- Does the copy of the painting have to cover the entire notebook?
- What are some other options if the image doesn't cover the entire notebook?
- What if the image is bigger than the notebook cover? What if it is smaller?

**Student Task Statement**

In real life, the Mona Lisa measures $2 \frac{1}{2}$ feet by $1 \frac{3}{4}$ feet. A company that makes office supplies wants to print a scaled copy of the Mona Lisa on the cover of a notebook that measures 11 inches by 9 inches.

1. What size should they use for the scaled copy of the Mona Lisa on the notebook cover?
2. What is the scale factor from the real painting to its copy on the notebook cover?
3. Discuss your thinking with your partner. Did you use the same scale factor? If not, is one more reasonable than the other?

**Student Response**

Answers vary. Sample responses:

- Converting to inches by multiplying the number of feet by 12, the dimensions $1 \frac{3}{4}$ feet by $2 \frac{1}{2}$ feet are equivalent to 21 inches by 30 inches. Multiplying both of these by $\frac{1}{3}$, the dimensions of the scaled copy would be 7 inches by 10 inches. This size would fit on the notebook with some space around the edges. (The scale factor is $\frac{1}{3}$.)

- The ratio of $1 \frac{3}{4}$ and $2 \frac{1}{2}$ is equivalent to 0.7 to 1, which is found by dividing $1 \frac{3}{4}$ by $2 \frac{1}{2}$. If we multiply both of these by 11, the dimensions could be 7.7 inches by 11 inches. (The scale factor needs to turn 30 inches into 11 inches, so the scale factor is $\frac{11}{30}$.)

**Activity Synthesis**

Select previously identified students to share the arguments they had with their partners. Some guiding questions:

- What scale factor did you and your partner agree upon? How did you both agree upon this?

- Were there any similarities between the methods you and your partner used? Were there any differences?

**Lesson Synthesis**

In this lesson, we worked with ratios of fractions.

- “What are strategies we can use to find solutions to ratio problems that involve fractions?”
  (double number line, tables, calculating unit rate)

- “How are those strategies different from and similar to ways we previously solved ratio problems that didn't involve fractions?” (They are structurally the same, but the arithmetic might take more time.)

**2.5 Comparing Orange Juice Recipes**

Cool Down: 5 minutes

**Addressing**

- 7.RP.A.1

**Student Task Statement**

- Clare mixes $2\frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of orange juice concentrate.

- Han mixes $1\frac{2}{3}$ cups of water with $\frac{1}{4}$ cup of orange juice concentrate.
Whose orange juice mixture tastes stronger? Explain or show your reasoning.

**Student Response**

Han's mixture tastes stronger. Clare uses \(7 \frac{1}{2}\) cups of water per cup of orange juice concentrate, because \(2 \frac{1}{2} ÷ \frac{1}{3} = 7 \frac{1}{2}\). Han uses \(6 \frac{2}{3}\) cups of water per cup of orange juice concentrate, because \(1 \frac{2}{3} ÷ \frac{1}{4} = 6 \frac{2}{3}\). Han's mixture has less water for the same amount of orange juice concentrate.

**Student Lesson Summary**

There are 12 inches in a foot, so we can say that for every 1 foot, there are 12 inches, or the ratio of feet to inches is \(1 : 12\). We can find the **unit rates** by dividing the numbers in the ratio:

\[
1 ÷ 12 = \frac{1}{12},
\]

so there is \(\frac{1}{12}\) foot per inch. \[
12 ÷ 1 = 12
\]

so there are 12 inches per foot.

The numbers in a ratio can be fractions, and we calculate the unit rates the same way: by dividing the numbers in the ratio. For example, if someone runs \(\frac{3}{4}\) mile in \(\frac{11}{2}\) minutes, the ratio of minutes to miles is \(\frac{11}{2} : \frac{3}{4}\).

\[
\frac{11}{2} ÷ \frac{3}{4} = \frac{22}{3},
\]

so the person's pace is \(\frac{22}{3}\) minutes per mile. \[
\frac{3}{4} ÷ \frac{11}{2} = \frac{3}{22},
\]

so the person's speed is \(\frac{3}{22}\) miles per minute.

**Glossary**

- unit rate
Lesson 2 Practice Problems

Problem 1

Statement
A cyclist rode 3.75 miles in 0.3 hours.

a. How fast was she going in miles per hour?

b. At that rate, how long will it take her to go 4.5 miles?

Solution
a. 12.5 miles per hour

b. 0.36 hours or 21.6 minutes

Problem 2

Statement
A recipe for sparkling grape juice calls for \( 1 \frac{1}{2} \) quarts of sparkling water and \( \frac{3}{4} \) quart of grape juice.

a. How much sparkling water would you need to mix with 9 quarts of grape juice?

b. How much grape juice would you need to mix with \( \frac{15}{4} \) quarts of sparkling water?

c. How much of each ingredient would you need to make 100 quarts of punch?

Solution
Notice that the ratio \( 1 \frac{1}{2} \) quarts of sparkling water to \( \frac{3}{4} \) quart of grape juice is equivalent to the ratio 2 quarts of sparkling water to 1 quart of grape juice. While not needed, this ratio with whole numbers can help answer all three questions.

a. 18 quarts

b. \( \frac{15}{8} \) quarts or equivalent

c. \( \frac{200}{3} \) quarts of sparkling water and \( \frac{100}{3} \) quarts of grape juice (or equivalent).

Problem 3

Statement
At a deli counter,

- Someone bought \( 1 \frac{3}{4} \) pounds of ham for $14.50.
Someone bought $\frac{3}{2}$ pounds of turkey for $26.25.

Someone bought $\frac{3}{8}$ pounds of roast beef for $5.50.

Which meat is the least expensive per pound? Which meat is the most expensive per pound? Explain how you know.

**Solution**

Ham is the least expensive. It costs about $8.29 per pound, because $14.50 \div 1 \frac{3}{4} = 8 \frac{2}{7} \approx 8.29$. Roast beef is the most expensive. It costs about $14.67 per pound, because $5.50 \div \frac{3}{8} = 14 \frac{2}{3} \approx 14.67$.

Turkey costs about $10.50 per pound, because $26.25 \div 2 \frac{1}{2} = 10.50$. While these prices per pound are not exact, they are far enough apart to put the costs in order with certainty.

**Problem 4**

**Statement**

a. Draw a scaled copy of the circle using a scale factor of 2.

b. How does the circumference of the scaled copy compare to the circumference of the original circle?

c. How does the area of the scaled copy compare to the area of the original circle?

**Solution**

a. The outer circle is a scaled copy of the inner circle using scale factor 2.
b. The circumference of the scaled copy is twice the circumference of the original.

c. The area of the scaled copy is four times the area of the original.

(From Unit 3, Lesson 10.)

Problem 5

Statement

Jada has a scale map of Kansas that fits on a page in her book. The page is 5 inches by 8 inches. Kansas is about 210 miles by 410 miles. Select all scales that could be a scale of the map. (There are 2.54 centimeters in an inch.)

A. 1 in to 1 mi
B. 1 cm to 1 km
C. 1 in to 10 mi
D. 1 ft to 100 mi
E. 1 cm to 200 km
F. 1 in to 100 mi
G. 1 cm to 1000 km

Solution

["E", "F"]

(From Unit 1, Lesson 11.)
Lesson 3: Revisiting Proportional Relationships

Goals

• Calculate and interpret (orally) the constant of proportionality for a proportional relationship involving fractional quantities.

• Explain (orally and in writing) how to use a table with only two rows to solve a problem involving a proportional relationship.

• Write an equation to represent a given proportional relationship with a fractional constant of proportionality.

Learning Targets

• I can use a table with 2 rows and 2 columns to find an unknown value in a proportional relationship.

• When there is a constant rate, I can identify the two quantities that are in a proportional relationship.

Lesson Narrative

In grade 6 students solved ratio problems by reasoning about scale factors or unit rates. In grade 7 they see the two quantities in a set of equivalent ratios as being in a proportional relationship and move towards using the constant of proportionality to find missing numbers. This is useful in the sorts of tasks they are studying in this unit because the tasks involve repeatedly applying the same number (for example, a unit price) to different amounts. The unit price is a constant of proportionality between the amount purchased and the amount paid. When students describe the proportional relationship behind the repeated operation of finding the amount paid, they are engaging in MP8.

In this lesson students move toward solving problems involving proportional relationships by more efficient methods, especially by setting up and reasoning about a two-row table of equivalent ratios. This method encourages them to use the constant of proportionality rather than equivalent ratios.

Alignments

Building On

• 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

• 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $\frac{1}{2}/\frac{1}{4}$ miles per hour, equivalently 2 miles per hour.
• 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards
• 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.

• 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines
• Anticipate, Monitor, Select, Sequence, Connect
• MLR1: Stronger and Clearer Each Time
• MLR2: Collect and Display
• MLR3: Clarify, Critique, Correct
• Think Pair Share

Student Learning Goals
Let's use constants of proportionality to solve more problems.

3.1 Recipe Ratios

Warm Up: 5 minutes
The purpose of this warm-up is to bring up two main methods for figuring out missing numbers in a table that represents a proportional relationship. The two methods students might use for this activity are:

• Using a scale factor to find equivalent ratios, e.g. multiplying the first row by \( 1 \frac{1}{2} \) to get the second row.
• Using the constant of proportionality, 2, between the first column and the second column.

This activity is to get students thinking about the second method as a more efficient method, since it works for every row. This lays the groundwork for solving problems using proportional relationships and for the activities in this lesson (specifically, the activity right after the warm up).

Building On
• 6.RP.A.3

Building Towards
• 7.RP.A.1
• 7.RP.A.2

Unit 4 Lesson 3
**Instructional Routines**

- Think Pair Share

**Launch**

In a previous unit, students worked extensively with sets of equivalent ratios that represented ingredients for different numbers of batches of a recipe. If necessary, remind them how this works. For example, you might say "This recipe calls for $\frac{1}{2}$ cup of sugar and 1 cup of flour. What if I wanted to make half a batch of the recipe? What if I wanted to make 5 batches?"

Arrange students in groups of 2. Give 1 minute of quiet work time followed by time to compare their table with a partner and a whole-class discussion.

**Anticipated Misconceptions**

Some students may assume the sugar column will continue to increase by the same amount without paying close attention to the values in the flour column. Ask these students what they notice about the values in the flour column and if it makes sense for the sugar amount to increase by the same amount each time.

**Student Task Statement**

A recipe calls for $\frac{1}{2}$ cup sugar and 1 cup flour. Complete the table to show how much sugar and flour to use in different numbers of batches of the recipe.

<table>
<thead>
<tr>
<th>sugar (cups)</th>
<th>flour (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>$1\frac{3}{4}$</td>
</tr>
<tr>
<td>1</td>
<td>$2\frac{1}{2}$</td>
</tr>
</tbody>
</table>
### Student Response

<table>
<thead>
<tr>
<th>sugar (cups)</th>
<th>flour (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{2})</td>
<td>1</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td>(1 \frac{1}{2}), because the recipe uses twice as much flour as sugar, and (2 \cdot \frac{3}{4} = 1 \frac{1}{2}).</td>
</tr>
<tr>
<td>(\frac{7}{8}), because the recipe uses half as much sugar as flour, and (\frac{1}{2} \cdot 1 \frac{3}{4} = \frac{1}{2} \cdot \frac{7}{4} = \frac{7}{8}).</td>
<td>(1 \frac{3}{4})</td>
</tr>
<tr>
<td>1</td>
<td>2, because the recipe uses twice as much flour as sugar, and (2 \cdot 1 = 2).</td>
</tr>
<tr>
<td>(1 \frac{1}{4}), because the recipe uses half as much sugar as flour, and (\frac{1}{2} \cdot 2 \frac{1}{2} = 1 \frac{1}{4}).</td>
<td>(2 \frac{1}{2})</td>
</tr>
</tbody>
</table>

### Activity Synthesis

Display the table for all to see and ask students to share the answers they calculated for each missing entry. After the table is complete, ask the students if they agree or disagree with the values in the table. Select students to share who used the scale factor or constant of proportionality methods to find the equivalent ratios to share their reasoning. Record their ideas directly on the table if possible and display for all to see.

### 3.2 The Price of Rope

**15 minutes**

The purpose of this activity is to ensure students understand how an abbreviated table can be used to solve a problem (the most abbreviated table consists of only two rows). Students may have done some work like this in grade 6, in which case this activity serves to reinforce and remind. For teachers accustomed to procedures for “setting up a proportion,” this approach is very similar, except that students have the column headings to help make sure they get the numbers in the right places. Also, they should get a better idea for why they are multiplying and dividing, because they are finding and using either a scale factor or the constant of proportionality. (Note that using the constant of proportionality is easier, and also the natural way you would think about calculating the price of any amount of something.)
**Launch**

Students should be comfortable with Kiran's method from their work with tables of equivalent ratios in grade 6. However, if needed, show them this even longer solution method first and let them examine it. Ask why Lin decided to multiply by $\frac{1}{3}$. Once students are comfortable with the reasoning shown, explain that you will be looking at more efficient ways of solving this problem with a table.

Lin's method:

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time and then time to discuss their solutions with their partner. Follow with a whole-class discussion around the method they think Priya used.
Access for English Language Learners

Speaking: MLR2 Collect and Display. While pairs are working, listen for and collect vocabulary and phrases students use to explain the similarities and differences between the scale factor method and the constant of proportionality method. On a display, organize the responses into two columns, one for each method. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language during paired and group discussions.

Design Principle(s): Optimize output (for explanation); Support sense-making

Anticipated Misconceptions

Some students may struggle to progress with Priya’s method because the arrows are not drawn in the image and none of the values given are easily divisible. There are many supporting questions that could be asked.

- What if we knew the price of 1 foot of rope?
- If 6 times something is 7.5, how can we find the something?

Student Task Statement

Two students are solving the same problem: At a hardware store, they can cut a length of rope off of a big roll, so you can buy any length you like. The cost for 6 feet of rope is $7.50. How much would you pay for 50 feet of rope, at this rate?

1. Kiran knows he can solve the problem this way.

What would be Kiran’s answer?

2. Kiran wants to know if there is a more efficient way of solving the problem. Priya says she can solve the problem with only 2 rows in the table.

<table>
<thead>
<tr>
<th>length of rope (feet)</th>
<th>price of rope (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7.50</td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Unit 4 Lesson 3
What do you think Priya's method is?

**Student Response**

1. $62.50, because $(1.25) \cdot 50 = 62.5$

2. Answers vary. Sample responses:
   - (preferred) 1 foot of rope costs $1.25 because $7.5 \div 6 = 1.25$. (Or, $6 \cdot 1.25 = 7.5$.) So multiply 50 by 1.25 to find the cost of 50 feet of rope. $50 \cdot (1.25) = 62.5$.
   - Since $50 \div 6 = 8 \frac{1}{3}$, that means $6 \cdot 8 \frac{1}{3} = 50$. So multiply 7.5 by $8 \frac{1}{3}$ to get 62.5.

**Activity Synthesis**

Ask selected students to show the way they solved the problem. If no students come up with one of these methods, display it for all to see.

Scale factor method:

<table>
<thead>
<tr>
<th>length of rope (feet)</th>
<th>price of rope (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>7.50</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Constant of proportionality method:

<table>
<thead>
<tr>
<th>length of rope (feet)</th>
<th>price of rope (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7.50</td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Ask students:

- “How did you find the scale factor?”
- “How did you find the constant of proportionality?”
- “What does the constant of proportionality (1.25) mean in this context?”
- “Which method is your preference to use? Why?”

Although either method will work, there are reasons to prefer using the constant of proportionality to approach problems like these. First, the constant of proportionality 1.25 means something important in the problem—it’s the price of 1 foot of rope. Because of that, the 1.25 could be easily
used to compute the price of any length of rope. If no students bring it up, point out that the equation \( y = 1.25x \) could be used to relate any length of rope, \( x \), to its price, \( y \).

### 3.3 Swimming, Manufacturing, and Painting

10 minutes

In this activity students use proportional relationships to solve problems. Although students might remember a few of the problems from previous lessons, students are asked to answer different questions. Students first need to recognize that the situation involves a proportional relationship and then use that knowledge to solve the problems. Notice that the problems decrease in the amount of scaffolding in order to build confidence for students as they work through them. Look out for students who have a systematic way of approaching these problems, and ask them to share their strategy during the discussion.

#### Addressing

- 7.RP.A.2

#### Instructional Routines

- MLR3: Clarify, Critique, Correct

#### Launch

Give students 3–5 minutes of quiet work time, followed by whole-class discussion.

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**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for: Organization; Attention*

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**Anticipated Misconceptions**

Some students may struggle to continue working as the scaffolding is decreased. Consider using these questions to prompt students:

- What are the two associated quantities in this problem?
- How many quarts of blue paint are needed for 1 quart of white paint?
Student Task Statement

1. Tyler swims at a constant speed, 5 meters every 4 seconds. How long does it take him to swim 114 meters?

<table>
<thead>
<tr>
<th>distance (meters)</th>
<th>time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>114</td>
<td>91.2</td>
</tr>
</tbody>
</table>

2. A factory produces 3 bottles of sparkling water for every 8 bottles of plain water. How many bottles of sparkling water does the company produce when it produces 600 bottles of plain water?

<table>
<thead>
<tr>
<th>number of bottles of sparkling water</th>
<th>number of bottles of plain water</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>600</td>
</tr>
</tbody>
</table>

3. A certain shade of light blue paint is made by mixing $1\frac{1}{2}$ quarts of blue paint with 5 quarts of white paint. How much white paint would you need to mix with 4 quarts of blue paint?

4. For each of the previous three situations, write an equation to represent the proportional relationship.

Student Response

1. 91.2 seconds or equivalent. Sample reasoning: Tyler swims 1 meters in 0.8 seconds because $4 \div 5 = 0.8$. It takes him 91.2 seconds to swim 114 meters, because $114 \cdot 0.8 = 91.2$

<table>
<thead>
<tr>
<th>distance (meters)</th>
<th>time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>114</td>
<td>91.2</td>
</tr>
</tbody>
</table>

2. 225. Sample reasoning: The factory produces 0.375 of a bottle of sparkling water per bottle of plain water because $3 \div 8 = 0.375$. The factory produces 225 bottles of sparkling water when it produces 600 bottles of plain water, because $600 \cdot 0.375 = 225$. 
3. $13\frac{1}{3}$ or equivalent. Sample reasoning: There are $3\frac{1}{3}$ quarts of white paint per quart of blue paint, because $5 \div 1\frac{1}{2} = 3\frac{1}{3}$. So you would need to mix $13\frac{1}{3}$ quarts of white paint with 4 quarts of blue paint, because $4 \cdot 3\frac{1}{3} = 4 \cdot \frac{10}{3} = 13\frac{1}{3}$.

<table>
<thead>
<tr>
<th>number of bottles of sparkling water</th>
<th>number of bottles of plain water</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>225</td>
<td>600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>blue paint (quarts)</th>
<th>white paint (quarts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\frac{1}{2}$</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>$13\frac{1}{3}$</td>
</tr>
</tbody>
</table>

4. Answers vary. Sample response:
   - $t = \frac{4}{5}d$
   - $p = \frac{8}{3}s$
   - $w = \frac{10}{3}b$

**Are You Ready for More?**

Different nerve signals travel at different speeds.

- Pressure and touch signals travel about 250 feet per second.
- Dull pain signals travel about 2 feet per second.

1. How long does it take you to feel an ant crawling on your foot?

2. How much longer does it take to feel a dull ache in your foot?

**Student Response**

Answers vary. Sample response: The distance between your foot and your brain depends on how tall you are. If you are 5.5 feet tall, then:

1. It takes about $5.5 \div 250 = 0.022$ seconds for the signal to reach your brain.

2. It takes about $5.5 \div 2 = 2.75$ seconds for the pain signal to reach your brain.
**Activity Synthesis**

Students may have used the same method for each problem. For this reason, it might not be necessary to go over every problem. For the second question, you might select 2 different students who used the different methods to share their strategy. For the third question, ask students to share how they knew which quantities to put into the table. You might also ask:

- Does it matter which heading goes in which column?
- Do you get a different answer if you switch them? Why or why not?
- Would your strategy change if you switched them? Why or why not?

Ask students where they got stuck and what helped them to move through the hard parts. (Making a table; using the given information to figure out new information.)

---

**Access for English Language Learners**

*Writing: MLR3 Clarify, Critique, Correct.* Present an incorrect response to the question about mixing 4 quarts of blue paint. For example, “Since there are 0.3 quarts of blue paint to white paint, you need 1.2 quarts of white paint.” Prompt students to identify the error (e.g., ask students, “Do you agree with the statement? Why or why not?”), and then write a correct version. This helps students evaluate, and improve on, the written mathematical arguments of others.

*Design Principle(s): Maximize meta-awareness*

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**3.4 Finishing the Race and More Orange Juice**

Optional: 10 minutes

The purpose of this activity is to give students an opportunity to solve proportional relationships with fractions without the scaffolded support as given in the previous activity (MP1). If students get stuck, here are some questions to ask:

- How many miles do they run in one hour?
- How many cups of orange juice concentrate are needed for one cup of water?

Monitor for students who are using the constant of proportionality strategy they learned in an earlier activity. These students should be asked to share during the discussion.

**Addressing**

- 7.RP.A.1

**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
Launch
Give students 3–5 minutes of quiet work time followed by whole-class discussion.
If time is limited, pick one of the problems to talk about during the discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using a table of equivalent ratios. If students are unsure where to begin, suggest that they draw a table of equivalent ratios to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. To help students refine their explanations for the second question, listeners should press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language (e.g., “How did you use the constant of proportionality?” “What steps did you take to find the amount of orange juice they each need?” “How do you know your answer is correct?” etc.). Students can borrow ideas and language from each partner to strengthen their final product.
Design Principle(s): Optimize output (for explanation)

Student Task Statement

1. Lin runs $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Tyler runs $8\frac{2}{3}$ miles in $\frac{4}{3}$ of an hour. How long does it take each of them to run 10 miles at that rate?

2. Priya mixes $2\frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of orange juice concentrate. Diego mixes $1\frac{2}{3}$ cups of water with $\frac{1}{4}$ cup orange juice concentrate. How much concentrate should each of them mix with 100 cups of water to make juice that tastes the same as their original recipe? Explain or show your reasoning.

Student Response

1. Lin takes $1\frac{5}{11}$ hours. Tyler takes $1\frac{7}{13}$ hours. Lin takes $\frac{8}{55}$ of an hour to run each mile because $\frac{2}{5} \div 2\frac{3}{4} = \frac{8}{55}$. Lin takes $1\frac{5}{11}$ of an hour to run 10 miles because $10 \cdot \frac{8}{55} = 1\frac{5}{11}$. Tyler takes $\frac{2}{13}$ of an hour to run each mile because $\frac{4}{3} \div 8\frac{2}{3} = \frac{2}{13}$. Tyler takes $1\frac{7}{13}$ of an hour to run 10 miles because $10 \cdot \frac{2}{13} = 1\frac{7}{13}$.
2. Priya should use $13\frac{1}{3}$ cups of concentrate. Sample explanation: Priya needs to make 40 batches of her recipe, because $2\frac{1}{2} \times 40 = 100$. This means she needs to use $40 \times \frac{1}{3}$, or $\frac{40}{3}$ cups of orange juice concentrate.

○ Diego should use 15 cups of orange juice concentrate. Sample explanation: Diego uses $\frac{3}{20}$ cups of concentrate per cup of water, because $\frac{1}{4} \div \frac{5}{3} = \frac{3}{20}$. For 100 cups of water, he would need to use $100 \times \frac{3}{20}$, or 15 cups of orange juice concentrate.

**Activity Synthesis**

The purpose of this discussion is for students to share how they solved proportion problems involving fractions. Select previously identified students to share their method for finding a solution. Ask students to describe why they chose to calculate the constant of proportionality for this problem and how that helped them with finding the solution.

**Lesson Synthesis**

In this lesson, we worked efficiently with tables.

“How can we use a table that only has two rows to solve a problem about a proportional relationship?” (Calculate either a scale factor or the constant of proportionality, use this as a multiplier.)

### 3.5 Walnuts in Bulk

Cool Down: 5 minutes

**Addressing**

- 7.RP.A.1

**Student Task Statement**

It costs $3.45 to buy $\frac{3}{4}$ lb of chopped walnuts. How much would it cost to purchase 7.5 lbs of walnuts? Explain or show your reasoning.

**Student Response**

$34.50$. Sample explanation: It costs 10 times as much to buy 7.5 lbs of walnuts as to buy $\frac{3}{4}$ lbs of walnuts since $\frac{3}{4} \times 10 = 7.5$. It costs $ because $3.45 \times 10 = 34.50$.

**Student Lesson Summary**

If we identify two quantities in a problem and one is proportional to the other, then we can calculate the constant of proportionality and use it to answer other questions about the situation. For example, Andre runs at a constant speed, 5 meters every 2 seconds. How long does it take him to run 91 meters at this rate?
In this problem there are two quantities, time (in seconds) and distance (in meters). Since Andre is running at a constant speed, time is proportional to distance. We can make a table with distance and time as column headers and fill in the given information.

<table>
<thead>
<tr>
<th>distance (meters)</th>
<th>time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>91</td>
<td></td>
</tr>
</tbody>
</table>

To find the value in the right column, we multiply the value in the left column by \(\frac{2}{5}\) because \(\frac{2}{5} \cdot 5 = 2\). This means that it takes Andre \(\frac{2}{5}\) seconds to run one meter.

At this rate, it would take Andre \(\frac{2}{5} \cdot 91 = \frac{182}{5}\), or 36.4 seconds to walk 91 meters. In general, if \(t\) is the time it takes to walk \(d\) meters at that pace, then \(t = \frac{2}{5}d\).
Lesson 3 Practice Problems

Problem 1

Statement
It takes an ant farm 3 days to consume $\frac{1}{2}$ of an apple. At that rate, in how many days will the ant farm consume 3 apples?

Solution
18 days

Problem 2

Statement
To make a shade of paint called jasper green, mix 4 quarts of green paint with $\frac{2}{3}$ cups of black paint. How much green paint should be mixed with 4 cups of black paint to make jasper green?

Solution
24 quarts

Problem 3

Statement
An airplane is flying from New York City to Los Angeles. The distance it travels in miles, $d$, is related to the time in seconds, $t$, by the equation $d = 0.15t$.

a. How fast is it flying? Be sure to include the units.

b. How far will it travel in 30 seconds?

c. How long will it take to go 12.75 miles?

Solution
a. It is traveling at 0.15 miles per second.

b. It will travel 4.5 miles in 30 seconds.

c. It will take 85 seconds to travel 12.75 miles.

Problem 4

Statement
A grocer can buy strawberries for $1.38 per pound.
a. Write an equation relating \( c \), the cost, and \( p \), the pounds of strawberries.

b. A strawberry order cost $241.50. How many pounds did the grocer order?

Solution
a. \( c = 1.38p \)

b. 175 pounds

Problem 5
Statement
Crater Lake in Oregon is shaped like a circle with a diameter of about 5.5 miles.

a. How far is it around the perimeter of Crater Lake?

b. What is the area of the surface of Crater Lake?

Solution
a. About 17 miles \((5.5\pi)\)

b. About 24 square miles \(\pi \cdot 2.75^2\)

(From Unit 3, Lesson 10.)

Problem 6
Statement
A 50-centimeter piece of wire is bent into a circle. What is the area of this circle?

Solution
\[
\frac{625}{\pi} \text{ or about } 199 \text{ cm}^2
\]

(From Unit 3, Lesson 8.)

Problem 7
Statement
Suppose Quadrilaterals A and B are both squares. Are A and B necessarily scaled copies of one another? Explain.
Solution
Yes. Since all four side lengths of a square are the same, whatever scale factor works to scale one edge of A to an edge of B takes all edges of A to all edges of B. Since scaling a square gives another square, B is a scaled copy of A.

(From Unit 1, Lesson 2.)
Lesson 4: Half as Much Again

Goals

- Apply the distributive property to generate algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value, and explain (orally) the reasoning.
- Coordinate tables, equations, tape diagrams, and verbal descriptions that represent a relationship involving adding or subtracting a fraction of the initial value.
- Generalize a process for finding the value that is “half as much again,” and justify (orally and in writing) why this can be abstracted as \( \frac{1}{2}x \) or equivalent.

Learning Targets

- I can use the distributive property to rewrite an expression like \( x + \frac{1}{2}x \) as \( (1 + \frac{1}{2})x \).
- I understand that “half as much again” and “multiply by \( \frac{3}{2} \)” mean the same thing.

Lesson Narrative

In this lesson students see how to use the distributive property to write a compact expression for situations where one quantity is described in relation to another quantity in language such as “half as much again” and “one third more than.” If \( y \) is half as much again as \( x \), then \( y = x + \frac{1}{2}x \). Using the distributive property, this can be written as \( y = (1 \frac{1}{2})x \). Students apply this sort of reasoning to various situations. A warm-up activity activates their prior knowledge of using the distributive property to write equivalent expressions. When students look for opportunities to use the distributive property to write equations in a simpler way, they are engaging in MP7.

In the next lesson they will consider similar situations involving fractions expressed as decimals. These two lessons prepare them for later study of situations involving percent increase and percent decrease.

Alignments

Building On

- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression \( 3(2 + x) \) to produce the equivalent expression \( 6 + 3x \); apply the distributive property to the expression \( 24x + 18y \) to produce the equivalent expression \( 6(4x + 3y) \); apply properties of operations to \( y + y + y \) to produce the equivalent expression \( 3y \).
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks
1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.

**Addressing**
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

**Building Towards**
- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

**Required Preparation**
Print and cut up slips from the Representations of Proportional Relationships Card Sort Instructional master. Prepare 1 copy for every 2 students. These can be re-used if you have more than one class. Consider making a few extra copies that are not cut up to serve as an answer key.

**Student Learning Goals**
Let’s use fractions to describe increases and decreases.

**4.1 Notice and Wonder: Tape Diagrams**

**Warm Up: 5 minutes**
The purpose of this warm-up is to elicit the idea that there are different ways to represent a value, which will be useful when students apply the distributive property to write expressions in a later activity. While students may notice and wonder many things about these images, the similarities and differences between the two tape diagrams are the important discussion points.

**Building On**
- 6.EE.A.3

**Building Towards**
- 7.RP.A.3
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?

Student Response

Answers vary. Samples responses:

I notice:

- In each diagram I am putting together two rectangles of different lengths.
- Each diagram has a blue rectangle and one other rectangle.
- The blue rectangles in each diagram look to be the same size.
- The yellow rectangle is the same length as the green and blue rectangles put together.
- The green rectangle looks to be three times as long as the blue rectangle.
- The yellow rectangle looks to be four times as long as the blue rectangle.
- The green rectangle looks to be three-fourths as long as the yellow rectangle.

I wonder:
• How many blue rectangles would it take to cover the green or yellow rectangle?
• What fraction does the blue rectangle represent?
• How long is the total length of the first diagram?
• What is the missing length in the second diagram?
• What situations do these diagrams represent?

Activity Synthesis
Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the total length of the first diagram does not come up during the conversation, ask students to discuss this idea.

4.2 Walking Half as Much Again
10 minutes
In this activity, students find patterns in situations to connect to the distributive property (MP7). These patterns build understanding of the equations \( x + 0.5x = (1 + 0.5)x = 1.5x \) or \( x + \frac{1}{2}x = (1 + \frac{1}{2})x = 1 \frac{1}{2}x \). Students should see that the expressions are all representations of the same thing. Students learn that multiplying a number by \( \frac{1}{2} \) and adding that product to the original numbers is the same as multiplying by 1.5. This idea is extended to percents in the following activity.

As students work on the task, monitor the reasoning they come up with for the question comparing Mai and Kiran’s representations (MP3). Identify students who agree with both Mai and Kiran; these students should be chosen to share during the discussion.

Building On
• 6.EE.A.3

Building Towards
• 7.EE.A.1
• 7.RP.A.3

Instructional Routines
• Anticipate, Monitor, Select, Sequence, Connect
• MLR8: Discussion Supports
**Launch**

Demonstrate, or ask students to demonstrate, walking a certain distance and then walking “half as much again”. Ask students how they could draw a tape diagram to represent the first situation. For example, they could draw something like this.

![Tape Diagram](image)

Give students 5 minutes of quiet work time followed by partner and then whole-class discussion.

**Anticipated Misconceptions**

Students might struggle coming up with an expression in terms of $x$. Ask students to describe how you would calculate it in words, then see if they can use that to write the expression.

When comparing Kiran and Mai’s equations, even though there is no coefficient written in front of the $x$ in Mai’s equation, $x$ is equivalent to $1x$.

**Student Task Statement**

1. Complete the table to show the total distance walked in each case.

<table>
<thead>
<tr>
<th>Initial Distance</th>
<th>Total Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x$</td>
<td>$x$</td>
</tr>
</tbody>
</table>

2. Explain how you computed the total distance in each case.
3. Two students each wrote an equation to represent the relationship between the initial distance walked \( (x) \) and the total distance walked \( (y) \).

- Mai wrote \( y = x + \frac{1}{2}x \).
- Kiran wrote \( y = \frac{3}{2}x \).

Do you agree with either of them? Explain your reasoning.

**Student Response**

<table>
<thead>
<tr>
<th>initial distance walked</th>
<th>total distance walked</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15 feet</td>
</tr>
<tr>
<td>3</td>
<td>4.5 yards</td>
</tr>
<tr>
<td>4.5</td>
<td>6.75 km</td>
</tr>
<tr>
<td>1</td>
<td>1.5 miles</td>
</tr>
<tr>
<td>( x )</td>
<td>( x + \frac{1}{2}x ) meters</td>
</tr>
</tbody>
</table>

2. Answers vary. Sample response: I took the half of the initial distance walked and added it to the initial distance walked to get the total distance walked.

3. They are both correct. Explanations vary. Sample explanations:

- The quotient of total distance walked and the corresponding initial distance walked is constant (it's 1.5).
- The total distance walked is always 1.5 times the initial distance walked.
- I agree with Kiran because \( 10 \cdot 1.5 = 15 \) and that works for every other entry in the table.

**Are You Ready for More?**

Zeno jumped 8 meters. Then he jumped half as far again (4 meters). Then he jumped half as far again (2 meters). So after 3 jumps, he was \( 8 + 4 + 2 = 14 \) meters from his starting place.

1. Zeno kept jumping half as far again. How far would he be after 4 jumps? 5 jumps? 6 jumps?

2. Before he started jumping, Zeno put a mark on the floor that was exactly 16 meters from his starting place. How close can Zeno get to the mark if he keeps jumping half as far again?

3. If you enjoyed thinking about this problem, consider researching Zeno's Paradox.
Student Response

1. 15, $15\frac{1}{2}$, $15\frac{3}{4}$

2. He can get as close as he wants, because he can always cut the distance between his current position and the mark in half.

3. No response expected.

Activity Synthesis

Ask selected students to share who they agree with (Mai or Kiran) and have them explain how they know. Highlight instances when students are reasoning with the distributive property, and encourage them to identify it by name. Suggested questions:

- “Are Mai and Kiran’s equations equivalent?”
- “What is the same and what is different about the two equations?”
- “How does each equation represent the “one and a half” relationship between the quantities?”
- “How else could you write an equation to represent the relationship?” ($y = (1 + 0.5)x$, $y = \frac{3}{2}x$, $y = 1x + 0.5x$) Students should see that the equations express the same relationship and understand why.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I _____ because...”, “I noticed ____ so I…”, “Why did you...?”, “I agree/disagree because....”

Supports accessibility for: Language; Social-emotional skills

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After selected students share their explanations for who they agree with (Mai or Kiran). Call on students to restate and/or revoice the explanations presented using mathematical language (e.g., distributive property, equivalent equation, etc.). Consider giving students time to practice restating what they heard to a partner, before selecting one or two students to share with the class. This will provide more students with an opportunity to produce language to explain whether two equations are equivalent.

Design Principle(s): Support sense-making; Maximize meta-awareness
4.3 More and Less

10 minutes
The purpose of this activity is to connect various representations of proportional relationships including images, equations, and descriptions. Students first match descriptions of a proportional relationship, involving a variable \( x \), to tape diagrams that represent the situations. Then they create equations that describe the proportion using only variables. Finally, they apply their understanding to create their own scenarios and describe a proportional relationship, which is already represented in the form of a tape diagram.

The focus of the discussion following the activity is on the numbers used to go from between significant quantities in a proportional relationship. For example, what numbers are relevant as we go from the quantity of blueberries eaten by Han to the quantity eaten by Mai?

**Building On**
- 6.RP.A.3

**Building Towards**
- 7.RP.A.3

**Instructional Routines**
- MLR7: Compare and Connect

**Launch**
Arrange students in groups of 2. Give students 1--2 minutes of quiet think time followed by partner discussion on the first problem. Then give students 3--5 minutes to complete the problems. Follow with whole-class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to show how \( \frac{1}{3} \) is represented in the situation, the tape diagram, and in the corresponding equation.

*Supports accessibility for: Visual-spatial processing*

**Anticipated Misconceptions**
Students may match the equation \( y = \frac{2}{3}x \) with the situation “Mai biked \( x \) miles, and Han biked \( \frac{2}{3} \) more than that.” Ask them who biked farther, Han or Mai? The equation they choose must result in Han biking a greater distance than Mai.
**Student Task Statement**

1. Match each situation with a diagram. A diagram may not have a match.

- Han ate $x$ ounces of blueberries. Mai ate $\frac{1}{3}$ less than that.
- Mai biked $x$ miles. Han biked $\frac{2}{3}$ more than that.
- Han bought $x$ pounds of apples. Mai bought $\frac{2}{3}$ of that.

2. For each diagram, write an equation that represents the relationship between $x$ and $y$.
   a. Diagram A:
   b. Diagram B:
   c. Diagram C:
   d. Diagram D:

3. Write a story for one of the diagrams that doesn't have a match.

**Student Response**

1.  
   - C. Han ate $x$ ounces of blueberries. Mai ate $\frac{1}{3}$ less than that. Diagram C shows that $y$ is less than $x$ by one third.
   - B. Mai biked $x$ miles. Han biked $\frac{2}{3}$ more than that. Diagram B shows that $y$ is more than $x$ by two-thirds.
   - C. Han bought $x$ pounds of apples. Mai bought $\frac{2}{3}$ of that. Diagram C shows that $y$ consists of two-thirds of $x$.

2. A: $y = \frac{4}{3}x$, B: $y = \frac{5}{3}x$, C: $y = \frac{2}{3}x$, D: $y = \frac{1}{3}x$ (or equivalent)

3. Answers vary. Sample responses: Mai slept $x$ hours. Han slept $\frac{1}{3}$ more than that (for diagram A). Han has $x$ quarters. Mai has $\frac{1}{3}$ of that (for diagram D).
**Activity Synthesis**

After students complete the task, ask students to share a few of their arguments for the matches they came up with (not all need to be shared). Questions you could ask them as they share:

- What is different about the numbers in the description and the numbers in the equation?
- How is the number in the equation related to the number in the description?
- What is different between a fractional amount more than the original versus less than the original?

**Access for English Language Learners**

*Representing, Listening: MLR7 Compare and Connect.* As students share their explanations for the matches they came up with with the class, call students’ attention to the different ways quantities are represented in the descriptions, tape diagrams and equations. Wherever possible, amplify student words and actions that describe the correspondence between specific features of one mathematical representation with a specific feature of another representation. This will help students interpret what is communicated by each type of representation.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

**4.4 Card Sort: Representations of Proportional Relationships**

*Optional: 10 minutes*

In this activity, students receive a set of cards that have 3 different representations of proportional relationships (table, equation, description). The purpose of this activity is for students to interpret wording like “y is \( \frac{1}{3} \) more than x” and figure out how that relationship can be expressed using different mathematical representations. Students may initially think that the wording means \( y = \frac{1}{3} x \), however the numbers in the table representation will help them realize that this is not true. They are engaging in this reasoning here in preparation for upcoming work on percent increase and decrease.

They will work with their partner to match the different representations with each other (each set should contain a table, equation and description) and explain their reasoning (MP3).

There are 8 groups of 3 to match up. Here is an example of one of the groups of 3:
Launch
Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the rest of the cards and place them face-up. Point out that each card contains a description, table, or equation. Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group pre-printed cut-up slips for matching. Place two copies of uncut Instructional masters in envelopes to serve as answer keys.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

*Supports accessibility for: Conceptual processing; Organization*

Anticipated Misconceptions

Students may match the equation $y = \frac{2}{3}x$ with the situation “Elena biked $x$ miles, and Noah biked $\frac{2}{3}$ more than that.” Ask them who biked farther, Noah or Elena? The equation they choose must result in Noah biking a greater distance than Elena.
**Student Task Statement**
Your teacher will give you a set of cards that have proportional relationships represented three different ways: as descriptions, equations, and tables. Mix up the cards and place them all face-up.

1. Take turns with a partner to match a description with an equation and a table.
   a. For each match you find, explain to your partner how you know it’s a match.
   b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.

2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

**Student Response**
The Instructional master shows the correct matches.

**Activity Synthesis**
After students complete the task, ask students to share a few of their arguments for the matches they came up with (not all need to be shared). Questions you could ask them as they share:

- What is different about the numbers in the description and the numbers in the equation?
- How is the number in the equation related to the number in the description?
- What is different between a fractional amount more than the original versus less than the original?

**Lesson Synthesis**
Students should understand the role the distributive property plays in making calculations more efficient. Ask students:

- “Give examples of how we can use the distributive property to create equivalent expressions that make it easier for us to calculate an amount plus (or minus) a fraction of that amount.” (e.g. \( x + \frac{1}{2}x = 1\frac{1}{2}x \))
- “What does this look like in different representations?” (refer to the card sort examples)

**4.5 Fruit Snacks and Skating**

**Cool Down: 5 minutes**
If students only write expressions, like \( \frac{1}{4}x \) and \( \frac{8}{5}x \), instead of equations, then they have shown they achieved the goal of the lesson. It’s not necessary or desirable to hold students accountable for correctly interpreting *expressions* versus *equations* at this time.
Building Towards

- 7.RP.A.3

Student Task Statement

1. Tyler ate \( x \) fruit snacks, and Han ate \( \frac{3}{4} \) less than that. Write an equation to represent the relationship between the number Tyler ate \((x)\) and the number Han ate \((y)\).

2. Mai skated \( x \) miles, and Clare skated \( \frac{3}{5} \) farther than that. Write an equation to represent the relationship between the distance Mai skated \((x)\) and the distance Clare skated \((y)\).

Student Response

1. \( y = \frac{1}{4}x \) (or equivalent). Han ate \( \frac{3}{4} \) less than the number of fruit snacks Tyler ate. Han ate \( \frac{1}{4} \) fruit snacks because \( x - \frac{3}{4}x = \frac{1}{4}x \).

2. \( y = \frac{8}{5}x \) (or equivalent). Clare skated \( \frac{3}{5} \) farther than the number of miles Mai skated. Clare skated \( \frac{8}{5}x \) miles because \( x + \frac{3}{5}x = \frac{8}{5}x \).

Student Lesson Summary

Using the distributive property provides a shortcut for calculating the final amount in situations that involve adding or subtracting a fraction of the original amount.

For example, one day Clare runs 4 miles. The next day, she plans to run that same distance plus half as much again. How far does she plan to run the next day?

Tomorrow she will run 4 miles plus \( \frac{1}{2} \) of 4 miles. We can use the distributive property to find this in one step: \( 1 \cdot 4 + \frac{1}{2} \cdot 4 = (1 + \frac{1}{2}) \cdot 4 \)

Clare plans to run \( 1\frac{1}{2} \cdot 4 \), or 6 miles.

This works when we decrease by a fraction, too. If Tyler spent \( x \) dollars on a new shirt, and Noah spent \( \frac{1}{3} \) less than Tyler, then Noah spent \( \frac{2}{3}x \) dollars since \( x - \frac{1}{3}x = \frac{2}{3}x \).

Glossary

- tape diagram
Lesson 4 Practice Problems

Problem 1

Statement

Match each situation with a diagram.

A. Diagram A

B. Diagram B

C. Diagram C

Solution

- A: 2
- B: 1
- C: 3

1. Diego drank $x$ ounces of juice. Lin drank $\frac{1}{4}$ less than that.

2. Lin ran $x$ miles. Diego ran $\frac{3}{4}$ more than that.

3. Diego bought $x$ pounds of almonds. Lin bought $\frac{1}{4}$ of that.
Problem 2

Statement
Elena walked 12 miles. Then she walked \( \frac{1}{4} \) that distance. How far did she walk all together?
Select all that apply.

A. \( 12 + \frac{1}{4} \)
B. \( 12 \cdot \frac{1}{4} \)
C. \( 12 + \frac{1}{4} \cdot 12 \)
D. \( 12 \left( 1 + \frac{1}{4} \right) \)
E. \( 12 \cdot \frac{3}{4} \)
F. \( 12 \cdot \frac{5}{4} \)

Solution
["C", "D", "F"]

Problem 3

Statement
Write a story that can be represented by the equation \( y = x + \frac{1}{4}x \).

Solution
Answers vary. Sample response: Andre slept \( x \) hours. Diego slept \( \frac{1}{4} \) more than that.

Problem 4

Statement
Select all ratios that are equivalent to 4 : 5.
A. 2 : 2.5
B. 2 : 3
C. 3 : 3.75
D. 7 : 8
E. 8 : 10
F. 14 : 27.5

Solution

["A", "C", "E"]
(From Unit 4, Lesson 1.)

Problem 5

Statement
Jada is making circular birthday invitations for her friends. The diameter of the circle is 12 cm. She bought 180 cm of ribbon to glue around the edge of each invitation. How many invitations can she make?

Solution
Each card needs $12\pi$ or about 37.7 cm of ribbon. She has enough ribbon for 4 cards since $180 \div 37.7 \approx 4.77$.

(From Unit 3, Lesson 10.)
Lesson 5: Say It with Decimals

Goals

- Comprehend and use the term “repeating” (in spoken language) and the notation — (in written language) to refer to a decimal expansion that keeps having the same number over and over forever.
- Coordinate fraction and decimal representations of situations involving adding or subtracting a fraction of the initial value.
- Use long division to generate a decimal representation of a fraction, and describe (in writing) the decimal that results.

Learning Targets

- I can use the distributive property to rewrite an equation like \( x + 0.5x = 1.5x \).
- I can write fractions as decimals.
- I understand that “half as much again” and “multiply by 1.5” mean the same thing.

Lesson Narrative

In this lesson students continue to study situations of fractional increase and decrease. They start to use decimal notation to express the situations. For example, they see that “one quarter less than \( x \)” can be expressed as \( \frac{3}{4}x \) or as 0.75\( x \).

Alignments

Addressing

- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Notice and Wonder

Unit 4 Lesson 5
**Required Preparation**
Print and cut up slips from the Representations of Proportional Relationships Card Sort Instructional master. Prepare 1 copy for every 2 students. These can be re-used if you have more than one class. Consider making a few extra copies that are not cut up to serve as an answer key.

**Student Learning Goals**
Let’s use decimals to describe increases and decreases.

**5.1 Notice and Wonder: Fractions to Decimals**

**Warm Up: 5 minutes**
The purpose of this warm-up is to get students ready to think about decimal expansions of fractions. This will be useful in the next activity when students use long division to find decimal expansions of different fractions and find out why some repeat and others don’t. In this activity, they are given calculator answers for different unit fractions (don’t have to be unit fractions, could be any fractions) and they are starting to notice and verbalize different patterns. By the end of this activity they should be curious about why the decimal expansions of different fractions behave so differently.

**Building Towards**
- 7.RP.A.3

**Instructional Routines**
- Notice and Wonder

**Launch**
Arrange students in groups of 2. Tell students that they will look at a collection of decimals, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

**Student Task Statement**
A calculator gives the following decimal representations for some unit fractions:
\[
\begin{align*}
\frac{1}{2} &= 0.5 & \frac{1}{7} &= 0.142857143 \\
\frac{1}{3} &= 0.3333333 & \frac{1}{8} &= 0.125 \\
\frac{1}{4} &= 0.25 & \frac{1}{9} &= 0.1111111 \\
\frac{1}{5} &= 0.2 & \frac{1}{10} &= 0.1 \\
\frac{1}{6} &= 0.1666667 & \frac{1}{11} &= 0.0909091
\end{align*}
\]

What do you notice? What do you wonder?

**Student Response**

Things students may notice:

- Different fractions have different numbers of digits
- Some decimals repeat
- Some decimals don't repeat (not really, but that's what it looks like)
- Some decimals almost repeat, except for the last digit (0.0909091)

Things students may wonder:

- Do some decimals really finish after 7 digits or do they keep going?
- Why are some decimals shorter than others?
- Are there other fractions that only have one digit after the decimal?

**Activity Synthesis**

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If rounding does not come up during the conversation, ask students to discuss this idea.

**5.2 Repeating Decimals**

15 minutes

This activity introduces students to the term *repeating* for describing the decimal they get after using long division to convert a fraction. If desired, the term *terminating* can also be introduced.

**Addressing**

- 7.NS.A.2.d

Unit 4 Lesson 5
Building Towards

• 7.RP.A.3

Instructional Routines

• MLR8: Discussion Supports

Launch

Explain to students that we can use long division to calculate the decimal representation of a fraction. For example, \( \frac{7}{8} \) is equal to \( 7 \div 8 \).

\[
\begin{align*}
8 & \overline{)7} \\
-0 & \\
7 & 0 \\
-64 & \\
60 & \\
-56 & \\
40 & \\
-40 & \\
0 & 
\end{align*}
\]

Using long division, we see that \( \frac{7}{8} \) is equal to 0.875. This process works for any fraction.

Here is another example: \( \frac{7}{12} \) is equal to \( 7 \div 12 \).

\[
\begin{align*}
12 & \overline{)73.3} \\
-0 & \\
70 & \\
-60 & \\
100 & \\
-96 & \\
40 & \\
-36 & \\
40 & \\
-36 & \\
40 & \\
-36 & \\
4 & 
\end{align*}
\]

In this case, the division will never result in a remainder of 0. Because we keep getting 3 over and over again, this is called a repeating decimal and can be written as 0.58\(\overline{3}\).

Arrange students in groups of 2. Give students 3–5 minutes of quiet think time on the first problem and 1–2 minutes to compare their responses and discuss the second question with their partner.
Then give students 2–3 minutes of quiet think time on the remaining question. Follow with whole-class discussion.

**Anticipated Misconceptions**
Some students may set up their long division with the divisor and dividend in the wrong places. They will get 2.7, 2.72, and 2.75 as their answers. Prompt them to think about what is being divided and what it is being divided by.

**Student Task Statement**

1. Use long division to express each fraction as a decimal.
   \[
   \frac{9}{25} \quad \frac{11}{30} \quad \frac{4}{11}
   \]

2. What is similar about your answers to the previous question? What is different?

3. Use the decimal representations to decide which of these fractions has the greatest value. Explain your reasoning.

**Student Response**

1. \( \frac{9}{25} = 0.36, \frac{11}{30} = 0.3\overline{6}, \frac{4}{11} = 0.\overline{36} \)

2. Answers vary. Sample response: All 3 of these decimals have the same two numbers in the tenths and hundredths places, but \( \frac{11}{30} \) and \( \frac{4}{11} \) are both repeating decimals while \( \frac{9}{25} \) is not.

3. \( \frac{11}{30} \) is the largest because it has a 6 in the thousandths place when written as a decimal. \( \frac{4}{11} \) has a 3 in the thousandths place and \( \frac{9}{25} \) would have a 0 in the thousandths place.

**Are You Ready for More?**

One common approximation for \( \pi \) is \( \frac{22}{7} \). Express this fraction as a decimal. How does this approximation compare to 3.14?

**Student Response**

\( 3.142857 \) is closer to \( \pi \) than 3.14 is.

**Activity Synthesis**

The purpose of this discussion is to help students make sense of the value of repeating decimals. Ask students to explain the strategies they used to answer question 3. If no students mention place value, ask them to use this language to describe their strategies.
**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* To support students in explaining their reasoning for the last question, provide sentence frames for students to use when they are comparing fractions. For example, “____ is (larger/smaller) because ____.” Revoice student ideas using mathematical language as necessary.

*Design Principle(s): Support sense-making; Optimize output for (comparison)*

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**5.3 More and Less with Decimals**

15 minutes

This activity continues work done in the previous lesson that connected various representations of proportional relationships including images, equations, and descriptions. In this activity students match diagrams, descriptions, and equations that represent a proportional relationship, involving variables \(x\) and \(y\). Students then create their own diagram to represent an equation.

Identify students that create diagrams representing each of the unmatched equations, so they can share their work during the discussion.

**Addressing**

- 7.RP.A.2

**Building Towards**

- 7.RP.A.3

**Instructional Routines**

- MLR3: Clarify, Critique, Correct

**Launch**

Give students 7–10 minutes of quiet think time on the questions. Follow with whole-class discussion.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Begin the activity with concrete or familiar contexts. For example, display a tape diagram with concrete values for \(x\) and \(y\) such as 20 and 15 respectively. Ask students “How much did 20 decrease by to get to 15?” Highlight the connections between the concrete and abstract tape diagrams. In the concrete tape diagram, 20 decreased by \(\frac{1}{4}\) to get 15. In the abstract tape diagram, \(x\) also decreased by \(\frac{1}{4}\) to get \(y\).

*Supports accessibility for: Conceptual processing; Memory*
Student Task Statement

1. Match each diagram with a description and an equation.

Diagrams:

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>An increase by $\frac{1}{4}$</td>
<td>$y = 1.6x$</td>
</tr>
<tr>
<td>B</td>
<td>An increase by $\frac{1}{3}$</td>
<td>$y = 1.33x$</td>
</tr>
<tr>
<td></td>
<td>An increase by $\frac{2}{3}$</td>
<td>$y = 0.75x$</td>
</tr>
<tr>
<td></td>
<td>A decrease by $\frac{1}{5}$</td>
<td>$y = 0.4x$</td>
</tr>
<tr>
<td></td>
<td>A decrease by $\frac{1}{4}$</td>
<td>$y = 1.25x$</td>
</tr>
</tbody>
</table>

2. Draw a diagram for one of the unmatched equations.

Student Response

1. Diagram A can match to an increase by $\frac{1}{3}$ or a decrease by $\frac{1}{4}$. Diagram A matches to $y = 0.75x$. Diagram B can match to an increase by $\frac{1}{4}$ or a decrease by $\frac{1}{5}$. Diagram B matches to $y = 1.25x$.

2. Answers vary.

Activity Synthesis

Ask previously identified students to share their diagrams and explain how they represent the equation they chose. Then, ask students that did not share their diagrams to explain whether $x$ (or $y$) is increased or decreased by some amount in the discussed equations.

Access for English Language Learners

Representing, Writing: MLR3 Clarify, Critique, Correct. Present an incorrect diagram to represent one of the unused equations that reflects a possible misunderstanding from the class. For example, for the equation $y = 0.75x$, draw a diagram where $y$ is greater than $x$. Prompt students to identify the error, and then write a correct diagram to represent the equation. This will support students to understand the relationship between equations and diagrams. Design Principle(s): Maximize meta-awareness

5.4 Card Sort: More Representations

Optional: 10 minutes
This activity continues the card sort from the previous lesson by including a fourth representation for each relationship, an equation with a decimal instead of a fraction.

**Addressing**
- 7.RP.A.2

**Building Towards**
- 7.RP.A.3

**Instructional Routines**
- MLR2: Collect and Display

**Launch**
Remind students of card sort activity of the earlier lesson. Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the rest of the cards and place them face-up. Point out that each card contains a description, table, or equation (with fractions or decimals). Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group pre-printed cut-up slips for matching. Place two copies of uncut Instructional masters in envelopes to serve as answer keys.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

*SUPPORTS accessibility for: Conceptual processing; Organization*

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### Access for English Language Learners

*Representing, Conversing: MLR2 Collect and Display.* As students work, circulate and listen to the language students use to justify their matches. Write down common or important phrases you hear students as they match each representation, and display for all to see. Remind students to borrow language from the display as needed, and continue to update the display during the whole-class discussion. Call students' any language that helps to make an explanation more convincing.

*Design Principle(s): Support sense-making; Optimize output (for justification)*
Student Task Statement

Your teacher will give you a set of cards that have proportional relationships represented 2 different ways: as descriptions and equations. Mix up the cards and place them all face-up.

Take turns with a partner to match a description with an equation.

1. For each match you find, explain to your partner how you know it’s a match.
2. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
3. When you have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

Student Response

The Instructional master shows the correct matches.

Activity Synthesis

After students complete the task, ask students to share a few of their arguments for the matches they came up with (not all need to be shared). Ask students to compare the equations with decimals and fractions.

Lesson Synthesis

This lesson was similar to the last one, except we focused on writing things with decimals.

“Give examples of how we can use the distributive property to create equivalent expressions that make it easier for us to calculate an amount plus (or minus) a fraction of that amount, but written with decimals” (e.g. $100x + 12x = 112x$)

5.5 Reading More

Cool Down: 5 minutes

Addressing

- 7.RP.A.2

Student Task Statement

Kiran read for $x$ minutes, and Andre read for $\frac{5}{8}$ more than that. Write an equation that relates the number of minutes Kiran read with $y$, the number of minutes that Andre read. Use decimals in your equation.

Student Response

$y = 1.625x$ or equivalent. Andre read $\frac{5}{8}x = 0.625x$ more minutes than Kiran read.

$x + 0.625x = 1.625x. y = 1.625x$
Student Lesson Summary

Long division gives us a way of finding decimal representations for fractions.

For example, to find a decimal representation for \( \frac{9}{8} \), we can divide 9 by 8.

\[
\begin{array}{c|c}
8 & 9.000 \\
\hline
8 & 10 \\
\hline
8 & 20 \\
& 16 \\
& 40 \\
& 40 \\
& 0 \\
\end{array}
\]

So \( \frac{9}{8} = 1.125 \).

Sometimes it is easier to work with the decimal representation of a number, and sometimes it is easier to work with its fraction representation. It is important to be able to work with both. For example, consider the following pair of problems:

- Priya earned \( x \) dollars doing chores, and Kiran earned \( \frac{6}{5} \) as much as Priya. How much did Kiran earn?
- Priya earned \( x \) dollars doing chores, and Kiran earned 1.2 times as much as Priya. How much did Kiran earn?

Since \( \frac{6}{5} = 1.2 \), these are both exactly the same problem, and the answer is \( \frac{6}{5}x \) or 1.2x. When we work with percentages in later lessons, the decimal representation will come in especially handy.

Glossary

- long division
- repeating decimal
Lesson 5 Practice Problems

Problem 1

Statement

a. Match each diagram with a description and an equation.

Descriptions:

- An increase by \( \frac{2}{3} \)
- An increase by \( \frac{5}{6} \)
- A decrease by \( \frac{2}{5} \)
- A decrease by \( \frac{5}{11} \)

Equations:

- \( y = 1.83x \)
- \( y = 1.6x \)
- \( y = 0.6x \)
- \( y = 0.4x \)

b. Draw a diagram for one of the unmatched equations.

Solution

a. Diagram A: A decrease by \( \frac{2}{5} \) and \( y = 0.6x \)

Diagram B: An increase by \( \frac{5}{6} \) and \( y = 1.83x \)

b. Answers vary.

Problem 2

Statement

At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, the family ate 0.3 of the peanut butter. How many ounces of peanut butter are in the pantry now?
Problem 3

Statement

a. On a hot day, a football team drank an entire 50-gallon cooler of water and half as much again. How much water did they drink?

b. Jada has 12 library books checked out and Han has $\frac{1}{3}$ less than that. How many books does Han have checked out?

Solution

a. 75 gallons

b. 8 books

(From Unit 4, Lesson 4.)

Problem 4

Statement

If $x$ represents a positive number, select all expressions whose value is greater than $x$.

A. $\left(1 - \frac{1}{2}\right)x$

B. $\left(1 + \frac{1}{2}\right)x$

C. $\frac{7}{8}x$

D. $\frac{9}{8}x$

Solution

["B", "D"]

(From Unit 4, Lesson 4.)
Problem 5

Statement

A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch, and counts 8 heartbeats in 10 seconds.

a. Is his heart rate typical? Explain how you know.

b. Write an equation for $h$, the number of times Noah's heart beats (at this rate) in $m$ minutes.

Solution

a. No. Noah's heart rate is 48 beats per minute, because $10 \cdot 6 = 60$, and $8 \cdot 6 = 48$.

b. $h = 48m$

(From Unit 2, Lesson 6.)
Section: Percent Increase and Decrease

Lesson 6: Increasing and Decreasing

Goals

- Coordinate statements about “percent increase” or “percent decrease” with comparisons to the original amount, e.g., a 20% increase means the new value is 120% of the original value.
- Draw and label a tape diagram to represent a situation that involves adding or subtracting a percentage of the initial value.
- Explain (orally and in writing) how to calculate the new amount given the original amount and a percentage of increase or decrease.

Learning Targets

- I can draw a tape diagram that represents a percent increase or decrease.
- When I know a starting amount and the percent increase or decrease, I can find the new amount.

Lesson Narrative

This is the first of four lessons about percent increase and percent decrease. The goal of this lesson is to understand what is meant by "20% more than" or "10% less than." Students relate this language to the previous two lessons where they talked about "half as much again" and "one third less than." They use tape diagrams to represent percent increase and percent decrease, and to solve problems. The contexts in this first lesson are all of the type where you are given the original amount and the percent increase or decrease and must calculate the final amount.

Students use tape diagrams and their understanding of the language of percent increase and decrease to reason about different contexts (MP3). Students should be able to interpret the meaning of a percent increase or percent decrease in the context of a problem (MP2).

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let’s use percentages to describe increases and decreases.

6.1 Improving Their Game

Warm Up: 10 minutes

The purpose of this warm-up is to encourage students to recognize that in some situations it is helpful to think about a multiplicative increase rather than an additive increase. These situations can be described in terms of percent increase (situations in which an increase is obtained by adding a certain percentage of a quantity to itself). In this situation, total points for each sports team increase from Game 1 to Game 2, however the increases are different percentages of the first scores. For example, an increase of 8 points from 100 to 108 is not as significant as an increase from 4 points to 12 points, because in the first increase is only 8% of the original value while the second is 200% of the original value.

As students share things they notice, listen for language students use to discuss the significance of the different increases. For example, students may say that the baseball team tripled their score, which would be like the basketball team going from 100 to 300 points in the next game.

Building Towards

- 7.RP.A.3

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students they will look at a table and think of at least one thing they notice and at least one thing they wonder. Display the table for all to see and give 1 minute of
quiet think time. Ask students to give a signal when they have noticed something about the teams’ scores. Invite students to share their ideas; record and display their responses for all to see. If no students wonder which team improved the most, direct them to the second question and give them 1 minute to work with a partner.

**Anticipated Misconceptions**

Students may say that the football team improved the least because the 8 points could have been scored from only 1 touchdown in football, but it would have to be 3 or 4 baskets in basketball and 8 separate runs in baseball. Prompt students to look at the significance of the 8 additional points in the context of each team’s score in the game 1, rather than the mechanics of scoring in each sport.

**Student Task Statement**

Here are the scores from 3 different sports teams from their last 2 games.

<table>
<thead>
<tr>
<th>sports team</th>
<th>total points in game 1</th>
<th>total points in game 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>football team</td>
<td>22</td>
<td>30</td>
</tr>
<tr>
<td>basketball team</td>
<td>100</td>
<td>108</td>
</tr>
<tr>
<td>baseball team</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

1. What do you notice about the teams’ scores? What do you wonder?

2. Which team improved the most? Explain your reasoning.

**Student Response**

1. Answers vary. Students may notice: each team improved by 8 points; the basketball team had the most points; and the baseball team’s score in game 2 was 3 times as much as in game 1, a three-fold increase.

2. Answers vary. Sample response: The baseball team improved the most because their 8-point increase tripled their previous score, while the other teams’ scores were multiplied by a smaller factor: about 1.1 for the basketball team and about 1.4 for the football team.

**Activity Synthesis**

Poll students on which team they think improved the most. Ask a student who thinks they all improved by the same amount to share their reasoning (each team increased its score by 8 points). Then, ask a few students who said the baseball team improved the most to share their reasoning. There is no need to invoke the phrase “percent increase“ or to express the change as a percent of the game 1 score at this time, but you want to plant the idea that it sometimes makes sense to describe a change *relative to a starting amount*, instead of just looking at absolute change. In the course of discussion, though, it may be natural to say things like the basketball team improved their score by 8% of their game 1 score, the football team improved by nearly $\frac{1}{3}$ of their game 1 score, while the baseball team tripled their game 1 score.
6.2 More Cereal and a Discounted Shirt

10 minutes
In this activity, students are given a percent increase and use it to calculate the new value, rather than being given the original and new values to calculate the percent increase. Students can solve the problems using the double number lines but the discussion that follows will be the connection to what the words specifically mean (MP6).

As students work, monitor for students who complete the double number line diagram and any student that uses other methods to correctly reason about the problem.

Addressing
- 7.RP.A.3

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

Launch
Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by partner then whole-class discussion.

Anticipated Misconceptions
Have students use the double number line diagram if they need help figuring out 20% more.

Student Task Statement
1. A cereal box says that now it contains 20% more. Originally, it came with 18.5 ounces of cereal. How much cereal does the box come with now?

2. The price of a shirt is $18.50, but you have a coupon that lowers the price by 20%. What is the price of the shirt after using the coupon?
**Student Response**

1. 22.2 ounces of cereal. Sample Explanations:
   - The cereal box gained 3.7 ounces, because 20% of 18.5 ounces is $0.2 \times 18.5$, or 3.7 ounces. That means the cereal box now has 22.2 ounces, because $18.5 + 3.7 = 22.2$.
   - The cereal box now has 120% as many ounces of cereal as it originally had, because $100 + 20 = 120$. Now it has 22.2 ounces of cereal because $18.5 \times 1.2 = 22.2$.

2. $14.80. Sample Explanations:
   - The price drops by $3.70, because 20% of $18.50 is $0.2 \times 18.5$, or $3.70$. That means the shirt will cost $14.80, because $18.50 - 3.70 = 14.80$.
   - The sale price is 80% of the original price, because $100 - 20 = 80$. The price of the shirt after using the coupon will be $14.80 because $18.50 \times 0.80 = 14.8$.

**Activity Synthesis**

Select students to share their reasoning for each problem. Start with a student that used the double number line diagram to solve the problem. Then, have students share other methods they used to solve the problem, such as multiplying by 1.2 and 0.8.

Ask students:

- Did the number of ounces of cereal in the cereal box increase or decrease?
- What percentage of the original amount of cereal is the new amount of cereal?
- Did the price of the shirt increase or decrease?
- What percentage of the original price of the shirt is the new price of the shirt?

Tell students that the change on the cereal box is an example of a percent increase. The discount on the shirt is an example of a percent decrease. You might want to mention that in both cases, 100% always corresponds to the original amount before the change; however, future activities will address this concept in depth.

**Access for Students with Disabilities**

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of percent increase and percent decrease.

*Supports accessibility for: Conceptual processing; Language*
Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Before students share their reasoning for each problem, present an incomplete strategy for finding the discount price for the shirt. For example, “The discount is 20%, so the price of the shirt is $3.70.” Ask students to critique the reasoning, and work with a partner to write an improved explanation. Listen for and amplify the language students use to make sense of what is displayed (e.g., “I think they ___ because ____.”), as well as the mathematical language that students use that strengthen their explanations. This will support student understanding of mathematical language related percent increase and percent decrease.

Design Principle(s): Maximize meta-awareness

6.3 Using Tape Diagrams

10 minutes
The purpose of this activity is for students to understand that a percent increase of, say, 15% corresponds to 115% of the original amount, and a percent decrease of, say, 30% corresponds to 70% of the original amount.

Building On
- 6.RP.A.3

Building Towards
- 7.RP.A.3

Instructional Routines
- MLR8: Discussion Supports
- Think Pair Share

Launch
Show students this image.

Say, “Explain how each of these is related to the diagram.”

1. \(x + \frac{1}{4}x\)
2. \( y = 1.25x \)
3. 125%
4. An increase of 25%

1 minute of quiet think time followed by partner and then whole group discussion.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, annotate the section of diagram B that represents 75% to emphasize that this section represents this year’s blueberry harvest.

*Supports accessibility for: Visual-spatial processing*

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**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* As students explain their reason for selecting diagram A or B, press for details in students’ explanations by requesting that students challenge an idea, elaborate on an idea, or give an example with values. Provide a sentence frame, such as: “Diagram (A/B) represents the situation because ______.” This will help students produce and make sense of the language needed to communicate their own ideas.

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

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**Student Task Statement**

1. Match each situation to a diagram. Be prepared to explain your reasoning.
   
   a. Compared with last year’s strawberry harvest, this year’s strawberry harvest is a 25% increase.
   
   b. This year’s blueberry harvest is 75% of last year’s.
   
   c. Compared with last year, this year’s peach harvest decreased 25%.
   
   d. This year’s plum harvest is 125% of last year’s plum harvest.
2. Draw a diagram to represent these situations.

   a. The number of ducks living at the pond increased by 40%.
   
   b. The number of mosquitoes decreased by 80%.

**Student Response**

1. 

   a. Diagram A. A 25% increase from last year means this year's harvest corresponds to 125% of last year's harvest. The rectangle for this year is 125% of the rectangle for last year.

   b. Diagram B. The rectangle for this year is 75% of the rectangle for last year.

   c. Diagram B. A 25% decrease from last year means this year's harvest corresponds to 75% of last year's harvest. The rectangle for this year is 75% of the rectangle for last year.

   d. Diagram A. The rectangle for this year is 125% of the rectangle for last year.

2. Answers vary. Sample diagrams:

   ![Diagram of ducks and mosquitoes](image)

**Are You Ready for More?**

What could it mean to say there is a 100% decrease in a quantity? Give an example of a quantity where this makes sense.

**Student Response**

It means the quantity is now zero. For example, a person could walk 1 mile one day and 0 miles the next day.

**Activity Synthesis**

If the amount of fruit increases by 40%, what percent of the original amount do you have?

If the amount of fruit decreases by 40%, what percent of the original amount do you have?

**6.4 Agree or Disagree: Percentages**

Optional: 10 minutes
The purpose of this activity is to for students to evaluate claims about percentages within contexts in which common misunderstandings occur (MP3). The first question prompts students to think about the original pay of each employee. Since we do not know the pay for each employee, a higher pay raise percentage does not necessarily mean a higher dollar amount. Students could disagree based on the reasoning that Employee A makes $10 per hour, so a 50% raise would be an increase of $5 per hour. However, if Employee B makes $20 per hour, a 45% raise would be an increase of $9 per hour. On the other hand, students could agree with this statement if they think both employees make the same amount. The second question prompts students to reason about the effect of trying to combine percentages. Ask students to discuss the following with their partner:

- Did you agree or disagree with one another?
- How did you test out your ideas?
- What did you notice to be true after testing out some numbers in the statement?

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time
- MLR3: Clarify, Critique, Correct
- Think Pair Share

**Launch**

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by partner and whole-class discussions. Refer to MLR 3 (Clarify, Critique, Correct) for prompts to build student language for evaluating a statement. One example is to use the "Always-Sometimes-Never" approach for helping students determine the validity of the statements.

**Access for English Language Learners**

*Writing, Speaking: MLR1 Stronger and Clearer Each Time.* To help students refine their justifications for whether they agree or disagree with the first statement, give students time to meet with 2-3 partners, sharing their responses. Encourage listeners to press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language (e.g., “Why do you think that?” “How could you use values to show your thinking?” etc.). Students can borrow ideas and language from each partner to strengthen their final product.

*Design Principle(s): Optimize output (for justification)*
Anticipated Misconceptions

Students may agree with both statements at first as the statements themselves are common misconceptions. Ask these students to assign values to the pieces of the statement and test them out. For example, assign how much each employee makes in the first statement or how much each shirt costs in the second statement.

Student Task Statement

Do you agree or disagree with each statement? Explain your reasoning.

1. Employee A gets a pay raise of 50%. Employee B gets a pay raise of 45%. So Employee A gets the bigger pay raise.

2. Shirts are on sale for 20% off. You buy two of them. As you pay, the cashier says, “20% off of each shirt means 40% off of the total price.”

Student Response

1. Agree if both employees make the same amount of money. Disagree if Employee A makes enough less than Employee B.

2. Disagree. It's still 20% off the total price. If we represent the discount from 20% off shirt A as $0.2a$ and the discount from 20% off shirt B as $0.2b$, then the amount off the total price would be $0.2a + 0.2b$ or by the distributive property, $0.2(a + b)$.

Activity Synthesis

Poll students if they agree or disagree with each statement and ask students to explain their reasoning. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- “Do you agree or disagree? Why?”
- “Did anyone think about the statement in the same way but would explain it differently?”
- “Does anyone want to add on to ______’s reasoning?”

Lesson Synthesis

Students should be able to apply their understanding of proportional increases and decreases, from previous lessons, to problems involving percents.

- “What is another way to describe a 25% percent increase or decrease?” (when we increase or decrease a quantity by adding or subtracting $\frac{1}{4}$ of the original quantity)
- “When a quantity is increased or decreased, what percent describes the original or starting value?” (100%)
- “What strategies have we used to help us calculate percent increase and decrease?” (double number line, table, equation)
6.5 Fish Population

Cool Down: 5 minutes

Addressing

• 7.RP.A.3

Anticipated Misconceptions

Some students may answer 15 fish, because the find 25% of 60, but don't realize this is the amount of the decrease, not the final amount.

Some students may answer 75 fish, because they calculate a 25% increase instead of a 25% decrease.

Some students may answer 80 fish, because they use 60 as the number after the 25% decrease instead of before.

Student Task Statement

The number of fish in a lake decreased by 25% between last year and this year. Last year there were 60 fish in the lake. What is the population this year? If you get stuck, consider drawing a diagram.

Student Response

There are 45 fish in the lake this year. Sample explanations:

• The number of fish decreased by 15, because $0.25 \cdot 60 = 15$. That means there are 45 fish left, because $60 - 15 = 45$.

• There are only 75% as many fish this year, because $100 - 25 = 75$. We can multiply $0.75 \cdot 60 = 45$.

• Here is a tape diagram that shows there are 45 fish left:

![Tape Diagram]

Student Lesson Summary

Imagine that it takes Andre $\frac{3}{4}$ more than the time it takes Jada to get to school. Then we know that Andre's time is $1 \frac{3}{4}$ or 1.75 times Jada's time. We can also describe this in terms of percentages:
We say that Andre’s time is 75% more than Jada’s time. We can also see that Andre’s time is 175% of Jada’s time. In general, the terms **percent increase** and **percent decrease** describe an increase or decrease in a quantity as a percentage of the starting amount.

For example, if there were 500 grams of cereal in the original package, then “20% more” means that 20% of 500 grams has been added to the initial amount, $500 + (0.2) \cdot 500 = 600$, so there are 600 grams of cereal in the new package.

We can see that the new amount is 120% of the initial amount because

$$500 + (0.2) \cdot 500 = (1 + 0.2) \cdot 500$$

**Glossary**

- percentage decrease
- percentage increase
Lesson 6 Practice Problems

Problem 1

**Statement**

For each diagram, decide if \( y \) is an increase or a decrease relative to \( x \). Then determine the percent increase or decrease.

**Solution**

For A, \( y \) is a 25% decrease of \( x \).

For B, \( y \) is a 25% increase of \( x \).

Problem 2

**Statement**

Draw diagrams to represent the following situations.

a. The amount of flour that the bakery used this month was a 50% increase relative to last month.

b. The amount of milk that the bakery used this month was a 75% decrease relative to last month.

**Solution**

Answers vary.

Problem 3

**Statement**

Write each percent increase or decrease as a percentage of the initial amount. The first one is done for you.
a. This year, there was 40% more snow than last year.

*The amount of snow this year is 140% of the amount of snow last year.*

b. This year, there were 25% fewer sunny days than last year.

c. Compared to last month, there was a 50% increase in the number of houses sold this month.

d. The runner's time to complete the marathon was a 10% less than the time to complete the last marathon.

**Solution**

a. The amount of snow this year is 140% of the amount of snow last year.

b. The number of sunny days this year is 75% of the number of sunny days last year.

c. The number of houses sold this month is 150% of the number of houses sold last month.

d. The runner's time to complete the marathon was 90% of the time to complete the last marathon.

**Problem 4**

**Statement**

The graph shows the relationship between the diameter and the circumference of a circle with the point (1, π) shown. Find 3 more points that are on the line.

**Solution**

Answers vary. Possible answers: (0, 0), (2, 2π), (3, 9.4)
Problem 5

Statement

Priya bought \( x \) grams of flour. Clare bought \( \frac{3}{8} \) more than that. Select all equations that represent the relationship between the amount of flour that Priya bought, \( x \), and the amount of flour that Clare bought, \( y \).

A. \( y = \frac{3}{8} x \)
B. \( y = \frac{5}{8} x \)
C. \( y = x + \frac{3}{8} x \)
D. \( y = x - \frac{3}{8} x \)
E. \( y = \frac{11}{8} x \)

Solution

["C", "E"]

(From Unit 4, Lesson 4.)
Lesson 7: One Hundred Percent

Goals

• Critique (orally and in writing) double number line diagrams that represent situations involving percent increase or decrease.

• Generalize (orally) that the original amount corresponds to 100% and the new amount corresponds to more or less than 100%, depending on whether the situation involves an increase or decrease.

• Interpret a description of a situation to identify the original amount, the new amount, the change, and corresponding percentages. Label these on a double number line diagram.

Learning Targets

• I can use a double number line diagram to help me solve percent increase and decrease problems.

• I understand that if I know how much a quantity has grown, then the original amount represents 100%.

• When I know the new amount and the percentage of increase or decrease, I can find the original amount.

Lesson Narrative

In this second lesson about percent increase and percent decrease, students work with problems where they are given the final amount after a percent increase or decrease and must calculate the original amount, or are given the final and original amounts and must calculate the percent increase or decrease. They use double number lines to visualize such situations in order to help see clearly which of the two amounts involved, the starting amount or the final amount, is to be regarded as the whole, or 100%. They explore common misconceptions resulting from getting confused about which amount is the whole. For example, if you are given the final amount after a 10% decrease, a common error is to regard that final amount as the whole and calculate the original amount by adding 10% of the final amount. Being clear about which quantity is the whole is a good example of attending to precision (MP6).

Alignments

7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let’s solve more problems about percent increase and percent decrease.

7.1 Notice and Wonder: Double Number Line

Warm Up: 5 minutes
The purpose of this warm-up is to elicit the idea that percent increases and decreases can be represented with double number lines, which will be useful when students use double number lines in a later activity. While students may notice and wonder many things about these images, recognizing the original amount, represented by 100%, is an important discussion point.

Building Towards

- 7.RP.A.3

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.
Student Task Statement

What do you notice? What do you wonder?

Student Response

Things students may notice:

- If 100% is the amount needed, then 10 cups of chocolate milk is needed.
- A cup is 10% of a quantity.

Things students may wonder:

- If the number lines say how much chocolate milk is needed for a recipe, could they be used to double or triple the recipe?
- What would it mean to have negative numbers on the number lines?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the original quantity, represented by 100%, does not come up during the conversation, ask students to discuss this idea.

7.2 Double Number Lines

15 minutes
The purpose of this activity is for students to use double number line diagrams to represent situations of percent increase and decrease. Additionally, students identify the original and new amount in the double number lines to reinforce that the original amount pertains to 100%.

As students work on the task, monitor for students who created various equations for the last question.

Addressing

- 7.RP.A.3
Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give 5–8 minutes of quiet work time. After 5 minutes allow students to work with a partner or to continue to work alone.

Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for:* Organization; Attention

Anticipated Misconceptions

Students may continue to struggle to recognize the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

Student Task Statement

For each problem, complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.

1. The gas tank in dad’s car holds 12 gallons. The gas tank in mom’s truck holds 50% more than that. How much gas does the truck’s tank hold?

2. At a movie theater, the size of popcorn bags decreased 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?
3. A school had 1,200 students last year and only 1,080 students this year. What was the percentage decrease in the number of students?

4. One week gas was $1.25 per gallon. The next week gas was $1.50 per gallon. By what percentage did the price increase?

5. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?

6. Compared to last year, the population of Boom Town has increased 25%. The population is now 6,600. What was the population last year?

Student Response

1. 18 gallons. We can put 12 on the top number line above 100% to represent the gallons of gas that dad's car tank holds. There are 2 steps on the number line to reach that number, so each step is equal to 6 gallons. It is one more step to get to the tick above 150%. Mom's truck tank holds 18 gallons because $12 + 6 = 18$.

2. 12 cups. We can write 15 on the top number line above 100% to represent the cups of popcorn the old bags held. There are 5 steps to reach 15 cups, so each step represents 3 cups. The new bags hold 80% as much popcorn, because $100 - 20 = 80$. That's one step back. So the new bags hold 12 cups of popcorn since $15 - 3 = 12$.

3. 10% decrease. We can write 100 on the bottom number line below 1,200 to represent the percentage of students the school had last year. There are 10 steps on the number line to
reach 100%, so each step represents 10%. The population of 1,080 people is one step backwards, so there was a 10% decrease.

4. 20% increase. We can write 100 on the bottom number line below 1.25 to represent the percentage of the price per gallon in the first week. There are 5 steps to reach 100%, so each step represents 20%. The price of $1.50 is one step after the price of gas the first week, so there was a 20% increase.

5. $16. After the discount, the price of the T-shirt was 75% of the price before the discount, because $100 - 25 = 75. We can write 12 on the top number line above 75 to represent the price of the T-shirt after the discount. It takes 3 steps to reach 12, so each step represents $4. The price before the discount would be 100% of the price, which is one step later. The price before the discount was $16 because $12 + 4 = 16.

6. 5,280 people. The population of Boom Town this year is 125% of the population last year, because $100 + 25 = 125. We can write the number 6,600 on the top number line above the 125 to represent the population this year. It takes 5 steps to reach 6,600, so each step represents 1,320 people. The population last year would correspond to the step above 100%, which is one step before 6,600. Last year the population was 5,280 people because $6,600 - 1,320 = 5,280.

**Activity Synthesis**

Select students to share the values they identified as original amount and the new amount for a few problems. Discuss how 100% always corresponds to the original value and when there is an increase in the value the new value corresponds to a percentage greater than the original 100%.

Select students to share the different equations they came up with. Discuss how the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.

Discuss how solving problems about percent change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how you can find the unknown number. Looking at the examples below, the first two require multiplication, but the others require division.

Using the structure A% of B is C:

- $(1.5) \cdot 12 = c$
- $(0.80) \cdot 15 = c$
- $a \cdot (1,200) = 1,080$
- $a \cdot (1.25) = 1.50$
- $(0.75) \cdot b = 12$
- $(1.25) \cdot b = 6,600$
Access for English Language Learners

**Representing, Speaking: MLR7 Compare and Connect.** Use this routine after selected students share the values they identified for the original amount and the new amount for a few problems. Ask students what is the same and what is different about the ways double number lines were used to represent percent increase and decrease for the different situations. Call students' attention to how the original amount is represented as 100% in the tape diagrams and in some of the equations. This will help strengthen students' mathematical language use and reasoning based on percent increase and decrease.

*Design Principle(s): Maximize meta-awareness*

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### 7.3 Representing More Juice

**5 minutes**
The purpose of this activity is to help students understand that *percent increase* should be interpreted as a percent of the original or starting amount. Students are presented with two different methods to reason about and decide with which one they agree (MP3).

As students work on the activity, look for students who agree with Clare and Priya, these students should be asked to share during the whole-class discussion. The focus of the discussion should be on why Priya’s number line is the correct one and how the term percent increase affects the original or starting amount (it goes over 100%).

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- MLR3: Clarify, Critique, Correct
- Think Pair Share

**Launch**
Give students 1 minute of quiet think time followed by partner and then whole-class discussion.

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**Access for Students with Disabilities**

*Action and Expression: Develop Expression and Communication.* To help get students started, display sentence frames such as “I agree with ____ because ____.”

*Supports accessibility for: Language; Organization*
Anticipated Misconceptions
If students do not see why Clare's number line is incorrect, ask them to think about the original amount of ounces the juice box comes with. If the packaging claims to be more than what it started with should it be more or less than the original amount (100%)?

Student Task Statement
Two students are working on the same problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How much juice does the new packaging hold?

- Here is how Priya set up her double number line.

- Here is how Clare set up her double number line.

Do you agree with either of them? Explain or show your reasoning.

Student Response
Priya is correct. Explanations vary. Sample response: I agree with Priya's double number line because if the juice box is getting 20% more than what it starts with that means it will be more than the full capacity (which is 100%). So the original amount is 100% and the new amount is 120%.

Are You Ready for More?
Clare's diagram could represent a percent decrease. Describe a situation that could be represented with Clare's diagram.

Student Response
Answers vary. Sample response: A juice company modifies their packaging to ship more containers in each shipment. Their original package holds 15 fluid ounces, and their new packages holds 12 fluid ounces. What is the percent decrease made with this package change?

Activity Synthesis
Ask selected students to share their reasoning. Ask students, When the packaging says “20% more juice,” that means 20% more than what? (The amount of juice in the original packaging.) Explain that phrases like “percent more” or “percent less” are expressing the percent of some original amount.
So, if we use a double number line strategy, it makes sense to associate the original or starting amount with 100%.

**Access for English Language Learners**

*Writing: MLR3 Clarify, Critique, Correct.* Present an incorrect justification for why Clare’s double line is correct that reflects a possible misunderstanding from the class. For example, “Clare is correct because 20% more would be 100%.” Prompt discussion by asking, “Do you agree with the statement? Why or why not?” Ask students to correct the statement. This will help students understand that, in percent increase and decrease problems, original amounts are represented by 100%.

*Design Principle(s): Maximize meta-awareness*

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### 7.4 Protecting the Green Sea Turtle

**Optional: 10 minutes**

The purpose of this activity is for students to encounter a situation where the quantity given is not the whole amount, but rather is the amount after a decrease. In this case, they are given the amount after a 10% decrease. They should make the connection from previous lessons that the amount given is 90% of the whole.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR8: Discussion Supports

**Launch**

Give students 3–5 minutes of quiet work time followed with whole-class discussion.

If time is limited pick the second problem to talk about during the discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using double number line diagrams. If students are unsure where to begin, suggest that they draw a double number line diagram to help organize the information provided.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*
Anticipated Misconceptions

For the percent decrease problem, students may calculate 10% of 234, getting a change of 23.4 turtles and an original number of 257.4 turtles. Remind them that the percent decrease describes the change as a percentage of the original value, not as a percentage of the new value. If needed, prompt students to represent the situation using a double number line, placing 234 to the left by 10% of the quantity they want to find, which should be associated with 100%.

Student Task Statement

Green sea turtles live most of their lives in the ocean, but come ashore to lay their eggs. Some beaches where turtles often come ashore have been made into protected sanctuaries so the eggs will not be disturbed.

1. One sanctuary had 180 green sea turtles come ashore to lay eggs last year. This year, the number of turtles increased by 10%. How many turtles came ashore to lay eggs in the sanctuary this year?

2. At another sanctuary, the number of nesting turtles decreased by 10%. This year there were 234 nesting turtles. How many nesting turtles were at this sanctuary last year?

Student Response

1. 198 turtles. This year there was 110% of the number of turtles that came ashore to lay eggs last year, because the full amount increased by 10%. So 198 turtles came ashore to lay eggs this year, because $180 \cdot 1.1 = 198$.

2. 260 turtles. There were only 90% as many nesting turtles this year as last year, because the number decreased by 10%. If there were $t$ nesting turtles last year, we can write the equation $0.9t = 234$. Thus, there were 260 nesting turtles last year because $t = 234 \div 0.9 = 260$.

Activity Synthesis

Ask students:

- How are the two problems the same? How are they different?
- What information is given in the first problem that is not given in the second? Second and not the first?
- In which problem were you given the quantity that represented 100%?

Connect the previous discussion about the original value being 100% by asking:

- For the percent increase question, what percentage value were you given? (students might answer 10% here but that is not actually the case)
- How did you use that to figure out the original number of green sea turtles?
Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Ask 1–2 students to share how they used the given percentage value to figure out the original number of green sea turtles. Ask students to use mathematical language to restate and/or revoice one of the shared explanations. Consider providing students time to first share with a partner, before selecting one or two students to share with the class. This will support student understanding of situations where the quantity given is not the whole amount.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

When solving problems about percent increase and percent decrease, it is very important to start by asking yourself, “What is 100% in this situation?” Then you can find a percentage of that amount.

• “How can you find a 30% increase to 50?” (Find 30% of 50 and add it to 50. Or multiply 50 by 1.3.)

• “If you know that a 30% increase in a quantity is 50, how can you find the original quantity?” (Use a double number line that has 50 aligned with 130.)

• “How can you find a 30% decrease to 50?” (Find 30% of 50 and subtract it from 50. Or multiply 50 by 0.7.)

• “If you know that a 30% decrease in a quantity is 50, how can you find the original quantity?” (Use a double number line that has 50 aligned with 70.)

7.5 More Laundry Soap

Cool Down: 5 minutes

Addressing

• 7.RP.A.3

Student Task Statement

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?

Student Response

66.25 fluid ounces. The new container holds 125% as much soap as the original container, because it holds 25% more. The new container holds 66.25 fluid ounces of laundry soap, because $53 \times 1.25 = 66.25$.

Unit 4 Lesson 7
We can use a double number line diagram to show information about percent increase and percent decrease:

The initial amount of cereal is 500 grams, which is lined up with 100% in the diagram. We can find a 20% increase to 500 by adding 20% of 500:

\[ 500 + (0.2) \cdot 500 = (1.20) \cdot 500 = 600 \]

In the diagram, we can see that 600 corresponds to 120%.

If the initial amount of 500 grams is decreased by 40%, we can find how much cereal there is by subtracting 40% of the 500 grams:

\[ 500 - (0.4) \cdot 500 = (0.6) \cdot 500 = 300 \]

So a 40% decrease is the same as 60% of the initial amount. In the diagram, we can see that 300 is lined up with 60%.

To solve percentage problems, we need to be clear about what corresponds to 100%. For example, suppose there are 20 students in a class, and we know this is an increase of 25% from last year. In this case, the number of students in the class last year corresponds to 100%. So the initial amount (100%) is unknown and the final amount (125%) is 20 students.

Looking at the double number line, if 20 students is a 25% increase from the previous year, then there were 16 students in the class last year.
Lesson 7 Practice Problems
Problem 1

Statement
A bakery used 25% more butter this month than last month. If the bakery used 240 kilograms of butter last month, how much did it use this month?

Solution
300 kilograms

Problem 2

Statement
Last week, the price of oranges at the farmer's market was $1.75 per pound. This week, the price has decreased by 20%. What is the price of oranges this week?

Solution
$1.40 per pound, because 20% of 1.75 is 0.35 and $1.75 − 0.35 = 1.40$

Problem 3

Statement
Noah thinks the answers to these two questions will be the same. Do you agree with him? Explain your reasoning.

○ This year, a herd of bison had a 10% increase in population. If there were 550 bison in the herd last year, how many are in the herd this year?

○ This year, another herd of bison had a 10% decrease in population. If there are 550 bison in the herd this year, how many bison were there last year?

Solution
No, the answers are different. Although the answer to both problems will be larger than 550, the number of bison in each 10% change is different because the original values are not the same.

Problem 4

Statement
Elena walked 12 miles. Then she walked 0.25 that distance. How far did she walk all together? Select all that apply.
Problem 5

Statement
A circle’s circumference is 600 m. What is a good approximation of the circle’s area?

A. 300 m²
B. 3,000 m²
C. 30,000 m²
D. 300,000 m²

Solution
C
(From Unit 3, Lesson 8.)

Problem 6

Statement
The equation \( d = 3t \) represents the relationship between the distance \( (d) \) in inches that a snail is from a certain rock and the time \( (t) \) in minutes.

a. What does the number 3 represent?

b. How many minutes does it take the snail to get 9 inches from the rock?

c. How far will the snail be from the rock after 9 minutes?

Solution
a. The constant of proportionality or the speed of the snail, 3 inches per minute.
b. 3 minutes

c. 27 inches

(From Unit 2, Lesson 6.)
Lesson 8: Percent Increase and Decrease with Equations

Goals

- Explain (orally and in writing) how to calculate the original amount given the new amount and a percentage of increase or decrease.
- Generate algebraic expressions that represent a situation involving percent increase or decrease, and justify (orally) the reasoning.

Learning Targets

- I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it.

Lesson Narrative

In this lesson, students represent situations involving percent increase and percent decrease using equations. They write equations like $y = 1.06x$ to represent growth of a bank account, and use the equation to answer questions about different starting amounts. They write equations like $t - 0.25t = 12$ or $0.75t = 12$ to represent the initial price $t$ of a T-shirt that was $12 after a 25% discount. The focus of this unit is writing equations and understanding their connection to the context. In a later unit on solving equations the focus will be more on using equations to solve problems about percent increase and percent decrease.

When students repeatedly apply a percent increase to a quantity and see that this operation be expressed generally by an equation, they engage in MP8.

Alignments

Building On

- 5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Student Learning Goals

Let's use equations to represent increases and decreases.

8.1 Number Talk: From 100 to 106

Warm Up: 5 minutes
In this lesson, students will be finding percent increase and percent decrease by multiplying by an appropriate factor. For example, to find a 36% increase in a quantity, we can multiply that quantity by 1.36. This warm-up prompts them to think about the scale factor needed to multiply one number to get another.

Building On

- 5.NF.B

Building Towards

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies. Follow with a whole-class discussion.

Student Task Statement

How do you get from one number to the next using multiplication or division?

- From 100 to 106
- From 100 to 90
- From 90 to 100

Unit 4 Lesson 8
Student Response

Answers vary. Sample responses:

- Multiply by 1.06
- Multiply by 0.90
- Divide by 0.90
- Divide by 1.06

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ______’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ______’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

8.2 Interest and Depreciation

10 minutes

In this activity, students are calculating percent increase and decrease in the context of interest and depreciation. They may have encountered these contexts before this unit, but be sure that everyone understands the basic idea of interest and depreciation before students begin work.

Students have enough experience to solve these problems using various strategies and representations. Look for students writing different but correct expressions in terms of x.
Addressing

• 7.RP.A.3

Instructional Routines

• MLR7: Compare and Connect
• Think Pair Share

Launch

Remind students what interest is, give examples of when interest is applied (savings accounts, credit cards, etc.), and discuss depreciation value of an item.

Students in groups of 2. Give students 3–5 minutes of quiet work time, followed by partner then whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organize the information provided.

.Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

If students have a hard time organizing their information, suggest that they use a table.

Students might struggle to write an expression in terms of $x$. Have students describe in words how they calculated the previous problems with numbers and use that to come up with an expression.

Student Task Statement

1. Money in a particular savings account increases by about 6% after a year. How much money will be in the account after one year if the initial amount is $100? $50? $200? $125? $x$ dollars? If you get stuck, consider using diagrams or a table to organize your work.
2. The value of a new car decreases by about 15% in the first year. How much will a car be worth after one year if its initial value was $1,000? $5,000? $5,020? x dollars? If you get stuck, consider using diagrams or a table to organize your work.

Student Response

1. $106, since 100 \cdot 1.06 = 106.
   $53. Since 6% of 100 is 6, we know 6% of 50 is 3. Increasing $50 by $3 gives $53.
   $212. Since 6% of 100 is 6, we know 6% of 200 is 12. Increasing $200 by $12 gives $212.
   $132.50. If we deposit $25, we would have $26.50 after one year, because it is half the amount we would have if we deposit $50. If we deposit $125, we will have $132.50 because $106+26.50=132.50.
   1.06x or another equivalent expression

2. $850. After one year, the car is worth 85% of its original value, because $100 - 15 = 85$. The car would be worth $ because $1,000 \cdot 0.85 = 850.
   $4,250, since 5,000 \cdot 0.85 = 4,250.
   $4,267, since 5,020 \cdot 0.85 = 4,267.
   0.85x or another equivalent expression

Activity Synthesis

Select students with different ways of writing correct expressions in terms of $x$ to share their expression. Connect both problems back to the previous work with the distributive property by asking students:

- How else can the first problem be written? $(x + 0.06x$ or $(1 + 0.06)x$ or $1.06x$)
- How else can the second problem be written? $(x - 0.15x$ or $(1 - 0.15)x$ or $0.85x$)
Access for English Language Learners

**Representing: MLR7 Compare and Connect.** Use this routine when students present their equations that represent the first problem. Ask students to what they notice about the different ways of writing correct expressions. For each expression, draw students’ attention to the meaning of each variable, and to the different ways the percent increase is represented. This will support students’ mathematical language use as they make sense of strategies used to solve problems about percent increase or decrease.

*Design Principle(s): Maximize meta-awareness*

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### 8.3 Matching Equations

**5 minutes**

In this activity, students match equations that represent a percent increase situation to the situations they represent.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR8: Discussion Supports

**Launch**

Arrange students in groups of 2. 2 minutes of quiet think time followed by 2 minutes of partner discussion.

**Student Task Statement**

Match an equation to each of these situations. Be prepared to share your reasoning.

1. The water level in a reservoir is now 52 meters. If this was a 23% increase, what was the initial depth?  
   
   \[0.23x = 52\]

2. The snow is now 52 inches deep. If this was a 77% decrease, what was the initial depth?  
   
   \[0.77x = 52\]

**Student Response**

1. \[1.23x = 52\]
2. \[0.23x = 52\]
Are You Ready for More?

An astronaut was exploring the moon of a distant planet, and found some glowing goo at the bottom of a very deep crater. She brought a 10-gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

1. How much goo will she have after 1 hour? After 2 hours? After 3 hours? After $n$ hours?

2. When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for $n$ hours to return to the original size?

Student Response

1. After 1 hour, there will be 20 grams. After 2 hours, there will be 40 grams. After 3 hours, there will be 80 grams. After $n$ hours, there will be $10 \cdot 2^n$.

2. $\frac{n}{3}$ hours. A 75% decrease is $\frac{1}{4}$ as much, so for every hour, the amount decreases to $\frac{1}{4}$ of what was there at the beginning of the hour. For example, after 2 hours of light exposure, there will be 40 grams of goo, but after only one hour in the dark, it will be back to 10 grams.

Activity Synthesis

Ask one or more students to share which equation they matched with each situation, and resolve any discrepancies. Once the matches are agreed upon, ask students how they would solve the equation to find the amount without actually solving the equations.

Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support whole-class discussion. Provide sentence frames for students to use when they share the equation they matched to each situation: “Situation ____ matches with equation ____ because ____.” Call students attention to how the percent increase is represented in the equation and the situation.

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

8.4 Representing Percent Increase and Decrease: Equations

Optional: 15 minutes

The purpose of this activity is for students to use equations to represent situations of percent increase and decrease. Additionally, students identify the original and new amount in the double number lines to reinforce what they learned in earlier lessons (that the original amount pertains to 100%).

As students work on the task, look for students who created various equations for the last question.
Addressing
  • 7.RP.A.3

Instructional Routines
  • MLR1: Stronger and Clearer Each Time
  • Think Pair Share

Launch
Show students the double number line from the activity in the previous lesson.

Arrange students in groups of 2. Give 5–8 minutes of quiet work time. After 5 minutes allow students to work with a partner or to continue to work alone.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using double number line diagrams. If students are unsure where to begin, suggest that they draw a double number line diagram to help organize the information provided.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions
Students may continue to struggle to recognize the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

Student Task Statement
1. The gas tank in dad's car holds 12 gallons. The gas tank in mom's truck holds 50% more than that. How much gas does the truck's tank hold? Explain why this situation can be represented by the equation \((1.5) \cdot 12 = t\). Make sure that you explain what \(t\) represents.

2. Write an equation to represent each of the following situations.
a. A movie theater decreased the size of its popcorn bags by 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?

b. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?

c. Compared to last year, the population of Boom Town has increased by 25%. The population is now 6,600. What was the population last year?

Student Response

1. 18 gallons. Let \( t \) represent the amount of gas that mom’s truck holds. Dad’s car holds 12 gallons, and mom’s truck holds 50% more or \( 0.5 \cdot 12 \) more, so it holds \( 12 + 0.5 \cdot 12 = 1.5 \cdot 12 \). So \( 1.5 \cdot 12 = t \).

2. a. \((0.8) \cdot 15 = p\), where \( p \) is the amount of popcorn the new bag holds
   
   b. \((0.75) \cdot t = 12\), where \( t \) is the price before the discount
   
   c. \((1.25) \cdot p = 6,600\), where \( p \) is the population last year

Activity Synthesis

Select students to share the values they identified as original amount and the new amount for a few problems. Discuss how 100% always corresponds to the original value and when there is an increase in the value the new value corresponds to a percentage greater than the original 100%.

Select students to share the different equations they came up with. Discuss how the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.

Discuss how solving problems about percent change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how you can find the unknown number. Looking at the examples below, the first two require multiplication, but the others require division.

Using the structure \( A\% \) of \( B \) is \( C \):

- \((1.5) \cdot 12 = c\)
- \((0.80) \cdot 15 = c\)
- \(a \cdot (1,200) = 1,080\)
- \(a \cdot (1.50) = 1.75\)
- \((0.75) \cdot b = 12\)
- \((1.25) \cdot b = 61,600\)
Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. To begin the whole-class discussion, use this routine to give students a structured opportunity to revise and refine their response to the first question, “Explain why this situation can be represented by the equation \((1.5) \cdot 12 = t\).” Give students time to meet with 2–3 partners, to share and get feedback on their initial explanations. Provide listeners with prompts for feedback such as, “What does \(t\) represent?” “Where is 50\%\) in the equation?” “Can you say that another way?” etc. Invite students to go back and refine their written explanation based on the peer feedback they receive. This will help students understand how situations of percent increase or decrease can be represented by an equation.

Design Principle(s): Optimize output (for generalization); Cultivate conversation

Lesson Synthesis

Students should feel confident calculating percent increase/decrease using a method of their choice. Ask students:

- “What are some ways we have learned to solve percent increase or percent decrease problems?” (double number lines, tables, equations)
- “Which representation do you prefer to use? Why?”

8.5 Tyler's Savings Bond

Cool Down: 5 minutes

Addressing

- 7.RP.A.3

Student Task Statement

Tyler's mom purchased a savings bond for Tyler. The value of the savings bond increases by 4\% each year. One year after it was purchased, the value of the savings bond was $156.

Find the value of the bond when Tyler's mom purchased it. Explain your reasoning.

Student Response

Sample reasoning: Represent the situation using the equation \(1.04x = 156\), where \(x\) represents the value of the savings bond when Tyler’s mom purchased it. The solution is \(x = 156 \div 1.04 = 150\), so the bond was originally worth $150.
Student Lesson Summary

We can use equations to express percent increase and percent decrease. For example, if $y$ is 15% more than $x$,

we can represent this using any of these equations:

\[ y = x + 0.15x \quad y = (1 + 0.15)x \quad y = 1.15x \]

So if someone makes an investment of $x$ dollars, and its value increases by 15% to $1250, then we can write and solve the equation $1.15x = 1250$ to find the value of the initial investment.

Here is another example: if $a$ is 7% less than $b$,

we can represent this using any of these equations:

\[ a = b - 0.07b \quad a = (1 - 0.07)b \quad a = 0.93b \]

So if the amount of water in a tank decreased 7% from its starting value of $b$ to its ending value of 348 gallons, then you can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percent increase or percent decrease.
Lesson 8 Practice Problems

Problem 1

Statement
A pair of designer sneakers was purchased for $120. Since they were purchased, their price has increased by 15%. What is the new price?

Solution
$138

Problem 2

Statement
Elena's aunt bought her a $150 savings bond when she was born. When Elena is 20 years old, the bond will have earned 105% in interest. How much will the bond be worth when Elena is 20 years old?

Solution
$307.50

Problem 3

Statement
In a video game, Clare scored 50% more points than Tyler. If \( c \) is the number of points that Clare scored and \( t \) is the number of points that Tyler scored, which equations are correct? Select all that apply.

A. \( c = 1.5t \)
B. \( c = t + 0.5 \)
C. \( c = t + 0.5t \)
D. \( c = t + 50 \)
E. \( c = (1 + 0.5)t \)

Solution
["A", "C", "E"]
**Problem 4**

**Statement**

Draw a diagram to represent each situation:

a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

**Solution**

Answers vary. Sample responses:

a. A tape diagram showing 10 equal pieces labeled “number of miles driven last month” on the top with one below it that is just 7 pieces long and is labeled, “number of miles driven this month.”

b. A tape diagram showing 4 equal pieces labeled “amount of paper they used last month” on the top with one below it that is 5 pieces long and is labeled, “amount of paper they used this month.”

(From Unit 4, Lesson 6.)

**Problem 5**

**Statement**

Which decimal is the best estimate of the fraction \(\frac{29}{40}\)?

A. 0.5

B. 0.6

C. 0.7

D. 0.8

**Solution**

C

(From Unit 4, Lesson 5.)
Problem 6

Statement
Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

Solution
No, since $7.2 \cdot \pi \approx 22.6$.

(From Unit 3, Lesson 3.)
Lesson 9: More and Less than 1%

Goals

- Comprehend that percentages do not have to be a whole number.
- Recognize that 0.1% of a number is 1/10 of 1% of the number.
- Use reasoning about place value to calculate percentages that are not whole numbers, and explain (orally) the strategy.

Learning Targets

- I can find percentages of quantities like 12.5% and 0.4%.
- I understand that to find 0.1% of an amount I have to multiply by 0.001.

Lesson Narrative

Until now, students have been working with whole number percentages when they solve percent increase and percent decrease problems. As they move towards more complex contexts such as interest rates, taxes, tips and measurement error, they will encounter percentages that are not necessarily whole numbers. A percentage is a rate per 100, and now that students are working with ratios of fractions and their associated rates, they can work with fractional amounts per 100. In this lesson students consider situations where fractional percentages arise naturally. They also consider how to calculate a fractional percentage using a whole number percentage as a reference and dividing by 10 or 100. For example, if you know that 1% of 200 is 2, you can use the structure of the base-ten system to reason that 0.1% of 200 is 0.2 and 0.01% of 200 is 0.02 (MP7).

This lesson gives students an opportunity to show that they can attend to precision (MP6) by being careful about the difference between a fractional percentage and a fraction, for example understanding that 0.4% of a quantity is not the same as 0.4 times the quantity.

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
**Building Towards**

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

**Student Learning Goals**

Let’s explore percentages smaller than 1%.

### 9.1 Number Talk: What Percentage?

**Warm Up: 5 minutes**

The purpose of this number talk is to reason about a progressive set of percentages from benchmark percentages to 1% to "unfriendly" percentages. The reasoning parallels the reasoning from earlier work where students are guided to find a unit rate and use the unit rate to solve generic percentage problems. In this activity, there are five problems, so in the interest of time it may not be possible to share all possible strategies for each problem. Instead, gather two different strategies for each.

**Building On**

- 6.RP.A.3

**Building Towards**

- 7.RP.A.3

**Instructional Routines**

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

**Launch**

Display each problem one at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Keep each problem displayed
throughout the discussion. Follow with a whole-class discussion. Students may have difficulty understanding the wording of the question “10 is what percentage of 50?” so when discussing strategies with the whole class, use MLR 7 (Compare and Connect) to see different ways (e.g., words, equations, double number-lines, etc.) to represent and solve these problems. Ask students "What is similar and what is different?" in their approaches.

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

### Anticipated Misconceptions

Students might think the question is asking them to calculate 10% of 50. Ask students a variation of the question: What percentage of 50 is 10?

#### Student Task Statement

Determine the percentage mentally.

10 is what percentage of 50?

5 is what percentage of 50?

1 is what percentage of 50?

17 is what percentage of 50?

#### Student Response

- 20%, because \( \frac{10}{50} = \frac{20}{100} \).
- 10%, because \( \frac{5}{50} = \frac{10}{100} \).
- 2%, because \( \frac{1}{50} = \frac{2}{100} \).
- 34%, because \( \frac{17}{50} = \frac{34}{100} \).

#### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
• “Does anyone want to add on to ____’s strategy?”
• “Do you agree or disagree? Why?”

Since students may not have encountered the idea of percent rate recently, take the time to show any representations of the relationship that come up.

For example, a double number line:

![Double Number Line Diagram]

A table:

<table>
<thead>
<tr>
<th>number</th>
<th>percentage of 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>17</td>
<td>34</td>
</tr>
<tr>
<td>75</td>
<td>150</td>
</tr>
</tbody>
</table>

An equation: If $x$ represents the number and $y$ represents its percentage of 50, then $y = 2x$ since 1 is 2% of 50.

A shortcut that they learned previously: For example, $17 \div 50 = 0.34$, and $0.34$ is $\frac{34}{100}$, so 17 is 34% of 50.

---

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*
9.2 Waiting Tables

10 minutes (there is a digital version of this activity)
This activity gives students an opportunity to put into practice some things they already know about finding percent rates. Additionally, the idea of a fraction of a percent appears for the first time. Encourage students to use any representation they would like to calculate the percentage of appetizers, entrees and desserts. Monitor for students who used various representations and ask them to share during the discussion. The main focus should be on the fractional percentages they encounter in this problem for the first time.

Addressing

• 7.RP.A.3

Instructional Routines

• MLR5: Co-Craft Questions

• Think Pair Share

Launch

Tell students they will be finding some more percentages. Encourage them to use any representation they understand, for example, a double number line or a table. Students in groups of 2. Give students 1–2 minutes of quiet work time, followed by partner then whole-class discussion.

If using the digital activity, students will use an applet to find and check percentages.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about diagrams that can be used to represent percent rates such as tables and double number line diagrams.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Writing and Listening: MLR5 Co-Craft Questions. Before students begin work, display the waiter’s situation without revealing the questions. Ask students to write down possible mathematical questions that might be asked about the situation. Invite pairs to compare their questions, and then ask for a few to be shared in a whole-class discussion. Reveal the actual questions about the waiter’s situation that students will answer. This will help students make sense of the problem before attempting to solve it.
Design Principle(s): Optimize output for explanation
**Anticipated Misconceptions**

If students round to the nearest percentage, they will get that 33% of the dishes were appetizers and 43% of the dishes were entrées. Along with the 25% desserts, their percentages will sum to 101%. Point out that all of the dishes taken as percentages should sum to 100% and encourage them to be critical of their method and try to figure out where the extra 1% came from.

**Student Task Statement**

During one waiter's shift, he delivered 13 appetizers, 17 entrées, and 10 desserts.

1. What percentage of the dishes he delivered were:
   a. desserts?
   b. appetizers?
   c. entrées?

2. What do your percentages add up to?

**Student Response**

Desserts: 25%. There are 40 total dishes because $13 + 17 + 10 = 40$. There are 10 desserts, and $10 \div 40 = 0.25$.

Appetizers: 32.5%. The 13 appetizers are 32.5% of the dishes, because $13 \div 40 = 0.325$.

Entrees: 42.5%. There are 17 entrées, and $17 \div 40 = 0.425$.

The total sums to 100%, because $32.5 + 42.5 + 25 = 100$.

**Activity Synthesis**

Select students to share the percentages they calculated for each type of dish the waiter delivered. Depending on the outcome of the warm-up, it may be appropriate once again to display different representations of percentages as rates per 100.

Double number line:

![Double number line diagram]

Table:
Equation: Students may have previously learned to represent relationships like this using an equation in a form $y = kx$. For example, to find what percent 13 is of 40, they might write $13 = k \cdot 40$, and find that $k$ is 0.325 by evaluating $13 \div 40$. 0.325 is the rate per 1, so 32.5 is the rate per 100.

Students may have never seen a percentage that was not a whole number. Spend a few minutes making sense of this. Ask students:

- What do you notice that is different about these percentages from the ones you have looked at before? (Some of these percentages are not whole numbers.)
- What do the percentages add up to? (Exactly 100.)
- What does 32.5% of 40 mean? (It’s halfway between 32% and 33% of 40.)

### 9.3 Fractions of a Percent

10 minutes
The purpose of this activity is to encourage students to look for efficient strategies while working with fractional percentages. Monitor for students using the following strategies:

- Using 1% to find 0.1%
- Making substitutions of known quantities to help compute unknown quantities

Select students to share these strategies during discussion.

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
Launch
Give students 5 minutes of quiet work time, followed by whole-class discussion.

Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for: Organization; Attention*

Anticipated Misconceptions

When students calculate the various percentages of 60 they may make mistakes in the place value of the answers. Refer students to the previous activity’s discussion. You may also want to ask students to calculate 10% of 60 and use that answer to calculate 30%.

If students get stuck calculating various percents of 5,000, recommend they use the double number line provided. Ask them:

- What percentages are visible in the bottom number line?
- How much is that 1% in reference to the top number line?
- How can we use that 1% to figure out the other percentages?

Use these same questions if students get stuck calculating 15.1% and 15.7%.

Student Task Statement

1. Find each percentage of 60. What do you notice about your answers?

   30% of 60 3% of 60 0.3% of 60 0.03% of 60

2. 20% of 5,000 is 1,000 and 21% of 5,000 is 1,050. Find each percentage of 5,000 and be prepared to explain your reasoning. If you get stuck, consider using the double number line diagram.

   a. 1% of 5,000
   b. 0.1% of 5,000
   c. 20.1% of 5,000
   d. 20.4% of 5,000
3. 15% of 80 is 12 and 16% of 80 is 12.8. Find each percentage of 80 and be prepared to explain your reasoning.

   a. 15.1% of 80
   b. 15.7% of 80

**Student Response**

1. Percentages of 60:
   a. 18 since $0.3 \cdot 60 = 18$.
   b. 1.8 since $0.03 \cdot 60 = 1.8$.
   c. 0.18 since $0.003 \cdot 60 = 0.18$.
   d. 0.018 since $0.0003 \cdot 60 = 0.018$.

   I notice that each percentage is $\frac{1}{10}$ of the previous percentage.

2. Percentages of 5,000:
   a. 50, because $21\% - 20\% = 1\%$ and $1,050 - 1,000 = 50$.
   b. 5, because 5 is $\frac{1}{10}$ of 50.
   c. 1,005, because $1,000 + 5 = 1005$.
   d. 1,020, because $1,000 + 4 \cdot 5 = 1020$.

3. Percentages of 80:
   a. 12.08. One percent of 80 is 0.8, because $16\% - 15\% = 1\%$ and $12.8 - 12 = 0.8$. So 0.1% of 80 is 0.08 because $\frac{1}{10} \cdot 0.8 = 0.08$. Then we add 15% of 80 to 0.1% of 80, which is $12 + 0.08$.
   b. 12.56, because $12 + 7 \cdot 0.08 = 12.56$.

**Are You Ready for More?**

To make Sierpinski's triangle,

- Start with an equilateral triangle. This is step 1.
- Connect the midpoints of every side, and remove the middle triangle, leaving three smaller triangles. This is step 2.
- Do the same to each of the remaining triangles. This is step 3.
• Keep repeating this process.

1. What percentage of the area of the original triangle is left after step 2? Step 3? Step 10?

2. At which step does the percentage first fall below 1%?

Student Response

1. Step 2: 75%. Step 3: 56.25%. Step 10: about 5.63%

2. Step 17

Activity Synthesis

Select previously identified students to share the different strategies used to solve the problems. For the first problem, select students who use the answer to 30% of 60 to calculate the answers to the other problems.

- One likely strategy is one where you keep dividing by 10.

- Another is to make substitutions into an expression. For example, I know that 30% of 60 is 18 and I want to find 3% of 60. I also know that 3% is \( \frac{1}{10} \) of 30%.

\[
\begin{align*}
\text{3\% of 60} &= \frac{1}{10} \\
\text{of 30\% of 60} &= \frac{1}{10} \\
\text{of 18} &= 1.8
\end{align*}
\]

Avoid using the terminology “moving the decimal...” and instead focus on the relationship between 30% and 3%. For the other problems, highlight strategies by students who recognized that they can use 1% of a number to calculate 0.1% of a number and make multiples of that to get, for example, 0.7% of a number.
Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to amplify mathematical uses of language to communicate about the relationship between quantities. As students share their strategies for the first question, revoice their statements to use appropriate mathematical language, such as, “10 times more” or “10 times less.” Invite students to use this language when describing their strategies.
*Design Principle(s): Optimize output (for explanation)*

9.4 Population Growth

Optional: 15 minutes
The purpose of this activity is for students to find a fractional percent increase.

Look for students who calculate the percentage first and then add them together, and students who multiply by 1.08 and 1.008, respectively.

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- MLR7: Compare and Connect

**Launch**
Students in groups of 2. 4 minutes of quiet work time followed by partner and then whole-class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Allow students to use calculators to ensure inclusive participation in the activity.
*Supports accessibility for: Memory; Conceptual processing*

**Anticipated Misconceptions**
Students who want to multiply by $1 + \frac{p}{100}$ may have trouble determining where to put the decimal.
Have them think about the problem in steps. How can you find 8%? (Multiply by 0.08.) How can you find 0.8%? (Multiply by 0.008.)
**Student Task Statement**

1. The population of City A was approximately 243,000 people, and it increased by 8% in one year. What was the new population?

2. The population of city B was approximately 7,150,000, and it increased by 0.8% in one year. What was the new population?

**Student Response**

1. Approximately 262,000. $1.08 \cdot 243,000 \approx 262,000$.

2. Approximately 7,210,000. $1.008 \cdot 7,150,000 \approx 7,210,000$.

**Activity Synthesis**

Have selected students show solutions, starting with a solution where the percentage is found first and then added to the initial amount, then the approach where one multiplies by 1.08 or 1.008, respectively. Make sure everyone understands both methods. Help students see the connections between these strategies.

**Access for English Language Learners**

*Representing, Speaking: MLR7 Compare and Connect.* Use this routine after students present their solutions for calculating the new population of each city. Ask students, "what is the same and what is different about these approaches?" Call students' attention to the different ways students represented the percent increase in their strategies. These exchanges strengthen students' mathematical language use and reasoning based on percent increases with and without fractional amounts.

*Design Principle(s): Maximize meta-awareness*

**Lesson Synthesis**

In this lesson, we worked with fractions of a percentage.

- “How are these percentages related to each other: 40%, 4%, 0.4%, 0.04%?” (Each is $\frac{1}{10}$ of the previous one.)

- “How can we use 40% to help calculate the other percentages?” (Use the fact that 4% is $\frac{1}{10}$ of 40% so if we know 40% of something we can reason to figure out 4%, 0.4% or others.)

- “If we know 1% of a number, how can we use that to help us calculate 0.5% of a number?” (Calculate 5% of that number (5 times 1%) and use same reasoning as above to figure out 0.5%. Alternatively, 0.5% of a number is half of 1% of that number.)
9.5 Percentages of 75

Cool Down: 5 minutes

Addressing
- 7.RP.A.3

**Student Task Statement**
Find each percentage of 75. Explain your reasoning.

1. What is 10% of 75?
2. What is 1% of 75?
3. What is 0.1% of 75?
4. What is 0.5% of 75?

**Student Response**

1. 7.5, because $0.1 \cdot 75 = 7.5$.
2. 0.75, because 1% is $\frac{1}{10}$ of 10%, and $\frac{1}{10} \cdot 7.5 = 0.75$.
3. 0.075, because 0.1% is $\frac{1}{100}$ of 1%, and $\frac{1}{100} \cdot 0.75 = 0.075$.
4. 0.375, because 0.5% is half of 1%, and $0.75 \div 2 = 0.375$.

**Student Lesson Summary**
A percentage, such as 30%, is a rate per 100. To find 30% of a quantity, we multiply it by $30 \div 100$, or 0.3.

The same method works for percentages that are not whole numbers, like 7.8% or 2.5%. In the square, 2.5% of the area is shaded.

To find 2.5% of a quantity, we multiply it by $2.5 \div 100$, or 0.025. For example, to calculate 2.5% interest on a bank balance of $80, we multiply $(0.025) \cdot 80 = 2$, so the interest is $2$.

We can sometimes find percentages like 2.5% mentally by using convenient whole number percents. For example, 25% of 80 is one fourth of 80, which is 20. Since 2.5 is one tenth of 25, we know that 2.5% of 80 is one tenth of 20, which is 2.
Lesson 9 Practice Problems

Problem 1

Statement
The student government snack shop sold 32 items this week. For each snack type, what percentage of all snacks sold were of that type?

<table>
<thead>
<tr>
<th>snack type</th>
<th>number of items sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>fruit cup</td>
<td>8</td>
</tr>
<tr>
<td>veggie sticks</td>
<td>6</td>
</tr>
<tr>
<td>chips</td>
<td>14</td>
</tr>
<tr>
<td>water</td>
<td>4</td>
</tr>
</tbody>
</table>

Solution
Fruit cup: 25%, veggie sticks: 18.75%, chips: 43.75%, water: 12.5%

Problem 2

Statement
Select all the options that have the same value as $3\frac{1}{2}$% of 20.

A. 3.5% of 20
B. $3\frac{1}{2} \cdot 20$
C. $(0.35) \cdot 20$
D. $(0.035) \cdot 20$
E. 7% of 10

Solution
["A", "D", "E"]

Problem 3

Statement
22% of 65 is 14.3. What is 22.6% of 65? Explain your reasoning.
Solution

14.69. \(22.6\% \text{ of } 65 \) is \(22\% \text{ of } 65 \text{ (or } 14.3)\) and an additional \(0.6\% \text{ of } 65\). \(1\% \text{ of } 65 \text{ is } 0.65\). \(0.1\% \text{ of } 65 \text{ is } 0.065\). \(0.6\% \text{ of } 65 \text{ is } 6 \cdot (0.065) = 0.39\). So \(22.6\% \text{ of } 65 \text{ is } 14.69\), because \(14.3 + 0.39 = 14.69\).

Problem 4

Statement

A bakery used 30% more sugar this month than last month. If the bakery used 560 kilograms of sugar last month, how much did it use this month?

Solution

728 kilograms

(From Unit 4, Lesson 7.)

Problem 5

Statement

Match each situation to a diagram. The diagrams can be used more than once.

A. The amount of apples this year decreased by 15% compared with last year's amount.

B. The amount of pears this year is 85% of last year's amount.

C. The amount of cherries this year increased by 15% compared with last year's amount.

D. The amount of oranges this year is 115% of last year's amount.

Solution

- A: 1
- B: 1
Problem 6

Statement
A certain type of car has room for 4 passengers.

a. Write an equation relating the number of cars \( n \) to the number of passengers \( p \).

b. How many passengers could fit in 78 cars?

c. How many cars would be needed to fit 78 passengers?

Solution

a. \( p = 4n \)

b. 312 passengers, because \( 4 \times 78 = 312 \)

c. 20 cars, because \( 78 \div 4 = 19.5 \) and you can't use half of a car.

(From Unit 2, Lesson 6.)

Unit 4 Lesson 9
Section: Applying Percentages

Lesson 10: Tax and Tip

Goals

• Comprehend “sales tax” and “tip” as two contexts that involve adding a percentage of the initial amount.

• Explain (orally) how to calculate the total cost including a tax or tip, given the “subtotal” and the percentage.

• Explain (orally) how to determine the percentage of the subtotal that a tax or tip is.

Learning Targets

• I understand and can solve problems about sales tax and tip.

Lesson Narrative

In this lesson students are introduced to contexts involving sales tax and tips. They can use tape diagrams and double number lines from their grade 6 work, but the lesson provides an opportunity to be more efficient by using an equation of the form \( y = kx \). For example, if the tax rate is 6.2% they can calculate the tax, \( T \), for any price, \( p \), using the equation \( T = 0.062p \). They do not necessarily write this equation out with variables, but rather repeatedly use it with specific values of \( p \). By repeatedly calculating the tax for different prices and then generalizing the process they are engaging in expressing regularity in repeated reasoning (MP8).

Questions about rounding naturally come up in this lesson. This lesson primarily involves dollar amounts, so it is sensible to round to the nearest cent (the nearest hundredth of a dollar). When students attend to precision and make decisions about what is the appropriate level of rounding, they engage in MP6.

Alignments

Addressing

• 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

• 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

• Anticipate, Monitor, Select, Sequence, Connect
• MLR2: Collect and Display
• MLR3: Clarify, Critique, Correct
• MLR6: Three Reads
• Notice and Wonder
• Think Pair Share

Required Materials
Four-function calculators

Required Preparation
It is recommended that students be provided access to four-function calculators so that they can focus on reasoning about how numbers are related to each other, representing those relationships, and deciding which operations are appropriate (rather than focusing on computation.)

Student Learning Goals
Let’s learn about sales tax and tips.

10.1 Notice and Wonder: The Price of Sunglasses

Warm Up: 5 minutes
The purpose of this warm-up is to introduce students to the meaning of sales tax.

Building Towards
• 7.RP.A.3

Instructional Routines
• Notice and Wonder

Launch
Arrange students in groups of 2. Tell students to think of at least one thing they notice or wonder. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have at least one thing they noticed or wondered.

Student Task Statement
You are on vacation and want to buy a pair of sunglasses for $10 or less. You find a pair with a price tag of $10. The cashier says the total cost will be $10.45.
What do you notice? What do you wonder?

**Student Response**

Answers vary. Sample responses:

Students may notice:

- The price of the sunglasses is $10
- The total cost is more than the price of the sunglasses.
- Sales tax was added to the cost of the sunglasses.

Students may wonder:

- If the cashier made a mistake.
- Why the total cost is more than the price listed on the sunglasses.
- What the tax was on the sunglasses.
- If something else was purchased to make the price more.

**Activity Synthesis**

Ask students to share what they noticed and wondered. Record and display the responses for all to see. Students are likely to notice that the total cost is more than the price listed on the sunglasses. Ask students to share why they think the amount shown on the cash register is more than the price of the glasses. Ask them if they have ever heard of sales tax before, and if some have, ask them to share their understanding.
Tell students that sales tax is a fee (an amount of money) paid to the government. The amount of tax is a percentage of the price of the item. Different states charge different sales tax percentages, and additionally some local governments like counties and cities also charge a sales tax.

To start to help make sense of how sales tax works, ask questions like:

- How much sales tax is being collected on the $10 sunglasses? ($0.45 or 45 cents)
- 45 cents is what percentage of $10? (It's 4.5%)
- What is the sales tax rate for our local area? (Varies based on location.)

### 10.2 Shopping in Two Different Cities

**20 minutes**

In this activity, students work with tax rates. Because students reason repeatedly about the same percentage of different quantities, they have the opportunity to represent this process as an equation of the form $y = kx$ (MP8). Students should see connections to their previous work with percent increase. As students work, monitor for different strategies, especially students who note that they can always multiply by the same factor and students who set up and use an equation.

In this activity, the tax rates are whole percentages so that students do not have to deal with rounding. The next activity deals with a tax rate that is a fractional percentage.

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

**Launch**

Provide access to calculators. Arrange students in groups of 2. Make sure students understand the situation by asking questions like, "How much would you have to pay for the paper towels in City 1? And for the lamp in City 2?" ($8.48 and $27)

Tell students, "In some places, there are different sales tax rates for different types of items (clothing, food, medicine, cars, etc.), but the cities in this question have a single sales tax rate for all items."

Give students 3–5 minutes of quiet work time. Afterwards, give students the option to work with a partner or to continue to work alone. Follow with a whole-class discussion.
Access for English Language Learners

Reading, Writing: MLR6 Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., There are two cities. Each city has a different tax rate.). If needed, discuss the meaning of unfamiliar terms at this time. Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., price, sales tax, and total cost). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the table. This will help students connect the language in the word problem, the table, and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Student Task Statement

Different cities have different sales tax rates. Here are the sales tax charges on the same items in two different cities. Complete the tables.

City 1

<table>
<thead>
<tr>
<th>item</th>
<th>price (dollars)</th>
<th>sales tax (dollars)</th>
<th>total cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>paper towels</td>
<td>8.00</td>
<td>0.48</td>
<td>8.48</td>
</tr>
<tr>
<td>lamp</td>
<td>25.00</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>pack of gum</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>laundry soap</td>
<td>12.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

City 2
<table>
<thead>
<tr>
<th>item</th>
<th>price (dollars)</th>
<th>sales tax (dollars)</th>
<th>total cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>paper towels</td>
<td>8.00</td>
<td>0.64</td>
<td>8.64</td>
</tr>
<tr>
<td>lamp</td>
<td>25.00</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>pack of gum</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>laundry soap</td>
<td>12.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>x</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Student Response

**City 1**

<table>
<thead>
<tr>
<th>item</th>
<th>price (dollars)</th>
<th>sales tax (dollars)</th>
<th>total cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>paper towels</td>
<td>8.00</td>
<td>0.48</td>
<td>8.48</td>
</tr>
<tr>
<td>lamp</td>
<td>25.00</td>
<td>1.50</td>
<td>26.50</td>
</tr>
<tr>
<td>pack of gum</td>
<td>1.00</td>
<td>0.06</td>
<td>1.06</td>
</tr>
<tr>
<td>laundry soap</td>
<td>12.00</td>
<td>0.72</td>
<td>12.72</td>
</tr>
<tr>
<td></td>
<td><strong>x</strong></td>
<td>0.06x</td>
<td>1.06x</td>
</tr>
</tbody>
</table>

**City 2**

<table>
<thead>
<tr>
<th>item</th>
<th>price (dollars)</th>
<th>sales tax (dollars)</th>
<th>total cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>paper towels</td>
<td>8.00</td>
<td>0.64</td>
<td>8.64</td>
</tr>
<tr>
<td>lamp</td>
<td>25.00</td>
<td>2.00</td>
<td>27.00</td>
</tr>
<tr>
<td>pack of gum</td>
<td>1.00</td>
<td>0.08</td>
<td>1.08</td>
</tr>
<tr>
<td>laundry soap</td>
<td>12.00</td>
<td>0.96</td>
<td>12.96</td>
</tr>
<tr>
<td></td>
<td><strong>x</strong></td>
<td>0.08x</td>
<td>1.08x</td>
</tr>
</tbody>
</table>

**Unit 4 Lesson 10**
Activity Synthesis
Select previously identified students to share the sales tax they calculated for laundry soap in each city. Have students share how they calculated the sales tax.

Have some students share the expressions for the last row of each tables. Make sure students see the connection between this row and their previous work on percent increase. Point out that sometimes we want to know just the amount of the tax, \(0.06x\), and sometimes we want to know the total cost, which is the price plus the cost, \(x + 0.06x = 1.06x\).

Tell students that, when there is a certain tax that gets applied to a class of goods, it is called a tax rate. Tax rates are usually described in terms of percentages.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I ____ because . . . “, “I noticed ____ so I...”, “Why did you . . . ?”, “I agree/disagree because....”
Supports accessibility for: Language; Social-emotional skills

10.3 Shopping in a Third City

Optional: 5 minutes
The purpose of this activity is for students to encounter a situation in which rounding error makes it look like the relationship between the price of an item and the sales tax is not quite proportional. Students should realize this is due to having a fractional percentage for the tax rate and the custom of rounding dollar amounts to the nearest cent.

Addressing
• 7.RP.A.3

Instructional Routines
• MLR3: Clarify, Critique, Correct
• Think Pair Share

Launch
Provide access to four-function calculators. Keep students in the same groups of 2. Allow students 2 minutes quiet work time followed by partner and whole-class discussions.

Again, tell students that the tax rate for items in City 3 is the same for all types of items.
Access for Students with Disabilities

**Action and Expression: Internalize Executive Functions.** To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students calculate the tax rate for each item before estimating the tax rate of City 3.

*Supports accessibility for: Memory; Organization*

### Anticipated Misconceptions

Some students may say that the relationship is not proportional. Remind them of the activity in a previous unit where they measured the length of the diagonal and the perimeter of several squares and determined that there was really a proportional relationship, even though measurement error made it look like there was not an exact constant of proportionality.

Some students may say that the tax rate is exactly 7%. Prompt them to calculate what the sales tax would have been for the paper towels and the lamp if the tax rate were exactly 7%.

Some students may use 7.25% as the tax rate since that is what comes from the first item (paper towels) without checking this number against the tax on the other items provided. Prompt students to use the additional information they have to check their answer before proceeding to solve the row with laundry soap.

### Student Task Statement

Here is the sales tax on the same items in City 3.

<table>
<thead>
<tr>
<th>item</th>
<th>price (dollars)</th>
<th>sales tax (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>paper towels</td>
<td>8.00</td>
<td>0.58</td>
</tr>
<tr>
<td>lamp</td>
<td>25.00</td>
<td>1.83</td>
</tr>
<tr>
<td>pack of gum</td>
<td>1.00</td>
<td>0.07</td>
</tr>
<tr>
<td>laundry soap</td>
<td>12.00</td>
<td></td>
</tr>
</tbody>
</table>

1. What is the tax rate in this city?

2. For the sales tax on the laundry soap, Kiran says it should be $0.84. Lin says it should be $0.87. Do you agree with either of them? Explain your reasoning.

### Student Response

1. The tax rate is about 7.3%, because this rate can give all the dollar amounts in the table when rounded to the nearest cent. (Note: 7.31% would also work.)
   - For the paper towels, $0.073 \times 8.00 = 0.584$, which rounds to $0.58$.
   - For the lamp, $0.073 \times 25.00 = 1.825$, which rounds to $1.83$. 

Unit 4 Lesson 10
For the pack of gum, \(0.073 \cdot 1.00 = 0.073\), which rounds to $0.07.

2. No, I don't agree with either of them. Since the sales tax on the lamp was $1.83, both Kiran and Lin's answers are too small.
   - To get Kiran's answer of $0.84, the tax rate would be just 7%, but then the tax on the lamp would have been only $1.75.
   - To get Lin's answer of $0.87, the tax rate could be 7.25%, but then the tax on the lamp would still have been only $1.81.
   - The sales tax on the laundry soap should be $0.88, because \(0.073 \cdot 12.00 = 0.876\), which rounds to $0.88.

**Activity Synthesis**

Remind students about when measurement error made it look like the relationship between the length of the diagonal and the perimeter of a square was not quite a proportional relationship.

Consider asking these discussion questions:

- "How did you determine the tax rates for the items in City 1 and City 2 from the previous activity?" (Divide the sales tax by the price.)
- "How did determining the tax rate for City 3 differ from the work you did for the other cities?" (Since the tax rates were not the same for each item, I had to determine what tax rate might give each value listed.)

Tell students that, when multiple pairs of values are known (as in this activity or the previous one), they should work to find an exact tax rate as they did here. If only 1 pair of values is known (as in the following activities), they may use the exact tax rate found from that pair of values. For example, if this activity had only given the price and sales tax for paper towels, we may assume that the tax rate is 7.25%.

**Access for English Language Learners**

*Writing: MLR3 Clarify, Critique, Correct.* Present an incorrect response to the second question that reflects a possible misunderstanding from the class. For example, “Kiran is right because the tax rate is 7%.” Ask students to identify the error (e.g., ask, “Do you agree with the author’s reasoning? Why or why not?”), critique the reasoning, and write a correct explanation. This can help students to further understand the ways rounding errors can influence the appearance of a tax rate.

*Design Principle(s): Maximize meta-awareness*

### 10.4 Dining at a Restaurant

10 minutes
In this activity students use previously learned strategies to solve problems involving fractional percentages. Monitor for students using various strategies (double number line, table, unit rate, equation) and identify students using an equation of the form $y = kx$, especially to solve the last problem.

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

**Launch**
Keep students in groups of 2. Tell students that in some restaurants, people pay the server a tip in addition to paying for the meal. Tips usually range between 10% and 20% of the cost of the meal. Provide access to calculators. Give students 3–5 minutes of quiet work time, followed by partner and whole-class discussion.

**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for:* Organization; Attention

**Anticipated Misconceptions**
Students may attempt to write an equation, but place numbers in the wrong place. Ask them what each piece of their equation means in this situation. In particular, monitor for students who struggle with understanding the second part the first question. Help these students understand by rephrasing the question as, “The total is what percentage of the subtotal?” and helping them to see that the answer should be greater than 100% since the total is greater than the subtotal.

Students might need a way to keep track of all the information. Suggest using a table that keeps track of original price and percent.

**Student Task Statement**
1. Jada has a meal in a restaurant. She adds up the prices listed on the menu for everything they ordered and gets a subtotal of $42.00.
a. When the check comes, it says they also need to pay $3.99 in sales tax. What percentage of the subtotal is the sales tax?

b. After tax, the total is $45.99. What percentage of the subtotal is the total?

c. They actually pay $52.99. The additional $7 is a tip for the server. What percentage of the subtotal is the tip?

2. The tax rate at this restaurant is 9.5%.

Another person's subtotal is $24.95. How much will their sales tax be? Some other person's sales tax is $1.61. How much was their subtotal?

Student Response

1. a. 9.5% because $3.99 ÷ $42 = 0.095.
   
b. 109.5% because $45.99 ÷ $42 = 1.095.
   
c. About 16.67% because 7 ÷ 42 ≈ 0.1667.

2. a. $2.37 because 24.95 × 0.095 = 2.37025.
   
b. Answers vary from $16.90 to $16.99. Possible response: $16.95 because 1.61 ÷ 0.095 ≈ 16.95.
Are You Ready for More?

Elena’s cousins went to a restaurant. The part of the entire cost of the meal that was tax and tip together was 25% of the cost of the food alone. What could the tax rate and tip rate be?

Student Response

Answers vary. Sample response: 5% tax rate and 20% tip.

Activity Synthesis

Select students to share who used different strategies to solve the problems. Sequence them to show solutions that use diagrams first, then an equation like $y = kx$.

If no student uses the equation strategy ask students:

- "How we might use an equation to solve the problem?"
- "What are the two quantities being used in these problems?" (Sales tax and subtotal or tip and subtotal.)

Demonstrate to students how to use the equation to solve a problem.

Help students connect the different strategies.

Access for English Language Learners

Representing, Conversing: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about their approaches for calculating sales tax, tips, subtotals, and total cost. Write down common or important phrases you hear students say to describe the relationships between the quantities onto a visual display. This will help students read and use mathematical language during their paired and whole group discussions. Design Principle(s): Support sense-making

Lesson Synthesis

Students should understand that we reason about fractional percentages like 0.8% and 110.5% using the same strategies we did with percentages that were whole numbers, like 37%. Ask students:

- “Where did we see and use fractional percentages in this lesson?” (Sales tax)
- “What are strategies we can use to calculate fractional percentages (including sales tax)?” (Double number lines, tables, an equation)

10.5 A Restaurant in a Different City

Cool Down: 5 minutes

Unit 4 Lesson 10
This cool-down assesses whether students understand how to calculate a tax rate based on a price before tax was added and the amount of tax added. Additionally, they must use proportional reasoning to determine the tax on another item with the same tax rate.

**Addressing**
- 7.RP.A.3

**Student Task Statement**
At a dinner, the meal cost $22 and a sales tax of $1.87 was added to the bill.

1. How much would the sales tax be on a $66 meal?
2. What is the tax rate for meals in this city?

**Student Response**
1. $5.61, because $22 \cdot 3 = 66$ and $1.87 \cdot 3 = 5.61$.
2. 8.5%, because $1.87 \div 22 = 0.085$.

**Student Lesson Summary**
Many places have sales tax. A sales tax is an amount of money that a government agency collects on the sale of certain items. For example, a state might charge a tax on all cars purchased in the state. Often the tax rate is given as a percentage of the cost. For example, a state's tax rate on car sales might be 2%, which means that for every car sold in that state, the buyer has to pay a tax that is 2% of the sales price of the car.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone buys something, they have to add 0.075 times the amount on the price tag to determine the total cost of the item.

For example, if the price tag on a T-shirt in Arizona says $11.50, then the sales tax is $(0.075) \cdot 11.5 = 0.8625$, which rounds to 86 cents. The customer pays $11.50 + 0.86$, or $12.36$ for the shirt.

The total cost to the customer is the item price plus the sales tax. We can think of this as a percent increase. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A tip is an amount of money that a person gives someone who provides a service. It is customary in many restaurants to give a tip to the server that is between 10% and 20% of the cost of the meal. If a person plans to leave a 15% tip on a meal, then the total cost will be 115% of the cost of the meal.
Lesson 10 Practice Problems

Problem 1

Statement

In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total price including tax for each item.

<table>
<thead>
<tr>
<th>item</th>
<th>price before tax ($)</th>
<th>sales tax ($)</th>
<th>price including tax ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pillow</td>
<td>8.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>blanket</td>
<td>22.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trash can</td>
<td>14.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>item</th>
<th>price before tax ($)</th>
<th>sales tax ($)</th>
<th>price including tax ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pillow</td>
<td>8.00</td>
<td>0.58</td>
<td>8.58</td>
</tr>
<tr>
<td>blanket</td>
<td>22.00</td>
<td>1.60</td>
<td>23.60</td>
</tr>
<tr>
<td>trash can</td>
<td>14.50</td>
<td>1.05</td>
<td>15.55</td>
</tr>
</tbody>
</table>

For the blanket and the trash can, the tax is rounded to the nearest cent: it is rounded up for the blanket and rounded down for the trash can.

Problem 2

Statement

The sales tax rate in New Mexico is 5.125%. Select all the equations that represent the sales tax, \( t \), you would pay in New Mexico for an item of cost \( c \)?

A. \( t = 5.125c \)

B. \( t = 0.5125c \)

C. \( t = 0.05125c \)

D. \( t = c \div 0.05125 \)

E. \( t = \dfrac{5.125}{100}c \)
Problem 3

Statement
Here are the prices of some items and the amount of sales tax charged on each in Nevada.

<table>
<thead>
<tr>
<th>cost of item ($)</th>
<th>sales tax ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.46</td>
</tr>
<tr>
<td>50</td>
<td>2.30</td>
</tr>
<tr>
<td>5</td>
<td>0.23</td>
</tr>
</tbody>
</table>

a. What is the sales tax rate in Nevada?

b. Write an expression for the amount of sales tax charged, in dollars, on an item that costs \( c \) dollars.

Solution

a. 4.6%

b. 0.046\( c \) or equivalent

Problem 4

Statement
Find each amount:

a. 3.8% of 25

b. 0.2% of 50

c. 180.5% of 99

Solution

a. 0.95

b. 0.1

c. 178.695

(From Unit 4, Lesson 9.)
Problem 5

Statement
On Monday, the high was 60 degrees Fahrenheit. On Tuesday, the high was 18% more. How much did the high increase from Monday to Tuesday?

Solution
10.8 degrees Fahrenheit.

(From Unit 4, Lesson 8.)

Problem 6

Statement
Complete the table. Explain or show your reasoning.

<table>
<thead>
<tr>
<th>object</th>
<th>radius</th>
<th>circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ceiling fan</td>
<td>2.8 ft</td>
<td></td>
</tr>
<tr>
<td>water bottle cap</td>
<td>13 mm</td>
<td></td>
</tr>
<tr>
<td>bowl</td>
<td></td>
<td>56.5 cm</td>
</tr>
<tr>
<td>drum</td>
<td></td>
<td>75.4 in</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>object</th>
<th>radius</th>
<th>circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ceiling fan</td>
<td>2.8 ft</td>
<td>17.6 ft</td>
</tr>
<tr>
<td>water bottle cap</td>
<td>13 mm</td>
<td>82 mm</td>
</tr>
<tr>
<td>bowl</td>
<td>9 cm</td>
<td>56.5 cm</td>
</tr>
<tr>
<td>drum</td>
<td>12 in</td>
<td>75.4 in</td>
</tr>
</tbody>
</table>

The constant of proportionality is $2 \cdot \pi$. The given radii is multiplied by 6.28 to find the missing circumferences, and the given circumferences is divided by 6.28 to find the missing radii.

(From Unit 3, Lesson 4.)
Lesson 11: Percentage Contexts

Goals
• Comprehend “interest,” “markup,” “markdown,” and “commission” as other contexts that involve adding or subtracting a percentage of the initial amount.

• Determine the original dollar amount before a markup, markdown, or commission.

• Explain (orally) how to calculate the new dollar amount after a markup, markdown, or commission.

Learning Targets
• I understand and can solve problems about commission, interest, markups, and discounts.

Lesson Narrative
In this lesson students are introduced to contexts involving markups, discounts, and commissions, and they continue to study contexts involving tax and tips.

Questions about rounding may naturally come up in this lesson. This lesson primarily involves dollar amounts, so it is sensible to round to the nearest cent (the nearest hundredth of a dollar). Percentages may be rounded to the nearest whole percent or fraction of a percent, depending on the situation.

Alignments
Building On
• 6.EE.A.2.b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

• 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Addressing
• 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards
• 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR6: Three Reads
- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Four-function calculators
Instructional master
Pre-printed slips, cut from copies of the

Required Preparation

Print and cut up slips from the Card Sort: Percentage Situations Instructional master. Prepare 1 copy for every 2 students. These may be re-used if you have multiple classes.

It is recommended that students be provided access to four-function calculators so that they can focus on reasoning about how numbers are related to each other, representing those relationships, and deciding which operations are appropriate (rather than focusing on computation.)

Student Learning Goals

Let’s learn about more situations that involve percentages.

11.1 Leaving a Tip

Warm Up: 5 minutes
The purpose of this warm-up is to help students connect their current work with percentage contexts to their prior work on percent increase and efficient ways of finding percent increase.

Building On

- 6.EE.A.2.b
- 6.RP.A.3.c

Building Towards

- 7.RP.A.3

Launch

Consider telling students that these questions may have more than one correct answer. Students in groups of 2. 2 minutes of quiet think time followed by partner and then whole-class discussion.

Student Task Statement

Which of these expressions represent a 15% tip on a $20 meal? Which represent the total bill?
Student Response

The second and third expressions represent the total bill while the last expression represents the tip.

Activity Synthesis

For each expression, ask a few students to explain whether they think it represents: the total bill, the tip, or neither. For each expression, select a student to explain their reasoning.

11.2 A Car Dealership

10 minutes

The purpose of this activity is to introduce students to a context involving markups and markdowns or discounts, and to connect this to the work on percent increase and percent decrease they did earlier. The first question helps set the stage for students to see the connection to markups and percent increase. Look for students who solve the second question by finding 90% of the retail price, and highlight this approach in the discussion.

Addressing

• 7.RP.A.3

Instructional Routines

• MLR6: Three Reads
• Think Pair Share

Launch

Tell students that a mark-up is a percentage that businesses often add to the price of an item they sell, and a mark-down is a percentage they take off of a given price. If helpful, review the meaning of wholesale (the price the dealership pays for the car) and retail price (the price the dealership charges to sell the car). Sometimes people call mark-downs discounts.

Provide access to calculators. Students in groups of 2. Give students 5 minutes of quiet work time, followed by partner then whole-class discussion.
Access for English Language Learners

Reading, Writing: MLR6 Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., A car dealership bought a car. The dealership wants to make a profit. They need to decide what price the car should be.). If needed, discuss the meaning of unfamiliar terms at this time (e.g., profit, wholesale, retail price, commission, etc.). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., wholesale price, profit or mark-up, and retail price). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the task. This will help students connect the language in the word problem and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Anticipated Misconceptions

It is important throughout that students attend to the meanings of particular words and remain clear on the meaning of the different values they find. For example, "wholesale price," "retail price," and "sale price" all refer to specific dollar amounts. Help students organize their work by labeling the different quantities they find or creating a graphic organizer.

Student Task Statement

A car dealership pays a wholesale price of $12,000 to purchase a vehicle.

1. The car dealership wants to make a 32% profit.
   a. By how much will they mark up the price of the vehicle?
   b. After the markup, what is the retail price of the vehicle?

2. During a special sales event, the dealership offers a 10% discount off of the retail price. After the discount, how much will a customer pay for this vehicle?

Student Response

1. a. $3,840, because $0.32 \times 12,000 = 3,840.
   b. $15,840. Possible explanations: $12,000 + 3,840 = 15,840$ or $1.32 \times 12,000 = 15,840$.

2. $14,256. Possible explanations: because $0.1 \times 15,840 = 1,584$, and $15,840 - 1,584 = 14,256$ or $0.9 \times 15,840 = 14,256$.

Unit 4 Lesson 11
Are You Ready for More?

This car dealership pays the salesperson a bonus for selling the car equal to 6.5% of the sale price. How much commission did the salesperson lose when they decided to offer a 10% discount on the price of the car?

Student Response

$102.96. Before the discount, the salesperson would have earned a bonus of $1,029.60 (15,840 · 0.065 = 1,029.6). After the discount, the salesperson only earned $926.64 (14,256 · 0.065 = 926.64), so the salesperson lost $102.96 (1029.6 − 926.64 = 102.96).

Activity Synthesis

For the first question, help students connect markups to percent increase.

Select students to share solutions to the second question. Highlight finding 90% of the retail price, and reinforce that a 10% discount is a 10% decrease.

Ask them to describe how they would find (but not actually find) . . .

- "The retail price after a 12% markup?" (Multiply the retail price by 0.12, then add that answer to the retail price. Alternatively, multiply the retail price by 1.12.)

- "The price after a 24% discount?" (Multiply the retail price by 0.24, then subtract that answer from the retail price. Alternatively, multiply the retail price by 0.76.)

11.3 Commission at a Gym

10 minutes

The purpose of this activity is to introduce students to the concept of a commission and to solve percentage problems in that context. Students continue to practice finding percentages of total prices in a new context of commission.

Monitor for students who use equations like $c = r \cdot p$ where $c$ is the commission, $r$ represents the percentage of the total that goes to the employee, and $p$ is the total price of the membership.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share
Launch

Tell students that a commission is the money a salesperson gets when they sell an item. It is usually used as an incentive for employees to try to sell more or higher priced items than they usually would. The commission is usually a percentage of the price of the item they sell.

Provide access to calculators. Students in groups of 2. Give students 2 minutes of quiet work time. Partner then whole-class discussion.

Anticipated Misconceptions

Students may find the percentage of an incorrect quantity. Ask them to state, in words, what they are finding a percentage of.

Students may not understand the first question. Tell them that a membership is sold for a certain price and the money is split with $42 going to the gym and $8 going to the employee.

Student Task Statement

1. For each gym membership sold, the gym keeps $42 and the employee who sold it gets $8. What is the commission the employee earned as a percentage of the total cost of the gym membership?

2. If an employee sells a family pass for $135, what is the amount of the commission they get to keep?

Student Response

1. 16%, because $42 + 8 = 50$ and $8 ÷ 50 = 0.16$.

2. $21.60$, because $0.16 \times 135 = 21.6$.

Activity Synthesis

Select students to share how they answered the questions.

During the discussion, draw attention to strategies for figuring out which operations to do with which numbers. In particular, strategies involving equations like $c = r \times p$ where $c$ is the commission, $r$ represents the percentage of the total that goes to the employee, and $p$ is the total price of the membership.
Access for English Language Learners

*Reading, Writing: MLR3 Clarify, Critique, Correct.* Present an incorrect response to the second question that reflects a possible misunderstanding from the class. For example, “For the family membership of $135, the employee would keep $8.” Prompt students to identify the error (e.g., ask, “Do you agree with the author’s reasoning? Why or why not?”), and then write a correct version. This will help students to understand that the employee’s commission is always a rate of 16% and not a flat amount of $8.  
*Design Principle(s): Maximize meta-awareness*

11.4 Card Sort: Percentage Situations

Optional: 10 minutes  
This activity gives students an opportunity to practice various vocabulary terms that come along with percentages. Students are asked to sort scenarios to different descriptors using the images, sentences or questions found on the scenario cards. The questions found on the scenario cards are intended to help students figure out which descriptor the scenario card belongs under.

As students work on the task, identify students that are using the vocabulary: tip, tax, gratuity, commission, markup/down, and discount. These students should be asked to share during the discussion.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR8: Discussion Supports
- Take Turns

**Launch**

Arrange students in groups of 2. Distribute the sorting cards, and explain that students will sort 8 scenarios into one of 6 categories. Demonstrate how students can take turns placing a scenario under a category and productive ways to disagree. Here are some questions they might find useful:

- Which category would you sort this under?
- What do you think this word means?
- What words can we use as clues about where to sort this card?
**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.  
*Supports accessibility for: Conceptual processing; Organization*

---

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gestures, and talking about the context of what is happening. This will help students to produce and make sense of the language needed to communicate their own ideas.  
*Design Principle(s): Optimize output (for explanation)*

---

**Anticipated Misconceptions**

Students should use the question at the bottom of the card to help them if they get stuck sorting the scenarios.

**Student Task Statement**

Your teacher will give you a set of cards. Take turns with your partner matching a situation with a descriptor. For each match, explain your reasoning to your partner. If you disagree, work to reach an agreement.

**Student Response**

Gratuity/Tip with "Kiran ate . . ."
Commission with "Diego’s uncle . . ."
Interest with "Andre is saving . . ." and "Clare’s aunt . . ."
Discount/Markdown with "Tyler bought . . ." and "Priya used . . ."
Sales Tax with "Lin bought . . ."
Markup with "A car dealership . . ."

**Activity Synthesis**

Ask identified students to share which situations they sorted under each word. Ask them:

- "What made you decide to put these situations under this descriptor?"
- "Were there any situations that you were really unsure of? What made you decide on where to sort them?"

---

**Unit 4 Lesson 11**
Consider asking some groups to order the situations from least to greatest in terms of the dollar amount of the increase or decrease and asking other groups to order them in terms of the percentage. Then, have them compare their results with a group that did the other ordering.

Answer students’ remaining questions about any of these contexts. Tell students there is a copy of this chart at the end of the lesson that they can use as a reference tool during future lessons. Allow them a space to take notes on their own to remember it or details from one of the activity examples.

<table>
<thead>
<tr>
<th>paid to:</th>
<th>how it works:</th>
</tr>
</thead>
<tbody>
<tr>
<td>sales tax</td>
<td>the government added to the price of the item</td>
</tr>
<tr>
<td>gratuity (tip)</td>
<td>the server added to the cost of the meal</td>
</tr>
<tr>
<td>interest</td>
<td>the lender (or account holder) added to the balance of the loan, credit card, or bank account</td>
</tr>
<tr>
<td>markup</td>
<td>the seller added to the price of an item so the seller can make a profit</td>
</tr>
<tr>
<td>markdown (discount)</td>
<td>the customer subtracted from the price of an item to encourage the customer to buy it</td>
</tr>
<tr>
<td>commission</td>
<td>the salesperson subtracted from the payment the store collects</td>
</tr>
</tbody>
</table>

**Lesson Synthesis**

In this lesson, we studied lots of different situations where people use percentages.

- “What are some situations in life in which people encounter percentages?”
- “Give examples of situations where you would encounter tax, tip, markup, markdown, commission.” (Lots of possible answers.)
- “When an item is marked down 10%, why does it make sense to multiply the price by 0.9?” (Since there is 10% off of the price, the new cost is 90% of the original.)
- “When an item is marked up 25%, why does it make sense to multiply the price by 1.25?” (Since the item now costs 100% plus an extra 25%, the new item costs 1.25 times the original.)
11.5 The Cost of a Bike

Cool Down: 5 minutes
The purpose of this activity is to check whether students can solve a problem involving a mark-up and a discount.

Addressing
• 7.RP.A.3

Student Task Statement
The bike store marks up the wholesale cost of all of the bikes they sell by 30%.

1. Andre wants to buy a bike that has a price tag of $125. What was the wholesale cost of this bike?

2. If the bike is discounted by 20%, how much will Andre pay (before tax)?

Student Response
1. $96.15, because $125 ÷ 1.3 = 96.15.

2. $100, because $125 · 0.8 = 100.

Student Lesson Summary
There are many everyday situations where a percentage of an amount of money is added to or subtracted from that amount, in order to be paid to some other person or organization:

<table>
<thead>
<tr>
<th></th>
<th>goes to</th>
<th>how it works</th>
</tr>
</thead>
<tbody>
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<td>markup</td>
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<td>added to the price of an item so the seller can make a profit</td>
</tr>
<tr>
<td>markdown (discount)</td>
<td>the customer</td>
<td>subtracted from the price of an item to encourage the customer to buy it</td>
</tr>
<tr>
<td>commission</td>
<td>the salesperson</td>
<td>subtracted from the payment that is collected</td>
</tr>
</tbody>
</table>

For example,
• If a restaurant bill is $34 and the customer pays $40, they left $6 dollars as a tip for the server. That is 18% of $34, so they left an 18% tip. From the customer’s perspective, we can think of this as an 18% increase of the restaurant bill.

• If a realtor helps a family sell their home for $200,000 and earns a 3% commission, then the realtor makes $6,000, because $(0.03) \cdot 200,000 = 6,000$, and the family gets $194,000$, because $200,000 - 6,000 = 194,000$. From the family’s perspective, we can think of this as a 3% decrease on the sale price of the home.
Lesson 11 Practice Problems

Problem 1

Statement
A car dealership pays $8,350 for a car. They mark up the price by 17.4% to get the retail price. What is the retail price of the car at this dealership?

Solution
$9802.90, although most dealerships round to the nearest 5 or 10.

Problem 2

Statement
A store has a 20% off sale on pants. With this discount, the price of one pair of pants before tax is $15.20. What was the original price of the pants?

A. $3.04
B. $12.16
C. $18.24
D. $19.00

Solution
D

Problem 3

Statement
Lin is shopping for a couch with her dad and hears him ask the salesperson, “How much is your commission?” The salesperson says that her commission is 5\(\frac{1}{2}\)% of the selling price.

a. How much commission will the salesperson earn by selling a couch for $495?

b. How much money will the store get from the sale of the couch?

Solution
a. $27.23. 5.5% of 495 is 27.225.

b. $467.77
Problem 4

Statement

A college student takes out a $7,500 loan from a bank. What will the balance of the loan be after one year (assuming the student has not made any payments yet):

a. if the bank charges 3.8% interest each year?

b. if the bank charges 5.3% interest each year?

Solution

a. $7,785.00

b. $7,897.50

(From Unit 4, Lesson 9.)

Problem 5

Statement

Match the situations with the equations.

a. Mai slept for $x$ hours, and Kiran slept for \frac{1}{10} less than that.  
   \[ y = 2.33x \]

b. Kiran practiced the piano for $x$ hours, and Mai practiced for \frac{2}{3} less than that.  
   \[ y = 1.375x \]

c. Mai drank $x$ oz of juice and Kiran drank \frac{4}{3} more than that.  
   \[ y = 0.9x \]

d. Kiran spent $x$ dollars and Mai spent \frac{1}{4} less than that.  
   \[ y = 1.6x \]

e. Mai ate $x$ grams of almonds and Kiran ate 1.5 times more than that.  
   \[ y = 0.7x \]

f. Kiran collected $x$ pounds of recycling and Mai collected \frac{3}{10} less than that.  
   \[ y = 2.5x \]

g. Mai walked $x$ kilometers and Kiran walked \frac{3}{5} more than that.  

h. Kiran completed $x$ puzzles and Mai completed \frac{3}{4} more than that.
Solution

a. \( y = 0.9x \)
b. \( y = 0.6x \)
c. \( y = 2.33x \)
d. \( y = 0.75x \)
e. \( y = 2.5x \)
f. \( y = 0.7x \)
g. \( y = 1.375x \)
h. \( y = 1.6x \)

(From Unit 4, Lesson 5.)
Lesson 12: Finding the Percentage

Goals

● Determine what information is needed to solve a problem involving sales tax and discounts. Ask questions to elicit that information.

● Explain (orally) how to calculate the percentage, given the dollar amounts before and after a sales tax, tip, or discount.

● Interpret (orally and in writing) tape diagrams that represent situations involving a sales tax, tip, or discount.

Learning Targets

● I can find the percentage increase or decrease when I know the original amount and the new amount.

Lesson Narrative

In this lesson, students consolidate what they have learned over the last few lessons and solve a variety of multi-step percentage problems involving taxes, tips, and discounts, including problems involving fractional percentages. They continue to move towards using equations to represent problems, which enable them to see the common underlying structure behind different problems (MP7). For example, $1.2x$ can represent

● a 20% increase in $x$.

● the total bill when 20% tax is added.

● the total bill when a 20% tip is added.

● the retail price when the wholesale price is marked up by 20%.

Alignments

Building On

● 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

● 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR4: Information Gap Cards
- Think Pair Share

Required Materials

Four-function calculators
Instructional master
Pre-printed slips, cut from copies of the

Required Preparation

Print and cut up slips from the Info Gap: Sporting Goods Instructional master. One copy of the Instructional master is needed for every 4 students. A class set could be re-used if you have more than one class.

Student Learning Goals

Let’s find unknown percentages.

12.1 Tax, Tip, and Discount

Warm Up: 5 minutes
In this warm-up, students are reminded of the tape diagram method for understanding parts of a whole. The tape diagrams are used in the context of tips, taxes, and discounts.

Building On

- 6.RP.A.3

Building Towards

- 7.RP.A.3

Instructional Routines

- Think Pair Share

Launch

Students in groups of 2. Allow students 2 minutes quiet work time followed by partner then whole-class discussion.
**Student Task Statement**
What percentage of the car price is the tax?

![Diagram of car price and tax]

What percentage of the food cost is the tip?

![Diagram of food cost and tip]

What percentage of the shirt cost is the discount?

![Diagram of shirt cost and discount]

**Student Response**
The tax is 25% since it is the same size as one fourth of the entire car price.

The tip is 20% since it is the same size as one fifth of the entire food cost.

The discount is about 33% since it is the same size as one third of the entire shirt cost.

**Activity Synthesis**
The purpose of the discussion is for students to recognize that a tape diagram can be useful for working with percentages as part of a whole.

Consider asking these discussion questions:

- "The tax on the car is what fraction of the car price before tax was added on?" (One fourth)
- "With the tip added on, how is the length of the entire bar related to the length of the bar that just represents the food cost?" (It is 1.2 times as long.)
- "How does the value \( \frac{2}{3} \) relate to the last tape diagram?" (It represents the fraction of the original cost of the shirt that is the price of the shirt after the discount.)
12.2 What Is the Percentage?

10 minutes
In this activity, students continue to practice finding percentages from dollar amounts including commission, tip, and markdown. Some of the questions require multiple steps to solve for the percentage needed.

As students work, monitor for students who take different approaches to solving the multiple steps involved in these problems. For example, some students may subtract the discounted price from the original price to find the amount that the item has been marked down before finding the percentage of markdown. Other students may find the percentage of the original price used for the discounted price and then subtract that from 100%.

Addressing
- 7.RP.A.3

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

Launch
Arrange students in groups of 2. Give students 3–5 minutes of quiet work time followed by partner and whole-class discussions.

Student Task Statement
1. A salesperson sold a car for $18,250 and their commission is $693.50. What percentage of the sale price is their commission?

2. The bill for a meal was $33.75. The customer left $40.00. What percentage of the bill was the tip?

3. The original price of a bicycle was $375. Now it is on sale for $295. What percentage of the original price was the markdown?

Student Response
1. 3.8% since $693.50 \div 18,250 = 0.038$.

2. about 18.52% since the tip is $6.25 and $6.25 \div 33.75 \approx 0.1852$.

3. about 21.33% since the price was marked down by $80 and $80 \div 375 \approx 0.2133$.

Are You Ready for More?
To make a Koch snowflake,
- Start with an equilateral triangle. This is step 1.
• Divide each side into 3 equal pieces. Construct a smaller equilateral triangle on the middle third. This is step 2.
• Do the same to each of the newly created sides. This is step 3.
• Keep repeating this process.

By what percentage does the perimeter increase at step 2? Step 3? Step 10?

**Student Response**

The perimeter increases by a factor of \(\frac{4}{3}\) at each step, so the percent increase at each step is \(33\frac{1}{3}\%\).

In step 2, the perimeter has increased by about 33.3%.

In step 3, the perimeter has increased by about 77.8% since the perimeter is \(\frac{4}{3}\) of the perimeter in step 2 which is, in turn, \(\frac{4}{3}\) of the perimeter in step 1, so the step 3 is \(\frac{4}{3} \cdot \frac{4}{3} = \frac{16}{9} \approx 1.778\) of the length of the perimeter in step 1.

In step 10, the perimeter has increased by about 1,675.8% since it increases by \(\frac{4}{3}\) at each step, so by step 10 the perimeter is \(\left(\frac{4}{3}\right)^{10} = \frac{1,048,576}{59,049} \approx 17.758\) of the length of the perimeter in step 1.

**Activity Synthesis**

The purpose of the discussion is for students to share methods for finding percentages in problems that require multiple steps.

Select the previously identified students to share their different methods for solving the last 2 questions. Sequence so strategies using diagrams come first, and those using equations come second.

**Access for Students with Disabilities**

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I ____ because . . .”, “I noticed ____ so I . . .”, “Why did you . . .?”, “I agree/disagree because . . .”

*Supports accessibility for: Language; Social-emotional skills*
Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present an incorrect response to the third question that reflects a possible misunderstanding from the class. For example, “It’s 27.12% because 375/295 is 1.2712.” To prompt students to identify the error, ask, “Do you agree with the author’s reasoning? Explain why or why not? and write a correct version.” Amplify mathematical language that students use, drawing attention to where precise language helps improve understanding. This can help students to further understand how to calculate the percentage of a markdown when given the original and sale prices.

Design Principle(s): Support sense-making

12.3 Info Gap: Sporting Goods

20 minutes
The purpose of this info gap activity is for students to identify the essential information needed to determine the total savings after various discounts are applied to different items.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Addressing
• 7.RP.A.3

Unit 4 Lesson 12
**Instructional Routines**

- MLR4: Information Gap Cards

**Launch**

Provide access to calculators. Tell students they will continue to work with percentages and their similarities to proportional relationships. Explain the Info Gap structure and consider demonstrating the protocol if students are unfamiliar with it. There are step-by-step instructions in the student task statement.

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

**Access for Students with Disabilities**

*Engagement: Develop Effort and Persistence.* Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

*Supports accessibility for: Memory; Organization*

**Access for English Language Learners**

*Conversing:* This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving discounts. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)?”, and “Why do you need to know . . . (that piece of information)?”

*Design Principle(s): Cultivate Conversation*

**Anticipated Misconceptions**

Students might fail to notice that Elena and Andre buy multiple cans of tennis balls and packages of socks, respectively. Ask students to figure out how much 2 packages of socks (or 3 cans of tennis balls) will cost.

If students automatically give the 15% discount on all of Elena's purchases, ask students which of Elena’s items fall under the discount.

Students might apply the discount after the adding in the sales tax. Remind students that the discount gets applied to the subtotal before the tax is calculated.

Some students may include the sales tax when calculating the percentage of Andre's savings. Remind them that the problem specifies “before tax.”
**Student Task Statement**

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “What specific information do you need?” and wait for them to ask for information.
   
   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask “Why do you need that information?” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

**Student Response**

1. Elena will pay $52.68. Sample explanation: Elena spent $36.55 on the tennis racket and $12 on the cans of tennis balls. The total price before tax was $48.55, because $36.55 + $12 = $48.55. The total cost including tax is $52.68, because $48.55 \times 1.085 = $52.68.

2. Andre saved about 8.7%. Sample explanation: The regular price for the baseball glove and two packages of socks is $46, because $34 + 6 + 6 = 46$. Andre saved $4, because each package of socks was discounted by $2, he bought 2 packages, and $2 \times 2 = 4$. This savings is about 8.7% of the regular price, because $4 \div 46 \approx 0.087$.

**Activity Synthesis**

The purpose of the discussion is for students to recognize what information may be needed to solve problems involving percentages for prices of items.

After students have completed their work, share the correct answers and ask students to discuss the different ways they solved this problem. Some guiding questions:
• "What information did you and your partner have to figure out?"

• "How did you determine the cost of Elena’s tennis racket?" (Multiply the original cost by 0.85 or multiply by 0.15 and subtract from the original cost.)

• "How did you determine the total cost after tax for Elena’s purchases?" (Multiply the total by 1.085 or multiply by 0.085 and add to the original cost.)

• "What different calculations did you have to make for Andre and Elena’s situations?"

• "Was there information given that you did not need to use?"

Lesson Synthesis
In this lesson, students found the percentage increase or decrease given the original and final amounts.

• “When the original price and discounted prices are known, how can we find the percent markdown?” (Find the difference and then find the percentage that difference is of the original price.)

• “When you know the original price of an item and the price you paid at a register, how can you find the tax rate?” (Find the difference and then find the percentage that difference is of the original price.)

12.4 Shoes on Sale

Cool Down: 5 minutes
In this cool-down, students are assessed on their ability to find the percentage discount on a pair of shoes given the original and discounted prices.

Addressing

• 7.RP.A.3

Student Task Statement
With a coupon, you can get a pair of shoes that normally costs $84 for only $72. What percentage was the discount?

Student Response
Approximately 14.3% since the discount takes $12 off the price of shoes and $12 ÷ $84 ≈ 0.143.

Student Lesson Summary
To find a 30% increase over 50, we can find 130% of 50. 1.3 • 50 = 65
To find a 30% decrease from 50, we can find 70% of 50. 0.7 • 50 = 35
If we know the initial amount and the final amount, we can also find the percent increase or percent decrease. For example, a plant was 12 inches tall and grew to be 15 inches tall. What percent increase is this? Here are two ways to solve this problem:

The plant grew 3 inches, because $15 - 12 = 3$. The plant’s new height is 125% of the original height, because $15 \div 12 = 1.25$. This means the height increased by 25%, because $125 - 100 = 25$.

Here are two ways to solve the problem: A rope was 2.4 meters long. Someone cut it down to 1.9 meters. What percent decrease is this?

The rope is now $2.4 - 1.9$, or 0.5 meters shorter. We can divide this decrease by the original length, $0.5 \div 2.4 = 0.2083$. So the length of the rope decreased by approximately 20.8%.

The rope’s new length is about 79.2% of the original length, because $1.9 \div 2.4 = 0.7916$. The length decreased by approximately 20.8%, because $100 - 79.2 = 20.8$. 
Lesson 12 Practice Problems

Problem 1

Statement

A music store marks up the instruments it sells by 30%.

a. If the store bought a guitar for $45, what will be its store price?

b. If the price tag on a trumpet says $104, how much did the store pay for it?

c. If the store paid $75 for a clarinet and sold it for $100, did the store mark up the price by 30%?

Solution

a. $58.50

b. $80.00

c. No. The store marked the price up by \( \frac{1}{3} \) or about 33.3% (rounded to the nearest tenth of a percent). The store needed to sell it for $97.50 to have a 30% markup.

Problem 2

Statement

A family eats at a restaurant. The bill is $42. The family leaves a tip and spends $49.77.

a. How much was the tip in dollars?

b. How much was the tip as a percentage of the bill?

Solution

a. $7.77

b. 18.5%

Problem 3

Statement

The price of gold is often reported per ounce. At the end of 2005, this price was $513. At the end of 2015, it was $1060. By what percentage did the price per ounce of gold increase?

Solution

About 107% \( (1060 - 513 = 547 \text{ and } 547 \div 513 \approx 1.07) \)
Problem 4

Statement
A phone keeps track of the number of steps taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your reasoning.

<table>
<thead>
<tr>
<th>number of steps</th>
<th>distance in kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>950</td>
<td>1</td>
</tr>
<tr>
<td>2,852</td>
<td>3</td>
</tr>
<tr>
<td>4,845</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Solution
No, there is not a proportional relationship. Since the first row shows that there are 950 steps in 1 kilometer, there should be 2,850 steps in 3 kilometers (since $950 \cdot 3 = 2,850$), but the table shows 2,852 steps.

(From Unit 2, Lesson 7.)

Problem 5

Statement
Noah picked 3 kg of cherries. Mai picked half as many cherries as Noah. How many total kg of cherries did Mai and Noah pick?

A. $3 + \frac{1}{2}$
B. $3 - \frac{1}{2}$
C. $(1 + \frac{1}{2}) \cdot 3$
D. $1 + \frac{1}{2} \cdot 3$

Solution
C
(From Unit 4, Lesson 4.)

Unit 4 Lesson 12
Lesson 13: Measurement Error

Goals

• Compare and contrast (orally) multiple measurements of the same length that result from using rulers with different levels of precision.

• Describe (orally) possible sources of “measurement error” when measuring lengths.

• Generalize a process for calculating measurement error and expressing it as a percentage of the actual length.

Learning Targets

• I can represent measurement error as a percentage of the correct measurement.

• I understand that all measurements include some error.

Lesson Narrative

This is the first of three lessons where students encounter the idea of percent error. Unlike situations involving percent increase and percent decrease, where there is an initial amount and a final amount, situations expressed with percent error involve a measured amount and a correct amount. The measurement error is the positive difference between the measured amount and the correct amount, and the percent error is the measurement error expressed as a percentage of the correct amount. In this first lesson students see how measurement error can arise in two different ways: from the level of precision in the measurement device, and from human error. In this lesson students encounter one of the important aspects of mathematical modeling, namely that mathematical representations are usually an approximation of the real situation (MP4).

Alignments

Building On

• 2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

• 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

• 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Preparation

Print the Measuring to the Nearest Instructional master. Prepare 1 copy for every 2 students. The Instructional master contains two versions of a centimeter ruler that students will cut out (or you may cut out ahead of time) and use to measure things, so card stock would be preferable if available. In the instructions, students are told to cut out the rulers they will use from the Instructional master, but to save class time you may want to do this for them ahead of time. These same rulers are also used in the Measuring Your Classroom activity in this lesson, so they should be used carefully during the warm-up.

Measure the height or length of several objects in your classroom to the nearest tenth of a centimeter. If possible, have at least one object per student in the class so that students don’t have to wait too long to measure things. Most of the items should be greater than 20 cm in length, but some can be less than or equal to 20 cm in length. Examples of such objects might be the width of the door, the length of the stick that holds a flag, the length of an eraser, or a side of a table or desk top.

Student Learning Goals

Let’s use percentages to describe how accurately we can measure.

13.1 Measuring to the Nearest

Warm Up: 10 minutes

The purpose of this task is to notice how differences in recorded measurements can result from the level of precision of your measuring device. Students use rulers that have varying levels of accuracy to measure the same lines. This warm-up gets the conversation started around measurement error that will continue in the follow activities.

Students will need to use the rulers again later in this lesson, so make sure they keep track of them.

Building On

- 2.MD.A.2

Unit 4 Lesson 13
5.NBT.B.7

Building Towards
5.RP.A.3

Instructional Routines
Think Pair Share

Launch
Arrange students in groups of 2. Give each group one copy of the Instructional master and access to scissors or cut out the rulers provided ahead of time. Remind students that they are to use the two different rulers to measure the line segments.

Give students 3–5 minutes of quiet work time to complete the task with their partner. Follow with whole-class discussion.

Anticipated Misconceptions
Students might not line up the edge of the ruler with the end of the line. Remind students that we need to line up the 0 mark on the ruler (in this case, the edge of the ruler) with the beginning edge of the line being measured.

Student Task Statement
Your teacher will give you two rulers and three line segments labeled A, B, and C.

1. Use the centimeter ruler to measure each line segment to the nearest centimeter. Record these lengths in the first column of the table.

2. Use the millimeter ruler to measure each line segment to the nearest tenth of a centimeter. Record these lengths in the second column of the table.

<table>
<thead>
<tr>
<th>line segment</th>
<th>length (cm) as measured with the first ruler</th>
<th>length (cm) as measured with the second ruler</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Student Response
Answers vary. Sample response:
<table>
<thead>
<tr>
<th>line segment</th>
<th>length (cm) as measured with the first ruler</th>
<th>length (cm) as measured with the second ruler</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>6.7</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>6.9</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>7.3</td>
</tr>
</tbody>
</table>

**Activity Synthesis**

Ask students if they noticed anything between the lengths they got using the two different measuring devices (resulted in different recorded measurements). Explain to students that measurement error can result from the precision level of your measuring device. Ask students, “Assuming the measurements to the nearest tenth are exact, by how much was each measurement in error when you used the centimeter scaled ruler?” (7 cm was 0.3 cm too long, 0.1 cm too long, and 0.3 cm too short respectively.)

**13.2 Measuring a Soccer Field**

10 minutes

In the warm-up, students learned that measurement error can result from the level of precision in a measuring device. In this activity, students learn about how real-world limitations on humans using measuring devices can introduce measurement errors. They discuss possible sources of error and express the error both as an amount and as a percentage. This is their first introduction into the concept of measurement error and how we use that to calculate percent error.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR7: Compare and Connect
- Think Pair Share

**Launch**

Keep students in groups of 2. Tell students that a soccer field is 120 yards long and ask them how they can measure that length using a 30-foot-long tape measure. (Note the use of two different units of measure, here.) If not mentioned by students, suggest measuring off 30 feet, making a mark, measuring off another 30 feet, and so on. Ask the class if they would all get exactly the same answer by this method. Tell students they are going to think more deeply about a specific measurement made by a person. Give the students 1–2 minutes of quiet work time to calculate the amount of the error and the percent error followed by partner and whole-group discussions.
Anticipated Misconceptions
If students fail to see the need for converting units of measure, ask them how many feet are in 120 yards? How many inches?

Student Task Statement
A soccer field is 120 yards long. Han measures the length of the field using a 30-foot-long tape measure and gets a measurement of 358 feet, 10 inches.

1. What is the amount of the error?

2. Express the error as a percentage of the actual length of the field.

Student Response
1. 1 foot, 2 inches or 14 inches. 120 yards is 360 feet. The amount of error is the difference between 360 feet and 358 feet, 10 inches.

2. The percent error is 0.32%. The soccer field is 120 yards long, which is 4,320 inches, because 120 \cdot 3 \cdot 12 = 4,320. Han's measurement error is 14 inches, and 14 \div 4,320 \approx 0.00324.

Activity Synthesis
Ask students, "What is a possible cause of the error?" Possible reasons include:

• He did not position the tape measure precisely every time he measured another 30 feet.

• He didn't go in a completely straight line. (Although this would result in a longer measurement.)

• Han did not correctly use the measuring tape.

Ask a few students to share their solutions for the last problem.

Explain to students, measurement error is the positive difference between the measurement and the actual value. The percent error is the error expressed as a percentage of the actual value. We always use a positive number to express percent error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value. In this case, we might say that the measured length was less than the actual length with a percent error of 0.32%.

Ask, "When might percent error be more useful than measurement error?" If needed, give an example of measuring a student's height and being incorrect by an inch versus measuring the height of a skyscraper and being incorrect by an inch. Although the measurement error is the same (1 inch), the percent error is very different since the 1 inch difference is significantly more important when measuring shorter distances.
Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: measurement error and percent error.
Supports accessibility for: Memory; Language

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine when students present their strategies for calculating the percent error. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the associations between the measurement, measurement error, and percent error. As students share what they noticed between the strategies, revoice their statements using the terms “measurement error” and “percent error.” These exchanges can strengthen students' mathematical language use and reasoning based on measurement errors.
Design Principle(s): Maximize meta-awareness

13.3 Measuring Your Classroom

20 minutes
This activity has students measuring things around the classroom to connect to the previous activity about measurement error. Students will work with their partner to measure 3 different things found in the classroom that the teacher has measured ahead of time (to obtain an “actual” measurement). They will use two different rulers (one with mm markings and one without).

After both students in a group have measured their three objects, provide them with the actual measurements of those items. They will calculate the measurement and percent errors after given the actual measurements.

Monitor for students who develop different procedures for computing percent error. For example: describing an algorithm in words (verbally or in writing), describing an algorithm more symbolically like \[
\frac{|\text{actual} - \text{measured}|}{\text{actual}}
\], or constructing a table of values.

Also monitor for a group whose two tables clearly shows how using the less-precise ruler results in greater percent error.

Addressing
- 7.RP.A.3

Unit 4 Lesson 13
**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

**Launch**

Keep students in the same groups of 2. Students will need the two rulers from the Instructional master from the warm-up of this lesson.

Assign each group 3 objects to measure (that you have measured ahead of time). If possible, select some items that are longer than the rulers provided so that students may encounter the issue raised in the previous activity.

Tell students that they will fill in only the first 2 columns of each table using their rulers to measure. After groups have completed measuring their 3 objects, tell students the measurements you made of these same objects for them to fill in the third column of the tables. Students should then work to fill in the last 2 columns of the table.

Give students 3–5 minutes of quiet work time and then allow time for them to discuss with a partner, followed by whole-class discussion.

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**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for: Organization; Attention*

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**Student Task Statement**

Your teacher will tell you which three items to measure. Keep using the paper rulers from the earlier activity.

1. Between you and your partner, decide who will use which ruler.

2. Measure the three items assigned by your teacher and record your measurements in the first column of the appropriate table.

Using the cm ruler:
3. After you finish measuring the items, share your data with your partner. Next, ask your teacher for the actual lengths.

4. Calculate the difference between your measurements and the actual lengths in both tables.

5. For each difference, what percentage of the actual length is this amount? Record your answers in the last column of the tables.

**Student Response**

Answers vary. Sample response:

<table>
<thead>
<tr>
<th>item</th>
<th>measured length (cm)</th>
<th>actual length (cm)</th>
<th>difference</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>eraser length</td>
<td>19</td>
<td>19</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>table width</td>
<td>106</td>
<td>110.4</td>
<td>4.4</td>
<td>4%</td>
</tr>
<tr>
<td>book height</td>
<td>28</td>
<td>27.5</td>
<td>0.5</td>
<td>1.8%</td>
</tr>
<tr>
<td>item</td>
<td>measured length (cm)</td>
<td>actual length (cm)</td>
<td>difference</td>
<td>percentage</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------------------</td>
<td>--------------------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>eraser length</td>
<td>19.1</td>
<td>19</td>
<td>0.1</td>
<td>0.5%</td>
</tr>
<tr>
<td>table width</td>
<td>106</td>
<td>110.4</td>
<td>4.4</td>
<td>4%</td>
</tr>
<tr>
<td>book height</td>
<td>27.7</td>
<td>27.5</td>
<td>0.2</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

**Are You Ready for More?**

Before there were standard units of measurement, people often measured things using their hands or feet.

1. Measure the length of your foot to the nearest centimeter with your shoe on.

2. How many foot-lengths long is your classroom? Try to determine this as precisely as possible by carefully placing your heel next to your toe as you pace off the room.

3. Use this information to estimate the length of your classroom in centimeters.

4. Use a tape measure to measure the length of your classroom. What is the difference between the two measurements? Which one do you think is more accurate?

**Student Response**

Answers vary. The tape measure should be more accurate.

**Activity Synthesis**

There are two desired outcomes of this activity: to develop a procedure that makes sense to students for computing percent error, and to reinforce that less-precise measuring devices result in greater percent error.

Select a few students who came up with different, correct procedures for computing percent error to explain their reasoning. A procedure might be described like, “Find the difference between the measured length and the actual length. Divide this difference by the actual length. Express the result as a percentage.” Other procedures may also be appropriate. For example, students might construct a table of values or a double number line to help them reason about expressing the error as a percentage of the actual length.

To highlight the effects of using a less-precise measuring device, select a pair of students whose tables clearly show that the measurements taken with the centimeter ruler had greater percent error than the measurements taken with the millimeter ruler. Display their work and ask them to explain why the measurements taken with the centimeter ruler had greater percent error.
Access for English Language Learners

Conversing: MLR8 Discussion Supports. Before selecting students to display their work, allow students think time to write and reflect on their understanding of measurement and percent error and describe how each is calculated. Then have a whole-class discussion pressing for detail about how less-precise measuring devices result in greater percent error. This will help students produce and make sense of the language needed to communicate their own ideas.

*Design Principle(s): Support sense-making; Cultivate conversation*

Lesson Synthesis

Students should have a basic understanding of what measurement error is and how to use it to calculate percent error. Ask students:

- “What is measurement error? What causes measurement error?” (The difference between a measurement of an object and its actual measure. It may exist due to human error in using a measurement tool or because the tool itself is not precise.)
- “How can we minimize the amount of error?” (Use precision tools and care when using them.)
- “What is the relationship between measurement error and percent error?” (Percent error is the measurement error divided by the actual quantity.)

13.4 Off by a Little Bit?

Cool Down: 5 minutes

In this cool-down, students are assessed on their ability to compute measurement error and percent error from two measurements, one estimate and another actual measurement.

Addressing

- 7.RP.A.3

Anticipated Misconceptions

If students do not think to put all the measurements in the same units of measure, they may divide 2 inches by 4 feet and come up with a percent error of 50%. Ask students to explain why an error of 50% is not reasonable in this situation.

Student Task Statement

Clare estimates that her brother is 4 feet tall. When they get measured at the doctor’s office, her brother’s height is 4 feet, 2 inches.

1. Should Clare’s or the doctor’s measurement be considered the actual height? Explain your reasoning.
2. What was the error, expressed in inches?

3. What was the error, expressed as a percentage of the actual height?

**Student Response**

1. The doctor's measurement, since Clare's is only an estimate, the doctor's is more precise, and the doctor is probably more skilled at measuring heights than Clare.

2. 2 inches

3. 4%, because 4 feet 2 inches is equivalent to 50 inches, and $2 \div 50 = 0.04$.

**Student Lesson Summary**

When we are measuring a length using a ruler or measuring tape, we can get a measurement that is different from the actual length. This could be because we positioned the ruler incorrectly, or it could be because the ruler is not very precise. There is always at least a small difference between the actual length and a measured length, even if it is a microscopic difference!

Here are two rulers with different markings.

The second ruler is marked in millimeters, so it is easier to get a measurement to the nearest tenth of a centimeter with this ruler than with the first. For example, a line that is actually 6.2 cm long might be measured to be 6 cm long by the first ruler, because we measure to the nearest centimeter.

The **measurement error** is the positive difference between the measurement and the actual value. Measurement error is often expressed as a percentage of the actual value. We always use a positive number to express measurement error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value.

For example, if we get 6 cm when we measure a line that is actually 6.2 cm long, then the measurement error is 0.2 cm, or about 3.2%, because $0.2 \div 6.2 \approx 0.032$.

**Glossary**

- measurement error
Lesson 13 Practice Problems

Problem 1

Statement

The depth of a lake is 15.8 m.

a. Jada accurately measured the depth of the lake to the nearest meter. What measurement did Jada get?

b. By how many meters does the measured depth differ from the actual depth?

c. Express the measurement error as a percentage of the actual depth.

Solution

a. 16 m

b. 0.2 m

c. 1.27%, because $0.2 \div 15.8 \approx 0.01265$.

Problem 2

Statement

A watermelon weighs 8,475 grams. A scale measured the weight with an error of 12% under the actual weight. What was the measured weight?

Solution

$7,458$ grams, $8,475 \times 0.88 = 7,458$

Problem 3

Statement

Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.

a. If the actual temperature is $325^\circ F$, what will the thermometer reading be?

b. If the thermometer reading is $76^\circ F$, what is the actual temperature?

Solution

a. 331.5 degrees Fahrenheit, $325 \times 1.02 = 331.5$

b. Approximately 74.5 degrees Fahrenheit, $76 \div 1.02 \approx 74.5$
Problem 4

Statement
At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Now, there is \( \frac{1}{3} \) less than that. How many ounces of peanut butter are in the pantry now?

A. \( \frac{2}{3} \cdot 80 \)

B. \( \frac{1}{3} \cdot 80 \)

C. \( 80 - \frac{1}{3} \)

D. \( (1 + \frac{1}{3}) \cdot 80 \)

Solution
A
(From Unit 4, Lesson 4.)

Problem 5

Statement
a. Fill in the table for side length and area of different squares.

<table>
<thead>
<tr>
<th>side length (cm)</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td></td>
</tr>
</tbody>
</table>

b. Is the relationship between the side length of a square and the area of a square proportional?
Solution

a.

<table>
<thead>
<tr>
<th>side length (cm)</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>100</td>
<td>10,000</td>
</tr>
<tr>
<td>25</td>
<td>625</td>
</tr>
<tr>
<td>$s$</td>
<td>$s^2$</td>
</tr>
</tbody>
</table>

b. No. There is no number the numbers in the first column of the table can be multiplied by to get the numbers in the second column.

(From Unit 3, Lesson 7.)
Lesson 14: Percent Error

Goals

- Calculate the percent error, correct amount, or erroneous amount, given the other two of these three quantities, and explain (orally and using other representations) the solution method.

- Compare and contrast (orally) strategies used for solving problems about percent error with strategies used for solving problems about percent increase or decrease.

Learning Targets

- I can solve problems that involve percent error.

Lesson Narrative

Situations involving percent error can be more difficult than situations involving percent increase or percent decrease because the student has to decide which amount represents the whole. In this lesson students get practice using the language of percent error in various different situations, and identifying the correct amount, which is the whole, and the incorrect amount (MP1). They work with a multi-step problem involving percent error. They also see a common usage of percent error to express a range of possible values by thinking about a scale that claims to be accurate to within 0.5%. Understanding and finding percent error is important for solving real-world problems (MP4).

Alignments

Building On

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR1: Stronger and Clearer Each Time

- MLR8: Discussion Supports
• Number Talk
• Think Pair Share

Required Materials
Four-function calculators

Required Preparation
For the Measuring in the Heat activity, students will need access to calculators.

Student Learning Goals
Let’s use percentages to describe other situations that involve error.

14.1 Number Talk: Estimating a Percentage of a Number

Warm Up: 5 minutes
The purpose of this number talk is for students to reason about a percentage of a number based on percentages they already know or could easily find. The percentages and numbers were purposefully chosen so that it would be cumbersome to calculate the exact answer and encourage making an estimate. During the whole-class discussion, highlight the percentages students found helpful and ask them to explain how they used these percentages. For example, if a student is estimating 9% of 38 and says, "I know 10% of 38 is 3.8..." ask the student to explain how they found 10% of 38.

Building On
• 6.RP.A.3.c

Building Towards
• 7.RP.A.3

Instructional Routines
• MLR8: Discussion Supports
• Number Talk

Launch
Display each problem one at a time. Give students 30 seconds of quiet think time followed by a whole-class discussion.
**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

**Anticipated Misconceptions**

If students try to figure out exact answers, encourage them to think about numbers that are close to the numbers in the problem, in order to estimate the percentage for each question.

**Student Task Statement**

Estimate.

- 25% of 15.8
- 9% of 38
- 1.2% of 127
- 0.53% of 6
- 0.06% of 202

**Student Response**

Answers vary. Sample responses:

- About 4, because 15.8 is close to 16, and 25% of 16 is 4.
- About 3.8, because 9% is close to 10% and 10% of 38 is 3.8.
- About 1.5, because 1.2% is close to 1% but a little bigger, and 1% of 127 is 1.27.
- About 0.03, because 0.53% is close to 0.5% which is half of 1%. Since 1% of 6 is 0.06, and half of that is 0.03.
- About 0.1, because 0.06% is close to 0.05% which is half of 0.1%. Since 0.1% of 202 is 0.202, and half of that is about 0.1.

**Activity Synthesis**

Ask students to share their responses for each question. Record and display student responses for all to see. After each response, ask students:

- "What benchmark percentages do you find it helpful to think about, when estimating?"
- "Is your estimate more or less than the actual answer? How do you know?"
14.2 Plants, Bicycles, and Crowds

10 minutes

The purpose of this activity is to give students practice using the language of percent error. The problems here are similar in structure to percent increase or decrease problems, but the language is different. Students may need some help interpreting the language used for percent error and drawing parallels to the language used for percentage increase and decrease. Students should use similar strategies they used to calculate percentage increase or decrease. This activity includes one of each type of problem:

- finding the erroneous amount given the correct amount and the percent error
- finding the correct amount given the erroneous amount and the percent error
- finding the percent error given the erroneous amount and the correct amount

Students may need help understanding that a different approach is needed for each question. As students work, monitor for those who use strategies similar to those used for percentage increase and decrease, and ask them to share during the whole-class discussion.

Addressing
- 7.RP.A.3

Instructional Routines
- MLR8: Discussion Supports
- Think Pair Share

Launch

Before students begin working, read the first question aloud. It is helpful to tie the language used in the task to the phrase "percentage increase." Ask several students to explain in their own words what information is given in the problem, and what it is asking them to find. The plant is supposed to get $\frac{3}{4}$ cup of water, but it is getting 25% more than that. We can think of this as $\frac{3}{4}$ cup increased by 25%.
Arrange students in groups of 2. Give students 3–5 minutes of quiet work time. Partner then by whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Represent the same information through different modalities. If students are unsure where to begin, suggest that they draw a double number line to help organize the information provided.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

Anticipated Misconceptions

Students might struggle with figuring out how much water the plant has been getting. Ask students, “How much more water has the plant been getting? How do you calculate that total?”

**Student Task Statement**

1. Instructions to care for a plant say to water it with \( \frac{3}{4} \) cup of water every day. The plant has been getting 25% too much water. How much water has the plant been getting?

2. The pressure on a bicycle tire is 63 psi. This is 5% higher than what the manual says is the correct pressure. What is the correct pressure?

3. The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,486 people. What is the percent error?

**Student Response**

1. \( \frac{15}{16} \) of a cup. 25% of \( \frac{3}{4} \) is one quarter of \( \frac{3}{4} \), which is \( \frac{3}{16} \). So the plant has been getting \( \frac{15}{16} \) cups of water each day, since \( \frac{3}{4} + \frac{3}{16} = \frac{15}{16} \). That is almost 1 cup.

2. 60 psi. We know that 63 psi is 5% more than the correct pressure. If the correct pressure is \( p \), this means that \( 63 = p + 0.05p \) or \( 1.05p \). Since \( 63 \div 1.05 = 60 \), the correct pressure is 60 psi.

3. Approximately 21%. The estimate is more than the exact amount by 514 people, so the percent error is \( \frac{514}{2,486} \approx 0.21 \).

**Are You Ready for More?**

A micrometer is an instrument that can measure lengths to the nearest micron (a micron is a millionth of a meter). Would this instrument be useful for measuring any of the following things? If so, what would the largest percent error be?

1. The thickness of an eyelash, which is typically about 0.1 millimeters.

2. The diameter of a red blood cell, which is typically about 8 microns.
3. The diameter of a hydrogen atom, which is about 100 picometers (a picometer is a trillionth of a meter).

**Student Response**

1. Yes. The biggest the error could be is half of a micron, or 0.0000005 meters. If we divide this by the thickness, which is 0.0001 meters, the percent error would be 0.5%.

2. Yes. The biggest the error could be is 0.5 microns. If we divide this by the length, which is 8 microns, the percent error would be 6.25%.

3. No. The diameter of a hydrogen atom is much smaller than a micron.

**Activity Synthesis**

Select students who used these strategies for each problem to share:

- Calculate a quarter of \( \frac{3}{4} \) and add it to \( \frac{3}{4} \).
- Use 1.05 to divide into 63.
- Calculate percent error using \( \frac{514}{2.486} \).

After each student has shared, ask the class if they agree or disagree or if they had a different way to calculate the solution. If students use strategies similar to ones they did calculating percentage increase or decrease, ask students if they see a connection. If no student brings it up, ask students how the solution strategies here are similar to the ones used with percentage increase and decrease.

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* As students describe their strategies for calculating the erroneous amount, correct amount, and percent error, revoice student ideas to demonstrate mathematical language use. Press for details in students’ explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. This will help students to produce and make sense of of the language needed to communicate their own ideas.

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

### 14.3 Measuring in the Heat

10 minutes

In this activity students use what they have learned about percent error in a multi-step problem.

Monitor for students who multiply 0.0000064 by 50 to answer the second part of the problem rather than using the calculation from the first part of the problem. These students should be asked to share during the whole-class discussion.
Display the image of the metal measuring tape for all to see. Ask if any students have used a tool like this before, and for what purpose. Tell them that many measuring tapes like this are made out of metal, and that some metals expand or contract slightly at warmer or colder temperatures.

In this problem, a metal measuring tape gets 0.00064% longer for every degree over 50°Fahrenheit. Ask students what would happen if a measuring tape was used to measure 10 feet, and then got 0.00064% longer. How much longer is that? Can they show the difference between two fingers? The difference would be very, very small, only 0.0000064 feet (about the width of a hair) . . . barely perceptible! For most uses, this difference wouldn't matter, but if someone needed a very, very precise measurement, they would want to know about it.

Arrange students in groups of 2. Give students 5 minutes of quiet work time. After 5 minutes, give students 3 minutes to discuss with a partner the ways they approached this problem.
Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have converted the percentage 0.00064 to its decimal form 0.0000064 prior to calculating how much longer the measuring tape is than its correct length of 30 feet.
*Supports accessibility for: Memory; Organization*

Access for English Language Learners

*Representing, Speaking: MLR1 Stronger and Clearer Each Time.* Use this routine to give students a structured opportunity to refine their explanation of their strategy for calculating the added length of the measuring tape and the percent error. Ask each student to meet with 2-3 other partners in a row for feedback. Provide listeners with prompts for feedback that will help teams strengthen their ideas and clarify their language. For example, “What did you do first?”, “How did the example help you?”, “How did you use 0.00064%?”, etc. Students can borrow ideas and language from each partner to strengthen their final response.
*Design Principle(s): Optimize output (for explanation)*

Anticipated Misconceptions

If students struggle with how to calculate the length increase, ask students

- "How many degrees over 50° is 100°?"
- "How do you calculate the length increase knowing that there is a 50° increase?"
- "What is the actual length of the tape measure?"

Student Task Statement

A metal measuring tape expands when the temperature goes above 50°F. For every degree Fahrenheit above 50, its length increases by 0.00064%.

1. The temperature is 100 degrees Fahrenheit. How much longer is a 30-foot measuring tape than its correct length?

2. What is the percent error?

Student Response

1. 0.0096 feet. The temperature is 50 degrees over the ideal temperature, because 100 — 50 = 50. So we need to compute 50 · (0.0000064) = 0.00032, and then we need that percentage of 30 feet. 30 · (0.00032) = 0.0096.

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2. 0.032%, because $0.0096 \div 30 = 0.00032$.

**Activity Synthesis**
Select previously identified students to share how they obtained their answer to the second part of the activity. Ask students who multiplied 0.0000064 by 50 why that method works and if there is another method (using the answer from the first part of the activity). Encourage students to make the connection between the two methods.

Ask students how big the error is in inches, and to show approximately how big they think it is with their thumb and forefinger. 0.0096 feet is almost 3 millimeters. When measuring 30 feet, this may not seem like very much, but the importance of the error may depend on what is being measured. The length of a car may not be as important as a precise scientific instrument to measure the speed of light, for example.

**Lesson Synthesis**
Students should feel confident calculating percent error given different contexts and information. Ask students:

- “What strategies did we use to solve percent error problems?” (diagrams, tables, equations)
- “How are these strategies similar to the ones we used while solving percent increase/decrease problems?” (the same)

### 14.4 Jumbo Eggs

**Cool Down: 5 minutes**
The cool-down assesses student ability to find percent error from an expected and an actual measurement.

**Addressing**
- 7.RP.A.3

**Student Task Statement**
To be labeled as a jumbo egg, the egg is supposed to weigh 2.5 oz. Priya buys a carton of jumbo eggs and measures one of the eggs as 2.4 oz. What is the percent error?

**Student Response**
4%. The correct weight is 2.5 oz and Priya's egg is off by 0.1 oz. so the percent error is $0.1 \div 2.5 = 0.04$ or 4%.

**Student Lesson Summary**
Percent error can be used to describe any situation where there is a correct value and an incorrect value, and we want to describe the relative difference between them. For example, if a milk carton is supposed to contain 16 fluid ounces and it only contains 15 fluid ounces:
• the measurement error is 1 oz, and
• the percent error is 6.25% because \( \frac{1}{16} = 0.0625 \).

We can also use percent error when talking about estimates. For example, a teacher estimates there are about 600 students at their school. If there are actually 625 students, then the percent error for this estimate was 4%, because \( 625 - 600 = 25 \) and \( \frac{25}{625} = 0.04 \).

**Glossary**

• percent error
Lesson 14 Practice Problems

Problem 1

Statement
A student estimated that it would take 3 hours to write a book report, but it actually took her 5 hours. What is the percent error for her estimate?

Solution
40%, because $5 - 3 = 2$ and $2 \div 5 = 0.4$

Problem 2

Statement
A radar gun measured the speed of a baseball at 103 miles per hour. If the baseball was actually going 102.8 miles per hour, what was the percent error in this measurement?

Solution
0.19%, because $103 - 102.8 = 0.2$ and $0.2 \div 102.8 \approx 0.0019$

Problem 3

Statement
It took 48 minutes to drive downtown. An app estimated it would be less than that. If the error was 20%, what was the app's estimate?

Solution
38.4 minutes, because $48 - (0.2)48 = 38.4$.

Problem 4

Statement
A farmer estimated that there were 25 gallons of water left in a tank. If this is an underestimate by 16%, how much water was actually in the tank?

Solution
About 29.8 gallons, because $25 \div 0.84 \approx 29.8$

Problem 5

Statement
For each story, write an equation that describes the relationship between the two quantities.
a. Diego collected $x$ kg of recycling. Lin collected $\frac{2}{5}$ more than that.

b. Lin biked $x$ km. Diego biked $\frac{3}{10}$ less than that.

c. Diego read for $x$ minutes. Lin read $\frac{4}{7}$ of that.

Solution

a. $y = \frac{7}{5}x$

b. $y = \frac{7}{10}x$

c. $y = \frac{4}{7}x$

(From Unit 4, Lesson 4.)

Problem 6

Statement

For each diagram, decide if $y$ is an increase or a decrease of $x$. Then determine the percentage.

Solution

A Decrease: $y$ is a 20% decrease of $x$

B Increase: $y$ is a 60% increase of $x$

(From Unit 4, Lesson 12.)

Problem 7

Statement

Lin is making a window covering for a window that has the shape of a half circle on top of a square of side length 3 feet. How much fabric does she need?
Solution

At least 12.5 square feet. The area of the square part of the window is 9 square feet, and the area of the half circle is about 3.5 square feet, because \( \frac{1}{2} \cdot \pi \cdot (1.5)^2 \approx 3.5 \). (Typically for sewing projects, you need more fabric than the area you are covering, so Lin would need a bit more fabric than that.)

(From Unit 3, Lesson 10.)
Lesson 15: Error Intervals

Goals

- Comprehend that manufacturers often define a maximum acceptable percent error for characteristics of their products.
- Determine what information is needed to solve a problem involving percent error. Ask questions to elicit that information.
- Generate values that fall within the acceptable range for a measurement, given a maximum percent error.

Learning Targets

- I can find a range of possible values for a quantity if I know the maximum percent error and the correct value.

Lesson Narrative

This lesson is optional. In this lesson, students find and analyze intervals of possible error based on maximum possible errors.

This material gives students a solid foundation for future work in statistics. It is not necessary at this stage to emphasize the idea of a margin of error which defines a range of possible values. It is enough for students to see the values falling into that range, as preparation for future learning.

Alignments

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR4: Information Gap Cards
- MLR6: Three Reads
- Poll the Class
- Think Pair Share

Required Materials

- Four-function calculators
- Instructional master
- Pre-printed slips, cut from copies of the
Required Preparation
Print and cut cards from the Info Gap: Quality Control activity Instructional master. One copy of the Instructional master will be used for every 4 students.

Student Learning Goals
Let's solve more problems about percent error.

15.1 A Lot of Iron Ore
Warm Up: 10 minutes
The purpose of this warm-up is to begin to understand the idea that a maximum percent error defines an interval of values that a quantity can lie within. Students are asked to give possible readings on a scale that has a possible error of up to 1%. Different answers are possible.

Addressing
• 7.RP.A.3

Instructional Routines
• Poll the Class

Launch
Give students 3 minutes quiet work time, followed by whole-class discussion.

Student Task Statement
An industrial scale is guaranteed by the manufacturer to have a percent error of no more than 1%. What is a possible reading on the scale if you put 500 kilograms of iron ore on it?

Student Response
Answers vary. The scale may show any value between 495 and 505.

Activity Synthesis
Draw a blank number line and then put three tick marks in the middle and label them 490, 500, 510. Poll the class for possible measurements, and plot each on the number line. Some students may give answers outside the error interval. Record those with the others and flag them mentally for discussion. Make sure everyone agrees with all of the possible measurements. Students might limit their answers to the extreme values 495 and 505, which have an error of exactly 1%. Be sure to solicit other answers if those are the only two offered.

Summarize the results using the greatest and least possible measurements (495 and 505, respectively) and point out how all of the possible measurements fall between these two values.

15.2 Saw Mill
Optional: 10 minutes
This activity uses a quality control situation to work with percent error. It is very common that products in a factory are checked to make sure that they meet certain specifications. In this case, boards are cut to a specific length. If they are too long or too short they are rejected. Students should make sense of the decision to be based on percent error and not on measurement error. If we know the correct length and an acceptable percent error, then we can find out which lengths are acceptable and which should be rejected.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR6: Three Reads
- Think Pair Share

**Launch**

Arrange students in groups of 2. Allow students 3--5 minutes of quiet work time followed by partner and whole-class discussion.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge of the meaning of percent error and how to convert a percentage to its decimal form. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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**Access for English Language Learners**

*Reading, Writing: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, ask students to read the problem with the goal of comprehending the situation (e.g., A saw mill cuts wood boards. Boards are inspected. Some boards are rejected.). If needed, discuss the meaning of unfamiliar terms at this time (e.g., saw mill, quality control, inspector, etc.). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., board length, percent error, tolerance interval). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the task. This will help students connect the language in the word problem and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.

*Design Principle(s): Support sense-making*
Student Task Statement

1. A saw mill cuts boards that are 16 ft long. After they are cut, the boards are inspected and rejected if the length has a percent error of 1.5% or more.

   a. List some board lengths that should be accepted.
   b. List some board lengths that should be rejected.

2. The saw mill also cuts boards that are 10, 12, and 14 feet long. An inspector rejects a board that was 2.3 inches too long. What was the intended length of the board?

Student Response

1. Answers vary. Acceptable responses:
   
   a. For a 16 ft board, 1.5% is 0.24 ft or 2.88 in. Acceptable lengths are between 15.76 ft and 16.24 ft.
   
   b. Unacceptable lengths are less than 15.76 ft or more than 16.24 ft.

2. The rejected board could have been an intended 10 ft or 12 ft board since 1.5% of 10 ft is 0.15 ft or 1.8 in and 1.5% of 12 ft is 0.18 ft or 2.16 in. It could not have been 14 ft since 1.5% of 14 ft is 0.21 ft or 2.52 in.

Activity Synthesis

The purpose of the discussion is for students to understand why a tolerance based on percent error may be acceptable.

Consider asking these discussion questions:

- "A 16 foot board is also 192 inches long. What is the maximum number of inches allowed for an acceptable board of this length?" (2.88 inches)

- "Why should the mill accept boards that are longer or shorter than 16 feet exactly?" (It may be difficult to cut the boards to that exact length, especially as quickly as a saw mill probably needs to cut them, so some range of acceptable lengths is probably allowed.)

- "Why does it make sense for the range of acceptable lengths to be listed as a percent error rather than on a fixed length?" (While 2.88 inches may be ok for a board that should be about 192 inches long, if the board was supposed to be 5 inches long and it was allowed to be 2.88 inches longer, it would be more than 50% longer than intended.)

15.3 Info Gap: Quality Control

Optional: 20 minutes

This info gap activity uses a quality control situation working with percent error. It is very common that products in a factory are checked to make sure that they meet certain specifications. In this case the odometer of a car is tested and the amount of liquid in a bottle that is automatically filled
is checked. It makes sense that the decision should be based on percent error and not on absolute error.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Addressing
• 7.RP.A.3

Instructional Routines
• MLR4: Information Gap Cards

Launch

Provide access to calculators. Tell students they will continue to work with percent errors in realistic scenarios. Explain the Info Gap structure and consider demonstrating the protocol if students are unfamiliar with it. There are step-by-step instructions in the student task statement.

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organization

Unit 4 Lesson 15
Access for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving percent error. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

Design Principle(s): Cultivate Conversation

Student Task Statement
Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:
1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.
4. Share the problem card and solve the problem independently.
5. Read the data card and discuss your reasoning.

If your teacher gives you the data card:
1. Silently read your card.
2. Ask your partner “What specific information do you need?” and wait for them to ask for information.
3. Before sharing the information, ask “Why do you need that information?” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the problem card and solve the problem independently.
5. Share the data card and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response
1. The speedometer must have shown a speed less than 73.5 miles per hour. The car was actually moving 1.25 miles per minute, because \( 10 \div 8 = 1.25 \). This is the same speed as 75 miles per hour, because there are 60 minutes in an hour, and \( 1.25 \cdot 60 = 75 \). Since 2% of 75 is 1.5, that means the speedometer must have been incorrect by more than 1.5 miles per hour.
The speedometer was showing slower than the car was actually moving, so the speedometer must have shown less than $75 - 1.5$, or 73.5 miles per hour.

2. The bottle must have more than 450 but less than 456.75 milliliters of juice in it. Since the bottle was slightly overfilled, we can multiply $1.015 \times 450$ to get the maximum amount of juice that could be in the bottle and still be acceptable.

**Activity Synthesis**

The purpose of the discussion is to recognize what information is needed when dealing with tolerances based on percent error.

After students have completed their work, share the correct answers and ask students to discuss the different ways they solved this problem. Some guiding questions:

- "What information did you and your partner have to figure out?"
- "What different calculations did you have to make for the two situations?"
- "Why might a car manufacturer not want to sell a car that shows a speed lower than what the car is actually going?" (It could be dangerous for the driver if they think they are going slower than they actually are. They are also more prone to getting speeding tickets if they think they are going under the speed limit based on the speedometer, but are actually going faster.)
- "Why might a juice manufacturer not want to have too much more juice in the bottle than it says on the label?" (If too many bottles are overfilled, the company may be losing money by giving away extra juice without charging the customer more.)

**Access for English Language Learners**

*Conversing:* This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving equivalent ratios.

*Design Principle(s): Cultivate Conversation*

**Lesson Synthesis**

This lesson was about error intervals.

- “Why might companies accept a percent error in measurement for their products?” (It may be very difficult to get the measurements exact in an efficient way, so some small error is allowed.)

- “Why might a percent error be used rather than an absolute measurement error?” (The same absolute measurement error may have a much greater impact for smaller measurements than for larger measurements. A percent error allows the company to use a single rule for multiple measurements rather than writing a new rule for each measurement.)
• “How do you find the range of values that are acceptable when you know the target measurement and a percent error that is allowed?” (Multiply the target measurement by the percent error, then add and subtract that value from the target measurement to get the two endpoints for the interval that is allowed.)

15.4 An Angler's Dilemma

Cool Down: 5 minutes
This cool-down assesses student's understand of a range of possible values for measurements based on a percent error tolerance.

Addressing
• 7.RP.A.3

Student Task Statement
A fisherman weighs an ahi tuna (a very large fish) on a scale and gets a reading of 135 pounds. The reading on the scale may have an error of up to 5%. What are two possible values for the actual weight of the fish?

Student Response
Acceptable answers are between 128.25 pounds and 141.75 pounds. 5% of 135 is 6.75 \((0.05 \cdot 135 = 6.75)\), so \(135 - 6.75 = 128.25\) and \(135 + 6.75 = 141.75\) define the range for the scale.

Student Lesson Summary
Percent error is often used to express a range of possible values. For example, if a box of cereal is guaranteed to have 750 grams of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as large as \((0.05) \cdot 750 = 37.5\) and could be either above or below than the correct amount.

Therefore, the box can have anywhere between 712.5 and 787.5 grams of cereal in it, but it should not have 700 grams or 800 grams, because both of those are more than 37.5 grams away from 750 grams.
Lesson 15 Practice Problems

Problem 1

Statement
Jada measured the height of a plant in a science experiment and finds that, to the nearest $\frac{1}{4}$ of an inch, it is $4\frac{3}{4}$ inches.

a. What is the largest the actual height of the plant could be?

b. What is the smallest the actual height of the plant could be?

c. How large could the percent error in Jada's measurement be?

Solution
a. At most $4\frac{7}{8}$ inches tall (if it were taller, then $4\frac{3}{4}$ would not be the nearest quarter inch measurement)

b. At least $4\frac{5}{8}$ inches tall

c. About 2.6% ($0.125 \div 4\frac{3}{4}$)

Problem 2

Statement
The reading on a car's speedometer has 1.6% maximum error. The speed limit on a road is 65 miles per hour.

a. The speedometer reads 64 miles per hour. Is it possible that the car is going over the speed limit?

b. The speedometer reads 66 miles per hour. Is the car definitely going over the speed limit?

Solution
a. Yes, the car might be going more than 65 mph. 1.6% of 64 is 1.024, so the car could be going 65.024 mph which is over the speed limit.

b. No, the car might be going less than 65 mph. 1.6% of 66 is 1.056, so the car could be going as slow as 64.944 mph which is less than the speed limit.
Problem 3

Statement
Water is running into a bathtub at a constant rate. After 2 minutes, the tub is filled with 2.5 gallons of water. Write two equations for this proportional relationship. Use $w$ for the amount of water (gallons) and $t$ for time (minutes). In each case, what does the constant of proportionality tell you about the situation?

Solution

$w = 1.25t$; Every minute the amount of water increases by 1.25 gallons.

$t = 0.8w$; Every 0.8 minutes the amount of water increases by 1 gallon.

(From Unit 2, Lesson 5.)

Problem 4

Statement
Noah picked 3 kg of cherries. Jada picked half as many cherries as Noah. How many total kg of cherries did Jada and Noah pick?

A. $3 + 0.5$
B. $3 - 0.5$
C. $(1 + 0.5) \cdot 3$
D. $1 + 0.5 \cdot 3$

Solution

C

(From Unit 4, Lesson 5.)

Problem 5

Statement
Here is a shape with some measurements in cm.
a. Complete the table showing the area of different scaled copies of the triangle.

<table>
<thead>
<tr>
<th>scale factor</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td></td>
</tr>
</tbody>
</table>

b. Is the relationship between the scale factor and the area of the scaled copy proportional?

**Solution**

a. Complete the table showing the area of different scaled copies of the triangle.

<table>
<thead>
<tr>
<th>scale factor</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>s</td>
<td>3s²</td>
</tr>
</tbody>
</table>

b. No, the relationship between the scale factor and the area of the scaled copy is not proportional.

(From Unit 3, Lesson 7.)
Section: Let's Put it to Work

Lesson 16: Posing Percentage Problems

Goals

- Generate questions (orally and in writing) about a real-world situation involving percent increase or decrease.
- Interpret news headlines or advertisements that include statements about percent increase and decrease.
- Solve a problem about a real-world situation involving percent increase or decrease and present the solution method (in writing and through other representations).

Learning Targets

- I can write and solve problems about real-world situations that involve percent increase and decrease.

Lesson Narrative

In this culminating lesson on percentages, students work in groups to collect news clippings that mention percentages and sort them according to whether they are about percent increase or percent decrease, formulate questions about them, and then share their questions with other groups in a gallery walk. The purpose for students to apply percentages in a real-world context (MP4).

Alignments

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Group Presentations
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Take Turns
Required Materials

**Grocery store circulars**
Grocery store advertisements from the newspaper or that are picked up at the store. If students have Internet access, you could substitute an online version of this.

**Tools for creating a visual display**
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

**Sticky notes**

**Required Preparation**

If possible, have students cut several clippings from newspapers or print advertisements that include percentages or bring in similar items from internet searches. If not possible, bring several examples yourself. Examples include coupons, news stories, and advertisements claiming an increase in product size (“Now with 33% more soap!”). Every group of 3–4 students should have a set including a variety of contexts as well as some that show a percentage increase of an amount and some that include a percentage decrease in an amount.

In the final activity, each group of 3–4 students create a visual display to be used in a gallery walk based on one of the situations. Provide materials to create these displays. During the gallery walk, students will leave feedback for each group on a sticky note they can attach to the displays. Provide several sticky notes for each group.

### Student Learning Goals

Let’s explore how percentages are used in the news.

#### 16.1 Sorting the News

**Warm Up:** 10 minutes

**Addressing**
- 7.RP.A.3

**Instructional Routines**
- Take Turns

**Launch**

Arrange students in groups of 3–4. Provide each group with a set of newspaper clippings involving percentage increase and decrease.

Tell students to take turns sorting the clippings into the piles representing percentage increase and percentage decrease and explaining the decision. If there is a disagreement, partners should discuss their ideas to try to agree about the correct sorting of the item. If an agreement cannot be made, the clipping can be put to the side to be discussed after the activity.
**Student Task Statement**

Your teacher will give you a variety of news clippings that include percentages.

1. Sort the clippings into two piles: those that are about increases and those that are about decreases.
2. Were there any clippings that you had trouble deciding which pile they should go in?

**Student Response**

Answers vary.

**Activity Synthesis**

The purpose of the discussion is to discuss any issues students may have had determining the sorting as well as to discuss any interesting contexts from the clippings.

Ask students, "Were there any clippings that were not easy to sort? What made it difficult?" Work with the class to try to determine the correct sorting, if possible. Ask each group to read the important parts of their most interesting newspaper clipping and explain their reasoning behind sorting it where they did.

### 16.2 Investigating

10 minutes

In this activity, students use the examples of percentage increase and decrease that they sorted in the previous activity to pose questions that arise from the different situations. They ask and answer questions based on the information given and present this information graphically. In the next activity they will make a poster using one of their news items. Then students will go on a gallery walk and use sticky notes to ask questions about the information presented on each poster.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- MLR5: Co-Craft Questions

**Launch**

Keep students in groups of 3–4. Display the following statement for all to see:

"Global human population growth amounts to around 75 million annually, or 1.1% per year."

Ask students what questions they could answer with this information. Sample responses:

- What were possible populations in the two years used to compute the annual increase?
- By how many people will the population grow next year?
Tell students they will use the clippings from the warm-up to write similar questions for different situations.

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge about diagrams that can be used to represent situations of percent increase or percent decrease such as tape diagrams and double number line diagrams.

*Supports accessibility for: Memory; Conceptual processing*

### Access for English Language Learners

*Writing, Conversing: MLR5 Co-Craft Questions.* Present a video or image that depicts population growth next to the display. Ask pairs of students to write possible mathematical questions about the situation. Then, invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the relationships between the two quantities in this task (e.g., a quantity and percent increase or decrease) prior to being asked to create and solve questions based on their newspaper clippings.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

### Student Task Statement

In the previous activity, you sorted news clippings into two piles.

1. For each pile, choose one example. Draw a diagram that shows how percentages are being used to describe the situation.
   
   a. Increase Example:
   
   b. Decrease Example:

2. For each example, write two questions that you can answer with the given information. Next, find the answers. Explain or show your reasoning.

### Student Response

Answers vary.

### 16.3 Displaying the News

20 minutes

In this activity, students work in groups and make a poster in their groups using one of their news items. Next, students go on a gallery walk and use sticky notes to ask questions about the information presented on each poster. They practice critiquing the reasoning of others as they
study information they have not themselves worked on. They then go back and study the feedback they received from their classmates and revise their own work.

**Addressing**

- 7.RP.A.3

**Instructional Routines**

- Group Presentations
- MLR8: Discussion Supports

**Launch**

Keep students in the same groups of 3–4. Give students supplies to make posters. Tell them that they will choose one of their news clippings and make a visual display for the information they worked on in the previous activity. The posters should include all necessary information so that somebody who has not extensively worked with the same information should be able to understand the work.

Allow students 10 minutes to work on creating their display. Review group work as they finish.

After all groups have finished, display each group’s work around the room for students to do a gallery walk. Tell students that they should leave feedback for each display on a sticky note attached to each group’s work. Feedback can include questions about the display or information as well as compliments or critiques. Comments and questions should be constructive with the goal to help the groups who made the poster improve their work.

Tell each group which poster to start with and in which direction they should move.

As groups finish viewing the displays, allow them time to view the feedback left on their own display and, if necessary, time to improve their display based on the feedback.

**Access for Students with Disabilities**

*Engagement: Develop Effort and Persistence.* Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a task checklist which makes all the required components of the poster explicit.

*Supports accessibility for: Attention; Social-emotional skills*
Access for English Language Learners

Writing: MLR8 Discussion Supports. Provide sentence frames for students to use when asking questions about features of the visuals displays they do not understand. For example, "Why did you ____?" or "How did you ____?"

Design Principle(s): Support sense-making

Student Task Statement

1. Choose the example that you find the most interesting. Create a visual display that includes:
   - a title that describes the situation
   - the news clipping
   - your diagram of the situation
   - the two questions you asked about the situation
   - the answers to each of your questions
   - an explanation of how you calculated each answer

   Pause here so your teacher can review your work.

2. Examine each display. Write one comment and one question for the group.

3. Next, read the comments and questions your classmates wrote for your group. Revise your display using the feedback from your classmates.

Student Response

Answers vary.

Student Lesson Summary

Statements about percentage increase or decrease need to specify what the whole is to be mathematically meaningful. Sometimes advertisements, media, etc. leave the whole ambiguous in order to make somewhat misleading claims. We should be careful to think critically about what mathematical claim is being made.

For example, if a disinfectant claims to "kill 99% of all bacteria," does it mean that:

- It kills 99% of the number of bacteria on a surface?
- Or is it 99% of the types of bacteria commonly found inside the house?
- Or 99% of the total mass or volume of bacteria?
- Does it even matter if the remaining 1% are the most harmful bacteria?
Resolving questions of this type is an important step in making informed decisions.
Family Support Materials
Family Support Materials

Proportional Relationships and Percentages

Here are the video lesson summaries for Grade 7, Unit 4: Proportional Relationships and Percentages. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

<table>
<thead>
<tr>
<th>Grade 7, Unit 4: Proportional Relationships and Percentages</th>
<th>Vimeo</th>
<th>YouTube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video 1: Proportional Relationships with Fractions &amp; Decimals (Lessons 4–5)</td>
<td>Link</td>
<td>Link</td>
</tr>
<tr>
<td>Video 2: Percent Increase and Decrease (Lessons 6–8)</td>
<td>Link</td>
<td>Link</td>
</tr>
<tr>
<td>Video 3: Applications of Percentages (Lessons 10–12)</td>
<td>Link</td>
<td>Link</td>
</tr>
<tr>
<td>Video 4: More Applications of Percentages (Lessons 14–15)</td>
<td>Link</td>
<td>Link</td>
</tr>
</tbody>
</table>

Video 1


Video 2
Video 'VLS G7U4V2 Percent Increase and Decrease (Lessons 6–8)' available here: https://player.vimeo.com/video/479533112.

Video 3


Video 4


Connecting to Other Units

• Coming soon
Proportional Relationships with Fractions

Family Support Materials 1

This week your student is learning about proportional relationships that involve fractions and decimals. For example, a baker decides to start using $\frac{1}{6}$ less than the amount of sugar called for in each recipe. If the recipe calls for 2 cups of sugar, the baker will leave out $\frac{1}{6} \cdot 2$, or $\frac{1}{3}$ cup of sugar. That means the baker will only use $2 - \frac{1}{3}$, or $1\frac{2}{3}$ cups of sugar.

<table>
<thead>
<tr>
<th>amount of sugar in the recipe (x)</th>
<th>amount of sugar the baker uses (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cup</td>
<td>$\frac{5}{6}$ cup</td>
</tr>
<tr>
<td>$1\frac{1}{2}$ cups</td>
<td>$1\frac{1}{4}$ cups</td>
</tr>
<tr>
<td>2 cups</td>
<td>$1\frac{2}{3}$ cups</td>
</tr>
</tbody>
</table>

The amount of sugar the baker actually uses, $y$, is proportional to the amount of sugar called for in the recipe, $x$. The constant of proportionality is $\frac{5}{6}$.

\[
y = x - \frac{1}{6}x \\
y = (1 - \frac{1}{6})x \\
y = \frac{5}{6}x
\]

Another way to write this equation is $y = 0.833x$. The line above the 3 tells us that if we use long division to divide $5 \div 6$, we will keep getting the answer 3 over and over. This is an example of a repeating decimal.

Here is a task to try with your student:

The baker also decides to start using $\frac{1}{6}$ more than the amount of liquid called for in each recipe.

1. How much of each ingredient will the baker use if the recipe calls for:
   a. $1\frac{1}{2}$ cups of milk?
   b. 3 tablespoons of oil?
2. What is the constant of proportionality for the relationship between the amount of liquid called for in the recipe and the amount this baker uses?

Solution:

1. a. $1 \frac{3}{4}$ cups.
   
   b. $3 \frac{1}{2}$ tablespoons.

2. $\frac{7}{6}$, 1.16, or equivalent.
Percent Increase and Decrease

Family Support Materials 2

This week, your student is learning to describe increases and decreases as a percentage of the starting amount. For example, two different school clubs can gain the same number of students, but have different percent increases.

The cooking club had 50 students. Then they gained 6 students.

This is a 12% increase, because

\[ 6 \div 50 = 0.12. \]

They now have 56 students, which is 112% of the starting amount.

\[ 1.12 \times 50 = 56 \]

The computer club had 8 students. Then they gained 6 students.

This is a 75% increase, because

\[ 6 \div 8 = 0.75. \]

They now have 14 students, which is 175% of the starting amount.

\[ 1.75 \times 8 = 14 \]

Here is a task to try with your student:

The photography club had 20 students. Then the number of students increased by 35%. How many students are in the photography club now?

Solution:

27 students. Possible strategies:

- The club gained 7 new students, because \( 0.35 \times 20 = 7 \). The club now has 27 students, because \( 20 + 7 = 27 \).

- The club now has 135% as many students as they started with, because \( 100 + 35 = 135 \). That means they have 27 students, because \( 1.35 \times 20 = 27 \).
Applying Percentages

Family Support Materials 3

This week, your student is learning about real-world situations that use percent increase and percent decrease, such as tax, interest, mark-up, and discounts.

For example, the price tag on a jacket says $24. The customer must also pay a sales tax equal to 7.5% of the price. What is the total cost of the jacket, including tax?

\[
24 \cdot 1.075 = 25.80
\]

The customer will pay 107.5% of the price listed on the tag, which is $25.80.

We can also find the percentage. For example, a backpack originally cost $22.50, but is on sale for $18.99. The discount is what percentage of the original price?

\[
22.50x = 18.99
\]
\[
x = 18.99 / 22.50
\]
\[
x = 0.844
\]

The sale price is 84.4% of the original price. The discount is \(100 - 84.4\), or 15.6% of the original price.

Here is a task to try with your student:

A restaurant bill is $18.75. If you paid $22, what percentage tip did you leave for the server?

Solution:

17.3%. Possible strategy: You paid \(117.3\)% of the bill, because \(22 / 18.75 = 1.17\)\(\bar{3}\). You left a 17.3% tip, because \(117.3 - 100 = 17.3\).
Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B
Proportional Relationships and Percentages: Check Your Readiness (A)

Do not use a calculator.

1. Diego made 6 boxes of spaghetti to feed the 20 people who attended his dinner party. Next time, 50 people are coming! How many boxes of spaghetti should Diego buy to feed all those people? Explain or show your reasoning.

2. Solve each problem. Explain or show your reasoning.
   a. Elena is feeding her neighbor's dogs. Each dog gets $\frac{2}{3}$ cup of dog food, and she uses $3\frac{1}{3}$ cups of food. How many dogs does her neighbor have?
   b. $5\frac{5}{8}$ cups of water fill $4\frac{1}{2}$ identical water bottles. How many cups fill each bottle?
3. Which of these expressions is equivalent to $0.05(x - 40)$?

A. $0.05x - 40.05$

B. $0.05x - 40$

C. $0.05x - 2.05$

D. $0.05x - 2$

4. a. Diego pours 6.8 ounces of water from a full bottle. He estimates that he poured out 20% of the water in the bottle. About how much water was in the full bottle? Explain or show your reasoning.

b. Diego pours out 25% of the remaining water. About how many ounces did he pour the second time? Explain or show your reasoning.

5. To solve the problem “56 is what percent of 70?”, Noah uses the double number line below. On the double number line, write the correct numbers at each unlabeled tick mark. Then solve the problem.
6. Solve each problem. Explain or show your reasoning.

   a. What is 25% of 160?

   b. What is 39% of 200?

   c. What is 150% of 32?

   d. 13 is 50% of what number?

   e. 18 is 120% of what number?

   f. 21 is what percentage of 30?
Proportional Relationships and Percentages: Check Your Readiness (B)

Do not use a calculator.

1. Jada volunteers at an animal shelter that holds picnics to introduce people to animals. For the first picnic, she brought 36 clementines for 16 people, which was the right amount. For the next picnic, there will be 40 people! How many clementines should Jada bring for all those people? Explain or show your reasoning. (Note: clementines are fruits that are like small oranges.)

2. Solve each problem. Explain or show your reasoning.
   a. Lin is baking cakes. Each cake needs \( \frac{3}{4} \) cup of brown sugar, and Lin uses \( 4 \frac{1}{2} \) cups of brown sugar for the entire batch of cakes. How many cakes does Lin bake?

   b. \( 4 \frac{1}{2} \) cups of salad dressing fill \( 3 \frac{3}{4} \) identical jars. How many cups fill each jar?
3. Which of these expressions is equivalent to $0.04(x - 25)$?

A. $0.04x - 1$
B. $0.04x - 1.04$
C. $0.04x - 25$
D. $0.04x - 25.04$

4. a. Tyler uses 7.5 ounces of barbeque sauce from a full bottle. He estimates that he used about 25% of the barbeque sauce in the bottle. About how much barbecue sauce was in the full bottle? Explain or show your reasoning.

b. Tyler uses 20% of the remaining barbecue sauce. About how many ounces did he use the second time? Explain or show your reasoning.

5. To solve the problem “48 is what percent of 80?”, Elena uses this double number line. On the double number line, write the correct numbers at each unlabeled tick mark. Then solve the problem.
6. Solve each problem. Explain or show your reasoning.

a. 14 is 50% of what number?

b. 24 is 120% of what number?

c. 18 is what percentage of 30?

d. What is 25% of 80?

e. What is 41% of 200?

f. What is 150% of 26?
Proportional Relationships and Percentages:
End-of-Unit Assessment (A)

1. Priya has a recipe for banana bread. She uses \(7\frac{1}{2}\) cups of flour to make 3 loaves of banana bread.

Andre will follow the same recipe. He will make \(b\) loaves of banana bread using \(f\) cups of flour.

Which of these equations represents the relationship between \(b\) and \(f\)?

A. \(b = \frac{2}{9}f\)

B. \(b = \frac{2}{5}f\)

C. \(b = \frac{5}{2}f\)

D. \(b = \frac{9}{2}f\)

2. Diego measured the length of a pen to be 22 cm. The actual length of the pen is 23 cm.

Which of these is closest to the percent error for Diego's measurement?

A. 4.3%

B. 4.5%

C. 95.7%

D. 104.5%
3. A car is 180 inches long. A truck is 75% longer than the car.

How long is the truck?

A. 135 inches
B. 240 inches
C. 255 inches
D. 315 inches

4. A circular running track is $\frac{1}{4}$ mile long. Elena runs on this track, completing each lap in $\frac{1}{20}$ of an hour.

What is Elena’s running speed? Include the unit of measure.

5. Today, everything at a store is on sale. The store offers a 20% discount.

a. The regular price of a T-shirt is $18. What is the discount price?

b. If the regular price of an item is $x$ dollars, what is the discount price in dollars?

c. The discount price of a hat is $18. What is the regular price?
6. Lin’s father is paying for a $20 meal. He has a 15%-off coupon for the meal. After the discount, a 7% sales tax is applied.

What does Lin’s father pay for the meal? Explain or show your reasoning.

7. Tyler’s brother works in a shoe store.

a. Tyler’s brother earns a commission. He makes 2.5% of the amount he sells. Last week, he sold $900 worth of shoes. How much was his commission?

b. The store bought a pair of shoes for $50, and sold it for $80. What percentage was the markup?

c. Tyler’s brother earns $12 per hour. The store offers him a raise—a 5% increase per hour. After the raise, how much will Tyler’s brother make per hour?
Proportional Relationships and Percentages: End-of-Unit Assessment (B)

1. Clare has a recipe for yellow cake. She uses $9 \frac{1}{3}$ cups of flour to make 4 cakes. Noah will follow the same recipe. He will make $c$ cakes using $f$ cups of flour.

Which of these equations represent the relationship between $c$ and $f$?

A. $c = \frac{16}{3} f$

B. $c = \frac{3}{16} f$

C. $c = \frac{3}{7} f$

D. $c = \frac{7}{3} f$

2. A graduated cylinder actually contains 7.5 milliliters of water. When Han measures the volume of the water inside the graduated cylinder, his measurement is 7 milliliters. Which of these is closest to the percent error for Han’s measurement?

A. 107.1%

B. 93.3%

C. 7.1%

D. 6.7%

3. It takes Diego $\frac{1}{24}$ of an hour to complete a lap on a circular bike track. The track is $\frac{1}{3}$ mile long. What is Diego’s bike speed?

A. $\frac{1}{8}$ mile per hour

B. $\frac{1}{8}$ hours per mile

C. 8 miles per hour

D. 8 hours per mile
4. A practice field is 250 feet long. The game field is 40% longer than the practice field. How long is the game field?

5. Noah has a coupon for 30% off at his favorite clothing store. He uses it to buy a hoodie and a pair of jeans.
   a. Noah paid $28 for the jeans after using the coupon. What is the regular price of the jeans?
   b. The regular price of a hoodie is $27. What did Noah pay for the hoodie?
   c. If the regular price of an item is $x dollars, what is the discount price in dollars?

6. Kiran’s mother gets a restaurant bill for $25. She has a coupon for 15% off. After the discount is applied, she adds 20% as a tip.
   What is the total after the discount and tip? Explain or show your reasoning.
7. Jada's sister works in a furniture store.

   a. Jada's sister earns $15 per hour. The store offers her a raise—a 9% increase per hour. After the raise, how much will Jada's sister make per hour?

   b. The store bought a table for $200, and sold it for $350. What percentage was the markup?

   c. Jada's sister earns a commission. She makes 3.5% of the amount she sells. Last week, she sold $7,000 worth of furniture. How much was her commission?
Assessment Answer Keys
Check Your Readiness A and B
End-of-Unit Assessment A and B
Assessment Answer Keys

Assessment : Check Your Readiness (A)

Teacher Instructions
Calculators should not be used.

Student Instructions
Do not use a calculator.

Problem 1
The content assessed in this problem is first encountered in Lesson 2: Ratios and Rates With Fractions.

In an earlier unit, students were introduced to proportional relationships and learned to use them to find equivalent ratios.

If most students struggle with this item, plan to use Lesson 2 Activity 2 to review strategies for working with ratios. While monitoring their work, notice which students are using each strategy listed in the lesson. During the synthesis of this activity, point out the connections between the shared strategies to support their understanding of the methods used. Students who struggle with the calculation may benefit from a more visual strategy using a double number line or a table.

Statement
Diego made 6 boxes of spaghetti to feed the 20 people who attended his dinner party. Next time, 50 people are coming! How many boxes of spaghetti should Diego buy to feed all those people? Explain or show your reasoning.

Solution
15 boxes.

Answers vary. Sample response:

That is \( \frac{6}{20} = 0.3 \) boxes per person. So if \( y \) is the number of boxes needed and \( x \) is the number of people attending, then \( y = 0.3x \) and \( x = 50 \) means \( y = (0.3) \cdot 50 \) or 15 boxes are needed.

Aligned Standards
6.RP.A.3.a
Problem 2

The content assessed in this problem is first encountered in Lesson 2: Ratios and Rates With Fractions.

In this unit, students will be asked to extend their understanding of ratios and unit rates to fractional quantities. This item assesses how well they are able to divide fractions, a grade 6 standard. In part a, students may find it easier to think visually or conceptually about how many times \( \frac{2}{3} \) goes into \( 3 \frac{1}{3} \). The division in part b is harder to do without using the standard algorithm.

If most students struggle with this item, plan to use the Number Talk in Lesson 2 Activity 1 to review dividing a fraction by a fraction. This lesson also offers opportunities to practice this calculation within a context. If students who struggled did not use diagrams for making sense of the problem, monitor for and display the work of students who used diagrams.

Statement

Solve each problem. Explain or show your reasoning.

1. Elena is feeding her neighbor’s dogs. Each dog gets \( \frac{2}{3} \) cup of dog food, and she uses \( 3 \frac{1}{3} \) cups of food. How many dogs does her neighbor have?

2. \( 5 \frac{5}{8} \) cups of water fill \( 4 \frac{1}{2} \) identical water bottles. How many cups fill each bottle?

Solution

1. Sample response: \( 3 \frac{1}{3} \) cups is the same as \( \frac{10}{3} \) cups, and \( \frac{10}{3} \div \frac{2}{3} = 5 \).

2. \( \frac{5}{4} \) cups or equivalent. Sample response: \( \frac{5}{8} \div 4 \frac{1}{2} = \frac{5}{4} \).

Aligned Standards

6.RP.A.3.b

Problem 3

The content assessed in this problem is first encountered in Lesson 4: Half as Much Again.

In this unit, students use the distributive property to analyze statements relating to percent increase and decrease, like “Kiran ran three miles, then half as much again.”

If most students struggle with this item, plan to use Lesson 4 and the lessons that follow to review and solidify using the distributive property in a variety of situations as students write equivalent expressions and find percentages.

Statement

Which of these expressions is equivalent to \( 0.05(x - 40) \)?
Problem 4

The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.

It is not required that students find exact solutions to this problem. However, the problem is designed to reveal the ways in which students are thinking about two common problem types: how to find a whole given a part and a percentage, and how to find a given percentage of a given value. Students will study questions like these more formally in this unit.

Students who solve the problem exactly will discover that the two pours are the same amount of water. A tape diagram can help them understand why.

If most students struggle with this item, plan to use Lesson 6 and the lessons that follow to study these two types of percentage problems. In Lesson 6 Activity 2 students will begin thinking about percent increase and percent decrease situations.

Statement

1. Diego pours 6.8 ounces of water from a full bottle. He estimates that he poured out 20% of the water in the bottle. About how much water was in the full bottle? Explain or show your reasoning.

2. Diego pours out 25% of the remaining water. About how many ounces did he pour the second time? Explain or show your reasoning.

Solution

Answers vary due to the estimation described.

1. 34 ounces. 6.8 is 20% of 34, so there are 34 ounces.

2. 6.8 ounces. The remaining amount of water after the first pour is 27.2 ounces, and 25% of 27.2 is 6.8.
Aligned Standards

6.RP.A.3, 7.RP.A.3

Problem 5

The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.

A student may accidentally use a different but correct scale for the tick marks: one such example is to label the top tick marks 42 and 56, and the bottom tick marks 60 and 80. This is unlikely but correct.

If most students struggle with this item, plan to monitor for students who use a double number line to solve Lesson 6 Activity 2. Then use the synthesis of Lesson 6 Activity 2 to review using double number lines to calculate the percent. These ideas will continue throughout the next several lessons.

Statement

To solve the problem “56 is what percent of 70?”, Noah uses the double number line below. On the double number line, write the correct numbers at each unlabeled tick mark. Then solve the problem.

Solution

Answers vary. Sample response:

The top tick marks are labeled 56 and 63, the bottom tick marks are labeled 80% and 90%.

56 is 80% of 70.

Because 7 is 10% of 70, tick marks on the top should be marked in multiples of 7, while tick marks on the bottom should be marked in multiples of 10. Working backwards from 70 and 100%, the first pair of marks are 63 and 90%, and the second pair are 56 and 80%. This means that 56 is 80% of 70.

Aligned Standards

6.RP.A.3

Problem 6

The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.
This problem provides an opportunity to assess students’ “go-to” strategies for solving percent problems. Some students who are able to reason their way through the previous two questions may have more difficulty with these problems, since no context is provided.

If most students struggle with this item, plan to review these strategies using this problem in the days leading up to Lesson 6. You can find similar problems in Grade 6 Unit 3, Lessons 10-16 Practice Problems.

**Statement**

Solve each problem. Explain or show your reasoning.

1. What is 25% of 160?
2. What is 39% of 200?
3. What is 150% of 32?
4. 13 is 50% of what number?
5. 18 is 120% of what number?
6. 21 is what percentage of 30?

**Solution**

1. 40
2. 78
3. 48
4. 26
5. 15
6. 70%

Explanations vary. Sample explanations:

1. 25% of a number is \( \frac{1}{4} \) of that number. Then \( 160 \cdot \frac{1}{4} = 40 \).
2. Multiply using a decimal: \( 200 \cdot (0.39) = 78 \). Alternatively: 39 is 39% of 100, so 78 must be 39% of 200.
3. 150% of a number is “the number, plus half again”: \( 32 + 16 = 48 \).
4. This question can be read as “13 is half of what number?” Doubling 13 gives the answer of 26.
5. Draw a double number line, with 3, 6, 9, 12, 15, 18 marked on the top line, and 20, 40, 60, 80, 100, 120 marked on the bottom line. 15 is marked above 100. Therefore, 18 is 120% of 15.

**Assessment: Check Your Readiness (A)**
6. \(21 \div 30 = 0.7\), so 21 is 70% of 30.

**Aligned Standards**

6.RP.A.3.c
Assessment: Check Your Readiness (B)

Teacher Instructions
Calculators should not be used.

Student Instructions
Do not use a calculator.

Problem 1
The content assessed in this problem is first encountered in Lesson 2: Ratios and Rates With Fractions.

In an earlier unit, students were introduced to proportional relationships, and learned to use them to find equivalent ratios.

If most students struggle with this item, plan to use Lesson 2 Activity 2 to review strategies for working with ratios. While monitoring their work, notice which students are using each strategy listed in the lesson. During the synthesis of this activity, point out the connections between the shared strategies to support their understanding of the methods used. Students who struggle with the calculation may benefit from a more visual strategy using a double number line or a table.

Statement
Jada volunteers at an animal shelter that holds picnics to introduce people to animals. For the first picnic, she brought 36 clementines for 16 people, which was the right amount. For the next picnic, there will be 40 people! How many clementines should Jada bring for all those people? Explain or show your reasoning. (Note: clementines are fruits that are like small oranges.)

Solution
90 clementines. That is \( \frac{36}{16} = 2.25 \) clementines per person. So if \( y \) is the number of clementines needed and \( x \) is the number of people attending, then \( y = 2.25x \) and \( x = 40 \) means that \( y = (2.25) \cdot 40 \) or 90 clementines are needed.

Aligned Standards
6.RP.A.3.a

Problem 2
The content assessed in this problem is first encountered in Lesson 2: Ratios and Rates With Fractions.
In this unit, students will be asked to extend their understanding of ratios and unit rates to fractional quantities. This item assesses how well they are able to divide fractions, a grade 6 standard. In part a, students may find it easier to think visually or conceptually about how many times $\frac{3}{4}$ goes into $4\frac{1}{2}$. The division in part b is harder to do without using the standard algorithm.

If most students struggle with this item, plan to use the Number Talk in Lesson 2 Activity 1 to review dividing a fraction by a fraction. This lesson also offers opportunities to practice this calculation within a context. If students who struggled did not use diagrams for making sense of the problem, monitor for and display the work of students who used diagrams.

**Statement**

Solve each problem. Explain or show your reasoning.

1. Lin is baking cakes. Each cake needs $\frac{3}{4}$ cup of brown sugar, and Lin uses $4\frac{1}{2}$ cups of brown sugar for the entire batch of cakes. How many cakes does Lin bake?

2. $4\frac{1}{2}$ cups of salad dressing fill $3\frac{3}{4}$ identical jars. How many cups fill each jar?

**Solution**

1. 6. Sample response: $4\frac{1}{2}$ cups is the same as $\frac{9}{2}$ or $\frac{18}{4}$ cups, and $\frac{18}{4} \div \frac{3}{4} = 6$.

2. $\frac{6}{5}$ cups or equivalent. Sample response: $4\frac{1}{2} \div 3\frac{3}{4} = \frac{6}{5}$

**Aligned Standards**

6.RP.A.3.b

**Problem 3**

The content assessed in this problem is first encountered in Lesson 4: Half as Much Again.

In this unit, students use the distributive property to analyze statements relating to percent increase and decrease, like “Kiran ran three miles, then half as much again.”

If most students struggle with this item, plan to use Lesson 4 and the lessons that follow to review and solidify using the distributive property in a variety of situations as students write equivalent expressions and find percentages.

**Statement**

Which of these expressions is equivalent to $0.04(x - 25)$?
Problem 4

The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.

It is not required that students find exact solutions to this problem. However, the problem is designed to reveal the ways in which students are thinking about two common problem types: how to find a whole, given a part and a percentage, and how to find a given percentage of a given value. Students will study questions like these more formally in this unit. Students who solve the problem exactly will discover that the two pours are the same amount of water. A tape diagram can help them understand why.

If most students struggle with this item, plan to use Lesson 6 and the lessons that follow to study these two types of percentage problems. In Lesson 6 Activity 2 students will begin thinking about percent increase and percent decrease situations.

Statement

1. Tyler uses 7.5 ounces of barbecue sauce from a full bottle. He estimates that he used about 25% of the barbecue sauce in the bottle. About how much barbecue sauce was in the full bottle? Explain or show your reasoning.

2. Tyler uses 20% of the remaining barbecue sauce. About how many ounces did he use the second time? Explain or show your reasoning.

Solution

Answers vary due to the estimation described.

1. 30 ounces. 7.5 is 25% of 30, so there are 30 ounces in a full bottle.

2. 4.5. The remaining amount of BBQ sauce after the first usage is 22.5 ounces, and 20% of 22.5 is 4.5.

Aligned Standards

6.RP.A.3, 7.RP.A.3

Assessment: Check Your Readiness (B)
Problem 5
The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.

A student may accidentally use a different but correct scale for the tick marks: one such example is to label the top tick marks 64 and 72, and the bottom tick marks 80 and 90. This is unlikely but correct.

If most students struggle with this item, plan to monitor for students who use a double number line to solve Lesson 6 Activity 2. Then use the synthesis of Lesson 6 Activity 2 to review using double number lines to calculate the percent. These ideas will continue throughout the next several lessons.

**Statement**
To solve the problem “48 is what percent of 80?”, Elena uses this double number line. On the double number line, write the correct numbers at each unlabeled tick mark. Then solve the problem.

**Solution**
Answers vary. Sample response: The top tick marks are labeled 48 and 64, the bottom tick marks are labeled 60% and 80%. 48 is 60% of 80. Because 16 is 20% of 80, tick marks on the top should be marked in multiples of 16, while tick marks on the bottom should be marked in multiples of 20. Working backwards from 80 and 100%, the first pair of marks are 64 and 80%, and the second pair are 48 and 60%. This means that 48 is 60% of 80.

**Aligned Standards**
6.RP.A.3

Problem 6
The content assessed in this problem is first encountered in Lesson 6: Increasing and Decreasing.

This problem provides an opportunity to assess students’ “go-to” strategies for solving percent problems. Some students who are able to reason their way through the previous two questions may have more difficulty with these problems, since no context is provided.

If most students struggle with this item, plan to review these strategies using this problem in the days leading up to Lesson 6. You can find similar problems in Grade 6 Unit 3, Lessons 10-16 Practice Problems.
**Statement**

Solve each problem. Explain or show your reasoning.

1. 14 is 50% of what number?
2. 24 is 120% of what number?
3. 18 is what percentage of 30?
4. What is 25% of 80?
5. What is 41% of 200?
6. What is 150% of 26?

**Solution**

1. 28
2. 20
3. 60%
4. 20
5. 82
6. 39

Explanations vary. Sample explanations:

1. This question can be read as “14 is half of what number?” Doubling 14 gives the answer of 28.
2. Draw a double number line, with 4, 8, 12, 16, 20, 24 marked on the top line, and 20, 40, 60, 80, 100, 120 marked on the bottom line. 20 is marked above 100. Therefore, 24 is 120% of 20.
3. $18 \div 30 = 0.6$, so 18 is 60% of 30.
4. 25% of a number is $\frac{1}{4}$ of that number. Then $80 \cdot \frac{1}{4} = 20$.
5. Multiply using a decimal: $200 \cdot (0.41) = 82$. Alternatively: 41 is 41% of 100, so 82 must be 41% of 200.
6. 150% of a number is “the number, plus half again”: $26 + 13 = 39$.

**Aligned Standards**

6.RP.A.3.c
Assessment : End-of-Unit Assessment (A)

Teacher Instructions
Consider allowing students to use a calculator.

Problem 1
Students selecting A may have subtracted $7 \frac{1}{2} - 3 = \frac{9}{2}$. Students selecting C reversed the relationship between the variables. Students selecting D may have reversed the variables, while also making the mistake of subtracting to determine $\frac{9}{2}$.

Statement
Priya has a recipe for banana bread. She uses $7 \frac{1}{2}$ cups of flour to make 3 loaves of banana bread.

Andre will follow the same recipe. He will make $b$ loaves of banana bread using $f$ cups of flour.

Which of these equations represents the relationship between $b$ and $f$?

A. $b = \frac{2}{9} f$
B. $b = \frac{2}{5} f$
C. $b = \frac{5}{2} f$
D. $b = \frac{9}{2} f$

Solution
B

Aligned Standards
7.RP.A.2

Problem 2
Students selecting B may be calculating error relative to the measurement and not the actual value. Students selecting C may be finding what percentage 22 is of 23. Students selecting D may be finding what percentage 23 is of 22.

Statement
Diego measured the length of a pen to be 22 cm. The actual length of the pen is 23 cm.

Which of these is closest to the percent error for Diego’s measurement?
A. 4.3%
B. 4.5%
C. 95.7%
D. 104.5%

Solution
A

Aligned Standards
7.RP.A.3

Problem 3
Students selecting A computed 75% of the car’s length, but did not add 180 to it. Students selecting B solved the problem “180 is 75% of what number?” instead of the actual problem. Students selecting C may have added $180 + 75$ to get 255.

Statement
A car is 180 inches long. A truck is 75% longer than the car.

How long is the truck?
A. 135 inches
B. 240 inches
C. 255 inches
D. 315 inches

Solution
D

Aligned Standards
7.RP.A.3

Problem 4
The most likely error is answering $\frac{1}{5}$ mile per hour.

Some students may answer $\frac{1}{5}$ hours per mile, which is correct, but is a pace and not a running speed; you can decide for yourself whether to mark this answer as correct, but it is not a correct answer to the question asked. Similarly, decide for yourself what to do if a student answers “5” instead of “5 miles per hour”; this is not a correct answer to the question asked.
Statement
A circular running track is $\frac{1}{4}$ mile long. Elena runs on this track, completing each lap in $\frac{1}{20}$ of an hour.

What is Elena’s running speed? Include the unit of measure.

Solution
5 miles per hour (We can divide the number of miles by the number of hours to find the number of miles per hour: $\frac{1}{4} \div \frac{1}{20}$.)

Aligned Standards
7.RP.A.1

Problem 5
The last question can be answered in several different ways, including a double number line, a table, or solving an equation.

Statement
Today, everything at a store is on sale. The store offers a 20% discount.

1. The regular price of a T-shirt is $18. What is the discount price?
2. If the regular price of an item is $x$ dollars, what is the discount price in dollars?
3. The discount price of a hat is $18. What is the regular price?

Solution
1. $14.40 (80% of $18)
2. $0.8x$ or $(x - 0.2x)$ or $\frac{4}{5}x$ (or equivalent)
3. $22.50 (0.8x = 18 \text{ so } x = 22.5)$

Aligned Standards
7.EE.B.3, 7.RP.A.3

Problem 6
Use this problem as a check on students’ ability to add and multiply with decimals, besides the check on their understanding of discounts and tax.

Statement
Lin’s father is paying for a $20 meal. He has a 15%-off coupon for the meal. After the discount, a 7% sales tax is applied.
What does Lin’s father pay for the meal? Explain or show your reasoning.

Solution

$18.19. Explanations vary. Sample explanation: The coupon takes away 15% of $20, which is $3. The cost after the coupon is $17. The added tax is $1.19, 7% of $17. The total is $18.19, the sum of $17 and $1.19.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $20 - 3 = 17$, and $17 \cdot (1.07) = 18.19$.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: taking the incorrect tax of $1.40 based on $20; answering $1.19, the amount of the tax; answering $18.19 without showing or explaining work; errors in computation but not in concept.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: ignoring the 15% discount or the 7% tax altogether; adding 15% to the total; using 15% of $20 as the base amount; errors based on major misunderstandings about operations, such as adding $7 as the tax instead of 7%.

Aligned Standards

7.RP.A.3

Problem 7

While this problem does not require students to know the term *commission* from this unit, students who know the term will be more successful here. The term *markup* is required for the second part.

Statement

Tyler’s brother works in a shoe store.

1. Tyler’s brother earns a commission. He makes 2.5% of the amount he sells. Last week, he sold $900 worth of shoes. How much was his commission?

2. The store bought a pair of shoes for $50, and sold it for $80. What percentage was the markup?

3. Tyler’s brother earns $12 per hour. The store offers him a raise—a 5% increase per hour. After the raise, how much will Tyler’s brother make per hour?

Assessment: End-of-Unit Assessment (A)
Solution

1. $22.50. The commission is $22.50, because $900 \cdot (0.025) = 22.5$.

2. 60%. The markup is $30, and $30 is 60% of $50.

3. $12.60. 5\% of $12 is $0.60, so Tyler’s brother now makes $12 + 0.60$ dollars per hour.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
  1. $900 \cdot (0.025) = 22.5$
  2. 60%, a $30 increase.
  3. 12.60. The raise is 0.60.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one or two calculation errors in executing the parts of the problem; stating 60 instead of 60% for the markup without a very clear description that the 60 refers to 60%; answering 0.60, the value of the raise, instead of the total after the raise.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: one major conceptual error, perhaps due to a misinterpretation of the meaning of one of the terms in the problem; omission of one part of the problem; repeated calculation errors in execution.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more parts of the problem omitted or with major conceptual errors.

Aligned Standards

7.EE.B.3, 7.RP.A.3
Assessment: End-of-Unit Assessment (B)

Teacher Instructions
Consider allowing students to use a calculator.

Problem 1
Students selecting A may have subtracted $9 \frac{1}{3} - 4 = \frac{16}{3}$. Students selecting C reversed the relationship between the variables. Students selecting E may have reversed the variables, while also making the mistake of subtracting to determine $\frac{16}{3}$.

Statement
Clare has a recipe for yellow cake. She uses $9 \frac{1}{3}$ cups of flour to make 4 cakes. Noah will follow the same recipe. He will make $c$ cakes using $f$ cups of flour.

Which of these equations represent the relationship between $c$ and $f$?

A. $c = \frac{16}{3} f$
B. $c = \frac{3}{16} f$
C. $c = \frac{3}{7} f$
D. $c = \frac{7}{3} f$

Solution
C

Aligned Standards
7.RP.A.2

Problem 2
Students selecting A may be finding what percentage 7.5 is of 7. Students selecting B may be finding what percentage 7 is of 7.5. Students selecting C may be calculating error relative to the measurement and not the actual value.

Statement
A graduated cylinder actually contains 7.5 milliliters of water. When Han measures the volume of the water inside the graduated cylinder, his measurement is 7 milliliters. Which of these is closest to the percent error for Han’s measurement?
Problem 3

Students selecting A found how many hours per mile \((\frac{1}{24} \div \frac{1}{3})\). Students selecting B, \(\frac{1}{8}\) hours per mile, divided correctly but found the pace and not a running speed. (You can decide for yourself if you want to consider B an alternate correct answer, or if you want to assign partial credit.) Students selecting D switched the order of the unit rate.

Statement

It takes Diego \(\frac{1}{24}\) of an hour to complete a lap on a circular bike track. The track is \(\frac{1}{3}\) mile long. What is Diego's bike speed?

A. \(\frac{1}{8}\) mile per hour
B. \(\frac{1}{8}\) hours per mile
C. 8 miles per hour
D. 8 hours per mile

Solution

C

Aligned Standards

7.RP.A.1

Problem 4

The most common error students answering 100 feet by calculating 40% of 250 without adding 100 to 250. Students may also miss the fact that 40 is a percentage and compute 250 + 40 feet. Students who miss this problem due to setting up a proportion incorrectly could benefit from using a double number line.
Statement
A practice field is 250 feet long. The game field is 40% longer than the practice field. How long is the game field?

Solution
350 feet

Aligned Standards
7.RP.A.3

Problem 5
The first question can be answered in several different ways, including a double number line, a table, or solving an equation.

Statement
Noah has a coupon for 30% off at his favorite clothing store. He uses it to buy a hoodie and a pair of jeans.

1. Noah paid $28 for the jeans after using the coupon. What is the regular price of the jeans?
2. The regular price of a hoodie is $27. What did Noah pay for the hoodie?
3. If the regular price of an item is \( x \) dollars, what is the discount price in dollars?

Solution
1. $40 (0.7\( x \) = 28 so \( x \) = 40)
2. $18.90 (70% of 27)
3. 0.7\( x \) or \( x - 0.3x \) or \( \frac{7}{10} x \) or equivalent

Aligned Standards
7.EE.B.3, 7.RP.A.3

Problem 6
Use this problem as a check on students' ability to add and multiply with decimals, besides the check on their understanding of discounts and tips.

Statement
Kiran's mother gets a restaurant bill for $25. She has a coupon for 15% off. After the discount is applied, she adds 20% as a tip.

What is the total after the discount and tip? Explain or show your reasoning.

Assessment: End-of-Unit Assessment (B)
Solution
$25.50. Explanations vary. Sample explanation: The coupon takes away 15% of $25, which is $3.75. The cost after the coupon is $21.25. The added tip is $4.25, 20% of $21.25. The total is $25.50, the sum of $21.25 and $4.25.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $25 \times 0.85 = 21.25$, and $21.25 \times 0.20 = 4.25$.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: taking the incorrect tip of $5 based on $25; answering $4.25, the amount of tip; answering $25.50 without showing or explaining work; errors in computation but not in concept.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: ignoring the 15% discount or the 20% tip altogether; adding 15% to the total; using 15% of $25 as the base amount; errors based on major misunderstandings about operations, such as subtracting $15 as the discount instead of 15% or subtracting only $0.15.

Aligned Standards
7.RP.A.3

Problem 7

While this problem does not require students to know the term *commission* from this unit, students who know the term will be more successful here. The term *markup* is required for the second part.

Statement

Jada’s sister works in a furniture store.

1. Jada’s sister earns $15 per hour. The store offers her a raise—a 9% increase per hour. After the raise, how much will Jada’s sister make per hour?

2. The store bought a table for $200, and sold it for $350. What percentage was the markup?

3. Jada’s sister earns a commission. She makes 3.5% of the amount she sells. Last week, she sold $7,000 worth of furniture. How much was her commission?
Solution

1. $16.35. 9% of $15 is $1.35, so Jada's sister now makes $15 + $1.35 = $16.35 dollars per hour.

2. 75%. The markup is $150, and $150 is 75% of $200.

3. $245. The commission is $245, because $7,000 \cdot (0.035) = 245.$

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification
- Sample:

  1. 16.35. The raise is 1.35, because 9% of $15 is $1.35.
  2. 75%. The markup is $150, and $150 is 75% of $200.
  3. $7,000 \cdot (0.035) = 245$

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one or two calculation errors in executing the parts of the problem; stating 75 instead of 75% for the markup without a very clear description that the 75 refers to 75%; answering 1.35, the value of the raise, instead of the total after the raise.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: one major conceptual error, perhaps due to a misinterpretation of the meaning of one of the terms in the problem; omission of one part of the problem; repeated calculation errors in execution.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more parts of the problem omitted or with major conceptual errors.

Aligned Standards

7.EE.B.3, 7.RP.A.3
Lesson
Cool Downs
Lesson 1: Lots of Flags

Cool Down: Colorado State Flag

The side lengths of the state flag of Colorado are in the ratio 2 : 3. If a flag is 12 feet long, what is its height?
Lesson 2: Ratios and Rates With Fractions

Cool Down: Comparing Orange Juice Recipes

- Clare mixes $2 \frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of orange juice concentrate.

- Han mixes $1 \frac{2}{3}$ cups of water with $\frac{1}{4}$ cup of orange juice concentrate.

Whose orange juice mixture tastes stronger? Explain or show your reasoning.
Lesson 3: Revisiting Proportional Relationships

Cool Down: Walnuts in Bulk

It costs $3.45 to buy $\frac{3}{4}$ lb of chopped walnuts. How much would it cost to purchase 7.5 lbs of walnuts? Explain or show your reasoning.
Lesson 4: Half as Much Again

Cool Down: Fruit Snacks and Skating

1. Tyler ate $x$ fruit snacks, and Han ate $\frac{3}{4}$ less than that. Write an equation to represent the relationship between the number Tyler ate ($x$) and the number Han ate ($y$).

2. Mai skated $x$ miles, and Clare skated $\frac{3}{5}$ farther than that. Write an equation to represent the relationship between the distance Mai skated ($x$) and the distance Clare skated ($y$).
Lesson 5: Say It with Decimals

Cool Down: Reading More

Kiran read for $x$ minutes, and Andre read for $\frac{5}{8}$ more than that. Write an equation that relates the number of minutes Kiran read with $y$, the number of minutes that Andre read. Use decimals in your equation.
Lesson 6: Increasing and Decreasing

Cool Down: Fish Population

The number of fish in a lake decreased by 25% between last year and this year. Last year there were 60 fish in the lake. What is the population this year? If you get stuck, consider drawing a diagram.
Lesson 7: One Hundred Percent

Cool Down: More Laundry Soap

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?
Lesson 8: Percent Increase and Decrease with Equations

Cool Down: Tyler's Savings Bond

Tyler's mom purchased a savings bond for Tyler. The value of the savings bond increases by 4% each year. One year after it was purchased, the value of the savings bond was $156.

Find the value of the bond when Tyler's mom purchased it. Explain your reasoning.
Lesson 9: More and Less than 1%

Cool Down: Percentages of 75

Find each percentage of 75. Explain your reasoning.

1. What is 10% of 75?

2. What is 1% of 75?

3. What is 0.1% of 75?

4. What is 0.5% of 75?
Lesson 10: Tax and Tip

Cool Down: A Restaurant in a Different City

At a dinner, the meal cost $22 and a sales tax of $1.87 was added to the bill.

1. How much would the sales tax be on a $66 meal?

2. What is the tax rate for meals in this city?
Lesson 11: Percentage Contexts

Cool Down: The Cost of a Bike

The bike store marks up the wholesale cost of all of the bikes they sell by 30%.

1. Andre wants to buy a bike that has a price tag of $125. What was the wholesale cost of this bike?

2. If the bike is discounted by 20%, how much will Andre pay (before tax)?
Lesson 12: Finding the Percentage

Cool Down: Shoes on Sale

With a coupon, you can get a pair of shoes that normally costs $84 for only $72. What percentage was the discount?
Lesson 13: Measurement Error

Cool Down: Off by a Little Bit?

Clare estimates that her brother is 4 feet tall. When they get measured at the doctor's office, her brother's height is 4 feet, 2 inches.

1. Should Clare's or the doctor's measurement be considered the actual height? Explain your reasoning.

2. What was the error, expressed in inches?

3. What was the error, expressed as a percentage of the actual height?
Lesson 14: Percent Error

Cool Down: Jumbo Eggs

To be labeled as a jumbo egg, the egg is supposed to weigh 2.5 oz. Priya buys a carton of jumbo eggs and measures one of the eggs as 2.4 oz. What is the percent error?
Lesson 15: Error Intervals

Cool Down: An Angler's Dilemma

A fisherman weighs an ahi tuna (a very large fish) on a scale and gets a reading of 135 pounds. The reading on the scale may have an error of up to 5%. What are two possible values for the actual weight of the fish?
## Instructional Masters for Proportional Relationships and Percentages

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Noah ate $x$ ounces of blueberries, and Elena ate $\frac{1}{3}$ less than that.

Elena biked $x$ miles, and Noah biked $\frac{2}{3}$ more than that.

Elena danced as long as Noah danced, $x$, and then she danced that same amount again.

Noah saved $x$ dollars, and Elena saved $\frac{1}{10}$ less than that.

Noah bought $x$ pounds of apples, and Elena bought $\frac{1}{4}$ less than that.
### Card Sort: Representations of Proportional Relationships

Elena swam $x$ laps in the pool, and Noah swam $\frac{2}{5}$ less than that.

$$y = \frac{3}{5}x$$

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<th>$x$</th>
<th>$y$</th>
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<tbody>
<tr>
<td>3</td>
<td>$\frac{9}{5}$</td>
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<td>5</td>
<td>3</td>
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</table>

Elena ate $x$ pretzels, and Noah ate $\frac{1}{3}$ more than that.

$$y = \frac{4}{3}x$$

<table>
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<th>$x$</th>
<th>$y$</th>
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<td>2</td>
<td>$2\frac{2}{3}$</td>
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<tr>
<td>6</td>
<td>8</td>
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</table>

Noah ran $x$ kilometers, and Elena ran $\frac{3}{10}$ more than that.

$$y = \frac{13}{10}x$$

<table>
<thead>
<tr>
<th>$x$</th>
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<tbody>
<tr>
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<td>7</td>
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<td>Card Sort: More Representations</td>
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<tr>
<td>Noah ate $x$ ounces of blueberries, and Elena ate $\frac{1}{3}$ less than that.</td>
<td>$y = 0.6x$</td>
</tr>
<tr>
<td>Elena biked $x$ miles, and Noah biked $\frac{2}{3}$ more than that.</td>
<td>$y = 1.6x$</td>
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<td>Noah saved $x$ dollars, and Elena saved $\frac{1}{10}$ less than that.</td>
<td>$y = 0.9x$</td>
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<tr>
<td>Noah bought $x$ pounds of apples, and Elena bought $\frac{1}{4}$ less than that.</td>
<td>$y = 0.75x$</td>
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<tr>
<td>Elena swam $x$ laps in the pool, and Noah swam $\frac{2}{5}$ less than that.</td>
<td>$y = 0.6x$</td>
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<tr>
<td>Elena ate $x$ pretzels, and Noah ate $\frac{1}{3}$ more than that.</td>
<td>$y = 1.3x$</td>
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<tr>
<td>Noah ran $x$ kilometers, and Elena ran $\frac{3}{10}$ more than that.</td>
<td>$y = 1.3x$</td>
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<td>Sales Tax</td>
<td>Markup</td>
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<tr>
<td>Gratuity (Tip)</td>
<td>Markdown (Discount)</td>
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Card Sort: Percentage Situations

Kiran ate breakfast at a café. The total check was $5. Here is the money he left.

Why did he pay $5.75?

Tyler bought a shirt from the 25% off rack. It normally costs $16. Here is the money he paid.

Why did he pay $12?

Card Sort: Percentage Situations

Clare’s aunt used a credit card to pay $40 for an outfit. Six weeks later, the credit card company added $0.60 to her bill.

Why does she owe $40.60 for the outfit now?

Lin bought 1 pretzel from a snack cart.

Why did she pay $4.83?
A car dealership paid $9,000 for a car. Here is the sticker price for a customer to buy the car.

Why are they charging $10,395?

Andre is saving money for college. He had $1,500 in his account. At the end of the year, the bank adds 3% of the balance to his account.

Why does his account have $1,545 now?

Priya used this coupon to buy a game that normally costs $24.50.

Why did she pay $22.05?

Diego's uncle sells computers. In addition to his wages, he gets paid 20% of the amount he sells.

Why did he earn the extra $600?
Elena went to a sporting goods store that was having a sale. She bought a tennis racket and 3 cans of tennis balls. How much will she pay for everything, including tax?

- The tennis racket normally costs $43.
- All tennis rackets are marked down 15%.
- One can of tennis balls normally costs $4.
- The tennis balls are not marked down.
- The sales tax rate is 8.5%.

Andre went to a sporting goods store that was having a different sale. He bought a baseball glove and 2 packages of socks. What percentage of the total regular price (before tax) was his savings?

- The baseball glove normally costs $34.
- The baseball glove is not discounted.
- One package of socks normally costs $6.
- On sale, one package of socks costs $4.
- The sale tax rate is 7.75%.
Measuring to the Nearest.
### Problem Card 1

A factory worker tested the speedometer of a new car by driving it a certain distance at a constant speed. The percent error was too large, so the car was sent back to be fixed. What speed did the speedometer show?

#### Data Card 1
- The car went 10 miles.
- The car drove for 8 minutes.
- To pass the test, the speedometer must show within 2% of the correct speed.
- The speedometer was showing a speed slower than the car was actually going.

### Problem Card 2

A machine is supposed to fill each bottle with the same amount of juice. A particular bottle was tested and the percent error was not too large. How much juice was in the bottle?

#### Data Card 2
- Each bottle is supposed to be filled with 450 milliliters of juice.
- To pass the test, the bottle must be filled to within 1.5% of the correct amount.
- The bottle was slightly overfilled.
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