Finding Volume

Teacher Guide
# Finding Volume

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Introduction to the CKMath Program

Welcome to the Core Knowledge Math™ (CKMath) program, based on the carefully researched and designed Illustrative Math™ (IM) instructional materials. IM K-12 Math is a problem-based core curriculum that believes all students are able to understand and use mathematics. Students learn about math by doing math. They bring their current understanding of math and their world experiences to the classroom. In these lessons, students take an active role in the learning process by building on their previous knowledge, and by exploration to develop conceptual understanding instead of being told how to solve problems. Doing math includes: understanding problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning. Encouraging students to participate in mathematical practices with other students gives the opportunity for them to perceive themselves as mathematical thinkers and as part of a mathematical community. By observing students’ understanding of concepts and their thought processes, teachers are able to direct student learning and guide them to recognize the connection between concepts and procedures.

Organization of Units and Lessons

Each unit is divided into sections. Each section revolves around specific goals.

- The Section Overview identifies the learning goals for each section of the unit and describes how students will work towards these goals. Sections are labeled by letters; e.g. Section A, Section B, and so on. Each section uses scaffolding to identify the Common Core Standards that apply to that section. In Fifth Grade, there are five areas covered by the Common Core Standards. They include Operations and Algebraic Thinking (5.OA), Number and Operations in Base Ten (5.NBT), Number and Operations – Fractions (5.NF), Measuring and Data (5.MD), and Geometry (5.G).

  The standards in each section are divided into three groups: Building On, Addressing, and Building Towards. A standard that reflects the work of prior grades and is being used to bridge to a grade-level standard is indicated as Building On. When the standard is focused on the grade-level work, the alignment is indicated as Addressing. A standard that is indicated as Building Towards means that the standard has not yet been achieved by the activities in that section.

- The Center Overview identifies the learning centers to be used in the unit. Each center has different stages, or levels. Students will progress through the stages as they master the objectives for each stage. Each center description includes the Common Core Standards that apply to that stage of the center, a stage narrative describing the activity with possible variations, and a list of materials needed for the center.

- The Standards for Mathematical Practice (MP) describe the types of thinking and behaviors students engage in as they are doing mathematics. Throughout the curriculum, the Teacher Guide identifies lessons and activities where different Mathematical Practices are likely to be observed.

Standards for Mathematical Practice Student Facing Learning Targets

MP1 I Can Make Sense of Problems and Persevere in Solving Them

- I can ask questions to make sure I understand the problem.
- I can say the problem in my own words.
- I can keep working when things aren’t going well and try again.
- I can show at least one try to figure out or solve the problem.
- I can check that my solution makes sense.
MP2 I Can Reason Abstractly and Quantitatively
- I can think about and show numbers in many ways.
- I can identify the things that can be counted in a problem.
- I can think about what the numbers in a problem mean and how to use them to solve the problem.
- I can make connections between real-world situations and objects, diagrams, numbers, expressions, or equations.

MP3 I Can Construct Viable Arguments and Critique the Reasoning of Others
- I can explain or show my reasoning in a way that makes sense to others.
- I can listen to and read the work of others and offer feedback to help clarify or improve the work.
- I can come up with an idea and explain whether that idea is true.

MP4 I Can Model with Mathematics
- I can wonder about what mathematics is involved in a situation.
- I can come up with mathematical questions that can be asked about a situation.
- I can identify what questions can be answered based on data I have.
- I can identify information I need to know and don’t need to know to answer a question.
- I can collect data or explain how it could be collected.
- I can model a situation using a representation such as a drawing, equation, line plot, picture graph, bar graph, or a building made of blocks.
- I can think about the real-world implications of my model.

MP5 I Can Use Appropriate Tools Strategically
- I can choose a tool that will help me make sense of a problem. These tools might include counters, base-ten blocks, tiles, a protractor, ruler, patty paper, graph, table, or external resources.
- I can use tools to help explain my thinking.
- I know how to use a variety of math tools to solve a problem.

MP6 I Can Attend to Precision
- I can use units or labels appropriately.
- I can communicate my reasoning using mathematical vocabulary and symbols.
- I can explain carefully so that others understand my thinking.
- I can decide if an answer makes sense for a problem.

MP7 I Can Look for and Make Use of Structure
- I can identify connections between problems I have already solved and new problems.
- I can compose and decompose numbers, expressions, and figures to make sense of the parts and of the whole.
- I can make connections between multiple mathematical representations.
- I can make use of patterns to help me solve a problem.

MP8 I Can Look for and Express Regularity in Repeated Reasoning
- I can identify and describe patterns and things that repeat.
- I can notice what changes and what stays the same when working with shapes, diagrams, or finding the value of expressions.
- I can use patterns to come up with a general rule.

- Each unit contains between 8 - 25 Lesson Plans.
Each lesson is designed to use 60 minutes. A typical lesson is divided into four phases; a warm-up activity, one or more instructional activities, the lesson synthesis, and a cool-down activity. Every activity within these phases is divided into three parts—the Launch, the Activity, and the Synthesis.

- **Warm-up Activity** - The warm-up activity is designed to strengthen the idea of mathematical community. In these activities, students work with their peers. Students use their personal experiences and mathematical knowledge to develop ideas, ask questions, defend their responses, and evaluate the reasoning of others. A warm-up activity might review a context students have seen before, have them reflect on where the previous lesson left off, or preview a context or idea that will come up in that lesson.

There are several **warm-up routines** that are used during the lessons.

- **Act It Out**—This routine is for kindergarten and first grade students. It encourages young children to understand the relationship between words and numbers. It provides opportunities for students to make sense of story problems. In this routine, students listen to a story problem and act it out through movement, using their fingers, or objects to represent the action in the story.

- **Choral Count**—This routine encourages students to make predictions and think about patterns. It also provides opportunities for students to justify their reasoning. In this routine, students count aloud starting from a given number. The count might be forwards or backwards. The teacher records the numbers on a chart as students say them. Students then stop and look at the written numbers to make predictions and look for patterns.

- **Estimation Exploration**—Estimation Exploration encourages students to use what they know and what they can see to problem-solve for a rough evaluation of a quantity rather than giving a "wild guess." The estimates can be in the context of measurement, computation, or numerosity—estimating about a large group of objects (MP2). In this routine, students make estimates in response to a question about an image. They first think about estimates that would be sensible, but too high or too low. Then they make a reasonable estimate and discuss why their estimate makes sense.

- **How Many Do You See?**—This routine encourages students to see groups when counting. Being able to see groups of objects in an organized way helps them visualize quantities and improves their ability to do mental computation. In this routine, students look at an image, which is typically an arrangement of dots or other shapes. Then students state how many dots or shapes they see. Also included in the discussion will be comments about the way they saw them or determined how many there were. This encourages students to see groups and patterns rather than count each item one by one.

- **Notice and Wonder**—This routine provides an opportunity for students to bring their understandings and experiences to a problem. They share their ideas and ask questions without any pressure to answer or solve a problem. This routine reinforces the importance of making sense of situations before solving a problem. In this routine, students look at an image related to the topic of the lesson and are asked, “What do you notice?” The teacher writes all comments on a chart. They are then asked, “What do you wonder?”, and their questions are also recorded on the chart.

- **Number Talk**—This routine provides an opportunity for students to practice mental math. It helps them solve problems and think about numbers in flexible
They not only justify their own reasoning, but critique the reasoning of others as they make sense of methods for solving problems. In this routine, a series of problems are presented one at a time. Students solve the problem in their head and signal when they have an answer. The teacher takes notes as they justify their answer and explain their method for solving.

- **Questions About Us**—This routine is used with kindergarten students. It provides them opportunities to learn more about their classmates and gives them practice asking questions, organizing quantities, counting, and analyzing data. In this routine, students ask their classmates a question with two choices. They keep track of the answers and count the responses. The teacher then asks follow up questions that students answer using the data that they collected.

- **True or False?**—This routine encourages students to make sense of equations, often without any computation. It provides another opportunity for students to justify their reasoning as they explain to others what they are thinking. In this routine, students are presented with a series of equations, one at a time. Some equations may be true, and some may be false. Students use what they know about place value, operations, and number relationships to decide if each is true or false. And then, students explain how they know.

- **What Do You Know About _____?**—This routine encourages students to share their experiences and understandings about a math topic. In this routine, students are presented with a number, expression, or are asked a general question about a math topic. They then list everything they know about that topic. The teacher writes what students say and then references the list later so that students can add more ideas.

- **Which One Doesn’t Belong?**—This routine provides an opportunity for students to reason about characteristics of shapes, math tools, or other images to decide which one doesn’t belong. Because any answer is correct, students are able to focus on communicating their reasoning and justifying their choice. In this routine, students are shown 4 different images, which may be numbers, equations, shapes, images, or diagrams. They decide which one doesn’t belong and explain why.

**Instructional Activities**—After the warm-up, lessons consist of one to three instructional activities.

Instructional Activities include:

- **5 Practices**—Lessons that include this routine are designed to allow students to solve problems in ways that make sense to them. During the activity, students engage in a problem in meaningful ways and teachers monitor to uncover and nurture conceptual understandings. During the activity synthesis, students collectively reveal multiple approaches to a problem and make connections between these approaches (MP3).

- **Card Sort**—A card sorting task gives students opportunities to analyze representations, statements, and structures closely, and make connections (MP2 and MP7). As students work, teachers monitor for the different ways groups choose their categories, and encourage increasingly precise mathematical language (MP6).

- **MLR1 Stronger and Clearer Each** *(MLR stands for Mathematics Learning Routine.)*—Provides students with a structured and interactive opportunity to revise and refine both
their ideas and their verbal and written output. *Embedded in grades 3–5.*

- **MLR2 Collect and Display**—Captures a variety of students’ oral words and phrases into a stable, collective reference. Output can be organized, re-voiced, or explicitly connected to other languages in a display that all students can refer to, build on, or make connections with during future discussion or writing. *Embedded in grades K–5.*

- **MLR3 Clarify, Critique, Correct**—Gives students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. *Embedded in grades 3–5.*

- **MLR4 Information Gap**—Creates an authentic need for students to communicate. Partners or team members are given different pieces of necessary information that must be used together to solve a problem. *Embedded in grades 3–5.*

- **MLR5 Co-craft Questions**—Allows students to get inside a context before feeling pressure to produce answers, and creates opportunities for students to produce the language of mathematical questions. *Embedded in grades 2–5.*

- **MLR6 Three Reads**—Supports reading comprehension, sense-making, and meta-awareness of mathematical language. Students take time to understand mathematical situations and story problems, and plan their strategies before finding solutions. *Embedded in grades K–5.*

- **MLR7 Compare and Connect**—Fosters students’ meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. *Embedded in grades K–5.*

- **MLR8 Discussion Supports**—Includes a large variety of teacher moves that support rich discussions about mathematical ideas, representations, contexts, and strategies. *Embedded in grades K–2.*

- **Lesson Synthesis**—After the instructional activities are completed, students take time to reflect on the knowledge they have gained during the instructional activities and incorporate this with their previous knowledge. The lesson synthesis activity should take 5–10 minutes. During this time, teachers help students with this process by asking questions verbally and having students respond orally or in a written journal, by asking students to add on to a graphic organizer or concept map, or some similar activity.

- **Cool-down Activity**—The cool-down activity is given to students at the end of the lesson. This activity should take about 5 minutes. Students work on the cool-down independently and turn it in. The teacher uses the cool-down as a formative assessment to determine if students understand the lesson and to adjust further instruction. *Note: The Cool-down activity is identified in the introduction to the lesson plan and not at the end of the lesson.*

- **Assessments**—There are several opportunities for assessment during each unit.
  
  - Pre-unit problems can be used as a pre-unit assessment.
  
  - Each instructional task includes expected student responses and suggestions to advance student thinking. Teachers will adjust their instruction depending on how the students respond to the task. Frequently there are suggested questions to help teachers better understand students’ thinking.

  - Practice problems are provided for each lesson that can be used for in-class practice, homework, or as a means to assess certain learning on a particular concept.
• Each section has a checklist to indicate that students are meeting the section goals.

• Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit.

## Unit Resources

### Teacher Components

**Teacher Guide:** The Teacher Guide for each unit contains an overview of the sections in which the unit is divided, a description of the centers students will use with the unit, detailed lesson plans, and teacher resources. Within the overview of the unit sections can be found suggested activities from each unit section that can be used as a PLC activity for teachers. PLCs, or Professional Learning Communities provide teachers the ability to work collaboratively in recurring cycles of collective inquiry and action research to achieve better results for students. PLCs give teachers the opportunity to discuss and plan instruction with peers.

The first few pages of each detailed lesson plan are directed to the teacher. Support notes to the teacher are in gray boxes throughout the lesson plan. On these first pages can be found:

- Alignment to the Common Core Standards
- Learning Goals
  - Teacher-facing learning goals appear at the top of lesson plans. They are directed to the teacher and describe the mathematical and pedagogical goals of the lesson.
  - Student-facing learning goals are directed to the student and start with the word “Let’s.” These learning goals can be written on the board before class begins. They are used to invite students into the work of that day without giving away too much and spoiling the problem-based instruction.
- Lesson Purpose
- Suggestions for instruction for English Learners and Students with Disabilities
- Instructional Routines
- List of materials needed for the lesson
- Lesson Timeline
- Description of the Cool-Down Activity
- Teacher Reflection Question – The purpose of this question is to provide a direction to the teachers to think critically about their teaching during the lesson.
- Sample Student Responses

At the back of the Teacher Guide are Teacher Resources for the unit.

- Family Support Materials
- Assessments
- Cool Downs
- Instructional Masters

These resources are also available for download from the CKF website and can be found in the section titled Individual Resources for each unit. Also included in the online resources are individual Printable Lesson Plans and the PowerPoint Slides for the unit.
**PowerPoint Slides**: Slides are provided that can be used in the classroom. These slides include questions and directions to be used during the lesson activities, as well as vocabulary, visuals, and other support to be used during the lesson.

**Student Component**

**Activity Book**: The Activity Book is used by the students during the lessons. It coordinates with the lesson plans. It displays the student-facing learning goals for each lesson as well as activity sheets for some activities. Not all activities will use the Activity Book.
Introduction to Grade 5

The big ideas in grade 5 include: developing fluency with addition and subtraction of fractions, developing understanding of multiplication and division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions), extending division to two-digit divisors, developing understanding of operations with decimals to hundredths, developing fluency with whole number and decimal operations, and developing understanding of volume.

Grade 5 is divided into eight units:

1. Finding Volume
2. Fractions as Quotients and Fraction Multiplication
3. Multiplying and Dividing Fractions
4. Wrapping Up Multiplication and Division with Multi-digit Numbers
5. Place Value Patterns and Decimal Operations
6. More Decimal and Fraction Operations
7. Shapes on the Coordinate Plane
8. Putting it All Together

Unit 1: Finding Volume

At a Glance

Unit 1 is estimated to be completed in 13-14 days including 2 days for assessment.

This unit is divided into three sections including 11 lessons and 1 optional lesson.

- Section A—Unit Cubes and Volume (Lessons 1-4)
- Section B—Expressions for Finding Volume (Lessons 5-7)
- Section C—Volume of Solid Figures (Lessons 8-12)

On page 6 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses four student centers.

- Can You Build It?
- Capture Squares
- Five in a Row: Multiplication
- Rolling for Fractions
Unit 1: Finding Volume

Unit Learning Goals

- Students find the volume of right rectangular prisms and solid figures composed of two right rectangular prisms.

This unit introduces students to the concept of volume by building on their understanding of area and multiplication.

In grade 3, students learned that the area of a two-dimensional figure is the number of square units that cover it without gaps or overlaps. They first found areas by counting squares and began to intuit that area is additive. Later, they recognized the area of a rectangle as a product of its side lengths and found the area of more-complex figures composed of rectangles.

Here, students learn that the volume of a solid figure is the number of unit cubes that fill it without gaps or overlaps. First, they measure volume by counting unit cubes and observe its additive nature. They also learn that different solid figures can have the same volume.

Next, they shift their focus to right rectangular prisms: building them using unit cubes, analyzing their structure, and finding their volume. They write numerical expressions to represent their reasoning strategies and work with increasingly abstract representations of prisms.

Later, students generalize that the volume of a rectangular prism can be found by multiplying its side measurements (length $\times$ width $\times$ height), or by multiplying the area of the base and its height (area of the base $\times$ height). As they analyze, write, and evaluate different expressions that represent the volume of the same prism, students revisit familiar properties of operations from earlier grades.

Later in the unit, students apply these understandings to find the volume of solid figures composed of two non-overlapping rectangular prisms and solve real-world problems involving such figures. In doing so, they also progress from using cubes to using standard units to measure volume.
Section A: Unit Cubes and Volume

Standards Alignments

Building On 3.OA.C.7
Addressing 5.MD.C.3, 5.MD.C.3.b, 5.MD.C.4, 5.MD.C.5.a, 5.OA.A.2
Building Towards 5.MD.C.3, 5.MD.C.5, 5.MD.C.5.a

Section Learning Goals

- Describe volume as the space taken up by a solid object.
- Measure the volume of a rectangular prism by finding the number of unit cubes needed to fill it.
- Use the layered structure in a rectangular prism to find volume.

In this section, students make sense of volume as a measurement of three-dimensional figures by building objects with unit cubes and counting the cubes. They experiment with different figures made from the same number of cubes and see them as having the same volume.

Students then build right rectangular prisms and analyze images of prisms constructed of unit cubes. To find the volume of these solids, students look at their structure and relate the number of horizontal and vertical layers to the total number of cubes (MP7). They engage with the commutative and associative properties of multiplication as they reason about the volume of rectangular prisms that are oriented in different ways.

![Images of rectangular prisms]

PLC: Lesson 4, Activity 1, Layers in Rectangular Prisms

Suggested Centers

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)
- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)
Section B: Expressions for Finding Volume

Standards Alignments
Addressing  5.MD.C.4, 5.MD.C.5.a, 5.MD.C.5.b, 5.OA.A.1, 5.OA.A.2
Building Towards  5.MD.C.5.b

Section Learning Goals
- Describe the calculations from the previous section as \( \text{length} \times \text{width} \times \text{height} \) or area of the base \( \times \) height.
- Find volume using \( \text{length} \times \text{width} \times \text{height} \) or area of the base \( \times \) height.

In this section, students continue to work with right rectangular prisms and to relate side measurements to volume. They observe that multiplying the number of layers of cubes in a prism by the number of cubes in one layer gives its volume. They also see that the number of cubes in one layer is in essence the area of a rectangle.

Students then generalize the volume of a right rectangular prism as the product of its side lengths, \( \text{length} \times \text{width} \times \text{height} \) and as the product of the area of its base and its height, base area \( \times \) height.

To promote flexible use of measurements and sense making in finding volume, students connect these mathematical terms to numerical expressions that represent volume, rather than relying on algebraic formulas. This work reinforces the associative property of multiplication and highlights that the volume of a rectangular prism can be represented with equivalent multiplication expressions.

PLC: Lesson 5, Activity 2, Growing Prism

Suggested Centers
- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
Section C: Volume of Solid Figures

Standards Alignments
Building On 5.MD.C.5.a
Addressing 5.MD.C, 5.MD.C.5, 5.MD.C.5.c, 5.OA.A.1, 5.OA.A.2
Building Towards 5.NBT.A.2

Section Learning Goals

- Find the volume of a figure composed of rectangular prisms.

In this section, students apply their understanding of volume to solve real-world and mathematical problems. They encounter solid figures that are composed of two or more right rectangular prisms, which reinforces their understanding of the additive nature of volume.

Students also work with side lengths that are larger than those in earlier sections, prompting them to activate multiplication strategies from earlier grades. The work reminds students that they can decompose multi-digit factors by place value to find their product, paving the way toward the standard algorithm for multiplication in a later unit.

PLC: Lesson 10, Activity 2, Find the Volume in Different Ways

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
Throughout the Unit

Students build on the multiplication and place value concepts from grades 3 and 4 to work toward fluency in multi-digit multiplication in grade 5. Because this unit focuses on the multiplication of three factors to find the volume, each Number Talk in this unit allows students to practice multiplying mentally with three factors. The product builds from simple factors such as $3 \times 2 \times 4$ to factors that include multiples of 10, such as $60 \times 20 \times 10$.

This progression intentionally incorporates multiples of 10 so that teachers can see how students apply place value understanding and the associative and distributive properties of operations to multiply. Teachers can use these warm-ups to build towards the grade 5 work with place value and multiplication.

Here is a sampling of Number Talk warm-ups in the unit.

<table>
<thead>
<tr>
<th>lesson 3</th>
<th>lesson 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 4$</td>
<td>$6 \times 2$</td>
</tr>
<tr>
<td>$3 \times 2 \times 4$</td>
<td>$6 \times 2 \times 10$</td>
</tr>
<tr>
<td>$3 \times 2 \times 5$</td>
<td>$6 \times 20 \times 10$</td>
</tr>
<tr>
<td>$3 \times 2 \times 6$</td>
<td>$60 \times 20 \times 10$</td>
</tr>
</tbody>
</table>
## Materials Needed

<table>
<thead>
<tr>
<th>LESSON</th>
<th>GATHER</th>
<th>COPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>• Connecting cubes</td>
<td>• Isometric Dot Paper Standard (groups of 1)</td>
</tr>
<tr>
<td>A.2</td>
<td>• Connecting cubes</td>
<td>• none</td>
</tr>
<tr>
<td>A.3</td>
<td>• Connecting cubes</td>
<td>• Card Sort Rectangular Prism Cards (groups of 2)</td>
</tr>
<tr>
<td></td>
<td>• Connecting cubes</td>
<td></td>
</tr>
<tr>
<td>A.4</td>
<td>• Connecting cubes</td>
<td>• none</td>
</tr>
<tr>
<td>B.5</td>
<td>• Connecting cubes</td>
<td>• none</td>
</tr>
<tr>
<td></td>
<td>• Materials from a previous activity</td>
<td></td>
</tr>
<tr>
<td>B.6</td>
<td>• Connecting cubes</td>
<td>• Matching Prisms and Expressions (groups of 2)</td>
</tr>
<tr>
<td>B.7</td>
<td>• Rulers (centimeters)</td>
<td>• Info Gap Volume Cards (groups of 2)</td>
</tr>
<tr>
<td></td>
<td>• Rulers (inches)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Yardsticks</td>
<td></td>
</tr>
<tr>
<td>C.8</td>
<td>• Connecting cubes</td>
<td>• none</td>
</tr>
<tr>
<td>C.9</td>
<td>• none</td>
<td>• Isometric Dot Paper Standard (groups of 1)</td>
</tr>
<tr>
<td>C.10</td>
<td>• none</td>
<td>• none</td>
</tr>
<tr>
<td>C.11</td>
<td>• none</td>
<td>• none</td>
</tr>
<tr>
<td>C.12</td>
<td>• Connecting cubes</td>
<td>• none</td>
</tr>
<tr>
<td></td>
<td>• Patty paper</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Tools for creating a visual display</td>
<td></td>
</tr>
</tbody>
</table>
Center: Can You Build It? (3–5)

Stage 2: Multiple Rectangles

Lessons
- Grade5.1.A1 (supporting)
- Grade5.1.A2 (supporting)

Stage Narrative
Before playing, students remove the cards that show 6 or higher and set them aside.

Students flip two number cards to get a number of tiles and then each partner tries to create as many rectangles as possible for that area. If both students each found all the rectangles, they get one point. A student gets two points for any rectangle they built that their partner did not. The player who gets the most points after eight rounds is the winner. Students may choose to draw their rectangle on grid paper, rather than use inch tiles.

This center stage is the first time Number Cards 0–10 are used in Grade 4, so they are provided as a Instructional master. Students will continue to use these throughout the year. Consider copying them on cardstock or laminating them and keeping them organized to be used repeatedly.

Standards Alignments
Addressing 4.OA.B.4

Materials to Gather
Folders, Grid paper, Inch tiles

Materials to Copy
Can You Build It Stage 2 Directions (groups of 6), Number Cards (0-10) (groups of 2)

Additional Information
Each group of 2 needs at least 120 inch tiles and a set of number cards.

Stage 3: Rectangular Prisms

Lessons
- Grade5.1.A1 (addressing)
- Grade5.1.A2 (addressing)
- Grade5.1.A3 (addressing)
- Grade5.1.A4 (addressing)
- Grade5.1.B5 (addressing)
Stage Narrative

Before playing, students remove the cards that show 6 or higher and set them aside.

Students flip two number cards to get a number of cubes and then each partner tries to create as many prisms as possible for that volume. If both students each found all the prisms, they each get one point. A student gets two points for any prism they built that their partner did not. The player who gets the most points after eight rounds is the winner.

Variation:

When students begin to play the game, use a subset of number cards (0–5) so they can build each shape. When they play subsequent times and use the full set of number cards, they may choose to record dimensions rather than build them.

This center stage is the first time Number Cards 0–10 are used in Grade 5, so they are provided as an Instructional master. Students will continue to use these throughout the year. Consider copying them on cardstock or laminating them and keeping them organized to be used repeatedly.

Standards Alignments

Addressing 5.MD.C.5

Materials to Gather

Connecting cubes, Folders

Materials to Copy

Can You Build It Stage 3 Directions (groups of 6), Number Cards (0-10) (groups of 2)

Additional Information

Each group of 2 needs at least 120 connecting cubes and a set of number cards.

Stages used in Grade 4

Stage 1

Supporting

• Grade4.1.A

Stage 2

Addressing

• Grade4.1.A
• Grade4.1.B
Center: Capture Squares (1–3)

Stage 7: Multiply with 6–9

Lessons
- Grade5.1.A1 (supporting)
- Grade5.1.A2 (supporting)
- Grade5.1.B6 (supporting)
- Grade5.1.B7 (supporting)
- Grade5.1.C8 (supporting)

Stage Narrative

Students roll a number cube and spin a spinner and find the product of the two numbers they generated. The spinner has the numbers 6–9.

Standards Alignments

Addressing 3.OA.C.7

Materials to Gather

Colored pencils or crayons, Number cubes, Paper clips

Materials to Copy

Capture Squares Stage 7 Gameboard (groups of 2), Capture Squares Stage 7 Spinner (groups of 2)

Additional Information

Each group of 2 needs one number cube.

Stages used in Grade 4

Stage 7

Supporting
- Grade4.1.A
Center: Five in a Row: Multiplication (3–5)

Stage 3: Two-digit Factors

Lessons
- Grade5.1.A3 (supporting)
- Grade5.1.A4 (supporting)

Stage Narrative
Students multiply using two-digit factors. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments
Addressing 4.NBT.B.5

Materials to Gather
Paper clips, Two-color counters

Materials to Copy
Five in a Row Multiplication and Division Stage 3 Gameboard (groups of 2)

Additional Information
Each group of 2 needs 25 two-color counters and 2 paper clips.

Stage 4: Three Factors

Lessons
- Grade5.1.B5 (addressing)
- Grade5.1.B6 (addressing)
- Grade5.1.B7 (addressing)
- Grade5.1.C8 (addressing)
- Grade5.1.C9 (addressing)
- Grade5.1.C10 (addressing)
- Grade5.1.C11 (addressing)
- Grade5.1.C12 (addressing)
Stage Narrative

Students multiply using 3 factors of 1–5. Partner A chooses three numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments
Addressing 5.MD.C.5.a

Materials to Gather
Paper clips, Two-color counters

Materials to Copy
Five in a Row Multiplication and Division Stage 4 Gameboard (groups of 2)

Additional Information
Each group of 2 needs 25 two-color counters and 3 paper clips.

Stages used in Grade 4

Stage 1
Supporting
• Grade4.1.A

Stage 2
Addressing
• Grade4.1.A
• Grade4.1.B
Supporting
• Grade4.5.A
• Grade4.6.A
• Grade4.6.B

Stage 3
Addressing
• Grade4.6.B
• Grade4.6.C
Center: Rolling for Fractions (3–5)

Stage 1: Equivalent Fractions

Lessons
- Grade 5.1.C9 (supporting)
- Grade 5.1.C10 (supporting)

Stage Narrative
One player rolls 6 number cubes and tries to use 4 of them to fill in a statement with 2 equivalent fractions. If the player cannot make a true statement, they can re-roll as many of the cubes as they like. Each player may re-roll twice. If the student can fill in a statement with 2 equivalent fractions, they get a point for the round. Students take turns for 6 rounds and the player with the most points at the end of the game wins.

Standards Alignments
Addressing 3.NF.A.3.b

Materials to Gather
- Number cubes

Materials to Copy
- Rolling for Fractions Stage 1 Recording Sheet (groups of 1)

Additional Information
Each group of 2 needs 6 number cubes.

Stage 2: Multiply a Fraction by a Whole Number

Lessons
- Grade 5.1.C11 (supporting)
- Grade 5.1.C12 (supporting)

Stage Narrative
Students roll 3 number cubes to generate a multiplication expression with a whole number and a fraction and compare the value of the expression to 1 in order to determine how many points are earned. Two recording sheets are provided, one where the fraction is a unit fraction and one where it can be any fraction.

Variation:
Students may choose a different target number to compare the value of their expression to.
Stage Description
Each group of 2 needs 3 number cubes.

Standards Alignments
Addressing 4.NF.B.4

Materials to Gather
Number cubes

Materials to Copy
Rolling for Fractions Stage 2 Recording Sheet (groups of 1)

Stages used in Grade 4

Stage 1
Supporting
• Grade4.3.A
• Grade4.4.A

Stage 2
Addressing
• Grade4.3.A
• Grade4.3.B
• Grade4.3.C

Supporting
• Grade4.6.C
• Grade4.7.A
Section A: Unit Cubes and Volume

Lesson 1: What Is Volume?

Standards Alignments
Addressing 5.MD.C.3
Building Towards 5.MD.C.3

Teacher-facing Learning Goals
- Explore volume by building objects with unit cubes and comparing them.

Student-facing Learning Goals
- Let's build and compare objects made of cubes.

Lesson Purpose

The purpose of this lesson is for students to understand that solid objects have measurable attributes. Volume is one of these attributes and is defined as the amount of space an object takes up.

In previous grades, students learned that they can count the number of square tiles that cover a plane shape without gaps or overlaps to find the area of the shape. In this lesson, students explore the concept of volume as they build and compare objects made of cubes. Students learn that objects can have different shapes but still take up the same amount of space and that we call this amount an object's volume.

In the next lesson, students learn that volume is measured in cubic units and use unit cubes to find the volume of different objects. This work builds a conceptual understanding of volume before the introduction of formal ways to calculate the volume of rectangular prisms.

In lessons with images of objects made of cubes, students may question if the objects are missing cubes they can't see. In these cases, tell students to assume the objects, including prisms, are completely packed with cubes.

Consider taking a community walk and noticing buildings and objects that are shaped like rectangular prisms in the community where your student's live. In future lessons, you can display a list of these building and objects and ask students to add to the display.

Math Community

Prepare a space, such as a piece of poster paper, titled “Mathematical Community” and a T-chart with the headers “Doing Math” and “Norms.” Partition each of the columns into two sections: students and teacher. The two sections encourage the students and teacher to be mindful that both respective parties are responsible for the way math is done in the classroom.
Access for:

- Students with Disabilities
  - Representation (Activity 2)

- English Learners
  - MLR2 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Connecting cubes: Activity 2

Materials to Copy

- Isometric Dot Paper Standard (groups of 1): Activity 2

Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
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<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
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<tr>
<td>Activity 2</td>
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<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
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</tbody>
</table>

Teacher Reflection Question

Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your student's ideas to support them in being seen and heard in tomorrow's math class?
Cool-down (to be completed at the end of the lesson) 5 min

Which Has More Volume?

Standards Alignments
Addressing 5.MD.C.3

Student-facing Task Statement
Which object has a greater volume? Explain or show your reasoning.

A

B

Student Responses
B, it is made of 9 cubes and A is made of 8 cubes.

Warm-up 10 min

Which One Doesn't Belong: Objects Made of Cubes

Standards Alignments
Building Towards 5.MD.C.3

The purpose of this warm-up is for students to compare four objects made of cubes and reason about their configuration. Responses related to the orientation of the object and the number of cubes in each will be applicable to the upcoming activities.
For all warm-up routines, consider establishing a small, discreet hand signal that students can display to indicate they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or a different subtle signal. This is a quick way to see if students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

**Math Community**

- After the warm-up, ask students to reflect on both individual and group actions while considering the question “What does it look and sound like to do math together as a mathematical community? What am I doing? What are you doing?”
- Record and display their responses under the “Doing Math” header. Students might mention things such as: we talked to each other and to the teacher; we had quiet time to think; we shared our ideas; we thought about the math ideas and words we knew; you were writing down our answers; you were waiting until we gave the answers.

**Instructional Routines**

**Which One Doesn't Belong?**

**Student-facing Task Statement**

Which one doesn't belong?

A

B

C

D

**Launch**

- Groups of 2
- Display image
- “Pick one that doesn't belong. Be ready to share why it doesn't belong.”
- 1 minute: quiet think time

**Activity**

- 2-3 minutes: partner discussion
- Share and record responses.

**Synthesis**

- “What do objects A, C, and D have in common?” (Each side or face of the shapes is a rectangle. They are all made from cubes of the same size.)

**Student Responses**

Sample responses:
A is the only one that is not 1 layer of cubes.
B is the only one that doesn't have the same number of cubes in each row.
C is the only one that is not 2 cubes tall.
D is the only one that doesn't have 8 cubes.

## Activity 1

Build Objects with Cubes

### Standards Alignments

Addressing 5.MD.C.3

The purpose of this activity is for students to recognize that objects with the same volume take up the same amount of space. The word *bigger* is intentionally vague to elicit ideas about length, width, and height and encourage students to reason about the number of cubes.

Monitor for the language students use to explain their choices such as longer, wider, taller or reference to the number of cubes. Students may choose to use connecting cubes to build the objects to compare and clarify their arguments (MP5). As students discuss and justify their decisions, they have opportunities to create viable arguments and critique one another's reasoning (MP3). The discussion and comparison of students' arguments helps illustrate the need for precise mathematical vocabulary and prepares students to learn the meaning of volume (MP6).

### Access for English Learners

*MLR2 Collect and Display.* Collect and record the language students use to compare objects. Display words and phrases such as: “bigger,” “longer,” “wider,” “taller,” “shorter,” “how many,” “more than,” “less than.” During the synthesis, invite students to suggest ways to update the display.

*Advances: Listening, Speaking*

### Student-facing Task Statement

1. Which is bigger? Explain or show your reasoning.

### Launch

- Groups of 2
- “Now you will compare objects to
2. Which is bigger? Explain or show your reasoning.

3. What does it mean for an object to be “bigger”?

**Student Responses**

1. A is wider, but they have the same amount of cubes.
2. B is taller, but they have the same number of cubes.
3. It has more cubes.

Activity

- 5 minutes: individual work time
- 5 minutes: partner work time
- As students work, monitor for students who discuss ‘bigger’ in terms of the number of cubes.

Synthesis

- Share responses for both problems.
- Encourage students who make different choices to explain their reasoning.
- Display the first pair of objects.
- “How would you describe the amount of space each object takes up?” (It’s the same for both shapes. They are made from 8 individual cubes.)
- “We call the amount of space an object takes up its **volume**.”
- Display the second pair of objects.
- “Which object has greater volume?” (They are the same. They are both made from 8 cubes.)
- “What is different about the two objects?” (One is a cube and the other is not. One is taller than the other.)

**Activity 2**

*Build and Order*

**Standards Alignments**

Addressing 5.MD.C.3
The purpose of this activity is for students to build solid objects and compare them by their volume. Students count the number of cubes in each object and may recognize that the shape and orientation of the object doesn't matter when comparing volumes. Encourage students to build any object in which the cubes connect, not just rectangular prisms.

If there is extra time, students could try drawing their objects on dot paper (Instructional master). It is not an expectation that students use the dot paper, but some students may like to try representing their objects with it.

Access for Students with Disabilities

*Representation: Access for Perception*. Read directions aloud. Students who both listen to and read the information will benefit from extra processing time.

*Supports accessibility for: Language, Attention*

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Materials to Gather

Connecting cubes

Materials to Copy

Isometric Dot Paper Standard (groups of 1)

Required Preparation

- Each group of 2 needs 24 connecting cubes.
- Have copies of centimeter dot paper available for students who would like to try drawing their objects.

Student-facing Task Statement

1. Each group member:
   a. Take a handful of connecting cubes.
   b. Build an object.
2. Order the objects by volume.
3. Repeat.
4. Each group member:
   a. Take 9 connecting cubes.
   b. Build an object.
5. Order the objects by volume

Launch

- Groups of 4
- Give each group a pile of connecting cubes.

Activity

- 10 minutes: group work time
- As students work, collect a group’s objects that all have a volume of 9 units but have different arrangements.

Synthesis

- Display 3 or 4 objects all built using 9 cubes.
- “What is the same? What is different?” (Some of the objects are taller, some are...
Student Responses

1 and 2: Sample responses. Students may build some objects with the same volume or all 4 objects may have different volumes.

4 and 5: The objects all have the same volume, but can be different shapes.

Lesson Synthesis

“Today, we built objects out of cubes and compared them by the amount of space they take up. We call this an object’s volume.”

Display the images from the activity or the objects made from cubes.

A

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Object A" /></td>
<td><img src="image2.png" alt="Object B" /></td>
</tr>
</tbody>
</table>

“How are the two objects the same?” (They are both made from 8 cubes. There are four cubes next to one another on the bottom layer.)

“How are the two objects different?” (One of them is a cube and the other looks like a tower. One of them is taller than the other.)

“These two objects look different, but take up the same amount of space, that is they have the same volume.”
Math Community

After the Cool-down, revisit the “Doing Math” list of actions. Ask students to discuss with a partner where they saw evidence of the actions during the rest of the day’s lesson. As a whole group, add any missing actions and revise earlier ideas.

Suggested Centers

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)
- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)

Response to Student Thinking

If students don’t recognize that object B has a volume of 9 cubes.

Next Day Support

- Give students access to connecting cubes to build the objects that are displayed in the book.
Lesson 2: Measure Volume

Standards Alignments
Addressing 5.MD.C.3, 5.MD.C.3.b, 5.MD.C.4

Teacher-facing Learning Goals
• Find the volume of solid figures, including rectangular prisms, in unit cubes, in a way that makes sense to them.
• Understand that volume is measured in unit cubes.

Student-facing Learning Goals
• Let’s count cubes.

Lesson Purpose
The purpose of this lesson is for students to understand that volume is the amount of unit cubes that fills a space.

Students are introduced to rectangular prisms as they begin to informally use the structure of rectangular prisms to find the number of cubes in the prism. In previous grades, students learned to measure area by counting unit squares, decomposed a rectangular region into rows and columns, and multiplied the number of unit squares in a row by the number of rows or the number of unit squares in a column by the number of columns. Similarly, in this lesson, students make use of the layered structure in a rectangular prism to count the cubes in a more systematic way (MP7). Students will have many opportunities to count the number of cubes in rectangular prisms in lessons 3 and 4 before the introduction of formulas in Lesson 5.

Math Community
Tell students they will have an opportunity to revise their “Mathematical Community” ideas at the end of this lesson, so as they work today they should think about actions that may be missing from the current list.

Access for:

Students with Disabilities
• Representation (Activity 2)

Instructional Routines
MLR2 Collect and Display (Activity 2), Which One Doesn't Belong? (Warm-up)
Lesson Timeline

<table>
<thead>
<tr>
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<td>Warm-up</td>
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</table>

Teacher Reflection Question

What did students understand about the volume of solid objects as they came into the lesson? In what ways did their understanding of volume change upon completing the lesson?

Cool-down (to be completed at the end of the lesson)

Volume of a Rectangular Prism

Standards Alignments

Addressing 5.MD.C.4

Student-facing Task Statement

Find the volume of the rectangular prism. Explain or show your reasoning.

Student Responses

- The volume is 30 cubes. Sample responses: $5 \times 6 = 30$, $5 \times 3 = 15$ and $15 \times 2 = 30$, or $3 \times 2 = 6$ and $6 \times 5 = 30$
Warm-up

Which One Doesn't Belong: Cubes

Standards Alignments
Addressing 5.MD.C.3

The purpose of this warm-up is for students to compare solid objects. Students may use informal language to describe the structure and orientation of the objects. Later in the lesson, students will have an opportunity to connect their informal language to more formal math vocabulary. During the synthesis, connect student’s descriptions of the objects to the strategies they would use to count the number of cubes in each object. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology when describing the structure and orientation of three dimensional objects. During the synthesis, ask students to explain the meaning of any terminology they use, such as volume.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn’t belong?

Launch

• Groups of 2
• Display image.
• “Pick one that doesn't belong. Be ready to share why it doesn't belong.”

Activity

• 1 minute: quiet think time
• 2–3 minutes: partner discussion
• Share and record responses.

Synthesis

• “We can count the number of unit cubes in an object to measure its volume.”
• “How many cubes are in object D? How did you count them?” (12, there are 6 on bottom and 6 on top)
• “What strategy would you use to count the cubes in the other objects?” (In object A, I can

Student Responses

• A doesn’t belong because it is the only one
that isn't a single object.

- B doesn't belong because it is the only one that doesn't use 12 cubes.
- C doesn't belong because it is the only one that isn't 2 cubes deep.
- D doesn't belong because it is the only one that is not 3 cubes tall.

also see 2 groups of 6. In object B, I can see 2 groups of 6 minus 1. In object C, there is only one group of 6 and then there are 4 cubes and 2 cubes.)

**Activity 1**

Finding Volume

**Standards Alignments**

Addressing 5.MD.C.3.b, 5.MD.C.4

The purpose of this activity is for students to build any object they want to. In the next activity, students focus only on rectangular prisms. As students count, they will need to make sure to count each cube once and only once. Monitor for these strategies:

- Touch each cube as it is counted
- Count the cubes by layer (including when there are different numbers of cubes in each layer)
- Use addition or multiplication when the cubes are organized in layers

**Required Preparation**

- Each group of 2 needs 24 connecting cubes.

**Student-facing Task Statement**

1. Partner A: Build an object using 8–12 cubes and give the object to Partner B.
2. Partner B: Explain how you would count the number of cubes in the object.
3. Partner A: Explain if you would count the cubes in the same way or in a different way.
4. Switch roles and repeat.

**Launch**

- Groups of 2
- Give 24 connecting cubes to each group.
- “In this activity, you will use unit cubes to build objects and describe how you would measure the volume.”
- 10 minutes: partner work time
5. Which objects were easiest to count? Why?

Student Responses

5. The ones with structure like layers, equal rows or equal columns were the easiest to count because I did not need to count each individual cube.

Activity

- As students work, monitor for students who build rectangular prisms to share during the synthesis.

Synthesis

- Discuss responses to the last problem.
- “Which objects were most challenging to count?” (Objects where I had to count each individual cube because it was a weird shape.)
- “Which objects were the simplest to count?” (Organized shapes like towers, rectangles, cubes, or boxes.)
- Invite selected students to share the rectangular prisms they built and how they counted the number of cubes.
- “How did you count the cubes?” (I knew that there were 4 cubes in each layer and 2 layers so that's 8 cubes.)
- “We call these objects rectangular prisms. What were some ways you used to count the cubes in rectangular prisms?” (I counted the cubes in groups and then skip counted. I used multiplication to find the total.)

Advancing Student Thinking

If students are counting all of the cubes one at a time, ask, “where do you see cubes that could be counted as a group?”

Activity 2

Guess My Prism

20 min
Standards Alignments
Addressing 5.MD.C.4

In the previous activity, students described how to count the number of unit cubes in an object and observed that for a special shape called a rectangular prism, the cubes are organized in a structure that makes counting them more systematic. The purpose of this activity is for students to continue to explore rectangular prisms, in particular their structure. As students describe the prisms, they communicate their ideas clearly and precisely (MP6). To describe an object requires students to identify key features of the object and communicate them in an unambiguous way to their partner.

This activity uses MLR2 Collect and Display. Advances: reading, writing

Access for Students with Disabilities

Representation: Internalize Comprehension. Use multiple examples and non-examples to emphasize attributes of a rectangular prism identified in the previous activity.
Supports accessibility for: Memory, Conceptual Processing

Instructional Routines
MLR2 Collect and Display

Required Preparation
- Each group of 2 needs 16-24 connecting cubes.

Student-facing Task Statement
What is the same? What is different?

1. The goal of the game is to get your partner to build the same prism.
   - Partner A: Use 16–24 cubes to build a prism. Describe it to your partner.

Launch
- Groups of 2
- Display images.
- “What is the same? What is different?” (There are 6 cubes in each shape. Each object is 2 cubes wide and 3 cubes long. One object is standing up on a side of 2 cubes and the other one is laying down on a side of 6 cubes.)
- “In this activity, you will build and describe rectangular prisms.”
- Give each student 16-24 connecting cubes.
Partner B: Build the prism your partner describes to you.

2. Place the two prisms next to each other and discuss what is the same and what is different about them.

3. Switch roles and repeat.

**Student Responses**

Sample responses:

1. Partner A builds:

   Partner A describes: “I used 12 cubes to build my prism. It has 2 layers. Each layer has 6 cubes.”

   Partner B builds:

2. What is the same about your prisms?
   - Both prisms have 12 cubes.
   - Both prisms have 2 layers with 6 cubes in each layer.
   - Both prisms have 3 layers with 4 cubes in each layer.

   What is different? Mine was taller and my partner’s was longer.

- Consider having students seated back to back or with an object between them that obstructs the view such as a folder.

**Activity**

**MLR2 Collect and Display**

- Circulate, listen for and collect the language students use to compare quantities. Listen for: layers, rows, columns, length, width, height, top, bottom, or side.
- Record students’ words and phrases on a visual display and update it throughout the remainder of the lesson.

**Synthesis**

- Invite partners who built different prisms to share their prisms and descriptions.
- “What language did your partner use that was most helpful for you to understand the prism they wanted you to build?” (My partner said there were 6 cubes on the bottom and 6 more on top. So I made a row of 6 cubes and put another row on top. But my partner had two rows of 3 instead of one row of 6.)
- Use this discussion to update the display, by adding (or removing) language, diagrams, or annotations.
- Invite groups who built the same prisms to share.
- “How did you describe your prism to your partner?” (I said how many cubes were on the bottom and how tall the prism was.)
- Use this discussion to update the display, by adding (or removing) language, diagrams, or annotations.
- Display any prism made by a student.
- “How can we describe this prism using language from our display?” (Highlight the use of language about layers or stacks.)
Advancing Student Thinking

If students do not see two prisms as the same when they are oriented differently, ask them to describe a prism, change the orientation of the prism, and ask them to describe it again. Emphasize that the total number of cubes stays the same, but the prism looks different.

Lesson Synthesis

“Today we described rectangular prisms and measured their volumes.”

Ask a student who has not shared in previous activities to display a prism they built.

Ask the class, “How can we find the volume of this rectangular prism?” (Find the number of cubes in a layer and multiply that number by how many layers there are.)

“How can we describe this rectangular prism so that our partner can build the same one?” (Describe the cubes on the bottom and how many layers there are.)

Math Community

After the cool-down, give students 2–3 minutes to discuss any revisions to the “Doing Math” actions in small groups. Share ideas as a whole group and record any revisions.

Suggested Centers

- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)

Response to Student Thinking

The work in this lesson builds from multiplication and division concepts and fluency developed in a prior course.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division
Lesson 3: Volumes of Prism Drawings

Standards Alignments
Building On 3.OA.C.7
Addressing 5.MD.C.4
Building Towards 5.MD.C.5

Teacher-facing Learning Goals
- Find the volume of a rectangular prism using its layered structure.

Student-facing Learning Goals
- Let's use layers to find volume.

Lesson Purpose
The purpose of this lesson is for students to find the volume of a rectangular prism using its layered structure.

In previous lessons, students built objects, including rectangular prisms, with unit cubes and counted the number of cubes. In this lesson, students continue to count the number of unit cubes needed to build a rectangular prism, but now they are presented with images of prisms instead of the objects themselves. To encourage students to develop a systematic way to count the cubes, they are shown prisms made from larger numbers of cubes. As students use horizontal or vertical layers to measure the volume, they make use of the layered structure of prisms (MP7).

Math Community
Tell students that, at the end of the lesson, they will be asked to identify specific actions from their “Doing Math” list (both teacher and student sections) they personally experienced.

Access for:

Students with Disabilities
- Representation (Activity 2)

English Learners
- MLR8 (Activity 1)

Instructional Routines
Number Talk (Warm-up)
**Materials to Gather**
- Connecting cubes: Activity 1
- Connecting cubes: Activity 2

**Materials to Copy**
- Card Sort Rectangular Prism Cards (groups of 2): Activity 1

**Lesson Timeline**

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<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

**Teacher Reflection Question**
What connections did students make between the different strategies they used to build the prisms and determine the volume? What questions did you ask to help make the connections more visible?

---

**Cool-down** (to be completed at the end of the lesson)

Jada's Prism

**Standards Alignments**
Addressing 5.MD.C.4

**Student-facing Task Statement**

Jada's prism has 4 layers and each layer has 9 cubes.

1. Circle the prism that is Jada's.

   ![Prism A](image)
   ![Prism B](image)
   ![Prism C](image)
   ![Prism D](image)

2. Find the volume of Jada's prism. Explain or show your reasoning.
Student Responses
1. B
2. 36 cubes. There are 4 layers with 9 cubes in each layer so there are 36 cubes, $4 \times 9 = 36$.

Warm-up

Number Talk: Multiplication

Standards Alignments
Building On 3.OA.C.7
Building Towards 5.MD.C.5

The purpose of this Number Talk is for students to multiply three factors. Strategies for multiplying three factors will be helpful as students find the volume of rectangular prisms in this lesson and upcoming lessons. Since the first problem is only 2 factors, it is not important to gather multiple strategies in order to leave more time for the other problems. Students may connect the third factor in the final three problems to ‘adding another layer of cubes’ in a prism when finding volume. Invite these students to share their observations during the synthesis.

Instructional Routines
Number Talk

Student-facing Task Statement
Find the value of each expression mentally.
- $6 \times 4$
- $3 \times 2 \times 4$
- $3 \times 2 \times 5$
- $3 \times 2 \times 6$

Launch
- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: individual think time

Activity
- Record answers and strategy.
**Student Responses**

- 24: I just know it, $6 \times 2 = 12$ and $12 \times 2 = 24$
- 24: It is equal to the first one, but instead of 6 it says $3 \times 2$
- 30: $6 \times 5 = 30$, $10 \times 3 = 30$
- 36: I just added another $3 \times 2$ or $6 \times 6 = 36$

**Synthesis**

- “How did changing one of the factors impact the product?” (Increasing one of the factors made the product get bigger by 6.)
- Consider asking:
  - “How are problems 2–4 like the work we did with prisms yesterday?”
  - “Who can restate ______’s reasoning in a different way?”
  - “Did anyone have the same strategy but would explain it differently?”

---

**Activity 1**

Build Rectangular Prisms

**Standards Alignments**

Addressing 5.MD.C.4

The purpose of this activity is for students to build and determine the volume of rectangular prisms from images. In the activity synthesis, students look at two related prisms to encourage them to think about 8 cubes as a layer.

**Access for English Learners**

*MLR8 Discussion Supports.* During small-group discussion, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say . . . .” Original speakers can agree or clarify for their partner.

*Advances: Listening, Speaking*

**Materials to Gather**

Connecting cubes

**Materials to Copy**

Card Sort Rectangular Prism Cards (groups of
Required Preparation

- Create a set of cards from the Instructional master for each group of 4. Each group of 2 needs 48 connecting cubes.

Student-facing Task Statement

The prisms on the cards are completely packed with unit cubes.

1. Pick a card.
2. Build the rectangular prism.
3. Find the volume. Explain how you found the volume to your partner.
4. Repeat.

Student Responses

<table>
<thead>
<tr>
<th>prism</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>24</td>
</tr>
<tr>
<td>B</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>16</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
</tr>
<tr>
<td>F</td>
<td>24</td>
</tr>
</tbody>
</table>

Launch

- Groups of 2
- Give students access to connecting cubes.
- “You are going to build rectangular prisms shown on cards. Before we look at the cards, what information in the image will help you build the prism?” (The number of cubes in each layer and the number layers.)
- Share responses.
- “Now, you will each pick a card, build the prism, and find its volume. Explain how you found the volume to your partner and then pick another card.”

Activity

- Monitor for students who discuss:
  - how many cubes are in each layer.
  - how the cubes are arranged in each layer.
  - how many layers are in the prism.
- Mathematical Community: As students work, monitor for examples of the “Doing Math” actions.

Synthesis

- Display Card E
  - “How did you build this rectangular prism?” (I counted 8 cubes and then made a stack of 8 cubes.)
  - “What is the volume of this rectangular prism? How do you know?” (8 cubes because I counted 8
Display Card A

- “How did you build this rectangular prism?” (I saw that there are 8 cubes on the top and bottom in 2 rows of 4. Then I built it up until it was 3 cubes high.)
- “What is the volume of this rectangular prism? How do you know?” (24 cubes because there are 3 sets of 8 cubes and $3 \times 8 = 24$)

**Advancing Student Thinking**

Some students may still count each individual cube to determine the volume of the prism. Ask, “How many cubes are in each layer? How many layers are there?”

**Activity 2**

Layers, layers, and more layers

**Standards Alignments**

Addressing 5.MD.C.4

The purpose of this activity is for students to find the volume of larger rectangular prisms shown in images. In previous lessons, students built rectangular prisms out of cubes and counted the cubes to determine the volume of the rectangular prisms. In this activity, the prisms were intentionally chosen to encourage students to use the layered structure of the prism to determine the volume of the prism (MP7). When students connect this structure to the operation of multiplication and use expressions and equations to find the volume, they decontextualize the geometric structure to solve the problems (MP2).
Access for Students with Disabilities

*Representation: Access for Perception.* Invite students to identify correspondences between the visual representation and the prism made of connecting cubes. Make connections between representations visible through gestures or labeled displays.

*Supports accessibility for: Visual-Spatial Processing, Conceptual Processing*

Materials to Gather

Connecting cubes

Required Preparation

- Give students access to connecting cubes.

Student-facing Task Statement

The prisms are completely packed with unit cubes. Determine the volume of each prism. Explain or show your reasoning.

1.

![Image](image1)

2.

![Image](image2)

3.

Launch

- Groups of 2
- Display the first prism.
- “How would you describe this prism?” (There are 5 layers and each layer has 8 cubes or there are 2 layers and each layer has 20 cubes in it.)
- Give students access to connecting cubes.

Activity

- 10 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who use the layered structure to determine the volume.

Synthesis

- Invite students to share how they found the volume of the rectangular prisms.
- Display image of the first prism.
- “The expression \(5 \times 8\) represents the volume of the prism. Where do you see 5 groups of 8 cubes in this prism?” (The top and bottom are 2 by 4 so there are 8 cubes on top and bottom. There are 5 of those
Student Responses

1. 40 cubes: $5 \times 4 \times 2$, $20 \times 2$, $8 \times 5$
2. 60 cubes: $20 \times 3$, $15 \times 4$
3. 80 cubes: $20 \times 4$, $16 \times 5$

Advancing Student Thinking

If students do not notice that each prism has one more layer of 20 cubes, consider asking, “What is the same about each of these prisms? What is different?”

Lesson Synthesis

Discussion or journal prompt: “What do you know about finding the volume of a prism made of cubes after today’s activities? Is there anything you have questions about?”

Share responses or read journals after class.

Math Community

- After the Cool-down, ask students to individually reflect on the question “Which ‘Doing Math’ action did you feel was most important in your work today, and why?” Have students write their responses on the bottom of their Cool-down paper, on a separate sheet of paper, or in a math journal.
- Collect and read their responses after class. These responses will offer insight into how students feel about their own mathematical work and help you make personal connections to the norms they will be creating during Days 4–6.
Suggested Centers

- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)

Response to Student Thinking

Students do not circle a prism that has 4 layers of 9 cubes.

Next Day Support

- During the warm-up of the next lesson, encourage students to use the number of layers and the number in each layer to create their estimate.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division
Lesson 4: Use Layers to Determine Volume

Standards Alignments
Addressing 5.MD.C.5.a, 5.OA.A.2
Building Towards 5.MD.C.5.a

Teacher-facing Learning Goals
- Describe and represent the volume of a rectangular prism as the product of the number of cubes in one layer and the number of layers.
- Write and interpret expressions and equations in the context of the volume of rectangular prisms.

Student-facing Learning Goals
- Let's relate multiplication to how we use layers to find volume.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of the layered structure of rectangular prisms to find the volume of a prism when they cannot see all of the cubes.

In previous lessons, students used the layered structure to determine the volume of rectangular prisms that were filled with cubes. The purpose of this lesson is to extend their understanding of the structure of rectangular prisms by conceptualizing a base layer in a rectangular prism and utilizing it to find the volume in a fully- or partially-filled prism. Students connect the layered structure to multiplication when they write and interpret expression that represent the volume of rectangular prisms (MP2, MP7). This is a conceptual way for students to make sense of the associative property of multiplication. Students do not, however, need to name or identify the property.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities
- Action and Expression (Activity 1)

English Learners
- MLR7 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)
Materials to Gather

- Connecting cubes: Activity 1, Activity 2

Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>20 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>15 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question

In the upcoming section, students will connect multiplying the area of the base and the height to multiplying all three edge lengths when finding volume. Based on this lesson, what understandings do your students now have that will allow them to make that connection?

Cool-down (to be completed at the end of the lesson)

Use Expressions

Standards Alignments

Addressing 5.MD.C.5.a

Student-facing Task Statement

1. If the rectangular prism was filled completely, how many cubes could it hold?

2. Explain or show how the expression $3 \times 8$ represents the volume of the prism.
Student Responses

1. The volume is 60 cubes.
2. One of the layers has 8 cubes in it and there are 3 of those layers.

Warm-up

Estimation Exploration: How Many Cubes?

Standards Alignments
Building Towards 5.MD.C.5.a

The purpose of this warm-up is for students to consider the information they need to find the volume of a rectangular prism and use the structure of a rectangular prism to think about a reasonable estimate. Students can see the 9 cubes on the front layer, but it is difficult to see how many layers there are.

The purpose of an Estimation Exploration is to think about reasonableness based on experience and known information. It gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it (MP3). Making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).
Instructional Routines

Estimation Exploration

Student-facing Task Statement

About how many cubes were used to build this prism?

Record an estimate that is:

<table>
<thead>
<tr>
<th>too low</th>
<th>about right</th>
<th>too high</th>
</tr>
</thead>
</table>

Student Responses

Sample responses:
- too low: 30 or fewer
- about right: 60 to 100
- too high: 200 to 300

Launch

- Groups of 2
- Display image
- “What is an estimate that’s too high?” “Too low?” “About right?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses

Synthesis

- “Why are multiples of 9 good estimates?” (We can see the layer of 9.)
- “What information would help you to find the exact number of cubes in the prism?” (How many layers of 9 cubes there are. How deep the prism goes.)
- “Based on this discussion does anyone want to revise their estimate?”
- Optional: Reveal a picture that shows the number of layers, 10.
Math Community

- Ask students to reflect on both individual and group actions while considering the question “What norms, or expectations, were we mindful of as we did math together in our mathematical community?”
- Record and display their responses under the “Norms” header.

Activity 1

Layers in Rectangular Prisms

Standards Alignments
Addressing 5.MD.C.5.a

In the previous lesson, students reasoned abstractly about the volume of rectangular prisms
when they considered the volume in terms of layers or equal groups of unit cubes. This activity continues to develop the idea of decomposing rectangular prisms into layers. Students explicitly multiply the number of cubes in a base layer by the number of layers. Students can use any layer in the prism as the base layer as long as the height is the number of those base layers.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Synthesis: Invite students to plan a strategy, including the tools they will use, for finding volume of partially filled prisms. If time allows, invite students to share their plan with a partner before they begin. Supports accessibility for: Conceptual Processing, Memory

Materials to Gather

Connecting cubes

Required Preparation

- Have connecting cubes available for students who need them.

Student-facing Task Statement

1. Complete the table. Be prepared to explain your reasoning.

<table>
<thead>
<tr>
<th>prism</th>
<th>number of cubes in one layer</th>
<th>number of layers</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Launch

- Groups of 2
- Display first image from student workbook
- “What do you know about the volume of this prism?”
- “What would you need to find out to find the exact volume of this prism?”
- “You are going to work with prisms that are only partially filled in this activity.”
- Give students access to connecting cubes.

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time
- As students work, monitor for:
  - students who notice that prisms A and D and prisms B and C are “the same” but they are sitting on different faces so the layers might
2. Find the volume of each prism. Explain or show your reasoning.

2. **Prism E**

3. How can you find the volume of any rectangular prism?

3. **Prism F**

be counted in different ways.
- students who reason about the partially filled prisms by referring to the cubes in one layer they would see if all of the cubes were shown.
- students who recognize that there are several different layers they can use to determine the volume of a prism, all of which result in the same volume.

**Synthesis**

- Display the expressions:
  - $2 \times 12$
  - $3 \times 8$
- “How do these expressions represent the volume of prism A?” (There are two layers of 12. We can also see 3 layers of 8.)
- “How does thinking about layers help us find the volume of prisms that are not completely filled?” (I know that all the layers have the same number of cubes even if they are not shown.)
### Student Responses

<table>
<thead>
<tr>
<th></th>
<th># of cubes in one layer</th>
<th># of layers</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6, 8, or 12</td>
<td>4, 3, or 2</td>
<td>24 cubes</td>
</tr>
<tr>
<td>B</td>
<td>15, 6, or 10</td>
<td>2, 5, or 3</td>
<td>30 cubes</td>
</tr>
<tr>
<td>C</td>
<td>6, 15, or 10</td>
<td>5, 2, or 3</td>
<td>30 cubes</td>
</tr>
<tr>
<td>D</td>
<td>8, 6, or 12</td>
<td>3, 4, or 2</td>
<td>24 cubes</td>
</tr>
</tbody>
</table>

2. Prism E: 10 cubes. \(2 \times 5 = 10\)
Prism F: 84 cubes. There are 2 columns of 6 (or 6 rows of 2) in one vertical layer. There are 7 layers so \(2 \times 6 \times 7\). There are 14 cubes in one horizontal layer. There are 6 total layers so \(6 \times 14\). There are 7 columns of 6 in one layer and I can multiply that by the 2 layers so \(7 \times 6 \times 2\).
3. Multiply the number of cubes in one layer by the height with that layer as the base.

### Activity 2

**Finding Volume in Different Ways**

#### Standards Alignments

Addressing 5.MD.C.5.a, 5.OA.A.2

In the previous activity, students saw that a rectangular prism is composed of layers and there are different ways to decompose a prism into layers, depending on how students view the prism and decompose the prism. Students recognize that the volume remains the same, regardless of the orientation of the prism. The goal of this activity is for students to identify how different expressions represent the volume of the same prism and correspond to the organization of the layers. Students have worked with parentheses in previous grades, so the lesson synthesis
provides an opportunity for students to revisit expressions with parentheses. Students will have more experience with evaluating expressions with grouping symbols in future lessons. Students go back and forth between numerical expressions and a geometric object whose volume is represented by the expression (MP2).

**Access for English Learners**

MLR7 Compare and Connect. Synthesis: Ask, “what is the same and what is different about the strategies used to calculate the volume of the rectangular prism?” Add labels or annotations to a visible display to support connections between approaches, and to amplify language such as layers, horizontal, vertical, volume and expression.

*Advances: Representing, Conversing.*

**Materials to Gather**

Connecting cubes

**Required Preparation**

- Have connecting cubes available for students who need them.

**Student-facing Task Statement**

1. Explain or show how the expression $5 \times 24$ represents the volume of this rectangular prism.
2. Explain or show how the expression $6 \times 20$ represents the volume of this rectangular prism.
3. Find a different way to calculate the volume of this rectangular prism. Explain or show your thinking.
4. Write an expression to represent the way you calculated the volume.

**Launch**

- Groups of 2
- “You are going to analyze different ways to find the volume of a rectangular prism.”
- Give students access to connecting cubes.

**Activity**

- 5 minutes: independent work time
- 5 minutes: group work time
- Monitor for students who identify the factor 5 or 6 as the number of layers and the second factor is then the number of cubes in each layer.

**Synthesis**

- “How does $5 \times 24$ represent the volume of the prism?” (There are 5 layers if I cut the prism horizontally and each layer has 24 cubes in it.)
**Student Responses**

1. The expression shows 5 horizontal layers of cubes in the prism. There are 24 or $6 \times 4$ cubes in each layer so that makes $5 \times 24$ cubes in the prism.

2. The expression shows 6 vertical layers of cubes in the prism. Each layer has $4 \times 5$ or $20$ cubes in it so that makes $6 \times 20$ cubes in the prism.

3. Sample response: $4 \times 30$ because there are 4 layers and each layer has 30 cubes in it.

4. Sample response: $4 \times 30$

**Advancing Student Thinking**

If students do not explain how the expression represents the volume of the prism, ask “Can you explain how you would find the volume of this prism?” Then, connect the student’s description to the numbers in the expressions.

**Lesson Synthesis**

Display the image from the warm-up showing all the layers of the prism.

“How describe the layers in the prism to a partner. What is a multiplication expression that would represent the volume of the prism? How does the expression represent the volume of the prism?” (There are 9 cubes in each layer and I can see 10 layers.)

**Math Community**

Revisit the “Norms” list. Ask students to discuss with a partner when a norm was helpful as they did math. Add any missing ideas or revise earlier ones.

**Suggested Centers**

- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)
Student Section Summary

We call the amount of space an object takes up volume. This prism has a volume of 120 cubes.

To find the volume of any prism, we can find the number of cubes in one layer and multiply that number by the number of layers. We can describe this prism as having 6 layers of 20 cubes, 4 layers of 30 cubes, or 5 layers of 24 cubes. We can use all of these expressions to represent the volume of the prism:

\[ 5 \times 24, \quad 5 \times (6 \times 4) \]
\[ 6 \times 20, \quad 6 \times (5 \times 4) \]
\[ 4 \times 30, \quad 4 \times (5 \times 6) \]

Response to Student Thinking

Students do not explain or show how the expression represents the volume of the prism.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division
Section B: Expressions for Finding Volume

Lesson 5: Side Lengths of Rectangular Prisms

Standards Alignments
Addressing 5.MD.C.5.b
Building Towards 5.MD.C.5.b

Teacher-facing Learning Goals
- Describe rectangular prisms in terms of their side lengths.
- Find the volume of a right rectangular prism by multiplying the side lengths and connect that to finding volume by multiplying the area of the base by the height.

Student-facing Learning Goals
- Let's describe the side lengths of a prism and find the volume.

Lesson Purpose
The purpose of this lesson is for students to formalize the language they use to describe the side lengths of a rectangular prism.

In previous lessons, students used the structure of a rectangular prism, namely layers, to find its volume. In this lesson, students describe the number of layers and the number of cubes in each layer in terms of the length, width, and height of the prism (MP6). Students choose a base for the prism and recognize that the number of cubes in each layer is the product of the length and width of the chosen base. They also recognize that the height that corresponds with the chosen base represents the number of layers in the prism. They connect this relationship to the product of the length, width, and height of a prism. With this understanding of how to find the volume of a rectangular prism, students do not always need to see the individual cubes that make up the prism. The poster from a previous lesson that displays language students used while building and describing prisms will be revised in this lesson synthesis to include the terms length, width, height, and area of a base.

Math Community
Tell students that, at the end of the lesson, they will be asked to identify specific examples of norms they experienced as they did math.
Access for:

 pedest Students with Disabilities
 - Engagement (Activity 1)

 English Learners
 - MLR2 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Connecting cubes: Activity 1, Activity 2

Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>15 min</td>
</tr>
<tr>
<td>Activity 3</td>
<td>10 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student’s ideas are a part of the collective learning?

Cool-down (to be completed at the end of the lesson)

Determine the Volume

Standards Alignments

Addressing 5.MD.C.5.b

Student-facing Task Statement

Here is a base of a rectangular prism.
What is the volume of the prism if it has a height of 3?

**Student Responses**

- 36 cubes

---

**Warm-up**

**Notice and Wonder: Prism Print**

**Standards Alignments**

- Building Towards 5.MD.C.5.b

The purpose of this warm-up is for students to notice that each face of a prism can be the base, which will be useful when students use a base of a prism to find the prism's volume in a later activity. While students may notice and wonder many things about these images, the relationship between the images of the prism and the images of the rectangles are the important discussion points.

**Instructional Routines**

- Notice and Wonder

**Student-facing Task Statement**

What do you notice? What do you wonder?

**Launch**

- Group size: 2
- Display the images for all to see.
### Student Responses

Students may notice:

- All the prisms have a volume of 12 cubes.
- The prisms match the rectangles.
- The prism on the left is 2 cubes long and so is the rectangle underneath it.

Students may wonder:

- Are they all the same prism?
- Is the prism supposed to be moving?
- Do the prisms fit on top of the rectangles?

### Activity

- Ask students to think of at least one thing they notice and at least one thing they wonder.
- 1 minute: quiet think time
- 2 minute: partner discussion
- Share and record responses.

### Synthesis

- “These rectangles show different faces of the prisms. Any face of a prism can be a base. We are going to learn more about this in the activities.”
- “Where do we see each base in the prism?”

### Activity 1

All About That Base

### Standards Alignments

Building Towards 5.MD.C.5.b

The purpose of this activity is for students to recognize that a base of a prism is a two-dimensional rectangle and any face of a prism can be a base. Students may start with a possible
rectangular base and try to visualize which face of a given prism matches the base or they may start with the prism, study the faces, and try to find an appropriate base to match. In either case, they need to persevere and think through all of the possible bases for each prism systematically in order to solve this problem (MP1).

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to various tools that can be used to solve the problem. For example, connecting cubes can be used to build the prisms and colored pencils can be used to shade the faces of each prism. Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Materials to Gather

Connecting cubes

Required Preparation

- Have connecting cubes available for students who need them.

Student-facing Task Statement

Here are 3 rectangular prisms.

1. Match each prism with a rectangle that represents its base. Note: Some prisms may match more than 1 rectangular base.
2. Find the volume of each prism. Explain or show your reasoning.

Launch

- Groups of 2
- Give students access to connecting cubes.

Activity

- 4 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who:
  - Match each prism to a rectangle that represents the base the prism is resting on.
  - Recognize that the 4 by 3 rectangle represents a base on each of the prisms.

Synthesis

- Invite previously identified students to share.
- Display student work that shows it is
Student Responses

1. Prism 1 matches base B, Prism 2 matches base A and B, Prism 3 matches base B and C.
2. Prism 1 has 48 cubes. Sample response: The base has 12 cubes and there are 4 layers. Prism 2 has 72 cubes. Sample response: The base has 18 cubes and there are 4 layers. Prism 3 has 60 cubes. Sample response: The base has 15 cubes and there are 4 layers.

Advancing Student Thinking

If students do not correctly match prisms to bases, ask them to build each prism and describe how a given face does or does not match each base that is represented with rectangles A, B, and C.

Activity 2

Growing Prism

Standards Alignments

Addressing 5.MD.C.5.b

The purpose of this activity is for students to describe the layered structure of rectangular prisms using the side lengths of the prism. Instead of diagrams of rectangular prisms built from cubes, students are shown a diagram of one of the bases of a prism and are asked to find the volume of the prism with different heights. Students may still use informal language, such as layers, to describe the prisms and find their volume. During the lesson synthesis, connect their informal language to the more formal math language of length, width, height, and area of the base.
Access for English Learners

**MLR2 Collect and Display.** Amplify words and phrases such as: length, width, taller, pattern, base, number of layers.
*Advances: Conversing, Reading*

Materials to Gather

Connecting cubes

Required Preparation

- Have connecting cubes available for students who need them.

Student-facing Task Statement

Here is a base of a rectangular prism.

![Prism Base]

1. Fill out the table for the volumes of rectangular prisms with this base and different heights.

<table>
<thead>
<tr>
<th>Height</th>
<th>Multiplication expression to represent the volume</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Student Responses

Sample responses:

1. 

Launch

- Groups of 2
- Give students access to connecting cubes.

Activity

- 2 minutes: independent work time
- 10 minutes: partner work time
- As students work, monitor for students who:
  - describe and use layers of the prism.
  - multiply 40 by the height of the prism to determine the volume of the prisms.

Synthesis

- Ask previously selected students to share their solutions for the first problem.
- Display a completed table or use the one given in student solutions:
<table>
<thead>
<tr>
<th>number of layers</th>
<th>multiplication expression for volume</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$4 \times 10 \times 2$ or $40 \times 2$</td>
<td>80 cubes</td>
</tr>
<tr>
<td>3</td>
<td>$4 \times 10 \times 3$ or $40 \times 3$</td>
<td>120 cubes</td>
</tr>
<tr>
<td>10</td>
<td>$4 \times 10 \times 10$ or $40 \times 10$</td>
<td>400 cubes</td>
</tr>
<tr>
<td>25</td>
<td>$4 \times 10 \times 25$ or $40 \times 25$</td>
<td>1,000 cubes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>number of layers</th>
<th>multiplication expression for volume</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$4 \times 10 \times 2$ or $40 \times 2$</td>
<td>80 cubes</td>
</tr>
<tr>
<td>3</td>
<td>$4 \times 10 \times 3$ or $40 \times 3$</td>
<td>120 cubes</td>
</tr>
<tr>
<td>10</td>
<td>$4 \times 10 \times 10$ or $40 \times 10$</td>
<td>400 cubes</td>
</tr>
<tr>
<td>25</td>
<td>$4 \times 10 \times 25$ or $40 \times 25$</td>
<td>1,000 cubes</td>
</tr>
</tbody>
</table>

- “How does the volume of the prism change in the table?” (The volume is increasing by 40 cubes for each layer added to the prism.)
- “How is the change in volume represented by the multiplication expressions in each row?” (The expressions in each row shows more groups of 40.)
- Display the expression $3 \times 40$.
- “How does the expression represent the volume of the prism?” (There are 40 cubes in the base layer of the prism or 4 rows of 10 cubes. Then there are 3 of these layers.)

## Activity 3

What is the Question?

### Standards Alignments

Addressing 5.MD.C.5.b

This activity provides students an opportunity to interpret a calculation in the context of the situation (MP2) when a scenario is given with an equation that shows solutions to unknown questions. Students have to interpret the equations and ask the question whose answer is given. Numbers are chosen specifically to prompt students to consider the structure of rectangular prisms.
Student-facing Task Statement

This is the base of a rectangular prism that has a height of 5 cubes.

These are answers to questions about the prism. Read each answer and determine what question it is answering about the prism.

1. 3 is the answer. What is the question?
2. 5 is the answer. What is the question?
3. $3 \times 4 = 12$. The answer is 12. What is the question?
4. $12 \times 5 = 60$. The answer is 60 cubes. What is the question?
5. 3 by 4 by 5 is the answer. What is the question?

Student Responses

1. What is the width of the prism?
2. What is the height of the prism?
3. What is the volume of one layer of cubes?
   What is the area of the base?
4. What is the volume of the entire prism?
5. What are the length, width, and height of the prism?

Launch

- groups of 2

Activity

- 2 minutes: quiet think time
- 4 minutes: partner work time
- Monitor for students who:
  - use informal language, such as layers.
  - use the terms length, width, height, and base in their questions.

Synthesis

- Ask previously selected students to share their solutions.
- Connect the informal language to the math terms length, width, height, and area of a base.
- “How does the expression $3 \times 4 \times 5$ represent the prism described in the second question?” (The area of the base is $3 \times 4 = 12$, and the height is 5, so $3 \times 4 \times 5$ represents the product of length, width, and height.)

Lesson Synthesis

Display the poster of language from a previous lesson.

“What information do you need to measure the volume of any rectangular prism?” (We need to know
the area of a base and how tall it is with that base or we need to know the length, width, and height.)

As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.

“What language can we add to our poster to explain how to find the volume of a prism when we can't see the cubes?” (We can multiply the area of the base and the height. We can multiply the length, width, and height.)

“What is the connection between the number of layers and the height of the prism?” (The number of layers is the number of cubes high, or the height.)

Update the display.

Math Community

After the Cool-down, give students 2–3 minutes to discuss in small groups any revisions to the “Norms” section. Collect and record any revisions.

Suggested Centers

- Can You Build It? (3–5), Stage 3: Rectangular Prisms (Addressing)
- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)

Response to Student Thinking

- Students write a volume that is not a multiple of 12 or the wrong multiple of 12.

- The work in this lesson builds from multiplication and division fluency developed in a prior course.

Next Day Support

- Give students access to connecting cubes during activities 1 and 2 of tomorrow’s lesson.

Prior Unit Support

Grade 4, Unit 1, Section A: Understand Factors and Multiples
Lesson 6: Expressions for Volume

Standards Alignments
Addressing 5.MD.C.5.a, 5.MD.C.5.b, 5.OA.A.1, 5.OA.A.2
Building Towards 5.MD.C.5.b

Teacher-facing Learning Goals
- Find the volume of a right rectangular prism by multiplying the side lengths and connect that to finding volume by multiplying the area of the base by the height.
- Write and interpret numerical expressions in the context of the volume of a rectangular prism.

Student-facing Learning Goals
- Let's write expressions for the volume of rectangular prisms.

Lesson Purpose
The purpose of this lesson is for students to use their understanding of the structure of rectangular prisms to find volume and write numerical expressions to represent volume.

In a previous lesson students learned to find the volume of a rectangular prism by multiplying the number of cubes in a base layer by the number of layers. They connected this understanding to multiplication of length, width and height to find volume. They also used a two-dimensional representation of a base and its corresponding height to find the volume of a rectangular prism.

In this lesson students write and interpret numerical expressions that represent the volume of rectangular prisms. Students use what they know about rectangular prisms to reason whether or not an expression represents its volume.

During the next lesson, students will describe which standard cubic unit they would use to measure the volume of objects that are shaped like rectangular prisms. Consider which objects students see on a regular basis in their communities that are shaped like rectangular prisms and bring picture of those objects to show students during the next lesson. If possible, consider going on a prism scavenger hunt around the community before the next lesson.

Math Community
Tell students they will reflect on their identified norms at the end of this lesson.
Access for:

- **Students with Disabilities**
  - Engagement (Activity 2)

- **English Learners**
  - MLR8 (Activity 1)

**Instructional Routines**
Card Sort (Activity 1), True or False (Warm-up)

**Materials to Gather**
- Connecting cubes: Activity 1

**Materials to Copy**
- Matching Prisms and Expressions (groups of 2): Activity 1

**Lesson Timeline**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>15 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 3</td>
<td>10 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

**Teacher Reflection Question**
As students matched expressions to images of prisms today, what evidence did you see that they are building on their understanding of the structure of rectangular prisms?

**Cool-down** (to be completed at the end of the lesson)

Choose the Expression

**Student-facing Task Statement**
1. Which of these expressions does not represent the volume of the rectangular prism in cubic units? Explain or show your reasoning.

\[ 4 \times 5 \times 8 \times 4 \quad 20 \times 8 \quad (4 \times 5) \times 8 \quad 4 \times 40 \]

2. Choose one of the expressions from above and explain why it represents the volume of the prism in cubic units.

**Student Responses**

1. \[ 4 \times 5 \times 8 \times 4 \] because once you find the area of the base \((4 \times 5)\), you only need to multiply it by the height, 8.

2. Sample responses: \[ 20 \times 8 \] because 20 represents the area of a base and 8 is the height of the prism with that base. \((4 \times 5) \times 8\) because \(4 \times 5\) are the side lengths of a base and 8 is the height of the prism with that base. \[ 4 \times 40 \] because 40 is the area of one of the bases and 4 is the height of the prism with that base.

---

**Warm-up**

10 min

True or False: Parentheses or No Parentheses

**Standards Alignments**

- Addressing: 5.OA.A.2
- Building Towards: 5.MD.C.5.b
The purpose of this True or False is for students to demonstrate strategies and understandings they have for determining equivalence of numerical expressions. These understandings help students deepen their understanding of the properties of operations and are helpful as students interpret expressions for volume. In this activity, students have an opportunity to notice and make use of structure (MP7) when they use the properties of operations to determine equivalence without having to calculate.

**Instructional Routines**

**True or False**

**Student-facing Task Statement**

Decide if each statement is true or false. Be prepared to explain your reasoning.

- \((4 \times 2) \times 5 = 4 \times (2 \times 5)\)
- \((2 \times 5) \times 4 = 2 \times 20\)
- \(5 \times 4 \times 2 = 10 \times 40\)

**Student Responses**

- True because the same numbers are being multiplied but grouped differently.
- True since \(2 \times (5 \times 4) = 2 \times 20, (2 \times 5) \times 4 = 2 \times 20\); both sides are equal to 40.
- False because \(5 \times 2\) is 10 but one side is multiplying that 10 by 4 and the other side by 40.

**Launch**

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

**Activity**

- Share and record answers and strategy.
- Repeat with each statement.

**Synthesis**

- Focus Question: “How can you justify your answer without evaluating both sides?” (I could see on the first equation that all of the factors are the same so it is true.)
- Consider asking:
  - “Who can restate ___’s reasoning in a different way?”
  - “Does anyone want to add on to ____’s reasoning?”
  - “Can we make any generalizations based on the statements?”
Activity 1
Card Sort: Match the Expression

Standards Alignments
Addressing 5.MD.C.5.a, 5.OA.A.1, 5.OA.A.2

The purpose of this activity is for students to interpret expressions that represent the volume of a rectangular prism. Students connect the structure in rectangular prisms to the symbols in their related expressions (MP2, MP7). If there is time and you would like to add student movement, have students make a poster to display the sorted cards. Students can walk around and add additional expressions to other posters to represent the volume of the prism.

Access for English Learners

MLR8 Discussion Supports. Invite students to take turns finding a match and explaining their reasoning. Display the following sentence frames for all to see: “I noticed _____, so I matched . . .” and “_____ and _____ match/do not match because . . .” Encourage students to challenge each other when they disagree.
Advances: Conversing, Representing

Instructional Routines
Card Sort

Materials to Gather
Connecting cubes

Materials to Copy
Matching Prisms and Expressions (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2. Have connecting cubes available for students who need them.

Student-facing Task Statement

1. Match each rectangular prism with the expression(s) that represents its volume in cubic units. Be prepared to explain your

Launch

- Groups of 2
- Distribute one set of pre-cut cards to each group of students.
reasoning.

2. For each prism write one additional expression, not in the card sort, that represents its volume in cubic units.

**Student Responses**

1. Matches:
   a. Prism A matches \((6 \times 5) \times 3, 15 \times 6, 6 \times (5 \times 3)\)
   b. Prism B matches \(2 \times 12, 24 \times 1\)
   c. Prism C matches \(3 \times 7 \times 4, (7 \times 4) \times 3\)
   d. Prism D matches \(3 \times 8 \times 3, 24 \times 3, 9 \times 8\)

2. Additional expressions:
   a. Prism A: \(18 \times 5, 30 \times 3, 3 \times 5 \times 6\) in any order
   b. Prism B matches \(2 \times 12 \times 1\) in any order
   c. Prism C matches \(21 \times 4, 12 \times 7, 28 \times 3, 7 \times 4 \times 3\) in any order
   d. Prism D matches \(8 \times 3 \times 3\) in any order

- “What do you notice about the prisms on these cards?” (They don’t have any cubes, It says “units”.)
- “When the measurements are in units, the cubes we use to fill the prism are called **cubic units**.”

**Activity**

- “In this activity, you will sort some cards into categories of your choosing. When you sort the cards, you should work with your partner to come up with categories.”
- 4 minutes: partner work time
- Select groups to share their categories and how they sorted their cards.
- “Now work with your partner to match each prism with the expressions that represent the volume.”
- 3 minutes: partner work time

**Synthesis**

- Select groups to share their matches.

- Display Prism A:

  - “How do these expressions represent the volume?”
    - \(6 \times (5 \times 3)\)
    - \((6 \times 5) \times 3\)
    - \(15 \times 6\)

- Display:
  - \((5 \times 3) \times 6 = 15 \times 6\)

- “How does the equation relate to Prism A?”
(Both expressions show that the prism has a height of 6. One expression shows the side lengths of the base. The other expression shows the area of the base.)

Advancing Student Thinking

If students do not correctly match expressions to the prisms, ask:

“How can we use the connecting cubes to help you match the expressions to the prisms?”

Activity 2

A Tale of Two Tables

Standards Alignments

Addressing 5.MD.C.5.b, 5.OA.A.1

The purpose of this activity is for students to compare and contrast two different ways to calculate the volume of a rectangular prism: multiplying the area of the base and its corresponding height, and multiplying all three side lengths. Students see that both of these strategies result in the same volume. It is a convention to consider a prism’s base the face it is resting on, however when calculating the volume of a rectangular prism, any face of the prism can be considered a base as long it is multiplied by the corresponding height. Similarly, when calculating the volume of a rectangular prism, any edge can be considered the length, width, or height.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to various tools that can be used to solve the problem. For example, colored pencils can be used to shade the base and different layers of the prisms.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing
Student-facing Task Statement

1. Work with your partner to complete the tables. One partner completes Table 1 and the other completes Table 2.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>length (units)</th>
<th>width (units)</th>
<th>height (units)</th>
<th>volume (cubic units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prism B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Compare your tables and discuss:
   a. What do the tables have in common?
   b. What is different about the tables?

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>area of the base (square units)</th>
<th>height (units)</th>
<th>volume (cubic units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prism B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Launch

- Groups of 2

Activity

- 1 minute: independent work time
- 8 minutes: partner work time

Synthesis

- Ask students to share responses to the second problem. Display the expression: $6 \times 3 \times 4$
- “How does this expression represent the volume of prism A?” (The prism's side lengths are 6, 4, and 3 and I multiply them to find the volume.)
- Display expression: $(6 \times 3) \times 4$
- “How does this expression represent the volume of prism A?” (One base has a length of 6 units and a width of 3 units and the height is 4 units.)
- Display equation: $(6 \times 3) \times 4 = (3 \times 4) \times 6$
- “How do you know the equation is true?” (Both expressions represent the volume of the prism and we can see both expressions in the prism. One of them represents a base with the side lengths 6 and 3 and a height of 4. The other expression represents a base with the side lengths 3 and 4 cubes and a height of 6.)

Student Responses

1. Table 1: The length, height and width of prism A are 6 units, 4 units, and 3 units listed in any order. The volume of prism A is 72 cubic units. The length, height, and width of prism B are 8 units, 5 units, and 4 units listed in any order. The volume of prism B is...
160 cubic units.

Table 2:

<table>
<thead>
<tr>
<th></th>
<th>area of the base (square units)</th>
<th>height (unit)</th>
<th>volume (cubic units)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prism A</strong></td>
<td>18 or 12 or 24</td>
<td>4 or 6 or 3</td>
<td>72</td>
</tr>
<tr>
<td><strong>Prism B</strong></td>
<td>40 or 20 or 32</td>
<td>4 or 8 or 5</td>
<td>160</td>
</tr>
</tbody>
</table>

2. a. We found the volume of the same prisms. We got the same volume. We used the height to find volume. The area of the base is found by using length and width so we used the same information (or numbers, or side lengths).

   b. One has length and width and the other uses the base. There are different numbers. The order of our numbers is different.

**Advancing Student Thinking**

If a student does not write the correct corresponding height for a given base, ask “How do the numbers in the table relate to the prism?” or “How did you decide which numbers to write in the table?”

**Activity 3 (optional)**

**Two Truths and a Lie**

**Standards Alignments**

Addressing 5.MD.C.5.b, 5.OA.A.1, 5.OA.A.2

This activity is optional if students need additional practice writing expressions to represent the
Students are given the opportunity to write and interpret expressions that show that the volume is the same when multiplying the edge lengths or multiplying the area of the base and height. In the second part of the activity, students reason abstractly and quantitatively when they interpret the meaning of expressions in the context of volume (MP2).

**Student-facing Task Statement**

Your teacher will assign you and your partner two prisms.

For each of your assigned prisms:

- Write 2 expressions to represent the volume in cubic units.
- Write 1 expression that does NOT represent the volume in cubic units.

Give your expressions to your partner:

1. For each prism, which expression does not represent its volume in cubic units? How do you know?
2. What other expressions represent the volume of this prism in cubic units?

**Launch**

- Groups of 2
- “You and your partner are going to play 2 truths and a lie with rectangular prisms.”
- “You will each write expressions, 2 true and one false, to represent the volume of two prisms and then trade to answer some questions.”
- “One partner writes 2 truths and a lie for Prisms A and C and the other partner writes about Prisms B and D.”

**Activity**

- 5 minutes: independent work time (create expressions)
- “Switch papers with your partner and see if you can figure out the expression that is a lie for each of their prisms.”
- 5 minutes: independent work time on partner’s problems (analyze expressions)

**Synthesis**

- Display each of the prisms.
- “Which expressions represent the volume of the prism in cubic units? Which do not?”
- “How did you decide the expressions that did not represent the volume of a rectangular prism?” (Looking at the different bases and heights and experimenting with expressions. Finding
Student Responses
Sample responses:

- Prism A Truth: $3 \times 4 \times 5$, $12 \times 5$. Lie: $20 \times 5$
- Prism B Truth: $4 \times 4 \times 4$, $4 \times 16$. Lie: $4 \times 4 \times 4 \times 4 \times 4$
- Prism C Truth: $60 \times 3$, $18 \times 10$. Lie: $10 \times 10 \times 6$
- Prism D Truth: $8 \times 10$, $2 \times 40$. Lie: $14 \times 2$

Advancing Student Thinking
If students do not write any correct expressions that represent the volume of the prism, refer to an expression that does represent the volume of the prism and ask, “Can you explain how this expression represents the volume of the prism?”

Lesson Synthesis

Display Prism C from activity 1:

```
```

“Which expressions could we write to represent the volume of this prism in cubic units?”

For each expression, ask students to explain how it represents the volume of the prism. As students explain, record expressions on a poster for all to see. Use parentheses to show which factors represent the area of a base and which factor represents the corresponding height. If not mentioned by students, display and discuss these expressions.

- $(7 \times 4) \times 3$
- $28 \times 3$
- $(7 \times 3) \times 4$
• 21 × 4
• 7 × (3 × 4)
• 7 × 12

Math Community

After the Cool-down, ask students to individually reflect on the following question: “Which one of the norms did you feel was most important in your work today, and why?” Students can write their responses on the bottom of their Cool-down paper, on a separate sheet of paper, or in a math journal.

Tell students that as their mathematical community works together over the course of the year, the group will continually add to and revise its “Doing Math” and “Norms” actions and expectations.

Suggested Centers

• Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
• Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)

Response to Student Thinking

Students do not correctly identify the expression that does not represent the volume of the prism.

Next Day Support

• During the warm-up in tomorrow’s lesson, ask students to name expressions that might represent the volume of the large cube the boy is sitting in.
Lesson 7: Cubic Units of Measure

Standards Alignments
Addressing 5.MD.C.4

Teacher-facing Learning Goals
- Find the volume of rectangular prisms with standard units of measure by multiplying the base times the height or multiplying the length times the width times the height.

Student-facing Learning Goals
- Let’s use different sized cubic units to measure volume.

Lesson Purpose
The purpose of this lesson is for students to find the volume of rectangular prisms in cubic centimeters, cubic inches, and cubic feet.

In previous lessons, students used unit cubes with a side length of 1 unit to determine the volume of right rectangular prisms. In this lesson, the units are now a specific unit of measure. In grade 5, students use words, not exponents, when recording the cubic unit of measure, such as “cubic centimeters (cm),” “cubic feet (ft)” or “cubic inches (in).” The exponents in Grade 5 are limited to powers of 10, which will be addressed in a later unit.

In this lesson, students distinguish between different standard unit measures of volume. They examine the distinction between cubic cm, cubic in, and cubic ft. Throughout the lesson, students share their rationale for choosing a unit to measure specific real-world objects and learn the importance of identifying the unit of measure when finding the volume of an object (MP6).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities
- Representation (Activity 2)

Instructional Routines
MLR4 Information Gap (Activity 2), Notice and Wonder (Warm-up)
Materials to Gather
- Rulers (centimeters): Activity 1
- Rulers (inches): Activity 1
- Yardsticks: Activity 1

Materials to Copy
- Info Gap Volume Cards (groups of 2): Activity 2

Lesson Timeline
<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>25 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question
During today’s lesson, students described which standard cubic measurement unit they would use to measure the volume of objects that are shaped like rectangular prisms. Consider which objects students see on a regular basis in their communities that are shaped like rectangular prisms. How did you incorporate these objects into the lesson to help students connect their lived experience with their developing math identity? How can you incorporate these objects into future lessons?

Cool-down (to be completed at the end of the lesson)

Find the Volume

Standards Alignments
Addressing 5.MD.C.4

Student-facing Task Statement
Priya’s family rented a moving truck to move their belongings to their new house. The space inside the back of the moving truck is 15 feet long, 5 feet wide, and 8 feet tall.

What is the volume of the back of the moving truck? Explain or show your reasoning. (Remember to include the cubic unit of measure.)

Student Responses
The volume of the moving truck is 600 cubic feet.
Sample response: $8 \times 5 \times 15 = 40 \times 15 = (40 \times 10) + (40 \times 5) = 400 + 200 = 600$
Warm-up
Notice and Wonder: Two Prisms

Standards Alignments
Addressing 5.MD.C.4

The purpose of this warm-up is for students to notice there are different sized cubic units. This will be useful as students use standard units to measure the volumes of right rectangular prisms in upcoming activities. While students may notice and wonder many things about these cubes, the characteristics of a cube and size of the cubes are the important discussion points.

Instructional Routines
Notice and Wonder

Student-facing Task Statement
What do you notice? What do you wonder?

Launch
- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”

Activity
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis
- “The little cube represents a cubic foot. The big cube represents a cubic yard. About how many of the big cubes could we fit in our classroom?” (Answers vary. If time permits, measure the lengths and widths of the classroom, round them to the nearest yard, and check the reasonableness of the estimates.)
- “We have been using cubic units to measure..."
They are different sizes.
• One of the cubes is taller than the boy.

Students may wonder:
• How big are those cubes?
• What are those cubes for?
• How many of the little cubes fit inside the big cube?

volume, but haven’t defined the size of the unit. The choice of the unit depends on the size of the object.”

Activity 1
What are the Units?

Standards Alignments
Addressing 5.MD.C.4

The purpose of this activity is for students to consider how the size of an object impacts the unit we use to measure the volume of that object. Since this is the students’ first experience with these cubic units of measure, it may be helpful for them to see the actual length of a centimeter, inch and foot. Have rulers or cubes available to provide extra support to visualize the size of the cubic units of measure. Because there are no mathematically correct or incorrect answers, this activity provides a rich opportunity for students to discuss and defend different points of view (MP3). The launch of the activity is an opportunity for students to share their experiences and ask questions about the objects to ensure each student has access to the context. If it is helpful, display images of the items for students to reference.

Materials to Gather
Rulers (centimeters), Rulers (inches), Yardsticks

Student-facing Task Statement
For each object, choose the cubic unit you would use to measure the volume: cubic centimeter, cubic inch, or cubic foot.

Launch
• Groups of 2
• Write the list of objects (moving truck, freezer, etc...) on a display for all students to see.
• “Take a minute to read this list. What do you know about these objects? What do you wonder?”
• 3 minutes: partner discussion
• Share a few responses.
• Display the image from the warm-up.
• “In this activity we are going to consider using different cubic units of measure to find the volume of different sized objects. There is no right or wrong answer in these questions, but be prepared to explain your choice.”
• Give students access to rulers and yardsticks.

Activity
• 2 minutes: independent work time
• 5 minutes: partner discussion
• As students work, monitor for students who discuss how big or small the object is when choosing the size of the unit of measure. Ask these students to share during the synthesis.
• If students finish early, ask them to find other objects they would measure the volume of using the different cubic units of measure. If the objects are in the room, they could estimate and check their estimates.

Synthesis
• “How did you decide which units made sense?” (I tried to pick a unit that was not much smaller or much larger than the object being measured.)
• “What if I only wrote 24 as the volume for the freezer? What questions would you have?” (I would wonder 24 what? If we don’t know the unit of measure, we don’t know how big the freezer actually is.)
• Display: 24 cubic centimeters, 24 cubic inches, 24 cubic yards
• “What is the same about these measurements?” (They are all 24 units. They are all cubic units.)
• “What is different about these measurements?” (They are different sizes. Cubic centimeters and cubic inches are a lot smaller than cubic yards.)

Activity 2
Info Gap: Sizing Up Cubic Units

This Info Gap activity gives students an opportunity to determine and request information needed to measure the volume of rectangular prisms using different sized units. The Info Gap routine requires students to make sense of problems by determining what information is necessary and then ask for information they need to solve them. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6). Since this is students’ first experience with the Info Gap routine, the Launch provides an opportunity to rehearse the routine together.

Access for Students with Disabilities

Representation: Access for Perception. Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content. Supports accessibility for: Language, Conceptual Processing

Instructional Routines

MLR4 Information Gap

Materials to Copy

Info Gap Volume Cards (groups of 2)
Student-facing Task Statement

This is a diagram of a freezer. What is the volume of the freezer?

Problem 1:
- Partner A has the problem card.
- Partner B has the data card.

Problem 2:
- Partner B has the problem card.
- Partner A has the data card.

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Launch

- Groups of 2
- Display Sample Problem Card from student workbook.
- “This is a diagram of a freezer. What is the volume of the freezer?”

- Read the problem aloud.
- Listen for and clarify any questions about the context.
- “Some of the information you need to solve this problem is missing, and I have it here. With your partner, decide what information you need to solve the problem, and create a list of questions you can ask to find out.”
- 1–2 minutes: quiet think time
- 2–3 minutes: partner discussion
- Invite students to share 1 question at a time.
- Record each question on a display, and respond with, “Why do you need to know ____ [restate the information requested]?” Students should provide a justification for how they will use the information before the information is revealed. For example, if students ask: “What is the width of the freezer?”, respond with, “Why do you need to know the width of the freezer?”
- Answer questions using only information stated on the Sample Data Card below (do not reveal):

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.
Student Responses

- Problem 1: Since 2 yards is 6 feet, the volume of the dumpster is $20 \times 6 \times 8 = 960$; 960 cubic feet
- Problem 2: Each box is $5 \times 4 \times 9 = 180$; that's 180 cubic centimeters for each juice box. $180 \times 10 = 1,800$ cubic centimeters. Jada cannot fit all ten boxes because 1,800 cubic centimeters is bigger than the 1,500 cubic centimeters of space that Jada has left in her backpack.

- The width of the freezer is 2 feet.
- The length of the freezer is 4 feet.

- Record information that is shared on the display. Give students time to decide whether they have enough information to solve the problem.
- Repeat until students decide they have enough information to solve.
- 2–4 minutes: independent work time

Activity

- In each group, distribute a problem card to one student and a data card to the other student.
- After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Synthesis

- Share the correct answers, including the units, and ask students to discuss the process of solving the problems.
- “What questions did you ask to help you find the volume?” (What is the height of the dumpster? How much space does Jada have in her backpack?)
- “What units were used for each problem? Did those units make sense?” (Cubic feet and cubic centimeters. Yes because a dumpster is big and a juice box is relatively small.)

Lesson Synthesis

Display image from notice/wonder.
Write the words cubic inches, cubic feet, cubic yards for all to see.

“Today we worked with these different sized cubic units. Turn and talk with your partner and try to think of an object for which you would use each unit of measure to find the volume.”

Partner discussion

“Now, pick one of the things you discussed and explain to your partner how you would find the volume of that object.” (I would multiply the area of a base by the corresponding height or I would multiply all three side lengths.)

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)

Student Section Summary

We find the volume of a right rectangular prism by multiplying the side lengths or by multiplying the area of a base by the corresponding height.
Each of these expressions represents the volume of this prism. The volume of this rectangular prism is 60 cubic units.

We can use different sized cubic units to measure the volume of different sized objects.

In this section, we used cubic inches, cubic feet, cubic yards, and cubic centimeters.

--- Complete Cool-Down ---

**Response to Student Thinking**

Students did not label the correct unit of measure or did not find the correct volume.

**Next Day Support**

- Before the warm-up, have students work in partners to discuss a correct response to this cool-down.
Section C: Volume of Solid Figures

Lesson 8: Figures Made of Prisms

Standards Alignments
Building On  5.MD.C.5.a
Addressing  5.MD.C.5.c

Teacher-facing Learning Goals
- Explain that the volume of a figure composed of rectangular prisms is the sum of the volumes of the prisms.
- Find the volume of a figure composed of rectangular prisms in which unit cubes are visible.

Student-facing Learning Goals
- Let’s find the volume of figures made of prisms.

Lesson Purpose
The purpose of this lesson is for students to recognize the structure of a solid figure made up of two non-overlapping right rectangular prisms and understand that its volume is the sum of the volumes of the two rectangular prisms.

In previous lessons, students found the volume of right rectangular prisms represented in images where the cubes were shown or not shown. In this lesson, they build separate rectangular prisms and put them together to emphasize the relationship between the volume of the individual smaller prisms and the volume of the whole figure. Then they find volumes of figures made from two rectangular prisms without first building each part. Students recognize that when the figure composed of two rectangular prisms is given, there are different ways to decompose the figure, leading to different calculations of the same volume.

Access for:

Students with Disabilities
- Engagement (Activity 1)

English Learners
- MLR7 (Activity 2)
Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Connecting cubes: Activity 1

Lesson Timeline

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<td>Cool-down</td>
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</table>

Teacher Reflection Question

What language did students use to demonstrate their understanding that the figures they worked with today were composed of rectangular prisms?

Cool-down (to be completed at the end of the lesson)

Volume of a Figure Made of Prisms

Standards Alignments

Addressing 5.MD.C.5.c

Student-facing Task Statement

Find the volume of the figure. Explain or show your reasoning.

Student Responses

56 cubic units
Sample response: I cut the shape horizontally to make a 2 by 2 by 4 set of cubes and a 4 by 2 by 5 set of cubes. So that’s 16 and 40 more, making 56 cubes altogether.

--- Begin Lesson ---

Warm-up

Which One Doesn't Belong: Different Figures

Standards Alignments
Addressing 5.MD.C.5.c

This warm-up prompts students to carefully analyze and compare features of figures built from rectangular prisms. In making comparisons, students have a reason to use language precisely (MP6), and refer to different measurements of the figures, their volume, or other characteristics.

Instructional Routines
Which One Doesn't Belong?

Student-facing Task Statement
Which one doesn't belong?

Launch
• Groups of 2
• Display the image.
• “Pick one that doesn't belong. Be ready to share why it doesn't belong.”
• 1 minute: quiet think time

Activity
• “Discuss your thinking with your partner.”
• 2–3 minutes: partner discussion
• Share and record responses.

Synthesis
• Refer to Figure C. “Today we are going to find
Student Responses

- A does not belong because it is not organized, it cannot be cut into two equal layers, and it cannot be cut into two rectangular prisms.
- B does not belong because it is the only rectangular prism. It doesn’t have pieces sticking out.
- C does not belong because it is not 3 cubes high.
- D does not belong because it does not have a volume of 12 cubes.

Activity 1

Put It Together

Standards Alignments

<table>
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<th>5.MD.C.5.a</th>
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The purpose of this activity is for students to combine rectangular prisms to make a new figure and understand that the volume of the new figure is the sum of the volumes of the two rectangular prisms. This will be true no matter how they put the two rectangular prisms together.
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity. Supports accessibility for: Attention, Social-Emotional Functioning

Materials to Gather

Connecting cubes

Required Preparation

- Each group of 2 needs at least 30 connecting cubes.

Student-facing Task Statement

1. Partner A, build a rectangular prism with 12 cubes.
2. Partner B, build a rectangular prism with 10 cubes.
3. Put your two rectangular prisms together to make one figure. What is the volume of the new figure? Explain or show your thinking.
4. Diego and Jada put together two rectangular prisms to make this figure:

   a. What is the volume of the figure Diego and Jada made?
   b. Which rectangular prisms could Diego and Jada each have built? Explain or show your thinking. Organize it so it can be followed by others.

Launch

- Groups of 2
- “We are going to build different rectangular prisms and put them together.”
- “Partner A, build a rectangular prism with 12 cubes. Partner B, build a rectangular prism with 10 cubes.”

Activity

- 10 minutes: partner work time building prisms and finding volume.
- 2 minutes: partner discussion time
- Monitor for students who build one figure out of two rectangular prisms in different ways.

Synthesis

- Ask previously identified students to share the individual prisms they built and how they put them together.
- “What was the volume of the shape you made when you put your prisms together? How do you know?” (22 cubic units because I used 12 cubes and my partner used 10 cubes, so that’s 22 cubes altogether.)
**Student Responses**

1. Sample responses include rectangular prisms built from 12 cubes.
2. Sample responses include rectangular prisms built from 10 cubes.
3. Sample response: The volume is 22 cubic units because one part has 10 cubic units and the other part has 12.
4. a. 30 cubic units
   b. Jada could have made the 2 by 3 by 4 prism on the bottom, and Diego could have made the 2 by 3 by 1 prism on top.

**Advancing Student Thinking**

If students do not see two rectangular prisms that compose the figure, ask them to build the figure with cubes and show how they could decompose the figure into two rectangular prisms.

**Activity 2**

**I See Two Prisms**

**Standards Alignments**

Addressing 5.MD.C.5.c

The purpose of this activity is for students to find the volume of a figure composed of two rectangular prisms and recognize that volume is additive. In the previous activity, students physically put two rectangular prisms together and then decomposed a figure into two rectangular prisms. In this activity, they are given an image of a figure composed of two rectangular prisms and asked to find the volume. They can do this in many ways. During the synthesis, connect the different strategies students used to the decomposition of the figure into two rectangular prisms.

As students experiment with different ways to group the cubes to efficiently count them, applying...
what they already know about the volume of rectangular prisms, they are looking for and making use of the structure of geometric objects (MP7). As students begin to generalize the idea that volume is additive, they are looking for and expressing regularity in repeated reasoning (MP8).

### Access for English Learners

**MLR7 Compare and Connect.** Synthesis: Invite students to prepare a visual display that shows the strategy they used to find the volume of each figure. Encourage students to include details that will help others interpret their thinking; for example, using specific language, or using different colors, shading, or arrows. Give students time to investigate each other’s work.

#### Student-facing Task Statement

Find the volume of each figure. Explain or show your reasoning.

**Launch**

- **Groups of 2**

#### Activity

- 5 minutes: individual work time
- 5 minutes: partner discussion
- As students work, monitor for students who:
  - find two (or more) rectangular prisms that can be put together to make the figure and adding their volumes.
  - find the number of cubes in a base layer and multiplying by the number of layers, even though the base layer is not a rectangle (figure d).

#### Synthesis

- Display Figures c and d.
- Ask selected students to share their way of splitting each figure.
- “Who broke the figure up the same way? Who broke it up differently?”
- “Can you think of other ways you could break up these figures?” (I can cut them into several layers—3 horizontal layers for...
b. 50 cubic units. Sample response: There is a 3 by 3 by 5 prism like in the first problem, so that's 45 cubes and there is a row of 5 more so that makes 50.

c. 60 cubic units. Sample response: There is a 3 by 3 by 6 prism, so that's 54 cubes and there is a row of 6 so that's 60.

d. 57 cubic units. Sample response: There is a 3 by 3 by 5 prism, so that's 45 cubes and then a 2 by 2 by 3 prism so that's 12 more.

Advancing Student Thinking

If students did not decompose the figures into rectangular prisms, encourage them to build the figures with connecting cubes and ask, “how can you decompose the figure into two rectangular prisms?”

Lesson Synthesis

“Today we found the volume of figures built from 2 or more rectangular prisms. When a figure is built from 2 or more rectangular prisms its volume can be found by adding the volumes of those rectangular prisms.”

“Where did you use multiplication in your work today? Where did you use addition?” (I used multiplication when I found the volume of a prism. I used addition when I added up the cubes in the different parts I broke the figure into.)

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
Response to Student Thinking

Students did not decompose figures into two rectangular prisms.

Next Day Support

- Give students access to connecting cubes to build the figures.
Lesson 9: Measure Figures Made From Prisms

Standards Alignments
Addressing 5.MD.C.5.c, 5.OA.A.2
Building Towards 5.NBT.A.2

Teacher-facing Learning Goals
• Find the volume of a figure composed of rectangular prisms in which unit cubes are not shown.

Student-facing Learning Goals
• Let's find the volume of more figures.

Lesson Purpose
The purpose of this lesson is for students to find the volume of figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts.

In the previous lesson, the figures students worked with showed the unit cubes, providing the opportunity to visualize the number of unit cubes and find volume in a variety of ways. In this lesson, the figures composed of two non-overlapping right rectangular prisms do not show the individual cubes. This encourages students to apply the volume formulas and make connections between expressions and the way the figures can be decomposed. Given an expression and a solid figure composed of two rectangular prisms, students determine how to decompose the figure to match the given expression (MP2, MP7).

Access for:

Students with Disabilities
• Representation (Activity 1)

English Learners
• MLR7 (Activity 2)

Instructional Routines
Number Talk (Warm-up)

Materials to Copy
• Isometric Dot Paper Standard (groups of 1): Activity 1
Lesson Timeline

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Teacher Reflection Question

What connections did students make between the different strategies shared? What questions did you ask to help make the connections more visible?

Cool-down (to be completed at the end of the lesson)

Find the Volume of a Figure

Student-facing Task Statement

Find the volume of the figure. Explain or show your reasoning.

Student Responses

Sample response 1: Cutting the figure vertically makes a 5 by 2 by 1 prism on the left and a 4 by 2 by 1 prism on the right. The total volume is $(5 \times 2 \times 1) + (4 \times 2 \times 1)$ cubic feet, which is $10 + 8$ or 18 cubic feet.

Sample response 2: Cutting the figure horizontally makes a 3 foot by 2 foot by 1 foot prism on top and a 6 foot by 2 foot by 1 foot prism on bottom. The volume is $(3 \times 2 \times 1) + (6 \times 2 \times 1)$ cubic feet, which is $6 + 12$ or 18 cubic feet.
Warm-up

Number Talk: Times Ten

Standards Alignments
Building Towards 5.NBT.A.2

The purpose of this Number Talk is for students to reason about place value relationships and the properties of multiplication. The elicited understandings and strategies will be helpful in later lessons and units when they multiply large numbers. In this unit, students produce and interpret multiplication expressions in terms of volume.

Instructional Routines
Number Talk

Student-facing Task Statement
Find the value of each expression mentally.
- $6 \times 2$
- $6 \times 2 \times 10$
- $6 \times 20 \times 10$
- $60 \times 20 \times 10$

Launch
- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis
- “What patterns do you notice in the problems we solved?” (There is a 6 and 2 in each product. There are also factors of 10, and the 6 and 2 are sometimes multiplied by a factor of 10.)
- Consider asking:
  - “Who can restate ____’s reasoning in a different way?”
“Did anyone have the same strategy but would explain it differently?”
“Did anyone approach the problem in a different way?”

Activity 1
Find the Volume of Figures

Standards Alignments
Addressing 5.MD.C.5.c

The purpose of this activity is for students to find the volume of figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. There are different ways to decompose the figures. Monitor for students who break the figures apart differently and find the same total volume. To reinforce earlier work with cubic units of measure, ask students for the unit of measure in their response if they state the volume as only a number (MP2). If students finish early, give them isometric grid paper to draw a figure composed of two rectangular prisms for their partner to find the volume.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were needed in order to solve the problem. Display the sentence frame: “The next time I need to find the volume of a figure composed of two rectangular prisms, I will . . .”

Supports accessibility for: Conceptual Processing, Organization, Memory

Materials to Copy
Isometric Dot Paper Standard (groups of 1)

Student-facing Task Statement

Launch
- Groups of 2
- Display the image from the student workbook with the missing side lengths.
- “How can we find the missing side lengths?”
- 1 minute: quiet think time
Partner A: Find the volume of Figure 1.

Figure 1

Sample response 1: Cutting the figure vertically makes a 7 by 2 by 2 prism on the left, and a 5 by 5 by 2 prism on the right. The total volume is $(2 \times 2 \times 7) + (5 \times 2 \times 5)$ cubic feet, which is 28 + 50 or 78 cubic feet.

Cutting the figure horizontally makes a 2 foot by 2 foot by 2 foot prism on top, and a 7 foot by 2 foot by 5 foot prism on bottom. The volume is $(2 \times 2 \times 2) + (7 \times 2 \times 5)$ cubic feet, which is 8 + 70 or 78 cubic feet.

Figure 2

Sample response 1: Cutting vertically gives a 3 inch by 4 inch by 7 inch rectangular

Activity

- “Now, each partner will find the volume of one of the figures and then switch papers. Your job is to then find the volume using a different decomposition strategy than your partner.”

- If there is time, the groups of 2 can make groups of 4 and share responses and strategies.

- 10 minutes: independent work time with partner discussion

- Give students access to isometric dot paper to draw the figures if they finish early.

Synthesis

- Display: Figure 2

- “How did you break up this shape to find the volume?” (I cut off the overhanging piece vertically to make two rectangular prisms. I cut off the bottom piece that the shape is resting on, horizontally.)

- “How did you find the side lengths of the rectangular prisms?” (Some of the lengths are provided, and I found the others by subtracting.)

- “Did you get the same volume when you broke up the figure differently? Why?” (Yes. The calculations were different but they both tell me how many cubic inches it takes to fill the shape.)
prism and a 5 inch by 5 inch by 7 inch rectangular prism. The volume is 
\((3 \times 4 \times 7) + (5 \times 5 \times 7)\) cubic inches, 
which is 84 + 175 or 259 cubic inches.

- Sample response 2: Cutting horizontally 
gives a 3 inch by 9 inch by 7 inch 
rectangular prism and a 2 inch by 5 inch by 
7 inch rectangular prism. The volume is 
\((3 \times 9 \times 7) + (2 \times 5 \times 7)\) cubic inches, 
which is 189 + 70 or 259 cubic inches.

**Advancing Student Thinking**

If students do not find the correct volume of the figure, ask, “Where do you see rectangular prisms in this figure?”

---

**Activity 2**

**Expressions for the Volume of Figures**

**Standards Alignments**

Addressing 5.MD.C.5.c, 5.OA.A.2

The goal of this activity is to represent expressions as decompositions of a figure made of two non-overlapping right rectangular prisms. This gives students an opportunity to interpret parentheses in expressions while also checking their understanding of different ways to represent the volume of rectangular prism; namely, length times width times height and area of a base times the corresponding height.

Students work abstractly and quantitatively in this problem (MP2) as they relate abstract expressions to decompositions of figures composed of two rectangular prisms.
Access for English Learners

MLR7 Compare and Connect. Invite students to think about details they can include that will help others understand their thinking. For example, specific language, using different colors, shading, arrows, labels, notes, diagrams or drawings. Give students time to investigate each others’ work. During the whole-class discussion, ask, “What kinds of details or language helped you understand your partner’s work?”

Advances: Representing, Conversing

Student-facing Task Statement

1. Explain how each expression represents the volume of the figure. Show your thinking. Organize it so it can be followed by others.
   a. \( (2 \times 3 \times 4) + (3 \times 3 \times 2) \)
   b. \( (5 \times 6) + (3 \times 4) \)

2. How does each expression represent the volume of the prism? Explain or show your thinking. Organize it so it can be followed by others.
   a. \( (5 \times 8 \times 6) + (5 \times 4 \times 9) \) cubic inches
   b. \( (5 \times 4 \times 3) + (5 \times 12 \times 6) \) cubic inches

Student Responses

Each student explanation shows that the figure is composed of 2 rectangular prisms with the properties described.

1. a. One prism has a base that is 2 inches by 3 inches and a height of 4 inches

Launch

- Groups of 2
- “Look at the first figure and think about how you might decompose it into 2 prisms.”
- 1 minute: quiet think time
- “Now you will consider how different expressions can represent the volume of figures.”

Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion time
- Monitor for students who
  - use the numbers in the expressions to determine how to break up the figure
  - break up the figure and use this to identify the expressions.

Synthesis

- Invite students to share how the expressions for the first prism represent its volume.
- “Why are there 3 factors in the expression \((2 \times 3) \times 4\) but only 2 factors in the expression \((5 \times 6)\)?” (The 2, 3, and 4 are the length, width, and height of the prism)
and the other prism has a base of 3 inches by 3 inches and a height of 2 inches.

b. One prism has a base that has area 6 square inches and the height for that base is 5 inches. The other prism has base with area 4 square inches and the height for that base is 3 inches.

2. a. One prism has a base that is 8 inches by 6 inches and a height of 5 inches and the other prism has a base that is 4 inches by 9 inches and a height of 5 inches.

b. One prism has a base that is 4 inches by 3 inches and a height of 5 inches and the other prism has a base that is 12 inches by 6 inches and a height of 5 inches.

Advancing Student Thinking

If students do not describe how the expressions represent the volume of the figure, show them part of the expression, the numbers inside the parentheses for example, and ask them to explain which part of the figure is represented.

Lesson Synthesis

“Today we represented the volumes of figures made of rectangular prisms with expressions.”

Display image from the first activity:
“Here is one of the figures we worked with today.”

Display the expression: $3 \times 7 \times 9$ cubic inches.

“Which part of the figure is represented by this expression? How do you know?” (The rectangular prism at the top of the shape. It is 3 inches tall, 9 inches wide, and 7 inches deep, so its volume is $3 \times 7 \times 9$ cubic inches.)

Draw a line to show the prism.

“What is the volume of the other rectangular prism? How do you know?” ($2 \times 5 \times 7$ cubic inches, since it is 2 inches tall, 5 inches wide, and 7 inches deep.)

Display the expression: $(3 \times 7 \times 9) + (2 \times 5 \times 7)$ cubic inches

“How does this expression represent the volume of the figure?” (It shows the addition of the volumes of the two rectangular prisms that the figure is made of.)

**Suggested Centers**

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)
Response to Student Thinking

Students do not find the correct volume of the figure.

Next Day Support

- Before the warm up, display the figure from the cool down from today's lesson with no numbers showing the side lengths and ask students, “Take turns describing to your partner what you would need to know in order to find the volume of this figure and why you would need to know it.”
Lesson 10: Represent Volume with Expressions

Standards Alignments
Addressing 5.MD.C.5, 5.MD.C.5.c, 5.OA.A.1, 5.OA.A.2

Teacher-facing Learning Goals
• Write and interpret numerical expressions to represent the volume of a figure decomposed in different ways.

Student-facing Learning Goals
• Let's write expressions for the volume of figures.

Lesson Purpose
The purpose of this lesson is for students to write, interpret, and evaluate numerical expressions that represent the volume of solid figures composed of two right rectangular prisms.

In previous lessons, students used formulas to find the volume of right rectangular prisms. They also learned to apply these formulas to find the volumes of figures made up of two non-overlapping right rectangular prisms. In this lesson, students use what they have learned to write and interpret numerical expressions in the context of finding the volume of figures composed of rectangular prisms (MP2).
Students recognize that subtraction can be used to find the volume of figures composed of two non-overlapping rectangular prisms.

Access for:

Students with Disabilities
• Action and Expression (Activity 2)

English Learners
• MLR8 (Activity 1)

Instructional Routines
5 Practices (Activity 2), Notice and Wonder (Warm-up)

Lesson Timeline

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Teacher Reflection Question
How did the work of the previous sections in the unit lay the foundation for students to be successful in this lesson?
Cool-down (to be completed at the end of the lesson)

Expressions as Volume

Standards Alignments
Addressing 5.MD.C.5.c, 5.OA.A.2

Student-facing Task Statement

1. Write an expression to represent the volume of the figure in cubic feet.
2. Find the volume of the figure.

Student Responses
1. \((4 \times 8 \times 2) + (2 \times 3 \times 2)\) or \((4 \times 5 \times 2) + (6 \times 3 \times 2)\) (or equivalent)
2. 76 cubic feet
Warm-up

Notice and Wonder: Prism Pieces

Standards Alignments
Addressing 5.MD.C.5

The purpose of this warm-up is for students to notice that figures composed of two right rectangular prisms can be decomposed in different ways which will be useful when students find the volume of figures composed of two right rectangular prisms in a later activity. While students may notice and wonder many things about these images, comparing the side lengths of the two figures is the important discussion point.

Launch
- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”

Activity
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis
- “Do you think the pictures show the same figure? Why or why not?” (Yes, they look the same. Yes, I can use the given side lengths to calculate that they are the same.)

Student Responses
Students may notice:
- Some of the side lengths are the same for the two figures.
- There is a blue prism and an orange prism in both figures.
- The blue prism is shorter in the figure on the left.
- The orange prism is longer in the prism on
the left.

Students may wonder:

- Are they the same figure?
- What is the volume of the figures?

**Activity 1**

Compare Expressions

**Standards Alignments**

Addressing 5.MD.C.5.c, 5.OA.A.1, 5.OA.A.2

The purpose of this activity is for students to find the volume of a figure in different ways. The given figure can be decomposed in two ways into rectangular prisms by making different cuts. However, it can also be found using a single, larger rectangular prism by removing a smaller rectangular prism. This provides an opportunity to express its volume as a difference of volumes of rectangular prisms. Students may notice this feature, and it is highlighted in the activity synthesis.

When students decide whether or not they have the same expressions, they need to reason carefully about what “the same” means. They consider if the order of the factors is different, is it the same expression and if the order of the addends is different, is it the same expression. Students use what they know about volume, geometric figures, and the properties of operations to justify the equivalence of the expressions and critique their peers' reasoning (MP2, MP3, MP7).

**Access for English Learners**

*MLR8 Discussion Supports.* Display sentence frames to support partner discussion: “First, I _____ because . . . ”, “_____ and _____ are the same because . . . ”, “_____ and _____ are different because . . .”

*Advances: Conversing, Representing*

**Student-facing Task Statement**

**Launch**

- Groups of 2
- “You are going to look for different
1. Write an expression to represent the volume of the figure in unit cubes.
2. Compare expressions with your partner.
   a. How are they the same?
   b. How are they different?
3. If they are the same, try to find another way to represent the volume.

Student Responses

1. Sample responses: \((5 \times 2 \times 5) + (3 \times 3 \times 5)\), \((3 \times 5 \times 5) + (2 \times 2 \times 5)\), \((5 \times 5 \times 5) - (2 \times 3 \times 5)\)
2. Sample response: The expressions have the same value but they are different.
3. Answers vary.

Advancing Student Thinking

If students did not see the figure as a cube missing a rectangular prism, ask them to build the figure and the missing piece that can be added to make it a cube.

Activity 2

Find the Volume in Different Ways  

25 min
Standards Alignments
Addressing 5.MD.C.5.c, 5.OA.A.1, 5.OA.A.2

The purpose of this activity is for students to write equivalent expressions in order to find the volume of a figure composed of two right rectangular prisms. Students decompose the figure in two different ways, and write matching expressions to find the volume. For extra support, provide students with colored pencils to shade the two parts of the prism before finding the side lengths they need to calculate the volume.

Monitor and select a student with each of the following strategies to share in the synthesis:

- decomposed the figure into a prism with the side lengths 4 ft by 4 ft by 3 ft and a prism with the side lengths 10 ft by 4 ft by 3 ft and wrote this expression (or one written in a different order) to represent the volume: \((4 \times 4 \times 3) + (10 \times 4 \times 3)\)
- decomposed the figure into a prism with the side lengths 4 ft by 8 ft by 3 ft and a prism with the side lengths 6 ft by 4 ft by 3 ft and wrote this expression (or one written in a different order) to represent the volume: \((4 \times 8 \times 3) + (6 \times 4 \times 3)\)

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to plan a strategy, including the tools they will use, for finding the volume of the figures. If time allows, invite students to share their plan with a partner before they begin.

Supports accessibility for: Organization, Memory, Attention

Instructional Routines

5 Practices

Student-facing Task Statement

1. Find the volume of the figure by decomposing the figure 2 different ways. Show your thinking. Organize it so it can be followed by others.

Launch

- Groups of 2

Activity

- 10 minutes: individual work time
- 5 minutes: partner discussion
- As students work, consider asking, “Why did you choose to decompose the prism that way?”
2. For each way you decomposed the figure, write an expression that represents the volume.

3. Mai used this expression to find the volume of the figure:

\[(10 \times 8 \times 3) - (6 \times 4 \times 3).\]

Use the diagram to interpret Mai’s expression. Show your thinking. Organize it so it can be followed by others.

**Synthesis**

- Ask the two selected students to display their work side by side for all to see.
- “How are the diagrams the same? How are they different?”
- “How do the expressions relate to the diagrams?”
- Display: \((10 \times 8 \times 3) - (6 \times 4 \times 3)\)
- “How does this expression represent the volume of the prism?” (The larger rectangular prism has the side lengths \(10 \times 8 \times 3\) cubic feet. We can subtract a rectangular prism with the side lengths \(6 \times 4 \times 3\) cubic feet.)
- “What is the value of \((10 \times 8 \times 3) - (6 \times 4 \times 3)\)?” \((168)\)

**Student Responses**

1. (4 \(\times\) 4 \(\times\) 3) + (10 \(\times\) 4 \(\times\) 3) cubic feet, (4 \(\times\) 8 \(\times\) 3) + (6 \(\times\) 4 \(\times\) 3) cubic feet, and (10 \(\times\) 8 \(\times\) 3) – (6 \(\times\) 4 \(\times\) 3) cubic feet

2. The figure is a 10 foot by 8 foot by 3 foot rectangular prism and then a 6 foot by 4
foot by 3 foot prism has been removed. So its volume is \((10 \times 8 \times 3) - (6 \times 4 \times 3)\) cubic feet.

Lesson Synthesis

“Today we decomposed the same figure in different ways and wrote expressions to represent the volume.”

“Which decomposition strategy did you prefer to use? Why?” (It depends on the numbers. I decompose the figure in the way that gives me the friendliest numbers.)

“Do you get the same expressions using either decomposition? Why?” (No, because the figure is broken into rectangular prisms with different side lengths.)

“The expressions are different, depending on how we decomposed the shape, but the volume is the same. Why is that?” (The volume doesn’t change. We just decompose the figure in different ways. The expressions are equal.)

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Rolling for Fractions (3–5), Stage 1: Equivalent Fractions (Supporting)

Response to Student Thinking

Students write an incorrect expression without showing decomposition of the figure.

Next Day Support

- Before the warm-up of the next lesson, have students meet with a partner to discuss a correct answer to the cool down from this lesson.
Lesson 11: All Kinds of Prisms

Standards Alignments
Addressing 5.MD.C, 5.MD.C.5

Teacher-facing Learning Goals
- Solve real-world and mathematical problems involving volume.

Student-facing Learning Goals
- Let’s find the volume of all different kinds of prisms.

Lesson Purpose
The mathematical purpose of this lesson is for students to apply what they have learned about finding the volumes of right rectangular prisms and figures composed of right rectangular prisms to solve real-world problems.

In previous lessons, students learned to find the volume of a right rectangular prism by multiplying the number of cubes in a layer by the number of layers. They found the volumes of rectangular prisms with and without the unit cubes showing. They used the associative property of multiplication to represent threefold whole-number products as volumes and learned to find the volume of rectangular prisms with whole-number side lengths by multiplying the length by the width by the height and multiplying the area of the base times the height. They also understand volume as additive. They found volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. In this lesson, students apply these understandings to solve real-world problems.

This lesson has a Student Section Summary.

Access for:
- Students with Disabilities
  - Action and Expression (Activity 2)
- English Learners
  - MLR6 (Activity 1)

Instructional Routines
Which One Doesn’t Belong? (Warm-up)

Lesson Timeline
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<thead>
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<th>Activity</th>
<th>Duration</th>
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<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
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</table>

Teacher Reflection Question
As you finish up this unit, reflect on the norms and activities that have supported each student.
Cool-down (to be completed at the end of the lesson)

The Volume of a Sandbox

Student-facing Task Statement

A preschool is building a sandbox. Below is a diagram that shows the side lengths of the sandbox.

What is the volume of the sandbox? Explain or show your reasoning.

Student Responses

The volume of the sandbox is 124 cubic feet. Sample response: It is a 10 foot by 5 foot by 2 foot prism and a 6 foot by 2 foot by 2 foot prism, so that's \((10 \times 5 \times 2) + (6 \times 2 \times 2)\) cubic feet.
Standards Alignments
Addressing S.MD.C

This warm-up prompts students to compare four images. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use such as volume, base, height, length, and width.

Instructional Routines
Which One Doesn't Belong?

Student-facing Task Statement
Which one doesn't belong?

Launch
- Groups of 2
- Display the image.
- “Pick one that doesn't belong. Be ready to share why it doesn't belong.”

Activity
- 1 minute: quiet think time
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis
- “What do Figures A, C, and D have in common?” (They all show individual cubes. They all have 5 as one of their side lengths.)

Student Responses
Sample responses
- A does not belong because it is not taller than it is wide or long.
- B does not belong because it does not show any individual cubes.
- C does not belong because you can’t tell how tall it is.
Activity 1

Prism Palooza

Standards Alignments

Addressing 5.MD.C.5

The purpose of this activity is for students to practice finding the volume of rectangular prisms given a real-world context. The first problem provides a diagram like students have seen in earlier lessons to illustrate the context. The other problems do not provide a picture so students will need to visualize or draw a sketch of the situation. Going from the words of the problem to a mental image to a solution strategy are all important aspects of making sense of and solving a problem (MP1).

Because these are real-world problems, each rectangular prism sits on a natural base. Monitor for students who use this structure and use the formula connecting volume to the area of the base and the height relative to that base.

Access for English Learners

MLR6 Three Reads. Keep books or devices closed. Display only the problem stem and picture, without revealing the question. “We are going to read this question 3 times.” After the 1st Read: “Tell your partner what this situation is about.” After the 2nd Read: “List the quantities. What can be counted or measured?” Reveal the question. After the 3rd Read: “What strategies can we use to solve this problem?”

Advances: Reading, Representing

Student-facing Task Statement

For each problem, explain or show your reasoning.

1. Han is filling a box with cubes. Below is a diagram of the box. How many cubes can fit in the box if Han completely packs it, without gaps between cubes?

Launch

- Groups of 2

Activity

- 8 minutes: individual work time
- 2 minutes: partner discussion
- Monitor for students who find the volume
2. Clare bought a storage container for her art supplies. The storage container was 4 feet wide, 9 feet long, and 5 feet high. What is the volume of her container?

3. Mai’s new bedroom has a walk-in closet with a floor that measures 30 square feet. Her closet ceiling is 9 feet from the floor. What is the volume of her closet?

**Student Responses**

1. 300 cubes. Sample response: there are 6 \( \times \) 10 or 60 cubes in one layer and there are 5 layers so \( 5 \times 60 = 300 \).

2. 180 cubic feet. Sample response: \( 4 \times 5 = 20 \) and \( 20 \times 9 = 180 \).

3. 270 cubic feet. Sample response: \( 30 \times 9 = 270 \)

**Synthesis**

- Ask selected students to share their solutions for the second problem.
- “How are the strategies the same? How are they different?” (They both got the same solution, but one person multiplied \( 4 \times 9 \) to get the area of the base and then multiplied the result by 5, but the other person chose to multiply \( 4 \times 5 \) first.)
- “How is the third problem different from the first two?” (It does not give us the length and width of the closet. It just gives the area of the floor.)

**Activity 2**

Problem Solving with Figures

**Standards Alignments**

Addressing 5.MD.C.5

The purpose of this activity is for students to solve a real-world problem that involves finding the volume of a figure composed of two right rectangular prisms. Unlike many other figures students have seen, this one can be decomposed into two rectangular prisms in only one way. Students may rearrange the two prisms to make a single, long rectangular prism.
Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Give students access to graph paper or connecting cubes as tools to use to design their own garden.

*Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization*

Student-facing Task Statement

The elementary school is going to build a raised bed garden like the one in the picture, but they will use a different design. Here is a diagram that shows the side lengths of the garden the school will build.

1. What is the volume of the garden? Explain or show your reasoning.
2. Write an expression to represent the volume of the garden.

Launch

- Display the picture of the garden from the student workbook:

```
10 feet
3 feet
4 feet
8 feet
3 feet
```

- “This type of garden is called a raised bed garden because the plants are not in the ground.”
- “If we planted a garden at our school, what vegetables would you want to grow?”

Activity

- 5 minutes: individual work time
- 5 minutes: partner discussion
- Monitor for students who break the garden into two rectangular prisms, finding the volume of each, and for students who put them together to form a single rectangular prism.

Synthesis

- Display the diagram of the garden from the task.
- Invite students to share how they found its
**Student Responses**

1. 216 cubic feet
   
   \[(3 \times 8 \times 4) + (3 \times 10 \times 4) = 96 + 120 = 216\]

2. \((3 \times 8 \times 4) + (3 \times 10 \times 4)\) with factors in any order

3. Sample response: \(3 \times 4 \times 18\)

4. Sample response: I like my garden better because it looks more like a square.

**Lesson Synthesis**

“In this unit, you did a lot of work with prisms and volume. What are some things you know about volume and rectangular prisms?”

Share and record students’ responses.

Display these images from the unit:

“Talk to your neighbor. Discuss which part of the unit was your favorite and why. How did working
Could the cubes help you with the rest of the volume work in the unit?"

**Suggested Centers**

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

**Student Section Summary**

Some figures are made from two rectangular prisms. We can decompose these figures and find the volume of each prism. Then, we add the volumes of the two prisms to find the total volume of the figure.

There is often more than one way to decompose figures made from 2 rectangular prisms. These expressions can be used to find the volume of the figure:

\[(3 \times 3 \times 5) + (5 \times 2 \times 5)\]

\[(3 \times 5 \times 5) + (2 \times 2 \times 5)\]
Lesson 12: Lots and Lots of Garbage (Optional)

Standards Alignments
Addressing 5.MD.C.5

Teacher-facing Learning Goals
• Find the volume of rectangular prisms with standard units of measure by multiplying the base times the height or multiplying the length times the width times the height.

Student-facing Learning Goals
• Let's investigate what happens to garbage.

Lesson Purpose
The purpose of this lesson is for students to use their understanding of volume of rectangular prisms to solve a real world problem. Students make a model to visualize large amounts of garbage.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling.

In previous lessons, students computed volumes. They packed unit cubes inside boxes and found the volume of the box by multiplying the side lengths of the box. In this lesson, they will investigate a real world problem using those ideas.

In the warm-up, students read a short paragraph from an article about garbage exports. Students are asked to visualize 3,300 tons or 60 containers of garbage. Mathematizing the world by recognizing math in everyday life is an important step of mathematical modeling.

In the first activity, students find different arrangements of 60 shipping containers, assuming that they are cubes. While real shipping containers are not cubes, we can simplify the situation by using figures that are easier to work with. This step of making simplified assumptions is an important part of mathematical modeling. Students are asked to draw a diagram of one of their arrangements.

In the second activity, students estimate the number of shipping containers on a fully loaded cargo ship from a picture. When students estimate quantities and make assumptions, they model with mathematics. Students circle back to the question of garbage by computing together how much garbage could fit on a fully loaded cargo ship. When students translate a mathematical answer back into the real-world situation, they model with mathematics (MP4).

Access for:

ิ Students with Disabilities
• Engagement (Activity 1)

English Learners
• MLR7 (Activity 1)
Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Connecting cubes: Activity 1
- Patty paper: Activity 1
- Tools for creating a visual display: Activity 1

Lesson Timeline

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<td>Activity 2</td>
<td>20 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
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Teacher Reflection Question

Who got to do math today in class and how do you know? Identify the norms or routines that allowed those students to engage in mathematics. How can you adjust these norms and routines so all students do math tomorrow?

---

Warm-up

Notice and Wonder: Garbage Truck

Standards Alignments

Addressing 5.MD.C.5

The purpose of this warm-up is to for students to discuss how much space a ton of garbage takes up, which will be useful when students consider 3,300 tons of plastic trash in shipping containers in a later activity. Some students may not know the precise definition of a ton. Encourage students to visualize what 2,000 pounds of trash might look like. The synthesis gives students an opportunity to consider the kind and the amount of trash produced in their community.

Instructional Routines

Notice and Wonder
Student-facing Task Statement
What do you notice? What do you wonder?

Launch
- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis
- “The average garbage truck can hold 12–14 tons of garbage.”
- If needed, “1 ton is the same as 2,000 pounds.”
- “If this truck collects trash only from our school (or neighborhood) what kinds of things might the truck be filled with?”
- “How long do you think it would take our school (or neighborhood) to completely fill the truck?”

Student Responses
Students may notice:
- There is a garbage truck.
- The truck looks like it is parked or waiting at a stop light.
- It looks like the truck is near a park.

Students may wonder:
- What is in the truck?
- How much garbage is in the truck?
- Where is the garbage truck?

Activity 1
60 Containers

In this activity students find different ways to arrange 60 shipping containers. They decide which arrangement is the best in the context of shipping 3,300 tons of garbage.
Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, “What did the representations have in common?”, “How were they different?”, and “Did anyone solve the problem the same way, but would explain it differently?”.
Advances: Representing, Conversing

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Use visible timers or audible alerts to help students monitor their progress, and make sure they have enough time to create a display.
Supports accessibility for: Attention, Organization

Materials to Gather

Connecting cubes, Patty paper, Tools for creating a visual display

Required Preparation

- Each group of 2 needs at least 20 connecting cubes.

Student-facing Task Statement

1. Find at least 5 different ways to arrange 60 containers. Represent each arrangement with an expression.

2. Create a visual display to show which is the best arrangement for shipping the 3,300 tons of garbage.

Student Responses

1. $1 \times 1 \times 60, 1 \times 2 \times 30, 1 \times 3 \times 20,$
   $1 \times 4 \times 15, 1 \times 5 \times 12, 1 \times 6 \times 10,$
   $2 \times 15 \times 2, 2 \times 3 \times 10, 2 \times 5 \times 6, 3 \times 4 \times 5$

Launch

- Groups of 2 or 4
- “Here is a short paragraph from a newspaper article from 2018."
- Display “Malaysia plans to send back roughly 3,300 tons of plastic trash to countries like the U.S. and Canada."
- “What are some questions you have after reading this?” (Where is Malaysia? Why is Malaysia sending it back? Why are countries sending their trash to places like Malaysia? Where else does our trash go?)
- If needed, show students Malaysia on a map.
- “The trash fits in 60 shipping containers and will be returned in a large cargo ship over water."
2. Students may make any visual model that makes sense to them.

- “What does a shipping container look like?”
- Display image of a shipping container.

- Give access to connecting cubes, poster paper, patty paper, and writing tools to each group.

Activity

- 1 minute: independent work time.
- 15 minutes: partner work time
- As students work, monitor for students who use different arrangements.

Synthesis

- Invite previously selected students to share their visual display.
- “What arrangement does this group’s visual display represent? Why might they think this is the best arrangement?”
- “What can you tell this group that may make them change their decision?” (It's too tall and might start falling in the water. It's too wide and might not fit. It's too wide and might not leave room for anything else.)
- Consider asking “How much trash do you think is in each of the containers?” (Students can estimate or make an educated guess.)
Activity 2

How Many Containers on the Ship?

Standards Alignments
Addressing 5.MD.C.5

The purpose of this activity is for students to estimate the number of shipping containers on a fully loaded cargo ship. Then they make many assumptions about the arrangement of the containers on the boat to come up with a better estimate.

Student-facing Task Statement

1. How many containers are on the cargo ship?

   Record an estimate that is:

   too low  about right  too high

2. How many containers are on the cargo ship?

Launch

• Groups of 2
• Display image.
• “How many containers are on the cargo ship?”
• “What is an estimate that's too high?” “Too low?” “About right?”
• 1 minute: quiet think time
• Record responses.

Activity

• “A cargo ship can hold many more than 60 containers. The second image is a picture of a cargo ship that is fully loaded."
• 5 minutes: independent work time
• “Discuss your estimates and assumptions with your partner.”
• 2 minutes: partner discussion
• Monitor for students who are making different assumptions:
  ○ The ship is fully loaded.
  ○ Some of the space is taken up with structures.
Record an estimate that is:

<table>
<thead>
<tr>
<th>too low</th>
<th>about right</th>
<th>too high</th>
</tr>
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</table>

3. What assumptions were you making when you came up with your estimates?

**Student Responses**

1. Sample responses:
   - Too low: 0-200
   - About right: 1,000-1,500
   - Too high: 5,000-6,000

2. Sample responses:
   - Too low: 1,000-2,000
   - About right: 4,000-5,000
   - Too high: 10,000-12,000

3. Assumptions:
   - The ship is fully loaded.
   - Some of the space is taken up with structures.
   - There are more containers below deck that we can't see.
   - All containers are the same size.

**Synthesis**

- Invite students to share their estimates and any assumptions they made.
- “How could you improve your estimate?” (Recreate the ship using a model. Check the height, width, and length of the container arrangement. See how many are on one side of the ship and multiply that to get a closer estimate.)

**Lesson Synthesis**

10 min
“We used math to investigate a real-world situation that each city and country has to think about: what to do with their garbage.” Ask students to write a journal response to a few of the following questions.

- “How much trash do you think our community makes and where do you think it goes?”
- “Where can the US and Canada put the 3,300 tons of garbage returned from Malaysia?”
- “What are some challenges about taking care of the 3,300 tons of garbage and putting it somewhere in the US or Canada?”
- “What are some challenges about sending the trash to another city or country?”

**Suggested Centers**

- Five in a Row: Multiplication (3–5), Stage 4: Three Factors (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
Family Support Materials
Family Support Materials

Finding Volume

In this unit, students find the volume of rectangular prisms and figures composed of two prisms.

Section A: Unit Cubes and Volume

In this section, students learn to call the amount of space an object takes up volume. Volume is measured in cubes. For example, this prism has a volume of 120 cubes.

To find the volume of any prism, students can find the number of cubes in one layer and multiply that number by the number of layers. In this example, students might describe this prism as having 5 layers of 24 cubes. They can find the number of cubes by multiplying 5 and 24. So, $5 \times 24 = 120$.

Section B: Expressions for Finding Volume

In this section, students find the volume of a rectangular prism by multiplying the side lengths or by multiplying the area of the base by the height.

For example they can multiply the length by width by height, or $3 \times 5 \times 6$ or they can find the bottom area by multiplying $3 \times 5$ to get 15 and then multiplying 15 by 6. The volume of this rectangular prism is 90 cubic units.

Section C: Volume of Solid Figures

In this section, students learn that some figures are made from two rectangular prisms. They break apart these figures and find the volume of each prism. Then, they add the volumes of the two prisms to find the total volume of the figure.
Depending on how they break it apart, they can find the volume in different ways. They could multiply in these ways to find the volume of the figure:

\[(3 \times 3 \times 5) + (5 \times 2 \times 5)\]
\[(3 \times 5 \times 5) + (2 \times 2 \times 5)\]

**Try it at home!**
Near the end of the unit, find the volume of these figures with your student.

Questions that may be helpful as they work:

- How are the 2 problems the same? How are they different?
- Can you explain or show me how you found the volume?
- How did you know you needed that number or piece of information?
Depending on how they break it apart, they can find the volume in different ways. They could multiply in these ways to find the volume of the figure:

Try it at home!

Near the end of the unit, find the volume of these figures with your student.

Questions that may be helpful as they work:
- How are the 2 problems the same? How are they different?
- Can you explain or show me how you found the volume?
- How did you know you needed that number or piece of information?

Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment
Finding Volume: Section A Checkpoint

1. Which figure has greater volume? Explain your reasoning.

A

B

2. Find the volume of each prism. Explain or show your reasoning.

a.
b.

3. Explain or show how the expression $4 \times 6$ represents the volume of the rectangular prism in cubes.
Finding Volume: Section B Checkpoint

1. Find the volume of the rectangular prism. Explain or show your reasoning.

2. Explain or show how the expression \(4 \times 48\) represents the volume of the rectangular prism in cubic units.
3. A box is shaped like a rectangular prism. Its measurements are 6 cm by 2 cm by 15 cm. Select all expressions that represent the volume of the box in cubic centimeters.

A. $6 \times 2 \times 15$
B. $2 \times 90$
C. $12 \times 30$
D. $8 \times 15$
E. $15 \times 12$
Finding Volume: Section C Checkpoint

1. Find the volume of each figure. Explain or show your reasoning.

a. 

b. 

2. A jewelry box is shaped like a rectangular prism. The base of the box has an area of 200 square centimeters and its height is 6 centimeters. What is the volume of the jewelry box? Explain or show your reasoning.
Finding Volume: End-of-Unit Assessment

1. Select all expressions that represent the volume of this rectangular prism in cubic units.

A. $3 \times 4 \times 5$
B. $3 + 4 + 5$
C. $20 + 20 + 20$
D. $15 \times 15 \times 15 \times 15$
E. $5 \times 12$

2. Find the volume of the figure. Explain or show your reasoning.
3. Select all expressions that represent a way to fill the rectangular prism with layers of centimeter cubes for a base.

   - 5 \times 24
   - 10 \times 12
   - 8 \times 15
   - 6 \times 20
   - 4 \times 30

4. Find the volume of a rectangular prism with the given side lengths.
   a. The length is 2 units, the width is 5 units, and the height is 7 units.
   b. The base has an area of 200 square inches and the height is 6 inches.
5. Find the volume of the prism. Explain or show your reasoning.

6. Which statement describes the volume of the rectangular prism in cubic units?

A. The volume is 7 cubic units.
B. The volume is less than 12 cubic units.
C. The volume is 12 cubic units.
D. The volume is greater than 12 cubic units.
7. Mai’s class is designing a garden with two levels and this general shape.

- The garden should have at least 200 square feet for the plants.
- The volume should be less than 500 cubic feet.

a. Recommend side lengths for the tiered garden that fit the needs of Mai’s class.

b. Label the diagram to show your choices for the side lengths.
Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment
Problem 1

Goals Assessed

- Describe volume as the space taken up by a three dimensional object.

Which figure has greater volume? Explain your reasoning.

A

B

Solution

A. Sample response: Figure A is 10 cubes and Figure B is only 9 cubes so Figure A takes up more space.

Problem 2

Goals Assessed

- Measure the volume of a rectangular prism by finding the number of unit cubes needed to fill it.
- Use the layered structure in a rectangular prism to find volume.

Find the volume of each prism. Explain or show your reasoning.

a.
Solution

a. 30 cubes or equivalent. There are 2 layers of 15 cubes.
b. 60 cubes or equivalent. The base layer has 20 cubes and there are 3 layers in the full prism.

Problem 3

**Goals Assessed**

- Use the layered structure in a rectangular prism to find volume.

Explain or show how the expression \(4 \times 6\) represents the volume of the rectangular prism in cubes.
Solution

The layer of cubes the prism sits on has 6 cubes. There are 4 of these layers in the prism. So the volume is $4 \times 6$ cubes.
Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Find volume using $\text{length} \times \text{width} \times \text{height}$ or area of the base $\times$ height.

Find the volume of the rectangular prism. Explain or show your reasoning.

Solution

140 cubic inches. The base is $7 \times 4$ or 28 square inches and I multiplied that by the height, $28 \times 5 = 140$. 

Problem 2

Goals Assessed

- Describe the calculations from the previous section as $\text{length} \times \text{width} \times \text{height}$ or area of the base $\times$ height.
- Find volume using $\text{length} \times \text{width} \times \text{height}$ or area of the base $\times$ height.

Explain or show how the expression $4 \times 48$ represents the volume of the rectangular prism in cubic units.
Solution

The base that the prism sits on has an area of 48 square units. There are 4 layers of 48 unit cubes in the prism so its volume is $4 \times 48$ cubic units.

Problem 3

**Goals Assessed**

- Find volume using $\text{length} \times \text{width} \times \text{height}$ or area of the base $\times$ height.

A box is shaped like a rectangular prism. Its measurements are 6 cm by 2 cm by 15 cm. Select all expressions that represent the volume of the box in cubic centimeters.

A. $6 \times 2 \times 15$
B. $2 \times 90$
C. $12 \times 30$
D. $8 \times 15$
E. $15 \times 12$

Solution

["A", "B", "E"]
Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Find the volume of a figure composed of rectangular prisms.

Find the volume of each figure. Explain or show your reasoning.

a.

b.

Solution

Sample responses:

a. 28 cubic units. Sample response: I cut this figure into a 2 cube by 3 cube by 4 cube rectangular prism that has volume 24 cubic units and there are 4 more cubic units so that's 28 cubic units.

b. 384 cubic inches. Sample response: I can cut this figure into two rectangular prisms. One has side lengths 8 inches by 6 inches by 4 inches and the other is 6 inches by 8 inches by 4 inches. They each have volume $8 \times 6 \times 4$ or 192 cubic inches. The total volume is 384 cubic inches.
Problem 2

**Goals Assessed**

- Find the volume of a figure composed of rectangular prisms.

A jewelry box is shaped like a rectangular prism. The base of the box has an area of 200 square centimeters and its height is 6 centimeters. What is the volume of the jewelry box? Explain or show your reasoning.

**Solution**

1,200 cubic centimeters. I multiplied the area of the base by the height, $200 \times 6 = 1,200$. 

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments
Addressing 5.MD.C.5.a, 5.OA.A.2

Narrative
Students identify different ways to find the volume of a rectangular prism, including:
- multiplying length, width, and height
- decomposing into layers that are one cube thick and multiplying the number of cubes in one layer by the number of layers
- choosing a face as the base and multiplying its area and the corresponding height

Students who select B or D are using the wrong operation. Students may not select C or E if they do not think about the different ways of decomposing the prism.

Select all expressions that represent the volume of this rectangular prism in cubic units.

A.  $3 \times 4 \times 5$
B.  $3 + 4 + 5$
C.  $20 + 20 + 20$
D.  $15 \times 15 \times 15 \times 15$
E.  $5 \times 12$
Problem 2

**Standards Alignments**
Addressing 5.MD.C.5.c

**Narrative**
Students find the volume of a figure. No strategy is suggested but students will likely cut the figure into two rectangular prisms and add the volumes of those prisms. But they may decompose the figure in any way that allows them to count the total number of cubes that make the figure.

Find the volume of the figure. Explain or show your reasoning.

![Diagram of a 3D figure]

**Solution**

90 cubes. Sample responses:

- A vertical cut makes a 2 by 4 by 5 rectangular prism and a 2 by 5 by 5 rectangular prism. The volume is \((2 \times 4 \times 5) + (2 \times 5 \times 5)\) or 90 cubes.
- A horizontal cut makes a 2 by 6 by 5 rectangular prism and a 3 by 2 by 5 rectangular prism. The volume is \((2 \times 6 \times 5) + (3 \times 2 \times 5)\) or 90 cubes.
- There are 5 vertical layers with 18 cubes in each layer. So there are \(5 \times 18\) or 90 total cubes in the figure.
Problem 3

**Standards Alignments**
Addressing 5.MD.C.5.a, 5.OA.A.2

**Narrative**
Students select different ways to fill a rectangular prism with centimeter cubes. The 3 correct options represent the different ways of decomposing the prism into layers. Options B and C both give the correct value but do not correspond to the structure of the rectangular prism. Students who select B and C likely understand how to find the volume of the prism but are using that value to assess the correctness of the answer rather than the structure of the expressions.

Select all expressions that represent a way to fill the rectangular prism with layers of centimeter cubes for a base.

A. \(5 \times 24\)
B. \(10 \times 12\)
C. \(8 \times 15\)
D. \(6 \times 20\)
E. \(4 \times 30\)

**Solution**

["A", "D", "E"]
Problem 4

**Standards Alignments**
Addressing 5.MD.C.5.b

**Narrative**
Students find the volume of rectangular prisms given their side lengths. For the first prism they are given the length, width, and height and for the second one they are given the area of the base and the height. They also need to pay attention to the units for the measurements which determine the units for the volume.

Find the volume of a rectangular prism with the given side lengths.

a. The length is 2 units, the width is 5 units, and the height is 7 units.
b. The base has an area of 200 square inches and the height is 6 inches.

**Solution**

a. 70 cubic units
b. 1,200 cubic inches

Problem 5

**Standards Alignments**
Addressing 5.MD.C.5.c

**Narrative**
Students find the volume of a figure composed of two rectangular prisms. Since one of the side lengths is 1 foot, the calculations are not difficult. Students need to decide how to decompose the figure and there are 3 main choices, namely a horizontal cut, a vertical cut, or completing the shape to make a rectangular prism and using subtraction.

Find the volume of the prism. Explain or show your reasoning.
Solution

18 cubic feet. Sample explanation: \((6 \times 2 \times 1) + (3 \times 2 \times 1) = 12 + 6 = 18\).

Problem 6

**Standards Alignments**
Addressing 5.MD.C.3.b, 5.MD.C.4

**Narrative**
Students see a rectangular prism that is partly filled with unit cubes, without gaps or overlaps, where the cubes do not, and cannot, fill the container exactly. Students identify that in this situation they cannot calculate the volume exactly, but they can say that it is at least the number of cubes that fit completely inside the prism. Students may choose response A if they count the cubes that are shown in the image. Students may select response C if they notice that 12 cubes will fit inside the prism but do not identify that they don’t fill up all of the space.

Which statement describes the volume of the rectangular prism in cubic units?
Problem 7

A. The volume is 7 cubic units.
B. The volume is less than 12 cubic units.
C. The volume is 12 cubic units.
D. The volume is greater than 12 cubic units.

Solution

D

Standards Alignments

Addressing 5.MD.C.5.c

Narrative

Students design a composite prism to meet certain criteria. The context is a tiered garden. While slightly different from the garden context that they worked on in the lesson, students have the scaffold of an image which indicates the overall shape. They choose the side lengths which amounts to choosing a length, two widths, and two heights. There are some constraints for the choices that they can make but there are many possibilities that meet the criteria and are realistic. Students may use unrealistic side lengths, such as 1 foot wide and 100 feet long, which meet the given criteria. The diagram will not necessarily reflect the measurements students choose (that is, it will not be drawn to scale).

Mai's class is designing a garden with two levels and this general shape.
The garden should have at least 200 square feet for the plants.
The volume should be less than 500 cubic feet.

a. Recommend side lengths for the tiered garden that fit the needs of Mai's class.

b. Label the diagram to show your choices for the side lengths.

Solution

a. Sample response: I decided to make each level 6 feet wide and 20 feet long. So that makes 240 square feet total for the plants. The lower section is 1 foot deep so that means that it uses 120 cubic feet of soil. The upper section is 2 feet deep so it uses 240 cubic feet of soil and that's 360 cubic feet altogether.

b.
Lesson 1: What Is Volume?

Cool Down: Which Has More Volume?

Which object has a greater volume? Explain or show your reasoning.

A

B

Grade 5 Unit 1
Lesson 1
Lesson 2: Measure Volume

Cool Down: Volume of a Rectangular Prism

Find the volume of the rectangular prism. Explain or show your reasoning.
Lesson 3: Volumes of Prism Drawings

Cool Down: Jada's Prism

Jada's prism has 4 layers and each layer has 9 cubes.

1. Circle the prism that is Jada's.

A  B  C  D

2. Find the volume of Jada's prism. Explain or show your reasoning.
Lesson 4: Use Layers to Determine Volume

Cool Down: Use Expressions

1. If the rectangular prism was filled completely, how many cubes could it hold?

2. Explain or show how the expression $3 \times 8$ represents the volume of the prism.
Lesson 5: Side Lengths of Rectangular Prisms

Cool Down: Determine the Volume

Here is a base of a rectangular prism.

What is the volume of the prism if it has a height of 3?
Lesson 6: Expressions for Volume

Cool Down: Choose the Expression

1. Which of these expressions does not represent the volume of the rectangular prism in cubic units? Explain or show your reasoning.

   \[ 4 \times 5 \times 8 \times 4 \quad 20 \times 8 \quad (4 \times 5) \times 8 \quad 4 \times 40 \]

2. Choose one of the expressions from above and explain why it represents the volume of the prism in cubic units.
Lesson 7: Cubic Units of Measure

Cool Down: Find the Volume

Priya's family rented a moving truck to move their belongings to their new house. The space inside the back of the moving truck is 15 feet long, 5 feet wide, and 8 feet tall.

What is the volume of the back of the moving truck? Explain or show your reasoning. (Remember to include the cubic unit of measure.)
Lesson 8: Figures Made of Prisms

Cool Down: Volume of a Figure Made of Prisms

Find the volume of the figure. Explain or show your reasoning.
Cool Down: Find the Volume of a Figure

Find the volume of the figure. Explain or show your reasoning.
1. Write an expression to represent the volume of the figure in cubic feet.

2. Find the volume of the figure.
Lesson 11: All Kinds of Prisms

Cool Down: The Volume of a Sandbox

A preschool is building a sandbox. Below is a diagram that shows the side lengths of the sandbox.

What is the volume of the sandbox? Explain or show your reasoning.
Instructional Masters
# Instructional Masters for Finding Volume

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Isometric Dot Paper Standard
Isometric Dot Paper Standard
Card Sort Rectangular Prism Cards

A

B

C

D

E

F
Problem C Card 1

Dumpper

What is the volume of the dumpper in cubic feet?

Outline a construction site.

Andre wants to measure the volume of the dumpper that is 8 feet tall. The dumpper is 20 feet long.

Data Card 1

Dumpper

Problem C Card 2

Juice Box

Can Jade fit all 10 Juice boxes in her backpack? Why or why not?

Jade wants to fit 10 Juice boxes in her backpack for the field trip.

Data Card 2

Juice Box

● Each juice box is 4 centimeters wide.
● Each juice box is 5 centimeters long.
● Jade’s backpack has space for up to 1,500 cubic centimeters.
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Matching Prisms and Expressions

A

B

C

D
Can You Build It Stage 2 Directions

Directions:
- Take 2 number cards to make a two-digit number.
- Both partners build as many rectangles as they can with that area.
- When both players are finished, compare rectangles.
- Each player gets 1 point if they both have all the same rectangles.
- A player gets 2 points if they build a rectangle with the given area that their partner does not have.
- The player with the most points after 8 rounds wins the game.
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Number Cards (0-10)

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Can You Build It Stage 3 Directions

Directions:

- Take 2 number cards to make a two-digit number.
- Both partners build as many prisms as they can with that volume.
- When both players are finished, compare prisms.
- Each player gets 1 point if they both have all the same prisms.
- A player gets 2 points if they build a prism with the given volume that their partner does not have.
- The player with the most points after 8 rounds wins the game.
Capture Squares Stage 7 Gameboard

Directions:

- On your turn:
  - Roll the number cube and spin the spinner. Find the product.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - If you can't draw a line, roll and spin again.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.
Capture Squares Stage 7 Spinner
Directions:
- Partner A:
  - Put a paper clip on 2 numbers in the grey rows. Multiply the numbers. Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

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<tr>
<td>21</td>
<td>24</td>
<td>27</td>
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</table>
Directions:

- **Partner A:**
  - Put a paper clip on 3 numbers in the grey row. Multiply the numbers. Cover the product of the numbers with a counter.

- **Partner B:**
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.

- Take turns. The first partner to cover 5 squares in a row wins.

<table>
<thead>
<tr>
<th>36</th>
<th>2</th>
<th>125</th>
<th>20</th>
<th>24</th>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Rolling for Fractions Stage 1 Recording Sheet

- Each partner:
  - Roll 6 number cubes. If you roll any fives, they count as a wild and can be any number you’d like.
  - See if you can fill in a statement to show equivalent fractions.
  - If you cannot make equivalent fractions, re-roll as many cubes as you’d like.
  - If you can make equivalent fractions, record your statement and show or explain how you know the fractions are equivalent. You get 1 point for each pair of equivalent fractions you write.
- Take turns. The partner who has the most points once the recording sheet is full wins the game.

| 1/2 | = | 2/4 |
| --- |  |
| 3/6 | = | 1/2 |
| 2/4 | = | 1/2 |
| 1/2 | = | 3/6 |
Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
  - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
  - Check your partner’s work to make sure you agree.
  - Determine the number of points each partner gets:
    - 2 points for creating an expression less than 1
    - 5 points for creating an expression greater than 1
    - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

<table>
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<tr>
<th>round</th>
<th>equation</th>
<th>points</th>
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<tr>
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<td>6</td>
<td><img src="image6" alt="Equation" /></td>
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Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
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  - Check your partner’s work to make sure you agree.
  - Determine the number of points each partner gets:
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Round equation points

1 2 3 4 5 6
Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
  - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
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Round equation points

1 2 3 4 5 6
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Units at this level include:

- **Finding Volume**
- Fractions as Quotients and Fraction Multiplication
- Multiplying and Dividing Fractions
- Wrapping Up Multiplication and Division with Multi-Digit Numbers
- Place Value Patterns and Decimal Operations
- More Decimal and Fraction Operations
- Shapes on the Coordinate Plane
- Putting it All Together

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