Rational Numbers

Teacher Guide

MON TUE WED THUR FRI SAT SUN
5 -1 -5 -2 3 4 0

Positive and Negative Numbers

High and Low

Drawing on the Coordinate Plane

six-pointed star Drawing

Four Quadrants
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Rational Numbers

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Teacher Guide
Core Knowledge Mathematics™
Rational Numbers

Unit 7 Narrative

In this unit, students are introduced to signed numbers and plot points in all four quadrants of the coordinate plane for the first time. They work with simple inequalities in one variable and learn to understand and use “common factor,” “greatest common factor,” “common multiple,” and “least common multiple.”

The first section of the unit introduces signed numbers. Students begin by considering examples of positive and negative temperatures, plotting each temperature on a vertical number line on which 0 is the only label. Next, they consider examples of positive and negative numbers used to denote height relative to sea level. In the second lesson, they plot positive and negative numbers on horizontal number lines, including “opposites”—pairs of numbers that are the same distance from zero. They use “less than,” “greater than,” and the corresponding symbols to describe the relationship of two signed numbers, noticing correspondences between the relative positions of two numbers on the number line and statements that use these symbols, e.g., 0.8 > -1.3 means that 0.8 is to the right of -1.3 on the number line. Students learn that the sign of a number indicates whether the number is positive or negative, and that zero has no sign. They learn that the absolute value of a number is its distance from zero, how to use absolute value notation, and that opposites have the same absolute value because they have the same distance from zero.

Previously, when students worked only with non-negative numbers, magnitude and order were indistinguishable: if one number was greater than another, then on the number line it was always to the right of the other number and always farther from zero. In comparing two signed numbers, students distinguish between magnitude (the absolute value of a number) and order (relative position on the number line), distinguishing between “greater than” and “greater absolute value,” and “less than” and “smaller absolute value.”

Students examine opposites of numbers, noticing that the opposite of a negative number is positive.

The second section of the unit concerns inequalities. Students graph simple inequalities in one variable on the number line, using a circle or disk to indicate when a given point is, respectively, excluded or included. In these materials, inequality symbols in grade 6 are limited to < and > rather than ≤ and ≥. However, in this unit students encounter situations when they need to represent statements such as 2 < x or 2 = x.

Students represent situations that involve inequalities, symbolically and with the number line, understanding that there may be infinitely many solutions for an inequality. They interpret and graph solutions in contexts (MP2), understanding that some results do not make sense in some contexts, and thus the graph of a solution might be different from the graph of the related symbolic inequality. For example, the graph describing the situation “A fishing boat can hold fewer than 9 people” omits values other than the whole numbers from 0 to 8, but the graph of x < 8 includes all numbers less than 8. Students encounter situations that require more than one inequality statement to describe, e.g., “It rained for more than 10 minutes but less than 30 minutes” (t > 10
and \( t < 30 \), where \( t \) is the amount of time that it rained in minutes) but which can be described by one number line graph.

The third section of the unit focuses on the coordinate plane. In grade 5, students learned to plot points in the coordinate plane, but they worked only with non-negative numbers, thus plotted points only in the first quadrant. In a previous unit, students again worked in the first quadrant of the coordinate plane, plotting points to represent ratio and other relationships between two quantities with positive values. In this unit, students work in all four quadrants of the coordinate plane, plotting pairs of signed number coordinates in the plane. They understand that for a given data set, there are more and less strategic choices for the scale and extent of a set of axes. They understand the correspondence between the signs of a pair of coordinates and the quadrant of the corresponding point. They interpret the meanings of plotted points in given contexts (MP2), and use coordinates to calculate horizontal and vertical distances between two points.

The last section of the unit returns to consideration of whole numbers. In the first lesson, students are introduced to “common factor” and “greatest common factor,” and solve problems that illustrate how the greatest common factor of two numbers can be used in real-world situations, e.g., determining the largest rectangular tile with whole-number dimensions that can tile a given rectangle with whole-number dimensions. The second lesson introduces “common multiple” and “least common multiple,” and students solve problems that involve listing common multiples or identifying common multiples of two or more numbers. In the third and last lesson, students solve problems that revisit situations similar to those in the first two lessons and identify which of the new concepts is involved in each problem. This lesson includes two optional classroom activities.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, interpreting, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe and interpret

- situations involving negative numbers (Lesson 1)
- features of a number line (Lessons 2, 4 and 6)
- situations involving elevation (Lesson 7)
- situations involving minimums and maximums (Lesson 8)
- points on a coordinate plane (Lessons 11 and 14)
- situations involving factors and multiples (Lesson 18)

Justify

- reasoning about magnitude (Lesson 3)
- reasoning about a situation involving negative numbers (Lesson 5)
• reasoning about solutions to inequalities (Lesson 9)
• that all possible pairs of factors have been identified (Lesson 16)

Generalize

• the meaning of integers for a specific context (Lesson 5)
• understandings of solutions to inequalities (Lesson 9)
• about the relationships between shapes (Lesson 10)
• about greatest common factors (Lesson 16)
• about least common multiples (Lesson 17)

In addition, students are expected to critique the reasoning of others, represent inequalities symbolically and in words, and explain how to order rational numbers and how to determine distances on the coordinate plane. Students also have opportunities to use language to compare magnitudes of positive and negative numbers, compare features of ordered pairs, and compare appropriate axes for different sets of coordinates.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.
Learning Targets

Rational Numbers

Lesson 1: Positive and Negative Numbers
- I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation.
- I can use positive and negative numbers to describe temperature and elevation.
- I know what positive and negative numbers are.

Lesson 2: Points on the Number Line
- I can determine or approximate the value of any point on a number line.
- I can represent negative numbers on a number line.
- I understand what it means for numbers to be opposites.

Lesson 3: Comparing Positive and Negative Numbers
- I can explain how to use the positions of numbers on a number line to compare them.
- I can explain what a rational number is.
- I can use inequalities to compare positive and negative numbers.

Lesson 4: Ordering Rational Numbers
- I can compare and order rational numbers.
- I can use phrases like “greater than,” “less than,” and “opposite” to compare rational numbers.

Lesson 5: Using Negative Numbers to Make Sense of Contexts
- I can explain and use negative numbers in situations involving money.
- I can interpret and use negative numbers in different contexts.
Lesson 6: Absolute Value of Numbers
• I can explain what the absolute value of a number is.
• I can find the absolute values of rational numbers.
• I can recognize and use the notation for absolute value.

Lesson 7: Comparing Numbers and Distance from Zero
• I can explain what absolute value means in situations involving elevation.
• I can use absolute values to describe elevations.
• I can use inequalities to compare rational numbers and the absolute values of rational numbers.

Lesson 8: Writing and Graphing Inequalities
• I can graph inequalities on a number line.
• I can write an inequality to represent a situation.

Lesson 9: Solutions of Inequalities
• I can determine if a particular number is a solution to an inequality.
• I can explain what it means for a number to be a solution to an inequality.
• I can graph the solutions to an inequality on a number line.

Lesson 10: Interpreting Inequalities
• I can explain what the solution to an inequality means in a situation.
• I can write inequalities that involves more than one variable.

Lesson 11: Points on the Coordinate Plane
• I can describe a coordinate plane that has four quadrants.
• I can plot points with negative coordinates in the coordinate plane.
• I know what negative numbers in coordinates tell us.

Lesson 12: Constructing the Coordinate Plane
• When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes.
Lesson 13: Interpreting Points on a Coordinate Plane

- I can explain how rational numbers represent balances in a money context.
- I can explain what points in a four-quadrant coordinate plane represent in a situation.
- I can plot points in a four-quadrant coordinate plane to represent situations and solve problems.

Lesson 14: Distances on a Coordinate Plane

- I can find horizontal and vertical distances between points on the coordinate plane.

Lesson 15: Shapes on the Coordinate Plane

- I can find the lengths of horizontal and vertical segments in the coordinate plane.
- I can plot polygons on the coordinate plane when I have the coordinates for the vertices.

Lesson 16: Common Factors

- I can explain what a common factor is.
- I can explain what the greatest common factor is.
- I can find the greatest common factor of two whole numbers.

Lesson 17: Common Multiples

- I can explain what a common multiple is.
- I can explain what the least common multiple is.
- I can find the least common multiple of two whole numbers.

Lesson 18: Using Common Multiples and Common Factors

- I can solve problems using common factors and multiples.

Lesson 19: Drawing on the Coordinate Plane

- I can use ordered pairs to draw a picture.
<table>
<thead>
<tr>
<th>lesson</th>
<th>new terminology</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>receptive                              productive</td>
</tr>
<tr>
<td>6.7.1</td>
<td>positive number                            negative number</td>
</tr>
<tr>
<td></td>
<td>temperature                              degrees Celsius</td>
</tr>
<tr>
<td></td>
<td>elevation                                 sea level</td>
</tr>
<tr>
<td></td>
<td>number line                               below zero</td>
</tr>
<tr>
<td>6.7.2</td>
<td>opposite (numbers)                         rational number</td>
</tr>
<tr>
<td></td>
<td>location                                  distance (away) from zero</td>
</tr>
<tr>
<td>6.7.3</td>
<td>sign                                       greater than less than</td>
</tr>
<tr>
<td></td>
<td>inequality                                closer to 0 farther from 0</td>
</tr>
<tr>
<td>6.7.4</td>
<td>from least to greatest                     temperature elevation sea level</td>
</tr>
<tr>
<td>6.7.5</td>
<td>positive change                           negative change context</td>
</tr>
<tr>
<td>6.7.6</td>
<td>absolute value                            positive number negative number distance (away) from zero</td>
</tr>
<tr>
<td>6.7.7</td>
<td>closer to 0 farther from 0</td>
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<tr>
<td>6.7.8</td>
<td>maximum                                   minimum</td>
</tr>
<tr>
<td>6.7.9</td>
<td>requirement                                solution to an inequality</td>
</tr>
<tr>
<td>6.7.10</td>
<td>unbalanced hanger                         inequality</td>
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<table>
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<th>lesson</th>
<th>new terminology</th>
<th>receptive</th>
<th>productive</th>
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<tr>
<td>6.7.11</td>
<td>quadrant</td>
<td>x-coordinate</td>
<td>y-coordinate</td>
</tr>
<tr>
<td>6.7.12</td>
<td>(line) segment</td>
<td>axis</td>
<td></td>
</tr>
<tr>
<td>6.7.13</td>
<td>degrees Fahrenheit</td>
<td>degrees Celsius</td>
<td></td>
</tr>
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<td>6.7.14</td>
<td></td>
<td></td>
<td>absolute value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>x-coordinate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>y-coordinate</td>
</tr>
<tr>
<td>6.7.16</td>
<td>common factor</td>
<td></td>
<td>factor</td>
</tr>
<tr>
<td></td>
<td>greatest common factor (GCF)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.7.17</td>
<td>common multiple</td>
<td></td>
<td>multiple</td>
</tr>
<tr>
<td></td>
<td>least common multiple (LCM)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Required Materials**

**Bingo chips**

**Colored pencils**

**Graphing technology**

Examples of graphing technology are: a handheld graphing calculator, a computer with a graphing calculator application installed, and an internet-enabled device with access to a site like desmos.com/calculator or geogebra.org/graphing. For students using the digital materials, a separate graphing calculator tool isn't necessary; interactive applets are embedded throughout, and a graphing calculator tool is accessible on the student digital toolkit page.

**Graph paper**

**Pre-printed cards, cut from copies of the Instructional master**

**Pre-printed slips, cut from copies of the Instructional master**

**Rulers**

**Rulers marked with centimeters**

**Snap cubes**

**Sticky notes**

**Tools for creating a visual display**

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

**Tracing paper**

Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.
Section: Negative Numbers and Absolute Value
Lesson 1: Positive and Negative Numbers

Goals

- Comprehend the words “positive” and “negative” (in spoken and written language) and the symbol “−” (in written language). Say “negative” when reading numbers written with the “−” symbol.
- Interpret positive and negative numbers that represent temperature or elevation, and understand the convention of what “below zero” typically means in each of these contexts.
- Recognize that the number line can be extended to represent negative numbers.

Learning Targets

- I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation.
- I can use positive and negative numbers to describe temperature and elevation.
- I know what positive and negative numbers are.

Lesson Narrative

Students in grade 6 have spent considerable time developing their understanding and fluency with positive numbers. In this lesson, students extend their thinking to negative numbers by exploring temperature and elevation. In these two contexts, zero represents a physical situation (freezing point of water, sea level) and numbers less than zero describe a physical state in the real world. Students abstract temperatures and elevations to positive and negative numbers on a number line (MP2).

Alignments

Addressing

- 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
Instructional Routines

- MLR5: Co-Craft Questions
- MLR6: Three Reads
- Notice and Wonder
- Think Pair Share

Required Materials

Rulers

Required Preparation

Rulers may be helpful to create number lines in the “High Places, Low Places” activity.

Student Learning Goals

Let’s explore how we represent temperatures and elevations.

1.1 Notice and Wonder: Memphis and Bangor

Warm Up: 5 minutes
The purpose of this task is to introduce students to temperatures measured in degrees Celsius. Many students have an intuitive understanding of temperature ranges in degrees Fahrenheit that are typical of the city or town in which they live, but many are unfamiliar with the Celsius scale.

Addressing

- 6.NS.C.5
- 6.NS.C.6

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.
Student Task Statement

What do you notice? What do you wonder?

Student Response

Things students may notice:

- The weather is different in the two cities.
- There are two temperatures for each city.
- The temperatures have different letters.
- The times are different.
- There is a minus sign on one of the temperatures.

Things students may wonder:

- What the two temperatures mean.
- Why it isn't snowing in the colder city.
- What the minus sign is doing there.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. Explain to students that temperatures are usually measured in either degrees Fahrenheit, which is what they are probably most familiar with, and degrees Celsius, which may be new for them. Tell them that many other countries measure temperature in degrees Celsius and that scientists use this temperature scale. One thing that is special about the Celsius scale is that water freezes at 0 degrees and boils at 100 degrees (at sea level).

1.2 Above and Below Zero

10 minutes (there is a digital version of this activity)
The purpose of this task is to understand that there are natural mathematical questions about certain contexts for which there are no answers if we restrict ourselves to positive numbers. The idea is to motivate the need for negative numbers and to see that there is a natural representation of them on the number line. This task is not about operations with signed numbers, but rather why we extend our number system beyond positive numbers. Students reason abstractly and quantitatively when they represent the change in temperature on a number line (MP2).

**Addressing**

- 6.NS.C.5

**Instructional Routines**

- MLR6: Three Reads
- Think Pair Share

**Launch**

Display this image for all to see.

Tell students, “The thermometer showed a temperature of 7 degrees Celsius one morning. Later, the temperature increased 4 degrees. We can represent this change in temperature using a number line, as shown in the picture.”

Arrange students in groups of 2. Give students 2 minutes of quiet work time for question 1. Give students 2 minutes to compare their responses to their partners and to work on question 2.

Students using the digital version of the curriculum can explore the changes in temperature with the dynamic applet.
Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support students’ comprehension of the temperature changes for the first situation. In the first read, students read the situation with the goal of comprehending the text (e.g., the thermometer was at a certain temperature earlier in the day and then changed later in the day). Delay asking “What was the temperature late in the afternoon?” In the second read, ask students to analyze the text to understand the quantities (e.g., at noon, the temperature was 7 degrees Celsius; it increases 6 degrees by late afternoon). In the third read, direct students’ attention to the question and ask students to brainstorm possible mathematical solution strategies to answer the question.

Design Principle(s): Support sense-making

Anticipated Misconceptions

Some students may have difficulty representing change on the number line. Students sometimes count tick marks rather than counting the space between tick marks when working on a number line. For example, in the original problem image, the arrow on the number line represents a change of 4 degrees. Some students may begin at tick mark 7 and count the tick marks to yield a temperature change of 5 degrees. When reviewing that task with the whole class, be sure to make this important point and demonstrate counting on a number line by highlighting the space between the tick marks while counting out loud.

Student Task Statement

1. Here are three situations involving changes in temperature and three number lines. Represent each change on a number line. Then, answer the question.

   a. At noon, the temperature was 5 degrees Celsius. By late afternoon, it has risen 6 degrees Celsius. What was the temperature late in the afternoon?

   b. The temperature was 8 degrees Celsius at midnight. By dawn, it has dropped 12 degrees Celsius. What was the temperature at dawn?

   c. Water freezes at 0 degrees Celsius, but the freezing temperature can be lowered by adding salt to the water. A student discovered that adding half a cup of salt to a gallon of water lowers its freezing temperature by 7 degrees Celsius. What is the freezing temperature of the gallon of salt water?
2. Discuss with a partner:

   a. How did each of you name the resulting temperature in each situation?
   b. What does it mean when the temperature is above 0? Below 0?
   c. Do numbers less than 0 make sense in other contexts? Give some specific examples to show how they do or do not make sense.

**Student Response**

1. a. 11 degrees Celsius
   b. 4 degrees below zero, -4 degrees Celsius
   c. 7 degrees below zero, -7 degrees Celsius

2. Answers vary. Sample responses:
   a. Temperatures below 0 may be marked with a signifier labeled “below,” or equivalent.
   b. Temperatures below 0 are colder than temperatures above 0.
   c. It’s possible to go underground, so if ground level is 0, underground would be below 0. If you are counting people, on the other hand, negative numbers don’t make sense. You can’t have negative people (not mathematically, anyway).

**Activity Synthesis**

Some students will use the phrase “degrees below zero.” Use this activity to introduce the term **negative** as a way to represent a quantity less than zero. In contrast, ask students how they would
describe a quantity that is greater than zero on the number line. Some students will have a pre-existing understanding of positive and negative numbers. Discuss the use of + and - as symbols to denote positive and negative numbers. Notation will be important throughout the rest of this unit. Students should understand that +7 and 7 both represent positive 7. Negative 7 is represented as -7.

Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: negative.
Supports accessibility for: Conceptual processing; Language; Memory

1.3 High Places, Low Places

20 minutes (there is a digital version of this activity)
The purpose of this task is to present a second, natural context for negative numbers and to start comparing positive and negative numbers in preparation for ordering them. Monitor for students who make connections between elevation and temperature or come up with strategies for deciding which points are lower or higher than other points. Students may use the structure of a vertical number line in order to compare the relative location of each elevation (MP7).

**Addressing**

- 6.NS.C.5

**Instructional Routines**

- MLR5: Co-Craft Questions

**Launch**

Display the table of elevations for all to see. Ask students to think of a way to explain in their own words what the numbers mean. Ask two or three students to share their ideas.

Tell students, “The term ‘elevation’ is commonly used to describe the height of a place (such as a city) or an object (such as an aircraft) compared to sea level. Denver, CO, is called ‘The Mile High City’ because its elevation is 1 mile or 5,280 feet above sea level.”

Arrange students in groups of 2 and give students 5 minutes of quiet work time to answer the first five questions. Ask students to be prepared to explain their thinking in a whole-class discussion.

Students using the digital activity are provided with an interactive map in addition to the questions about elevation. After they complete the questions in the task, they can drag each point to the elevation on the number line for the landmark it represents.
Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time. Check to make sure students have attended to all parts of the original figures.
Supports accessibility for: Organization; Attention

Access for English Language Learners

Reading, Writing, Speaking: MLR5 Co-Craft Questions. To help students use language related to positive and negative numbers within the context of elevation, show students the table and ask pairs to write down mathematical questions to ask about the situation. Have students share their questions with a partner and then share out with the class.
Design Principle(s): Support sense-making

Anticipated Misconceptions
Some students may have difficulty comparing negative elevations. For example, when students are asked to find a higher elevation than Coachella, CA, they may think that -35 feet is a higher elevation than -22 feet because 35 > 22. Encourage students to create a vertical number line and plot elevations before comparing them. Alternatively, provide them with a pre-made number line to use.

Student Task Statement

1. Here is a table that shows elevations of various cities.

<table>
<thead>
<tr>
<th>city</th>
<th>elevation (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harrisburg, PA</td>
<td>320</td>
</tr>
<tr>
<td>Bethell, IN</td>
<td>1,211</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>5,280</td>
</tr>
<tr>
<td>Coachella, CA</td>
<td>-22</td>
</tr>
<tr>
<td>Death Valley, CA</td>
<td>-282</td>
</tr>
<tr>
<td>New York City, NY</td>
<td>33</td>
</tr>
<tr>
<td>Miami, FL</td>
<td>0</td>
</tr>
</tbody>
</table>

a. On the list of cities, which city has the second highest elevation?
b. How would you describe the elevation of Coachella, CA in relation to sea level?

c. How would you describe the elevation of Death Valley, CA in relation to sea level?

d. If you are standing on a beach right next to the ocean, what is your elevation?

e. How would you describe the elevation of Miami, FL?

f. A city has a higher elevation than Coachella, CA. Select all numbers that could represent the city’s elevation. Be prepared to explain your reasoning.

- -11 feet
- -35 feet
- 4 feet
- -8 feet
- 0 feet

2. Here are two tables that show the elevations of highest points on land and lowest points in the ocean. Distances are measured from sea level.

<table>
<thead>
<tr>
<th>mountain</th>
<th>continent</th>
<th>elevation (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everest</td>
<td>Asia</td>
<td>8,848</td>
</tr>
<tr>
<td>Kilimanjaro</td>
<td>Africa</td>
<td>5,895</td>
</tr>
<tr>
<td>Denali</td>
<td>North America</td>
<td>6,168</td>
</tr>
<tr>
<td>Pikchu Pikchu</td>
<td>South America</td>
<td>5,664</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>trench</th>
<th>ocean</th>
<th>elevation (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mariana Trench</td>
<td>Pacific</td>
<td>-11,033</td>
</tr>
<tr>
<td>Puerto Rico Trench</td>
<td>Atlantic</td>
<td>-8,600</td>
</tr>
<tr>
<td>Tonga Trench</td>
<td>Pacific</td>
<td>-10,882</td>
</tr>
<tr>
<td>Sunda Trench</td>
<td>Indian</td>
<td>-7,725</td>
</tr>
</tbody>
</table>

a. Which point in the ocean is the lowest in the world? What is its elevation?

b. Which mountain is the highest in the world? What is its elevation?
c. If you plot the elevations of the mountains and trenches on a vertical number line, what would 0 represent? What would points above 0 represent? What about points below 0?

d. Which is farther from sea level: the deepest point in the ocean, or the top of the highest mountain in the world? Explain.

Student Response
1. a. Bethell, IN: 1,211 feet has the second highest elevation.
   b. 22 feet below sea level
   c. 282 feet below sea level
   d. My elevation would be 0 at the beach next to the sea
   e. Miami, FL, is at sea level
   f. A, C, D, and E because A is 11 feet below sea level, so it is higher than 22 feet below. C is above sea level, so it is higher. D is 8 feet below sea level, so it is higher than 22 feet below. E because it is 0 feet and that is sea level, so that is higher than 22 feet below.

2. a. Mariana Trench in the Pacific Ocean; -11,033 meters
   b. Mt. Everest in Asia; 8,848 meters
   c. 0 would represent sea level; the points above 0 would represent elevations above sea level; the points below 0 would represent elevations below sea level.
   d. The deepest point in the ocean because -11,033 meters is farther from zero on a number line than 8,848 meters.

Are You Ready for More?
A spider spins a web in the following way:

- It starts at sea level.
- It moves up one inch in the first minute.
- It moves down two inches in the second minute.
- It moves up three inches in the third minute.
- It moves down four inches in the fourth minute.

Assuming that the pattern continues, what will the spider’s elevation be after an hour has passed?

Student Response
30 inches below sea level. (The pattern after each minute is +1, -1, +2, -2, etc.)
Activity Synthesis

The important concept is that elevation measures how far below or above sea level something is. Positive elevation tells us that something is above sea level, whereas negative elevation tells us that something is below sea level. In the same way, positive numbers are greater than zero and negative numbers are less than zero. Zero is neither greater than or less than zero; therefore, it is neither positive or negative. Invite selected students to share their thinking about how they compared different elevations and any similarities they may have noticed between elevation and temperature.

Lesson Synthesis

In this lesson, students considered two contexts that motivate the need for numbers less than zero. Focus their attention on what zero represents in each situation, since that choice affects the interpretation of positive and negative numbers in the context.

- What does zero represent in each situation? (freezing point of water, sea level)
- What does a positive number represent in each context? (temperatures above freezing, elevations above sea level)
- What does a negative number represent in each context? (temperatures below freezing, elevations below sea level)

- Is -30 degrees warmer or colder than -40 degrees?
- Is an elevation of -20 feet higher or lower than an elevation of -10 feet?
- In general, what is a positive number? Where are they located on a number line? (a number that is greater than zero; on the same side of 0 as 1, which is usually to the right of zero or above zero)
- In general, what is a negative number? Where are they located on a number line? (a number that is less than zero; on the opposite side of 0 as 1, which is usually to the left of zero or below zero)

1.4 Agree or Disagree?

Cool Down: 5 minutes

The purpose of this cool-down is to review the previous contexts used for working with positive and negative numbers. Students will consider distances from zero as a preview for an upcoming lesson. Students are asked to make comparisons in relation to these contexts and consider relative positions on the number line.

Addressing

- 6.NS.C.5

Unit 7 Lesson 1
Student Task Statement
State whether you agree with each of the following statements. Explain your reasoning.

1. A temperature of 35 degrees Fahrenheit is as cold as a temperature of -35 degrees Fahrenheit.

2. A city that has an elevation of 15 meters is closer to sea level than a city that has an elevation of -10 meters.

3. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.

Student Response

1. Disagree. 35 degrees Fahrenheit is above 0 degrees Fahrenheit and -35 degrees Fahrenheit is below 0 degrees Fahrenheit. -35 degrees is 70 degrees colder than 35 degrees.

2. Disagree. -10 meters is 10 meters from sea level. 15 meters is 15 meters from sea level. -10 meters is closer to sea level.

3. Agree. -17 meters is 17 meters from sea level. -40 meters is 40 meters from sea level. -17 meters is closer to sea level.

Student Lesson Summary

Positive numbers are numbers that are greater than 0. Negative numbers are numbers that are less than zero. The meaning of a negative number in a context depends on the meaning of zero in that context.

For example, if we measure temperatures in degrees Celsius, then 0 degrees Celsius corresponds to the temperature at which water freezes.

In this context, positive temperatures are warmer than the freezing point and negative temperatures are colder than the freezing point. A temperature of -6 degrees Celsius means that it is 6 degrees away from 0 and it is less than 0. This thermometer shows a temperature of -6 degrees Celsius.

If the temperature rises a few degrees and gets very close to 0 degrees without reaching it, the temperature is still a negative number.
Another example is elevation, which is a distance above or below sea level. An elevation of 0 refers to the sea level. Positive elevations are higher than sea level, and negative elevations are lower than sea level.

Glossary

- negative number
- positive number
Lesson 1 Practice Problems

Problem 1

Statement

a. Is a temperature of -11 degrees warmer or colder than a temperature of -15 degrees?

b. Is an elevation of -10 feet closer or farther from the surface of the ocean than an elevation of -8 feet?

c. It was 8 degrees at nightfall. The temperature dropped 10 degrees by midnight. What was the temperature at midnight?

d. A diver is 25 feet below sea level. After he swims up 15 feet toward the surface, what is his elevation?

Solution

a. Warmer

b. Farther

c. -2 degrees

d. -10 feet or 10 feet below sea level

Problem 2

Statement

a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?

b. The whale swims down 300 feet to feed. What is the whale's elevation now?

c. The whale swims down 150 more feet more. What is the whale's elevation now?

d. Plot each of the three elevations as a point on a vertical number line. Label each point with its numeric value.

Solution

a. 0. (Sea level is 0 feet above or below sea level.)

b. -300 feet. (The whale is 300 feet below sea level.)

c. -450 feet. (The whale was 300 feet below sea level, and now it is an additional 150 feet below sea level.)

d. A number line with 0, -300, and -450 marked.
Problem 3

Statement
Explain how to calculate a number that is equal to \(\frac{21}{15}\).

Solution
Answers vary. Sample response: \(\frac{21}{15}\) means \(2.1 \div 1.5\). This can be done by long division. (The question doesn't require it, but the quotient is 1.4.)

(From Unit 6, Lesson 5.)

Problem 4

Statement
Write an equation to represent each situation and then solve the equation.

a. Andre drinks 15 ounces of water, which is \(\frac{3}{5}\) of a bottle. How much does the bottle hold?
   Use \(x\) for the number of ounces of water the bottle holds.

b. A bottle holds 15 ounces of water. Jada drank 8.5 ounces of water. How many ounces of water are left in the bottle? Use \(y\) for the number of ounces of water left in the bottle.

c. A bottle holds \(z\) ounces of water. A second bottle holds 16 ounces, which is \(\frac{8}{5}\) times as much water. How much does the first bottle hold?

Solution
a. \(\frac{3}{5}x = 15\). Solution: 25.

b. \(y + 8.5 = 15\). Solution: 6.5.

c. \(\frac{8}{5}z = 16\) Solution: 10. Equations equivalent to these are also acceptable.

(From Unit 6, Lesson 4.)

Problem 5

Statement
A rectangle has an area of 24 square units and a side length of \(2 \frac{3}{4}\) units. Find the other side length of the rectangle. Show your reasoning.

Solution
\(8 \frac{8}{11}\). Sample reasoning: \(24 \div \frac{11}{4} = 24 \cdot \frac{4}{11} = \frac{96}{11} = 8 \frac{8}{11}\).

Unit 7 Lesson 1
(From Unit 4, Lesson 13.)
Lesson 2: Points on the Number Line

Goals

- Comprehend that two numbers are called “opposites” when they are the same distance from zero, but on different sides of the number line.
- Interpret a point on the number line that represents a positive or negative rational number.
- Plot a point on a number line to represent a positive or negative rational number.

Learning Targets

- I can determine or approximate the value of any point on a number line.
- I can represent negative numbers on a number line.
- I understand what it means for numbers to be opposites.

Lesson Narrative

In this second lesson on signed numbers, students learn about opposites. First they revisit the context of temperature, represented on a vertical number line, extending previous work with interpreting equally spaced divisions to the negative part of the number line. The purpose of this activity is to reestablish the interpretation of distance on the number line in the context of negative numbers. They then create folded number lines to reason about opposites, which are numbers that are on opposite sides of 0 but the same distance from zero. Students will have more practice placing rational numbers of all kinds on the number line in future lessons. In this lesson, it is more important to focus on the concept of opposites than plotting different kinds of rational numbers.

Alignments

Building On
- 3.NF.A: Develop understanding of fractions as numbers.
- 4.NF.C: Understand decimal notation for fractions, and compare decimal fractions.

Addressing
- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.C.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.
- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
Building Towards

- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Rulers marked with centimeters

Tracing paper

Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

Required Preparation

Each student needs access to a ruler marked with centimeters and at least 1 sheet of tracing paper. If the tracing paper is less than 20 cm wide, then students will need to construct their number lines in the “Folded Number Lines” activity to go from -7 to 7, or otherwise construct their number line on the diagonal of the tracing paper.

Student Learning Goals

Let's plot positive and negative numbers on the number line.

2.1 A Point on the Number Line

Warm Up: 5 minutes
The purpose of this activity is to prime students for locating negative fractions on a number line. Students discern the value of a number by analyzing its position relative to landmarks on the number line. In this case, students estimate that the point is halfway between 2 and 3 and use their understanding about fractions and decimals to identify numbers equal or close to 2.5. In later activities, students do the same process when describing negative rational numbers, except with those numbers increasing in magnitude going from right to left.

Notice students who argue that 2.49 is correct or incorrect.

Building On

- 3.NF.A
- 4.NF.C

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time and then 2 minutes for partner discussion.
**Student Task Statement**

Which of the following numbers could be \( B \)?

\[
\begin{array}{cccc}
2.5 & \frac{2}{5} & \frac{5}{2} & \frac{25}{10} & 2.49
\end{array}
\]

**Student Response**

All but \( \frac{2}{5} \) could be \( B \). The 2.5 could be \( B \) because the point looks to be halfway between 2 and 3. But 2.49 could also be \( B \) because it is hard to tell by looking whether the point is exactly halfway between 2 and 3 or only close to it. The point could also be \( \frac{5}{2} \), since it is equivalent to 2.5.

**Activity Synthesis**

The goal of this discussion is for students to understand that they can use landmarks on the number line (in this case, 2 and 3) and their knowledge of fractions to identify equivalent expressions of a number on the number line. Ask students:

- “Were there any responses you could tell right away were not correct? How?” (Sample response: \( \frac{2}{5} \) is less than 1, but \( B \) is between 2 and 3.)

- “Were there any responses you had to think harder about? How did you decide those ones?” (Sample response: \( \frac{25}{10} \) seemed too large at first because the numbers are bigger, but after thinking, I saw it is equivalent to \( \frac{5}{2} \), which I already knew to be correct.)

If time allows, select students to share their thinking about whether 2.49 could represent \( B \).

**2.2 What’s the Temperature?**

**10 minutes**

The purpose of this task is to use the previously introduced context of temperature to build understanding of the negative side of the number line, both by reading values and assigning values to equally spaced divisions. Non-integer negative numbers are also used. Students reason abstractly and quantitatively as they interpret positive and negative numbers in context (MP2).

Notice the arguments students make to decide whether Elena or Jada are correct in question 2. Some students may defend Elena because they see the liquid is above -2 and conclude that the temperature is -2.5 degrees. Other students will defend Jada by noting the temperature is halfway between -1 and -2 degrees, concluding that it must be -1.5 degrees.

**Addressing**

- 6.NS.C.6

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**Unit 7 Lesson 2**
**Instructional Routines**

- MLR8: Discussion Supports

**Launch**

Allow students 5–6 minutes quiet work time followed by whole-class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Begin the activity with concrete or familiar contexts. Revisit a display that represents temperature on a number line from the previous lesson. *Supports accessibility for: Conceptual processing; Memory*

**Anticipated Misconceptions**

Some students may have difficulty identifying the non-integer temperatures on the thermometers. This difficulty arises when students are unable to identify the scale on a number line. This may be significantly more challenging on the negative side of the number line as students are accustomed to the numbers increasing in magnitude on the positive side as you go up. This issue is addressed in task item number 2. It may be helpful to draw attention to the tick mark between 1 and 2 and its label. This previews the idea of opposites addressed in the next activity.

**Student Task Statement**

1. Here are five thermometers. The first four thermometers show temperatures in Celsius. Write the temperatures in the blanks.
The last thermometer is missing some numbers. Write them in the boxes.

2. Elena says that the thermometer shown here reads -2.5°C because the line of the liquid is above -2°C. Jada says that it is -1.5°C. Do you agree with either one of them? Explain your reasoning.

3. One morning, the temperature in Phoenix, Arizona, was 8°C and the temperature in Portland, Maine, was 12°C cooler. What was the temperature in Portland?
Student Response

1. a. 1°C.
   b. -2°C.
   c. 3.5°C.
   d. -0.5°C.
   e. Missing numbers from low to high: -15, -10, 0, 5, 15, 20.

2. Jada is correct. Sample response: The temperature should read -1.5°C because the line of the liquid is 1.5 units below zero on the thermometer. Just as 1.5°C is 1.5 units above zero on the thermometer.

3. The temperature in Portland is -4°C.

Activity Synthesis

The purpose of the discussion is to use temperature to explore the concept of negative numbers and introduce the vocabulary of rational numbers. Select students to share their reasoning as to whether they agreed with Jada or Elena in question 2. If not mentioned by students, connect this question to the warm-up by pointing out that the temperature is halfway between -1 and -2 on the number line, and so it must be -1.5 degrees.

Tell students that rational numbers are like fractions except they can also be negative. So rational numbers are all fractions and their opposites. The term “RATIOnal number” comes from the fact that ratios and fractions are closely related ideas. Display some examples of rational numbers like 4, -3.8, -4/3, and 1/2 for all to see. Ask students whether they agree 4 and 3.8 are fractions. Tell them these might not look like fractions, but they actually are fractions because they can be written as 16/4 and 38/10. All rational numbers can be plotted as points on the number line and can be positive, zero, or negative just like temperature.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support all students to participate in the end of class discussion, provide a sentence frame such as “I agree with _______ because I notice _______” as students construct an argument to support Elena or Jada’s reasoning. Have students share their response with a partner before a whole-class discussion. Encourage students to explain how they are reading the thermometer.

Design Principle(s): Optimize output (for explanation)

2.3 Folded Number Lines

20 minutes
The purpose of this task is both to build an understanding of the symmetry across zero on the number line and to start introducing the notion that we can compare the distance from zero, or the absolute value, of numbers (MP7). Though this activity does not explicitly introduce the vocabulary of absolute value, it seeds the idea that a positive and a negative number can each have the same absolute value.

This is also the first time students work with negative numbers on a horizontal number line. If students have difficulty, remind them of the previous activities where they worked on a vertical number line. It might be helpful to have a vertical number line to display in order to compare and connect.

**Addressing**
- 6.NS.C.6.a

**Building Towards**
- 6.NS.C.6.c

**Instructional Routines**
- MLR1: Stronger and Clearer Each Time

**Launch**
Provide access to tracing paper and rulers marked by centimeters. If the tracing paper is less than 20 cm wide, instruct students to make their number lines from -7 to 7 instead of -10 to 10 or instruct them to make their number line on the diagonal of the tracing paper. Allow 10 minutes for students to construct their folded number line and answer question 2. Check student work on question 2 and allow 5 more minutes for all to complete question 3, followed by whole-class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension. Activate or supply background knowledge.* Some students may benefit from access to partially completed or blank number lines. Consider preparing blank number lines with just the tick marks to get students started.

*Supports accessibility for: Visual-spatial processing; Fine-motor skills*

**Anticipated Misconceptions**
Some students may have difficulty creating a number line. This includes creating equal interval tick marks. Also, students may label the space between the tick marks rather than the tick marks. Have students compare their number line to a peer's or the previous activities in which a number line was used.

**Student Task Statement**
Your teacher will give you a sheet of tracing paper on which to draw a number line.
1. Follow the steps to make your own number line.
   ○ Use a straightedge or a ruler to draw a horizontal line. Mark the middle point of the line and label it 0.
   ○ To the right of 0, draw tick marks that are 1 centimeter apart. Label the tick marks 1, 2, 3... 10. This represents the positive side of your number line.
   ○ Fold your paper so that a vertical crease goes through 0 and the two sides of the number line match up perfectly.
   ○ Use the fold to help you trace the tick marks that you already drew onto the opposite side of the number line. Unfold and label the tick marks -1, -2, -3... -10. This represents the negative side of your number line.

2. Use your number line to answer these questions:
   a. Which number is the same distance away from zero as is the number 4?
   b. Which number is the same distance away from zero as is the number -7?
   c. Two numbers that are the same distance from zero on the number line are called **opposites**. Find another pair of opposites on the number line.
   d. Determine how far away the number 5 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number 5.
   e. Determine how far away the number -2 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number -2.

Pause here so your teacher can review your work.

3. Here is a number line with some points labeled with letters. Determine the location of points P, X, and Y.

If you get stuck, trace the number line and points onto a sheet of tracing paper, fold it so that a vertical crease goes through 0, and use the folded number line to help you find the unknown values.

**Student Response**

1. No answers required.
2. a. -4
3. Point \( P \) is located at \(-2\); \( X \) is at \(-4.5\); and \( Y \) is at \(-7.5\).

**Are You Ready for More?**

At noon, the temperatures in Portland, Maine, and Phoenix, Arizona, had opposite values. The temperature in Portland was 18°C lower than in Phoenix. What was the temperature in each city? Explain your reasoning.

**Student Response**

The temperature in Portland was \(-9°C\). The temperature in Phoenix was \(9°C\).

**Activity Synthesis**

With the new language of opposites, return to the definition of a rational number. We can now think of a rational number as a fraction or the opposite of a fraction. So 6, -6, \( \frac{2}{7} \), \(-\frac{2}{7}\), 5.8, and -5.8 are all examples of rational numbers. In future grades, students will encounter numbers that are not rational.

The main goal of the discussion is to check that students understand what it means for numbers to be opposites, and to take that a step further in thinking about opposites of opposites. During discussion, it may be useful to provide these sentence frames:

- “The opposite of \(_\) is \(_\).”
- “The opposite of the opposite of \(_\) is \(_\).”

Ask students to identify and name a point on their folded number line and find the opposite of that number. Challenge students to find fractions like \( \frac{5}{2} \) and their opposites on the number line. Then ask them to find the opposite of the opposite. Do this for positive and negative numbers, including numbers written as fractions and decimals. Connect those sentence frames to equations. For example, the opposite of -4 is 4, so \( -(\ldots) \ldots \) = 4. Point out that the opposite of the opposite of a number is always the number itself. We can write, for example, \( -(\ldots) \ldots = \ldots \) to express that the opposite of the opposite of \( \frac{2}{3} \) is itself and verify this fact using the number line.
Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. To help students bridge the everyday meaning versus mathematical meaning of the word “opposite,” ask students to write a response to the question “How do you know if two numbers are opposites?” Use successive pair shares to give students a structured opportunity to revise and refine their response. Provide prompts for feedback that will help students strengthen their ideas and clarify their language (e.g. “Can you explain how . . .,” “You should expand on . . .”).

Design Principle(s): Support sense-making; Optimize output (for explanation)

Lesson Synthesis

To help students solidify plotting rational numbers in the correct order on both sides of zero and that opposites are the same distance from zero, have them use their folded number lines or draw a new number line to plot some or all of the numbers below:

- Two opposite numbers are 4 units away from each other. What are the numbers? (-2 and 2)
- Two opposite numbers are 7 units away from each other. What are the numbers? (-3.5 and 3.5)
- Two opposite numbers are 10.8 units away from each other. What are the numbers? (-5.4 and 5.4)
- Think about two numbers that are opposites and 106 units away from each other. Describe what they would look like on a large number line. What are the numbers? (Since they are opposites, they are the same distance from 0. So each would be half of 106 units away from 0, which makes the numbers -53 and 53.)

Students should have each successive positive number to the right of the one before it, while the negative numbers move to the left of the one before.

Tell students that they have spent most of their mathematical careers studying positive numbers called fractions. Now that we can find their opposites, we are studying rational numbers, which are fractions and their opposites. The “ratio” in “rational number” comes from the fact that ratios and fractions are closely related.

2.4 Positive, Negative, and Opposite

Cool Down: 5 minutes

For upcoming work in this unit, students must be able to correctly place positive and negative rational numbers on a number line and compare positive and negative rational numbers. If any students do poorly on this cool-down, they will have plenty of practice with placing positive and negative numbers on a number line in the next several lessons, but they may need more support in doing so.
Addressing

- 6.NS.C.6.a
- 6.NS.C.6.c

**Student Task Statement**

1. Put these numbers in order, from least to greatest. If you get stuck, consider using the number line.

3.5 -1 4.8 -1.5 -0.5 -4.2 0.5 -2.1 -3.5

2. Write two numbers that are opposites and each more than 6 units away from 0.

**Student Response**

1. -4.2, -3.5, -2.1, -1.5, -1, -0.5, 0.5, 3.5, 4.8

2. Answers vary. Sample response: -6.5 and 6.5.

**Student Lesson Summary**

Here is a number line labeled with positive and negative numbers. The number 4 is positive, so its location is 4 units to the right of 0 on the number line. The number -1.1 is negative, so its location is 1.1 units to the left of 0 on the number line.

We say that the *opposite* of 8.3 is -8.3, and that the *opposite* of \(-\frac{3}{2}\) is \(\frac{3}{2}\). Any pair of numbers that are equally far from 0 are called *opposites*.

Points \(A\) and \(B\) are opposites because they are both 2.5 units away from 0, even though \(A\) is to the left of 0 and \(B\) is to the right of 0.

A positive number has a negative number for its opposite. A negative number has a positive number for its opposite. The opposite of 0 is itself.
You have worked with positive numbers for many years. All of the positive numbers you have seen—whole and non-whole numbers—can be thought of as fractions and can be located on a the number line.

To locate a non-whole number on a number line, we can divide the distance between two whole numbers into fractional parts and then count the number of parts. For example, 2.7 can be written as $2\frac{7}{10}$. The segment between 2 and 3 can be partitioned into 10 equal parts or 10 tenths. From 2, we can count 7 of the tenths to locate 2.7 on the number line.

All of the fractions and their opposites are what we call rational numbers. For example, 4, -1.1, 8.3, -8.3, $-\frac{3}{2}$, and $\frac{3}{2}$ are all rational numbers.

Glossary

- opposite
- rational number
Lesson 2 Practice Problems

Problem 1

Statement

For each number, name its opposite.

\begin{align*}
  \text{a.} & \ -5 & \text{a.} & \ 0.875 \\
  \text{b.} & \ 28 & \text{b.} & \ 0 \\
  \text{c.} & \ -10.4 & \text{c.} & \ -8,003 \\
\end{align*}

Solution

\begin{align*}
  \text{a.} & \ 5 \\
  \text{b.} & \ -28 \\
  \text{c.} & \ 10.4 \\
  \text{d.} & \ -0.875 \\
  \text{e.} & \ 0 \\
  \text{f.} & \ 8,003 \\
\end{align*}

Problem 2

Statement

Plot the numbers -1.5, \( \frac{3}{2} \), \( -\frac{3}{2} \), and \( \frac{4}{3} \) on the number line. Label each point with its numeric value.

Solution

Problem 3

Statement

Plot these points on a number line.

Unit 7 Lesson 2
Problem 4
Statement

a. Represent each of these temperatures in degrees Fahrenheit with a positive or negative number.

- 5 degrees above zero
- 3 degrees below zero
- 6 degrees above zero
- $2\frac{3}{4}$ degrees below zero

b. Order the temperatures above from the coldest to the warmest.

Solution

a. 5, -3, 6, $-2\frac{3}{4}$

b. -3, $-2\frac{3}{4}$, 5, 6

(From Unit 7, Lesson 1.)

Problem 5
Statement

Solve each equation.

a. $8x = \frac{2}{3}$

b. $1\frac{1}{2} = 2x$

c. $5x = \frac{2}{7}$
Problem 6

Solution

a. \( x = \frac{2}{24} \) (or equivalent)

b. \( x = \frac{3}{4} \) (or equivalent)

c. \( x = \frac{2}{35} \) (or equivalent)

d. \( x = 20 \)

e. \( x = \frac{3}{10} \) (or equivalent)

(From Unit 6, Lesson 5.)

Problem 7

Statement

There are 15.24 centimeters in 6 inches.

a. How many centimeters are in 1 foot?

b. How many centimeters are in 1 yard?
Solution

a. 30.48 centimeters

b. 91.44 centimeters

(From Unit 3, Lesson 4.)
Lesson 3: Comparing Positive and Negative Numbers

Goals

- Compare rational numbers in the context of temperature or elevation, and express the comparisons (in writing) using the symbols > and <.
- Comprehend the word “sign” (in spoken language) to refer to whether a number is positive or negative.
- Critique (orally and in writing) statements comparing rational numbers, including claims about relative position and claims about distance from zero.

Learning Targets

- I can explain how to use the positions of numbers on a number line to compare them.
- I can explain what a rational number is.
- I can use inequalities to compare positive and negative numbers.

Lesson Narrative

Returning to the temperature context, students compare rational numbers representing temperatures and learn to write inequality statements that include negative numbers. Students then consider rational numbers in all forms (fractions, decimals) and learn to compare them by plotting on a number line and considering their relative positions. Students abstract from “hotter” and “colder” to “greater” and “less,” so if a number $a$ is to the right of a number $b$, we can write the inequality statements $a > b$ and $b < a$. Students also find that the greatest number is not always the one farthest from zero, which was the case before students encountered negative numbers. For example, -100 is much farther away from zero than $-\frac{1}{100}$, but since $-\frac{1}{100}$ is to the right of -100, it is larger and we can write $-\frac{1}{100} > -100$. Students are briefly introduced to the word sign (i.e., algebraic sign) since it is often used to talk about whether numbers are positive or negative. Students use the structure of the number line to reason about relationships between numbers (MP7).

Alignments

**Building On**

- 4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- 5.NBT.A.3.b: Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Addressing

- 6.NS.C.7.a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a number line oriented from left to right.

- 6.NS.C.7.b: Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.

Building Towards

- 6.NS.C.7.a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a number line oriented from left to right.

- 6.NS.C.7.d: Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than $-30$ dollars represents a debt greater than $30$ dollars.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Which One Doesn’t Belong?

Student Learning Goals

Let’s compare numbers on the number line.

3.1 Which One Doesn’t Belong: Inequalities

Warm Up: 5 minutes

This warm-up prompts students to compare four expressions that will prime them for writing inequality statements involving signed numbers in later activities. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the inequalities in comparison to one another. To allow all students to access the activity, each inequality has one obvious reason it does not belong. During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the unit.

At this time, students might explain the direction of the inequality symbol in terms of the size of the numbers. This explanation is acceptable for students to give during this warm-up, but the next activity introduces a more correct concept of ordering that includes negative numbers.

Building On

- 4.NBT.A.2
- 5.NBT.A.3.b
**Building Towards**
- 6.NS.C.7.a

**Instructional Routines**
- Which One Doesn’t Belong?

**Launch**
Arrange students in groups of 2–4. Display the questions for all to see. Ask students to indicate when they have noticed one question that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. In their groups, tell each student to share their reasoning why a particular question does not belong and together find at least one reason each question doesn’t belong.

**Student Task Statement**
Which inequality doesn’t belong?
- $\frac{5}{4} < 2$
- $8.5 > 0.95$
- $8.5 < 7$
- $10.00 < 100$

**Student Response**
Answers vary. Sample responses:
- $\frac{5}{4} < 2$ doesn't belong because it is the only one that has a fraction.
- $8.5 > 0.95$ is the only one with a greater than symbol or doesn't belong because it is the only one with two decimals.
- $8.5 < 7$ doesn't belong because it is the only one that is false.
- $10.00 < 100$ doesn't belong because it is the only one comparing two whole numbers.

**Activity Synthesis**
Ask each group to share one reason why a particular image does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students’ explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as “greater than” or “less than.” Also, press students on unsubstantiated claims.

Unit 7 Lesson 3
3.2 Comparing Temperatures

10 minutes
The purpose of the task is for students to compare signed numbers in a real-world context and then use inequality signs accurately with negative numbers (MP2). The context should help students understand “less than” or “greater than” language. Students evaluate and critique another’s reasoning (MP3).

Addressing
- 6.NS.C.7.a
- 6.NS.C.7.b

Instructional Routines
- MLR7: Compare and Connect

Launch
Allow students 5–6 minutes quiet work time followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Some students may benefit from access to partially completed or blank number lines. Consider preparing blank number lines with just the tick marks to get students started.

Supports accessibility for: Visual-spatial processing; Organization

Anticipated Misconceptions
Some students may have difficulty comparing numbers on the negative side of the number line. Have students plot the numbers on a number line in order to sequence them from least to greatest. Make it evident that temperatures become warmer (i.e., greater) as they move from left to right on the number line.

Student Task Statement
Here are the low temperatures, in degrees Celsius, for a week in Anchorage, Alaska.

<table>
<thead>
<tr>
<th>day</th>
<th>Mon</th>
<th>Tues</th>
<th>Weds</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature</td>
<td>5</td>
<td>-1</td>
<td>-5.5</td>
<td>-2</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

a. Plot the temperatures on a number line. Which day of the week had the lowest low temperature?
b. The lowest temperature ever recorded in the United States was -62 degrees Celsius, in Prospect Creek Camp, Alaska. The average temperature on Mars is about -55 degrees Celsius.

i. Which is warmer, the coldest temperature recorded in the USA, or the average temperature on Mars? Explain how you know.

ii. Write an inequality to show your answer.

c. On a winter day the low temperature in Anchorage, Alaska, was -21 degrees Celsius and the low temperature in Minneapolis, Minnesota, was -14 degrees Celsius.

Jada said, “I know that 14 is less than 21, so -14 is also less than -21. This means that it was colder in Minneapolis than in Anchorage.”

Do you agree? Explain your reasoning.

**Student Response**

a. Wednesday

b.  
   i. The average temperature on Mars is warmer. On the number line, -55 is farther to the right and closer to positive numbers than -62. This means that -55 degrees Celsius is warmer than -62 degrees Celsius.

   ii. $-55 > -62$

c. Disagree. Explanations vary. Sample response: On the number line, -14 is closer to the positive numbers than -21. Positive temperatures are warmer than negative temperatures. This means that -14 degrees Celsius is warmer than -21 degrees Celsius.

**Are You Ready for More?**

Another temperature scale frequently used in science is the Kelvin scale. In this scale, 0 is the lowest possible temperature of anything in the universe, and it is -273.15 degrees in the Celsius scale. Each 1 K is the same as 1°C, so 10 K is the same as 263.15°C.

a. Water boils at 100°C. What is this temperature in K?

b. Ammonia boils at -35.5°C. What is the boiling point of ammonia in K?

c. Explain why only positive numbers (and 0) are needed to record temperature in K.

**Student Response**

a. 373.15. Water boils at 100 degrees Celsius, and 0 Kelvin is the same as 273.15 degrees below zero Celsius, so these two numbers are added together to get the temperature in Kelvin.
b. 237.65. Since 0 Kelvin is 273.15 degrees Celsius below zero and the boiling point of ammonia is 35.5 degrees Celsius below zero, in Kelvin this will be 273.15 − 35.5.

c. The temperature cannot go below absolute zero so, in Kelvin, all temperatures will be 0 or positive.

Activity Synthesis

Ask students to explain how they could tell which of the two negative numbers was greater in problem 2. After gathering 1 or 2 responses, display the following image for all to see:

Explain to students that as we go to the right on the number line, we can think of the temperature as getting hotter, and as we go to the left, we can think of the temperature as getting colder. Numbers don’t always describe temperature, though. So we use the word “greater” to describe a number that is farther to the right, and “less” to describe numbers that are farther to the left. For example, we would write 6 > -50 and say “6 is greater than -50 because it is farther to the right on the number line.” Equivalently, we could write -50 < 6 and say “-50 is less than 6 because -50 is farther to the left on the number line.”

Ask students to make up their own list of negative numbers and plot on a number line. Then ask them to write several inequality statements with their numbers. As time allows, have students share one inequality statement with the class to see if all agree that it is true. If time allows, have students swap their list with a partner to plot and check their partner’s number lines and inequality statements.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine to support students’ understanding of both the language and the symbolic notation of inequalities. Ask students to consider what is the same and what is different between the following sentences: “6 degrees is warmer than -50 degrees,” “6 is greater than -50,” and “6 > -50.” This helps students make the connection between the language of the context, the written inequality statement and the symbolic inequality statement.

Design Principle(s): Optimize output (for comparison); Maximize meta-awareness
3.3 Rational Numbers on a Number Line

15 minutes (there is a digital version of this activity)
The purpose of this task is for students to understand that for a given number, numbers to the left are always less than the number, and numbers to the right are always greater than the number. The precise use of the term “absolute value” is not expected at this time.

Addressing
- 6.NS.C.7.a

Building Towards
- 6.NS.C.7.d

Instructional Routines
- MLR2: Collect and Display

Launch
Allow 10 minutes quiet work time followed by whole-class discussion.

Students using the digital materials can graph the points and check them with the applet. Marks at each half, quarter, and eighth of a unit can be shown to help plot the points or to self-check for accuracy.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about equivalent fractions. Some students may benefit from access to pre-prepared number lines to compare fractions for the final two problems.

Supports accessibility for: Conceptual processing; Organization

Anticipated Misconceptions
Some students may have difficulty comparing numbers on the negative side of the number line. Have students plot the numbers on a number line from least to greatest. It may be helpful to provide an example that students can use as a visual aid while they are working independently.

Some students may have difficulty comparing fractions in question 4. Remind them that comparing fractions is easier using a common denominator. Suggest that they subdivide the interval from 0 to 1 into fourths or eighths.

Student Task Statement
a. Plot the numbers -2, 4, -7, and 10 on the number line. Label each point with its numeric value.
b. Decide whether each inequality statement is true or false. Be prepared to explain your reasoning.
   
   i. -2 < 4
   ii. -2 < -7
   iii. 4 > -7
   iv. -7 > 10

c. Andre says that $\frac{1}{4}$ is less than $-\frac{3}{4}$ because, of the two numbers, $\frac{1}{4}$ is closer to 0. Do you agree? Explain your reasoning.

d. Answer each question. Be prepared to explain how you know.
   
   i. Which number is greater: $\frac{1}{4}$ or $\frac{5}{4}$? 
   ii. Which is farther from 0: $\frac{1}{4}$ or $\frac{5}{4}$? 
   iii. Which number is greater: $-\frac{3}{4}$ or $\frac{5}{8}$? 
   iv. Which is farther from 0: $-\frac{3}{4}$ or $\frac{5}{8}$? 
   v. Is the number that is farther from 0 always the greater number? Explain your reasoning.

**Student Response**

b. i. -2 < 4 true 
ii. -2 < -7 false 
iii. 4 > -7 true 
iv. -7 > 10 false 

c. Andre is incorrect. Explanations vary. Sample response: $\frac{1}{4}$ is to the right of $-\frac{3}{4}$ so it is greater.

d. i. $\frac{5}{4}$
   
   ii. $\frac{5}{4}$
   
   iii. $\frac{5}{8}$
iv. \(-\frac{3}{4}\)

v. No. Explanations vary. Sample response: It depends on their positions relative to each other. Numbers to the right are greater than numbers to the left.

**Activity Synthesis**

The key takeaway from the discussion is that now that we have numbers on both sides of 0, distance from 0 isn't enough to compare two numbers. Instead, we call numbers farther to the left on the number line “less” and numbers farther to the right “greater.” Display the following number line for all to see:

One inequality at a time, ask students to indicate whether they think each of the following is true or false:

- \(\frac{5}{4} > -\frac{3}{2}\) (true)
- \(\frac{5}{4}\) is farther from 0 than \(-\frac{3}{2}\) (false)
- \(-\frac{3}{2} < -\frac{3}{4}\) (true)
- \(-\frac{3}{2}\) is farther from 0 than \(-\frac{3}{4}\) (true)

Invite students to share their reasoning. Here are some sentence frames that might be helpful:

- “\(\_\) is greater (less) than \(\_\) because \(\_\).”
- “\(\_\) is farther from 0 than \(\_\) because \(\_\).”

**Access for English Language Learners**

*Speaking: MLR2 Collect and Display.* During the class discussion, record and display words and phrases that students use to explain why they decided certain inequality statements are true or false. Highlight phrases that include a reference to “to the right of,” “to the left of,” and a distance from zero. If students use gestures to support their reasoning, do your best to connect words to the gestures.

*Design Principle(s): Optimize output (for explanation); Support sense-making*
Lesson Synthesis

Introduce the word sign to mean whether a number is positive or negative and give a few examples like “The sign of -3 is negative. The sign of 5 is positive.” Explain that 0 has no sign because it is neither positive nor negative. Then display the number line for all to see.

- What is the sign of A? B? C? Which number is closest to 0? (negative; negative; positive; B and C both look equally close but it is hard to be sure)
- Is A greater than B? How can we write an inequality statement comparing A and B? (no; \( A < B \))
- Is A less than C? How can we write an inequality statement comparing A and C? (yes; \( A < C \))
- Is B equal to C? Write a statement that correctly compares B and C. (no, \( B < C \) or \( C > B \))
- If we plot two numbers on the number line, how can we tell which one is greater? (We call the one to the right “greater”)

3.4 Making More Comparisons

Cool Down: 10 minutes

Addressing

- 6.NS.C.7.a
- 6.NS.C.7.b

Student Task Statement

a. The elevation of Death Valley, California, is -282 feet. The elevation of Tallahassee, Florida, is 203 feet. The elevation of Westmorland, California, is -157 feet.
   i. Compare the elevations of Death Valley and Tallahassee using < or >.
   ii. Compare the elevations of Death Valley and Westmorland.

b. Here are the points \( A, B, C, \) and 0 plotted on a number line.

The points \( B \) and \( C \) are opposites. Decide whether each of the following statements is true.

i. \( A \) is greater than \( B \).

ii. \( A \) is farther from 0 than \( C \).

iii. \( A \) is less than \( C \).
iv. \( B \) and \( C \) are equally far away from 0.

v. \( B \) and \( C \) are equal.

**Student Response**

a.  
i. \(-282 < 203 \) or \( 203 > -282 \).

ii. \(-157 > -282 \) or \(-282 < -157 \).

b.  
i. False. \( B \) is greater than \( A \).

ii. True

iii. True

iv. True

v. False. One is positive and one is negative.

**Student Lesson Summary**

We use the words *greater than* and *less than* to compare numbers on the number line. For example, the numbers \(-2.7, 0.8, \) and \(-1.3, \) are shown on the number line.

Because \(-2.7 \) is to the left of \(-1.3, \) we say that \(-2.7 \) is less than \(-1.3. \) We write:

\[-2.7 < -1.3\]

In general, any number that is to the left of a number \( n \) is less than \( n. \)

We can see that \(-1.3 \) is greater than \(-2.7 \) because \(-1.3 \) is to the right of \(-2.7. \) We write:

\[-1.3 > -2.7\]

In general, any number that is to the right of a number \( n \) is greater than \( n. \)

We can also see that \(0.8 > -1.3 \) and \(0.8 > -2.7. \) In general, any positive number is greater than any negative number.

**Glossary**

○ *sign*
Lesson 3 Practice Problems

Problem 1

Statement
Decide whether each inequality statement is true or false. Explain your reasoning.

i. -5 > 2
ii. 3 > -8
iii. -12 > -15
iv. -12.5 > -12

Solution
i. False, -5 is to the left of 2.
ii. True, 3 is to the right of -8.
iii. True, -12 is to the right of -15.
iv. False, -12.5 is to the left of -12.

Problem 2

Statement
Here is a true statement: -8.7 < -8.4. Select all of the statements that are equivalent to -8.7 < -8.4.

A. -8.7 is further to the right on the number line than -8.4.
B. -8.7 is further to the left on the number line than -8.4.
C. -8.7 is less than -8.4.
D. -8.7 is greater than -8.4.
E. -8.4 is less than -8.7.
F. -8.4 is greater than -8.7.

Solution
["B", "C", "F"]
**Problem 3**

**Statement**
Plot each of the following numbers on the number line. Label each point with its numeric value: 0.4, -1.5, \(-1\frac{7}{10}\), \(-\frac{11}{10}\).

![Number line with points labeled](number_line.png)

**Solution**
A correct solution has four points plotted in the following order from left to right: \(-1\frac{7}{10}\), -1.5, \(-\frac{11}{10}\), 0.4 (between 0 and 1).

(From Unit 7, Lesson 2.)

**Problem 4**

**Statement**
The table shows five states and the lowest point in each state.

<table>
<thead>
<tr>
<th>state</th>
<th>lowest elevation (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>-282</td>
</tr>
<tr>
<td>Colorado</td>
<td>3350</td>
</tr>
<tr>
<td>Louisiana</td>
<td>-8</td>
</tr>
<tr>
<td>New Mexico</td>
<td>2842</td>
</tr>
<tr>
<td>Wyoming</td>
<td>3099</td>
</tr>
</tbody>
</table>

Put the states in order by their lowest elevation, from least to greatest.

**Solution**
California, Louisiana, New Mexico, Wyoming, Colorado

(From Unit 7, Lesson 4.)

**Problem 5**

**Statement**
Each lap around the track is 400 meters.

i. How many meters does someone run if they run:
2 laps? 5 laps? x laps?

ii. If Noah ran 14 laps, how many meters did he run?

iii. If Noah ran 7,600 meters, how many laps did he run?

**Solution**

i. 800 meters \((400 \cdot 2 = 800)\), 2,000 meters \((400 \cdot 5 = 2,000)\), \(400x\) meters or equivalent

ii. 5,600 \((400 \cdot 14 = 5,600)\)

iii. 19 \((7600 \div 400 = 19)\)

(From Unit 6, Lesson 6.)

**Problem 6**

**Statement**

A stadium can seat 16,000 people at full capacity.

i. If there are 13,920 people in the stadium, what percentage of the capacity is filled?

   Explain or show your reasoning.

   ii. What percentage of the capacity is not filled?

**Solution**

i. 87% is filled, because \(13,920 \div 16,000 = 0.87\).

ii. 13% remains, because \(100 - 87 = 13\).

(From Unit 3, Lesson 16.)
Lesson 4: Ordering Rational Numbers

Goals

○ Compare rational numbers without a context and express the comparisons using the terms “greater than,” “less than,” and “opposite” (orally and in writing).

○ Comprehend that all negative numbers are less than all positive numbers.

○ Order rational numbers from least to greatest, and explain (orally and through other representations) the reasoning.

Learning Targets

○ I can compare and order rational numbers.

○ I can use phrases like “greater than,” “less than,” and “opposite” to compare rational numbers.

Lesson Narrative

This lesson solidifies what students have learned in the past several lessons about the ordering of rational numbers on the number line. Students practice ordering rational numbers and use precise language to describe the relationships between numbers plotted on a number line (MP6). These phrases include “greater than,” “less than,” “negative,” and “opposite.”

Alignments

Building On

○ 4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

○ 5.NBT.A.3.b: Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Addressing

○ 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

○ 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

○ 6.NS.C.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.

○ 6.NS.C.7: Understand ordering and absolute value of rational numbers.
Building Towards

○ 6.NS.C.7: Understand ordering and absolute value of rational numbers.

Instructional Routines

○ Anticipate, Monitor, Select, Sequence, Connect
○ MLR2: Collect and Display
○ MLR8: Discussion Supports
○ Take Turns
○ Think Pair Share

Required Materials

Pre-printed cards, cut from copies of the Instructional master

Required Preparation

Make 1 copy of the “Ordering Rational Number Cards” activity Instructional master for every 2 students, and cut them up ahead of time. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let’s order rational numbers.

4.1 How Do They Compare?

Warm Up: 10 minutes
The purpose of this warm-up is for students to review strategies for comparing whole numbers, decimal numbers, and fractions as well as the use of inequality symbols. The numbers in each pair have been purposefully chosen based on misunderstandings students typically have when comparing. Since there are many pairs of numbers to compare, it may not be possible to share all of the students’ strategies for each pair. Consider sharing only one strategy for each pair if all of the students agree and more than one if there is a disagreement among the students.

Building On

○ 4.NBT.A.2
○ 5.NBT.A.3.b

Building Towards

○ 6.NS.C.7

Launch

Give students 3 minutes of quiet work time followed by a whole-class discussion.
Anticipated Misconceptions
Some students may not remember the inequality symbols that represent the phrases: greater than, less than, and equal to. Show these students each of the inequality symbols in an example that they can refer back to as they work.

Student Task Statement
Use the symbols >, <, or = to compare each pair of numbers. Be prepared to explain your reasoning.

- 12 _____ 19
- 15 _____ 1.5
- 6.050 _____ 6.05
- \( \frac{19}{24} \) _____ \( \frac{19}{21} \)
- 212 _____ 190
- 9.02 _____ 9.2
- 0.4 _____ \( \frac{9}{40} \)
- \( \frac{16}{17} \) _____ \( \frac{11}{12} \)

Student Response
- 12 < 19 because 12 is farther left on the number line than 19.
- 212 > 190 because 212 is farther right on the number line than 190.
- 15 > 1.5 because 15 is 10 times farther to the right than 1.5 on the number line.
- 9.02 < 9.2 because both numbers have 9 wholes and 9.2 has 2 tenths and 9.02 doesn’t have any.
- 6.050 = 6.05 because both numbers have the same number of ones, tenths, hundredths, and thousandths.
- 0.4 > \( \frac{9}{40} \) because 0.4 is greater than \( \frac{1}{4} \) and \( \frac{9}{40} \) is less than \( \frac{1}{4} \).
- \( \frac{19}{24} \) < \( \frac{19}{21} \) because both fractions are the same number of pieces and \( \frac{1}{21} \) is greater than \( \frac{1}{24} \).
- \( \frac{16}{17} \) > \( \frac{11}{12} \) because both fractions are 1 unit from a whole and \( \frac{1}{17} \) is less than \( \frac{1}{12} \).

Activity Synthesis
For each pair of numbers, ask one or two students to share their reasoning. Record and display their reasoning for all to see. If the whole class agrees, move on to the next question, but if there is a disagreement, ask students to explain their thinking until an agreement is reached. If possible, spend more time on the questions with numbers expressed with decimals and fractions.

4.2 Ordering Rational Number Cards
15 minutes
In this activity, students order rational numbers from least to greatest in 2 steps. They first order positive rational numbers, and then negative ones. The numbers are written as fractions, decimals, and integers. By manipulating physical cards, students get a tangible sense of how rational numbers relate to each other.

Notice conversations students have when deciding how to place fractions and decimals, especially on the negative side of the number line. Pay attention to proper use and understanding of “less” or “greater,” and improper use of “bigger” or “smaller.” One strategy to look for is fitting new numbers between known numbers (e.g., $-\frac{9}{8}$ is between -1 and -2).

**Addressing**
- 6.NS.C
- 6.NS.C.7

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- Take Turns

**Launch**

Arrange students in groups of 2. Distribute the first set of cards to each group. Give students 5 minutes to order the first set of cards, taking turns to place each number. When a group finishes ordering, check their ordering before giving them the second set of cards. To speed up the checking process, consider referring groups finishing the first set to compare their ordering to a group you have already checked. Give students 5 minutes to order the second set of cards followed by whole-class discussion. When collecting the cards, ask groups to separate the negative set of numbers and randomize each set for the next class.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have placed their initial cards in the correct order.

*Supports accessibility for: Conceptual processing; Organization*

**Anticipated Misconceptions**

Some students may place negative numbers in order of increasing absolute value on the left side of 0. Ask these students to draw a number line that goes 5 units right and 5 units left of 0. Point out that negative numbers progress as -1, -2, -3, -4, -5 as they move outward from 0.
Student Task Statement

Your teacher will give you a set of number cards. Order them from least to greatest.

Your teacher will give you a second set of number cards. Add these to the correct places in the ordered set.

Student Response

-23, -22 1/2, -22, -10, -9, -8, -7.5, -7, -5 1/2, -5, -3, -2.5, -2, -9/8, -1, -1/4, 0, 1/4, 1, 9/8, 2, 2.5,
8/3, 3, 4, 5, 5 1/2, 6, 7, 7.5, 8, 9, 10, 11, 14, 15, 16, 17, 22 1/2, 25, 29, 30, 53, 62, 78, 87, 99, 100.

Activity Synthesis

The purpose of the discussion is to solidify students’ understanding of the order of rational numbers. Select previously identified students to share how they decided how to place numbers like -9/8, 9/8, 8/3, and -22 1/2. Here are some questions to consider:

- Which numbers were hardest to place and which were the least difficult?
- How does placing negative numbers compare to placing positive numbers?
- How did you use numbers you had already placed to reason about where to place new numbers?

Access for English Language Learners

Speaking: MLR2 Collect and Display. During the class discussion record words and phrases that students use to explain how to order negative numbers. Highlight phrases that include a reference to “greater than,” “less than,” “to the right of” and “to the left of.” As a class, discuss how to order -23, -22 and one half, and -22. Record student words on a visual display of a number line. This will help students read and use mathematical language during future coursework.

Design Principle(s): Support sense-making

4.3 Comparing Points on A Line

15 minutes

Students practice using relational language “greater than” and “less than” to describe order and position on number line.

Addressing

- 6.NS.C.6
- 6.NS.C.6.a

Unit 7 Lesson 4
Instructional Routines

⊙ MLR8: Discussion Supports
⊙ Think Pair Share

Launch

Arrange students in groups of 2. Give students 7 minutes of quiet work time for both problems before 3–5 minutes for partner discussion, followed by whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Some students may benefit from access to blank or partially completed number lines for the final question.

*Supports accessibility for: Visual-spatial processing; Organization*

Access for English Language Learners

*Speaking, Listening: MLR8 Discussion Supports.* To support students to produce statements with justification during the conversation between partners, provide student sentence frames such as “___ is greater than ___ because ___.”; “___ is the opposite of ___ because ___.”; “I know that ___ is a negative number because ___.”

*Design Principle(s): Optimize output (for justification)*

Student Task Statement

a.

Use each of the following terms at least once to describe or compare the values of points \( M, N, P, R \).

- greater than
- less than
- opposite of (or opposites)
- negative number

b. Tell what the value of each point would be if:

i. \( P \) is \( 2 \frac{1}{2} \)
ii. $N$ is -0.4  
iii. $R$ is 200  
iv. $M$ is -15  

**Student Response**  

a. Responses vary. Sample response: $R$ is greater than $N$. $M$ is less than $P$. $M$ and $P$ are opposites. $N$ is a negative number.

b. i. $M = -2 \frac{1}{2}$, $N = -1$, $P = 2 \frac{1}{2}$, $R = 4$.  
   ii. $M = -1$, $N = -0.4$, $P = 1$, $R = 1.6$.  
   iii. $M = -125$, $N = -50$, $P = 125$, $R = 200$.  

**Are You Ready for More?**  
The list of fractions between 0 and 1 with denominators between 1 and 3 looks like this:  

$$\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}$$

We can put them in order like this:  

$$0 < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{1}{1}$$

Now let’s expand the list to include fractions with denominators of 4. We won't include $\frac{2}{4}$, because $\frac{1}{2}$ is already on the list.

$$0 < \frac{1}{4} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{1}{1}$$

a. Expand the list again to include fractions that have denominators of 5.

b. Expand the list you made to include fractions have have denominators of 6.

c. When you add a new fraction to the list, you put it in between two “neighbors.” Go back and look at your work. Do you see a relationship between a new fraction and its two neighbors?

**Student Response**  
a. $0 < \frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{1}{1}$  
b. $0 < \frac{1}{6} < \frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{5}{6} < \frac{1}{1}$

c. The numerator of a new fraction is always the sum of the two numerators of its neighbors. The denominator of a new fraction is always the sum of the two denominators of its neighbors.
**Activity Synthesis**

The purpose for discussion is to give students the opportunity to use precise language as they compare the relative positions of rational numbers. Give students 3–5 minutes to discuss their responses with a partner before whole-class discussion. Ask students to share their partner’s reasoning, especially if it was different than their own. Here are some questions to consider for whole-class discussion:

- “Did you ever have a different answer than your partner? If so, were you both correct? If not, how did you work to reach agreement?”
- “How did your partner decide the value of each unit on the number line in problem 2? Did you think of it a different way?”
- “How can we tell if two numbers are opposites?” (They are the same distance from 0.)
- “How can we tell if one number is greater or less than another number?” (Numbers toward the right are considered greater, and numbers toward the left are considered less.)

**Lesson Synthesis**

Ask students to summarize the ideas they have developed in the last few lessons about plotting and comparing rational numbers. Here are some questions to consider:

- “What are some situations where negative numbers make sense? What do the words ‘positive,’ ‘negative,’ and 0 mean in those situations?” (Elevation: 0 represents sea level, negative represents below sea level, and positive represents above sea level. Temperature: 0°C represents the standard freezing point of water, positive represents temperatures warmer than freezing, and negative represents temperatures below freezing.)
- “What about on the number line? What do ‘positive’ and ‘negative’ mean on the number line? Is 0 positive or negative?” (Negative numbers are numbers left of 0 on the number line, and positive numbers are to the right of 0. The number 0 is neither positive nor negative.)
- “What are some ideas you have about ‘opposites?’ What is the opposite of 0?” (Opposites are numbers that are the same distance from 0. They come in pairs—one positive, one negative—except for 0, which is its own opposite.)
- “How can you tell if one number is greater than or less than another? How do you write it?” (Given two rational numbers, the number toward the right on the number line is considered “greater,” and the number toward the left is considered “less.” We use the < and > symbols to indicate “less than” and “greater than,” respectively.)

**4.4 Getting Them in Order**

**Cool Down:** 5 minutes

**Addressing**

- 6.NS.C.6
Student Task Statement

a. Place these numbers in order from least to greatest:

\[
\frac{16}{5}, -3, 6, 3.1, -2.5, \frac{1}{4}, -\frac{3}{4}, -\frac{3}{8}
\]

b. Write a sentence to compare the two points shown on the number line.

Student Response

a. -3, -2.5, -\frac{3}{4}, -\frac{3}{8}, \frac{1}{4}, 3.1, \frac{16}{5}, 6

b. -2.7 is less than 4.5, or 4.5 is greater than -2.7.

Student Lesson Summary

To order rational numbers from least to greatest, we list them in the order they appear on the number line from left to right. For example, we can see that the numbers

-2.7, -1.3, 0.8

are listed from least to greatest because of the order they appear on the number line.
Lesson 4 Practice Problems
Problem 1

Statement
Select all of the numbers that are greater than -5.

A. 1.3
B. -6
C. -12
D. \(\frac{1}{7}\)
E. -1
F. -4

Solution
["A", "D", "E", "F"]

Problem 2

Statement
Order these numbers from least to greatest: \(\frac{1}{2}\), 0, 1, \(-1\frac{1}{2}\), \(-\frac{1}{2}\), -1

Solution
\(-1\frac{1}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1\)

Problem 3

Statement
Here are the boiling points of certain elements in degrees Celsius:

- Argon: -185.8
- Chlorine: -34
- Fluorine: -188.1
- Hydrogen: -252.87
- Krypton: -153.2

List the elements from least to greatest boiling points.
Solution
Hydrogen, fluorine, argon, krypton, chlorine

Problem 4

Statement
Explain why zero is considered its own opposite.

Solution
Answer vary. Sample response: Opposites are equally distant from 0. Since 0 is the only number that is 0 units from 0, it has to be its own opposite. \(0 + 0 = 0\).

(From Unit 7, Lesson 2.)

Problem 5

Statement
Explain how to make these calculations mentally.

i. \(99 + 54\)

ii. \(244 - 99\)

iii. \(99 \cdot 6\)

iv. \(99 \cdot 15\)

Solution
Answers vary. Sample responses:

i. 153; this is one less than \(100 + 54 \equiv 154\).

ii. 145; this is one more than \(244 - 100 \equiv 144\).

iii. 594; this is one 6 short of 100 sixes or 600.

iv. 1485; this is one 15 short of 100 fifteens or 1500.

(From Unit 6, Lesson 9.)

Problem 6

Statement
Find the quotients.

i. \(\frac{1}{2} \div 2\)
Problem 7

Statement

Over several months, the weight of a baby measured in pounds doubles. Does its weight measured in kilograms also double? Explain.

Solution

Yes. Explanations vary. Sample explanation: The weight itself doubles, so any measurement of the weight using the same units will also double. We can also see that by saying if the weight is \( x \) pounds, then double that weight would be \( 2x \) pounds. The weight in kilograms will be \( x \div 2.2 \), and the double weight will be \( (2x) \div 2.2 \) or \( 2(x \div 2.2) \), which is also double.
Lesson 5: Using Negative Numbers to Make Sense of Contexts

Goals
- Interpret a table of signed numbers that represent how a quantity changed.
- Recognize that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received and money paid, inventory bought and inventory sold, etc.

Learning Targets
- I can explain and use negative numbers in situations involving money.
- I can interpret and use negative numbers in different contexts.

Lesson Narrative
In this lesson, students are introduced to conventions for using signed numbers to represent money spent and received, as well as inventory gained and lost. While money contexts can be represented without signed numbers, there are many situations that are more efficiently modeled by signed numbers. For example, if a person has $50 in the bank and writes a $20 check, we can represent the balance as $50 - 20. If they had written an $80 check, we can still write the balance as $50 - 80, as long as we have adopted the convention that negative numbers represent what the person owes the bank (and assuming the bank allows overdrafts). Since students do not operate on signed numbers in this grade, this lesson is simply an introduction to the convention of using signed numbers to represent a change in money or a change in inventory, an important convention in modeling financial situations with mathematics (MP4). In a later lesson, students will be introduced to the idea of an account balance. In grade 7, students will study addition and subtraction of signed numbers and apply those concepts in accounting situations.

Alignments
Addressing
- 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Instructional Routines
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share
**Student Learning Goals**
Let's make sense of negative amounts of money.

### 5.1 Notice and Wonder: It Comes and Goes

**Warm Up: 5 minutes**
The purpose of this warm-up is to elicit the idea that we can represent money we get with positive numbers and money we spend with negative numbers, which will be useful when students make sense of data about money and inventory in later activities. While students may notice and wonder many things about this table, interpreting the meaning of positive and negative numbers in this context is most important.

**Addressing**
- 6.NS.C.5

**Instructional Routines**
- Notice and Wonder

**Launch**
Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

**Student Task Statement**

<table>
<thead>
<tr>
<th>activity</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>do my chores</td>
<td>30.00</td>
</tr>
<tr>
<td>babysit cousin</td>
<td>45.00</td>
</tr>
<tr>
<td>buy my lunch</td>
<td>-10.80</td>
</tr>
<tr>
<td>get my allowance</td>
<td>15.00</td>
</tr>
<tr>
<td>buy a shirt</td>
<td>-18.69</td>
</tr>
<tr>
<td>pet my dog</td>
<td>0.00</td>
</tr>
</tbody>
</table>

What do you notice? What do you wonder?

**Student Response**
Things students may notice:
- When I buy things, the numbers are negative numbers.
- When I do things to get money, the numbers are positive.
- When no money is involved, the value is zero.

Things students may wonder:

- How much I have left.
- Why there are negative numbers.
- Why there is a 0.

**Activity Synthesis**

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the table. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If representing money that we receive with positive numbers and money that we spend with negative numbers does not come up during the conversation, ask students to discuss this idea.

**5.2 The Concession Stand**

15 minutes

The purpose of this activity is to interpret signed numbers in a situation involving money and inventory. Students reason abstractly and quantitatively when they think about change in inventory and money using positive and negative numbers (MP2).

**Addressing**

- 6.NS.C.5

**Instructional Routines**

- MLR8: Discussion Supports
- Think Pair Share

**Launch**

Arrange students in groups of 2. Give students 8 minutes of quiet work time, 3 minutes for partner discussion, followed by whole-class discussion.
**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
*SUPPORTS ACCESSIBILITY FOR: Memory; Conceptual processing*

**Student Task Statement**

The manager of the concession stand keeps records of all of the supplies she buys and all of the items she sells. The table shows some of her records for Tuesday.

<table>
<thead>
<tr>
<th>item</th>
<th>quantity</th>
<th>value in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>doughnuts</td>
<td>-58</td>
<td>37.70</td>
</tr>
<tr>
<td>straws</td>
<td>3,000</td>
<td>-10.35</td>
</tr>
<tr>
<td>hot dogs</td>
<td>-39</td>
<td>48.75</td>
</tr>
<tr>
<td>pizza</td>
<td>13</td>
<td>-116.87</td>
</tr>
<tr>
<td>apples</td>
<td>-40</td>
<td>14.00</td>
</tr>
<tr>
<td>french fries</td>
<td>-88</td>
<td>132.00</td>
</tr>
</tbody>
</table>

a. Which items did she sell? Explain your reasoning.

b. How can we interpret -58 in this situation?

c. How can we interpret -10.35 in this situation?

d. On which item did she spend the most amount of money? Explain your reasoning.

**Student Response**

a. She sold doughnuts, hot dogs, apples, and french fries. Sample explanation: Those items are associated with negative quantities, which are less than 0.

b. 58 doughnuts went out, or she sold 58 doughnuts.

c. -10.35 means that she spent $10.35 on straws.

d. She spent the most money on pizza. Sample explanation: The dollar amount for pizza is negative or less than 0, which suggests money being paid out. The digits represent a number in the hundreds, which is more than the other payment for straws.
Activity Synthesis

Ask students to compare their responses with their partner and work to reach agreement. Monitor student discussions and select students to share their partner’s reasoning in whole-class discussion. The key takeaway in the discussion is that positive and negative numbers are useful for describing change. If an amount goes up, then the change is positive. If an amount goes down, that means the change is negative.

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. To support students to interpret this table and to produce statements about the values in the table, provide sentence frames such as “In this situation, $-58$ means ___” and “$-58$ is related to $37.70$ because ___. This will help students make the connection that a negative number in the “quantity” column means that the items have been sold which generates a positive number in the “value in dollars” column.

Design Principle(s): Support sense-making

5.3 Drinks for Sale

15 minutes
Students interpret positive and negative numbers in the context of a changing inventory (MP2).

Addressing

◦ 6.NS.C.5

Instructional Routines

◦ MLR2: Collect and Display
◦ Think Pair Share

Launch

Keep students in the same groups of 2. Give students 8 minutes of quiet work time and 3 minutes of partner discussion. Follow with whole-class discussion.

Student Task Statement

A vending machine in an office building sells bottled beverages. The machine keeps track of all changes in the number of bottles from sales and from machine refills and maintenance. This record shows the changes for every 5-minute period over one hour.
a. What might a positive number mean in this context? What about a negative number?

b. What would a “0” in the second column mean in this context?

c. Which numbers—positive or negative—result in fewer bottles in the machine?

d. At what time was there the greatest change to the number of bottles in the machine? How did that change affect the number of remaining bottles in the machine?

e. At which time period, 8:05–8:09 or 8:25–8:29, was there a greater change to the number of bottles in the machine? Explain your reasoning.

f. The machine must be emptied to be serviced. If there are 40 bottles in the machine when it is to be serviced, what number will go in the second column in the table?

<table>
<thead>
<tr>
<th>time</th>
<th>number of bottles</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00–8:04</td>
<td>-1</td>
</tr>
<tr>
<td>8:05–8:09</td>
<td>+12</td>
</tr>
<tr>
<td>8:10–8:14</td>
<td>-4</td>
</tr>
<tr>
<td>8:15–8:19</td>
<td>-1</td>
</tr>
<tr>
<td>8:20–8:24</td>
<td>-5</td>
</tr>
<tr>
<td>8:25–8:29</td>
<td>-12</td>
</tr>
<tr>
<td>8:30–8:34</td>
<td>-2</td>
</tr>
<tr>
<td>8:35–8:39</td>
<td>0</td>
</tr>
<tr>
<td>8:40–8:44</td>
<td>0</td>
</tr>
<tr>
<td>8:45–8:49</td>
<td>-6</td>
</tr>
<tr>
<td>8:50–8:54</td>
<td>+24</td>
</tr>
<tr>
<td>8:55–8:59</td>
<td>0</td>
</tr>
<tr>
<td>service</td>
<td></td>
</tr>
</tbody>
</table>

**Student Response**

a. A positive number means bottles being added to the machine. A negative number means bottles being dispensed or otherwise being removed from the machine.

b. A “0” would mean no activity, i.e., the machine is not being stocked with new bottles and not dispensing any bottles (or the amount stocked is equal to the amount dispensed, but this idea is not expected at this time).

c. Negative numbers lead to fewer bottles in the machine, because they mean bottles are being removed.

d. The greatest change happened at 8:50–8:54. The number of bottles in the machine increased by 24.

e. There was the same amount of change to the number of bottles in the machine. In both cases, the number of bottles changed by 12, but at 8:05–8:09 a.m. it increased by 12 and at 8:25–8:29 it decreased by 12.

f. -40
Are You Ready for More?

Priya, Mai, and Lin went to a cafe on a weekend. Their shared bill came to $25. Each student gave the server a $10 bill. The server took this $30 and brought back five $1 bills in change. Each student took $1 back, leaving the rest, $2, as a tip for the server.

As she walked away from the cafe, Lin thought, “Wait—this doesn’t make sense. Since I put in $10 and got $1 back, I wound up paying $9. So did Mai and Priya. Together, we paid $27. Then we left a $2 tip. That makes $29 total. And yet we originally gave the waiter $30. Where did the extra dollar go?”

Think about the situation and about Lin’s question. Do you agree that the numbers didn’t add up properly? Explain your reasoning.

Student Response

Disagree. Sample explanations:

- It doesn’t matter that the students originally paid $30. Between the bill, which is $25, and the $2 tip, the students paid $27 in total.
- Lin mistakenly thought that the $2 was in addition to the $27 the three of them paid, but the $27 actually already included the $2 tip, since the original bill was $25.
- Since the bill was $25 for three people, each person’s share was $\frac{25}{3}$ dollars. Together they left a tip of $2, which means each person pitched in $\frac{2}{3}$. Each person then paid a total of $\frac{25}{3} + \frac{2}{3}$, which is $\frac{27}{3}$ or $9$. This matches the fact that they each gave a $10 bill and took $1 for the change.

Activity Synthesis

The goal of the discussion is to allow students to share their thoughts on the meaning of positive and negative numbers in context. Ask students to share their responses with their partner and work to reach agreement. Monitor groups’ discussions and select students to share their partner’s reasoning about what the numbers in the table mean. Ask students to summarize the story the numbers in the table tell. Tell students that tables like this (but perhaps more complicated) are used all the time to tell stories about what is happening in the world.
Access for English Language Learners

Speaking, Listening: MLR2 Collect and Display. To highlight words students use while speaking about positive and negative numbers, record on a visual display common or important phrases you hear students say related to the values in the table (e.g., “put into the machine”, “added to the machine”, “taken out”, “removed”). Make sure to include informal descriptions as well as more precise language. This will help students connect the everyday language that is used in mathematical situation to the more precise academic language.

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, students interpreted situations where positive and negative numbers were used to show changes in inventory or changes in money. Here are some questions to consider while closing the lesson:

○ “What did the positive and negative numbers mean in this lesson?” (Positive numbers represented a gain, like receiving money or adding bottles to the machine. Negative numbers represented a loss, like spending money by buying something or removing bottles from the machine.)

○ “We saw that we could use positive and negative numbers to represent gaining and losing money. What other situations can you think of where you gain or lose an amount that you could use negative numbers to talk about? What would positive and negative changes mean in those situations?” (Responses vary. Some examples include weight, speed, number of subscribers, field position in football.)

5.4 Bakery Owner

Cool Down: 5 minutes

Addressing

○ 6.NS.C.5

Student Task Statement

The table shows records of money-related activities of a bakery owner over a period of a week.
<table>
<thead>
<tr>
<th>date</th>
<th>items</th>
<th>amount in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 1</td>
<td>rent</td>
<td>-850.00</td>
</tr>
<tr>
<td>May 2</td>
<td>order (birthday cake and cookies)</td>
<td>106.75</td>
</tr>
<tr>
<td>May 3</td>
<td>utilities (electricity, gas, phone)</td>
<td>-294.50</td>
</tr>
<tr>
<td>May 5</td>
<td>order (wedding cake and desserts)</td>
<td>240.55</td>
</tr>
<tr>
<td>May 5</td>
<td>baking supplies</td>
<td>-147.95</td>
</tr>
<tr>
<td>May 6</td>
<td>order (anniversary cake)</td>
<td>158.20</td>
</tr>
<tr>
<td>May 7</td>
<td>order (breads and desserts for a conference)</td>
<td>482.30</td>
</tr>
<tr>
<td>May 7</td>
<td>bakery sales</td>
<td>415.65</td>
</tr>
</tbody>
</table>

a. For which items did she receive money?

b. What does the number -147.95 mean in this context?

c. Did the bakery owner receive more or spend more money on May 5? Explain how you know.

**Student Response**

a. She received money on the orders for different events (birthday, weddings, etc.) and sales of bakery items.

b. The number -147.95 means she spent $147.95 on baking supplies.

c. She received more money than she spent. She received over $240 and spent under $150.

**Student Lesson Summary**

Sometimes we represent changes in a quantity with positive and negative numbers. If the quantity increases, the change is positive. If it decreases, the change is negative.

- Suppose 5 gallons of water is put in a washing machine. We can represent the change in the number of gallons as +5. If 3 gallons is emptied from the machine, we can represent the change as -3.

It is especially common to represent money we receive with positive numbers and money we spend with negative numbers.
Suppose Clare gets $30.00 for her birthday and spends $18.00 buying lunch for herself and a friend. To her, the value of the gift can be represented as +30.00 and the value of the lunch as -18.00.

Whether a number is considered positive or negative depends on a person's perspective. If Clare's grandmother gives her $20 for her birthday, Clare might see this as +20, because to her, the amount of money she has increased. But her grandmother might see it as -20, because to her, the amount of money she has decreased.

In general, when using positive and negative numbers to represent changes, we have to be very clear about what it means when the change is positive and what it means when the change is negative.
Lesson 5 Practice Problems

Problem 1

**Statement**
Write a positive or negative number to represent each change in the high temperature.

i. Tuesday's high temperature was 4 degrees less than Monday's high temperature.

ii. Wednesday's high temperature was 3.5 degrees less than Tuesday's high temperature.

iii. Thursday's high temperature was 6.5 degrees more than Wednesday's high temperature.

iv. Friday's high temperature was 2 degrees less than Thursday's high temperature.

**Solution**

i. -4

ii. -3.5

iii. +6.5 or 6.5

iv. -2

Problem 2

**Statement**
Decide which of the following quantities can be represented by a positive number and which can be represented by a negative number. Give an example of a quantity with the opposite sign in the same situation.

i. Tyler’s puppy gained 5 pounds.

ii. The aquarium leaked 2 gallons of water.

iii. Andre received a gift of $10.


v. A climber descended 550 feet.

**Solution**

Answers vary. Sample responses:

i. Positive. Tyler’s puppy lost 5 pounds.

ii. Negative. 2 gallons of water was added to the aquarium.


### Problem 3

**Statement**

Make up a situation where a quantity is changing.

i. Explain what it means to have a negative change.

ii. Explain what it means to have a positive change.

iii. Give an example of each.

**Solution**

Answers vary. Sample response: They were selling candy at the concession stand.

i. When they sell candy, the change is negative.

ii. When they get more candy to sell, the change is positive.

iii. For example, in one hour the number of packages of candy changed by -5 because they sold 5, and in the next hour it changed by 20 because they got 20 more to sell.

### Problem 4

**Statement**

i. On the number line, label the points that are 4 units away from 0.

![Number line]

ii. If you fold the number line so that a vertical crease goes through 0, the points you label would match up. Explain why this happens.

iii. On the number line, label the points that are \( \frac{3}{2} \) units from 0. What is the distance between these points?

**Solution**

i. On the number line, -4 and 4 should be labeled.

ii. The two points match up because they are opposites; they are the same distance from 0.
iii. 2.5 and -2.5 should be labeled. The distance between them is 5 units, because each one is 2.5 units away from 0.

(From Unit 7, Lesson 2.)

**Problem 5**

**Statement**

Evaluate each expression.

- $2^3 \cdot 3$
- $\frac{4^2}{2}$
- $3^1$
- $6^2 \div 4$
- $2^3 - 2$
- $10^2 + 5^2$

**Solution**

i. 24

ii. 8

iii. 3

iv. 9

v. 6

vi. 125

(From Unit 6, Lesson 12.)
Lesson 6: Absolute Value of Numbers

Goals

○ Compare rational numbers and their absolute values, and explain (orally and in writing) the reasoning.

○ Comprehend the phrase “absolute value” and the symbol || to refer to a number's distance from zero on the number line.

○ Interpret rational numbers and their absolute values in the context of elevation or temperature.

Learning Targets

○ I can explain what the absolute value of a number is.

○ I can find the absolute values of rational numbers.

○ I can recognize and use the notation for absolute value.

Lesson Narrative

In the past several lessons, students have reasoned about the structure of rational numbers by plotting them on a number line and noting their relative positions and distances from zero. They learned that opposite numbers have the same distance from zero. Students now formalize the concept of a number's magnitude with the term absolute value. They learn that the absolute value of a number is its distance from zero, which means that opposite numbers have the same absolute value. Students reason abstractly about the familiar contexts of temperature and elevation using the concept and notation of absolute value (MP2).

Alignments

Building On

○ 4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

○ 5.NBT.A: Understand the place value system.

Addressing

○ 6.NS.C.7: Understand ordering and absolute value of rational numbers.

○ 6.NS.C.7.c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of −30 dollars, write \(|−30| = 30\) to describe the size of the debt in dollars.
6.NS.C.7.d: Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than $-30$ dollars represents a debt greater than 30 dollars.

**Building Towards**

6.NS.C.7.c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of $-30$ dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

**Student Learning Goals**

Let’s explore distances from zero more closely.

### 6.1 Number Talk: Closer to Zero

**Warm Up: 5 minutes**

The purpose of this Number Talk is to elicit strategies and understandings students have about the distance from 0 on the number line. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to think about distance from 0 for various rational numbers. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem.

**Building On**

- 4.NF.A.2
- 5.NBT.A

**Building Towards**

- 6.NS.C.7.c

**Instructional Routines**

- MLR8: Discussion Supports
- Number Talk
Launch
Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organization*

Student Task Statement
For each pair of expressions, decide mentally which one has a value that is closer to 0.

\[
\frac{9}{11} \text{ or } \frac{15}{11}
\]

\[
\frac{1}{5} \text{ or } \frac{1}{9}
\]

\[
1.25 \text{ or } \frac{5}{4}
\]

\[
0.01 \text{ or } 0.001
\]

Student Response
- \(\frac{9}{11}\). Sample explanation: \(\frac{9}{11}\) is positive and less than 1, whereas \(\frac{15}{11}\) is greater than 1, so \(\frac{9}{11}\) is closer to 0.
- \(\frac{1}{5}\). Sample explanation: Ninths are smaller than fifths, so \(\frac{1}{9}\) is closer to zero.
- They are equal, so equally close to zero.
- 0.001 is 10 times closer to zero than 0.01.

Activity Synthesis
Ask students to share their reasoning for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same answer but would explain it differently?”
- “Did anyone reason about the problem in a different way?”
- “Does anyone want to add on to ____’s reasoning?”
- “Do you agree or disagree? Why?”
Access for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

6.2 Jumping Flea

15 minutes (there is a digital version of this activity)
The purpose of this task is to help students understand the absolute value of a number as its distance from 0 on the number line. The context is not realistic, but helps students visualize relationships on the number line in a more concrete way. Students have used the concept of absolute value informally in previous lessons, but this is where the term is formally introduced and used precisely (MP6).

Addressing

- 6.NS.C.7.c

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Allow students 10 minutes quiet work time followed by whole-class discussion.

Students using the digital materials, will use an applet to visualize the bug jumping. Students can pick a starting point for the bug, choose the direction it jumps, and then check where it lands.

Anticipated Misconceptions

Students may confuse absolute value with opposites, thinking that absolute value changes the sign of the number. Remind students that absolute value represents the distance of the number from 0 without worrying about the sign or direction. Use some concrete examples: in a board game, 3 moves forward and 3 moves backward both involve 3 jumps but you land in different places; traveling 5 miles east or 5 miles west would put the same number of miles on your car's odometer, but you end up in different places depending on the direction; earning $10 and spending $10 both involve the amount $10 but in one case you gain it and in the other you lose it. Absolute value is just the amount involved, not including the sign.

Student Task Statement

a. A flea is jumping around on a number line.
i. If the flea starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

ii. If the flea starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

iii. If the flea starts at 0 and jumps 3 units away, where might it land?

iv. If the flea jumps 7 units and lands at 0, where could it have started?

v. The absolute value of a number is the distance it is from 0. The flea is currently to the left of 0 and the absolute value of its location is 4. Where on the number line is it?

vi. If the flea is to the left of 0 and the absolute value of its location is 5, where on the number line is it?

vii. If the flea is to the right of 0 and the absolute value of its location is 2.5, where on the number line is it?

b. We use the notation $|-2|$ to say “the absolute value of -2,” which means “the distance of -2 from 0 on the number line.”

i. What does $|-7|$ mean and what is its value?

ii. What does $|1.8|$ mean and what is its value?

Student Response

a. 
   i. Counting 4 spaces to the right gets to 5; this is 5 units from 0.
   ii. Counting 4 spaces to the left gets to -3; this is 3 units from 0.
   iii. Counting 3 spaces to the right gets to 3, or counting 3 spaces to the left gets to -3.
   iv. Counting 7 spaces to the right of zero gets to 7, or counting 7 spaces to the left of 0 gets to -7.
   v. 4 units to the left of 0 is -4.
   vi. 5 units to the left of 0 is -5.
   vii. 2.5 units to the right of 0 is 2.5.

b. 
   i. $|-7|$ means the absolute value of -7 or the distance of -7 from 0. Its value is 7.
   ii. $|1.8|$ means the absolute value of 1.8 or the distance of 1.8 from 0. Its value is 1.8.
Activity Synthesis

Define the absolute value of a number as its distance from 0. Ask students to contrast |−8| and |8| to come to the conclusion that they have the same value but represent the distance between two distinct points and zero. Also ask for situations where they think the absolute value might be useful (example: your car’s odometer tracks the miles you drove, but if you make a round trip—the same distance in two opposite directions—the difference between where you started and where you ended is zero).

To help clear up misconceptions related to opposites and absolute values, ask:

○ “What is the difference between a number’s opposite and a number’s absolute value?” (Opposite is another number on the number line whose distance from zero is the same; absolute value is a number that describes that distance.)

○ “Does finding a number’s absolute value always mean changing the sign?” (No, absolute value represents the distance from zero. If the number is positive, the number and its absolute value are the same. If the number is negative, the distance is represented by the number without its negative sign.)

○ “If \( n \) is any number that can be positive or negative, what is the sign of the absolute value of \( n \)?”

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students share and explain their reasoning, scribe their thinking on a display of each problem so it is visible for all students. Use color and annotations to illustrate the difference between absolute value and opposites.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. To support students to recognize the similarities and differences between of “absolute value” and “opposite,” ask students to respond to the prompt “How is a number’s absolute value the same and different than its opposite?” Encourage students to include diagrams with their explanations. Ask each student to meet with 2–3 other partners for feedback. Provide student with prompts for feedback “Can you show on your diagram . . .” or “Why are the values the same for this number?” Students can borrow ideas and language from each partner to strengthen their final response. This will help student to refine their understanding of absolute value and improve their ability to discuss rational numbers.

Design Principle(s): Support sense-making, Cultivate conversation

Unit 7 Lesson 6
6.3 Absolute Elevation and Temperature

15 minutes
The purpose of this task is for students to develop their understanding of the $|x|$ notation in familiar contexts. They should build their understanding that $|x|$ represents the distance from zero to $x$ and that $|-x|$ and $|x|$ are equal.

Addressing

- 6.NS.C.7.c
- 6.NS.C.7.d

Instructional Routines

- MLR7: Compare and Connect

Launch

Allow students 10 minutes quiet work time followed by whole-class discussion.

Student Task Statement

a. A part of the city of New Orleans is 6 feet below sea level. We can use “-6 feet” to describe its elevation, and “|-6| feet” to describe its vertical distance from sea level. In the context of elevation, what would each of the following numbers describe?

   i. 25 feet
   ii. |25| feet
   iii. -8 feet
   iv. |-8| feet

b. The elevation of a city is different from sea level by 10 feet. Name the two elevations that the city could have.

c. We write “-5°C” to describe a temperature that is 5 degrees Celsius below freezing point and “5°C” for a temperature that is 5 degrees above freezing. In this context, what do each of the following numbers describe?

   i. 1°C
   ii. -4°C
   iii. |12|°C
   iv. |-7|°C

d.  i. Which temperature is colder: -6°C or 3°C?
   ii. Which temperature is closer to freezing temperature: -6°C or 3°C?
iii. Which temperature has a smaller absolute value? Explain how you know.

**Student Response**

a.  
   i. 25 feet above sea level.  
   ii. The distance in feet between a point 25 feet above sea level and sea level.  
   iii. 8 feet below sea level.  
   iv. The distance in feet between a point 8 feet below sea level and sea level.

b. -10 feet or 10 feet.

c.  
   i. 1 degree Celsius above freezing point.  
   ii. 4 degrees Celsius below freezing point.  
   iii. The distance in degrees between freezing point and 12 degrees Celsius.  
   iv. The distance in degrees between freezing point and -7 degrees Celsius.

d.  
   i. -6°C  
   ii. 3°C  
   iii. 3°C, because it has a closer distance to 0 than does -6°C.

**Are You Ready for More?**

At a certain time, the difference between the temperature in New York City and in Boston was 7 degrees Celsius. The difference between the temperature in Boston and in Chicago was also 7 degrees Celsius. Was the temperature in New York City the same as the temperature in Chicago? Explain your answer.

**Student Response**

Answers vary. Sample response: It is not possible to say for sure. There are the two temperatures that differ from the temperature in Boston by 7 degrees. The temperature in New York City could be either 7 degrees above or below the temperature in Boston. The temperature in Chicago could also be either 7 degrees above or below the temperature in Boston. So the temperatures in New York City and Chicago could be equal, or they could differ by 14 degrees.

**Activity Synthesis**

To summarize students’ work, consider displaying the following diagram and these four expressions. Give students a minute to study the diagram and match each letter on the diagram to an appropriate expression.

- -4 feet
- |15| feet
- |-4| feet

Unit 7 Lesson 6
○ 15 feet

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: absolute value of a number, elevation, sea level, less than, greater than. 
Supports accessibility for: Conceptual processing; Language; Memory

Access for English Language Learners

Representing, Speaking: MLR7 Compare and Connect. To help students connect the numerical values with the diagram in the activity synthesis, provide students phrases that also connect to the situation (e.g., “4 feet below sea level,” “15 feet from sea level,” “4 feet from sea level,” and “15 feet above sea level”). Emphasize the language needed to say what the numerical values mean and how those values connect to the diagram and the phrases. 
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

In this lesson, students learned the definition of absolute value and its relationship to rational numbers. Ask students to do the following:

○ Order the numbers 1, 3, and 6 from least to greatest. (1, 3, 6.)
Order $|1|, |3|$, and $|6|$ from least to greatest. ($|1|, |3|, |6|.$)

Order the numbers -1, -3, and -6 from least to greatest. (-6, -3, -1.)

Order $|-1|, |-3|$, and $|-6|$ from least to greatest. ($|-1|, |-3|, |-6|.$)

What do you notice about the order of numbers after taking absolute value? Explain why this happens.

Consider asking students to sketch a number line if they get stuck. Students should see that the order remained the same for the positive numbers but reversed for the negative numbers. They should be able to explain that as numbers move to the left on the number line, their absolute value gets larger because they are further from 0. This realization should help solidify the thinking that has been building for the past several lessons about the ordering and magnitude of rational numbers.

6.4 Greater, Less, the Same

Cool Down: 5 minutes

Addressing

6.NS.C.7

Student Task Statement

a. Write a number that has the same value as each expression:
   
i. $|5|$
   
ii. $|-12.9|$

b. Write a number that has a value less than $|4.7|$

c. Write a number that has a value greater than $|-2.6|$

Student Response

a. i. 5 or $|-5|$
   
   ii. 12.9 or $|12.9|$

b. Answers vary. Sample responses: 4.5, $|-4.5|$, or -10

c. Answers vary. Sample responses: 2.7 or $|-2.7|$

Student Lesson Summary

We compare numbers by comparing their positions on the number line: the one farther to the right is greater; the one farther to the left is less.
Sometimes we wish to compare which one is closer to or farther from 0. For example, we may want to know how far away the temperature is from the freezing point of $0^\circ C$, regardless of whether it is above or below freezing.

The **absolute value** of a number tells us its distance from 0.

The absolute value of -4 is 4, because -4 is 4 units to the left of 0. The absolute value of 4 is also 4, because 4 is 4 units to the right of 0. Opposites always have the same absolute value because they both have the same distance from 0.

The distance from 0 to itself is 0, so the absolute value of 0 is 0. Zero is the *only* number whose distance to 0 is 0. For all other absolute values, there are always two numbers—one positive and one negative—that have that distance from 0.

To say “the absolute value of 4,” we write:

$$|4|$$

To say that “the absolute value of -8 is 8,” we write:

$$|{-8}| = 8$$

**Glossary**

- absolute value
Lesson 6 Practice Problems

Problem 1

Statement
On the number line, plot and label all numbers with an absolute value of $\frac{3}{2}$.

Solution
Points $\frac{3}{2}$ and $-\frac{3}{2}$, $1 \frac{1}{2}$ and $-1 \frac{1}{2}$, or 1.5 and -1.5 should be plotted.

Problem 2

Statement
The temperature at dawn is 6°C away from 0. Select all the temperatures that are possible.

A. -12°C
B. -6°C
C. 0°C
D. 6°C
E. 12°C

Solution
["B", "D"]

Problem 3

Statement
Put these numbers in order, from least to greatest.

Solution
0  |-1|  1.3  2  |-2.7|
Problem 4

**Statement**

Lin’s family needs to travel 325 miles to reach her grandmother’s house.

i. At 26 miles, what percentage of the trip’s distance have they completed?

ii. How far have they traveled when they have completed 72% of the trip’s distance?

iii. At 377 miles, what percentage of the trip’s distance have they completed?

**Solution**

i. 8% of the trip, because $26 \div 325 = 0.08$.

ii. 234 miles, because $0.72 \cdot 325 = 234$.

iii. 116% of the trip, because $377 \div 325 = 1.16$.

(From Unit 5, Lesson 11.)

Problem 5

**Statement**

Elena donates some money to charity whenever she earns money as a babysitter. The table shows how much money, $d$, she donates for different amounts of money, $m$, that she earns.

<table>
<thead>
<tr>
<th>$d$</th>
<th>4.44</th>
<th>1.80</th>
<th>3.12</th>
<th>3.60</th>
<th>2.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>37</td>
<td>15</td>
<td>26</td>
<td>30</td>
<td>18</td>
</tr>
</tbody>
</table>

i. What percent of her income does Elena donate to charity? Explain or show your work.

ii. Which quantity, $m$ or $d$, would be the better choice for the dependent variable in an equation describing the relationship between $m$ and $d$? Explain your reasoning.

iii. Use your choice from the second question to write an equation that relates $m$ and $d$.

**Solution**

i. Elena donates 12% of her income to charity. Sample reasoning: We want to know what percent of 30 is 3.6, so we can write $30p = 3.6$. To solve this, divide 3.6 by 30, which is 0.12. So 12% of 30 is 3.6.

ii. Answers vary. Sample response: Since the amount of the donation depends on how much money she earns, $d$ would be better as the dependent variable. If she wants to donate a certain amount and needs to figure out how much she needs to earn to achieve that donation, then $m$ would be better as the dependent variable.
iii. \( d = .12m \) or equivalent \( m = \frac{100}{12} d \) or \( m = d \div .12 \) or equivalent

(From Unit 6, Lesson 16.)

**Problem 6**

**Statement**

How many times larger is the first number in the pair than the second?

i. \( 3^4 \) is ____ times larger than \( 3^3 \).

ii. \( 5^3 \) is ____ times larger than \( 5^2 \).

iii. \( 7^{10} \) is ____ times larger than \( 7^8 \).

iv. \( 17^6 \) is ____ times larger than \( 17^4 \).

v. \( 5^{10} \) is ____ times larger than \( 5^4 \).

**Solution**

i. 3

ii. 5

iii. \( 7^2 \) or 49

iv. \( 17^2 \) or 289

v. \( 5^6 \) or 15,625

(From Unit 6, Lesson 12.)
Lesson 7: Comparing Numbers and Distance from Zero

Goals

○ Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values.

○ Generate values that meet given conditions for their relative position and absolute value, and justify the comparisons (using words and symbols).

○ Recognize that the value of \(-a\) can be positive or negative, depending on the value of \(a\).

Learning Targets

○ I can explain what absolute value means in situations involving elevation.

○ I can use absolute values to describe elevations.

○ I can use inequalities to compare rational numbers and the absolute values of rational numbers.

Lesson Narrative

In this lesson, students use precise language to distinguish between order and absolute value of rational numbers (MP6). It is a common mistake for students to mix up “greater” or “less” with absolute value. A confused student might say that -18 is greater than 4 because they see 18 as being the “bigger” number. What this student means to express is $$-18 > 4$$. The absolute value of -18 is greater than 4 because -18 is more than 4 units away from 0. In the “Submarine” activity, students visualize possible elevations of characters with sticky notes on a vertical number line. The freedom to move a sticky note within a specified range anticipates the concept of a solution to an inequality in the next section.

Alignments

Addressing

○ 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

○ 6.NS.C.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $$-(-3) = 3$$, and that 0 is its own opposite.

○ 6.NS.C.7: Understand ordering and absolute value of rational numbers.

○ 6.NS.C.7.d: Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than \(-30\) dollars represents a debt greater than 30 dollars.
Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR4: Information Gap Cards
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the Instructional master
Sticky notes

Required Preparation

For every 4 students, create a set of 5 sticky notes that read Clare, Andre, Han, Lin, and Priya. These are for the launch of the “Submarine” activity.

Student Learning Goals

Let’s use absolute value and negative numbers to think about elevation.

7.1 Opposites

Warm Up: 10 minutes
The purpose of this warm-up is to use opposites to get students to think about distance from 0. Problem 3 also reminds students that the opposite of a negative number is positive.

Notice students who choose 0 or a negative number for \(a\) and how they reason about \(-a\).

Addressing

- 6.NS.C.6.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet think time, then 2 minutes of partner discussion. Follow with whole-class discussion.

Anticipated Misconceptions

For problem 3, students might assume that \(-a\) is always a negative number. Ask these students to start with a negative number and find its opposite. For example, starting with \(a = -3\), we can find its opposite, \(-(-3)\), to be equal to 3.
**Student Task Statement**

a. \(a\) is a rational number. Choose a value for \(a\) and plot it on the number line.

b. 
   i. Based on where you plotted \(a\), plot \(-a\) on the same number line.
   
   ii. What is the value of \(-a\) that you plotted?

c. Noah said, “If \(a\) is a rational number, \(-a\) will always be a negative number.” Do you agree with Noah? Explain your reasoning.

**Student Response**

a. Responses vary.

b. 
   i. The point \(-a\) will be plotted the same distance from 0 as \(a\), but on the opposite side of 0.

   ii. Responses vary. The value of \(-a\) has the opposite sign as the value for \(a\).

c. Noah is incorrect. Sample response: If \(a\) itself is negative, then \(-a\) will be its opposite, which will be positive. If \(a\) is 0, then \(-a\) is also 0 and neither is positive or negative.

**Activity Synthesis**

The main idea of discussion is that opposites have the same distance to 0 (i.e., same absolute value) and that the opposite of a negative number is positive. Ask students to discuss their reasoning with a partner. In a whole-class discussion, ask a student who chose \(a\) to be positive to share their reasoning about how to plot \(-a\) and whether they agreed with Noah in problem 3. Then, select previously identified students who chose \(a\) to be negative to share their thinking. If not mentioned by students, emphasize both symbolic and geometric statements of the fact that the opposite of a negative number is positive. For example, if \(a = -3\), write \(-(-3) = 3\) and show that 3 is the opposite of -3 on the number line because they are the same distance to 0. If time allows, select a student who chose \(a\) to be 0 and compare to cases where \(a\) is negative or positive. The number 0 is its own opposite because no other number is 0 units away from 0. Sequencing the discussion to look at positive, negative, and 0 values of \(a\) helps students to visualize and generalize the concept of opposites for rational numbers.

### 7.2 Submarine

15 minutes

Students distinguish between absolute value and order in the context of elevation. Students express their ideas carefully using symbols, verbally, and using a vertical number line. Placing possible elevations on the number line serves as a transition to thinking about solutions to inequalities. Look for students who choose positive and negative elevations for Han and Lin to compare in the discussion.
Addressing
- 6.NS.C.7

Instructional Routines
- MLR8: Discussion Supports

Launch
Arrange students in groups of 4. Distribute one set of sticky notes to each group, where each note contains one name: Clare, Andre, Han, Lin, and Priya. Display the image for all to see throughout the activity.

Ask students to read the instructions for the task and the description of each person's elevation. Give them a few minutes to use their sticky notes, as a group, to decide where each person (except Priya) could be located.

Place Clare’s sticky note on the number line according to the completed first row of the table. Explain the completed first row of the table to students as it pertains to Clare's description. Use precise language when explaining the symbols in the table:

- One possible elevation for Clare is 150 feet because 150 is greater than -100, and it is also farther from sea level.
- 150 is greater than -100.
- The absolute value of 150 is greater than the absolute value of -100.

Ask groups to complete the rest of the table for the other people (except Priya), and then answer the question about Priya. Note that it is possible to come up with different, correct responses that fit the descriptions. Give students 10 minutes to work followed by whole-class discussion.
Student Task Statement

A submarine is at an elevation of -100 feet (100 feet below sea level). Let’s compare the elevations of these four people to that of the submarine:

- Clare’s elevation is greater than the elevation of the submarine. Clare is farther from sea level than the submarine.
- Andre’s elevation is less than the elevation of the submarine. Andre is farther away from sea level than the submarine.
- Han’s elevation is greater than the elevation of the submarine. Han is closer to sea level than the submarine.
- Lin’s elevation is the same distance away from sea level as the submarine’s.

a. Complete the table as follows.

i. Write a possible elevation for each person.

ii. Use $<, >, \text{ or } =$ to compare the elevation of that person to that of the submarine.

iii. Use absolute value to tell how far away the person is from sea level (elevation 0).

As an example, the first row has been filled with a possible elevation for Clare.

<table>
<thead>
<tr>
<th></th>
<th>possible elevation</th>
<th>compare to submarine</th>
<th>distance from sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clare</td>
<td>150 feet</td>
<td>150 &gt; -100</td>
<td>$</td>
</tr>
<tr>
<td>Andre</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Han</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lin</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b. Priya says her elevation is less than the submarine’s and she is closer to sea level. Is this possible? Explain your reasoning.

Student Response

a. Responses vary. Andre could have any elevation less than -100 feet. Han could have any elevation between -100 and 100 feet. Lin could be at either -100 feet or 100 feet. Sample response:

<table>
<thead>
<tr>
<th></th>
<th>possible elevation</th>
<th>compare to submarine</th>
<th>distance from sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clare</td>
<td>150 feet</td>
<td>150 &gt; -100</td>
<td></td>
</tr>
<tr>
<td>Andre</td>
<td>-250 feet</td>
<td>-250 &lt; -100</td>
<td></td>
</tr>
<tr>
<td>Han</td>
<td>20</td>
<td>20 &gt; -100</td>
<td></td>
</tr>
<tr>
<td>Lin</td>
<td>100</td>
<td>100 &gt; -100</td>
<td></td>
</tr>
</tbody>
</table>

Sample explanations:
- Andre could have an elevation of -250 feet because -250 < -100 and |-250| > |-100|. We can write |-250| = 250 because Andre is 250 feet away from sea level.
- Han could have an elevation of 20 feet because 20 > -100 and |20| < |-100|. We can write |20| = 20 because Han is 20 feet away from sea level.
- Lin could have an elevation of 100 feet because |100| = |-100|. We can write |100| = 100 because Lin is 100 feet away from sea level.

b. It is not possible. Sample response: Priya’s description is impossible because any elevation that is less than the elevation of the submarine (below the submarine) must also be farther away from sea level.

Activity Synthesis

The purpose of the discussion is to let students practice using proper vocabulary to express ideas that distinguish order from absolute value with positive and negative numbers. Select previously identified students to share different elevations for Han and for Lin that show both positive and negative possibilities. Encourage students to explain why the elevation they chose satisfies the description in the problem. As students speak, record their statements using <, >, = and |·|. Allow students to rearrange sticky notes on the vertical number line display. If time allows, use the sticky notes to show the range of possible solutions for each character; this will help to further prepare students for the concept of graphing solutions of an inequality on the number line.
Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* To support students’ use of vocabulary related to absolute value and positive and negative numbers, provide sentence frames related to each column heading. Some examples include: “_____ could have an elevation of _____ because _____,” “Comparing _____’s elevation to the submarine’s, I notice _____,” or “_____’s distance from sea level is _____ because ______.”

*Design Principle(s): Cultivate conversation*

### 7.3 Info Gap: Points on the Number Line

**Optional: 15 minutes**

In this info gap activity, students use comparisons of order and absolute value of rational numbers to determine the location of unknown points on the number line. In doing so students reinforce their understanding that a number and its absolute value are different properties. Students will also begin to understand that the distance between two numbers, while being positive, could be in either direction between the numbers. This concept is expanded on further when students study arithmetic with rational numbers in grade 7.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:
Addressing

- 6.NS.C.6
- 6.NS.C.7

Instructional Routines

- MLR4: Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute the first problem card to one student and a data card to the other student. After debriefing on the first problem, distribute the cards for the second problem, in which students switch roles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organization
Access for English Language Learners

**Conversing**: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to determine the location of unknown points on the number line. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

*Design Principle(s): Cultivate Conversation*

**Anticipated Misconceptions**

Students may struggle to make sense of the abstract information they are given if they don’t choose to draw a number line. Rather than specifically instructing them to use this strategy, consider asking them a question like “How could you keep track of the information you've learned about the points so far?”

**Student Task Statement**

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

a. Silently read your card and think about what information you need to be able to answer the question.

b. Ask your partner for the specific information that you need.

c. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.

d. Share the *problem card* and solve the problem independently.

e. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

a. Silently read your card.

b. Ask your partner “What specific information do you need?” and wait for them to ask for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

c. Before sharing the information, ask “Why do you need that information?” Listen to your partner’s reasoning and ask clarifying questions.

d. Read the *problem card* and solve the problem independently.

e. Share the *data card* and discuss your reasoning.
Student Response

a. Point $A$ is at $-\frac{1}{2}$.

b. Point $Z$ is at 3.

Activity Synthesis

Select students with different strategies to share their approaches. Invite them to share which of the clues they thought were more helpful and which were least helpful. Ask students to explain how drawing a number line helped them and how they decided on the appropriate order for the unknown numbers.

### 7.4 Inequality Mix and Match

Optional: 15 minutes

The goal of this activity is for students to practice comparing rational numbers.

Notice students who compare fractions to decimals, fractions to integers, or who compare absolute values to negative numbers.

**Addressing**

- 6.NS.C.7.d

**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions

**Launch**

Arrange students in groups of 2. Give students 10 minutes to work before whole-class discussion.

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**Access for Students with Disabilities**

*Action and Expression: Provide Access for Physical Action.* Create alternatives for physically interacting with materials. Consider creating a set of cards for each of the numbers and inequality symbols that students can select from and sequence to create true comparison statements. Invite students to talk about their statements before writing them down.

*Supports accessibility for: Visual-spatial processing; Conceptual processing*
Access for English Language Learners

Speaking: MLR5 Co-Craft Questions. To create space for students to produce the language of mathematical questions themselves, display only the array of numbers that the students will be using in this activity. Ask students to think about the values of the numbers and write a mathematical question using two or more numbers from the array. Students may generate questions such as “How many values are greater than zero?” or “Which numbers are opposites?” Notice students that have questions about comparing and ordering the numbers and ask them to share their questions. This will help students use conversation skills to generate, choose, and improve their questions as well as develop meta-awareness of the language used in mathematical questions.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Here are some numbers and inequality symbols. Work with your partner to write true comparison statements.

-0.7 \[ \frac{3}{5} \] 1 4 \[ |-8| \] <  
\[ \frac{5}{3} \] -2.5 2.5 8 \[ |0.7| \] =  
-4 0 \[ \frac{7}{2} \] \[ |3| \] \[ |\frac{5}{2}| \] >

One partner should select two numbers and one comparison symbol and use them to write a true statement using symbols. The other partner should write a sentence in words with the same meaning, using the following phrases:

- is equal to
- is the absolute value of
- is greater than
- is less than

For example, one partner could write 4 < 8 and the other would write, “4 is less than 8.” Switch roles until each partner has three true mathematical statements and three sentences written down.

Student Response

Responses vary. Sample mathematical responses:

- -0.7 > -\[ \frac{6}{3} \]
- \[ \frac{7}{2} \] < 4
Sample sentence responses:

-0.7 is greater than $-\frac{6}{3}$.

$\frac{7}{2}$ is less than 4.

The absolute value of -8 is 8.

**Are You Ready for More?**

For each question, choose a value for each variable to make the whole statement true. (When the word *and* is used in math, both parts have to be true for the whole statement to be true.) Can you do it if one variable is negative and one is positive? Can you do it if both values are negative?

a. $x < y$ and $|x| < y$.

b. $a < b$ and $|a| < |b|$.

c. $c < d$ and $|c| > d$.

d. $t < u$ and $|t| > |u|$.

**Student Response**

Answers vary. Sample responses:

-1 < 5 and |-1| < 5.

-2 < 3 and |-2| < |3|.

-12 < -8 and |-12| > -8.

-10 < -1 and |-10| > |-1|.

**Activity Synthesis**

The goal of discussion is to allow students to use precise language when comparing rational numbers and absolute values verbally. Select previously identified students to share their responses that compare fractions to decimals, fractions to integers, or absolute values to negative numbers. Display their responses using absolute value and >, <, = symbols for all to see. Ask students to indicate whether they agree that each response is true, and ask students to share their reasoning about whether they agree or disagree.

**Lesson Synthesis**

During this lesson, students have used precise language to distinguish absolute value from order of rational numbers. Display $|-8|$ and 3 questions for all to see:
“How do you say this?” (The absolute value of -8.)

“What does it mean in an elevation situation?” (It's the distance from 8 feet below sea level to sea level.)

“What does it mean on a number line?” (It's the distance from -8 to 0 on the number line.)

“What is its value?” (8.)

Next, display \(|-8| < 5\) and two questions for all to see:

“How do you say this?” (The absolute value of -8 is less than 5.)

“What does it mean on a number line?” (-8 is less than 5 units away from 0.)

“Is it true?” (No, -8 is more than 5 units away from 0.)

7.5 True or False?

Cool Down: 5 minutes
This cool-down asks students to think about the differences between absolute value and order. The numbers -5 and 3 illustrate this difference because while -5 is greater in absolute value than 3, it is also less than 3. Encourage students who struggle with this cool-down to review absolute value, “greater than,” “less than,” and to use a number line to reason about the statements.

Addressing

6.NS.C.7

Student Task Statement

Mark each of the following as true or false and explain how you know.

a. \(-5 < 3\)

b. \(-5 > 3\)

c. \(|-5| < 3\)

d. \(|-5| > 3\)

Student Response

a. True, because -5 is farther to the left on the number line than 3.

b. False, because -5 is farther to the left on the number line than 3.

c. False, because \(|-5| = 5\) and \(5 > 3\).

d. True, because \(|-5| = 5\) and \(5 > 3\).
Student Lesson Summary

We can use elevation to help us compare two rational numbers or two absolute values.

- Suppose an anchor has an elevation of -10 meters and a house has an elevation of 12 meters. To describe the anchor having a lower elevation than the house, we can write $-10 < 12$ and say “-10 is less than 12.”

- The anchor is closer to sea level than the house is to sea level (or elevation of 0). To describe this, we can write $|-10| < |12|$ and say “the distance between -10 and 0 is less than the distance between 12 and 0.”

We can use similar descriptions to compare rational numbers and their absolute values outside of the context of elevation.

- To compare the distance of -47.5 and 5.2 from 0, we can say: $|-47.5|$ is 47.5 units away from 0, and $|5.2|$ is 5.2 units away from 0, so $|-47.5| > |5.2|$.

- $|-18| > 4$ means that the absolute value of -18 is greater than 4. This is true because 18 is greater than 4.
Lesson 7 Practice Problems

Problem 1

Statement
In the context of elevation, what would \(|-7|\) feet mean?

Solution
The vertical distance between the point at -7 feet and sea level (0 feet).

Problem 2

Statement
Match the the statements written in English with the mathematical statements.

A. The number -4 is a distance of 4 units away from 0 on the number line.
B. The number -63 is more than 4 units away from 0 on the number line.
C. The number 4 is greater than the number -4.
D. The numbers 4 and -4 are the same distance away from 0 on the number line.
E. The number -63 is less than the number 4.
F. The number -63 is further away from 0 than the number 4 on the number line.

Solution

- A: 4
- B: 1
- C: 5
- D: 6
- E: 2
- F: 3
Problem 3

Statement

Compare each pair of expressions using >, <, or =.

-32 ___ 15
|-32| ___ |15|
5 ___ -5
|5| ___ |-5|

Solution

i. -32 < 15 because 15 is further right on the number line.

ii. |-32| > |15| because -32 is further from zero than 15.

iii. 5 > -5 because 5 is further right on the number line than -5.

iv. |5| = |-5|, because 5 and -5 are the same distance away from zero.

v. 2 > -17 because 2 is further right on the number line than -17.

vi. 2 < |-17| because -17 is more than 2 units away from zero.

vii. |-27| < |-45| because -45 is further from zero than -27.


Problem 4

Statement

Mai received and spent money in the following ways last month. For each example, write a signed number to represent the change in money from her perspective.

i. Her grandmother gave her $25 in a birthday card.

ii. She earned $14 dollars babysitting.

iii. She spent $10 on a ticket to the concert.

iv. She donated $3 to a local charity

v. She got $2 interest on money that was in her savings account.

Solution

i. +25 or 25

ii. +14 or 14
Problem 5

Statement

Here are the lowest temperatures recorded in the last 2 centuries for some US cities.

- Death Valley, CA was \(-45^\circ F\) in January of 1937.
- Danbury, CT was \(-37^\circ F\) in February of 1943.
- Monticello, FL was \(-2^\circ F\) in February of 1899.
- East Saint Louis, IL was \(-36^\circ F\) in January of 1999.
- Greenville, GA was \(-17^\circ F\) in January of 1940.

i. Which of these states has the lowest record temperature?

ii. Which state has a lower record temperature, FL or GA?

iii. Which state has a lower record temperature, CT or IL?

iv. How many more degrees colder is the record temperature for GA than for FL?

Solution

i. CA

ii. GA

iii. CT

iv. 15 degrees

Problem 6

Statement

Find the quotients.

i. \[0.024 \div 0.015\]

ii. \[0.24 \div 0.015\]

iii. \[0.024 \div 0.15\]
iv. \( \frac{24}{15} \)

**Solution**

i. 1.4

ii. 14

iii. 0.14

iv. 1.4

(From Unit 5, Lesson 13.)
Section: Inequalities

Lesson 8: Writing and Graphing Inequalities

Goals

- Coordinate verbal, algebraic, and number line representations of inequalities.
- Critique (orally and in writing) possible values given for a situation with a constraint, including determining whether the boundary value is included and making sense of situations with discrete quantities.
- Interpret phrases that describe a quantity constrained by a maximum or minimum acceptable value, e.g. “at least,” “at most,” “up to,” “more than,” “less than”, etc., and write an inequality statement to represent the constraint.

Learning Targets

- I can graph inequalities on a number line.
- I can write an inequality to represent a situation.

Lesson Narrative

In extending their concept of numbers to all rational numbers, students began writing inequality statements that compared two numbers. In this lesson, students extend their work with inequality statements by considering comparisons with an unknown quantity. These quantities, represented by variables, often describe real-world situations, and their value is usually constrained by minimum or maximum allowable values. Students represent these situations with inequality statements and reason about possible values that make them true (MP2). As there are often many, even infinite, possibilities for the value of the variable that satisfy the constraint, students use the number line as a helpful tool to show all the possible values.

The activities in this lesson present students with two types of scenarios. When the variable represents a measurement, the possible values can usually be any number within the range satisfied by the constraint. When the variable represents a count of people or objects, the possible values are restricted to whole numbers within the range. Students also consider whether the constraint itself is included or excluded in the set of possible values, and learn how to indicate this result on the number line representation.

After writing inequality statements to represent situations, students test values to see if they make the statement true.
Alignments

Addressing

- **6.EE.B.6:** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- **6.EE.B.8:** Write an inequality of the form \( x > c \) or \( x < c \) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form \( x > c \) or \( x < c \) have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

- **6.NS.C.7.b:** Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write \(-3^\circ C > -7^\circ C\) to express the fact that \(-3^\circ C\) is warmer than \(-7^\circ C\).

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Take Turns
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the Instructional master

Required Preparation

Print and cut up cards from the Instructional master. Prepare 1 set of 16 cards for each group of 2 students.

Student Learning Goals

Let's write inequalities.

8.1 Estimate Heights of People

Warm Up: 5 minutes

The purpose of this warm-up is for students to estimate heights and discuss their estimates using the language of inequalities. As students discuss their estimates with a partner, monitor the discussions and identify students who use different estimation strategies to share during the whole-class discussion.

Addressing

- **6.NS.C.7.b**

Unit 7 Lesson 8
Instructional Routines

Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time to complete the first question. Pause after the first question and ask students to share their responses. Record and display them for all to see. Tell students the height of the man in the pictures is 5 feet 10 inches and give them 1 minute to discuss the second question with their partner. Follow with a whole-class discussion focused on the second question.

Student Task Statement

a. Here is a picture of a man.

i. Name a number, in feet, that is clearly too high for this man’s height.

ii. Name a number, in feet, that is clearly too low for his height.

iii. Make an estimate of his height.

Pause here for a class discussion.

b. Here is a picture of the same man standing next to a child.

If the man’s actual height is 5 feet 10 inches, what can you say about the height of the child in this picture?

Be prepared to explain your reasoning.
**Student Response**

Answers vary. Sample responses:

a. i. 12 feet
   ii. 4 feet
   iii. 6 feet (or another number between the responses for the first two questions).

b. The child is shorter than the man, so the child's height is less than 5 feet 10 inches.

**Activity Synthesis**

During the whole-class discussion, encourage students to use language such as “his height is more than . . .” and “her height is less than . . .” instead of “taller than . . .” and “shorter than . . .” so they become familiar with the more precise language of inequalities.

Ask students to share their responses to the second question. As students share, ask them how to write their explanation as a sentence and inequality. For example, if students say, “The child is shorter than 5’10”, ask them how we could write that statement using “greater” or “less” and using an inequality. Record and display their responses for all to see. If students do not say, “The child’s height is less than 5’10” and \( h < 5'10'' \), where \( h \) represents the child’s height, then make these statements explicit.

If there is time, ask students for the difference between their estimates and the actual heights.

**8.2 Stories about 9**

15 minutes (there is a digital version of this activity)

In this activity, students extend their understanding of inequality statements by considering an unknown quantity with one or more constraints in a real-world situation. Students represent situations with inequality statements and reason about possible values that make them true, showing the possibilities on a number line. They realize that these situations often have many
values that make the inequality true, and often even an infinite number, making the number line representation of possible values very useful (MP7). Students examine the difference between solutions that are continuous (can take on any value, usually involving measurement of an amount) and those that are discrete (require whole number solutions because of the context, usually involving a count of people or objects) (MP2). They also consider when to include or exclude the endpoints. They learn how to represent this on the number line with a closed (include) or open (exclude) circle at the boundary of the constraint. Note that these materials don't introduce \( \geq \) or \( \leq \) symbols until grade 7, but some individual teachers prefer to introduce them earlier.

**Addressing**
- 6.EE.B.8

**Instructional Routines**
- MLR7: Compare and Connect
- Take Turns

**Launch**
Copy and cut up the Instructional master. Make enough for each group of 2 students to have a set. Arrange students in groups of 2. Give each group 16 pre-cut slips and 6–8 minutes to complete the first question, which is to match each story and question to three ways to represent the solutions: a list or description, a number line, or an inequality statement. Ask each group to compare their matching decisions with another group and come to an agreement before recording their sorted representations.

Classes using the digital materials have an applet to match the story and question to two ways to represent the solutions: a list or description and a number line. Representation with an inequality is left to class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

*Supports accessibility for: Conceptual processing; Organization*

**Student Task Statement**

a. Your teacher will give you a set of paper slips with four stories and questions involving the number 9. Match each question to three representations of the solution: a description or a list, a number line, or an inequality statement.
b. Compare your matching decisions with another group's. If there are disagreements, discuss until both groups come to an agreement. Then, record your final matching decisions here.

i. A fishing boat can hold fewer than 9 people. How many people (x) can it hold?

- Description or list:
- Number line:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

- Inequality:

ii. Lin needs more than 9 ounces of butter to make cookies for her party. How many ounces of butter (x) would be enough?

- Description or list:
- Number line:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

- Inequality:

iii. A magician will perform her magic tricks only if there are at least 9 people in the audience. For how many people (x) will she perform her magic tricks?

- Description or list:
- Number line:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

- Inequality:

iv. A food scale can measure up to 9 kilograms of weight. What weights (x) can the scale measure?

- Description or list:
- Number line:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

- Inequality:

**Student Response**

See Instructional master.

_Unit 7 Lesson 8_
Activity Synthesis

Here are some topics for discussion:

- Differences between solutions to the people problem and the weight problem (continuous v. discrete).
- Inclusion and exclusion of 9 and how to distinguish them on the number line (including open and closed circle).
- The idea that there are many values that make these inequalities true. At times, there are an infinite number of values that make the inequality true.
- Advantages of using inequality statements or graphs instead of listing or describing the solutions.

There are some additional restrictions in some of the scenarios. For example, when counting people in a boat or measuring weight, zero is the least value that makes sense. For the people at the magic show and the amount of butter, very large numbers satisfy the inequalities; however, they don't necessarily make sense in the contexts presented. These restrictions are not a focus for this activity, but they are likely to come up during the discussion.

Toward the end of the discussion, display the statement “Jada built a robot that pushes small boxes from one place to another. The robot is able to push up to 3 pounds. For what box weights \( w \) can the robot push the box?” Ask students to represent all possible answers in three ways:

- By listing or describing all numbers that answer the question. (Any weight that is equal to or less than 3 pounds.)
- By plotting or graphing the values that work on a number line. (Closed circle at 3 and shaded to the left.)
- By writing an inequality. (Using the given variable) \( w < 3 \) and \( w = 3. \)

Access for English Language Learners

Representing, Conversing, Reading: MLR7 Compare and Connect. To foster students’ meta-awareness of language as they compare and contrast different mathematical situations, display these two sentences: “A fishing boat can hold fewer than 9 people” and “A magician will perform her magic tricks only if there are at least 9 people in the audience.” Ask students what is the same and what is different about these two situations. Invite students to work with a partner to find values can be true for each statement. This discussion makes explicit the language of inequalities (e.g., fewer, at least), and offers an opportunity to connect verbal descriptions to solution sets of numbers.

Design Principle(s): Maximize meta-awareness
8.3 How High and How Low Can It Be?

15 minutes
The purpose of this task is to extend the use of inequalities to describe maximum and minimum possible values, using symbols and determining whether or not a particular value makes an inequality true. Though students are thinking about whether or not a particular value makes an inequality true, the term “solution” will not be formally introduced until a future lesson.

Students will estimate the maximum and minimum height of a basketball hoop in a given picture. They will then explore how inequalities with these 2 values, the maximum and minimum, can describe the restrictions and possible heights for the hoop. The inequalities are represented as symbolic statements as well as on number lines.

Once they have created inequalities, students will decide if a given value could be a possible height for the hoop. For the last question, students work with a partner so each will have a chance to check a possible solution to an inequality that they didn't estimate and write themselves.

Addressing
○ 6.EE.B.8

Instructional Routines
○ MLR5: Co-Craft Questions

Launch
Arrange students in groups of 2. Give students 8 minutes of quiet work time to complete questions 1 through 4 followed by 2 minutes partner discussion for question 5. Follow with whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students to check for understanding within the first 2-3 minutes of work time. Look for students who are making reasonable estimates, filling in blanks and beginning to plot estimates on number lines.
Supports accessibility for: Memory; Organization
Access for English Language Learners

Writing, Speaking: MLR5 Co-Craft Questions. Before students begin work, use this routine to help students interpret the situation before being asked to make statements about the basketball hoop. Display just the photograph for all to see, and arrange students in groups of 2. Invite groups to discuss what they notice and to write mathematical questions they could ask about the situation depicted in the photograph. Once 2–3 pairs share their questions with the class, reveal the rest of the activity.

Design Principle(s): Supporting sense-making

Anticipated Misconceptions

Students may understand that maximum means the largest value but may have trouble with the concept that all other values are therefore less than the maximum. A similar misunderstanding may occur with the concept of minimum value. For these students, have a short conversation about their maximum and minimum height estimates and how those values restrict the possible heights of the hoop.

Student Task Statement

Here is a picture of a person and a basketball hoop. Based on the picture, what do you think are reasonable estimates for the maximum and minimum heights of the basketball hoop?

a. Complete the first blank in each sentence with an estimate, and the second blank with “taller” or “shorter.”

i. I estimate the minimum height of the basketball hoop to be _________ feet; this means the hoop cannot be _________ than this height.

ii. I estimate the maximum height of the basketball hoop to be _________ feet; this means the hoop cannot be _________ than this height.
b. Write two inequalities—one to show your estimate for the *minimum* height of the basketball hoop, and another for the *maximum* height. Use an inequality symbol and the variable \( h \) to represent the unknown height.

c. Plot each estimate for minimum or maximum value on a number line.

\[
\text{Minimum:} \quad \text{Maximum:}
\]

d. Suppose a classmate estimated the value of \( h \) to be 19 feet. Does this estimate agree with your inequality for the maximum height? Does it agree with your inequality for the minimum height? Explain or show how you know.

e. Ask a partner for an estimate of \( h \). Record the estimate and check if it agrees with your inequalities for maximum and minimum heights.

**Student Response**

a. i. Estimates vary. Sample response: 6 ft, shorter, because the word “minimum” means the lowest possible value, so the hoop cannot be shorter than 6 ft.

ii. Estimates vary. Sample response: 12 ft, taller, because the word “maximum” means the highest possible value, so the hoop cannot be taller than 12 ft.

b. Answers vary based on the estimates in previous question. Sample responses:

\[
\text{Minimum: } h > 6 \text{ and } h = 6 \quad \text{Maximum: } h < 12 \text{ and } h = 12
\]

c. Answers vary. Sample response:

Minimum:

\[
\text{Maximum:}
\]

d. Answers vary depending on previous question. Sample response: Since 19 ft is greater than 12 ft, this estimate does not agree with my inequality for the maximum height of the hoop. Using the inequality, \( h < 12 \), when \( h = 19 \), the inequality would be \( 19 < 12 \), which is false. Since 19 ft is greater than 6 ft, this answer does agree with my inequality for the minimum height of the hoop. Using the inequality, \( h > 6 \), when \( h = 19 \), the inequality would be \( 19 > 6 \), which is true. Students may also show that 19 falls within or outside of the shaded regions on their number lines.
e. Answers vary. Students should use similar reasoning to what they did in the previous question.

**Are You Ready for More?**

a. Find 3 different numbers that \(a\) could be if \(|a| < 5\). Plot these points on the number line. Then plot as many other possibilities for \(a\) as you can.

b. Find 3 different numbers that \(b\) could be if \(|b| > 3\). Plot these points on the number line. Then plot as many other possibilities for \(b\) as you can.

**Student Response**

a. Answers vary. Sample answers: \(a = 1, a = 0, a = -3\)

b. Answers vary. Sample answers: \(b = 4, b = -4, b = -7\)

**Activity Synthesis**

The discussion should focus on the concepts of minimum and maximum. While students are not yet using the formal definition of “solutions to an inequality,” they are reasoning about whether a value makes an inequality statement true. A student’s estimated minimum is the least value that makes their inequality true. Similarly, their maximum is the greatest value that makes their inequality true.

Ask students to share their responses to question 5. Select one group to share an estimate that makes a particular inequality true and an estimate that makes the same inequality false. Ask students to describe the values that make both inequalities true at the same time. Record their responses and display for all to see. Include both inequality symbols and number line representations.

**Lesson Synthesis**

Ask students to think about situations where a quantity might have a maximum or minimum value (for example, the number of students in a class, the pounds of fruit purchased by the school cafeteria, the budget for a trip). Ask students to create a variable and represent the situation with an inequality statement using their variable. Ask them whether the maximum or minimum is included in the range of possible values of the variable (for example, does the number of students in a class have to be less than 30, or can it also be equal to 30?). Then ask them to graph the
possible values on a number line. Invite selected students to share their situations, inequalities, and graphs.

8.4 A Box of Paper Clips

Cool Down: 5 minutes

Addressing

6.EE.B.6

Student Task Statement

Andre looks at a box of paper clips. He says: “I think the number of paper clips in the box is less than 1,000.”

Lin also looks at the box. She says: “I think the number of paper clips in the box is more than 500.”

a. Write an inequality to show Andre’s statement, using \( p \) for the number of paper clips.

b. Write another inequality to show Lin’s statement, also using \( p \) for the number of paper clips.

c. Do you think both Lin and Andre would agree that there could be 487 paperclips in the box? Explain your reasoning.

d. Do you think both Lin and Andre would agree that there could be 742 paperclips in the box? Explain your reasoning.

Student Response

a. \( p < 1,000 \)

b. \( p > 500 \)

c. No. Andre would agree because the inequality, \( 487 < 1,000 \) is a true statement. However, Lin would not agree because the inequality \( 487 > 500 \) is a false statement.

d. Yes. Both inequalities are true for 742 paper clips: \( 742 < 1,000 \) and \( 742 > 500 \). This means that according to Lin and Andre, there could be 742 paperclips in the box.

Student Lesson Summary

An inequality tells us that one value is less than or greater than another value.

Suppose we knew the temperature is less than \( 3^\circ F \), but we don’t know exactly what it is. To represent what we know about the temperature \( t \) in \( ^\circ F \) we can write the inequality:

\[ t < 3 \]
The temperature can also be graphed on a number line. Any point to the left of 3 is a possible value for $t$. The open circle at 3 means that $t$ cannot be equal to 3, because the temperature is less than 3.

Here is another example. Suppose a young traveler has to be at least 16 years old to fly on an airplane without an accompanying adult.

If $a$ represents the age of the traveler, any number greater than 16 is a possible value for $a$, and 16 itself is also a possible value of $a$. We can show this on a number line by drawing a closed circle at 16 to show that it meets the requirement (a 16-year-old person can travel alone). From there, we draw a line that points to the right.

We can also write an inequality and equation to show possible values for $a$:

\[
\begin{align*}
    a &> 16 \\
    a &= 16
\end{align*}
\]
Lesson 8 Practice Problems

Problem 1

Statement
At the book sale, all books cost less than $5.

i. What is the most expensive a book could be?

ii. Write an inequality to represent costs of books at the sale.

iii. Draw a number line to represent the inequality.

Solution
i. $4.99

ii. Answer varies. Sample response: If $p$ is the price of a book, then $p < 5$.

iii. The number line has an open circle and an arrow drawn to the left starting with 5.

Problem 2

Statement
Kiran started his homework before 7:00 p.m. and finished his homework after 8:00 p.m. Let $h$ represent the number of hours Kiran worked on his homework.

Decide if each statement it is definitely true, definitely not true, or possibly true. Explain your reasoning.

i. $h > 1$

ii. $h > 2$

iii. $h < 1$

iv. $h < 2$

Solution
i. Definitely true. Kiran worked from 7:00 until 8:00 and some additional time.

ii. Possibly true. It is true if Kiran started his homework at 6:15 and stopped at 8:30. It could also be false if Kiran started his work at 6:45 and finished at 8:15.

iii. Definitely false. $h > 1$ is true.

iv. Possibly true. $h > 2$ might be true, but it might also be false.
Problem 3

**Statement**

Consider a rectangular prism with length 4 and width and height \( d \).

i. Find an expression for the volume of the prism in terms of \( d \).

ii. Compute the volume of the prism when \( d = 1 \), when \( d = 2 \), and when \( d = \frac{1}{2} \).

**Solution**

i. \( 4d^2 \)

ii. When \( d = 1 \), the volume is 4. When \( d = 2 \), the volume is 16. When \( d = \frac{1}{2} \), the volume is 1.

(From Unit 6, Lesson 14.)

Problem 4

**Statement**

Match the statements written in English with the mathematical statements. All of these statements are true.

A. The number -15 is further away from 0 than the number -12 on the number line.

B. The number -12 is a distance of 12 units away from 0 on the number line.

C. The distance between -12 and 0 on the number line is greater than -15.

D. The numbers 12 and -12 are the same distance away from 0 on the number line.

E. The number -15 is less than the number -12.

F. The number 12 is greater than the number -12.

1. \( |-12| > -15 \)

2. \( -15 < -12 \)

3. \( |-15| > |-12| \)

4. \( |-12| = 12 \)

5. \( 12 > -12 \)

6. \( |12| = |-12| \)
Solution

- A: 3
- B: 4
- C: 1
- D: 6
- E: 2
- F: 5

(From Unit 7, Lesson 7.)

Problem 5

Statement

Here are five sums. Use the distributive property to write each sum as a product with two factors.

i. \(2a + 7a\)

ii. \(5z - 10\)

iii. \(c - 2cd\)

iv. \(r + r + r + r\)

v. \(2x - \frac{1}{2}\)

Solution

Answers vary. Sample responses:

i. \((2 + 7)a\) or \(9a\)

ii. \(5(z - 2)\)

iii. \(c(1 - 2d)\)

iv. \((1 + 1 + 1 + 1)r\) or \(4r\)

v. \(2(x - \frac{1}{4})\) or \(\frac{1}{2}(4x - 1)\)

(From Unit 6, Lesson 11.)
Lesson 9: Solutions of Inequalities

Goals

- Draw and label a number line diagram to represent the solutions to an inequality.
- Recognize and explain (orally and in writing) that an inequality may have infinitely many solutions.
- Use substitution to justify (orally) whether a given value is a “solution” to a given inequality.

Learning Targets

- I can determine if a particular number is a solution to an inequality.
- I can explain what it means for a number to be a solution to an inequality.
- I can graph the solutions to an inequality on a number line.

Lesson Narrative

In this lesson, students consider situations where there might be more than one condition. Students have already learned “solution to an equation” to mean a value of the variable that makes the equation true. Here, they learn a similar definition about inequalities: a solution to an inequality is a value of the variable that makes the inequality true. But while the equations students solved in the last unit generally had one solution, the inequalities they solve in this unit have many, sometimes infinitely many, solutions.

Constraints in real-world situations reduce the range of possible solutions. Students reason abstractly by using inequalities or graphs of inequalities to represent those situations and interpreting the solutions, (MP2). Students think carefully about whether to include boundary values as solutions of inequalities in various contexts.

Alignments

Addressing

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- 6.NS.C.7.a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a number line oriented from left to right.
Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

- Colored pencils
- Instructional master
- Pre-printed slips, cut from copies of the

Required Preparation

The included Instructional master is for the optional activity, “What Number Am I?” Print and cut up slips from the Instructional master. Prepare 1 set of inequalities and 1 set of numbers for each group of 4 students. Colored pencils are only needed for an “Are You Ready for More” problem.

Student Learning Goals

Let’s think about the solutions to inequalities.

9.1 Unknowns on a Number Line

Warm Up: 10 minutes

The purpose of this warm-up is for students to compare and name values on a number line based on their relative position to one another and 0. Students also review completing inequality statements based on their comparisons.

Since there are many ways to make each inequality true, it may not be possible for students to share all of the possible ways due to time. Consider sharing 2 possibilities for each before moving on to the next question.

Addressing

- 6.NS.C.7.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time. Tell students there are many possible answers for the questions. Give students 1 minute to compare their responses to their partners and decide if they are both correct, even if they are different. Follow with whole-class discussion.

Student Task Statement

The number line shows several points, each labeled with a letter.
a. Fill in each blank with a letter so that the inequality statements are true.
   
   i. _____ > _____
   
   ii. _____ < _____

b. Jada says that she found three different ways to complete the first question correctly. Do you think this is possible? Explain your reasoning.

c. List a possible value for each letter on the number line based on its location.

**Student Response**

a.  
   i. Answers vary. The first point should be to the right of the second point. Sample response: $B > A$

   ii. Answers vary. The first point should be to the left of the second point. Sample response: $A < B$

b. Jada is correct, there are many ways to answer question 1 correctly. Here are 3 possible answers, though there are more: $B > A, C > B, C > A$.

c. Answers vary. Sample response: $A = -11, B = -5, C = -2, D = 0, E = 5, F = 9$

**Activity Synthesis**

Ask students to share their responses and explanations for each question. Record and display their responses for all to see. If possible, as students share, record their reasoning directly on the displayed number line and reference the points’ locations.

If there is time after students share the possible values of each point in the last question, ask students how they could complete the inequality: __+__>__. Ask students to share their responses and explanations for how they know the inequality is true based on their assigned values for each point.

**9.2 Amusement Park Rides**

25 minutes

The purpose of this activity is for students to represent situations with inequalities and investigate whether values are solutions to multiple inequalities at the same time. Students are formally introduced to the term solution to an inequality and are given the opportunity to use it precisely during discussion (MP6). A solution to an inequality is a value of the variable that makes the inequality true. Students explore these ideas using given height restrictions for a variety of amusement park rides. Students represent the height restrictions as inequality statements and
graph those inequalities on the number line. Students reason abstractly when determining whether a value is a solution to one or more of the inequalities and what that means in context (MP2).

Question 3 will likely lead to a discussion of whether or not the endpoints of the inequality are included. As students work, take note that students’ inequalities and graphs should match their reasoning on the inclusion or exclusion of the endpoints 55 and 72. Monitor for one student who included 55 and 72 as a solution and one student who did not.

**Addressing**
- 6.EE.B.5
- 6.EE.B.8

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

**Launch**
Prior to beginning this activity, remind students that in their previous work with inequalities, they considered which values made an inequality true and which values did not. Introduce the more formal definition of solution here by using students’ previous work with solutions to equations as a starting point. Just as a solution to an equation was a value of the variable that made the equation true, a **solution to an inequality** is a value of the variable that makes the inequality true. But while the equations students solved in the last unit generally had one solution, inequalities have many, sometimes infinitely many, solutions.

Arrange students in groups of 2. Give students 5 minutes of quiet think time for questions 1 and 2. Pause after question 2 to tell students to work with their partner for questions 3 through 6. Tell students that if there is disagreement, work to reach agreement. Give students 5 minutes of work time. Follow with a whole-class discussion.

**Student Task Statement**

Priya finds these height requirements for some of the rides at an amusement park.

<table>
<thead>
<tr>
<th>To ride the . . .</th>
<th>you must be . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Bounce</td>
<td>between 55 and 72 inches tall</td>
</tr>
<tr>
<td>Climb-A-Thon</td>
<td>under 60 inches tall</td>
</tr>
<tr>
<td>Twirl-O-Coaster</td>
<td>58 inches minimum</td>
</tr>
</tbody>
</table>

a. Write an inequality for each of the three height requirements. Use $h$ for the unknown height. Then, represent each height requirement on a number line.

■ High Bounce
b. Han’s cousin is 55 inches tall. Han doesn’t think she is tall enough to ride the High Bounce, but Kiran believes that she is tall enough. Do you agree with Han or Kiran? Be prepared to explain your reasoning.

c. Priya can ride the Climb-A-Thon, but she cannot ride the High Bounce or the Twirl-O-Coaster. Which, if any, of the following could be Priya’s height? Be prepared to explain your reasoning.

- 59 inches
- 53 inches
- 56 inches

d. Jada is 56 inches tall. Which rides can she go on?

e. Kiran is 60 inches tall. Which rides can he go on?

f. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, of another ride. Write three values that are solutions to both of these inequalities.

### Student Response

a. High Bounce: $h > 55$ and $h < 72$. Students may also interpret the height description to include $h = 55$ and $h = 72$. This is a point to discuss. Optional notation for that interpretation: $h \geq 55$ and $h \leq 72$. Additionally, students may write: $55 \leq h \leq 72$ though this is not required notation. Climb-A-Thon: $h < 60$ Twirl-O-Coaster: $h > 58$ or $h = 58$.

High Bounce:

(Some students may opt to shade in 55 and 72.)

Climb-A-Thon:

Twirl-O-Coaster:
b. Answers vary due to ambiguity of the stated restriction. If one interprets the restriction to include the endpoints, then Han’s cousin will be able to go on the High Bounce. If one believes the endpoints are not included, then Han's cousin cannot go on this ride.

c. 53 inches. Explanations vary. Sample response: The lowest restriction is for the High Bounce, which Priya cannot ride. Therefore, Priya’s height is less than 55 inches. The only choice that meets this criteria is 53.

d. High Bounce and Climb-A-Thon. Explanations vary. Sample response: Jada can go on the High Bounce because $56 > 55$ and $56 < 72$ are both true. Jada can go on the Climb-A-Thon because $56 < 60$ is true. Jada cannot go on the Twirl-O-Coaster because $56 > 58$ is false.

e. High Bounce and Twirl-O-Coaster. Kiran can go on the High Bounce because $60 > 55$ and $60 < 72$ are both true. Kiran can also go on the Twirl-O-Coaster because $60 > 58$ is true. Kiran cannot go on the Climb-A-Thon because $60 < 60$ is false.

f. Answers vary. Three sample responses: 65.5, 65, and 74 inches. Any heights between 64 and 75 inches, not including 64 and 75, are possible responses.

**Are You Ready for More?**

a. Represent the height restrictions for all three rides on a single number line, using a different color for each ride.

---

b. Which part of the number line is shaded with all 3 colors?

---

c. Name one possible height a person could be in order to go on all three rides.

**Student Response**

a. High Bounce: Number line with open circles at 55 and 72. Shaded in between these circles. (Note to teacher: students may opt to shade in 55 and 72.)

Climb-A-Thon: Number line with open circle at 60 and shaded to the left. Bold left arrow.

Twirl-O-Coaster: Number line with closed circle at 58 and shaded to the right. Bold right arrow.

b. The space between 58 and 60, not including 58 and 60.

c. Answers vary. One possible response is: 59 inches. This is the only whole number response possible, but other numbers, such as $59\frac{1}{4}$ are possible.

**Activity Synthesis**

The discussion should include these topics:

a. What does it mean to be a solution to an inequality? (A value of the variable that makes the statement true.)
b. What are some ways to find solutions to an inequality? (A substitution can be used to check if values make the inequality true, or the inequality can be graphed on a number line and the value checked against the shaded part.)

c. How many solutions are there to the inequality for the high bounce? In other words, how many possible heights are there that are allowed? (There are infinitely many solutions because 58.1, 58.01, 58.001, 58.0001, etc. are all allowable heights in inches.)

d. Continue the discussion started in the last lesson on the meaning of an open circle vs. a closed circle on the number line. This will come up when students are writing inequalities for the High Bounce ride and then trying to determine if Han's cousin can go on this ride. There is ambiguity in the language “between x and y,” so both interpretations should be included in the discussion. Take time to make sure students' inequalities and number lines match their reasoning. Select previously identified students to share their graphs of the height restrictions for the High Bounce ride.

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**Access for Students with Disabilities**

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: inequality, solution to an inequality.

*SUPPORTS ACCESSIBILITY FOR:* Conceptual processing; Language; Memory

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**Access for English Language Learners**

*Writing, Speaking, Listening: MLR1 Stronger and Clearer.* To help students strengthen their understanding of solutions to inequalities, ask students to write a response to the prompt “What does it mean to be a solution to an inequality?” and ask them to provide an example of an inequality and a solution to it. Have students read their writing to a partner. Partner can ask clarifying questions as well as confirm that their partner has written an inequality and has a correct solution. Have students share with two partners, reminding students to capture new ideas and language after each time they share. For students needing extra support, provide a sentence frame such as “If a number is a solution to an inequality, it means that ____.”

*DESIGN PRINCIPLES:* Optimize output (for explanation); Cultivate conversation

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**9.3 What Number Am I?**

*Optional: 15 minutes*

This activity is optional due to time considerations. The purpose of this activity is for students to reason about whether given values make an inequality true and justify their answers using inequality statements and graphs (MP3). Students explored this concept in the previous activity, so
they should work to articulate how they check if a number is a solution to an inequality statement and an inequality on a number line.

Students will play a game to practice this skill. The game’s goal is for one student to guess a mystery number using as few inequalities as possible. To create a class competition, keep track of how many inequalities each group uses for each number and display scores after each round. The winning group is the group with the lowest score.

**Addressing**
- 6.EE.B.5

**Instructional Routines**
- MLR8: Discussion Supports

**Launch**
Arrange students in groups of 4. As students read the game instructions with their group, give each group 1 set of inequalities and 1 set of numbers, pre-cut from the Instructional master. Review the game instructions as a whole class. Students will play 1 round and then pause to reflect and plan strategies for the next rounds. Allow students 10 minutes of game time followed by a whole-class discussion.

Once student groups have completed one round of game play, pause the game and ask groups to reflect on their strategies. Groups should make a plan before continuing to the second round of the game. Post or ask these questions to guide a short discussion:

- “Clue givers, how did you decide which inequalities would be the most helpful for your detective?”
- “Clue givers, did you work together to decide which 3 clues to give or did you decide independently?”
- “Detective, were some inequalities more helpful than others as you tried to guess the mystery number? If so, what made an inequality more helpful?”

**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Begin with a small-group or whole-class demonstration of how to play the game. Check for understanding by inviting students to rephrase directions in their own words.

*Supports accessibility for: Memory; Conceptual processing*

**Anticipated Misconceptions**
Some groups may misunderstand the directions, thinking that each person giving clues is supposed to take a different number card from the stack. Explain to them that there is only one unknown
number per round and everyone gives clues about this same number. When a new person becomes the detective, that is when a new number card is drawn.

**Student Task Statement**

Your teacher will give your group two sets of cards—one set shows inequalities and the other shows numbers. Place the inequality cards face up where everyone can see them. Shuffle the number cards and stack them face down.

To play:

- One person in your group is the detective. The other people will give clues.
- Pick one number card from the stack and show it to everyone except the detective.
- The people giving clues each choose an inequality that will help the detective identify the unknown number.
- The detective studies the inequalities and makes three guesses.
  - If the detective does not guess the right number, each person chooses another inequality to help.
  - When the detective does guess the right number, a new person becomes the detective.
- Repeat the game until everyone has had a turn being the detective.

**Student Response**

Responses vary depending on what values are randomly chosen.

**Activity Synthesis**

Once all students have had a turn as the detective, start a whole-class discussion. Ask groups to share their most successful strategies for choosing helpful clue inequalities as well as strategies for using those clues to guess the mystery number. Emphasize how students can check that a value is or is not a solution to an inequality: both symbolic statements (substitute the value and check that the resulting statement is true) and on the number line (plot the value to test and make sure that it falls within a shaded region).

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* To support all students to participate in the discussion, provide students with sentence frames, such as “One strategy I used to choose a clue was _____” or “One way that I checked that a value was a solution to the inequality (number line representation) was to ____.” Ask them to complete one of the sentences in writing or share a completed response verbally with a partner.

*Design Principle(s): Maximize meta-awareness*
Lesson Synthesis

Ask students to think about situations where a quantity can take on a range of values (for example, the ages of students eligible for a certain program, the salary range that applies to a particular tax rate, the speed you can drive on the highway). Ask students to decide on a variable and represent their situation with two inequality statements. Ask whether the maximum and minimum are included in the range of possible values of the variable (for example, can your maximum speed on a highway be equal to 65 miles per hour or does it have to be less than 65?). Then ask them to graph the solutions on one or two number lines. Invite selected students to share their situations, inequalities, and graphs.

9.4 Solutions of Inequalities

Cool Down: 5 minutes

Addressing

- 6.EE.B.5
- 6.EE.B.8

Student Task Statement

a. i. Select all numbers that are solutions to the inequality \( w < 1 \).
   - 5
   - -5
   - 0
   - 0.9
   - -1.3
   ii. Draw a number line to represent this inequality.

b. i. Write an inequality for which 3, -4, 0, and 2,300 are solutions.
ii. How many total solutions are there to your inequality?

Student Response

a. i. -5, 0, 0.9, -1.3
ii. 

b. i. Answers vary. One possible response \( x > -5 \).
ii. There are infinitely many solutions.
Let's say a movie ticket costs less than $10. If \( c \) represents the cost of a movie ticket, we can use \( c < 10 \) to express what we know about the cost of a ticket.

Any value of \( c \) that makes the inequality true is called a solution to the inequality. For example, 5 is a solution to the inequality \( c < 10 \) because 5 < 10 (or “5 is less than 10”) is a true statement, but 12 is not a solution because 12 < 10 (“12 is less than 10”) is not a true statement.

If a situation involves more than one boundary or limit, we will need more than one inequality to express it. For example, if we knew that it rained for more than 10 minutes but less than 30 minutes, we can describe the number of minutes that it rained (\( r \)) with the following inequalities and number lines.

\[
\begin{align*}
r &> 10 \\
r &< 30
\end{align*}
\]

Any number of minutes greater than 10 is a solution to \( r > 10 \), and any number less than 30 is a solution to \( r < 30 \). But to meet the condition of “more than 10 but less than 30,” the solutions are limited to the numbers between 10 and 30 minutes, not including 10 and 30.

We can show the solutions visually by graphing the two inequalities on one number line.

**Glossary**

- solution to an inequality
Lesson 9 Practice Problems

Problem 1

Statement
   i. Select all numbers that are solutions to the inequality $k > 5$.
      
      \[
      4 \quad 5 \quad 6 \quad 5.2 \quad 5.01 \quad 0.5
      \]
      
   ii. Draw a number line to represent this inequality.

Solution
   i. 6, 5.2, 5.01
   ii. The number line should show an open circle above the number 5 and an arrow pointing to the right.

Problem 2

Statement
   A sign on the road says: “Speed limit, 60 miles per hour.”
   
   i. Let $s$ be the speed of a car. Write an inequality that matches the information on the sign.
   ii. Draw a number line to represent the solutions to the inequality.
   iii. Could 60 be a value of $s$? Explain your reasoning.

Solution
   i. $s < 60$ or $s = 60$, or equivalent
   ii. The number line should show a closed circle at 60 and an arrow pointing to the left.
   iii. Yes, 60 is the limit.

Problem 3

Statement
   One day in Boston, MA, the high temperature was 60 degrees Fahrenheit, and the low temperature was 52 degrees.
   
   i. Write one or more inequalities to describe the temperatures $T$ that are between the high and low temperature on that day.
   ii. Show the possible temperatures on a number line.
Solution

i. $52 < T$ and $T < 60$ or equivalent

ii. A graph showing empty circles at 52 and 60 and all of the numbers between.

Problem 4

Statement
Select all the true statements.

A. $-5 < |-5|$
B. $|-6| < -5$
C. $|-6| < 3$
D. $4 < |-7|$
E. $|-7| < |-8|$

Solution

["A", "D", "E"]
(From Unit 7, Lesson 7.)

Problem 5

Statement
Match each equation to its solution.

i. $x^4 = 81$  ■ 2
ii. $x^2 = 100$  ■ 3
iii. $x^3 = 64$  ■ 4
iv. $x^5 = 32$  ■ 10

Solution

i. 3
ii. 10
iii. 4
iv. 2

(From Unit 6, Lesson 15.)
Problem 6

Statement

i. The price of a cell phone is usually $250. Elena’s mom buys one of these cell phones for $
150. What percentage of the usual price did she pay?

ii. Elena’s dad buys another type of cell phone that also usually sells for $250. He pays 75% of
the usual price. How much did he pay?

Solution

i. 60%

ii. $187.50

Sample reasoning:

<table>
<thead>
<tr>
<th>number</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td>62.5</td>
<td>25</td>
</tr>
<tr>
<td>187.5</td>
<td>75</td>
</tr>
</tbody>
</table>

(From Unit 3, Lesson 14.)
Lesson 10: Interpreting Inequalities

Goals

◦ Critique (orally and in writing) possible values given for a situation with more than one constraint, including whether fractional or negative values are reasonable.

◦ Interpret unbalanced hanger diagrams (orally and in writing) and write inequality statements to represent relationships between the weights on an unbalanced hanger diagram.

◦ Write and interpret inequality statements that include more than one variable.

Learning Targets

◦ I can explain what the solution to an inequality means in a situation.

◦ I can write inequalities that involves more than one variable.

Lesson Narrative

In this final lesson on inequalities, students explore situations in which some of the solutions to inequalities do not make sense in the situation's context. Students learn to think carefully about a situation's constraints when coming up with reasonable solutions to an inequality. Students also see that inequalities can represent a comparison of two or more unknown quantities.

Alignments

Addressing

◦ 6.EE.A.2.b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

◦ 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

◦ 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

◦ 6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Instructional Routines

◦ MLR3: Clarify, Critique, Correct
10.1 True or False: Fractions and Decimals

Warm Up: 5 minutes
The purpose of this warm-up is to encourage students to reason about properties of operations in equivalent expressions. While students may evaluate each side of the equation to determine if it is true or false, encourage students to think about the properties of arithmetic operations in their reasoning (MP7).

Addressing
- 6.EE.A.2.b

Launch
Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 1 minute of quiet think time and follow with a whole-class discussion.

Student Task Statement
Is each equation true or false? Be prepared to explain your reasoning.

a. $3(12 + 5) = (3 \cdot 12) \cdot (3 \cdot 5)$

b. $\frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4} \cdot \frac{2}{6}$

c. $2 \cdot (1.5) \cdot 12 = 4 \cdot (0.75) \cdot 6$

Student Response
a. False. Possible responses: 51 does not equal 540; When we use the distributive property, the 2 terms on the right side of the equation should be added not multiplied.

b. True. Possible responses: Both sides equal $\frac{1}{4}$; Since $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, we can see that the left side and right side of this equation show the same numbers being multiplied just in a different order. By the commutative property of multiplication, both sides of this equation are equal.

c. False. Possible responses: 36 does not equal 18; If we change the order and the groupings of the numbers on both sides using the commutative and associative properties of
multiplication, we can group the whole numbers together like this:

\((2 \cdot 12) \cdot 1.5 = (4 \cdot 6) \cdot 0.75\), so that the equation becomes: \(24 \cdot 1.5 = 24 \cdot 0.75\). Since 1.5 does not equal 0.75, we see that the two sides of this equation are not equal.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. After each true equation, ask students if they could rely on that same reasoning to think about or solve other problems that are similar in type. After each false equation, ask students how we could make the equation true.

To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s reasoning?”

10.2 Basketball Game

15 minutes

Students interpret inequalities that represent constraints or conditions in a real-world problem. They find solutions to an inequality and reason about the context’s limitations on solutions (MP2).

Addressing

- 6.EE.B.5
- 6.EE.B.8

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Allow students 10 minutes quiet work time to complete all questions followed by whole-class discussion.

Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Provide students with access to blank number lines. Encourage students to attempt more than one strategy for at least one of the problems.

*Supports accessibility for: Visual-spatial processing; Organization*
Anticipated Misconceptions

Students might have trouble interpreting \( 15 < n \) because of the placement of the variable on the right side of the inequality. Encourage students to reason about the possible values of \( n \) that would make this inequality true.

Student Task Statement

Noah scored \( n \) points in a basketball game.

a. What does \( 15 < n \) mean in the context of the basketball game?

b. What does \( n < 25 \) mean in the context of the basketball game?

c. Draw two number lines to represent the solutions to the two inequalities.

d. Name a possible value for \( n \) that is a solution to both inequalities.

e. Name a possible value for \( n \) that is a solution to \( 15 < n \), but not a solution to \( n < 25 \).

f. Can \( -8 \) be a solution to \( n \) in this context? Explain your reasoning.

Student Response

a. Noah scored more than 15 points.

b. Noah scored less than 25 points.

c. \( n > 15 \): open circle at 15, shaded line and arrow to the right, \( n < 25 \): open circle at 25, shaded line and arrow to the left.

d. Answers vary. Sample response: \( n = 17 \).

e. Answers vary. Sample response: \( n = 30 \).

f. No, \( -8 \) cannot be a solution in this context because the score of a basketball game cannot be below 0.

Activity Synthesis

Invite selected students to justify their answers. Extend the discussion of the basketball game to consider how scoring works and whether any number could represent the points scored by a player. For example, could a player have scored 1 point? \( 2 \frac{1}{2} \) points? 0 points? -3 points? Is it reasonable for a player to score 200 points in a game?
Access for English Language Learners

Reading, Speaking, Representing: MLR3 Clarify, Critique, Correct. To support students in their ability to read inequalities and their number line representations as well as to critique the reasoning of others, present an incorrect response to the prompt “Draw two number lines that represent the two inequalities.” For example, use the first number line representing $15 < n$ to contain an error by shading the line and arrow to the left instead of the right. Then correctly display the second number line. Tell students the response is not drawn correctly even though the numbers and open circles are correct. Ask students to identify the error, explain how they know that the response is incorrect, and revise the incorrect number line. This helps prompt student reflection with an incorrect written mathematical statement, and for students to improve upon the written work by correcting errors and clarifying meaning.

Design Principle(s): Optimize output (for explanation)

10.3 Unbalanced Hangers

15 minutes
In this activity, students describe unbalanced hanger diagrams with inequalities. Students construct viable arguments and critique the reasoning of others during partner and whole-class discussions about how unknown values relate to each other (MP3).

Addressing
○ 6.EE.B.6
○ 6.EE.B.8

Instructional Routines
○ MLR8: Discussion Supports
○ Think Pair Share

Launch
Arrange students in groups of 2. Give students 7 minutes quiet work time, followed by 3–5 minutes for partner discussion. Tell students to check in with their partners and, if there are disagreements, work to come to an agreement. Follow with whole-class discussion.
**Student Task Statement**

a. Here is a diagram of an unbalanced hanger.

i. Jada says that the weight of one circle is greater than the weight of one pentagon. Write an inequality to represent her statement. Let $p$ be the weight of one pentagon and $c$ be the weight of one circle.

ii. A circle weighs 12 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

b. Here is another diagram of an unbalanced hanger.

i. Write an inequality to represent the relationship of the weights. Let $p$ be the weight of one pentagon and $s$ be the weight of one square.

ii. One pentagon weighs 8 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

iii. Graph the solutions to this inequality on a number line.

c. Based on your work so far, can you tell the relationship between the weight of a square and the weight of a circle? If so, write an inequality to represent that relationship. If not, explain your reasoning.

d. This is another diagram of an unbalanced hanger.
Andre writes the following inequality: \( c + p < s \). Do you agree with his inequality? Explain your reasoning.

Student Response

a. i. \( p < c \)

ii. \( p < 12 \). This means that the pentagon weighs less than 12 ounces.

b. i. \( s < p \)

ii. \( s < 8 \). This means that the square weighs less than 8 ounces.

iii. On the graph, there is an open circle at 8 and all values less than 8 are shaded.

c. Yes, the square weighs less than the circle, so \( s < c \). Explanations vary. Sample response: The square weighs less than the pentagon, which in turn weighs less than the circle. So the square weighs less than the circle.

d. No. Explanations vary. Sample response: The combined weight of the circle and the pentagon is greater than the weight of the square, so the inequality should be \( c + p > s \).

e. The diagram should show a circle and a square on one side and two triangles on the other. The side with the circle and square outweighs the side with the triangles so it hangs lower.

Are You Ready for More?

Here is a picture of a balanced hanger. It shows that the total weight of the three triangles is the same as the total weight of the four squares.

a. What does this tell you about the weight of one square when compared to one triangle? Explain how you know.

b. Write an equation or an inequality to describe the relationship between the weight of a square and that of a triangle. Let \( s \) be the weight of a square and \( t \) be the weight of a triangle.
**Student Response**

1. The weight of a square is less than the weight of a triangle, or the weight of a triangle is more than the weight of a square. Sample reasoning: It takes 4 squares to weigh the same as 3 triangles, so each square must be lighter than each triangle.

2. \( s < t; t > s; \) or \( 3t = 4s. \)

**Activity Synthesis**

The purpose of the discussion is to let students explain how they used inequalities to compare the weights of different shapes on the hanger diagrams. Invite groups to describe any disagreements or difficulties they had and how they resolved them. Select students to share how they reasoned about the quantities when there were two or more unknowns. Ask students if they can think of other situations comparing two or more unknown quantities (people’s heights, weights of backpacks). Invite them to represent the quantities with variables and write inequality statements to compare them.

If time allows, display a circle opposite a pentagon and square for all to see. Ask students which side they think would be heavier. In this case, which side is heavier depends on how much the square weighs. Since the circle is 12 ounces and the pentagon is 8 ounces, the square would have to be less than 4 ounces for the circle to be heavier and greater than 4 ounces for the pentagon and square to be heavier.

---

**Access for English Language Learners**

*Representing, Speaking: MLR8 Discussion Supports.* To help students be more precise in their use of language related representations of inequalities, use the sentence frames to support their discussion. Some examples include “I know the circle is heavier because _____,” “The inequality _____ represents the hanger because _____,” or “If the circle weighs 12 ounces, I know ____.”

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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**Lesson Synthesis**

Ask students to think about situations where limits or ranges of values can be important to public health or safety (e.g., weight limitations on an elevator, safe dosage for medication, tire pressure, speed limit, temperature for growing carrots, etc.). Ask them to define variables and write inequalities to represent these situations. Select 2 or 3 students to share their responses. Record and display those responses for all to see using the appropriate symbols. Here are some questions to consider during discussion:

- “Do solutions that are not whole numbers make sense in this situation?”
- “Do solutions that are negative numbers make sense in this situation?”

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Unit 7 Lesson 10
“Do the numbers on the boundary count as solutions? For example, if an elevator has a maximum capacity of 2,500 pounds, can it handle exactly 2,500 pounds?”

10.4 Lin and Andre’s Heights

Cool Down: 5 minutes

Addressing
- 6.EE.B.6
- 6.EE.B.8

Student Task Statement

a. Lin says that the inequalities \( h > 150 \) and \( h < 160 \) describe her height in centimeters. What do the inequalities tell us about her height?

b. Andre notices that he is a little taller than Lin but is shorter than their math teacher, who is 164 centimeters tall. Write two inequalities to describe Andre’s height. Let \( a \) be Andre’s height in centimeters.

c. Select all heights that could be Andre’s height in centimeters. If you get stuck, consider drawing a number line to help you.

   i. 150
   ii. 154.5
   iii. 160
   iv. 162.5
   v. 164

Student Response

a. These inequalities tell us that Lin is between 150 and 160 cm tall.

b. \( a < 164 \) and \( a > h \) (or \( h < a \)).

c. B, C, D

Student Lesson Summary

When we find the solutions to an inequality, we should think about its context carefully. A number may be a solution to an inequality outside of a context, but may not make sense when considered in context.

- Suppose a basketball player scored more than 11 points in a game, and we represent the number of points she scored, \( s \), with the inequality \( s > 11 \). By looking only
at \( s > 11 \), we can say that numbers such as 12, \( 14 \frac{1}{2} \), and 130.25 are all solutions to the inequality because they each make the inequality true.

\[
12 > 11 \quad 14 \frac{1}{2} > 11 \quad 130.25 > 11
\]

In a basketball game, however, it is only possible to score a whole number of points, so fractional and decimal scores are not possible. It is also highly unlikely that one person would score more than 130 points in a single game.

In other words, the context of an inequality may limit its solutions.

Here is another example:

- The solutions to \( r < 30 \) can include numbers such as \( 27 \frac{3}{4} \), 18.5, 0, and -7. But if \( r \) represents the number of minutes of rain yesterday (and it did rain), then our solutions are limited to positive numbers. Zero or negative number of minutes would not make sense in this context.

To show the upper and lower boundaries, we can write two inequalities:

\[
0 < r \quad r < 30
\]

Inequalities can also represent comparison of two unknown numbers.

- Let’s say we knew that a puppy weighs more than a kitten, but we did not know the weight of either animal. We can represent the weight of the puppy, in pounds, with \( p \) and the weight of the kitten, in pounds, with \( k \), and write this inequality:

\[
p > k
\]
Lesson 10 Practice Problems

Problem 1

Statement
There is a closed carton of eggs in Mai’s refrigerator. The carton contains $e$ eggs and it can hold 12 eggs.

i. What does the inequality $e < 12$ mean in this context?

ii. What does the inequality $e > 0$ mean in this context?

iii. What are some possible values of $e$ that will make both $e < 12$ and $e > 0$ true?

Solution

i. There are fewer than 12 eggs in the carton; the carton is not full.

ii. There are more than 0 eggs in the carton; the carton is not empty.

iii. There could be as few as 1 egg or as many as 11 eggs in the carton: any whole number of eggs from 1 up to 11.

Problem 2

Statement
Here is a diagram of an unbalanced hanger.

Solution

i. Write an inequality to represent the relationship of the weights. Use $s$ to represent the weight of the square in grams and $c$ to represent the weight of the circle in grams.

ii. One red circle weighs 12 grams. Write an inequality to represent the weight of one blue square.

iii. Could 0 be a value of $s$? Explain your reasoning.

Solution

i. $s < c$

ii. $s < 12$

iii. No, 0 could not be a value of $s$ because the square represents an object. It must have some weight, even if it is very small.
Problem 3

Statement

i. Jada is taller than Diego. Diego is 54 inches tall (4 feet, 6 inches). Write an inequality that compares Jada’s height in inches, $j$, to Diego’s height.

ii. Jada is shorter than Elena. Elena is 5 feet tall. Write an inequality that compares Jada’s height in inches, $j$, to Elena’s height.

Solution

i. $j > 54$

ii. $j < 60$

(From Unit 7, Lesson 8.)

Problem 4

Statement

Tyler has more than $10. Elena has more money than Tyler. Mai has more money than Elena. Let $t$ be the amount of money that Tyler has, let $e$ be the amount of money that Elena has, and let $m$ be the amount of money that Mai has. Select all statements that are true:

A. $t < j$

B. $m > 10$

C. $e > 10$

D. $t > 10$

E. $e > m$

F. $t < e$

Solution

["A", "B", "C", "F"]

Problem 5

Statement

Which is greater, $\frac{9}{20}$ or -0.5? Explain how you know. If you get stuck, consider plotting the numbers on a number line.

Unit 7 Lesson 10
Solution

$\frac{9}{20}$ is larger. Explanations vary. Sample explanation: $\frac{9}{20} = -0.45$, and this is to the right of -0.5 on the number line. So, $\frac{9}{20}$ is larger.

(From Unit 7, Lesson 3.)

Problem 6

Statement

Select all the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.

A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

B. $\frac{1}{2^3}$

C. $\left(\frac{1}{3}\right)^2$

D. $\frac{1}{6}$

E. $\frac{1}{8}$

Solution

["A", "B", "E"]

(From Unit 6, Lesson 13.)
Section: The Coordinate Plane
Lesson 11: Points on the Coordinate Plane

Goals
- Generalize about the signs of coordinates that represent locations in each “quadrant” of the coordinate plane.
- Plot a point given its coordinates or identify the coordinates of a given point on the coordinate plane.
- Recognize that the axes of the coordinate plane can be extended to represent negative numbers.

Learning Targets
- I can describe a coordinate plane that has four quadrants.
- I can plot points with negative coordinates in the coordinate plane.
- I know what negative numbers in coordinates tell us.

Lesson Narrative
In earlier lessons, students extended the number line to include negative numbers. In this lesson, students extend the coordinate axes to expand the coordinate plane. In a previous unit, students worked in the coordinate plane when they examined ratio and other relationships between two quantities with positive values. They now consider an expanded coordinate plane where negative numbers appear on both the vertical and horizontal axes. The crossing axes create the four regions of the coordinate plane, called quadrants. In this first lesson on the coordinate plane, students extend their understanding of the coordinate plane to points with negative coordinates. They gain experience by choosing and plotting points in order to hit targets or to maneuver through mazes in all four quadrants of the coordinate plane.

Alignments
Building On
- 5.G.A.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
Addressing

○ 6.NS.C.6.b: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

○ 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

○ 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards

○ 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

○ 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

○ MLR2: Collect and Display

○ MLR8: Discussion Supports

Student Learning Goals

Let’s explore and extend the coordinate plane.

11.1 Guess My Line

Warm Up: 10 minutes

The purpose of this warm-up is for students to review graphing and locating points in the first quadrant of the coordinate plane. Students observe the structure of horizontal and vertical lines when they compare points on the same line and notice which coordinate of the ordered pair changes and why (MP7).

Building On

○ 5.G.A.1

Building Towards

○ 6.NS.C.6

○ 6.NS.C.8
Launch

Arrange students in groups of 2. Display the coordinate plane for all to see and ask, “What do you notice about the plane? What do you wonder?” Invite a few students to share what they notice and wonder until a student has noticed there are no labels on the axes. Ask, “How should we label the axes?” Otherwise, point out that the axes aren’t labeled and ask them how they should be labeled. This lesson plan refers to the axes and coordinates with the standard $x$ and $y$ variables. Invite a student to read the directions for both questions in the task. Once confident that groups understand the directions, give groups 4 minutes to guess each other’s points followed by a whole-class discussion.

Anticipated Misconceptions

Some students may not remember that the first coordinate in an ordered pair corresponds to the horizontal coordinate and the second coordinate in the ordered pair corresponds to the vertical coordinate. Display the ordered pair $(x, y)$ or $(\text{horizontal}, \text{vertical})$ to remind students of the order.

Student Task Statement

a. Choose a horizontal or a vertical line on the grid. Draw 4 points on the line and label each point with its coordinates.

![Coordinate Plane]

b. Tell your partner whether your line is horizontal or vertical, and have your partner guess the locations of your points by naming coordinates.

If a guess is correct, put an X through the point. If your partner guessed a point that is on your line but not the point that you plotted, say, “That point is on my line, but is not one of my points.”

Take turns guessing each other’s points, 3 guesses per turn.
**Student Response**

a. Answers vary. Possible response: Vertical line: (2, 3), (2, 4), (2, 7), (2, 8)

b. Answers vary.

**Activity Synthesis**

The key takeaway of this discussion is that points on the same horizontal line share the same $y$ coordinate and points on the same vertical line share the same $x$ coordinate. Ask 3 or 4 students to share the coordinates of their 4 points. After each student shares, ask the rest of the class if the given points are on the same horizontal or vertical line and to explain how they know. To help guide the conversation, consider asking some of the following questions:

- “How do you know the points fall on the same line?”
- “How do you know the points are on a horizontal or vertical line?”
- “Could you name other points on the same line?”
- “How far is each of the points from one another?”
- “How far is each point from the $x$-axis and $y$-axis?”

**11.2 The Coordinate Plane**

15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to extend the vertical and horizontal axes to include 4 quadrants just as they extended the number line to include negative numbers. Students are introduced to the term **quadrant**. Students plot and label coordinates using ordered pairs and identify their quadrants.

**Addressing**

- 6.NS.C.6.b
- 6.NS.C.6.c

**Instructional Routines**

- MLR8: Discussion Supports

**Launch**

Introduce the concept of the 4-region coordinate plane by explaining that, just like we extended the number line include negative numbers, we can extend both the number lines of the coordinate plane (the axes) to include negative coordinates. Use the word **quadrant** to describe the four regions of the coordinate plane. It may be helpful to explain that the prefix “quad-” means 4 and give other examples from English and other languages that use the prefix (quadriceps, quadrilateral, cuatro). Give students 10 minutes to work followed by whole-class discussion.
Classes using the digital version have an interactive applet to use. Instead of naming the coordinates, students enter them into the Input Bar. If correct, the new points will hit the targets and turn them black.

### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Activate or supply background knowledge. During the launch, take time to review terms that students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of: vertical number line, horizontal number line.

*Supports accessibility for: Conceptual processing; Language*

### Anticipated Misconceptions

Students may confuse the order of the coordinate pairs when plotting points. Direct students to look at their work with vertical and horizontal lines in the warm up to remind them which coordinate is vertical.

Students may locate -4.5 between -3 and -4, rather than between -4 and -5. Direct them to the number lines used in earlier lessons to remind them of locating non integer numbers on number lines.

### Student Task Statement

- a. Label each point on the coordinate plane with an ordered pair.

- b. What do you notice about the locations and ordered pairs of $B$, $C$, and $D$? How are they different from those for point $A$?
c. Plot a point at (-2, 5). Label it $E$. Plot another point at (3, -4.5). Label it $F$.

d. The coordinate plane is divided into four quadrants, I, II, III, and IV, as shown here.

![Coordinate Plane Diagram]

- $G = (5, 2)$
- $H = (-1, -5)$
- $I = (7, -4)$

e. In which quadrant is point $G$ located? Point $H$? Point $I$?

f. A point has a positive $y$-coordinate. In which quadrant could it be?

**Student Response**

a. i. $A = (3, 4)$
   
i. $B = (-4, 2)$
   
ii. $C = (-5, -3)$
   
iv. $D = (4, -2)$

b. Answers vary. Sample response: They all have at least one negative coordinate.

c.

![Graph with Points]

d. i. Point $G$ is in quadrant I, $H$ is in quadrant III, and $I$ is in quadrant IV.
ii. Quadrant I or quadrant II.

**Activity Synthesis**

The most important idea for students to understand is that by extending the two number lines that form the coordinate axes for the first quadrant, we now have 4 quadrants. We describe points in these quadrants using negative and positive numbers as the $x$ and $y$ coordinates. Focus on responses from question 4 for discussion. Invite students to share their reasoning about how to identify the quadrants for the points $G$, $H$, and $I$. As time allows, consider asking the following questions:

- “If a point has a negative $x$-coordinate, what quadrant could it be in?”
- “If a point has a negative $y$-coordinate, what quadrant could it be in?”
- “If a point has a positive $x$-coordinate, what quadrant could it be in?”

To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s reasoning?”

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* To support students in producing statements about features of the quadrants that have negative and/or positive numbers provide sentence frames for students to use such as: “Point ____ is in quadrant ____ because ____.” This will support class discussion in providing low entry points for students to start speaking about their reasoning.

*Design Principle(s): Support sense-making*

---

### 11.3 Coordinated Archery

15 minutes (there is a digital version of this activity)

In this activity, students select and describe points in different regions of the coordinate plane using ordered pairs. Students must name specific coordinates in order to hit different parts of an archery target embedded in a coordinate plane. All points within the archery target contain negative coordinates, allowing students to practice and build on what they were introduced to in the previous activity.

This activity was inspired by one created by Nathan Kraft [https://teacher.desmos.com/activitybuilder/custom/563d705f36a7843710aba2ce](https://teacher.desmos.com/activitybuilder/custom/563d705f36a7843710aba2ce).
Addressing

- 6.NS.C.8

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2. It may be necessary to introduce students to what an archery target looks like and how it is scored. More points are scored the closer to the center the arrow lands. Remind students to label the axes with x and y so that they can accurately describe the coordinates as x-coordinates or y-coordinates. Give students 8 minutes of quiet work time followed by 2 minutes of partner discussion for students to check whether their partner has made valid choices for coordinates. Follow with a whole-class discussion.

Students using the digital activity will be able to plot points using an applet and then determine if their points are in the desired area. To enter points, students enter an ordered pair of coordinates in the table.

Access for English Language Learners

Listening, Representing: MLR2 Collect and Display. As students work, listen for, collect and display vocabulary and phrases students use to describe the coordinates, such as x-coordinate, y-coordinate, and quadrant. Update the list with student language and questions from student discussions that describe where to place the points on the graph. Remind students to borrow language from the display as needed. Teachers can use the list to check for student progress and misunderstandings. This will help students develop mathematical language for the class discussions that describe the location of points.

Design Principle(s): Support sense-making

Student Task Statement

Here is an image of an archery target on a coordinate plane. The scores for landing an arrow in the colored regions are shown.
Name the coordinates for a possible landing point to score:

a. 6 points
b. 10 points
c. 2 points
d. No points
e. 4 points
f. 8 points

**Student Response**

Answers vary. Sample responses:

a. (-2, -4)
b. (-4.5, -4)
c. (0, -3)
d. (-10, -10)
e. (-2, -2)
f. (-3, -4)

**Are You Ready for More?**

Pretend you are stuck in a coordinate plane. You can only take vertical and horizontal steps that are one unit long.
a. How many ways are there to get from the point (-3, 2) to (-1, -1) if you will only step down and to the right?
b. How many ways are there to get from the point (-1, -2) to (4, 0) if you can only step up and to the right?
c. Make up some more problems like this and see what patterns you notice.

Student Response
a. 10 paths
b. 21 paths
c. Answers vary. This problem is mostly an exercise in careful counting, although some students may realize that, for example, the first problem amounts to counting how many ways there are to arrange two steps to the right and three steps down. For some, this may be an easier representation of the problem.

Activity Synthesis
The main goal of discussion is to allow students to describe points in the plane that involve negative coordinates. Display the archery target for all to see. Ask students to share their responses for coordinates in the various regions of the target and record them for all to see. Record the points exactly as students describe them and push students to be precise if there was a mistake. Ask whether it's possible to get even closer to the exact center of the target. If not mentioned by students, suggest using decimals or fractions as coordinates.

Lesson Synthesis
In this lesson, students extended the axes of the coordinate plane to include negative coordinates. Consider asking students the following questions to summarize the main ideas of the lesson:

- What are the names of the quadrants and where are they in the coordinate plane? (The quadrants are called I, II, III, and IV. They go in counterclockwise order from top right to top left to bottom left to bottom right.)
- What quadrant is the point (-4, 5) in? How do you know? (That point is in quadrant II. The $x$-value is negative, so the point is left of the $y$-axis, and the $y$-value is positive so the point is above the $x$-axis.)
- What quadrant is the point (5, -4) in? How do you know? (That point is in quadrant IV. The $x$-value is positive, so the point is right of the $y$-axis, and the $y$-value is negative, so the point is below the $x$-axis.)

If time allows, ask students to make up their own challenges for the class with the targets. Invite other students to pick points that meet the requirement of the challenges. For example, one challenge could be to hit a point that is exactly between two colors. Another might be to hit the target in the bullseye 3 times on a horizontal line.
11.4 Target Practice

Cool Down: 5 minutes

Addressing
○ 6.NS.C.8

Student Task Statement
Here are the scores for landing an arrow in the colored regions of the archery target.

○ Yellow: 10 points
○ Red: 8 points
○ Blue: 6 points
○ Green: 4 points
○ White: 2 points

a. Andre shot three arrows and they landed at (-5, 4), (-8, 7) and (1, 6). What is his total score? Show your reasoning.

b. Jada shot an arrow and scored 10 points. She shot a second arrow that landed directly below the first one but scored only 2 points. Name two coordinates that could be the landing points of her two arrows.

Student Response
a. (-5, 4) is 6 points, (-8, 7) is 8 points, (1, 6) is 0 points. The total is 14 points.

b. Answers vary. Sample responses:
   ■ (-7, 5) and (-7, 1)
   ■ (-6.5, 5.5) and (-6.5, 1.1)

Student Lesson Summary
Just as the number line can be extended to the left to include negative numbers, the $x$- and $y$-axis of a coordinate plane can also be extended to include negative values.
The ordered pair \((x, y)\) can have negative \(x\)- and \(y\)-values. For \(B = (-4, 1)\), the \(x\)-value of -4 tells us that the point is 4 units to the left of the \(y\)-axis. The \(y\)-value of 1 tells us that the point is one unit above the \(x\)-axis.

The same reasoning applies to the points \(A\) and \(C\). The \(x\)- and \(y\)-coordinates for point \(A\) are positive, so \(A\) is to the right of the \(y\)-axis and above the \(x\)-axis. The \(x\)- and \(y\)-coordinates for point \(C\) are negative, so \(C\) is to the left of the \(y\)-axis and below the \(x\)-axis.

**Glossary**

- quadrant
Lesson 11 Practice Problems
Problem 1

**Statement**

i. Graph these points in the coordinate plane: (-2, 3), (2, 3), (-2, -3), (2, -3).

ii. Connect all of the points. Describe the figure.

**Solution**

i. The points $A = (-2, 3)$, $B = (2, 3)$, $C = (-2, -3)$, $D = (2, -3)$ are shown on a coordinate plane with segments between them.

ii. Graph
Problem 2

Statement
Write the coordinates of each point.

Solution
\[ A = (1, 0), \ B = (0, -3), \ C = (-6, 5), \ D = (-3, -5), \ E = (-1, 4) \]

Problem 3

Statement
These three points form a horizontal line: \((-3.5, 4), (0, 4), \text{ and } (6.2, 4)\). Name two additional points that fall on this line.
Solution
Answers vary. Any answer that has a \( y \)-coordinate of 4 is on the line.

**Problem 4**

**Statement**
One night, it is 24°C warmer in Tucson than it was in Minneapolis. If the temperatures in Tucson and Minneapolis are opposites, what is the temperature in Tucson?

A. -24°C  
B. -12°C  
C. 12°C  
D. 24°C

**Solution**
C  
(From Unit 7, Lesson 2.)

**Problem 5**

**Statement**
Lin ran 29 meters in 10 seconds. She ran at a constant speed.

i. How far did Lin run every second?  
ii. At this rate, how far can she run in 1 minute?

**Solution**

i. 2.9 meters every second, because \( 29 \div 10 = 2.9 \).  
ii. 174 meters, because \( (2.9) \cdot 60 = 174 \).

(From Unit 2, Lesson 9.)

**Problem 6**

**Statement**
Noah is helping his band sell boxes of chocolate to fund a field trip. Each box contains 20 bars and each bar sells for $1.50.

i. Complete the table for values of \( m \).
ii. Write an equation for the amount of money, \( m \), that will be collected if \( b \) boxes of chocolate bars are sold. Which is the independent variable and which is the dependent variable in your equation?

iii. Write an equation for the number of boxes, \( b \), that were sold if \( m \) dollars were collected. Which is the independent variable and which is the dependent variable in your equation?

**Solution**

i. Values for \( m \): 30, 60, 90, 120, 150, 180, 210, 240

ii. \( m = 30b \), \( b \) is independent, \( m \) is dependent

iii. \( b = \frac{m}{30} \), \( m \) is independent, \( b \) is dependent

(From Unit 6, Lesson 16.)
Lesson 12: Constructing the Coordinate Plane

Goals

- Choose and label appropriate scales for the axes of the coordinate plane, based on the coordinates to be plotted, and explain (orally and in writing) the choice.
- Compare and contrast different scales for the axes of the coordinate plane.

Learning Targets

- When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes.

Lesson Narrative

In this lesson, students explore the idea of scaling axes appropriately to accommodate data where coordinates are rational numbers. Students attend to precision as they plan where to place axes on a grid and how to scale them to represent data clearly (MP6). In an optional activity, students practice working with coordinates in all 4 quadrants as they navigate a maze on a coordinate grid. This lesson gives students the opportunity to develop fluency with plotting coordinates in all 4 quadrants and scaling axes to fit data that is essential for the context-driven work over the next few lessons.

Alignments

Building On

- 5.G.A.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Addressing

- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Building Towards

- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
Student Learning Goals
Let's investigate different ways of creating a coordinate plane.

12.1 English Winter

Warm Up: 5 minutes
The purpose of this warm-up is for students to reason about the need for quadrants beyond the first quadrant in the coordinate plane when representing data within a situation's context. When choosing an appropriate set of axes, students should also notice that the scale of the axes is important for the given data. Both of these ideas will be important for students' reasoning in upcoming activities.

While option B is the preferred response, it is more important that students explain and support whatever choice they make.

Building On
- 5.G.A.1

Building Towards
- 6.NS.C.6

Instructional Routines
- Poll the Class

Launch
Give students 2 minutes of quiet work time followed by a whole-class discussion. If needed, clarify that the term “noon” refers to 12 p.m.
Student Task Statement

The following data were collected over one December afternoon in England.

<table>
<thead>
<tr>
<th>time after noon (hours)</th>
<th>temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
</tr>
<tr>
<td>7</td>
<td>-4</td>
</tr>
<tr>
<td>8</td>
<td>-4</td>
</tr>
</tbody>
</table>

a. Which set of axes would you choose to represent these data? Explain your reasoning.

b. Explain why the other two sets of axes did not seem as appropriate as the one you chose.

Student Response

a. Responses vary. Sample response: Option B represents the data the best because all of the data points will fit on on the graph nicely where the grid lines meet.

b. Explanations vary. Sample explanation: Option A is not the most helpful because there is nowhere to plot points with negative coordinates. Option C is not the most helpful because the grid lines are spaced 10 units apart, so it is hard to plot coordinates that are not a multiple of 10.
**Activity Synthesis**

Poll the class on which set of axes they chose to represent the data. Ask selected students to explain why they chose one set of axes and did not choose the other two. Record and display the responses for all to see. If possible, display and reference the three sets of axes as students explain their reasoning.

If there is time, ask students what kind of data would make the other sets of axes appropriate choices. For example, set A would be appropriate if the temperatures were all positive and set C would be appropriate if the data were collected at 10 hour intervals and happened to be close to multiples of 10.

### 12.2 Axes Drawing Decisions

25 minutes

The purpose of this activity is for students to draw their own axes for different sets of coordinates. They must decide which of the four quadrants they need to use and how to scale the axes. Some students may use logic such as “the largest/smallest point is this, so my axes must go at least that far.” Identify these strategies for the discussion. Monitor for differences in scales and axes where the points were still able to be plotted correctly to highlight during discussion.

**Addressing**

- 6.NS.C.6.c

**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

**Launch**

Arrange students in groups of 2. Allow 10 minutes for students to construct their graphs and discuss them with their partners. Follow with a whole-class discussion.

**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time. Invite students to share their reasoning about where they placed each axis and how they determined an appropriate scale. Some students may benefit from access to partially-created graphs with varying degrees of completion—for example, axes with or without unlabeled tick marks.

*Supports accessibility for: Memory; Organization*
Access for English Language Learners

Listening, Speaking: MLR2 Collect and Display. Listen for and display vocabulary and phrases students use to justify their choice of axes (e.g., “minimum/maximum x- or y-coordinate” or “appropriate units”). Continue to update collected language students used to explain their reasoning to their peers. Remind students to borrow language from the display during paired and whole-class discussions.

Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

Make sure that students label distances on their axes consistently. For example, if the first tick mark after 0 is 3, then the next must be 6 in order for the spacing to be consistent.

Student Task Statement

a. Here are three sets of coordinates. For each set, draw and label an appropriate pair of axes and plot the points.

i. (1, 2), (3, -4), (-5, -2), (0, 2.5)

ii. (50, 50), (0, 0), (-10, -30), (-35, 40)
b. Discuss with a partner:

- How are the axes and labels of your three drawings different?
- How did the coordinates affect the way you drew the axes and label the numbers?

**Student Response**
Check student work to ensure they made reasonable choices about axes and scale that allowed them to clearly plot all the points.
Activity Synthesis
The key takeaway from this discussion is that defining axes and scale is a process of reasoning, not an exact science. Ask students to share their strategies about how to place and scale their axes. First, display previously selected student responses that capture the same data on their axes, but with slightly different origins or scales. Ask students which axes they think better represent the data. If not mentioned by students, point out that axes with a lot of empty space probably could benefit from either a different scale or a different origin. Select previously identified students to demonstrate a reliable strategy for finding the needed maximum or minimum. Link back to the warm-up when talking about how to scale the axes: if there are larger numbers, then a bigger scale makes more sense. Also draw attention to the fractional coordinates and how using decimal equivalents might make it easier to scale.

12.3 Positively A-maze-ing

Optional: 10 minutes (there is a digital version of this activity)
The purpose of this task is for students to locate and express coordinates in all four quadrants as they navigate around a maze. Students plan their route through the maze and strategically choose coordinates to correctly execute their plans (MP1).

This activity was inspired by one created by Nathan Kraft https://teacher.desmos.com/activitybuilder/custom/563c039dccdd442e107a0ce2.

Addressing
○ 6.NS.C.6.c

Instructional Routines
○ MLR7: Compare and Connect

Launch
Give students 8 minutes quiet work time followed by a brief whole-class discussion.

If using digital materials, students will navigate around a maze using a digital applet.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities. Provide students with access to mazes superimposed over a set of axes demarcated by single units instead of two units.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions
Students might disregard the fact that the side of each square grid is 2 units and just count boxes. Redirect students' attention to the relevant instruction, and ask how they will address it.
Student Task Statement

Here is a maze on a coordinate plane. The black point in the center is (0, 0). The side of each grid square is 2 units long.

(a) Enter the above maze at the location marked with a green segment. Draw line segments to show your way through and out of the maze. Label each turning point with a letter. Then, list all the letters and write their coordinates.

(b) Choose any 2 turning points that share the same line segment. What is the same about their coordinates? Explain why they share that feature.
Student Response

a. Start: (-11, 9), A: (-7, 9), B: (-7, 5), C: (1, 5), D: (1, 3), E: (7, 3), F: (7, -9), G: (-1, -9), H: (-1, -5), I: (-5, -5), J: (-5, -15)

b. Responses vary. Sample response: I chose points D and E. They have the same second coordinate. This is because I am moving along a straight line horizontally so my vertical position hasn't changed

Are You Ready for More?
To get from the point (2, 1) to (-4, 3) you can go two units up and six units to the left, for a total distance of eight units. This is called the “taxicab distance,” because a taxi driver would have to drive eight blocks to get between those two points on a map.
Find as many points as you can that have a taxicab distance of eight units away from \((2, 1)\). What shape do these points make?

**Student Response**

The points form a square with vertices \((10, 1)\), \((2, 9)\), \((-6, 1)\), and \((2, -7)\).

**Activity Synthesis**

Ask students to share how they planned which points to plot and how they determined the coordinates for each point with the given information. Invite students to share their responses for
question 2 to review the idea that points on a horizontal line share the same $y$-coordinate and points on the same vertical line share the same $x$-coordinate. Explain that in modern navigation, directions are precisely given in terms of coordinates. Navigation programs process coordinate data and translate it into a visual display for the driver, for example. Precise coordinates are also used to navigate through virtual space in computer simulations.

**Access for English Language Learners**

*Speaking: MLR7 Compare and Connect.* Use this routine to help students compare the various approaches used for the question, “How did you plan which points to plot and how you determined the coordinates for each point with the given information?” Prepare by looking for distinct strategies that highlight the difference between the horizontal and vertical lines and their coordinates. As students discuss their strategies, ask students to consider what information they used to decide on their coordinates and the path they chose to plot. These exchanges strengthen students' mathematical language use and reasoning about the coordinate grid.

*Design Principle(s): Maximize meta-awareness*

**Lesson Synthesis**

In this lesson, students placed and scaled axes to accommodate a variety of points with rational coordinates. To get students thinking about proper scaling, it may be helpful to start with two points plotted with axes that have been scaled improperly. Display an empty grid where both axes are labeled by 100 units from -500 to 500 and attempt to plot the points $(-2, 3)$ and $(5, 7)$. Emphasize that it is difficult to communicate information on a coordinate plane if the axes are labeled poorly. Here are some questions to consolidate what students have learned:

- When plotting $(-2, 3)$ and $(5, 7)$, how many units across would you make the $x$- and $y$-axes? How would you label the axes? (The $x$-axis needs to be at least 7 units across to go from -2 to 5. The $y$-axis needs to include values from 3 to 7, but in order to meet the $x$-axis, it should go at least from 0 to 7. It might look nicer to give some space in either direction, for example going from -4 to 7 in the $x$ direction and -1 to 8 in the $y$ direction. In this case, the grid lines could be labeled by 1 unit.)

- When plotting $(1.75, -0.5)$ and $(-2.25, 1.5)$, how many units across would you make the $x$- and $y$-axes? How would you label the axes? (The coordinates all look like multiples of 0.25, so the grid lines could be labeled by multiples of 0.25 units. The $x$-axis could go from -2.5 to 2 and the $y$-axis could go from -0.75 to 1.75.)

- When plotting $(-3, 40)$ and $(4, -60)$, how many units across would you make the $x$- and $y$-axes? How would you label the axes? (The $x$ axis could be labeled by 1 unit and the $y$ axis could be labeled by 10 units. The $x$-axis could go from -5 to 5 and the $y$-axis could go from -70 to 50.)
It may be helpful to display an empty grid to place and label the axes and plot the points for each example.

### 12.4 What Went Wrong: Graphing Edition

Cool Down: 5 minutes  
**Addressing**  
- 6.NS.C.6.c

#### Student Task Statement
Lin drew this set of axes and plotted the points $A = (1, 2)$, $B = (-3, -5)$, $C = (5, 7)$, $D = (-4, -3)$, and $E = (-4, 6)$ on them.

Identify as many mistakes as you notice in Lin's graph.

#### Student Response
Point $C$ is plotted at $(5, 8)$ instead of $(5, 7)$. Point $D$ is plotted at $(-3, -4)$ instead of $(-4, -3)$.  

[Diagram of the graph with points plotted]
Student Lesson Summary

The coordinate plane can be used to show information involving pairs of numbers.

When using the coordinate plane, we should pay close attention to what each axis represents and what scale each uses.

Suppose we want to plot the following data about the temperatures in Minneapolis one evening.

<table>
<thead>
<tr>
<th>time (hours from midnight)</th>
<th>temperature (degrees C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-8</td>
</tr>
</tbody>
</table>

We can decide that the $x$-axis represents number of hours in relation to midnight and the $y$-axis represents temperatures in degrees Celsius.

- In this case, $x$-values less than 0 represent hours before midnight, and $x$-values greater than 0 represent hours after midnight.
- On the $y$-axis, the values represent temperatures above and below the freezing point of 0 degrees Celsius.

The data involve whole numbers, so it is appropriate that the each square on the grid represents a whole number.

- On the left of the origin, the $x$-axis needs to go as far as -4 or less (farther to the left). On the right, it needs to go to 3 or greater.
- Below the origin, the $y$-axis has to go as far as -8 or lower. Above the origin, it needs to go to 3 or higher.

Here is a graph of the data with the axes labeled appropriately.
On this coordinate plane, a point at (0, 0) would mean a temperature of 0 degrees Celsius at midnight. The point at (-4, 3) means a temperature of 3 degrees Celsius at 4 hours before midnight (or 8 p.m.).
Lesson 12 Practice Problems

Problem 1

Statement

Draw and label an appropriate pair of axes and plot the points.

\((\frac{1}{5} \cdot \frac{4}{5})\)

\((-\frac{3}{5}, \frac{2}{5})\)

\((-1 \frac{1}{5}, -\frac{4}{5})\)

\((\frac{1}{5}, -\frac{3}{5})\)

Solution

Answers vary. Check student work to ensure they made reasonable choices about axes and scale that allowed them to clearly plot all the points.

Problem 2

Statement

Diego was asked to plot these points: \((-50, 0), (150, 100), (200, -100), (350, 50), (-250, 0)\).

What interval could he use for each axis? Explain your reasoning.

Solution

Answers vary. Sample response: Use an interval of 50, because all the coordinates involve points that are greater than 50 and multiples of 50.

Problem 3

Statement

i. Name 4 points that would form a square with the origin at its center.

ii. Graph these points to check if they form a square.

Solution

Answers vary. Sample response: \(A = (3, 3), B = (3, -3), C = (-3, 3), D = (-3, -3)\)
Problem 4

Problem 4

Statement

Which of the following changes would you represent using a negative number? Explain what a positive number would represent in that situation.

i. A loss of 4 points
ii. A gain of 50 yards
iii. A loss of $10
iv. An elevation above sea level

Solution

Answers vary. Sample response:

1 and 3 can be represented with negative numbers. A loss of 4 points is -4, but if any points are gained, the value becomes positive. A loss of $10 is -10, but if any money is earned, the value becomes positive.

(From Unit 7, Lesson 5.)

Problem 5

Problem 5

Statement

Jada is buying notebooks for school. The cost of each notebook is $1.75.
i. Write an equation that shows the cost of Jada's notebooks, $c$, in terms of the number of notebooks, $n$, that she buys.

ii. Which of the following could be points on the graph of your equation?

    (1.75, 1)    (2, 3.50)    (5, 8.75)    (17.50, 10)    (9, 15.35)

**Solution**

i. $c = 1.75n$

ii. b and c

(From Unit 6, Lesson 16.)

**Problem 6**

**Statement**

A corn field has an area of 28.6 acres. It requires about 15,000,000 gallons of water. About how many gallons of water per acre is that?

A. 5,000  
B. 50,000  
C. 500,000  
D. 5,000,000

**Solution**

C

(From Unit 5, Lesson 13.)
Lesson 13: Interpreting Points on a Coordinate Plane

Goals

○ Compare points on a graph, including statements about relative position and the vertical distance between points.

○ Describe (using words and inequality symbols) and interpret the range of coordinates on a graph, including the meaning of y-values that are negative.

○ Identify and interpret points on a graph to answer questions about situations involving temperature or money.

Learning Targets

○ I can explain how rational numbers represent balances in a money context.

○ I can explain what points in a four-quadrant coordinate plane represent in a situation.

○ I can plot points in a four-quadrant coordinate plane to represent situations and solve problems.

Lesson Narrative

This lesson pays particular attention to choices about what axes represent and the scale used on each axis. Graphs need to present information clearly and legibly to be useful for visualizing relationships between quantities. Students learn to make these choices purposefully when plotting points and to consider the decisions that have been made when reading and interpreting the coordinates of points from a graph. They interpret and label axes appropriately to clearly communicate their correspondence with the quantities in a problem. They reason abstractly and quantitatively as they interpret vertical distance in a coordinate plane in context (MP2).

Alignments

Building On

○ 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Addressing

○ 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.C.7.c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of $-30$ dollars, write $| - 30| = 30$ to describe the size of the debt in dollars.

6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards
6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines
MLR5: Co-Craft Questions
MLR8: Discussion Supports

Student Learning Goals
Let's examine what points on the coordinate plane can tell us.

13.1 Unlabeled Points

Warm Up: 5 minutes
In this warm-up, students practice skills that they have developed for plotting points in all 4 quadrants of the coordinate plane. This warm-up also gives students the opportunity to describe points that do not fall nicely on the intersection of grid lines. In the next few activities, students apply these skills to answer questions in context.

Addressing
6.NS.C.6.c

Building Towards
6.NS.C.8

Launch
Give students 3 minutes of quiet work time followed by whole-class discussion.

Anticipated Misconceptions
Some students may have trouble locating decimal values on a coordinate plane. (They have placed decimals on horizontal number lines before, but up to this point have mostly seen coordinates that are integers or 0.5's.) Demonstrate how point $B$ is 4 units above the $x$-axis by tracing a pencil 4 units vertically to land on point $B$. Then ask, "What if the point was only 0.2 units above the $x$-axis? Where would it go?"

Unit 7 Lesson 13
Student Task Statement

Label each point on the coordinate plane with the appropriate letter and ordered pair.

\[ A = (7, -5.5) \quad B = (-8, 4) \quad C = (3, 2) \quad D = (-3.5, 0.2) \]
Student Response

Activity Synthesis
The main goal of discussion is to review the order of ordered pairs and make sense of points that don't fall on the intersection of grid lines. Invite students to explain how they knew which points matched with which coordinates. Ask students how they would make sense of point \( D \), since it doesn't fall nicely where grid lines cross.

13.2 Account Balance

15 minutes
In this activity, students interpret points in the coordinate plane that correspond to the balance in a bank account (MP2). Since bank accounts are not likely to be familiar to students in grade 6, they will need to be oriented to the context.

Building On
- 6.NS.C.5

Addressing
- 6.NS.C.6.c
- 6.NS.C.7.c

Building Towards
- 6.NS.C.8

Instructional Routines
- MLR5: Co-Craft Questions

Unit 7 Lesson 13
Launch

Arrange students in groups of 2. Tell students that when someone opens a bank account, they have to put money into the account. The "account balance" is the amount of money in the account at any given time. For example, they might put $350 into the account when they open it, and then the account balance will be 350. However, sometimes they have to borrow money from the bank and then their account balance is a negative value. For instance, if they have no money in the account and borrow $200, then the account balance is -200. The graph they see shows the account balance for a person's account at the start of each day for two weeks.

Give students 10 minutes of work time. Encourage students to check in with their partner after each problem and work to reach agreement if they disagree. Follow with whole-class discussion.

Access for Students with Disabilities

**Action and Expression:** Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "I noticed ____ so I....", "If ____ then ____ because...."

Supports accessibility for: Language; Organization

Access for English Language Learners

**Conversing:** MLR5 Co-craft Questions. In order to help students read and interpret the graph in this activity, first display the opening paragraph and graph without showing the questions that follow. Ask students to consider the graph and generate mathematical questions that could be answered by using the graph, such as “How much money was spent on Day 5?” or “What happened on Day 13?” Then, invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the meaning behind the positive and negative coordinates in this task (e.g., owing money vs. putting money into the bank) prior to being asked to analyze the situation.

*Design Principle(s): Cultivate conversation*

**Student Task Statement**

The graph shows the balance in a bank account over a period of 14 days. The axis labeled $b$ represents account balance in dollars. The axis labeled $d$ represents the day.
a. Estimate the greatest account balance. On which day did it occur?

b. Estimate the least account balance. On which day did it occur?

c. What does the point (6, -50) tell you about the account balance?

d. How can we interpret |-50| in the context?

**Student Response**

a. The greatest balance was about $375 and it occurred on the 14th day.

b. The least balance was about $-90 and it occurred on the 11th day.

c. The point (6, -50) tells us that the account balance was -$50 on the 6th day.

d. |-50| = 50 is the amount of money the person owes the bank on day 6.

**Activity Synthesis**

The purpose of the discussion is to check how comfortable students are with the concept of an account balance that included negative numbers and how to interpret the coordinate plane in this context. Ask for students to explain their responses to each question. To include more students in the discussion, consider asking:

- "Do you agree or disagree? Why?"
- "Who can restate ___’s reasoning in a different way?"
“Does anyone want to add on to ____’s reasoning?”

Bring attention to the days when the account balance changed. Ask students to come up with a story of what might have happened on those days.

13.3 High and Low Temperatures

15 minutes
Students reason abstractly and quantitatively about temperatures over time graphed on coordinate axes (MP2). The goal of this activity is for students to use inequalities to describe the location of points on a coordinate grid in one direction. This activity also introduces the idea of vertical difference on the coordinate plane using a familiar context. Students may use previous strategies such as counting squares, but are not expected to explicitly add or subtract using negative numbers.

Building On
○ 6.NS.C.5

Addressing
○ 6.NS.C.6.c
○ 6.NS.C.8

Instructional Routines
○ MLR8: Discussion Supports

Launch
Arrange students in groups of 2. Allow students 3–4 minutes of quiet work time and 1–2 minutes to check results with their partner. Follow with a whole-class discussion.

---

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students about the sea levels and elevation number line representations.

Supports accessibility for: Social-emotional skills; Conceptual processing

---

Anticipated Misconceptions

Because the high temperature on day 7 was positive and the low temperature was negative, some students may not notice that the difference was 10 degrees on this day as well. Consider prompting them to use tracing paper to compare the temperature differences on days 6 and 7.
Student Task Statement

The coordinate plane shows the high and low temperatures in Nome, Alaska over a period of 8 days. The axis labeled \( T \) represents temperatures in degrees Fahrenheit. The axis labeled \( d \) represents the day.

a. i. What was the warmest high temperature?
   ii. Write an inequality to describe the high temperatures, \( H \), over the 8-day period.

b. i. What was the coldest low temperature?
   ii. Write an inequality to describe the low temperatures, \( L \), over the 8-day period.

c. i. On which day(s) did the largest difference between the high and low temperatures occur? Write down this difference.
   ii. On which day(s) did the smallest difference between the high and low temperatures occur? Write down this difference.

Student Response

a. i. The warmest high temperature was 28°F.
   ii. \( H < 28 \) or \( H = 28 \). Additionally, students may write \( H > 2 \) or \( H = 2 \).

b. i. The coldest low temperature was -3°F.
   ii. \( L > -3 \) or \( L = -3 \). Additionally, students may write \( L < 26 \) or \( L = 26 \).
c. i. Days 3, 6, and 7. The difference was $10^\circ F$.

ii. Day 1. The difference was $2^\circ F$.

**Are You Ready for More?**

Before doing this problem, do the problem about taxicab distance in an earlier lesson.

The point $(0, 3)$ is 4 taxicab units away from $(-4, 3)$ and 4 taxicab units away from $(2, 1)$.

a. Find as many other points as you can that are 4 taxicab units away from $(-4, 3)$ and $(2, 1)$.

b. Are there any points that are 3 taxicab units away from both points?

**Student Response**

a. $(-2, 1)$ and $(-1, 2)$

b. There are no such points. This is because $(-4, 3)$ and $(2, 1)$ are 8 taxicab units apart, so their combined distance to any other point must be at least 8.

**Activity Synthesis**

This discussion should lead to two key takeaways. First, students express the range of values for the low and high temperatures using inequalities. Second, students share strategies for finding a difference between two values on the coordinate plane.

Ask students to share their inequalities for $H$ and $L$. It is expected that students have inequalities that describe the maximum high temperature for $H$ and the minimum low temperature for $L$, but the discussion should bring out that each variable has 4 statements that capture its possible values: $L > -3$, $L = -3$, $L < 26$, and $L = 26$ for the variable $L$ and $H > 2$, $H = 2$, $H < 28$, and $H = 28$ for the variable $H$.

Ask students to share their strategies for finding the vertical distance between points. Push them to explain how they took the scale of the vertical axis into account. Invite students to explain how they used the context to make sense of their answers.

**Access for English Language Learners**

*Listening, Speaking: MLR8 Discussion Supports.* To support students in producing statements about details in the inequalities, provide sentence frames for students to use when they explain their reasoning to their peers, such as: “This inequality should/should not also be equal to because _____”, “A situation uses an equal sign when _____”, “I know that the inequality ____ is correct because ____.”

*Design Principle(s): Support sense-making*
Lesson Synthesis

In this lesson, students graphed temperature and account balance over time on coordinate axes and interpreted questions involving vertical distance. Consider asking some of the following questions:

- “Sketch a graph of (4, 450) and (4, -47). Which of the situations we looked at today would make these points make the most sense? What would the x- and y-axes represent?” (The account balance over the course of several days makes most sense in this situation. Then x would represent the number of days and y would represent the account balance. A temperature of 450 degrees Celsius doesn't make as much sense if we are talking about Earth.)

- “Suppose two people open their own bank accounts on the same day. Graphing their account balances over several days, one person's situation is represented by (4, 450) and the other person's is represented by (4, -47). What does this mean in the situation? How do their account balances compare?” (This means the first person has $450 on day 4 and the other person owes $47 by that same day. The first person's balance is $497 higher than the second person's.)

- “The high temperature on day 6 of a 10 day period is 30 degrees Celsius and the low temperature on that same day is 12 degrees Celsius. Sketch a graph, label the axes, and plot the high and low temperatures on day 6. How much warmer is the high temperature than the low temperature?”

Invite students to display their sketches for all to see. Highlight the vertical distance between points and compare students' strategies for finding that distance.

13.4 Time and Temperature

Cool Down: 5 minutes

This cool-down checks whether students can represent data on a coordinate plane and interpret points in context. Students who label time as the vertical axis and temperature as the horizontal axis need to be reminded of the convention that the first column in a table represents values of the first (horizontal) coordinate and the second column represents values of the second (vertical) coordinate. Given the way the axes are scaled, students need to estimate the location of points that don't fall precisely on the intersection of grid lines.

Addressing

- 6.NS.C.8

Student Task Statement

The temperature in Princeton was recorded at various times during the day. The times and temperatures are shown in the table.

Unit 7 Lesson 13
<table>
<thead>
<tr>
<th>time (hours before or after midnight)</th>
<th>temperature (degrees C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>1.2</td>
</tr>
<tr>
<td>-2</td>
<td>-1.6</td>
</tr>
<tr>
<td>0</td>
<td>-3.5</td>
</tr>
<tr>
<td>8</td>
<td>-6.7</td>
</tr>
</tbody>
</table>

a. Plot points that represent the data. Be sure to label the axes.

b. In the town of New Haven, the temperature at midnight was 1.2°C. Plot and label this point. Which town was warmer at midnight, Princeton or New Haven? How many degrees warmer was it?

c. If the point (3, -2.5) were also plotted on the diagram, what would it mean?

**Student Response**

b. The point (0, 1.2) should be added to the plot from the previous question. New Haven is warmer by 4.7 degrees Celsius.

c. (3, -2.5) means 3 hours after midnight the temperature was -2.5 degrees.
**Student Lesson Summary**

Points on the coordinate plane can give us information about a context or a situation. One of those contexts is about money.

To open a bank account, we have to put money into the account. The account balance is the amount of money in the account at any given time. If we put in $350 when opening the account, then the account balance will be 350.

Sometimes we may have no money in the account and need to borrow money from the bank. In that situation, the account balance would have a negative value. If we borrow $200, then the account balance is -200.

A coordinate grid can be used to display both the balance and the day or time for any balance. This allows to see how the balance changes over time or to compare the balances of different days.

Similarly, if we plot on the coordinate plane data such as temperature over time, we can see how temperature changes over time or compare temperatures of different times.
Lesson 13 Practice Problems

Problem 1

Statement

The elevation of a submarine is shown in the table. Draw and label coordinate axes with an appropriate scale and plot the points.

<table>
<thead>
<tr>
<th>time after noon (hours)</th>
<th>elevation (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-567</td>
</tr>
<tr>
<td>1</td>
<td>-892</td>
</tr>
<tr>
<td>2</td>
<td>-1,606</td>
</tr>
<tr>
<td>3</td>
<td>-1,289</td>
</tr>
<tr>
<td>4</td>
<td>-990</td>
</tr>
<tr>
<td>5</td>
<td>-702</td>
</tr>
<tr>
<td>6</td>
<td>-365</td>
</tr>
</tbody>
</table>

Solution
Problem 2

Statement
The inequalities \( h > 42 \) and \( h < 60 \) represent the height requirements for an amusement park ride, where \( h \) represents a person's height in inches.

Write a sentence or draw a sign that describes these rules as clearly as possible.

Solution
Answers vary. Sample response: To ride, a person must be more than 3 feet 6 inches tall, and no taller than 5 feet.

(From Unit 7, Lesson 8.)

Problem 3

Statement
The \( x \)-axis represents the number of hours before or after noon, and the \( y \)-axis represents the temperature in degrees Celsius.

\[
\begin{array}{c}
\text{quadrant II} \\
\text{quadrant I} \\
\text{quadrant III} \\
\text{quadrant IV}
\end{array}
\]

i. At 9 a.m., it was below freezing. In what quadrant would this point be plotted?

ii. At 11 a.m., it was 10°C. In what quadrant would this point be plotted?

iii. Choose another time and temperature. Then tell the quadrant where the point should be plotted.

iv. What does the point \((0, 0)\) represent in this context?

Solution
i. Quadrant III

ii. Quadrant II

iii. Answers vary. Sample response: At 11 p.m., the temperature was -5°C. This point would be plotted in Quadrant IV.
iv. A point at (0, 0) would represent a freezing temperature (0°C) at noon.

Problem 4

Statement

Solve each equation.

\[
\begin{align*}
3a &= 12 \\
b + 3.3 &= 8.9 \\
n &= \frac{1}{4}c \\
5 \frac{1}{2} &= d + \frac{1}{4} \\
2e &= 6.4
\end{align*}
\]

Solution

i. \( a = 4 \)

ii. \( b = 5.6 \)

iii. \( c = 4 \)

iv. \( d = 5 \frac{1}{4} \)

v. \( e = 3.2 \)

(From Unit 6, Lesson 4.)
Lesson 14: Distances on a Coordinate Plane

Goals

- Compare and contrast (orally and in writing) the coordinates for points in different locations on the coordinate plane.
- Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.
- Generalize (orally) about the coordinates of points that are reflected across the x- or y-axis.

Learning Targets

- I can find horizontal and vertical distances between points on the coordinate plane.

Lesson Narrative

In this lesson, students explore ways to find vertical and horizontal distances in the coordinate plane. In the first activity, students use repeated reasoning to explore the relationship between points with opposite coordinates (MP8). In the second activity, students develop strategies for finding the distance between two points where the coordinates might not be integers. Students can use previous strategies, such as considering the distance of a point from zero, or counting squares. Students will use these skills in Grade 7 to find distances on maps. In Grade 8, they will use these skills to draw slope triangles in the coordinate plane and find the lengths of their sides when considering graphs of proportional and non-proportional relationships.

Alignments

Addressing

- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.C.6.b: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share
Required Preparation

It may be useful, but not required, to provide access to tracing paper or rulers to help students think through the misconception that the length of a diagonal is equal to the length of a related horizontal or vertical distance in the coordinate plane.

Student Learning Goals

Let’s explore distance on the coordinate plane.

14.1 Coordinate Patterns

Warm Up: 10 minutes (there is a digital version of this activity)

The purpose of this warm-up is for students to review plotting and labeling points that include negative coordinates and use repeated reasoning to generalize patterns in the coordinates of points in each quadrant (MP8).

Addressing

- 6.NS.C.6

Launch

Arrange students in groups of 4. Assign each person in a group a different quadrant. Tell students you will give them 2 minutes to plot and label at least three points in their assigned quadrant, and up to six if they have time. Give students 2 minutes of quiet work time. Give students 4 minutes to share their points and their coordinates with their small group and look for patterns in the coordinates of points in each quadrant. Follow with whole-class discussion.

Though the paper and pencil version may be preferred, a digital applet is available for this activity.

Student Task Statement

Plot points in your assigned quadrant and label them with their coordinates.
Student Response

Responses vary. Sample response: The points (-5, 5), (-3, 6), and (-2, 4) are all in quadrant II. If the first coordinate is negative and the second coordinate is positive, then the point is in quadrant II.

Activity Synthesis

The focus of the discussion is for students to explain why the following patterns emerge:

- In quadrants I and IV, the x-coordinate of a point (or the first number in an ordered pair) is positive.
- In quadrants II and III, the x-coordinate of a point is negative.
- In quadrants I and II, the y-coordinate of a point (or the second number in an ordered pair) is positive.
- In quadrants III and IV, the y-coordinate of a point is negative.

Ask the students to share any patterns they noticed among the coordinates of the points in each quadrant. After each student shares, ask the rest of the class if they noticed the same pattern within their small group. Record and display these patterns for all to see. If possible, plot and label a few example points in each quadrant based on students’ observations.

14.2 Signs of Numbers in Coordinates

15 minutes (there is a digital version of this activity)

The purpose of this task is for students to connect opposite signs in coordinates with reflections across one or both axes. Students investigate relationships between several pairs of points in order.
to make this connection more generally (MP8). The square grid, spaced in units, means that students can use counting squares as a strategy for finding distances.

The use of the word "reflection" is used informally to describe the effect of opposite signs in coordinates. In grade 8, students learn a more precise, technical definition of the word "reflection" as it pertains to rigid transformations of the plane.

**Addressing**

- 6.NS.C.6.b
- 6.NS.C.8

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

**Launch**

Arrange students in groups of 2. Allow students 3-4 minutes of quiet work time and 1-2 minutes to check results with their partner for questions 1 and 2. Tell students to pause after question 2 for whole-class discussion. At that time, briefly check that students have the correct coordinates for points A, B, C, D, and E before moving on to the rest of the questions. Give students 4 minutes to answer the rest of the questions with their partner, followed by whole-class discussion.

Though the paper and pencil version may be preferred, a digital applet is available for this activity.

**Access for English Language Learners**

*Speaking: MLR1 Stronger and Clearer Each Time.* Use this routine with successive pair shares to give students a structured opportunity to revise and refine their response to “How far away are the points from the x-axis and y-axis?” Ask each student to meet with 2-3 other partners in a row for feedback. Provide students with prompts for feedback that will help teams strengthen their ideas and clarify their language (e.g., “How did you use the horizontal/vertical distance to help?”) Students can borrow ideas and language from each partner to strengthen their final response.

*Design Principle(s): Cultivate conversation*

**Student Task Statement**

a. Write the coordinates of each point.
b. Answer these questions for each pair of points.

- How are the coordinates the same? How are they different?
- How far away are they from the y-axis? To the left or to the right of it?
- How far away are they from the x-axis? Above or below it?

i. $A$ and $B$

ii. $B$ and $D$

iii. $A$ and $D$

Pause here for a class discussion.

c. Point $F$ has the same coordinates as point $C$, except its $y$-coordinate has the opposite sign.

i. Plot point $F$ on the coordinate plane and label it with its coordinates.

ii. How far away are $F$ and $C$ from the $x$-axis?

iii. What is the distance between $F$ and $C$?

d. Point $G$ has the same coordinates as point $E$, except its $x$-coordinate has the opposite sign.

i. Plot point $G$ on the coordinate plane and label it with its coordinates.

ii. How far away are $G$ and $E$ from the $y$-axis?

iii. What is the distance between $G$ and $E$?
e. Point $H$ has the same coordinates as point $B$, except its both coordinates have the opposite sign. In which quadrant is point $H$?

**Student Response**

a. $A = (4, 3), B = (4, -3), C = (3, -5), D = (-4, -3), E = (-5, 3)$

b. Answers vary. Sample response:

   i. $A$ and $B$ have the same x-coordinate (4) but opposite y-coordinates (3 and -3). Both points are 4 units to the right of the y-axis. Both are 3 units from the x-axis, but $A$ is above the x-axis and $B$ is below it.

   ii. $B$ and $D$ have the opposite x-coordinates (4 and -4) but the same y-coordinate of -3. Both points are 4 units away from the y-axis but one is to the left of it and the other to the right of it. Both are 3 units below the x-axis.

   iii. $A$ and $D$ have opposite x-coordinates (4 and -4) and y-coordinates (3 and -3). Both are 4 units away from y-axis and 3 units away from the x-axis, but in opposite directions.

c. i. $F = (3, 5)$

   ii. Each point is 5 units from the x-axis.

   iii. 10 units

d. i. $G = (5, 3)$

   ii. Each point is 5 units from the y-axis.

   iii. 10 units

e. Quadrant II

**Activity Synthesis**

The key takeaway is that coordinates with opposite signs correspond to reflections across the axes. Encourage students to share ideas that they discussed with their partner. Ask students first about what patterns they noticed for pairs of points whose x-coordinates had opposite signs. Push students to give specific examples of pairs of points and their coordinates, and to describe what opposite x-coordinates mean in terms of the coordinate plane. Record students' explanations for all to see. Students may use phrasing like “the point flips across the y-axis.” This would be a good opportunity to use the word “reflection” and discuss the similarities between reflections across the y-axis and reflections in a mirror.

Repeat this discussion for pairs of points where the y-coordinates had opposite signs to get to the idea that they are reflections across the x-axis. Close by discussing the relationship between points $H$ and $B$, where both the x- and y-coordinates have opposite signs. Ask students how they might describe the relationship between $H$ and $B$ visually on the coordinate plane. While students may describe the relationship in terms of 2 reflections (once across the x-axis and again across the y-axis or vice versa), it is not expected that students see their relationship in terms of rotation.
14.3 Finding Distances on a Coordinate Plane

15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to develop strategies for finding the distance between two points in the coordinate plane when the coordinates might not be integers. These distances are restricted to horizontal and vertical distances; use of the general two-dimensional distance formula is not expected, nor are students expected to add or subtract negative numbers fluently. More general strategies for finding distance in the coordinate plane are developed in grade 8, and rational number arithmetic is developed more completely in grade 7.

Addressing

6.NS.C.8

Instructional Routines

MLR5: Co-Craft Questions

Think Pair Share

Launch

Arrange students in groups of 2. Allow students 5–6 minutes of quiet work time followed by 3–4 minutes to check results with their partner. Follow with a whole-class discussion.

Though the paper and pencil version may be preferred, a digital applet is available for this activity.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students within the first 2–3 minutes of work time to ensure they understood the directions. If students are unsure how to begin, suggest that they use previous strategies, such as considering the distance of a point from zero.

Supports accessibility for: Organization; Attention
Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. Display only the first line of the first question with the coordinate grid and ask pairs of students to write possible mathematical questions about the coordinate grid. Then, invite pairs to share their questions with the class. This helps students begin the conversation of horizontal and vertical distance and even possibly talking about diagonal distances between two points.

Design Principle(s): Cultivate conversation

Anticipated Misconceptions

Some students may assume that a diagonal line across a number of squares has the same length. Ask students to use a ruler or tracing paper to compare the length of the diagonal distance in question against the horizontal or vertical distance the student claims is equal.

Student Task Statement

a. Label each point with its coordinates.

b. Find the distance between each of the following pairs of points.
   
   i. Point $B$ and $C$
   
   ii. Point $D$ and $B$
   
   iii. Point $D$ and $E$

   c. Which of the points are 5 units from (-1.5, -3)?

   d. Which of the points are 2 units from (0.5, -4.5)?
e. Plot a point that is both 2.5 units from \( A \) and 9 units from \( E \). Label that point \( M \) and write down its coordinates.

**Student Response**

a. \( A = (-1.5, 2), \ B = (3.5, 2), \ C = (3.5, -3), \ D = (3.5, -4.5), \ E = (-1.5, -4.5) \)

b.  
   i. 5 units  
   ii. 6.5 units  
   iii. 5 units

c. Points \( A \) and \( C \)

d. Point \( E \)

e. \( M = (-1.5, 4.5) \)

**Are You Ready for More?**

Priya says, “There are exactly four points that are 3 units away from \((-5, 0)\).” Lin says, “I think there are a whole bunch of points that are 3 units away from \((-5, 0)\).”

Do you agree with either of them? Explain your reasoning.

**Student Response**

Answers vary. Possible response: I agree with Lin, because I can measure a length of 3 units from the point \((-5, 0)\) in any direction. I don’t have to just go up, down, left, or right.

(Students will learn in grade 7 that a circle is the (infinite) set of points that are equidistant from a center point. For grade 6 we focus on vertical and horizontal distances where naming the coordinates of points at a given distance is clear.)

**Activity Synthesis**

The important idea students should come away with is that they can continue to use strategies they have developed for finding horizontal and vertical distances even without a context. Here are some questions for discussion:

- How did finding lengths in this activity compare to the previous activity? How were they the same? How were they different?

- Were any of the points reflections across the axes? How could you tell by looking at the coordinate plane? How could you tell by looking at the coordinates?

- Were there any points of disagreement with your partner? How did you come to agreement?
Lesson Synthesis

The activities in this lesson asked students to analyze the effect of replacing coordinates with their opposites and to find horizontal and vertical distances in the coordinate plane. Here are some questions for discussion:

- “Without graphing, what can you say about the points (5, -3) and (5, 3) on the coordinate plane?” (Sample responses: The first point is in quadrant IV and the second point is in quadrant I. They are both 3 units from the x-axis. They are reflections across the x-axis. They are 6 units apart. They are both on the same vertical line.)

- “Without graphing, what can you say about (-6, 4) and (6, 4) on the coordinate plane?” (Sample responses: The first point is in quadrant II and the second point is in quadrant I. They are both 6 units from the y-axis. They are reflections across the y-axis. They are 12 units apart. They are both on the same horizontal line)

- “Without graphing, what can you say about (2.5, 1) and (6, 1) on the coordinate plane?” (Sample responses: they are both in quadrant I. They are both on the same horizontal line because they have the same y-value. The second point is 3.5 units to the right of the first point.)

If time allows, challenge students to draw, for example, a rectangle with given side lengths, and identify its vertices. This will lead nicely into the next lesson, where students will explore shapes in the coordinate plane. Select student responses to display for all to see.

14.4 Points and Distances

Cool Down: 5 minutes
Addressing

- 6.NS.C.8

Student Task Statement

Here are four points on a coordinate plane.
a. What is the distance between points A and B?

b. What is the distance between points C and D?

c. Plot the point (-3, 2). Label it E.

d. Plot the point (-4.5, -4.5). Label it F.

**Student Response**

a. About 9.5 units

b. About 6.5 units

c. 
**Student Lesson Summary**

The points $A = (5, 2)$, $B = (-5, 2)$, $C = (-5, -2)$, and $D = (5, -2)$ are shown in the plane. Notice that they all have almost the same coordinates, except the signs are different. They are all the same distance from each axis but are in different quadrants.

Notice that the vertical distance between points $A$ and $D$ is 4 units, because point $A$ is 2 units above the horizontal axis and point $D$ is 2 units below the horizontal axis. The horizontal distance between points $A$ and $B$ is 10 units, because point $B$ is 5 units to the left of the vertical axis and point $A$ is 5 units to the right of the vertical axis.

We can always tell which quadrant a point is located in by the signs of its coordinates.

<table>
<thead>
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<th>$y$</th>
<th>quadrant</th>
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<tr>
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<td>positive</td>
<td>II</td>
</tr>
<tr>
<td>negative</td>
<td>negative</td>
<td>III</td>
</tr>
<tr>
<td>positive</td>
<td>negative</td>
<td>IV</td>
</tr>
</tbody>
</table>

In general:

- If two points have $x$-coordinates that are opposites (like 5 and -5), they are the same distance away from the vertical axis, but one is to the left and the other to the right.
- If two points have $y$-coordinates that are opposites (like 2 and -2), they are the same distance away from the horizontal axis, but one is above and the other below.
When two points have the same value for the first or second coordinate, we can find the distance between them by subtracting the coordinates that are different. For example, consider \((1, 3)\) and \((5, 3)\):

They have the same \(y\)-coordinate. If we subtract the \(x\)-coordinates, we get \(5 - 1 = 4\). These points are 4 units apart.
Lesson 14 Practice Problems

Problem 1

Statement
Here are 4 points on a coordinate plane.

Solution
i. Label each point with its coordinates.

ii. Plot a point that is 3 units from point K. Label it P.

iii. Plot a point that is 2 units from point M. Label it W.

Problem 2

Statement
Each set of points are connected to form a line segment. What is the length of each?

i. A = (3, 5) and B = (3, 6)

ii. C = (-2, -3) and D = (-2, -6)
Problem 3

**Statement**

On the coordinate plane, plot four points that are each 3 units away from point \( P = (-2, -1) \). Write the coordinates of each point.

**Solution**

Answers vary. Sample response: \( A = (-5, -1) \), \( B = (1, -1) \), \( C = (-2, 2) \), \( D = (-2, -4) \). (Students are unlikely to come up with other possible solutions at this stage.)

Problem 4

**Statement**

Noah’s recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

i. Noah prepares large batches of sparkling orange juice for school parties. He usually knows the total number of liters, \( t \), that he needs to prepare. Write an equation that shows how Noah can find \( s \), the number of liters of soda water, if he knows \( t \).
ii. Sometimes the school purchases a certain number, \( j \), of liters of orange juice and Noah needs to figure out how much sparkling orange juice he can make. Write an equation that Noah can use to find \( t \) if he knows \( j \).

**Solution**

i. \( s = \frac{5}{9} t \)

ii. \( t = \frac{9}{4} j \)

(From Unit 6, Lesson 16.)

**Problem 5**

**Statement**

For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 pounds. Andre’s suitcase weighs 23 kilograms. Can Andre check his suitcase? Explain or show your reasoning. (Note: 10 kilograms \( \approx \) 22 pounds)

**Solution**

No, Andre will not be able to check his suitcase if they are strict about following the rule. Possible explanation:

1 kg weighs 2.2 pounds, so 23 kg weighs \((2.2) \times 23 = 50.6\) pounds.

<table>
<thead>
<tr>
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<tr>
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<tr>
<td>3</td>
<td>6.6</td>
</tr>
<tr>
<td>23</td>
<td>50.6</td>
</tr>
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</table>

(From Unit 3, Lesson 4.)
Lesson 15: Shapes on the Coordinate Plane

Goals

- Determine the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.
- Draw a polygon in the coordinate plane given the coordinates for its vertices.
- Explain (orally) that coordinates can be a useful way of describing geometric figures or modeling real-world locations.

Learning Targets

- I can find the lengths of horizontal and vertical segments in the coordinate plane.
- I can plot polygons on the coordinate plane when I have the coordinates for the vertices.

Lesson Narrative

In this lesson, students apply their understanding of rational coordinates and distance in the coordinate plane to construct polygons and navigate a maze. Students plot coordinates in all four quadrants and find horizontal and vertical distances.

Alignments

Building On

- 5.G.A.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
- 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
• 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards
• 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

• 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines
• Anticipate, Monitor, Select, Sequence, Connect
• MLR5: Co-Craft Questions
• MLR8: Discussion Supports
• Think Pair Share

Required Materials
Colored pencils  Graph paper

Required Preparation
Provide students access to graph paper and colored pencils for the lesson synthesis or upon request during the lesson.

Student Learning Goals
Let’s use the coordinate plane to solve problems and puzzles.

15.1 Figuring Out The Coordinate Plane

Warm Up: 5 minutes (there is a digital version of this activity)
The purpose of this warm-up is for students to review properties of figures and polygons within the context of graphing points in the coordinate plane.

As the students work, monitor and select students with different figures, some that are polygons and some that are not, to share during the whole-class discussion. The focus of the whole-class discussion should be on the properties of a polygon.

Building On
• 5.G.A.1
Building Towards

- 6.G.A.3
- 6.NS.C.8

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time. After the 2 minutes, tell students to share their figure with their partner to check if it has at least three of the listed properties. Follow with a whole group discussion.

Student Task Statement

a. Draw a figure in the coordinate plane with at least three of the following properties:

- 6 vertices
- Exactly 1 pair of parallel sides
- At least 1 right angle
- 2 sides with the same length

b. Is your figure a polygon? Explain how you know.

Student Response

a. Answers vary. Sample strategy: Start with two segments of equal lengths that meet at a right angle. This takes care of 2 requirements. To reach a third requirement, just choose 3 other points and connect them to the existing vertices so that there are 6 total vertices.

Unit 7 Lesson 15
**Activity Synthesis**

Ask selected students to share their figure and its properties. Display these figures for all to see. After each student shares, ask the class if it is a polygon and how they know.

Defining characteristics of a polygon that should be emphasized during the discussion are:

- It is composed of line segments.
- Each line segment meets one and only one other line segment at each end.
- The line segments never intersect each other except at their endpoints.
- It lays flat on the coordinate plane.

**15.2 Plotting Polygons**

15 minutes (there is a digital version of this activity)

The purpose of this task is for students to practice plotting points in the coordinate plane to make polygons.

**Building On**

- 6.G.A.3

**Addressing**

- 6.NS.C.6.c

**Instructional Routines**

- MLR8: Discussion Supports

**Launch**

Arrange students in groups of 2. Give students 8 minutes quiet work time, 4 minutes for partner discussion, followed by whole-class discussion.

Students using digital materials will plot points and create polygons with a digital applet.

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**Access for Students with Disabilities**

*Engagement: Provide Access by Recruiting Interest.* Leverage choice around perceived challenge. Invite students to select 2–3 of the polygons to plot on the coordinate plane. Chunking this task into more manageable parts may also benefit students who benefit from additional processing time.

*Supports accessibility for: Organization; Attention; Social-emotional skills*
**Student Task Statement**

Here are the coordinates for four polygons. Plot them on the coordinate plane, connect the points in the order that they are listed, and label each polygon with its letter name.

a. Polygon A: (-7, 4), (-8, 5), (-8, 6), (-7, 7), (-5, 7), (-5, 5), (-7, 4)

b. Polygon B: (4, 3), (3, 3), (2, 2), (2, 1), (3, 0), (4, 0), (5, 1), (5, 2), (4, 3)

c. Polygon C: (-8, -5), (-8, -8), (-5, -8), (-5, -5), (-8, -5)

d. Polygon D: (-5, 1), (-3, -3), (-1, -2), (0, 3), (-3, 3), (-5, 1)
Are You Ready for More?

Find the area of Polygon D in this activity.

Student Response

19.5 square units. There are many possible approaches to this problem: students can partition the polygon into squares and right triangles, or they can draw a rectangle around the polygon, then subtract the area outside the polygon from the area of the rectangle.

Activity Synthesis

The purpose of the discussion is to emphasize the connection between numbers, the coordinate plane, and geometry. To highlight these connections, ask:

- “How is the coordinate plane related to the number line?” (The coordinate plane has two axes that are both number lines.)
- “How are we able to make polygons in the coordinate plane?” (The vertices of a polygon are plotted as points in the coordinate plane.)

Complete the connection by explaining to students that the coordinate plane allows us to describe shapes and geometry in terms of numbers. This is how computers are able to create 2 and 3 dimensional images even though they can only interpret numbers.
Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about the relationship between between numbers, the coordinate plane, and geometry. As students share the connections they notice, revoice their statements using appropriate mathematical language, such as “points in the coordinate plane” or “the two axes of the coordinate plane.”
Design Principle(s): Cultivate conversation

15.3 Four Quadrants of A-Maze-ing

15 minutes (there is a digital version of this activity)
The purpose of this task is for students to practice plotting coordinates in all four quadrants and find horizontal and vertical distances between coordinates in a puzzle. In past activities, students have been told the scale for the distance between grid lines, but here they must determine that each grid square has length 2 from the information given.

Addressing
○ 6.G.A.3
○ 6.NS.C.8

Instructional Routines
○ MLR5: Co-Craft Questions

Launch
Arrange students in groups of 2. Tell students that they should not assume that each grid box is 1 unit. Give students 10 minutes quiet work time and 2 minutes for partner discussion. Follow with whole-class discussion.

Students using digital materials will be able to create a path through the maze by plotting points with an applet.

Access for Students with Disabilities

Representation: Internalize Comprehension. Check in with students after the first 2–3 minutes of work time. Check to make sure students have selected appropriate coordinates for the first points of Andre’s route through the maze.
Supports accessibility for: Conceptual processing; Organization
Access for English Language Learners

Conversing: MLR5 Co-craft Questions. Display only the picture of the maze and ask pairs of students to write possible mathematical questions about the situation. This is an opportunity for students to think about and relate to previous questions from previous lessons. Then, invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the coordinate grid and the points on the maze.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Student Task Statement

a. The following diagram shows Andre’s route through a maze. He started from the lower right entrance.

i. What are the coordinates of the first two and the last two points of his route?

ii. How far did he walk from his starting point to his ending point? Show how you know.

b. Jada went into the maze and stopped at (-7, 2).

i. Plot that point and other points that would lead her out of the maze (through the exit on the upper left side).

ii. How far from (-7, 2) must she walk to exit the maze? Show how you know.
Student Response

a.  
   i. The coordinates of the first two points are (12, -8) and (4, -8). The coordinates of the last two points are (-4, 8) and (-12, 8).
   
ii. 80 units. Sample explanation: Counting grid squares as steps, Andre went 40 steps. Each step was 2 units, so the journey was a total of 80 units.

b.  
   i. Jada's path is (-7, 2), (-4, 2), (-4, 8), (-12, 8).
   
ii. Jada's path is 17 units. Sample explanation: Counting grid squares as steps, Jada went 8.5 steps. Each step was 2 units, so the journey was a total of 17 units.

Activity Synthesis

The key idea is that it is possible to find distances and describe situations involving movement using the coordinate plane. This abstraction is important to appreciate because it means we can use numbers (in this case, pairs of numbers in the coordinate plane) to model situations that involve distance or movement. This will play a key role in later studies. To highlight these ideas, consider asking:

- How were you or your partner able to find the coordinates in the maze? Did you come up with any strategies or shortcuts?
- How did you find the distances that Andre and Jada traveled?
- What other situations involving movement could be represented with a coordinate plane?

Students may come up with examples like board games, maps, and perhaps even 3-dimensional examples.

Lesson Synthesis

Challenge students to create a drawing with a perimeter of 30 units using a continuous path of horizontal and vertical line segments. Ask students to identify the coordinates of vertices and justify that the perimeter is the given length. If time allows, arrange students in groups of 2 and ask them to draw their partner’s figure in a coordinate plane with only verbal information and no coordinates. Students can check their drawing by asking for the exact coordinates. Ask students to explain why coordinates are useful for communicating information about flat space. Consider displaying student work for all to see throughout the rest of the unit. It may be interesting for students to see the variety of figures that all have a perimeter of 30 units.

15.4 Perimeter of A Polygon

Cool Down: 5 minutes

Addressing

- 6.G.A.3
- 6.NS.C.8

Unit 7 Lesson 15
Student Task Statement

a. Plot the following points on the coordinate plane and connect them to create a polygon.

- $A = (1, 3)$
- $B = (3, 3)$
- $C = (3, -2)$
- $D = (-2, -2)$
- $E = (-2, 0)$
- $F = (0, 0)$
- $G = (0, 2)$
- $H = (1, 2)$
- $I = (1, 3)$

b. Find the perimeter of the polygon.

Student Response

a.

b. The perimeter is 20 units.

Student Lesson Summary

We can use coordinates to find lengths of segments in the coordinate plane.
For example, we can find the perimeter of this polygon by finding the sum of its side lengths. Starting from (-2, 2) and moving clockwise, we can see that the lengths of the segments are 6, 3, 3, 3, 3, and 6 units. The perimeter is therefore 24 units.

In general:

- If two points have the same x-coordinate, they will be on the same vertical line, and we can find the distance between them.
- If two points have the same y-coordinate, they will be on the same horizontal line, and we can find the distance between them.
Lesson 15 Practice Problems
Problem 1

Statement
The coordinates of a rectangle are (3, 0), (3, -5), (-4, 0) and (-4, -5)

i. What is the length and width of this rectangle?
ii. What is the perimeter of the rectangle?
iii. What is the area of the rectangle?

Solution
i. The length is 7 units and the width is 5 units.
ii. The perimeter is 24 units, because \(7 + 7 + 5 + 5 = 24\).
iii. The area is 35 square units, because \(7 \cdot 5 = 35\).

Problem 2

Statement
Draw a square with one vertex on the point (-3, 5) and a perimeter of 20 units. Write the coordinates of each other vertex.

Solution
Answers vary. Sample response: The coordinates of each point are (-3, 5), (-3, 0), (2, 5), (2, 0).
Problem 3

Statement

i. Plot and connect the following points to form a polygon.
   (-3, 2), (2, 2), (2, -4), (-1, -4), (-1, -2), (-3, -2), (-3, 2)

ii. Find the perimeter of the polygon.

Solution

i. The plotted polygon is a hexagon.

ii. 22 units. (Going in the same order as the points listed, the sides of the polygon have lengths 5, 6, 3, 2, 2, and 4 units.)

Problem 4

Statement

For each situation, select all the equations that represent it. Choose one equation and solve it.

i. Jada's cat weighs 3.45 kg. Andre's cat weighs 1.2 kg more than Jada's cat. How much does Andre's cat weigh?
   
   \[ x = 3.45 + 1.2 \quad \quad x = 3.45 - 1.2 \quad \quad x + 1.2 = 3.45 \quad \quad x - 1.2 = 3.45 \]

ii. Apples cost $1.60 per pound at the farmer's market. They cost 1.5 times as much at the grocery store. How much do the apples cost per pound at the grocery store?
\[ y = (1.5) \cdot (1.60) \quad y = 1.60 \div 1.5 \quad (1.5)y = 1.60 \quad \frac{y}{1.5} = 1.60 \]

**Solution**

i. \[ x = 3.45 + 1.2, \quad x - 1.2 = 3.45; \quad x = 4.65 \]

ii. \[ y = (1.5) \cdot (1.60), \quad \frac{y}{1.5} = 1.60; \quad y = 2.40 \]

(From Unit 6, Lesson 4.)
Section: Common Factors and Common Multiples

Lesson 16: Common Factors

Goals
- Comprehend (orally and in writing) the terms “factor,” “common factor,” and “greatest common factor.”
- Explain (orally and in writing) how to determine the greatest common factor of two whole numbers less than 100.
- List the factors of a number and identify common factors for two numbers in a real-world situation.

Learning Targets
- I can explain what a common factor is.
- I can explain what the greatest common factor is.
- I can find the greatest common factor of two whole numbers.

Lesson Narrative
In this lesson, students use contextual situations to learn about common factors and the greatest common factor of two whole numbers. They develop strategies for finding common multiples and least common multiples. They develop a definition of the terms common factor and greatest common factor for two whole numbers (MP6).

Alignments

Addressing
- 6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR6: Three Reads

Required Materials
- Graph paper
- Snap cubes
Required Preparation

For the first classroom activity, "Diego's Bake Sale," provide access to two different colors of snap cubes (48 of one color and 64 of the other) for students who would benefit from manipulatives. For students with visual impairment, provide access to manipulatives that are distinguished by their shape rather than color.

In the second classroom activity, "Greatest Common Factor," it may be helpful for some students to have access to graph paper to make rectangles that will help them find all possible factors of a whole number.

Student Learning Goals

Let’s use factors to solve problems.

16.1 Figures Made of Squares

Warm Up: 5 minutes

The purpose of this warm-up is for students to notice factors and common factors of 6 and 10 based on rectangles that can be made with the corresponding number of squares. Students may notice that the height of each pair of images changes, but they might not connect this to factors. The whole-class discussion should focus on how the images reflect factors of 6 and 10.

Addressing

- 6.NS.B.4

Launch

Tell students you will show them four pairs of images and their job is to find something that is similar and different about the pairs of images. Tell them to give a signal when they have at least one thing that is similar and one thing that is different. Give students 1 minute of quiet think time followed by a whole-class discussion.

Student Task Statement

How are the pairs of figures alike? How are they different?

![Images of figures made of squares]
**Student Response**

Answers vary. Sample responses:

**Similarities:**
- Each pair has a blue figure and a yellow figure.
- Each figure is made of small squares.
- Each yellow figure is made up of 6 squares. Each blue figure is made up of 10 squares.

**Differences:**
- Some pairs have rectangles some do not.
- Some pairs have "L"-shaped figures and some do not.
- The heights of each pair is going up by 1 every time: 2, 3, 4, 5
- The first pair with a height of 2 is the only one with two rectangles.
- The pair with the height of 4 is the only one with not at least one rectangle.

**Activity Synthesis**

Ask students to share the things that are alike and different among the pairs of images. Record and display their responses for all to see. If possible, reference the images as the students share and record their responses on the images where appropriate.

If the following two ideas do not come up in the conversation, ask students these questions:

- “2 and 3 are both factors of 6. How is this reflected in the diagram?”
- “2 is a factor of both 6 and 10. How is this reflected in the diagram?”
- “4 is not a factor of either 6 or 10. How is this reflected in the diagram?”

End the discussion defining the term factor as one of two or more numbers, that when multiplied together result in a given product. In this particular case, a factor is the height that will make a rectangle have a given area.

**16.2 Diego’s Bake Sale**

15 minutes

Students begin to think about common factors and the greatest common factor in the context of finding ways to group equal amounts of baked goods into bags. Students find all common factors of 2 whole numbers, one representing the number of brownies and another representing cookies. They then compare these factors to determine the greatest common factor.
Monitor for strategies and representations students use to make sure they account for all possible combinations. Some students may organize their work by number of bags, checking each time if the total number can be divided into those bags evenly without remainder. Other students may notice that combinations come in pairs. For example, 4 bags of 12 brownies can be paired with 12 bags of 4 brownies.

**Addressing**

- 6.NS.B.4

**Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

**Launch**

Arrange students in groups of 2. Give students 10 minutes work time followed by whole-class discussion. Encourage students to check in with their partner after each question to make sure they get every possible combination of bags.

**Access for Students with Disabilities**

*Representation: Develop Language and Symbols.* Use virtual or concrete manipulatives to connect symbols to concrete objects. Provide students with snap cubes and containers to model brownie and cookie combinations.

*Supports accessibility for: Conceptual processing*

**Access for English Language Learners**

*Reading: MLR6 Three Reads.* Only reveal the situation, hiding the questions below until the third read. In the first read, done as a whole class, the goal for students to understand the situation (e.g., Diego makes both brownies and cookies for his bake sale). Make sure students understand the meaning of “equal-size” in this context. In the second read, ask students to name the quantities in the problem, noting exact numbers are not necessary at this time for all quantities (e.g., the number of brownies, the number of cookies, the number of bags). In the third read, done in pairs, reveal the questions and instruct students to discuss a plan to solve each part before doing any calculations. This will help students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.

*Design Principle(s): Support sense-making*
Anticipated Misconceptions
Students might not find all combinations of factor pairs for each number. If this is the case, ask them to use snap cubes and prompt them to find more combinations. For example, “Is there a way to place 64 snap cubes into 4 groups with no snap cubes left over? How many are in each group?”

Student Task Statement
Diego is preparing brownies and cookies for a bake sale. He would like to make equal-size bags for selling all of the 48 brownies and 64 cookies that he has. Organize your answer to each question so that it can be followed by others.

a. How can Diego package all the 48 brownies so that each bag has the same number of them? How many bags can he make, and how many brownies will be in each bag? Find all the possible ways to package the brownies.

b. How can Diego package all the 64 cookies so that each bag has the same number of them? How many bags can he make, and how many cookies will be in each bag? Find all the possible ways to package the cookies.

c. How can Diego package all the 48 brownies and 64 cookies so that each bag has the same combination of items? How many bags can he make, and how many of each will be in each bag? Find all the possible ways to package both items.

d. What is the largest number of combination bags that Diego can make with no left over? Explain to your partner how you know that it is the largest possible number of bags.

Student Response

a. Diego can make the following bags with brownies:
   - 1 bag of 48 brownies or 48 bags containing 1 brownie
   - 2 bags with 24 brownies or 24 bags with 2 brownies
   - 3 bags with 16 brownies or 16 bags with 3 brownies
   - 4 bags with 12 brownies or 12 bags with 4 brownies
   - 6 bags with 8 brownies or 8 bags with 6 brownies

b. Diego can make the following bags with cookies:
   - 1 bag of 64 cookies or 64 bags containing 1 cookie
   - 2 bags with 32 cookies or 32 bags with 2 cookies
   - 4 bags with 16 cookies or 16 bags with 4 cookies
   - 8 bags with 8 cookies

c. Diego can make the following bags with both brownies and cookies:
   - 1 bag with 48 brownies and 64 cookies
2 bags with 24 brownies and 32 cookies
- 4 bags with 12 brownies and 16 cookies
- 8 bags with 6 brownies and 8 cookies
- 16 bags with 3 brownies and 4 cookies

d. The largest amount of combination bags that Diego can make is 16 bags with 3 brownies and 4 cookies.

Activity Synthesis
For questions 1 and 2, invite students to share how they organized the different combinations of bags, and highlight their different strategies. Sequence responses by first selecting students who found all combinations using pictorial representations, then students who made an organized list or table, and finally students who were able to highlight the fact that factors come in pairs (i.e. the number of bags and the number of brownies can always be reversed). During this discussion, ask students how they know that they have found all possible bag combinations for each number. Confirm that there should be 10 different bag arrangements for the brownies and 7 different bag arrangements for the chocolate chip cookies.

For questions 3 and 4, ask students to compare answers with a partner. Did they find all the combinations? Confirm that there are 5 different bag arrangements, and the greatest number of bags that can be made is 16. Select a group that used snap cubes to share what this arrangement looks like when represented with the two different colors. If possible, create a visual representation of this arrangement that can be displayed for all to see throughout the rest of the unit.

16.3 Greatest Common Factor

15 minutes
In the last activity, students worked with the concept of common factors in the context of distributing two kinds of baked goods equally. In this activity, students explore common factors of numbers more generally and are introduced to the term greatest common factor. The final question connects the concept of greatest common factor to geometry. They describe what the greatest common factor is and how it applies to a geometric context.

Addressing
- 6.NS.B.4

Instructional Routines
- MLR1: Stronger and Clearer Each Time

Launch
Arrange students in groups of 2. Ask students to discuss what they think a common factor of two numbers is with a partner and select for 1--2 groups to share their thinking. Give 10 minutes of quiet work time followed by whole-class discussion.
Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts. After students have solved the first two problems, check in with either select groups of students or the whole class. Invite students to share the strategies they have used so far, as well as any questions they have before continuing.  
*Supports accessibility for: Organization; Attention*

Access for English Language Learners

*Speaking, Listening, Writing: MLR1 Stronger and Clearer Each Time.* Use this routine to support students to respond to the question “What do you think the term ‘greatest common factor’ means?” Give students time to meet with 2-3 partners, sharing their responses. Encourage the listener can provide feedback that will help teams strengthen their ideas and clarify their language (e.g., “What other details about greatest common factors are important?”). After 2–3 successive shares, individuals can refine and revise their original draft. This helps students use mathematical language to strengthen their understanding of greatest common factor.  
*Design Principle(s): Cultivate conversation; Support sense-making*

Anticipated Misconceptions

Some students may not list all of the factors of a number. Prompt these students to try to find more factors. If additional support is needed, provide graph paper and ask the student if it's possible to make a rectangle area equal to the number with a height of 1, 2, 3, 4, etc. until they are convinced they have all the factors.

**Student Task Statement**

a. The greatest common factor of 30 and 18 is 6. What do you think the term “greatest common factor” means?

b. Find all of the factors of 21 and 6. Then, identify the greatest common factor of 21 and 6.

c. Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

d. A rectangular bulletin board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.
i. What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your reasoning.

ii. How is the solution to this problem related to greatest common factor?

Student Response

a. Answers vary. Possible response: The greatest common factor is the largest factor that numbers share.

b. The factors of 21 are 1, 3, 7, 21. The factors of 6 are 1, 2, 3, and 6. The greatest common factor is 3.

c. The factors of 28 are 1, 2, 4, 7, 14, 28. The factors of 12 are 1, 2, 3, 4, 6, and 12. The greatest common factor is 4.

d. 
   i. The square is 3 inches wide. You could fit 4 squares by 9 squares within the rectangle.
   
   ii. Explanations vary. Sample explanation: The side length of the square must divide 12 since the squares must stack vertically to reach exactly 12 inches. The side length of the square must also divide 27 since the squares must stack horizontally to reach exactly 27 inches. So the side length of the square in inches is a common factor of 12 and 27. That means the side length (in inches) of the largest such square is the greatest common factor of 12 and 27, which is 3.

Are You Ready for More?

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then . . .

○ One student goes down the hall and opens each locker.
○ A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
○ A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
○ A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

Student Response

The lockers that are open are 1, 4, 9, 16, 25... all of the square numbers up to 1,000. This is because most numbers have factor pairs and therefore have an even number of factors. For example, $6 = 1 \cdot 6$ and $2 \cdot 3$. So the locker is opened twice and shut twice, meaning that it is closed at the end of the process. The exceptions are the square numbers, which have an odd number of factors. For example, $25 = 1 \cdot 25$ and $5 \cdot 5$, which means that it is only touched three times: opened, closed, and then opened again.
**Activity Synthesis**

Ask students to share their thinking to question 4. Record and display their responses for all to see. Consider asking how their responses would change if the bulletin board was 18 inches tall and 63 inches wide instead. Encourage students to use the terms “common factor” and “greatest common factor” in their explanations.

**Lesson Synthesis**

In this lesson, students learned about common factors of 2 whole numbers, as well as the greatest common factor. Discuss:

- “What are some situations when finding greatest common factor is helpful?” (When forming the largest amount of equal mixed groups with no items left over, or when determining the largest side length of a square that can be used to tile a rectangle)
- “Explain what greatest common factor means.” (It is the largest factor that numbers share.)
- “How can we can determine the greatest common factor?” (List the factors of each number, circle the ones that are the same, and then find the largest number that is the same.)

**16.4 In Your Own Words**

Cool Down: 5 minutes

**Addressing**

- 6.NS.B.4

**Student Task Statement**

a. What is the greatest common factor of 24 and 64? Show your reasoning.

b. In your own words, what is the greatest common factor of two whole numbers? How can you find it?

**Student Response**

a. 8. The common factors of 24 and 64 are 1, 2, 4, and 8, and 8 is the greatest.

b. Answers vary. Sample response: The greatest common factor of 2 whole numbers is the largest number that divides evenly into both numbers. You can find the greatest common factor by listing the factors of each number and then finding the greatest one that is the same for both numbers.

**Student Lesson Summary**

A factor of a whole number \( n \) is a whole number that divides \( n \) evenly without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly and without a remainder.
A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45; they are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **greatest common factor** (sometimes written as GCF) of two whole numbers is the greatest of all of the common factors. For example, 15 is the greatest common factor for 45 and 60.

One way to find the greatest common factor of two whole numbers is to list all of the factors for each, and then look for the greatest factor they have in common. Let's try to find the greatest common factor of 18 and 24. First, we list all the factors of each number.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

**Glossary**
- **common factor**
- **greatest common factor**
Lesson 16 Practice Problems

Problem 1

Statement

A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers each.

What are the other possibilities? Explain or show your reasoning.

Solution

3 bags with 8 pencils and 12 stickers ($3 \cdot 8 = 24$ and $3 \cdot 12 = 36$)
4 bags with 6 pencils and 9 stickers ($4 \cdot 6 = 24$ and $4 \cdot 9 = 36$)
6 bags with 4 pencils and 6 stickers ($6 \cdot 4 = 24$ and $6 \cdot 6 = 36$)
12 bags with 2 pencils and 3 stickers ($12 \cdot 2 = 24$ and $12 \cdot 3 = 36$)

Problem 2

Statement

i. List all the factors of 42.

ii. What is the greatest common factor of 42 and 15?

iii. What is the greatest common factor of 42 and 50?

Solution

i. 1, 2, 3, 6, 7, 14, 21, 42

ii. 3

iii. 2

Problem 3

Statement

A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers, with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

i. What is the largest number of groups that could be formed? Explain or show your reasoning.
ii. If that many groups are formed, how many students of each grade level would be in each group?

Solution

i. 15 groups. The greatest common factor of 75 and 90 is 15.

ii. 6 sixth-grade students and 5 seventh-grade students (6 \times 15 = 90 and 5 \times 15 = 75)

Problem 4

Statement

Here are some bank transactions from a bank account last week. Which transactions represent negative values?

Monday: $650 paycheck deposited
Tuesday: $40 withdrawal from the ATM at the gas pump
Wednesday: $20 credit for returned merchandise
Thursday: $125 deducted for cell phone charges
Friday: $45 check written to pay for book order
Saturday: $80 withdrawal for weekend spending money
Sunday: $10 cash-back reward deposited from a credit card company

Solution

Tuesday, Thursday, Friday, and Saturday
(From Unit 7, Lesson 13.)

Problem 5

Statement

Find the quotients.

i. \frac{1}{7} \div \frac{1}{8}

ii. \frac{12}{5} \div \frac{6}{5}

iii. \frac{1}{10} \div 10

iv. \frac{9}{10} \div \frac{10}{9}
Problem 6

Statement
An elephant can travel at a constant speed of 25 miles per hour, while a giraffe can travel at a constant speed of 16 miles in $\frac{1}{2}$ hour.

i. Which animal runs faster? Explain your reasoning.

ii. How far can each animal run in 3 hours?

Solution
i. The giraffe is faster, because it covers more distance in the same amount of time.

ii. The elephant can run 75 miles ($25 \times 3 = 75$), and the giraffe can run 96 miles ($16 \times 3 \times 2 = 96$).
Lesson 17: Common Multiples

Goals

○ Comprehend (orally and in writing) the terms “multiple,” “common multiple,” and “least common multiple.”

○ Explain (orally and in writing) how to calculate the least common multiple of 2 whole numbers.

○ List the multiples of a number and identify common multiples for two numbers in a real-world situation.

Learning Targets

○ I can explain what a common multiple is.

○ I can explain what the least common multiple is.

○ I can find the least common multiple of two whole numbers.

Lesson Narrative

In this lesson, students use contextual situations to learn about common multiples and the least common multiples of two whole numbers. They develop strategies for finding common multiples and least common multiples.

Alignments

Addressing

○ 6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

Instructional Routines

○ Anticipate, Monitor, Select, Sequence, Connect

○ MLR2: Collect and Display

○ MLR6: Three Reads

○ MLR8: Discussion Supports

○ Notice and Wonder

○ Think Pair Share

Required Materials

Snap cubes
Required Preparation
For the first classroom activity, "The Florist's Order," provide access to two different colors of snap cubes (100 of each color) to students who would benefit from manipulatives. For students with visual impairment, provide access to manipulatives that are distinguished by their shape rather than by color.

Student Learning Goals
Let's use multiples to solve problems.

17.1 Notice and Wonder: Multiples

Warm Up: 5 minutes
The purpose of this warm-up is to review factors and multiples while eliciting ideas on common factors and common multiples that will be useful in the activities of this lesson. While students may notice and wonder many things about the numbers they have circled, it is important for students to notice the multiples 4 and 6 have in common and wonder what other multiples they would have in common if the counting sequence continued.

Addressing
◦ 6.NS.B.4

Instructional Routines
◦ Notice and Wonder

Launch
Arrange students in groups of 2. Tell students 10 is a multiple of 5 because $10 = 5 \cdot 2$. One way you can find multiples of a number is by skip counting. For example, the multiples of 5 are 5, 10, 15, 20, . . . and so on. Give students 1 minute of quiet work time to circle the multiples of 4 and 6. Give students 1 minute to discuss the things they notice and wonder with a partner, followed by a whole-class discussion.

Student Task Statement
Circle all the multiples of 4 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Circle all the multiples of 6 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

What do you notice? What do you wonder?

Student Response
Answers vary. Possible responses:
Students may notice:

- 4 and 6 have common multiples of 12, 24
- 4 has multiples not in common with 6 like 8, 16, 20...etc.
- 6 has multiples not in common with 4 like 6, 18, 30...etc.
- All of the common multiples of 4 and 6 are multiples of 12

Students may wonder:

- What other multiples would 4 and 6 have in common if we kept counting?
- Why are all of the common multiples of 4 and 6 also multiples of 12?
- Could we multiply 4 by any numbers to match all of the multiples of 6?

**Activity Synthesis**

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If there is time, ask students what other multiples 4 and 6 would have in common if the counting sequence continued. Ask them to explain their reasoning. Record and display their responses for all to see.

**17.2 The Florist’s Order**

10 minutes

Students begin to think about common multiples and the least common multiple when finding ways to place two types of flowers into groups that contain the same number of each flower. Students find all multiples up to 100 for two different numbers. They compare these multiples to determine which ones are the same, and then they determine the least common multiple.

Look for different strategies and representations students use to describe the situation. Some students may draw pictures of groups of flowers, other students may use tables or lists, and other students may do a combination of these.

**Addressing**

- 6.NS.B.4

**Instructional Routines**

- MLR8: Discussion Supports
- Think Pair Share
Launch
Arrange students into groups of 2. Give students 5–7 minutes of quiet work time, then 2 minutes of partner discussion. Follow with whole-class discussion.

Student Task Statement
A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

a. If she ordered no more than 100 of each kind of flower, how many bunches of each could she have ordered? Find all the possible combinations.

b. What is the smallest number of bunches of roses that she could have ordered? What about the smallest number of bunches of lilies? Explain your reasoning.

Student Response
a. She could have placed 4 possible orders:
   - 2 bunches of roses and 3 bunches of lilies (24 of each kind of flower)
   - 4 bunches of roses and 6 bunches of lilies (48 of each kind of flower)
   - 6 bunches of roses and 9 bunches of lilies (72 of each kind of flower)
   - 8 bunches of roses and 12 bunches of lilies (96 of each kind of flower)

b. The smallest amount of bunches she could have ordered is 2 bunches of roses and 3 bunches of lilies (24 of each kind of flower).

Activity Synthesis
Invite students to share how they organized the different combinations of flowers, and highlight the different strategies they used. Strategies to highlight include tables, lists, snap cubes, and other pictorial representations. Confirm that there are 4 different order combinations, and each time there are 24 more of each flower added. Discuss why the smallest number of flowers of each type is 24, and that it takes 2 bunches of roses to equal 24, and 3 bunches of lilies to equal 24. If there was a group that used snap cubes, ask them to share what this arrangement looks like when represented with two different colors. If possible, use student responses to create a visual display of the concept of least common multiple and display it for all to see throughout the unit.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to amplify mathematical uses of language to communicate the multiple strategies used by different students. Provide sentence frames for students to use when they are comparing and contrasting different representations such as: “All ___ have ___ except ___.“, “What makes ___ different from the others is ___.“

Design Principle(s): Support sense-making; Cultivate conversation
17.3 Least Common Multiple

10 minutes
In this activity, students are introduced to the terms common multiple and least common multiple.

Addressing

○ 6.NS.B.4

Instructional Routines

○ MLR2: Collect and Display
○ Think Pair Share

Launch

Arrange students in groups of 2. Ask students to discuss what they think a common multiple of two whole numbers is with their partner. Give 5 minutes of quiet work time followed by 2 minutes of partner discussion. Follow with whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators for finding greatest common factors and least common multiples to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Conversing, Representing: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about the meaning of and reason about least common multiple. Write down phrases (e.g., “the same,” “the smallest,” “multiply”) and representations (e.g., number lines, lists of multiples) you observe students using. Write these on a visual display, as this will help students use mathematical language as they represent least common multiples.

Design Principle(s): Support sense-making

Student Task Statement

The least common multiple of 6 and 8 is 24.

a. What do you think the term “least common multiple” means?

b. Find all of the multiples of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.
c. Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.

Student Response
a. Answers vary. Possible response: Least common multiple is the smallest multiple that numbers share.

b. Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.
Common multiples: 40, 80.
The least common multiple of 10 and 8 is 40.

c. Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.
Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.
Common multiples: only 63.
The least common multiple of 7 and 9 is 63.

Are You Ready for More?
a. What is the least common multiple of 10 and 20?
b. What is the least common multiple of 4 and 24?
c. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

Student Response
a. The multiples of 10 are 10, 20, 30, 40, . . . The multiples of 20 are 20, 40, 60, 80, . . . 20 is the least common multiple.
b. The multiples of 4 are 4, 8, 12, 16, 20, 24, . . . The multiples of 12 are 12, 24, 36, 48, 60, 72, . . . 12 is the least common multiple.
c. If one number is a multiple of the other, then it is a common multiple. For example, in the previous question with 4 and 12, we know that 12 is a multiple of 4 (12 = 4 \times 3) and 12 is a multiple of 12 (12 = 12 \times 1). If one number is a multiple of another, then it must also be the least common multiple, because no multiples of a number are less than the number itself. In the previous question, there are no multiples of 12 that are less than 12.

Activity Synthesis
The purpose of discussion is to clarify the process of finding common multiples and identifying the least common multiple. Ask students to discuss a way to find the least common multiple of any two numbers with a partner. Then in whole-class discussion, invite students to describe their process.
for finding least common multiples. Display several pairs of numbers and ask students to describe their process for finding common multiples and the least common multiple. Encourage students to use the terms “common multiple” and “least common multiple” in their responses.

17.4 Prizes on Grand Opening Day

15 minutes
In this activity, students continue to explore common multiples in context. Prizes are being given away to every 5th, 9th, and 15th customer. Students list the multiples of each number when determining which customers get prizes and when customers get more than one prize. Customers who get more than one prize represent pairwise least common multiples. It is also true that the first customer who gets all 3 prizes represents the least common multiple of all three numbers, but this idea goes beyond the standards being addressed, and there aren't enough customers for this to happen. Students reason abstractly about common multiples and least common multiple to solve problems in context (MP2).

Monitor for students using these strategies:

- List numbers from 1 to 50 and skip count to identify common multiples.
- Analyze common multiples of pairs of numbers, rather than all three numbers at once.
- Denote multiples of different numbers with different shapes, colors, or other notations. Identify common multiples as numbers that have multiple designations.

Addressing
- 6.NS.B.4

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

Launch
Arrange students in groups of 2. Encourage students to discuss their reasoning with their partner as they work. Give students 10 minutes work time followed by a whole-class discussion.
**Access for English Language Learners**

*Reading: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., The bakery is giving away free prizes on opening day.). In the second read, ask students to describe quantities in the story (e.g., total of first 50 people get prizes, every fifth person gets a bagel, every 9th person a muffin, and every 12th person a slice of cake). In the third read, ask students to draw representations of the situation, and then brainstorm possible mathematical solution strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem, keeping the intended level of cognitive demand in the task.

*Design Principle(s): Support sense-making*

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**Student Task Statement**

Lin’s uncle is opening a bakery. On the bakery’s grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every fifth customer will get a free bagel. Every ninth customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

a. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Is he right? If so, how far back should he go to get at least one prize? Explain your reasoning.

b. Jada is the 36th customer.
   
i. Will she get a prize? If so, what prize will she get?
   
   ii. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

   c. How many prizes total will Lin’s uncle give away? Explain your reasoning.

**Student Response**

a. Diego is correct that he will not get a prize because 23 is not a multiple of 5, 9, or 12, and he could get a prize if he went backward in line. If he goes back 1 spot to be the 24th customer, he will get a slice of carrot cake, because 24 is a multiple of 12. If he goes back 2 spots, he will be the 25th customer and get a bagel, because 25 is a multiple of 5. If he goes back 4 spots, he will be the 27th customer and blueberry muffin, because 27 is a multiple of 9.

b.  
i. Jada will get a muffin and a slice of carrot cake, because 36 is a multiple of 9 and of 12.
   
   ii. Yes, it’s possible. Her number in line is a common multiple of both 9 and 12, so she will get both prizes.

---

**Unit 7 Lesson 17**
c. Lin’s uncle will need to give away 10 bagels (to customers 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50), 5 blueberry muffins (to customers 9, 18, 27, 36, and 45), and 4 slices of carrot cake (to customers 12, 24, 36, and 48). Altogether, he will give away 19 prizes \((10 + 5 + 4 = 19)\).

Activity Synthesis

For each unique strategy, select one group to display their work for all to see and explain their reasoning. Sequence their presentations in the order they are presented in the Activity Narrative. If one or more of these strategies isn't brought up by students, demonstrate them. Encourage students to use the terms “common multiple” and “least common multiple.”

Lesson Synthesis

In this lesson, students learned what a common multiple for two whole numbers is as well as the least common multiple. They also developed strategies for how to find the least common multiple of two whole numbers. Discuss:

- “What are some situations when finding least common multiple is helpful?” (When forming the smallest amount of equal groups, or when events both first happen at the same time.)
- “Explain what least common multiple means.” (It is the smallest multiple that numbers share.)
- “How can we can determine least common multiple?” (List multiples of each number until you find the first one that is common to both lists.)

17.5 In Your Own Words Again

Cool Down: 5 minutes

Addressing

- 6.NS.B.4

Student Task Statement

a. What is the least common multiple of 6 and 9? Show your reasoning.

b. In your own words, what is the least common multiple of two whole numbers? How can you find it?

Student Response

a. The least common multiple of 6 and 9 is 18. The first few multiples of 6 are 6, 12, 18, 24, 30, 36. The first few multiples of 9 are 9, 18, 27, 36. The number 18 is the first to appear on both lists.

b. Answers vary. Possible response: The least common multiple of two numbers is the smallest multiple that the numbers share. You can find least common multiple by listing the multiples of each number until you find one that is common to both lists. The first multiple that is common to both lists is the least common multiple.
Student Lesson Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \cdot 4$.

A common multiple for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The least common multiple (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 . . .
- Multiples of 10: 10, 20, 30, 40, 50, . . .

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, . . . ), but 20 is the smallest number that is on both lists, so 20 is the least common multiple.

Glossary

- common multiple
- least common multiple
Lesson 17 Practice Problems

Problem 1

Statement

i. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

ii. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?

iii. Explain how to determine when 2 lights blink together.

Solution

i. 20, 40, 60, 80, 100 . . . seconds, because these are common multiples of 4 and 5.

ii. 36, 72, 108 . . . seconds, because these are common multiples of 15 and 9.

iii. Answers vary. Sample response: They blink together every common multiple.

Problem 2

Statement

i. List all multiples of 10 up to 100.

ii. List all multiples of 15 up to 100.

iii. What is the least common multiple of 10 and 15?

Solution

i. 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

ii. 15, 30, 45, 60, 75, 90

iii. 30

Problem 3

Statement

Cups are sold in packages of 8. Napkins are sold in packages of 12.

i. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

ii. How many sets of cups and napkins will there be?
Solution
i. 3 packages of cups \((3 \cdot 8 = 24)\) and 2 packages of napkins \((2 \cdot 12 = 24)\)
ii. 24 sets

Problem 4
Statement
i. Plot and connect these points to form a polygon.
\((-5, 3), (3, 3), (1, -2), (-3, -2)\)

ii. Find the lengths of the two horizontal sides of the polygon.

Solution
i. A trapezoid is plotted.

ii. The longer horizontal side is 8 units long, and the shorter horizontal side is 4 units long.

(From Unit 7, Lesson 15.)

Problem 5
Statement
Rectangle ABCD is drawn on a coordinate plane. \(A = (-6, 9)\) and \(B = (5, 9)\). What could be the locations of points C and D?

Solution
Answers vary. Sample responses: C could be \((-6, -9)\) and D could be \((5, -9)\), or C could be \((-6, 18)\), and D could be \((5, 18)\).
Problem 6

Statement
A school wants to raise $2,500 to support its music program.

i. If it has met 20% of its goal so far, how much money has it raised?

ii. If it raises 175% of its goal, how much money will the music program receive? Show your reasoning.

Solution

i. $500 (2,500 \cdot 0.2 = 500)

ii. $4,375 (2,500 \cdot 1.75 = 4,375)
Lesson 18: Using Common Multiples and Common Factors

Goals

○ Choose to calculate the greatest common factor or least common multiple to solve a problem about a real-world situation, and justify (orally) the choice.

○ Present (orally, in writing, and using other representations) the solution method for a problem involving greatest common factor or least common multiple.

Learning Targets

○ I can solve problems using common factors and multiples.

Lesson Narrative

In this lesson, students apply what they have learned about factors and multiples to solve a variety of problems. In the first activity, students use what they have learned about common factors and common multiples to solve less structured problems in context (MP1). The two activities that follow are optional. The optional activity "More Factors and Multiples" allows students to explore common factors and common multiples of 3 whole numbers and present their findings. The optional activity "Factors and Multiples Bingo" allows students to practice finding multiples and factors.

Alignments

Addressing

○ 6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.

Instructional Routines

○ Group Presentations

○ MLR2: Collect and Display

○ MLR6: Three Reads

○ MLR8: Discussion Supports
Required Materials
Bingo chips
Pre-printed slips, cut from copies of the Instructional master
Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart

Required Preparation
If using the optional activity, "More Factors and Multiples," provide access to tools for creating a visual display.

If using the optional "Factors and Multiples Bingo" activity, prepare one set of pre-printed slips, one set of pre-printed Bingo boards cut from the Instructional master, and enough Bingo chips for the boards. Each group of 2 students will share 1 Bingo board. It is suggested that the Bingo boards are copied onto cardstock for durability.

Student Learning Goals
Let's use common factors and common multiple to solve problems.

18.1 Keeping a Steady Beat

Warm Up: 5 minutes
In this warm-up, students explore the concept of least common multiple using rhythm.

Addressing
○ 6.NS.B.4

Launch
Tell the class they will be establishing a steady beat as a class. Tell half the class to clap on every other beat and tell the other half to say “yeah!” on every third beat. If time permits, repeat this activity for 3 and 4, 2 and 4, 4 and 6. Follow with a whole-class discussion.

Student Task Statement
Your teacher will give you instructions for playing a rhythm game. As you play the game, think about these questions:

○ When will the two sounds happen at the same time?
○ How does this game relate to common factors or common multiples?

Student Response
When claps happen every 2 beats and yeahs happen every 3 beats, they happen at the same time after 6, 12, 18, and 24 beats. These are all common multiples of 2 and 3. The first time it happens is the least common multiple, which is after 6 beats.
Activity Synthesis
Select 1--2 students to explain what they noticed about the activity and how it is related to least common multiples. Tell students that during the activities in this lesson, they will look for opportunities to solve problems using common factors and common multiples.

18.2 Factors and Multiples

20 minutes
In this activity, students work in pairs to solve problems that involve thinking about factors and multiples as well as the greatest common factor and least common multiple. After solving the problems, they reflect on what type of mathematical work was part of each problem and then record this information into a table. Students begin to notice similarities in the types of problems that involve factors and and in those that involve multiples. Students must make sense and persevere as they decide how the problems relate to common factors and common multiples (MP1).

Addressing
○ 6.NS.B.4

Instructional Routines
○ MLR2: Collect and Display

Launch
Arrange students in groups of 2. Give students 15 minutes of work time followed by whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Look for students to find common factors and common multiples to solve problems involving combinations of quantities.
Supports accessibility for: Memory; Organization
Access for English Language Learners

Writing, Speaking, Listening: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about the strategies they used to determine whether they are answering problems about the least common multiple or the greatest common factor. Write down common phrases you hear students use such as “smallest,” “common,” “largest,” “show up twice,” etc. Write the students’ words on a visual display, and continue to use it as a reference throughout the lesson. This will help students use mathematical language as they make sense of how the problems relate to common factors and common multiples.

Design Principle(s): Support sense-making

Student Task Statement

Work with your partner to solve the following problems.

a. Party. Elena is buying cups and plates for her party. Cups are sold in packs of 8 and plates are sold in packs of 6. She wants to have the same number of plates and cups.
   i. Find a number of plates and cups that meets her requirement.
   ii. How many packs of each supply will she need to buy to get that number?
   iii. Name two other quantities of plates and cups she could get to meet her requirement.

b. Tiles. A restaurant owner is replacing the restaurant’s bathroom floor with square tiles. The tiles will be laid side-by-side to cover the entire bathroom with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 24 feet by 18 feet.
   i. What is the largest possible tile size she could use? Write the side length in feet. Explain how you know it’s the largest possible tile.
   ii. How many of these largest size tiles are needed?
   iii. Name more tile sizes that are whole number of feet that she could use to cover the bathroom floor. Write the side lengths (in feet) of the square tiles.

c. Stickers. To celebrate the first day of spring, Lin is putting stickers on some of the 100 lockers along one side of her middle school’s hallway. She puts a skateboard sticker on every 4th locker (starting with locker 4), and a kite sticker on every 5th locker (starting with locker 5).
   i. Name three lockers that will get both stickers.
ii. After Lin makes her way down the hall, will the 30th locker have no stickers, 1 sticker, or 2 stickers? Explain how you know.

d. **Kits.** The school nurse is assembling first-aid kits for the teachers. She has 75 bandages and 90 throat lozenges. All the kits must have the same number of each supply, and all supplies must be used.

   i. What is the largest number of kits the nurse can make?

   ii. How many bandages and lozenges will be in each kit?

   e. What kind of mathematical work was involved in each of the previous problems? Put a checkmark to show what the questions were about.

<table>
<thead>
<tr>
<th>problem</th>
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<th>finding factors</th>
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<td>Kits</td>
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**Student Response**

a.  
   i. Answers vary. Sample response: Elena could have 24 cups and 24 plates.

   ii. Answers vary. Sample response: Elena could get 3 packages of cups and 4 packages of plates to get 24 of each.

   iii. Answers vary. Sample response: She could get 48 of each if she gets 6 packages of cups and 8 packages of plates. She could also get 72 of each if she gets 9 packages of cups and 12 packages of plates.

b.  
   i. The largest square that will fit has a length of 6 ft.

   ii. 12 tiles are needed to cover the floor.

   iii. She could also use tiles of length 1, 2, or 3 ft.

c.  
   i. Answers vary. Any 3 lockers from 20, 40, 60, 80, and 100 are valid responses. The least common multiple of 4 and 5 is 20. All common multiples of 4 and 5 are multiples of 20.

   ii. Since 30 is not a multiple of 4, the 30th locker will not get a flower sticker. It will only get 1 sticker.

d.  
   i. The greatest amount of kits that can be made is 15.
ii. Each kit will have 5 bandages and 6 lozenges.

e. Are You Ready for More?
You probably know how to draw a five-pointed star without lifting your pencil. One way to do this is to start with five dots arranged in a circle, then connect every second dot.

If you try the same thing with six dots arranged in a circle, you will have to lift your pencil. Once you make the first triangle, you’ll have to find an empty dot and start the process over. Your six-pointed star has two pieces that are each drawn without lifting the pencil.

With twelve dots arranged in a circle, we can make some twelve-pointed stars.

a. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?
b. This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?

c. What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?

d. Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.

e. Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

**Student Response**

a. No, you have to lift up your pencil. There are two pieces.

b. No, you have to lift up your pencil here, too. This time, there are three pieces.

c. No. There are four pieces.

d. Yes. If you connect every fifth dot, you can draw the star in one motion. This also works if you connect every seventh dot. (or every "first" or 11th dot).

e. Answers vary. You can only draw the star without picking up your pencil if the total number of dots and the "skip number" have no factors in common.
Activity Synthesis

The purpose of the discussion for students to express what kinds of problems have to do with least common multiples and what kinds have to do with greatest common factor. For each problem, ask students to indicate whether they think the problem had to do with common multiples or common factors, and invite a few to share their reasoning. Select students to explain their reasoning about how they solved the problems as time allows.

18.3 More Factors and Multiples

Optional: 40 minutes

This activity is optional because it asks students to think about greatest common factor and least common multiple for sets of 3 whole numbers, where the standards only call for students to analyze pairs of whole numbers. In this activity, students work in groups to predict whether a problem involves common factors or common multiples. Groups then solve 1 assigned problem, create a visual display to represent their work, and prepare a brief presentation.

Addressing

- 6.NS.B.4

Instructional Routines

- Group Presentations
- MLR6: Three Reads

Launch

Arrange students in groups of 4. Provide access to tools for creating a visual display. Tell students that they will first read through each problem and discuss whether its solution has to do with finding common factors or common multiples. Give students 5 minutes to discuss questions 1–5. When finished discussing questions 1–5, assign each group 1 of those questions to solve. Give students 10 minutes of work time to solve and 10 minutes to create their visual display and prepare a short presentation.

Access for Students with Disabilities

*Engagement: Internalize Self Regulation.* Provide a project checklist that chunks the various steps of the project into a set of manageable tasks.

*Supports accessibility for: Organization; Attention*
Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of this problem. In the first read, students read the problem with the goal of comprehending the situation (e.g., #1, We have the number of soccer games in a month). In the second read, ask students to analyze the mathematical structure of the story by naming quantities (e.g., #1, every 3rd day and every 4th day). In the third read, ask students to brainstorm possible mathematical solution strategies to answer the questions. Remind students to apply strategies from the previous activity, and encourage students to draw diagrams if necessary. This helps students connect the language in the situations to start identifying when to look for the least common multiple or the greatest common factor.

Design Principle(s): Support sense-making

Student Task Statement

Here are five more problems. Read and discuss each one with your group. Without solving, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

a. Soccer. Diego and Andre are both in a summer soccer league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.

- common multiples
- least common multiple
- greatest common factor

i. What is the first date that both boys will have a game?

ii. How many of their games fall on the same date?

b. Performances. During a performing arts festival, students from elementary and middle schools will be grouped together for various performances. There are 32 elementary students and 40 middle-school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student can be a part of only one group.

- common multiples
- least common multiple
- greatest common factor

i. Name all possible groupings.

ii. What is the largest number of groups that can be formed? How many elementary school students and how many middle school students will be in each group?
c. **Lights.** There is a string of holiday lights with red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midnight.

- common multiples
- least common multiple
- common factors
- greatest common factor

i. After how much time will all 3 lights blink at the exact same time?

ii. How many times total will this happen in one day?

d. **Banners.** Noah has two pieces of cloth. He is making square banners for students to hold during the opening day game. One piece of cloth is 72 inches wide. The other is 90 inches wide. He wants to use all the cloth, and each square banner must be of equal width and as wide as possible.

- common multiples
- least common multiple
- common factors
- greatest common factor

i. How wide should he cut the banners?

ii. How many banners can he cut?

e. **Dancers.** At Elena’s dance recital her performance begins with a line of 48 dancers that perform in the dark with a black light that illuminates white clothing. All 48 dancers enter the stage in a straight line. Every 3rd dancer wears a white headband, every 5th dancer wears a white belt, and every 9th dancer wears a set of white gloves.

- common multiples
- least common multiple
- common factors
- greatest common factor

i. If Elena is the 30th dancer, what accessories will she wear?

ii. Will any of the dancers wear all 3 accessories? If so, which one(s)?

iii. How many of each accessory will the dance teacher need to order?

f. Your teacher will assign your group a problem. Work with your group to solve the problem. Show your reasoning. Pause here so your teacher can review your work.

g. Work with your group to create a visual display that includes a diagram, an equation, and a math vocabulary word that would help to explain your mathematical thinking while solving the problem.

h. Prepare a short presentation in which all group members are involved. Your presentation should include: the problem (read aloud), your group’s prediction of what mathematical concept the problem involved, and an explanation of each step of the solving process.
Student Response

a. This problem involves multiples and least common multiple.
   i. August 12th. They will also have a game together on August 24th.
   ii. Two dates: August 12th and 24th. $3 \times 8 = 24$ and $4 \times 6 = 24$. There are no more dates because the next multiple of 12 is 36 and there are only 31 days in August.

b. This problem involves factors and greatest common factor.
   i. There could be 1 group with 32 elementary school students and 40 middle school students, 2 groups with 16 elementary school students and 20 middle school students, 4 groups with 8 elementary school students and 10 middle school students, or 8 groups with 4 elementary school students and 5 middle school students.
   ii. 8 groups, with 9 students in each group.

c. This problem involves multiples and least common multiple.
   i. Every 24 seconds.
   ii. In 5 hours there are 300 minutes and 1,800 seconds. $1,800 \div 24 = 75$. The 3 lights will blink together 75 times each day.

d. This problem involves factors and greatest common factor.
   i. 18 inches wide.
   ii. 9 banners, $72 \div 18 = 4$, $90 \div 18 = 5$, and $4 + 5 = 9$

e. This problem involves multiples and least common multiple.
   i. Elena will wear a headband and belt. 30 is a multiple of 3 and 5 but not of 9.
   ii. The 45th dancer will have all 3 accessories because it is the least common multiple of 3, 5, and 9.
   iii. 16 headbands (there are 16 multiples of 3 up to 48 and $48 \div 3 = 16$), 9 belts (there are 9 multiples of 5 up to 48), and 5 sets of gloves (there are 5 multiples of 9 up to 48).

Activity Synthesis
Give each group the opportunity to briefly present their visual display and approach to their problem. If time allows, highlight the different ways in which students used diagrams, equations, and vocabulary to represent their work.

18.4 Factors and Multiples Bingo

Optional: 20 minutes
This game provides an opportunity for students to review factors and multiples. There are two versions of the game:

○ Version A, “10 Anywhere”: The teacher mixes the calling cards and randomly selects one at a time. Upon selecting the card, the teacher reads the statement out loud and records it on the
board. Students cover all numbers that fit the statement. When the next card is called, all numbers that fit this next statement are covered. If a number has already been covered, it gets a second bingo chip stacked onto it. (For example, let's say the first card called is “common factors of 25 and 75” and the second card called is “odd multiples of 5.” The numbers 5 and 25 fit both statements, so these numbers would then be double stacked.) The first group that gets 10 chips anywhere on the board calls “bingo”. The teacher listens for the first voice heard. This student will have the opportunity to prove that they have bingo by calling out each number covered on their board and referring to the corresponding statements listed on the board. If a student makes an error, the teacher says, “out” and the game proceeds.

- Version B, “4 in a Row”: In this game, stacking chips is not allowed. Instead, there need to be 4 chips in a row (horizontally, vertically, or diagonally). The same process of calling cards and waiting to hear a winner takes place. In between games, have students switch either bingo cards or partners.

**Addressing**
- 6.NS.B.4

**Instructional Routines**
- MLR8: Discussion Supports

**Launch**

Arrange students in groups of 2. Distribute bingo chips and 1 pre-cut Bingo board from backline master to each group. Explain how both versions of Bingo are played. Play at least one round of each version with the class. Play as many rounds as time allows before whole-class discussion.

**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Begin with a physical demonstration of one example round of each version of the game to support connections between new situations involving bingo and prior understandings of factors and multiples.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

**Anticipated Misconceptions**

Preview the calling cards and decide if students are familiar with all the vocabulary used in the statements. Some of the words may need to be reviewed ahead of time. While playing the game, if you notice students making errors in identifying numbers that fit a particular statement, stop and discuss the meaning of the statement's math vocabulary.

**Student Task Statement**

Your teacher will explain the directions for a bingo game. Here are some things to keep in mind:
Share one bingo board and some bingo chips with a partner.

To play the game, your teacher will read statements aloud. You may help one another decide what numbers fit each statement, but speak only in a whisper. If the teacher hears anything above a whisper, you are out.

The first person to call bingo needs to call out each number and identify the statement that it corresponds to. If there is an error in identifying statements, that player is out and the round continues.

Good luck, and have fun!

Student Response

Answers vary.

Activity Synthesis

After playing a few rounds, discuss:

“Is this a game of luck, strategy, or a combination of both? Explain how you know.” (Luck is part of the game, but students must also be familiar with the math vocabulary words and must also be accurate in identifying all the factors and multiples of various numbers.)

“Are some numbers better to have on a game board than others? Explain how you know.” (Yes, the number zero is not a good number to have because there not many statements fit this value. Larger composite numbers are probably more likely to get covered, because they have many factors and could also be multiples of the smaller numbers that precede them.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. If students need review on vocabulary words, use this to amplify mathematical uses of language to communicate about finding factors and multiples. Invite students to use these words when stating students’ ideas. Provide sentence frames for students to use such as: “To find the factors, I can ___.”, or “To find the multiples, I can ___.”.

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, students solved more challenging problems that involved least common multiple and greatest common factor. Here are some questions for discussion:

“What is greatest common factor? How is it determined?” (The greatest common factor of 2 whole numbers is the greatest number that evenly divides both numbers without remainder. It’s possible to find the greatest common factor by listing all factors for both number and choosing the largest that appears in both lists.)
“What types of situations involve finding greatest common factor?” (Situations that involve having to divide two different numbers into equal groups with no remainders.)

“What is least common multiple? How is it determined?” (The least common multiple of 2 whole numbers is the least number that is a multiple of both numbers. It is possible to find the least common multiple by listing multiples of each number in order and choosing the first multiple that appears on both lists.)

“What types of situations involve finding least common multiple?” (Situations that involve different numbers that need to be multiplied to make the same number.)

18.5 What Kind of Problem?

Cool Down: 5 minutes
Addressing

6.NS.B.4

Student Task Statement

a. For each problem, tell whether finding the answer requires finding a greatest common factor or a least common multiple. You do not need solve the problems.

i. Elena has 20 apples and 35 crackers for making snack bags. She wants to make as many snack bags as possible and wants each bag to have the same combination of apples and crackers. What is the largest number of snack bags she could make?

ii. A string of holiday lights at a store have three colors that flash at different times. Red lights flash every fifth second. Blue lights flash every third seconds. Green light flashes every four seconds. The store owner turns on the lights. After how many seconds will all three lights flash at the same time for the first time?

iii. A florist orders sunflowers every 6 days, starting from the sixth day of the year, and daisies every 4 days, starting from the fourth day of the year. When (or on which day) will she orders both kinds of flowers on the same day?

iv. Noah has 12 yellow square cards and 18 green ones. All the cards are the same size. He would like to arrange the square cards into two rectangles—one of each color. He wants both the yellow and green rectangles to have the same height and to be as tall as possible. What is the tallest possible height for the two rectangles?

b. Explain how you know which problem(s) involves finding the greatest common factor.

Student Response

a. i. Greatest common factor
   ii. Least common multiple
   iii. Least common multiple
iv. Greatest common factor

b. Answers vary. Sample responses:

■ A problem is about finding the greatest common factor when it involves looking for the greatest number of equal sets that can be made from a certain number of items (with no left over), or looking for the greatest length of a rectangle given a certain number of square units.

■ A problem is about finding the greatest common factor when it involves finding the largest possible number that divides into two whole numbers without a remainder.

Student Lesson Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the largest number that can divide into the two whole numbers, we are looking for the greatest common factor.

Suppose we have 12 bagels and 18 muffins and want to make bags so each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 12 and 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

- 1 bag: 12 bagels and 18 muffins
- 2 bags: 6 bagels and 9 muffins
- 3 bags: 4 bagels and 6 muffins
- 6 bags: 2 bagels and 3 muffins

We can see that the largest number of bags that can be made, 6, is the greatest common factor.

If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the first instance the two numbers share a multiple, we are looking for the least common multiple.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy, as shown here.

- Forks: 9, 18, 27, 36, 45, 54, 63, 72, 90 . . .
- Spoons: 15, 30, 45, 60, 75, 90 . . .
If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of utensils we could buy is 45, the least common multiple. This means buying 5 boxes of forks \((5 \cdot 9 = 45)\) and 3 boxes of spoons \((3 \cdot 15 = 45)\).
Lesson 18 Practice Problems

Problem 1

Statement

Mai, Clare, and Noah are making signs to advertise the school dance. It takes Mai 6 minutes to complete a sign, it takes Clare 8 minutes to complete a sign, and it takes Noah 5 minutes to complete a sign. They keep working at the same rate for a half hour.

i. Will Mai and Clare complete a sign at the same time? Explain your reasoning.

ii. Will Mai and Noah complete a sign at the same time? Explain your reasoning.

iii. Will Clare and Noah complete a sign at the same time? Explain your reasoning

iv. Will all three students complete a sign at the same time? Explain your reasoning

Solution

i. Answers vary. Sample response: Yes, they will both finish at 24 minutes, because 24 is a common multiple of 6 and 8.

ii. Answers vary. Sample response: Yes, they will both finish at 30 minutes, because 30 is a common multiple of 6 and 5.

iii. Answers vary. Sample response: No. The first common multiple of 8 and 5 is 40, and 40 minutes is longer than a half hour.

iv. Answers vary. Sample response: No. If Clare and Noah will not finish together, then all three won't, either.

Problem 2

Statement

Diego has 48 chocolate chip cookies, 64 vanilla cookies, and 100 raisin cookies for a bake sale. He wants to make bags that have all three cookie flavors and the same number of each flavor per bag.

i. How many bags can he make without having any cookies left over?

ii. Find the another solution to this problem.

Solution

(The two solutions could swap places.)

i. 4 bags with 12 chocolate chip cookies, 16 vanilla cookies, and 25 raisin cookies.

ii. 2 bags with 24 chocolate chip cookies, 32 vanilla cookies, and 50 raisin cookies.
Problem 3

Statement
i. Find the product of 12 and 8.
ii. Find the greatest common factor of 12 and 8.
iii. Find the least common multiple of 12 and 8.
iv. Find the product of the greatest common factor and the least common multiple of 12 and 8.
v. What do you notice about the answers to question 1 and question 4?
vi. Choose 2 other numbers and repeat the previous steps. Do you get the same results?

Solution
i. $12 \cdot 8 = 96$
ii. 4 is the greatest common factor.
iii. 24 is the least common multiple.
iv. $4 \cdot 24 = 96$.
v. The answers are the same.
vi. Answers vary. Yes, the results are the same.

Problem 4

Statement
i. Given the point (5.5, -7), name a second point so that the two points form a vertical segment. What is the length of the segment?
ii. Given the point (3, 3.5), name a second point so that the two points form a horizontal segment. What is the length of the segment?

Solution
i. Answers vary. Any answer that has an $x$-coordinate of 5.5 will form a vertical segment. The length will be the distance between -7 and the other $y$-coordinate.

ii. Answers vary. Any answer that has a $y$-coordinate of 3.5 will form a horizontal segment. The length will be the distance between 3 and the other $x$-coordinate.
Problem 5

Statement
Find the value of each expression mentally.

i. $\frac{1}{2} \cdot 37 - \frac{1}{2} \cdot 7$

ii. $3.5 \cdot 40 + 3.5 \cdot 60$

iii. $999 \cdot 5$

Solution

i. 15

ii. 350

iii. 4995

(From Unit 6, Lesson 9.)
Section: Let's Put it to Work

Lesson 19: Drawing on the Coordinate Plane

Goals
- Generate a list of ordered pairs to create an image in the coordinate plane, and explain (orally) the reasoning.

Learning Targets
- I can use ordered pairs to draw a picture.

Lesson Narrative
In this optional culminating lesson, students use graphing technology to plot ordered pairs and create images. Using graphing technology for this lesson is highly recommended over doing it with pencil and paper. In the first activity, students recreate a given image to help familiarize them with how the graphing technology works. The second activity, where students design their own image, could be lengthened or shortened as needed by instructing students to create an image with more or less detail. Using graphing technology gives students an opportunity to attend to precision (MP6) because the program graphs exactly what they enter and will not guess at what they might have meant if they input a point incorrectly.

Alignments

Addressing
- 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines
- Group Presentations
- MLR8: Discussion Supports
Required Materials

Graphing technology
Examples of graphing technology are:
a handheld graphing calculator, a computer with
a graphing calculator application installed, and
an internet-enabled device with access to a site
like desmos.com/calculator or geogebra.org/
graphing. For students using the digital
materials, a separate graphing calculator tool
isn't necessary; interactive applets are
embedded throughout, and a graphing
calculator tool is accessible on the student
digital toolkit page.

Graph paper

Required Preparation

Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if
each student has their own device. It is highly recommended to use graphing technology for this
lesson. If graphing technology is not available, using graph paper is a possibility or consider skipping
this lesson.

Student Learning Goals

- Let's draw on the coordinate plane.

19.1 Cat Pictures

Optional: 15 minutes (there is a digital version of this activity)
This activity introduces students to using graphing technology to plot ordered pairs and create
images.

Addressing

- 6.G.A.3
- 6.NS.C.8

Launch

Demonstrate how to use the technology available in the classroom to plot coordinate pairs. If using
the applet or using Desmos in a web browser, consider using these instructions:

- On a blank graph, add a new table by clicking on the “+” icon in the upper left and select “table”
  from the drop-down menu.
- Enter pairs of x- and y-values in the table. Corresponding points should appear on the graph.
- Click on the wheel icon on the upper right corner of left sidebar. The circle next to the y label
  in the table will turn into a solid circle.
- Click on the circle next to y and turn on the “Lines” option in the drop-down menu.
  Consecutive points on the graph will now be connected by line segments.

Provide access to graphing technology. Give students 10–12 minutes of quiet work time followed by
a whole-class discussion.

Unit 7 Lesson 19
**Access for Students with Disabilities**

*Engagement: Internalize Self Regulation.* Provide a project checklist that chunks the various steps of the project into a set of manageable tasks. Consider providing students with a starting point and ordered pairs for select key details such as tips of the ears.

*Supports accessibility for: Organization; Attention*

**Anticipated Misconceptions**

Some students may struggle with getting the graphing technology to do what they want. Consider having a peer show them how to use it or help them identify their mistakes.

Students who are doing the activity on paper may not realize that they should continue listing ordered pairs when they get to the part about adding more details. Encourage them to list the ordered pairs as appropriate.

**Student Task Statement**

Use graphing technology to recreate this image. If graphing technology is not available, list the ordered pairs that make up this image. Then compare your list with a partner.

If you have time, consider adding more details to your image such as whiskers, the inside of the ears, a bow, or a body.
Are You Ready for More?
If you are using graphing technology, add these statements to the list of things being graphed:

\[
\begin{align*}
&x > 6 \\
&y > 5 \\
&x < -4 \\
&y < -6
\end{align*}
\]

Describe the result. Why do you think that happened?

Student Response
Answers vary. Sample response: There is a frame around the cat’s face and everything outside it is shaded. This happened because these are the places where it is true that \( x > 6 \) or \( y > 5 \) or \( x < -4 \) or \( y < -6 \).
Activity Synthesis

Briefly invite students to share what challenges they experienced while graphing the image and how they overcame them. If not brought up by students, highlight the fact that the image's line of symmetry was not on the axis and ask them to share how this affected the coordinates.

19.2 Design Your Own Image

Optional: 30 minutes (there is a digital version of this activity)
In this activity, students use graphing technology to create an image of their own design. While determining the ordered pairs needed to make their image, they have opportunities to think about distances on the coordinate plane, signs in ordered pairs, reflections across an axis, and distance from zero. Students make use of structure as they adjust their ordered pairs until the image looks just the way they want it (MP7).

Addressing

- 6.G.A.3
- 6.NS.C.8

Instructional Routines

- Group Presentations
- MLR8: Discussion Supports

Launch

If needed, review how to use the graphing technology to make sure every student feels capable of plotting points and connecting them.

Student Task Statement

Use graphing technology to create an image of your own design. You could draw a different animal, a vehicle, a building, or something else. Make sure your image includes at least 4 points in each quadrant of the coordinate plane.

If graphing technology is not available, create your image on graph paper, and then list the ordered pairs that make up your image. Trade lists with a partner but do not show them your image. Graph your partner's ordered pairs and see if your images match.

Student Response

Answers vary.

Activity Synthesis

Invite students to share what they thought about while they were figuring out the ordered pairs for their image. Consider doing a gallery walk so students can see each other's designs.
Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their image and their process with a partner. Display sentence frames to support student conversation such as: “First, I ____ because . . .”, “I noticed ____ so I . . .”, “How did you . . .?”, “A different step I used was ____ because . . .”

*Design Principle(s): Support sense-making; Optimize output*
Family Support Materials
Family Support Materials

Rational Numbers

Here are the video lesson summaries for Grade 6, Unit 7: Rational Numbers. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

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Video 1

Video 2


Video 3


Video 4


Video 5


Connecting to Other Units

• Coming soon
Negative Numbers and Absolute Value

Family Support Materials 1

This week, your student will work with signed numbers, or positive and negative numbers. We often compare signed numbers when talking about temperatures. For example, -30 degrees Fahrenheit is colder than -10 degrees Fahrenheit. We say “-30 is less than -10” and write: -30 < -10.

We also use signed numbers when referring to elevation, or height relative to the sea level. An elevation of 2 feet (which means 2 feet above sea level) is higher than an elevation of -4 feet (which means 4 feet below sea level). We say “2 is greater than -4” and write 2 > -4.

We can plot positive and negative numbers on the number line. Numbers to the left are always less than numbers to the right.

We can see that -1.3 is less than 0.8 because -1.3 is to the left of 0.8, but -1.3 is greater than -2.7 because it is to the right of -2.7.

We can also talk about a number in terms of its absolute value, or its distance from zero on the number line. For example, 0.8 is 0.8 units away from zero, which we can write as |0.8| = 0.8, and -2.7 is 2.7 units away from zero, which we can write as |-2.7| = 2.7. The numbers -3 and 3 are both 3 units from 0, which we can write as |3| = 3 and |-3| = 3.
Here is a task to try with your student:

1. A diver is at the surface of the ocean, getting ready to make a dive. What is the diver’s elevation in relation to sea level?

2. The diver descends 100 feet to the top of a wrecked ship. What is the diver’s elevation now?

3. The diver descends 25 feet more toward the ocean floor. What is the absolute value of the diver’s elevation now?

4. Plot each of the three elevations as a point on a number line. Label each point with its numeric value.

Solution:

1. 0, because sea level is 0 feet above or below sea level

2. -100, because the diver is 100 feet below sea level

3. The new elevation is -125 feet or 125 feet below sea level, so its absolute value is 125 feet.

4. A number line with 0, -100, and -125 marked, as shown:
Inequalities

Family Support Materials 2

This week, your student will compare positive and negative numbers with inequalities symbols ($<$ and $>$). They will also graph inequalities in one variable, such as $x < 1$ or $1 > x$, on the number line.

For example, to represent the statement “the temperature in Celsius ($x$) is less than 1 degree,” we can write the inequality $x < 1$ and draw a number line like this:

The diagram shows all numbers to the left of 1 (or less than 1) being possible values of $x$.

We call any value of $x$ that makes an inequality true a solution to the inequality.

This means $x$ values that are greater than -8 are solutions to the inequality $x > -8$. Likewise, $x$ values that are less than 15 could be a solution to the inequality $x < 15$. Depending on the context, however, the solutions may include only positive whole numbers (for example, if $x$ represents the number of students in a class), or any positive and negative numbers, not limited to whole numbers (for example, if $x$ represents temperatures).

Here is a task to try with your student:

A sign at a fair says, “You must be taller than 32 inches to ride the Ferris wheel.” Write and graph an inequality that shows the heights of people who are tall enough to ride the Ferris wheel.

Solution:

If $x$ represents the height of a person in inches, then the inequality $x > 32$ represents the heights of people who can ride the ferris wheel. We can also write the inequality $32 < x$.

The graph of the inequality is:
The Coordinate Plane

Family Support Materials 3

This week, your student will plot and interpret points on the coordinate plane. In earlier grades, they plotted points where both coordinates are positive, such as point $A$ in the figure. They will now plot points that have positive and negative coordinates, such as points $B$ and $C$.

To find the distance between two points that share the same horizontal line or the same vertical lines, we can simply count the grid units between them. For example, if we plot the point $(2, -4)$ on the grid above (try it!), we can tell that the point will be 7 units away from point $A = (2, 3)$.

Points on a coordinate plane can also represent situations that involve positive and negative numbers. For instance, the points on this coordinate plane shows the temperature in degrees Celsius every hour before and after noon on a winter day. Times before noon are negative and times after noon are positive.
For example, the point (5, 10) tells us that 5 hours after noon, or 5:00 p.m, the temperature was 10 degrees Celsius.

Here is a task to try with your student:

In the graph of temperatures above:

1. What was the temperature at 7 a.m.?

2. For which recorded times was it colder than 5 degrees Celsius?

Solution:

1. It was -5 degrees Celsius at 7:00 a.m. You can see this at the point (-5, -5).

2. It was 5 degrees Celsius right at noon, and for the times recorded before that, it was colder.
Common Factors and Common Multiples

Family Support Materials 4

This week, your student will solve problems that involve factors and multiples. Because \(2 \cdot 6 = 12\), we say that 2 and 6 are factors of 12, and that 12 is a multiple of both 2 and 6. The number 12 has other factors: 1, 3, 4, and 12 itself.

Factors and multiples were studied in earlier grades. The focus here is on common factors and common multiples of two whole numbers. For example, 4 is a factor of 8 and a factor of 20, so 4 is a common factor of 8 and 20. 80 is a multiple of 8 and a multiple of 20, so 80 is a common multiple of those two numbers.

One way to find the common factors of two numbers is to list all of the factors for each number and see which factors they have in common. Sometimes we want to find the greatest common factor. To find the greatest common factor of 18 and 24, we first list all the factors of each number and look for the greatest one they have in common.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

To find the common multiples of two numbers, we can do the same. Sometimes we want to find the least common multiple. Let's find the least common multiple of 18 and 24.

- Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, \ldots
- Multiples of 24: 24, 48, 72, 96, 120, 144, 168, 192, \ldots

The first two common multiples are 72 and 144. We can see that 72 is the least common multiple.

Here is a task to try with your student:

A cook is making cheese sandwiches to sell. A loaf of bread can make 10 sandwiches. A package of cheese can make 15 sandwiches. How many loaves of bread and how many packages of cheese should the cook buy so that he can make cheese sandwiches without having any bread or any cheese left over?
Solution:

If he is using up the entire loaf of bread, then the number of sandwiches he can make will be a multiple of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, . . .

If he is using up all of the cheese in each package, then the number of sandwiches he can make will be a multiple of 15: 15, 30, 45, 60, 75, 90, 105, . . .

30, 60, and 90 are some of the common multiples.

- To make 30 sandwiches, he will need 3 loaves of bread (3 \( \cdot \) 10 = 30) and 2 packages of cheese (2 \( \cdot \) 15 = 30).
- To make 60 sandwiches, he will need 6 loaves of bread and 4 packages of cheese.
- To make 90 sandwiches, he will need 9 loaves of bread and 6 packages of cheese.

There are other solutions as well! If he wants to buy the fewest number of loaves and cheese packages, then the first solution is the least.
Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B
Rational Numbers: Check Your Readiness (A)

Do not use a calculator.

1. A ribbon is 24 inches long. You need to cut it into pieces so that all the pieces are the same length.
   
a. Can you cut the ribbon into 5-inch pieces?
   
b. What are all the possible whole-number lengths of the equal-length pieces?

2. Select all the numbers that point \( A \) could reasonably represent on this number line.

   - A. 0.65
   - B. 1.3
   - C. 1.5
   - D. 1.6
   - E. 1.65

3. Here is a number line with some points labeled.

   a. Write the number at \( A \) as a fraction.
   
b. Write the number at \( B \) as a decimal.
   
c. Write the number at \( C \) in two different ways: as a fraction and as a decimal.
4. For each pair of numbers, fill in the blank with <, =, or >.

   a. 1.41     1.397
   b. 1.5     \frac{3}{2}
   c. \frac{4}{3}     \frac{5}{3}
   d. \frac{4}{5}     \frac{4}{7}
   e. \frac{3}{4}     \frac{6}{8}
   f. \frac{4}{9}     \frac{3}{5}

5. Andre and Mai leave the post office at the same time. They walk in opposite directions. Andre walks 50 feet, and Mai walks 30 feet. When they stop walking, how far apart are they? Explain or show your reasoning.

6. Find the coordinates of each point.
7. On a coordinate grid, the \( x \)-axis represents the age of a plant in weeks, and the \( y \)-axis represents the height of the plant in inches.

a. After 1 week, the plant was 2 inches tall. Plot and label this point \( A \).

b. After 2 weeks, the plant was 4 inches tall. Plot and label this point \( B \).

c. After 3 weeks, the plant was 4.5 inches tall. Plot and label this point \( C \).

d. Plot a point \( D \) that you think could represent the plant's age and height after 4 weeks. Give the plant's age and height at that time.
Rational Numbers: Check Your Readiness (B)

Do not use a calculator.

1. A ribbon is 30 inches long. You need to cut it into pieces so that all the pieces are the same length.
   a. Can you cut the ribbon into 4-inch pieces?
   b. What are all the possible whole-number lengths of the equal-length pieces?

2. Select all the numbers that point A could reasonably represent on this number line.
   
   A. 2.5
   B. 2.25
   C. 2.2
   D. 2.0
   E. 1.2

3. Here is a number line with some points labeled.

   a. Write the number A as a fraction.
   b. Write the number B as a decimal.
   c. Write the number C in two different ways: as a fraction and as a decimal.
4. For each pair of numbers, fill in the blank with <, =, or >.

   a. \(0.75 \underline{\quad} \frac{3}{4}\)
   
   b. \(1.5923 \underline{\quad} 1.62\)
   
   c. \(\frac{3}{5} \underline{\quad} \frac{3}{7}\)
   
   d. \(\frac{5}{4} \underline{\quad} \frac{6}{4}\)
   
   e. \(\frac{5}{9} \underline{\quad} \frac{7}{8}\)
   
   f. \(\frac{1}{2} \underline{\quad} \frac{3}{6}\)

5. A lizard and a snail start out next to each other on a drain pipe. The lizard climbs 10 inches up, and the snail climbs 4 inches down. After they stop climbing, how far apart are they? Explain or show your reasoning.

6. Find the coordinates of each point.
7. On a coordinate grid, the horizontal axis represents the number of days since someone bought a gallon of milk. The vertical axis represents the weight of the milk jug, in pounds.

- a. After 1 day, the milk jug weighed 8 pounds. Plot and label this point \textit{A}.
- b. After 2 days, the milk jug weighed 6 pounds. Plot and label this point \textit{B}.
- c. After 3 days, the milk jug weighed 3.5 pounds. Plot and label this point \textit{C}.
- d. Plot a point \textit{D} that you think could represent the weight of the milk jug after 4 days. Give the weight of the milk jug and the number of days.
Rational Numbers: End-of-Unit Assessment (A)

Do not use a calculator.

1. These four numbers are plotted on a number line: $-\frac{2}{3}$, $\frac{5}{8}$, $-\frac{3}{5}$, $-\frac{1}{2}$

Which is the correct ordering on the number line, from left to right?

A. $-\frac{1}{2}, -\frac{3}{5}, -\frac{2}{3}, \frac{5}{8}$

B. $\frac{1}{2}, -\frac{3}{5}, \frac{5}{8}, -\frac{2}{3}$

C. $-\frac{2}{3}, -\frac{3}{5}, -\frac{1}{2}, \frac{5}{8}$

D. $-\frac{3}{5}, -\frac{2}{3}, -\frac{1}{2}, \frac{5}{8}$

2. Diego’s dog weighs more than 10 kilograms and less than 15 kilograms. Select all the inequalities that must be true if $w$ is the weight of Diego’s dog in kilograms.

A. $w > 10$

B. $w < 10$

C. $w > 11$

D. $w < 11$

E. $w > 15$

F. $w < 15$
3. Select all the numbers that are common multiples of 4 and 6.
   A. 1
   B. 2
   C. 10
   D. 12
   E. 24
   F. 40
   G. 60

4. Given \( x = -2 \), mark and place these expressions on the same number line.
   \( x, -x, |-1.5|, -4, |5|, |-6| \)

5. a. Which temperature is warmer, -2 degrees Celsius, or -5 degrees Celsius?

   b. Write an inequality to express the relationship between -2 and -5.

   c. On this number line, graph all the temperatures that are warmer than -2 degrees Celsius.
6. Draw polygon ABCDEF in this coordinate plane, given its vertices $A = (-2, -3)$, $B = (0, -3)$, $C = (0, 1)$, $D = (3, 1)$, $E = (3, 3)$, $F = (-2, 3)$.

7. Starting at 7:00 a.m., Lin spent a day hiking through a canyon. This graph shows her elevation (in meters) at some different times. Negative values of $x$ represent times earlier than noon, and positive values of $x$ represent times later than noon.
a. What was Lin’s elevation at noon? Explain how you know.

b. At 10:00 a.m., Lin’s elevation was 7 meters. Add this point to the graph.

c. At 1:00 p.m., Lin was at sea level. Add this point to the graph.

d. Did Lin’s elevation increase or decrease between 7:00 a.m. and 2:00 p.m.? Explain how you know.

e. Lin climbed downward from 2:00 p.m. to 3:00 p.m. Add a point to the graph that shows her possible elevation at 3:00 p.m. Explain your reasoning.
Rational Numbers: End-of-Unit Assessment (B)

Do not use a calculator.

1. These four numbers are plotted on a number line: \(-\frac{4}{5}, \frac{1}{2}, \frac{1}{8}, -\frac{3}{4}\)

Which is the correct ordering on the number line, from left to right?

A. \(\frac{1}{2}, \frac{1}{8}, -\frac{3}{4}, -\frac{4}{5}\)

B. \(-\frac{4}{5}, -\frac{3}{4}, \frac{1}{2}, \frac{1}{8}\)

C. \(-\frac{3}{4}, -\frac{4}{5}, \frac{1}{8}, \frac{1}{2}\)

D. \(-\frac{4}{5}, -\frac{3}{4}, \frac{1}{8}, \frac{1}{2}\)

2. Select all the numbers that are a common multiple of 8 and 12.

A. 96

B. 80

C. 48

D. 32

E. 24

F. 20

G. 4
3. Given that \( x \) is -4, select all the true statements.

A. Point \( A \) is at \(|6|\)
B. Point \( B \) is at \(-x\)
C. Point \( C \) is at \(|-2.5|\)
D. Point \( D \) is at \(|3|\)
E. Point \( E \) is at \(|x|\)
F. Point \( F \) is at \(-|6|\)

4. A kid’s size small T-shirt is designed to fit children who weigh between 43 and 55 pounds.

a. Write an inequality to describe \( w \), the weight of a child who has outgrown the small T-shirt.

b. Write an inequality to describe \( y \), the weight of a child who is not ready for the small T-shirt yet.

5. a. Which elevation is higher: 8 feet below sea level or 13 feet below sea level?

b. Write an inequality to express the relationship between -8 and -13.

c. On this number line, graph all the elevations that are higher than 8 feet below sea level.
6. Draw polygon $UVWXYZ$ in this coordinate plane, given its vertices $U = (1, 2)$, $V = (4, 5)$, $W = (-1, 5)$, $X = (-3, 3)$, $Y = (-3, -4)$, $Z = (1, -2)$. 
7. This graph shows the temperature (in degrees Celsius) at different times during one winter day. Negative values of $x$ represent times earlier than noon, and positive values of $x$ represent times later than noon.

![Graph of temperature over time]

a. What was the temperature at noon? Explain how you know.

b. At 3:00 p.m., the temperature was $7^\circ C$. Add this point to the graph.

c. At 8:00 a.m., the temperature was $-1^\circ C$. Add this point to the graph.

d. Did the temperature increase or decrease between 8:00 a.m. and noon? Explain how you know.

e. From 4:00 p.m. to 5:00 p.m., it got colder. Add a point to the graph that shows a possible temperature at 5:00 p.m. Explain your reasoning.
Assessment Answer Keys

Check Your Readiness A and B
End-of-Unit Assessment A and B
Assessment Answer Keys

Assessment: Check Your Readiness (A)

Teacher Instructions
Calculators should not be used.

Student Instructions
Do not use a calculator.

Problem 1
The content assessed in this problem is first encountered in Lesson 16: Common Factors.

If most students struggle with this item, plan to spend additional time on Lesson 16 Activity 2, and provide manipulatives to help students act out the problem to understand the context. Teachers will have opportunities throughout Lessons 16 and 17 to help students understand common factors and multiples through contexts. These lessons are designed to use contextual situations to develop strategies and definitions for common factor, greatest common factor, common multiple, and least common multiple.

Statement
A ribbon is 24 inches long. You need to cut it into pieces so that all the pieces are the same length.

1. Can you cut the ribbon into 5-inch pieces?
2. What are all the possible whole-number lengths of the equal-length pieces?

Solution
1. No.
2. 1, 2, 3, 4, 6, 8, and 12 inches.

Aligned Standards
4.OA.B.4

Problem 2
The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.
This problem requires understanding that the value must be larger than 1.5 because it is closer to 2 than to 1, but also that it is not possible to identify, by sight, the exact position of a point on the number line.

If most students struggle with this item, plan to allow for more practice with number lines without tick marks. Lesson 2 Activity 1 provides an opportunity to use landmark numbers in reasoning about placement.

**Statement**

Select all the numbers that point A could reasonably represent on this number line.

- A. 0.65
- B. 1.3
- C. 1.5
- D. 1.6
- E. 1.65

**Solution**

["D", "E"]

**Aligned Standards**

4.NF.C.6

**Problem 3**

The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.

This problem will confirm whether students can identify numbers at tick marks when they must be interpreted. Watch for students giving incorrect denominators or accidentally labeling points as integers.

If most students struggle with this item, plan to allow for more practice with number lines including lines with and without tick marks. Lesson 2 Activity 1 provides an opportunity to use landmark numbers in reasoning about placement. Lessons 3, 4, and 5 incorporate this skill and provide many opportunities for discussion.

**Statement**

Here is a number line with some points labeled.

**Assessment: Check Your Readiness (A)**
1. Write the number at $A$ as a fraction.
2. Write the number at $B$ as a decimal.
3. Write the number at $C$ in two different ways: as a fraction and as a decimal.

**Solution**

1. $\frac{1}{2}$ (or equivalent)
2. 1.25
3. 2.75 or $\frac{11}{4}$ (or equivalent)

**Aligned Standards**

3.NF.A.2

**Problem 4**

The content assessed in this problem is first encountered in Lesson 3: Comparing Positive and Negative Numbers.

Students compare numbers written as decimals and fractions. They can apply a variety of techniques. They might compare with a benchmark fraction, or they might think about the meaning of the numerator and denominator in terms of determining the size of the fraction.

If most students struggle with this item, plan to share examples of common incorrect responses and analyze the errors after doing Lesson 3 Activity 1, Which One Doesn't Belong?

**Statement**

For each pair of numbers, fill in the blank with $<$, $=$, or $>$. 

1. $1.41 \underline{\quad} 1.397$
2. $1.5 \underline{\quad} \frac{3}{2}$
3. $\frac{4}{3} \underline{\quad} \frac{5}{3}$
4. $\frac{4}{5} \underline{\quad} \frac{4}{7}$
5. $\frac{3}{4} \underline{\quad} \frac{6}{8}$
6. $\frac{4}{9} \underline{\quad} \frac{3}{5}$
Solution

1. $1.41 > 1.397$

2. $1.5 = \frac{3}{2}$

3. $\frac{4}{3} < \frac{5}{3}$

4. $\frac{4}{5} > \frac{4}{7}$

5. $\frac{3}{4} = \frac{6}{8}$

6. $\frac{4}{9} < \frac{3}{5}$

Aligned Standards

3.NF.A.3.d, 4.NF.A.2, 5.NBT.A.3.b

Problem 5

The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.

While arithmetic with signed numbers is not part of unit 6.7, students will consider many contexts where numbers represent opposite directions. They will also find the distance between two points that have the same $x$-coordinate or same $y$-coordinate. This item assesses students’ capacity to reason about distances in opposite directions. Students may create a diagram to support their reasoning.

If most students struggle with this item, plan to revisit this item during the synthesis of Lesson 2, and invite students to share how they could draw a diagram to illustrate this situation. Invite students to share how they connect "opposite direction" to "opposites" and to share why a number line is helpful for thinking about this situation. Call on students to share how they reasoned about how far apart Andre and Mai were, using the number line. This is also an opportunity to ask students how they thought about how to scale the number line. Students will have additional opportunities throughout the unit to practice illustrating situations and finding distances using number lines.

Statement

Andre and Mai leave the post office at the same time. They walk in opposite directions. Andre walks 50 feet, and Mai walks 30 feet. When they stop walking, how far apart are they? Explain or show your reasoning.

Solution

80 feet. Sample reasoning: Since they walk in opposite directions, we need to add the distances they each travel.

Assessment: Check Your Readiness (A)
Problem 6
The content assessed in this problem is first encountered in Lesson 11: Points on the Coordinate Plane.

Watch for students getting the coordinates backwards, notably with points $B$ and $C$, but also with $D$ and $E$.

If most students struggle with this item, plan to address any misconceptions during Lesson 11 Activity 1. During the synthesis, share several student responses with an emphasis on the order of an ordered pair. Share the solutions from this problem and ask students if these points could all be on a vertical or horizontal line.

Statement
Find the coordinates of each point.

Solution
$A = (3, 3), B = (1, 4), C = (4, 1), D = (2, 0), E = (0, 1.5)$

Problem 7
The content assessed in this problem is first encountered in Lesson 12: Constructing the Coordinate Plane.
Plane.

Students plot points representing a situation on a coordinate grid.

If most students struggle with this item, plan to spend additional time after Lesson 12 Activity 1 asking students where the points would be plotted on the axes they chose, and ensuring that students understand how to plot a point from a context on a coordinate plane. Students will have more practice interpreting points in a context in Lesson 13.

**Statement**

On a coordinate grid, the $x$-axis represents the age of a plant in weeks, and the $y$-axis represents the height of the plant in inches.

![Coordinate Grid](image)

1. After 1 week, the plant was 2 inches tall. Plot and label this point $A$.
2. After 2 weeks, the plant was 4 inches tall. Plot and label this point $B$.
3. After 3 weeks, the plant was 4.5 inches tall. Plot and label this point $C$.
4. Plot a point $D$ that you think could represent the plant’s age and height after 4 weeks. Give the plant’s age and height at that time.

**Assessment: Check Your Readiness (A)**
Solution

Answers vary. Sample response: \( D = (4, 5) \) indicates that after 4 weeks, the plant is 5 inches tall. Point \( D \)'s x-coordinate must be 4.

Aligned Standards

5.G.A.2
**Assessment : Check Your Readiness (B)**

**Teacher Instructions**
Calculators should not be used.

**Student Instructions**
Do not use a calculator.

**Problem 1**
The content assessed in this problem is first encountered in Lesson 16: Common Factors.

If most students struggle with this item, plan to spend additional time on Lesson 16 Activity 2, and provide manipulatives to help students act out the problem to understand the context. Teachers will have opportunities throughout Lessons 16 and 17 to help students understand common factors and multiples through contexts. These lessons are designed to use contextual situations to develop strategies and definitions for common factor, greatest common factor, common multiple, and least common multiple.

**Statement**
A ribbon is 30 inches long. You need to cut it into pieces so that all the pieces are the same length.

1. Can you cut the ribbon into 4-inch pieces?
2. What are all the possible whole-number lengths of the equal-length pieces?

**Solution**
1. No.
2. 1, 2, 3, 5, 6, 10, and 15 inches.

**Aligned Standards**
4.OA.B.4

**Problem 2**
The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.

This problem requires understanding that the value must be smaller than 2.5 because it is closer to 2 than to 3, but also that it is not possible to identify, by sight, the exact position of a point on the number line.
If most students struggle with this item, plan to allow for more practice with number lines without tick marks. Lesson 2 Activity 1 provides an opportunity to use landmark numbers in reasoning about placement.

**Statement**

Select all the numbers that point \( A \) could reasonably represent on this number line.

- A. 2.5
- B. 2.25
- C. 2.2
- D. 2.0
- E. 1.2

**Solution**

["B", "C"]

**Aligned Standards**

4.NF.C.6

**Problem 3**

The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.

This problem will confirm whether students can identify numbers at tick marks when they must be interpreted. Watch for students giving incorrect denominators or accidentally labeling points as integers.

If most students struggle with this item, plan to allow for more practice with number lines including lines with and without tick marks. Lesson 2 Activity 1 provides an opportunity to use landmark numbers in reasoning about placement. Lessons 3, 4, and 5 incorporate this skill and provide many opportunities for discussion.

**Statement**

Here is a number line with some points labeled.

1. Write the number \( A \) as a fraction.
2. Write the number \( B \) as a decimal.
3. Write the number \( C \) in two different ways: as a fraction and as a decimal.

**Solution**

1. \( \frac{3}{4} \) (or equivalent)
2. 1.5
3. 2.25 and \( \frac{9}{4} \) (or equivalent)

**Aligned Standards**

3.NF.A.2

**Problem 4**

The content assessed in this problem is first encountered in Lesson 3: Comparing Positive and Negative Numbers.

Students compare numbers written as decimals and fractions. They can apply a variety of techniques. They might compare with a benchmark fraction, or they might think about the meaning of the numerator and denominator in terms of determining the size of the fraction.

If most students struggle with this item, plan to share examples of common incorrect responses and analyze the errors after doing Lesson 3 Activity 1, Which One Doesn't Belong?

**Statement**

For each pair of numbers, fill in the blank with \( <, =, \text{ or } > \).

1. 0.75 ________ \( \frac{3}{4} \)
2. 1.5923 ________ 1.62
3. \( \frac{3}{5} \) ________ \( \frac{3}{7} \)
4. \( \frac{5}{4} \) ________ \( \frac{6}{4} \)
5. \( \frac{5}{9} \) ________ \( \frac{7}{8} \)
6. \( \frac{1}{2} \) ________ \( \frac{3}{6} \)

**Solution**

1. 0.75 \( = \) \( \frac{3}{4} \)
2. 1.5923 \( < \) 1.62
3. \( \frac{3}{5} \) \( > \) \( \frac{3}{7} \)
4. \( \frac{5}{4} \) \( < \) \( \frac{6}{4} \)

**Assessment: Check Your Readiness (B)**
5. $\frac{5}{9} < \frac{7}{8}$
6. $\frac{1}{2} = \frac{3}{6}$

**Aligned Standards**

3.NF.A.3.d, 4.NF.A.2, 5.NBT.A.3.b

**Problem 5**

The content assessed in this problem is first encountered in Lesson 2: Points on the Number Line.

While arithmetic with signed numbers is not part of unit 6.7, students will consider many contexts where numbers represent opposite directions. They will also find the distance between two points that have the same $x$-coordinate or same $y$-coordinate. This item assesses students’ capacity to reason about distances in opposite directions. Students may create a diagram to support their reasoning.

If most students struggle with this item, plan to revisit this item during the synthesis of Lesson 2, and invite students to share how they could draw a diagram to illustrate this situation. Invite students to share how they connect "opposite direction" to "opposites" and to share why a number line is helpful for thinking about this situation. Call on students to share how they reasoned about how far apart Andre and Mai were, using the number line. This is also an opportunity to ask students how they thought about how to scale the number line. Students will have additional opportunities throughout the unit to practice illustrating situations and finding distances using number lines.

**Statement**

A lizard and a snail start out next to each other on a drain pipe. The lizard climbs 10 inches up, and the snail climbs 4 inches down. After they stop climbing, how far apart are they? Explain or show your reasoning.

**Solution**

14 inches. Sample reasoning: Since they climb in opposite directions, we need to add the distances they each travel.

**Aligned Standards**

6.NS.C.8

**Problem 6**

The content assessed in this problem is first encountered in Lesson 11: Points on the Coordinate Plane.

Watch for students getting the coordinates backwards, notably with points $A$ and $B$.

If most students struggle with this item, plan to address any misconceptions during Lesson 11 Activity 1. During the synthesis, share several student responses with an emphasis on the order of
an ordered pair. Share the solutions from this problem and ask students if these points could all be
on a vertical or horizontal line.

**Statement**

Find the coordinates of each point.

**Solution**

\[ A = (3, 5), \quad B = (5, 3), \quad C = (4, 1), \quad D = (1, 0), \quad E = (3.5, 0) \]

**Aligned Standards**

5.G.A.1

**Problem 7**

The content assessed in this problem is first encountered in Lesson 12: Constructing the Coordinate Plane.

Students plot points representing a situation on a coordinate grid.

If most students struggle with this item, plan to spend additional time after Lesson 12 Activity 1 asking students where the points would be plotted on the axes they chose, and ensuring that students understand how to plot a point from a context on a coordinate plane. Students will have more practice interpreting points in a context in Lesson 13.

**Statement**

On a coordinate grid, the horizontal axis represents the number of days since someone bought a gallon of milk. The vertical axis represents the weight of the milk jug, in pounds.
1. After 1 day, the milk jug weighed 8 pounds. Plot and label this point $A$.
2. After 2 days, the milk jug weighed 6 pounds. Plot and label this point $B$.
3. After 3 days, the milk jug weighed 3.5 pounds. Plot and label this point $C$.
4. Plot a point $D$ that you think could represent the weight of the milk jug after 4 days. Give the weight of the milk jug and the number of days.

**Solution**

Sample response: $D = (4, 1)$. This indicates that after 4 days, the milk jug weighed 1 pound. Point $D$’s $x$-coordinate must be 4.
Aligned Standards

5.G.A.2
Assessment : End-of-Unit Assessment (A)

Teacher Instructions
Calculators should not be used.

Student Instructions
Do not use a calculator.

Problem 1
Students selecting A have listed the negative fractions in the reverse order. Students selecting B have listed all fractions in increasing order by magnitude, ignoring signs. Students selecting D may believe that fractions are larger in magnitude if their numerators are larger (making the error that \( \frac{3}{5} < \frac{2}{3} \)).

Statement
These four numbers are plotted on a number line: \(-\frac{2}{3}, \frac{5}{8}, -\frac{3}{5}, -\frac{1}{2}\)

Which is the correct ordering on the number line, from left to right?

A. \(-\frac{1}{2}, -\frac{3}{5}, -\frac{2}{3}, \frac{5}{8}\)
B. \(-\frac{1}{2}, -\frac{3}{5}, \frac{5}{8}, -\frac{2}{3}\)
C. \(-\frac{2}{3}, -\frac{3}{5}, -\frac{1}{2}, \frac{5}{8}\)
D. \(-\frac{3}{5}, -\frac{2}{3}, -\frac{1}{2}, \frac{5}{8}\)

Solution
C

Aligned Standards
6.NS.C.6.c

Problem 2
In this problem, there is both a bound from above and from below. These bounds lead to the two correct answers. In addition, students need to think about whether or not inequalities not stated in the problem stem are also valid.

Students selecting B or E have a misunderstanding of the inequality symbols, or perhaps a misunderstanding of how “more” and “less” apply to them. Students selecting C have a misunderstanding about the meaning of the inequality—while \( w \) may be more than 11, it is not...
definitely true. The same is true for D, though this is less likely to be selected. Check in with any student selecting a contradictory pair of options, such as A and B together.

**Statement**

Diego’s dog weighs more than 10 kilograms and less than 15 kilograms. Select all the inequalities that must be true if \( w \) is the weight of Diego’s dog in kilograms.

- A. \( w > 10 \)
- B. \( w < 10 \)
- C. \( w > 11 \)
- D. \( w < 11 \)
- E. \( w > 15 \)
- F. \( w < 15 \)

**Solution**

["A", "F"]

**Aligned Standards**
6.EE.B.8

**Problem 3**

Students selecting A or B may have confused common multiples for common factors. Students selecting C probably calculated \( 4 + 6 \) and have a deep misunderstanding. Students selecting F did not check whether the number was a common multiple, since it is a multiple of 4 only. Students failing to select G may only be looking for common multiples in the range up to the product \( 4 \cdot 6 \).

Check in with any student who does not select D, which is the least common multiple of 4 and 6.

**Statement**

Select all the numbers that are common multiples of 4 and 6.
Problem 4

One likely source of error here is students mistakenly using the absolute value of -4. Another potential source of error is mistakenly replacing |5| with -5. The use of x and -x tests whether students recognize the use and meaning of opposites.

Statement

Given x = -2, mark and place these expressions on the same number line.

\[ x, -x, |\text{-}1.5|, -4, |5|, |\text{-}6| \]

Solution

(The absolute value of a number is its distance from 0 on the number line. The values of the expressions are, in increasing order, -4, -2, 1.5, 2, 5, 6.)

Aligned Standards

6.NS.C.7
Problem 5
The various parts of this problem are interconnected; students may be able to use part c to aid their thinking about parts a and b.

Statement
1. Which temperature is warmer, -2 degrees Celsius, or -5 degrees Celsius?
2. Write an inequality to express the relationship between -2 and -5.
3. On this number line, graph all the temperatures that are warmer than -2 degrees Celsius.

Solution
1. -2 degrees Celsius
2. -2 > -5 (or equivalent)
3. Graph has an open circle at -2 and an arrow pointing to the right

Minimal Tier 1 response:
- Work is complete and correct.
- Acceptable errors: no arrow on the graph, provided that the graph extends all the way to the right on the number line.

Sample:
1. -2
2. -5 > -2
3. Graph includes an open circle at -2 and extends fully to the right.

Tier 2 response:
- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: graph has a closed circle or no circle at -2; answers to parts a and b are incorrect but consistent with one another.

Tier 3 response:
- Significant errors in work demonstrate lack of conceptual understanding or mastery.

Assessment: End-of-Unit Assessment (A)
Sample errors: no graph or badly incorrect graph; work in parts a and b shows extreme confusion about negative numbers.

**Aligned Standards**
6.EE.B.8, 6.NS.C.5, 6.NS.C.7.b

**Problem 6**
Watch for students drawing just the vertices and not the polygon; also watch for students failing to connect point F to point A to complete the polygon.

**Statement**
Draw polygon ABCDEF in this coordinate plane, given its vertices \( A = (-2, -3), \ B = (0, -3), \ C = (0, 1), \ D = (3, 1), \ E = (3, 3), \ F = (-2, 3). \)
Solution

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Polygon drawn as above. Vertices do not need to be marked as discrete points or labeled.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

Assessment: End-of-Unit Assessment (A)
• Sample errors: one or two vertices are incorrect; vertices are correct but connected in the wrong order, e.g., A to C to B; sides A and F are not connected though the rest of the sides of the polygon are present.

Tier 3 response:

• Significant errors in work demonstrate lack of conceptual understanding or mastery.

• Sample errors: consistent errors such as reversing the $x$- and $y$-coordinates; several missing vertices; vertices are plotted but not the sides of the polygon.

Aligned Standards
6.G.A.3, 6.NS.C.6.c, 6.NS.C.8

Problem 7

Students plot points and interpret the meaning of points in all quadrants in context. They will need to think carefully about the meaning of the numbers on both axes, especially the $x$-axis—this has time relative to noon, with negative numbers being before noon and positive numbers being after.

Statement

Starting at 7:00 a.m., Lin spent a day hiking through a canyon. This graph shows her elevation (in meters) at some different times. Negative values of $x$ represent times earlier than noon, and positive values of $x$ represent times later than noon.

1. What was Lin's elevation at noon? Explain how you know.

2. At 10:00 a.m., Lin's elevation was 7 meters. Add this point to the graph.

3. At 1:00 p.m., Lin was at sea level. Add this point to the graph.
4. Did Lin’s elevation increase or decrease between 7:00 a.m. and 2:00 p.m.? Explain how you know.

5. Lin climbed downward from 2:00 p.m. to 3:00 p.m. Add a point to the graph that shows her possible elevation at 3:00 p.m. Explain your reasoning.

Solution

1. 3 meters, because the point (0, 3) is on the graph.

2. The graph should include the point (-2, 7).

3. The graph should include the point (1, 0).

4. Lin’s elevation decreased. Her elevation changed from 10 meters to -2 meters.

5. Answers vary. Sample response: a point at (3, -5). Lin climbed downward so her elevation will be more negative. A correct point will be (3, y) with y < -2.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
  1. 3, because at noon, the point is 3 up.
  2. (-2, 7) plotted
  3. (1, 0) plotted
  4. Decreased. She went down 12 meters.
  5. (3, -5) plotted. Lin dropped further, so the new point is lower.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: misinterpretation of the scale so that each grid line is treated as one unit; one problem part is incorrect.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: failure to understand that the zero mark is noon results in difficulty making headway, but understanding of interpreting of points on graphs is shown; x- and y-coordinates are consistently reversed.

Tier 4 response:

Assessment: End-of-Unit Assessment (A)
• Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

• Sample errors: work does not show understanding of the relationship between points on the graph and the situation.

**Aligned Standards**

6.NS.C.5, 6.NS.C.6.c, 6.NS.C.8
Teacher Instructions
Calculators should not be used.

Student Instructions
Do not use a calculator.

Problem 1
Students selecting A have listed the fractions in the reverse order. Students selecting B may believe that unit fractions with larger denominators are bigger than fractions with smaller denominators (making the error that $\frac{1}{2} < \frac{1}{8}$). Students selecting C may have had trouble with the non-unit fractions or may have forgotten that “more negative” numbers are smallest.

Statement
These four numbers are plotted on a number line: $-, \frac{4}{5}, \frac{1}{2}, \frac{1}{8}, -\frac{3}{4}$

Which is the correct ordering on the number line, from left to right?

A. $\frac{1}{2}, \frac{1}{8}, -\frac{3}{4}, -\frac{4}{5}$

B. $-\frac{4}{5}, -\frac{3}{4}, \frac{1}{2}, \frac{1}{8}$

C. $-\frac{3}{4}, -\frac{4}{5}, \frac{1}{8}, \frac{1}{2}$

D. $-\frac{4}{5}, -\frac{3}{4}, \frac{1}{8}, \frac{1}{2}$

Solution
D

Aligned Standards
6.NS.C.6.c

Problem 2
Students selecting B or D did not check whether the number was a common multiple, since those are multiples of 8 only. Check in with any student who does not select E, which is the least common multiple of 8 and 12. Students selecting F probably calculated $8 + 12$ and have a deep misunderstanding. Students selecting G may have confused common multiples for common factors.
Statement
Select all the numbers that are a common multiple of 8 and 12.

A. 96  
B. 80  
C. 48  
D. 32  
E. 24  
F. 20  
G. 4

Solution
["A", "C", "E"]

Aligned Standards
6.NS.B.4

Problem 3
Students selecting A or failing to select C may believe that taking the absolute value always switches the sign of a number. Students selecting B may be confused about the meaning of opposites. Students selecting F are confusing -|6| with |-6|.

Statement
Given that \( x = -4 \), select all the true statements.

A. Point \( A \) is at \(|6|\)  
B. Point \( B \) is at \(-x\)  
C. Point \( C \) is at \(|-2.5|\)  
D. Point \( D \) is at \(|3|\)  
E. Point \( E \) is at \(|x|\)  
F. Point \( F \) is at \(-|6|\)
Problem 4

Watch for students who may have a misunderstanding of “outgrown” or “not ready” in this situation. Students who reverse the order of the symbols may have a misunderstanding about the meaning of the inequality.

**Statement**

A kid’s size small T-shirt is designed to fit children who weigh between 43 and 55 pounds.

1. Write an inequality to describe \( w \), the weight of a child who has outgrown the small T-shirt.
2. Write an inequality to describe \( y \), the weight of a child who is not ready for the small T-shirt yet.

**Solution**

1. \( 55 < w \) or \( w > 55 \)
2. \( y < 43 \) or \( 43 > y \)

**Aligned Standards**

6.NS.C.7

Problem 5

The various parts of this problem are interconnected; students may be able to use part c to aid their thinking about parts a and b.

**Statement**

1. Which elevation is higher: 8 feet below sea level or 13 feet below sea level?
2. Write an inequality to express the relationship between -8 and -13.
3. On this number line, graph all the elevations that are higher than 8 feet below sea level.

**Solution**

1. 8 feet below sea level
2. \(-8 > -13\) (or equivalent)

**Assessment: End-of-Unit Assessment (B)**
3. Graph has an open circle at -8 and an arrow pointing to the right.

Minimal Tier 1 response:

- Work is complete and correct.
- Acceptable errors: no arrow on the graph, provided that the graph extends all the way to the right on the number line.

Sample:
- 8 feet below sea level
- \(-8 > -13\)
- Graph includes an open circle at -8 and extends fully to the right.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: graph has a closed circle or no circle at -8; answers to parts a and b are incorrect but consistent with one another.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: no graph or badly incorrect graph; work in parts a and b shows extreme confusion about negative numbers.

Aligned Standards
6.EE.B.8, 6.NS.C.5, 6.NS.C.7.b

Problem 6
Watch for students drawing just the vertices and not the polygon; also watch for students failing to connect point \(G\) to point \(A\) to complete the polygon.

Statement
Draw polygon \(U VWXYZ\) in this coordinate plane, given its vertices \(U = (1, 2), V = (4, 5), W = (-1, 5), X = (-3, 3), Y = (-3, -4), Z = (1, -2)\).
Solution

Mineral Tier 1 response:

- Work is complete and correct.

Assessment: End-of-Unit Assessment (B)
• Sample: Polygon drawn as above. Vertices do not need to be marked as discrete points or labeled.

Tier 2 response:

• Work shows general conceptual understanding and mastery, with some errors.

• Sample errors: one or two vertices are incorrect; vertices are correct but connected in the wrong order, e.g., $X$ to $Z$ to $Y$; points $U$ and $Z$ are not connected though the rest of the sides of the polygon are present.

Tier 3 response:

• Significant errors in work demonstrate lack of conceptual understanding or mastery.

• Sample errors: consistent errors such as reversing the $x$- and $y$-coordinates; several missing vertices; vertices are plotted but the sides of the polygon are not.

**Aligned Standards**

6.G.A.3, 6.NS.C.6.c, 6.NS.C.8

**Problem 7**

Students plot points and interpret the meaning of points in all quadrants in context. They will need to think carefully about the meaning of the numbers on both axes, especially the $x$-axis—this has time relative to noon, with negative numbers being before noon and positive numbers being after.

**Statement**

This graph shows the temperature (in degrees Celsius) at different times during one winter day. Negative values of $x$ represent times earlier than noon, and positive values of $x$ represent times later than noon.
1. What was the temperature at noon? Explain how you know.

2. At 3:00 p.m., the temperature was 7°C. Add this point to the graph.

3. At 8:00 a.m., the temperature was -1°C. Add this point to the graph.

4. Did the temperature increase or decrease between 8:00 a.m. and noon? Explain how you know.

5. From 4:00 p.m. to 5:00 p.m., it got colder. Add a point to the graph that shows a possible temperature at 5:00 p.m. Explain your reasoning.

**Solution**

1. 10 degrees Celsius, because the point (0, 10) is on the graph.

2. The graph should include the point (3, 7).

3. The graph should include the point (-4, -1).

4. The temperature increased. The temperature changed from -1 to 10 degrees Celsius.

5. Answers vary. Sample response: a point at (5, 1). It got colder so the temperature will be lower than 4 degrees Celsius. A correct point will be (5, y) with y < 4.

Minimal Tier 1 response:
• Work is complete and correct, with complete explanation or justification.

• Sample:
  ○ 10 degrees Celsius, because the point (0, 10) is on the graph.
  ○ (3, 7) plotted
  ○ (-4, -1) plotted
  ○ Increased. It went up 11 degrees Celsius.
  ○ (5, 1) plotted. A correct point will be (5, y) with y < 4.

Tier 2 response:

• Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

• Sample errors: misinterpretation of the scale so that each grid line is treated as one unit; one problem part is incorrect.

Tier 3 response:

• Work shows a developing but incomplete conceptual understanding, with significant errors.

• Sample errors: failure to understand that the zero mark is noon results in difficulty making headway, but understanding of interpreting points on graphs is shown; x- and y-coordinates are consistently reversed.

Tier 4 response:

• Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

• Sample errors: work does not show understanding of the relationship between points on the graph and the situation.

**Aligned Standards**

6.NS.C.5, 6.NS.C.6.c, 6.NS.C.8
Lesson
Cool Downs
Lesson 1: Positive and Negative Numbers

Cool Down: Agree or Disagree?

State whether you agree with each of the following statements. Explain your reasoning.

1. A temperature of 35 degrees Fahrenheit is as cold as a temperature of -35 degrees Fahrenheit.

2. A city that has an elevation of 15 meters is closer to sea level than a city that has an elevation of -10 meters.

3. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.
Lesson 2: Points on the Number Line

Cool Down: Positive, Negative, and Opposite

1. Put these numbers in order, from least to greatest. If you get stuck, consider using the number line.

   3.5  -1  4.8  -1.5  -0.5  -4.2  0.5  -2.1  -3.5

2. Write two numbers that are opposites and each more than 6 units away from 0.
Lesson 3: Comparing Positive and Negative Numbers

Cool Down: Making More Comparisons

1. The elevation of Death Valley, California, is -282 feet. The elevation of Tallahassee, Florida, is 203 feet. The elevation of Westmorland, California, is -157 feet.
   
a. Compare the elevations of Death Valley and Tallahassee using < or >.

   
b. Compare the elevations of Death Valley and Westmorland.

2. Here are the points $A$, $B$, $C$, and 0 plotted on a number line.

   
   The points $B$ and $C$ are opposites. Decide whether each of the following statements is true.

   
a. $A$ is greater than $B$.

   b. $A$ is farther from 0 than $C$.

   c. $A$ is less than $C$.

   d. $B$ and $C$ are equally far away from 0.

   e. $B$ and $C$ are equal.
Lesson 4: Ordering Rational Numbers

Cool Down: Getting Them in Order

1. Place these numbers in order from least to greatest:

\( \frac{16}{5} \quad -3 \quad 6 \quad 3.1 \quad -2.5 \quad \frac{1}{4} \quad \frac{3}{4} \quad -\frac{3}{8} \)

2. Write a sentence to compare the two points shown on the number line.

Grade 6 Unit 7
Lesson 4
Lesson 5: Using Negative Numbers to Make Sense of Contexts

Cool Down: Bakery Owner

The table shows records of money-related activities of a bakery owner over a period of a week.

<table>
<thead>
<tr>
<th>date</th>
<th>items</th>
<th>amount in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 1</td>
<td>rent</td>
<td>-850.00</td>
</tr>
<tr>
<td>May 2</td>
<td>order (birthday cake and cookies)</td>
<td>106.75</td>
</tr>
<tr>
<td>May 3</td>
<td>utilities (electricity, gas, phone)</td>
<td>-294.50</td>
</tr>
<tr>
<td>May 5</td>
<td>order (wedding cake and desserts)</td>
<td>240.55</td>
</tr>
<tr>
<td>May 5</td>
<td>baking supplies</td>
<td>-147.95</td>
</tr>
<tr>
<td>May 6</td>
<td>order (anniversary cake)</td>
<td>158.20</td>
</tr>
<tr>
<td>May 7</td>
<td>order (breads and desserts for a conference)</td>
<td>482.30</td>
</tr>
<tr>
<td>May 7</td>
<td>bakery sales</td>
<td>415.65</td>
</tr>
</tbody>
</table>

1. For which items did she receive money?

2. What does the number -147.95 mean in this context?

3. Did the bakery owner receive more or spend more money on May 5? Explain how you know.
Lesson 6: Absolute Value of Numbers

Cool Down: Greater, Less, the Same

1. Write a number that has the same value as each expression:
   a. $|5|$
   b. $|-12.9|$

2. Write a number that has a value less than $|4.7|$.

3. Write a number that has a value greater than $|-2.6|$.
Lesson 7: Comparing Numbers and Distance from Zero

Cool Down: True or False?
Mark each of the following as true or false and explain how you know.

1. \(-5 < 3\)

2. \(-5 > 3\)

3. \(|-5| < 3\)

4. \(|-5| > 3\)
Lesson 8: Writing and Graphing Inequalities

Cool Down: A Box of Paper Clips

Andre looks at a box of paper clips. He says: “I think the number of paper clips in the box is less than 1,000.”

Lin also looks at the box. She says: “I think the number of paper clips in the box is more than 500.”

1. Write an inequality to show Andre's statement, using $p$ for the number of paper clips.

2. Write another inequality to show Lin's statement, also using $p$ for the number of paper clips.

3. Do you think both Lin and Andre would agree that there could be 487 paperclips in the box? Explain your reasoning.

4. Do you think both Lin and Andre would agree that there could be 742 paperclips in the box? Explain your reasoning.
Lesson 9: Solutions of Inequalities

Cool Down: Solutions of Inequalities

1. a. Select all numbers that are solutions to the inequality \( w < 1 \).
   - 5
   - -5
   - 0
   - 0.9
   - -1.3

   b. Draw a number line to represent this inequality.

2. a. Write an inequality for which 3, -4, 0, and 2,300 are solutions.

   b. How many total solutions are there to your inequality?
Lesson 10: Interpreting Inequalities

Cool Down: Lin and Andre’s Heights

1. Lin says that the inequalities \( h > 150 \) and \( h < 160 \) describe her height in centimeters. What do the inequalities tell us about her height?

2. Andre notices that he is a little taller than Lin but is shorter than their math teacher, who is 164 centimeters tall. Write two inequalities to describe Andre's height. Let \( a \) be Andre's height in centimeters.

3. Select all heights that could be Andre's height in centimeters. If you get stuck, consider drawing a number line to help you.

   a. 150
   b. 154.5
   c. 160
   d. 162.5
   e. 164
Lesson 11: Points on the Coordinate Plane

Cool Down: Target Practice

Here are the scores for landing an arrow in the colored regions of the archery target.

- Yellow: 10 points
- Red: 8 points
- Blue: 6 points
- Green: 4 points
- White: 2 points

1. Andre shot three arrows and they landed at (-5, 4), (-8, 7) and (1, 6). What is his total score? Show your reasoning.

2. Jada shot an arrow and scored 10 points. She shot a second arrow that landed directly below the first one but scored only 2 points. Name two coordinates that could be the landing points of her two arrows.
Lesson 12: Constructing the Coordinate Plane

Cool Down: What Went Wrong: Graphing Edition

Lin drew this set of axes and plotted the points $A = (1, 2)$, $B = (-3, -5)$, $C = (5, 7)$, $D = (-4, -3)$, and $E = (-4, 6)$ on them.

Identify as many mistakes as you notice in Lin's graph.
Lesson 13: Interpreting Points on a Coordinate Plane

Cool Down: Time and Temperature

The temperature in Princeton was recorded at various times during the day. The times and temperatures are shown in the table.

<table>
<thead>
<tr>
<th>time (hours before or after midnight)</th>
<th>temperature (degrees C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>1.2</td>
</tr>
<tr>
<td>-2</td>
<td>-1.6</td>
</tr>
<tr>
<td>0</td>
<td>-3.5</td>
</tr>
<tr>
<td>8</td>
<td>-6.7</td>
</tr>
</tbody>
</table>

1. Plot points that represent the data. Be sure to label the axes.

2. In the town of New Haven, the temperature at midnight was 1.2°C. Plot and label this point. Which town was warmer at midnight, Princeton or New Haven? How many degrees warmer was it?

3. If the point (3, -2.5) were also plotted on the diagram, what would it mean?
Here are four points on a coordinate plane.

1. What is the distance between points $A$ and $B$?

2. What is the distance between points $C$ and $D$?

3. Plot the point $(-3, 2)$. Label it $E$.

4. Plot the point $(-4.5, -4.5)$. Label it $F$. 

Grade 6 Unit 7
Lesson 14
Lesson 15: Shapes on the Coordinate Plane

Cool Down: Perimeter of A Polygon

1. Plot the following points on the coordinate plane and connect them to create a polygon.

   \[ A = (1, 3) \]
   \[ B = (3, 3) \]
   \[ C = (3, -2) \]
   \[ D = (-2, -2) \]
   \[ E = (-2, 0) \]
   \[ F = (0, 0) \]
   \[ G = (0, 2) \]
   \[ H = (1, 2) \]
   \[ I = (1, 3) \]

2. Find the perimeter of the polygon.
Lesson 16: Common Factors

Cool Down: In Your Own Words

1. What is the greatest common factor of 24 and 64? Show your reasoning.

2. In your own words, what is the greatest common factor of two whole numbers? How can you find it?
Lesson 17: Common Multiples

Cool Down: In Your Own Words Again

1. What is the least common multiple of 6 and 9? Show your reasoning.

2. In your own words, what is the least common multiple of two whole numbers? How can you find it?
Lesson 18: Using Common Multiples and Common Factors

Cool Down: What Kind of Problem?

1. For each problem, tell whether finding the answer requires finding a greatest common factor or a least common multiple. You do not need solve the problems.

   a. Elena has 20 apples and 35 crackers for making snack bags. She wants to make as many snack bags as possible and wants each bag to have the same combination of apples and crackers. What is the largest number of snack bags she could make?

   b. A string of holiday lights at a store have three colors that flash at different times. Red lights flash every fifth second. Blue lights flash every third seconds. Green light flashes every four seconds. The store owner turns on the lights. After how many seconds will all three lights flash at the same time for the first time?

   c. A florist orders sunflowers every 6 days, starting from the sixth day of the year, and daisies every 4 days, starting from the fourth day of the year. When (or on which day) will she orders both kinds of flowers on the same day?

   d. Noah has 12 yellow square cards and 18 green ones. All the cards are the same size. He would like to arrange the square cards into two rectangles—one of each color. He wants both the yellow and green rectangles to have the same height and to be as tall as possible. What is the tallest possible height for the two rectangles?

2. Explain how you know which problem(s) involves finding the greatest common factor.
# Instructional Masters for Rational Numbers

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<th>address</th>
<th>title</th>
<th>students per copy</th>
<th>written on?</th>
<th>requires cutting?</th>
<th>card stock recommended?</th>
<th>color paper recommended?</th>
</tr>
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<td>6.7.A7.BlacklineMaster</td>
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<td>no</td>
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<td>What Number Am I?</td>
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<td>no</td>
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<td>Ordering Rational Number Cards</td>
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<td>yes</td>
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<td>Activity Grade6.7.8.2</td>
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<td>yes</td>
<td>no</td>
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<td>1</td>
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<td>yes</td>
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### 6.7.4.2 Ordering Rational Number Cards.

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<td>4</td>
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<td>7</td>
<td>7.5</td>
<td>8</td>
<td>9</td>
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<td>11</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>22(\frac{1}{2})</td>
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<td>29</td>
<td>30</td>
<td>53</td>
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<tr>
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<td>-22 $\frac{1}{2}$</td>
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<td>-2.5</td>
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<td>Ordering Rational Number Cards - Set 2</td>
<td>Ordering Rational Number Cards - Set 2</td>
<td>Ordering Rational Number Cards - Set 2</td>
</tr>
<tr>
<td>$-\frac{9}{8}$</td>
<td>-2</td>
<td>-1</td>
<td>$-\frac{1}{4}$</td>
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Info Gap: Points on the Number Line
Problem Card 1
The points A, B, C, and D are located on the number line. What is the location of point A?

Data Card 1
- Point A has the same absolute value as B, but a different sign.
- B is less than D.
- Point C is located at -2.
- D is the opposite of C.
- The distance between B and D is 1\frac{1}{2}.

Info Gap: Points on the Number Line
Problem Card 2
The points X, Y, and Z are located on the number line. What is the location of point Z?

Data Card 2
- The absolute value of X is 2.
- Y is greater than X.
- Point Y is closer to zero than point X is.
- Z is positive.
- The distance between X and Y is 1.
- The distance between Y and Z is 4.

Info Gap: Points on the Number Line
Problem Card 1
The points A, B, C, and D are located on the number line. What is the location of point A?

Data Card 1
- Point A has the same absolute value as B, but a different sign.
- B is less than D.
- Point C is located at -2.
- D is the opposite of C.
- The distance between B and D is 1\frac{1}{2}.

Info Gap: Points on the Number Line
Problem Card 2
The points X, Y, and Z are located on the number line. What is the location of point Z?

Data Card 2
- The absolute value of X is 2.
- Y is greater than X.
- Point Y is closer to zero than point X is.
- Z is positive.
- The distance between X and Y is 1.
- The distance between Y and Z is 4.
Stories about 9

A fishing boat can hold fewer than 9 people. How many people can it hold?

{0, 1, 2, 3, 4, 5, 6, 7, 8}
Whole numbers that are at least 0 and are less than 9.

Lin needs more than 9 ounces of butter to make cookies for her party. How many ounces of butter would be enough for the cookies?

Any value higher than 9, including non-whole numbers.

$x > 0$ or $x = 0$, and $x < 9$
A magician will perform her magic tricks only if there are at least 9 people in the audience. For how many people will she perform her magic tricks?

\{9, 10, 11, 12 \ldots \}
Whole numbers that are 9 or higher.

A food scale can measure up to 9 kilograms of weight. What weights can the scale measure?

Any value from 0 to 9, including non-whole numbers.

\ x > 0 \quad \text{or} \quad x = 0, \quad \text{and} \quad x < 9 \quad \text{or} \quad x = 9
<table>
<thead>
<tr>
<th>What Number Am I?</th>
<th>What Number Am I?</th>
<th>What Number Am I?</th>
<th>What Number Am I?</th>
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[Diagram 1]

[Diagram 2]

[Diagram 3]

[Diagram 4]
<table>
<thead>
<tr>
<th>Factors and Multiples Bingo</th>
<th>Factors and Multiples Bingo</th>
<th>Factors and Multiples Bingo</th>
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<tbody>
<tr>
<td>All common factors of 24 and 36</td>
<td>All numbers with 4 or more factors</td>
<td>All even square numbers</td>
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<td>All even multiples of 3</td>
<td>All numbers divisible by 8</td>
<td>All two-digit numbers with digits that sum to 6</td>
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<td>All odd square numbers</td>
<td>All prime numbers greater than 17</td>
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<td>All single-digit composite numbers</td>
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<td>All multiples of 15</td>
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<td>All numbers with exactly 3 factors</td>
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<tr>
<td>All odd multiples of 3</td>
<td>All factors of 10</td>
<td>All odd factors of 180</td>
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6.7.18.4 Factors and Multiples Bingo.
6.7.18.4 Factors and Multiples Bingo.

<table>
<thead>
<tr>
<th>Factors and Multiples Bingo</th>
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<tr>
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### Factors and Multiples Bingo

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