Linear Equations and Linear Systems

Teacher Guide
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# Linear Equations and Linear Systems

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Linear Equations and Linear Systems

Unit Narrative

In this unit, students build on their grades 6 and 7 work with equivalent expressions and equations with one occurrence of one variable, learning algebraic methods to solve linear equations with multiple occurrences of one variable. Students learn to use algebraic methods to solve systems of linear equations in two variables, building on their grades 7 and 8 work with graphs and equations of linear relationships. Understanding of linear relationships is, in turn, built on the understanding of proportional relationships developed in grade 7 that connected ratios and rates with lines and triangles.

The unit begins with a lesson on “number puzzles” in which students are shown a number line diagram that displays numerical changes (e.g., as in grade 7 work with signed numbers) and asked to write descriptions of situations and equations that the diagram could represent. Students are then given descriptions of situations in which an unknown quantity is linearly related to a combination of known quantities and asked to determine the unknown quantities in any way they can, e.g., using diagrams or writing equations.

In the second and third sections of the unit, students write and solve equations, abstracting from contexts (MP2) to represent a problem situation, stating the meanings of symbols that represent unknowns (MP6), identifying assumptions such as constant rate (MP4), selecting methods and representations to use in obtaining a solution (MP5), reasoning to obtain a solution (MP1), interpreting solutions in the contexts from which they arose (MP2) and writing them with appropriate units (MP6), communicating their reasoning to others (MP3), and identifying correspondences between verbal descriptions, tables, diagrams, equations, and graphs, and between different solution approaches (MP1).

The second section focuses on linear equations in one variable. Students analyze “hanger diagrams” that depict two collections of shapes that balance each other. Assuming that identical shapes have the same weight, they decide which actions of adding or removing weights preserve that balance. Given a hanger diagram that shows one type of shape with unknown weight, they use the diagram and their understanding of balance to find the unknown weight. Abstracting actions of adding or removing weights that preserve balance (MP7), students formulate the analogous actions for equations, using these along with their understanding of equivalent expressions to develop algebraic methods for solving linear equations in one variable. They analyze groups of linear equations in one unknown, noting that they fall into three categories: no solution, exactly one solution, and infinitely many solutions. They learn that any one such equation is false, true for one value of the variable, or (using properties of operations) true for all values of the variable. Given descriptions of real-world situations, students write and solve linear equations in one variable, interpreting solutions in the contexts from which the equations arose.
The third section focuses on systems of linear equations in two variables. It begins with activities intended to remind students that a point lies on the graph of a linear equation if and only if its coordinates make the equation true. Given descriptions of two linear relationships students interpret points on their graphs, including points on both graphs. Students categorize pairs of linear equations graphed on the same axes, noting that there are three categories: no intersection (lines distinct and parallel, no solution), exactly one intersection (lines not parallel, exactly one solution), and same line (infinitely many solutions).

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Critique**

- strategies for solving puzzles (Lesson 1)
- reasoning about maintaining balance in equations (Lesson 3)
- solutions of linear equations (Lessons 4 and 5)
- reasoning about structures of systems of equations (Lesson 14)
- explanations of solutions (Lesson 16)

**Justify**

- strategies for solving puzzles (Lessons 1 and 5)
- predictions about maintaining balance (Lesson 2)
- predictions about solutions of linear equations (Lesson 6)

**Generalize**

- about the structures of equations that have one, infinite, and no solutions (Lessons 7 and 8)
- about the structures of systems of equations (Lessons 14 and 15)

In addition, students are expected to use language to represent and interpret situations involving systems of linear equations, compare solutions of linear equations, and describe graphs of systems of linear equations.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow the one in which it was first introduced.
Learning Targets

Linear Equations and Linear Systems

Lesson 1: Number Puzzles
- I can solve puzzle problems using diagrams, equations, or other representations.

Lesson 2: Keeping the Equation Balanced
- I can add or remove blocks from a hanger and keep the hanger balanced.
- I can represent balanced hangers with equations.

Lesson 3: Balanced Moves
- I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution.

Lesson 4: More Balanced Moves
- I can make sense of multiple ways to solve an equation.

Lesson 5: Solving Any Linear Equation
- I can solve an equation where the variable appears on both sides.

Lesson 6: Strategic Solving
- I can solve linear equations in one variable.

Lesson 7: All, Some, or No Solutions
- I can determine whether an equation has no solutions, one solution, or infinitely many solutions.

Lesson 8: How Many Solutions?
- I can solve equations with different numbers of solutions.

Lesson 9: When Are They the Same?
- I can use an expression to find when two things, like height, are the same in a real-world situation.
Lesson 10: On or Off the Line?
• I can identify ordered pairs that are solutions to an equation.
• I can interpret ordered pairs that are solutions to an equation.

Lesson 11: On Both of the Lines
• I can use graphs to find an ordered pair that two real-world situations have in common.

Lesson 12: Systems of Equations
• I can explain the solution to a system of equations in a real-world context.
• I can explain what a system of equations is.
• I can make graphs to find an ordered pair that two real-world situations have in common.

Lesson 13: Solving Systems of Equations
• I can graph a system of equations.
• I can solve systems of equations using algebra.

Lesson 14: Solving More Systems
• I can use the structure of equations to help me figure out how many solutions a system of equations has.

Lesson 15: Writing Systems of Equations
• I can write a system of equations from a real-world situation.

Lesson 16: Solving Problems with Systems of Equations
• I can use a system of equations to represent a real-world situation and answer questions about the situation.
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Required Materials

Copies of blackline master
Pre-printed cards, cut from copies of the blackline master
Pre-printed slips, cut from copies of the blackline master
Scissors
Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.
Section: Puzzle Problems

Lesson 1: Number Puzzles

Goals

- Calculate a missing value for a number puzzle that can be represented by a linear equation in one variable, and explain (orally and in writing) the solution method.
- Create a number puzzle that can be represented by a linear equation in one variable.

Learning Targets

- I can solve puzzle problems using diagrams, equations, or other representations.

Lesson Narrative

In this introductory lesson, students solve and write number puzzles of the sort where you are given a series of operations on a number, and the final result, and have to find out the original number. These puzzles are good preparation for solving linear equations, where students have to perform operations on each side of the equation to isolate the variable. Students use representations of their choosing, such as line diagrams, tape diagrams, and equations.

Alignments

Building On

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Building Towards

- 8.EE.C.7: Solve linear equations in one variable.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Notice and Wonder

Student Learning Goals

Let’s solve some puzzles!

1.1 Notice and Wonder: A Number Line

Warm Up: 5 minutes
The purpose of this warm-up is to introduce students to a number line diagram they will be using to represent addition and subtraction of integers in future lessons. To introduce this idea, students write a story and equation that a given number line diagram could represent.

**Building On**
- 7.NS.A.1

**Instructional Routines**
- Notice and Wonder

**Launch**
Tell students they are going to see a number line diagram and that their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

**Student Task Statement**
What do you notice? What do you wonder?

![Number Line Diagram]

**Student Response**
Things students might notice:
- There are 3 arrows above the number line and 1 arrow below it.
- The blue arrow could represent -4.
- The green arrow could represent +9 or +4.
- The black arrow goes from 0 to 12.

Things students might wonder:
- What do the arrows represent?
- Why do the arrows above and below both start at 0 and end at 12?
Activity Synthesis
After students’ notice and wonder ideas are displayed, tell them that the diagram is about money, and invite students to share a possible story and equation that the diagram represents. For example, someone owes $4. Then they earn $7 doing chores and another $9 helping out the neighbor with their yard. The equation would be $-4 + 7 + 9 = 12$.

1.2 Telling Temperatures

15 minutes
The purpose of this activity is for students to solve number puzzles using any representation they choose. Students then make sense of other representations for the same problems, starting with those of a partner. The whole-class discussion should focus on the strengths and weaknesses of different representations (MP5). For example, tape diagrams only work for problems with all positive values, so you could use one for the distance puzzle, but a tape diagram would not work for the temperature puzzle.

Identify students using different strategies, such as number line diagrams, tape diagrams, written out reasoning, and equations, to share during the whole-class discussion.

Building Towards
• 8.EE.C.7

Instructional Routines
• MLR2: Collect and Display

Launch
Arrange students in groups of 2. Give 5–6 minutes of quiet work time followed by partner discussion. During their discussion, partners explain their representations of the problems to one another, including any representations they started out with that didn’t work. If partners used the same representation, ask them to find another representation they could use to solve the puzzle. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use color to highlight connections between changing temperatures and positive and negative numbers on a number line.
Supports accessibility for: Visual–spatial processing
Support for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. During partner discussion, circulate and listen to students explain their representations of the problems to one another. Listen for the variety of ways students describe their number line diagrams, tape diagrams, and equations. Record examples of student language and diagrams on a visual display. Continue to update the display, introducing mathematical vocabulary next to student language as students move through the activity. Remind students that they can borrow words, phrases or representations from the display to support their work. This will help students develop mathematical language about each representation.

Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Students who write the equation \(2x - 10 + 40 = 16\) for the temperature puzzle may incorrectly follow that up with \(2x - 50 = 16\). By comparing their process with a different representation, help them notice that a decrease of 10 and an increase of 40 is, overall, an increase of 30.

Student Task Statement

Solve each puzzle. Show your thinking. Organize it so it can be followed by others.

1. The temperature was very cold. Then the temperature doubled. Then the temperature dropped by 10 degrees. Then the temperature increased by 40 degrees. The temperature is now 16 degrees. What was the starting temperature?

2. Lin ran twice as far as Diego. Diego ran 300 m farther than Jada. Jada ran \(\frac{1}{3}\) the distance that Noah ran. Noah ran 1200 m. How far did Lin run?

Student Response

1. -7 degrees Celsius. Sample responses:

(Or \(2t - 10 + 40 = 16\). Or Starting temperature = \((16 - 40 + 10) \cdot \frac{1}{2}\)).

1. 1400 meters. Sample response:
| Noah's distance | 400 | 400 | 400 |
| Jada's distance | 400 |     |     |
| Diego's distance | 400 | 300 |     |
| Lin's distance  | 700 | 700 |     |

(Or \( \left( \frac{1}{2}x - 300 \right) \cdot 3 = 1200 \). Or Lin's distance = \( 2 \left( \frac{1}{3} \cdot 1200 + 300 \right) \).

**Activity Synthesis**

The goal of this discussion is for students to make connections between different representations of problems and, more centrally to this unit as a whole, to see equations as an efficient way to represent problems.

Select previously identified students to share their representations. Record and display a visual of the representations for each problem. If not shared by students, make sure at least one equation representation for each problem is included. Once multiple representations from the class are displayed, ask 2–3 students to explain which one(s) they prefer and why. If not brought up in discussion, note that some representations, such as tape diagrams, do not work all the time. For example, a tape diagram is not possible for the temperature puzzle due to the negative values in the problem. Other representations, such as equations, can work for almost any type of problem.

### 1.3 Making a Puzzle

**15 minutes**

In this task, students create their own number puzzle to trade with a partner to solve. The purpose of this task is for students to practice writing and solving multi-step number puzzles and compare their representations with the representations of others to decide which are more efficient. While these problems are phrased using the words “number puzzle,” it is important to note the mathematical work students are doing here thinking about, creating, and solving situations that are, essentially, linear equations in one variable, even if not all students are using equations to represent them.

While students are sharing representations, identify partners who solved at least one of their puzzles using different representations to share during the whole-class discussion. If possible, select groups who used an equation.

**Building Towards**

- 8.EE.C.7

**Instructional Routines**

- MLR7: Compare and Connect
Launch

Keep students in the same groups. Give 5 minutes for students to write their own puzzle and make a representation of their solution before trading their puzzle with a partner to solve. Make sure students write their puzzle and solution in such a way that when they trade, their partner cannot see the solution.

If both partners created the same (or very similar) representation for their solutions, ask them to work together to create a different representation. If they created different representations, ask partners to discuss which one they prefer and to be ready to explain why during the whole-class discussion.

Support for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* Before students create their own number puzzle, use this routine to give students the opportunity to identify and explain the correspondences between different strategies for representing the equation \(3d + 5 = 2 = 15\). Invite students to demonstrate their strategy using a visual or numerical representation. Display one example of each representation to discuss. In pairs or groups, ask students to compare their strategies. Ask students to discuss how the strategies are the same or different, and then share with the whole class. This will increase students' meta-awareness and use of language for comparisons of mathematical representations.

*Design Principle(s): Optimize output (for comparison); Maximize meta-awareness*

Student Task Statement

Write another number puzzle with at least three steps. On a different piece of paper, write a solution to your puzzle.

Trade puzzles with your partner and solve theirs. Make sure to show your thinking.

With your partner, compare your solutions to each puzzle. Did they solve them the same way you did? Be prepared to share with the class which solution strategy you like best.

Student Response

Answers vary. Sample response: The temperature was very cold. Then the temperature dropped by 5 degrees. Then the temperature doubled. The temperature is now 40 degrees. What was the starting temperature?

Equation: \(2(x - 5) = 40\)

Answer: 25 degrees
Are You Ready for More?
Here is a number puzzle that uses math. Some might call it a magic trick!

1. Think of a number.
2. Double the number.
3. Add 9.
4. Subtract 3.
5. Divide by 2.
6. Subtract the number you started with.
7. The answer should be 3.

Why does this always work? Can you think of a different number puzzle that uses math (like this one) that will always result in 5?

Student Response
Explanations vary. Sample response: Using \( x \) to represent any number, the steps in the puzzle form the expression \( ((2x + 9 - 3)/2) - x \), which equals 3.

Activity Synthesis
Select partners previously identified to share a puzzle with the class and the two representations they created. Ask which representation they prefer and why. If students do not bring it up in their explanations, ask which of their representations was the most efficient one for solving the puzzle.

During this discussion, students may ask you to state which representation is best and, if so, it is important to note that there is no one correct answer for the “best representation.” The “best representation” is the one that makes sense to the student and helps them solve the problem. However, as problems grow more and more complex, students are likely to find that certain representations are more useful for solving problems than others.

Lesson Synthesis
Ask students to think about what their number line diagrams, tape diagrams, and equations represented in each of the activities. Guide them in seeing that stories with an unknown quantity usually involve actions, like the temperature rising, earning money by doing chores, or relationships, like Diego’s distance being half of Lin’s distance and 300 m more than Jada’s. Ask them to think about the puzzle they wrote in the last activity and whether they described actions or relationships. Invite their opinions about which representations best represent actions and which best represent relationships.

Tell students to think about the expression \( x + 5 \). Ask: “What could this mean in terms of two numbers being related to each other? What could this represent as an action?” (One number is 5 more than the other. A sample action: the temperature increases by 5 degrees.)
If time allows, ask students to make up another puzzle that describes actions if they previously chose relationships, and vice versa, and to represent their puzzles with a diagram and an equation. Display diagrams and equations for all to see.

1.4 Seeing the Puzzle

Cool Down: 5 minutes
Building Towards

- 8.EE.C.7

Student Task Statement

André and Elena are reading the same book over the summer. André says he has read \( \frac{1}{5} \) of the book. Elena says she has read 20 more pages than André. If Elena is on page 55, how many pages are in the book?

Lin has drawn a diagram to solve this question. Find her error.

\[
\begin{array}{c}
a \quad 20 \\
\end{array}
\]

Student Response

Answers vary. Sample response: The diagram shows Elena read \( \frac{1}{5} \) of the book. In order to represent that André read \( \frac{1}{5} \) of the book, subtract 20 from Elena's 55 pages to get that André has read 35 pages. If 35 pages is \( \frac{1}{5} \) of the book, then the book must be \( 5 \times 35 \), or 175 pages long.

Student Lesson Summary

Here is an example of a puzzle problem:

Twice a number plus 4 is 18. What is the number?

There are many different ways to represent and solve puzzle problems.

- We can reason through it.

  Twice a number plus 4 is 18.
  Then twice the number is \( 18 - 4 = 14 \).
  That means the number is 7.

- We can draw a diagram.
• We can write and solve an equation.

\[
2x + 4 = 18
\]
\[
2x = 14
\]
\[
x = 7
\]

Reasoning and diagrams help us see what is going on and why the answer is what it is. But as number puzzles and story problems get more complex, those methods get harder, and equations get more and more helpful. We will use different kinds of diagrams to help us understand problems and strategies in future lessons, but we will also see the power of writing and solving equations to answer increasingly more complex mathematical problems.

Lesson 1 Practice Problems
Problem 1

Statement

Tyler reads \(\frac{2}{15}\) of a book on Monday, \(\frac{1}{3}\) of it on Tuesday, \(\frac{2}{9}\) of it on Wednesday, and \(\frac{3}{4}\) of the remainder on Thursday. If he still has 14 pages left to read on Friday, how many pages are there in the book?

Solution

180 pages

Problem 2

Statement

Clare asks Andre to play the following number puzzle:

- Pick a number
- Add 2
- Multiply by 3
- Subtract 7
- Add your original number

Andre's final result is 27. Which number did he start with?
Solution
Andre's starting number was 7.

\[3(x + 2) - 7 + x \text{ simplifies to } 4x - 1. 4x - 1 = 27 \text{ has solution } x = 7.\]

Problem 3

Statement
In a basketball game, Elena scores twice as many points as Tyler. Tyler scores four points fewer than Noah, and Noah scores three times as many points as Mai. If Mai scores 5 points, how many points did Elena score? Explain your reasoning.

Solution
22 points. Noah scores 15 points, which means Tyler scores 11 points, and Elena scores twice as many points as Tyler.

Problem 4

Statement
Select all of the given points in the coordinate plane that lie on the graph of the linear equation \(4x - y = 3\).

A. \((-1, -7)\)
B. \((0, 3)\)
C. \((\frac{3}{4}, 0)\)
D. \((1, 1)\)
E. \((2, 5)\)
F. \((4, -1)\)

Solution
["A", "C", "D", "E"]
(From Unit 3, Lesson 12.)
Problem 5

Statement

A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 feet long, followed by the nested carts (so 0 nested carts means there’s just the starting cart). The store measured a row of 13 nested carts to be 23.5 feet long, and a row of 18 nested carts to be 31 feet long.

a. Create a graph of the situation.

b. How much does each nested cart add to the length of the row? Explain your reasoning.

c. If the store design allows for 43 feet for each row, how many total carts fit in a row?

Solution

a.

b. 1.5 feet. Explanations vary. Sample response: The slope, which can be found with the calculation \( \frac{31 - 23.5}{18 - 13} \), tells the rate of change, or amount that each nested cart adds.

c. 26 nested carts, or 27 carts total. Explanations vary. Sample response: We can subtract 4 feet from 43 feet for the starting cart and then divide by 1.5 to find the number of nested carts that
will fit. We can use a table and repeatedly add 1.5. There are 12 more feet from 31 to 43, so $12 + 1.5$, or 8 more carts, can be added to 18.

(From Unit 3, Lesson 5.)

Problem 6

Statement
Triangle $A$ is an isosceles triangle with two angles of measure $x$ degrees and one angle of measure $y$ degrees.

a. Find three combinations of $x$ and $y$ that make this sentence true.

b. Write an equation relating $x$ and $y$.

c. If you were to sketch the graph of this linear equation, what would its slope be? How can you interpret the slope in the context of the triangle?

Solution
a. Answers vary; the key constraint is that the three angles must sum to 180 ($x + x + y = 180$). For example, $x = y = 60$, or $x = 30$ and $y = 120$ or $x = 45$ and $y = 90$.

b. $2x + y = 180$

c. -2. In the context of the triangle, for every 1 degree increase of $x$, $y$ decreases by 2 degrees.

(From Unit 3, Lesson 13.)

Problem 7

Statement
Consider the following graphs of linear equations. Decide which line has a positive slope, and which has a negative slope. Then calculate each line's exact slope.
Solution

The line $\ell$ moves up on the y-axis as it moves to the right, so it has a positive slope. $m$ has a negative slope since it moves downwards. The slope of $\ell$ is $\frac{80-20}{8-5} = \frac{60}{3} = 20$. The slope of $m$ is $\frac{-40-20}{8-5} = \frac{-60}{3} = -20$.

(From Unit 3, Lesson 10.)
Section: Linear Equations in One Variable
Lesson 2: Keeping the Equation Balanced

Goals
- Calculate the weight of an unknown object using a hanger diagram, and explain (orally) the solution method.
- Comprehend that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger by the same amount are moves that keep the hanger balanced.

Learning Targets
- I can add or remove blocks from a hanger and keep the hanger balanced.
- I can represent balanced hangers with equations.

Lesson Narrative
This lesson is the first of a sequence of eight lessons where students learn to work with equations that have variables on each side. In this lesson, students recall a representation that they have seen in prior grades: the balanced hanger. The hanger is balanced because the total weight on each side, hanging at the same distance from the center, is equal in measure to the total weight on the other side.

In the warm-up, students encounter two real hangers, one balanced and one slanted, and notice and wonder about what could cause the hangers' appearance. This leads into the first activity where students consider two questions about a balanced hanger: first, whether a change of the number of weights keeps the hanger in balance, and second, how to find the unknown weight of one of the shapes if the weight of the other shape is known. Students learn that adding or removing the same weight from each side is analogous to writing an equation to represent the hanger and adding or subtracting the same amount from each side of the equation. They reason similarly about how halving the weight on each side of the hanger is analogous to multiplying by $\frac{1}{2}$ or dividing by 2. In both the hanger and the equation, these kinds of moves will produce new balanced hangers and equations that ultimately reveal the value of the unknown quantity.

In the second activity, students encounter a hanger with an unknown weight that cannot be determined. This situation parallels the situation of an equation where the variable can take on any value and the equation will always be true, which is a topic explored in more depth in later lessons.

As students use concrete quantities to develop their power of abstract reasoning about equations, they engage in MP2.
Alignments

Addressing

• 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

Building Towards

• 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

Instructional Routines

• MLR1: Stronger and Clearer Each Time
• MLR2: Collect and Display
• MLR8: Discussion Supports
• Notice and Wonder

Student Learning Goals

Let’s figure out unknown weights on balanced hangers.

2.1 Notice and Wonder: Hanging Socks

Warm Up: 5 minutes

The purpose of this warm-up is to give students an opportunity to ground their understanding of equality in the context of weight, which is a context that will be used throughout the lesson.

Building Towards

• 8.EE.C

Instructional Routines

• MLR2: Collect and Display
• Notice and Wonder

Launch

Tell students they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

Student Task Statement

What do you notice? What do you wonder?
Student Response

Things students might notice:

- There are four socks / four clips / two hangers.
- One hanger is hanging diagonally and one is straight.
- Half of the socks are blue and half are pink.
- One of the socks looks heavier because it is weighing down that side of its hanger.
- You could fit 20 toes inside of those socks.

Things students might wonder:

- Are the hangers a number line and the socks numbers?
- Is this representing a multiplication problem?
- Would the crooked hanger straighten out if there were two socks on its right side?
- Why is one of the hangers slanted when the socks look identical?
- Did they put something in one blue sock that is making it weigh more than the other sock?

Activity Synthesis

Ask students to share their ideas. Record and display the responses for all to see. In the interest of time, you can ask if anything students wondered was a “why” question, meaning the question begins with the word why. Refer to MLR 2 (Collect and Display).

If not brought up during the first part of the discussion, ask students why they think the left hanger is unbalanced while the right hanger is balanced. Students should understand that a hanger will
only balance if the weight of the unknown objects in both socks is the same. If they are not the same, then the heavier side is lower than the lighter side.

2.2 Hanging Blocks

10 minutes (there is a digital version of this activity)
The purpose of this task is for students to understand and explain why they can add or subtract expressions from each side of an equation and still maintain the equality, even if the value of those expressions are not known. Both problems have shapes with unknown weight on each side to promote students thinking about unknown values in this way before the transition to equations.

While the focus of this activity is on the relationship between both sides of the hanger and not equations, some students may start the second problem by writing and solving an equation to find the weight of a square. While students are working, identify those using equations and those not using equations to answer the second problem during the whole-class discussion.

Addressing
• 8.EE.C

Instructional Routines
• MLR2: Collect and Display
• MLR8: Discussion Supports

Launch
Display the problem image for all to see. Tell students that this is a hanger problem similar to the one in the warm-up, only instead of the weights hidden inside socks, each block type represents a different weight. Give 5 minutes of quiet work time followed by a whole-class discussion.

If using the digital activity, introduce the hanger problem to set the context and connection to the warm-up. Give students individual work time to figure out the weights and use the applet to check their work.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using concrete representations. Create a physical model of the hanger diagrams using a clothes hanger and weighted objects. Demonstrate how the weights of objects on either side impact whether the hanger is balanced or unbalanced.
Supports accessibility for: Conceptual processing; Visual-spatial processing
Anticipated Misconceptions

In the first question, students may think the hanger will stay in balance since removing the 5 triangles results in three shapes on each side.

Student Task Statement

This picture represents a hanger that is balanced because the weight on each side is the same.

1. Elena takes two triangles off of the left side and three triangles off of the right side. Will the hanger still be in balance, or will it tip to one side? Which side? Explain how you know.

2. If a triangle weighs 1 gram, how much does a square weigh?

Student Response

1. The hanger will tip to the left since only 2 triangles were taken off the left while 3 triangles were taken off the right, which means more weight was taken off the right side making it lighter than the left side.

2. A square weighs $\frac{3}{2}$ grams or equivalent. The hanger can be represented by the equation $3x + 2(1) = x + 5(1)$.

Activity Synthesis

Begin the discussion by asking if students think the hanger will stay in balance, tip to the left, or tip to the right. Select 2–3 students to explain their vote. Make sure the class understands that removing unequal amounts of weight from the two sides results in the hanger tipping before moving on. Use MLR 2 (Collect and Display) to capture student reasoning about it being okay to add or remove terms of the same “size” from both sides of an equation.

For the second question, select previously identified students to explain their answers, with the students who used equations going last. Record and display the specific equations the selected students wrote for all to see, such as $x + x + x + 1 + 1 = x + 1 + 1 + 1 + 1 + 1$ or $3x + 2 = x + 5$, and use it to help the class visualize how that student solved for the weight of a square.

The outcome of this discussion should be that it is okay to add or remove terms of the same “size” from both sides of an equation, and the sides will still be equal. This can be thought of in terms of
shapes hanging on hangers, where you can remove one square from both sides or add two triangles to both sides, and the hanger will stay in balance. Equations are a more abstract representation of this, but the same concept holds: you can remove one \( x \) from both sides or add two 3s to both sides and the equation is still true with the left side equal to the right side. Removing equal weights from both sides can leave the hanger with 2 squares on the left and 3 triangles (or just 3) on the right. In equation form, this is the same as \( 2x = 3 \). Finally, you can halve the amount of weight on both sides of the hanger and keep it in balance, which is the same as multiplying \( 2x = 3 \) by \( \frac{1}{2} \) (or dividing both sides by 2).

**Support for English Language Learners**

*Speaking: MLR8 Discussion Supports.* During the discussion, use this routine to amplify mathematical uses of language to explain how to balance the hanger. To begin the discussion, ask students, “How do you know that the hanger is balanced? Explain how this relates to solving an equation.” Emphasize words and phrases such as: “each side of the equation,” “balance,” “same size,” and/or “equal weights.” Invite students to use these sentence frames in their response: “After the triangles are removed....” and “I can keep the hanger/equation balanced by....” This will help students reason and explain that the balancing of an equation removes equal amounts from each side of the equation.

*Design Principle(s): Support sense-making*

### 2.3 More Hanging Blocks

15 minutes (there is a digital version of this activity)

Building on the previous activity, students now solve two more hanger problems and write equations to represent each hanger. In the first problem, the solution is not an integer, which will challenge any student who has been using guess-and-check in the previous activities to look for a more efficient method. In the second problem, the solution is any weight, which is a preview of future lessons when students purposefully study equations with one solution, no solution, and infinite solutions. The goal of this activity is for students to transition their reasoning about solving hangers by maintaining the equality of each side to solving equations using the same logic. In future lessons, students will continue to develop this skill as equations grow more complex culminating in solving systems of equations at the end of this unit.

As students work, identify those using strategies to find the weight of one square/pentagon that do not involve an equation. For example, some students may cross out pairs of shapes that are on each side (such as one circle and one square from each side of hanger A) to reason about a simpler problem while others may replace triangles with 3s and circles with 6s first before focusing on the value of 1 square. This type of reasoning should be encouraged and built upon using the language of equations.
Addressing

- 8.EE.C

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give 5 minutes of quiet work time followed by partner discussion. Let students know that they should be prepared to share during the whole-class discussion, so they should make sure their partner understands and agrees with their solution.

If students use the digital activity, the applet provides a way for students to check solutions. Encourage students to work individually (most likely they will need paper/pencil to work these problems) and then check their thinking using the digital applet. After students have had 5 minutes to work alone and with the applet, give them time to discuss their thinking with a partner before the whole-class discussion.

Support for Students with Disabilities

*Representation: Internalize Comprehension.* Differentiate the degree of difficulty or complexity by beginning with more accessible values. Provide students with a simplified hanger, with fewer shapes, to solve first. Encourage students to begin by labeling values they know.
*Supports accessibility for: Conceptual processing*

Anticipated Misconceptions

Triangles weigh 3 grams in this activity instead of 1 gram as in the previous activity.

Student Task Statement

A triangle weighs 3 grams and a circle weighs 6 grams.
1. Find the weight of a square in Hanger A and the weight of a pentagon in Hanger B.

2. Write an equation to represent each hanger.

Student Response

1. In Diagram A, each square weighs $\frac{15}{4}$ grams or 3.75 grams or equivalent. In Diagram B, the pentagon's weight cannot be determined. It could be any possible weight.

2. Answers vary. Sample responses: $3 + 18 + x = 5x + 6$ and $12 + 2x = 2x + 3 + 3 + 6$

Are You Ready for More?

What is the weight of a square on this hanger if a triangle weighs 3 grams?

Student Response

This hanger is not possible since the squares would have to weigh -3 grams for the hanger to balance. If the square's weight were a positive value, then the left side would have to be hanging lower than the right side.

Activity Synthesis

Select previously identified students to share their strategies for finding the unknown weight without using an equation. Ask students to be clear how they are changing each side of the hanger equally as they share their solutions.
Next, record the equations written by students for each hanger and display for all to see in two lists. Assign half the class to the list for Hanger A and the other half to the list for Hangar B. Give students 1–2 minutes to examine the equations for their assigned hanger and be prepared to explain how different pairs of equations are related. The goal here is for students to use the language they developed with the hangers (e.g., “remove 6 from each side”) on equations.

For example, for Hanger A, you might contrast $3 + 6 + 6 + 6 = 4x + 6$ with $21 + x = 6 + 5x$. Possible student responses:

- Removing an $x$ from each side of the second equation would result in the first equation.
- $x = 3.75$ grams makes both equations true.
- You can subtract $6$s from the sides of each equation and they are still both true.

For Hanger B, examining equations should illuminate why it is impossible to know the weight of the unknown shape. If we start with $6 + 6 + x + x = x + x + 3 + 3 + 6$ and keep removing things of equal weight from each side, we might end up with an equation like $2x = 2x$. Any value of $x$ will work to make this equation true. For example if $x$ is 10, then the equation is $20 = 20$. It is also possible to keep removing things of equal weight from each side and end up with an equation like $6 = 6$, which is always true.

### Support for English Language Learners

_Writing, Conversing: MLR1 Stronger and Clearer Each Time._ Use this routine to give students and opportunity to describe how they wrote equations to represent the Hangers. Give students 4 minutes of quiet time to write a response to: “Explain how you created your equation for Hanger A or Hanger B.” Invite to meet with 2–3 partners, to share and get feedback on their responses. Encourage each listener to ask clarifying questions such as, “Why did you subtract ___ from both sides?” or “How did you represent the red circles in your equation?” Invite students to write a final draft based on their peer feedback. This will help students reason about solving equations with balancing variables, and prepare them for the whole-class discussion.

_Design Principle(s): Optimize output; Cultivate conversation_

### Lesson Synthesis

The purpose of this discussion is to have students revisit the warm-up and connect it to the activities, reflecting on why the hanger is an appropriate and helpful analogy for an equation.

Ask these questions:

- “In the warm-up we wondered why one hanger was slanted, whether there were weights in one blue sock that made it heavier than the other, whether the crooked hanger would
straighten out if another sock was added to the other side (add any other pertinent things your students wondered). How would you answer these questions now?"

- "What is an equation? What does the equal sign in an equation tell you?" (An equation is a statement that two expressions have the same value. The equal sign tells you that the expressions on either side must have the same value, however that value is measured—as a count of objects, a measurement like 10 miles or 6 seconds, or numbers without units.)

- "What features do balanced hangers and equations have in common?" (Both representations have sides that are equal in value, even if the actual value of a side is unknown. Each side can contain numbers we do not know in the form of either shapes or variables. Changing the value of one side of a hanger or equations means changing the value of the other side by the same amount.)

- "You saw an example of a hanger where the unknown weight could not be determined. Can you design your own hanger like this one? How would you think about the weights needed on each side?" (If students completed the extension, you might ask them to also design a hanger with no solution.)

### 2.4 Changing Blocks

Cool Down: 5 minutes

**Addressing**

- B.EE.C

**Student Task Statement**

Here is a hanger that is in balance. We don't know how much any of its shapes weigh. How could you change the number of shapes on it, but keep it in balance? Describe in words or draw a new diagram.
Student Response
Answers vary. Possible solution: I could remove 2 circles, 2 squares, or all 4 of these shapes from each side of the equation and the hanger would still balance. I could also add any number of a specific shape to the left side so long as I added the same amount to the right side and the hanger would stay in balance. I could also remove half each type of shape from each side, since there is an even number of each type of shape.

Student Lesson Summary
If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than the other, the hanger will tilt to the heavier side.

We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on each side have equal value, just like a balanced hanger has equal weights on each side.

If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance.

We can do these moves with equations as well: adding or subtracting the same amount from each side of an equation maintains the equality.

Lesson 2 Practice Problems
Problem 1
Statement
Which of the changes would keep the hanger in balance?
Select all that apply.
A. Adding two circles on the left and a square on the right
B. Adding 2 triangles to each side
C. Adding two circles on the right and a square on the left
D. Adding a circle on the left and a square on the right
E. Adding a triangle on the left and a square on the right

Solution
["A", "B", "C"]

Problem 2

Statement
Here is a balanced hanger diagram.

Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and \( x \) represents the weight of each square. Select all equations that represent the hanger.
A. \( x + x + x + x + 11 = x + 11.5 \)
B. \( 2x = 0.5 \)
C. \( 4x + 5 + 6 = 2x + 2.5 + 6 \)
D. \( 2x + 2.5 = 3 \)
E. \( 4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3 \)

**Solution**

["B", "D", "E"]

**Problem 3**

**Statement**

What is the weight of a square if a triangle weighs 4 grams?

Explain your reasoning.

**Solution**

8 grams. There is one more square on the left than on the right and two more triangles on the right than on the left. So the square on the left balances with two triangles on the right.

**Problem 4**

**Statement**

Andre came up with the following puzzle. “I am three years younger than my brother, and I am 2 years older than my sister. My mom’s age is one less than three times my brother’s age. When you add all our ages, you get 87. What are our ages?”

a. Try to solve the puzzle.
b. Jada writes this equation for the sum of the ages:
   \((x) + (x + 3) + (x - 2) + 3(x + 3) - 1 = 87.\)

   Explain the meaning of the variable and each term of the equation.

   c. Write the equation with fewer terms.

   d. Solve the puzzle if you haven't already.

Solution

a. Answers vary.

b. \(x\) is the age of Andre; \(x + 3\) is the age of Andre's brother; \(x - 2\) is the age of Andre's sister; \(3(x + 3) - 1\) is the age of Andre's mother; 87 is the total of all the ages.

c. Use the distributive property and combine like terms to get \(6x + 9 = 87.\)

d. Since \(6x + 9 = 87\), we also know that \(6x = 78\) and \(x = 13\) are true. So, Andre is 13, his brother is 16, his sister is 11, and his mom is 47.

(From Unit 4, Lesson 1.)

Problem 5

Statement

These two lines are parallel. Write an equation for each.

Solution

Answers vary. Possible responses:

- \(y = \frac{4}{5}x\) (or \(\frac{y}{x} = \frac{4}{5}\), or \(\frac{y - 4}{x - 5} = \frac{4}{5}\))
- \(y = \frac{4}{3}(x - 4)\) (or \(\frac{y}{x - 4} = \frac{4}{3}\))
(From Unit 3, Lesson 8.)
Lesson 3: Balanced Moves

Goals

- Compare and contrast (orally and in writing) solution paths to solve an equation in one variable by performing the same operation on each side.
- Correlate (orally and in writing) changes on hanger diagrams with moves that create equivalent equations.

Learning Targets

- I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution.

Lesson Narrative

In this lesson students move from using hangers to using equations in order to represent a problem. In the warm-up they match a series of hangers with the corresponding series of equations. They see how moves that maintain the balance of a hanger correspond to moves that maintain the equality of an equation, such as halving the value of each side or subtracting the same unknown value from each side. In the next activity students match pairs of equations with the corresponding equation move—performing the same operation on each side—that produces the second from the first. In the activity after that, they compare different choices of moves that lead to the same solution. In this activity the solution is negative, which would not have been representable with hangers. Students can check that it is a solution by substituting into the equation, reinforcing the idea that a solution is a number that makes the equality in an equation true, and that different moves maintain the equality. As students reason about why the steps in solving an equation maintain the equality and compare different solution methods, they engage in MP3.

Alignments

Addressing

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
- 8.EE.C.7: Solve linear equations in one variable.

Instructional Routines

- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Pre-printed cards, cut from copies of the blackline master
Required Preparation
Print and cut up the Matching Equation Moves blackline master for the matching activity. Prepare one set of cards for every 2 students.

Student Learning Goals
Let's rewrite equations while keeping the same solutions.

3.1 Matching Hangers

Warm Up: 10 minutes
The purpose of this warm-up is for students to revisit ideas they learned in the previous lesson about balanced hangers:

- You can add or subtract the same thing on each side and the hanger stays in balance.
- You can divide each side by the same number and the hanger stays in balance.

Addressing
- 8.EE.C

Instructional Routines
- MLR2: Collect and Display

Launch
Give students 2 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions
Some students may think a variable stands for more than one object. Tell these students that a variable only stands for one object, as it also only represents one number.

Student Task Statement
Figures A, B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.
Here are some equations. Each equation represents one of the hanger diagrams.

\[
\begin{align*}
2(x + 3y) &= 4x + 2y \\
2y &= x \\
2(x + 3y) + 2z &= 2z + 4x + 2y \\
x + 3y &= 2x + y
\end{align*}
\]

1. Write the equation that goes with each figure:
   
   A:
   
   B:
   
   C:
   
   D:

2. Each variable (x, y, and z) represents the weight of one shape. Which goes with which?

3. Explain what was done to each equation to create the next equation. If you get stuck, think about how the hangers changed.

**Student Response**

1.
   
   A: \(2(x + 3y) + 2z = 2z + 4x + 2y\)
   
   B: \(2(x + 3y) = 4x + 2y\)
   
   C: \(x + 3y = 2x + y\)
   
   D: \(2y = x\)

2. \(x\) is the blue square. \(y\) is the green triangle. \(z\) is the red circle.
3. The same type and number of objects were removed from each side.

**Activity Synthesis**

Ask students to explain how they decided on the matching equation. As students discuss the final questions, highlight responses that emphasize the same objects being removed from each side creates the next figure in line. The exception to this is the move from Hanger B to Hanger C, where the number of objects on each side is halved. Ask students to explain why this is an okay move, even though different objects are being removed from each side (1 square and 3 triangles on the left, 2 squares and 1 triangle on the right). Refer to MLR 2 (Collect and Display).

### 3.2 Matching Equation Moves

**15 minutes**

In this activity, students match a card with two equations to another card describing the move that turns the first equation into the second. The goal is to help students think about equations the same way they have been thinking about hangers: objects where equality is maintained so long as the same move is made on each side. Additionally, this is the first activity where students encounter equation moves involving negative numbers, which is not possible when using hangers.

**Addressing**

- 8.EE.C.7

**Instructional Routines**

- MLR3: Clarify, Critique, Correct

**Launch**

Review with students what we know about equations based on reasoning about hangers:

- We can add the same quantity to each side, and the equation is still true (the hanger is still in balance).
- We can subtract the same quantity from each side, and the equation is still true.
- We can double or triple or halve or third the things that appear on each side, and the equation is still true. More generally, we can multiply the number of things on each side by the same number.

Tell students that hanger diagrams are really only useful for reasoning about positive numbers, but the processes above also work for negative numbers. Negative numbers are just numbers, and they have to follow the same rules as positive numbers. In fact, if we allow negative numbers into the mix, we can express any maneuver with one of two types of moves:

- Add the same thing to each side. (The “thing” could be negative.)
- Multiply each side by the same thing. (The “thing” could be a fraction less than 1.)
Arrange students in groups of 2. Give each group 12 pre-cut slips from the blackline master. Give 3-4 minutes for partners to match the numbered slips with the lettered slips then 1-2 minutes to trade places with another group and review each other’s work. Ask partners who finish early to write down on a separate sheet of paper what the next move would be for each of the numbered cards if the goal were to solve for \( x \). Follow with a whole-class discussion.

**Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

*Supports accessibility for: Conceptual processing; Organization*

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**Student Task Statement**

Your teacher will give you some cards. Each of the cards 1 through 6 show two equations. Each of the cards A through E describe a move that turns one equation into another.

1. Match each number card with a letter card.
2. One of the letter cards will not have a match. For this card, write two equations showing the described move.

**Student Response**

1. B
2. E
3. D
4. F
5. A

6. C. Answers vary. Possible response: \( 5 - 3x = 2x + 8 \), \( 5 = 5x + 8 \).

**Activity Synthesis**

The goal of this discussion is to get students using the language of equations and describing the changes happening on each side when solving. Ask:

- “What is a move you could do to the equation \( 7 = 2x \) on card 1 that would result in an equation of the form \( x = \)? What is another move that would also work?” (Multiply each side by \( \frac{1}{2} \). Divide each side by 2.)

- “Which numbered card was the most challenging to match?” (Card 2, because it at first I only looked at the \( x \)-terms and thought the move involved a change of \( 8x \).)
• “Does anyone have a value for $x$ that would solve one of the numbered cards? How did you figure it out?” ($x = 2$ is a solution for card 5. I added 3 to each side and then multiplied each side by $\frac{1}{4}$.)

End the discussion by inviting groups to share the equations they wrote for card 6 and describe how they match the move "add $3x$ to each side."

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**Support for English Language Learners**

*Speaking: MLR3 Clarify, Critique, Correct.* Display the statements: “When we add to both sides, it is the same.” and “When we multiply both sides, it stays the same.” Ask students to clarify or improve these statements in a way that is more specific. Prompt students to think about positive and negative numbers as well as fractions. This will help students to use the language of equations to explain why you can add (or subtract) and multiply (or divide) each side of an equation by an expression involving rational numbers and still have an equivalent equation.  
*Design Principle(s): Optimize output (for generalization)*

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### 3.3 Keeping Equality

10 minutes  
The purpose of this activity is to get students thinking about strategically solving equations by paying attention to their structure. Distribution first versus dividing first is a common point of divergence for students as they start solving.

Identify students who choose different solution paths to solve the last two problems.

**Addressing**

- 8.EE.C.7

**Instructional Routines**

- MLR7: Compare and Connect

- Think Pair Share

**Launch**

Arrange students in groups of 2. Give students 2 minutes quiet think time for problem 1, then 3-5 minutes partner time to discuss problem 1 and complete the other problems. Follow with a whole-class discussion.
Support for English Language Learners

Conversing: MLR7 Compare and Connect. Display Noah’s and Lin’s solution methods side by side. Once students have determined that they are both correct, ask students to explain the differences between the approaches. Amplify mathematical language students use to distinguish between distributing or dividing. This will help students reflect on and produce mathematical language to explain why they choose to take either approach when answering the remaining questions.

Design Principle(s): Maximize meta-awareness; Cultivate conversation

Anticipated Misconceptions

Some students may not distribute or collect like terms before performing the same operation on each side.

Student Task Statement

1. Noah and Lin both solved the equation \(14a = 2(a - 3)\).
   
   Do you agree with either of them? Why?  
   
   Noah’s solution:  
   
   \[
   14a = 2(a - 3) \\
   14a = 2a - 6 \\
   12a = -6 \\
   a = -\frac{1}{2}
   \]

   Lin’s solution:  

   \[
   14a = 2(a - 3) \\
   7a = a - 3 \\
   6a = -3 \\
   a = \frac{1}{2}
   \]

2. Elena is asked to solve \(15 - 10x = 5(x + 9)\). What do you recommend she does to each side first?

3. Diego is asked to solve \(3x - 8 = 4(x + 5)\). What do you recommend he does to each side first?

Student Response

1. Both Noah and Lin have correct solutions. Explanations vary. Sample response: Both Noah and Lin followed valid solution paths. Substituting \(a = -\frac{1}{2}\) into the original equation yields a true statement, so their solutions are correct.

2. Answers vary. There are at least two solution paths to this equation: you can divide each side by 5 first, then collect like terms, or you can distribute and collect like terms, then continue to solve.

3. Answers vary. There are still two solution paths to this equation, but one is much simpler than the other. Since not all the terms are multiples of 4, dividing first by 4 will give a fractional
coefficient of \( x \) on one side. Therefore, distributing first and then collecting like terms and solving is the simpler solution path.

**Are You Ready for More?**
In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

\[
\text{HANGER} + \text{HANGER} + \text{HANGER} = \text{ALGEBRA}
\]

**Student Response**
A:2, B:8, E:1, G:6, H:9, L:7, N:0, R:4

**Activity Synthesis**
Have previously identified groups share the different solution paths they chose for solving the last two questions.

To highlight the different strategies, ask:

- “What are the advantages of choosing to distribute first? To divide first?” (Answers vary. Distributing first eliminates confusion about which terms can be subtracted from each side. Dividing first makes the numbers smaller and easier to mentally calculate.)

- “What makes it easier to distribute versus divide first on the last question?” (Dividing by 4 before distributing will result in non-integer terms, which can be harder to add and subtract mentally.)

- “Is one path more ‘right’ than another?” (No. As long as we follow valid steps, like adding or multiplying the same thing to each side of an equation, the steps are right and will give a correct solution.)

**Lesson Synthesis**
Display the equation \( 6x + 12 = 10x - 4 \) for all to see. Tell students to think of three different things they could do to each side of the question but still maintain equality. Invite students to share their moves. Possible responses include:

- subtract 6x from each side
- add 4 to each side
- divide each side by 2

Ask students, “If you made a mistake when solving this equation and thought that \( x = 2 \), how would you be able to tell?” (If I put 2 into the equation, I would get that 24 = 16, which isn’t true.)
3.4 More Matching Moves

Cool Down: 5 minutes

Addressing

• 8.EE.C.7

Student Task Statement

Match these equation balancing steps with the description of what was done in each step.

Step 1:  
12x - 6 = 10  
6x - 3 = 5

Step 2:  
6x = 8

Step 3:  
x = \frac{4}{3}

Descriptions to match with each step:

A: Add 3 to both sides

B: Multiply both sides by \frac{1}{6}

C: Divide both sides by 2

Student Response

In Step 1, we did C, divide both sides by 2.

In Step 2, we did A, add 3 to both sides.

In Step 3, we did B, multiply both sides by \frac{1}{6}.

Student Lesson Summary

An equation tells us that two expressions have equal value. For example, if 4x + 9 and -2x - 3 have equal value, we can write the equation

\[ 4x + 9 = -2x - 3 \]

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add negative numbers! For example, we can add -9 to each side of the equation.

\[
4x + 9 + (-9) = -2x - 3 + (-9) \quad \text{add -9 to each side}
\]

\[
4x = -2x - 12 \quad \text{combine like terms}
\]

Since expressions represent numbers, we can also add expressions to each side of an equation. For example, we can add 2x to each side and still maintain equality.
4x + 2x = -2x - 12 + 2x  \quad \text{add 2x to each side}

6x = -12  \quad \text{combine like terms}

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (so long as we do not divide by zero).

\[
6x \cdot \frac{1}{6} = -12 \cdot \frac{1}{6} \quad \text{multiply each side by } \frac{1}{6}
\]

or

\[
6x \div 6 = -12 \div 6 \quad \text{divide each side by 6}
\]

Now we can see that \( x = -2 \) is the solution to our equation.

We will use these moves in systematic ways to solve equations in future lessons.

**Lesson 3 Practice Problems**

**Problem 1**

**Statement**

In this hanger, the weight of the triangle is \( x \) and the weight of the square is \( y \).

![Diagram of a hanger with triangles and squares]

a. Write an equation using \( x \) and \( y \) to represent the hanger.

b. If \( x \) is 6, what is \( y \)?

**Solution**

a. \( x + 3y = 4x + y \)

b. \( y = 9 \)

**Problem 2**

**Statement**

Andre and Diego were each trying to solve \( 2x + 6 = 3x - 8 \). Describe the first step they each make to the equation.
a. The result of Andre’s first step was \(-x + 6 = -8\).

b. The result of Diego’s first step was \(6 = x - 8\).

Solution
a. Andre subtracted 3x from each side.
b. Diego subtracted 2x from each side.

Problem 3
Statement
a. Complete the table with values for \(x\) or \(y\) that make this equation true: \(3x + y = 15\).

\[
\begin{array}{cccccc}
  x & 2 & 6 & 0 & 3 & \quad \\
  y & 3 & \quad & \quad & 0 & 8
\end{array}
\]

b. Create a graph, plot these points, and find the slope of the line that goes through them.
Solution

a. 

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>( \frac{7}{3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>9</td>
<td>3</td>
<td>-3</td>
<td>15</td>
<td>6</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

b. Slope = -3

From Unit 3, Lesson 11.

Problem 4

Statement

Match each set of equations with the move that turned the first equation into the second.

A. \(6x + 9 = 4x - 3\)  
   \(2x + 9 = -3\)
   1. Multiply both sides by \(-\frac{1}{4}\)

B. \(-4(5x - 7) = -18\)
   \(5x - 7 = 4.5\)
   2. Multiply both sides by -4
   3. Multiply both sides by \(-\frac{1}{4}\)

C. \(8 - 10x = 7 + 5x\)
   \(4 - 10x = 3 + 5x\)
   4. Add \(-4x\) to both sides
   5. Add -4 to both sides

D. \(-\frac{5x}{4} = 4\)
   \(5x = -16\)

E. \(12x + 4 = 20x + 24\)
   \(3x + 1 = 5x + 6\)

Solution

- A: 4
- B: 1
- C: 5
- D: 2
- E: 3

Problem 5

Statement

Select all the situations for which only zero or positive solutions make sense.
A. Measuring temperature in degrees Celsius at an Arctic outpost each day in January.
B. The height of a candle as it burns over an hour.
C. The elevation above sea level of a hiker descending into a canyon.
D. The number of students remaining in school after 6:00 p.m.
E. A bank account balance over a year.
F. The temperature in degrees Fahrenheit of an oven used on a hot summer day.

Solution

["B", "D", "F"]
(From Unit 3, Lesson 14.)
Lesson 4: More Balanced Moves

Goals

- Calculate a value that is a solution for a linear equation in one variable, and compare and contrast (orally) solution strategies with others.
- Critique (in writing) the reasoning of others in solving a linear equation in one variable.

Learning Targets

- I can make sense of multiple ways to solve an equation.

Lesson Narrative

In this lesson, students continue to reinforce the circle connecting three fundamental ideas: a solution to an equation is a number that makes the equation true, performing the same operation on each side of an equation maintains the equality in the equation, and therefore two equations related by such a move have the same solutions. In the warm-up, students are given an equation and then asked whether each of four other equations has the same solution as the given one. They see the move connecting each of the four to the original and conclude that they all have the same solutions. In the next activity, they compare two correct solution paths for an equation, and then two incorrect solution paths, identifying the mistakes made. Then they get some practice solving equations choosing their own paths.

Alignments

Addressing

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
- 8.EE.C.7: Solve linear equations in one variable.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

Student Learning Goals

Let's rewrite some more equations while keeping the same solutions.

4.1 Different Equations?

Warm Up: 5 minutes
The purpose of this warm-up is to for students to use the structure of equations to recognize when they are the same without having to solve for the specific $x$ value that makes the equations true.

Monitor for students who:

- solve each equation for $x$, then compare the solutions
- solve Equation 1 for $x$, then substitute it into Equations A-D
- manipulate Equations A-D to look like Equation 1 and vice versa

**Addressing**
- 8.EE.C

**Instructional Routines**
- Anticipate, Monitor, Select, Sequence, Connect

**Launch**
Give students 2–3 minutes quiet think time, then whole-class discussion.

**Student Task Statement**

Equation 1

$$x - 3 = 2 - 4x$$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

<table>
<thead>
<tr>
<th>Equation A</th>
<th>Equation B</th>
<th>Equation C</th>
<th>Equation D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 6 = 4 - 8x$</td>
<td>$x - 5 = -4x$</td>
<td>$2(1 - 2x) = x - 3$</td>
<td>$-3 = 2 - 5x$</td>
</tr>
</tbody>
</table>

**Student Response**

All of the other equations have the same solutions as the first equation, $x = 1$.

Equation A: If you multiply each side of Equation 1 by 2, the result is Equation A. So if $x$ makes Equation 1 true then it makes Equation A true as well.

Equation B: If you subtract 2 from each side of Equation 1, the result is Equation B, so if Equation 1 is true then Equation B is true.

Equation C: If you switch everything to the left of the equal sign and everything to the right of the equal sign on Equation C, then rewrite the expression $2(1 - 2x)$ as $2 - 4x$ using the distributive property, then the result is Equation 1, so if Equation 1 is true then Equation C is true.
Equation D: If you subtract x from each side of Equation 1, the result is Equation D, so if Equation 1 is true then Equation D is true.

**Activity Synthesis**

Select students previously identified to share how they determined whether each equation had the same solution as Equation 1 in the sequence listed in the Activity Narrative. Point out that the question did not ask students what the solution was, only whether each equation had the same solution.

To help students make connections between the different methods their classmates used to solve the warm-up, ask:

- "Which method of answering the question was most efficient? After seeing all these ways to answer the question, which would you choose?"

- "What is an advantage of changing the equation to look like Equation 1? What is a disadvantage?" (An advantage is that I could see quickly whether it would be the same as Equation 1, and I didn't have to keep going to actually figure out the value of x. A disadvantage would be that I never discovered what the value for x is that makes the equations true.)

- "How is this method (manipulating the equation to look like Equation 1) similar to what we did in previous lessons with the balance hangers?" (In order to keep the hangers balanced, I had to make sure to do the same thing to each side of the hanger. In order to have each equation still be true, I have to make sure to do the same thing to each side of an equation.)

By showing that two equations are related by a move (or series of moves), we know they must have the same solution.

If time allows, have students create another equation with the same solution as Equation 1 and trade with a partner. They should then explain the step(s) necessary to make it look like Equation 1 to each other.

**4.2 Step by Step by Step by Step**

15 minutes

Before students work on solving complex equations on their own, in this activity they examine the work (both good and bad) of others. The purpose of this activity is to build student fluency solving equations by examining the solutions of others for both appropriate and inappropriate strategies (MP3).

Encourage students to use precise language when discussing the different steps made by the four students in the problem (MP6). For example, if a student says Clare distributed to move from \(12x + 3 = 3(5x + 9)\) to \(3(4x + 1) = 3(5x + 9)\), ask them to be more specific about how Clare used the distributive property to help the whole class follow along. (Clare used the distributive property to re-write \(12x + 3\) as \(3(4x + 1)\).)
Addressing

- 8.EE.C.7

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give 4–5 minutes of quiet work time and ask students to pause after the first two problems for a partner discussion. Give 2–3 minutes for partners to work together on the final problem followed by a whole-class discussion. Refer to MLR 3 (Clarify, Critique, Correct) to guide students in using language to describe the wrong steps.

Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Demonstrate for students how to use an index card or scrap piece of paper to cover and then unveil the steps one at a time. Invite students to make comparisons at each step.

Supports accessibility for: Organization; Attention

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**Student Task Statement**

Here is an equation, and then all the steps Clare wrote to solve it:

\[
14x - 2x + 3 = 3(5x + 9) \\
12x + 3 = 3(3x + 9) \\
3(4x + 1) = 3(5x + 9) \\
4x + 1 = 5x + 9 \\
1 = x + 9 \\
-8 = x
\]

Here is the same equation, and the steps Lin wrote to solve it:

\[
14x - 2x + 3 = 3(5x + 9) \\
12x + 3 = 3(5x + 9) \\
12x + 3 = 15x + 27 \\
12x = 15x + 24 \\
-3x = 24 \\
x = -8
\]

1. Are both of their solutions correct? Explain your reasoning.

2. Describe some ways the steps they took are alike and different.

3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.
Mai:
\[ 14x - 2x + 3 = 3(5x + 9) \]
\[ 12x + 3 = 3(5x + 9) \]
\[ 7x + 3 = 3(9) \]
\[ 7x + 3 = 27 \]
\[ 7x = 24 \]
\[ x = \frac{24}{7} \]

Noah:
\[ 14x - 2x + 3 = 3(5x + 9) \]
\[ 12x + 3 = 15x + 27 \]
\[ 27x + 3 = 27 \]
\[ 27x = 24 \]
\[ x = \frac{24}{27} \]

**Student Response**

1. Both solutions are correct. The solution of \( x = -8 \) is the only value of \( x \) that makes the equation true.

2. Answers vary. Sample response: Both students combined like terms from line one to line two. Clare used the distributive property to re-write the left side as \( 3(4x + 1) \) moving from line two to line three, while Lin used the same property to distribute the 3 on the left side moving from line two to line three.

3. Mai made an error moving from line two to line three by subtracting \( 5x \) from each side of the equation before multiplying by 3 on the right hand side of the equation. Noah made an error moving from line two to line three by adding \( 15x \) to each side of the equation instead of adding \( -15x \) to each side of the equation.

**Activity Synthesis**

Begin the discussion by asking, “How do you know when a solution to an equation is correct?” (One way to know it is correct is by substituting the value of \( x \) into the original equation and seeing if it makes the equation true.)

Display Clare and Lin’s solutions for all to see. Poll the class to see which solution they prefer. It is important to draw out that neither solution is better than the other, they are two ways of accomplishing the same task: solving for \( x \). Invite groups to share ways the steps Clare and Lin took are alike and different while annotating the two solutions with students’ observations. If none of the groups say it, point out that while the final steps may look different for Clare and Lin, their later steps worked to reduce the total number of terms until only an \( x \)-term and a number remained on either side of the equal sign.

Display Mai and Noah’s incorrect solutions for all to see. Invite groups to share an incorrect step they found and what advice they would give to Mai and Noah for checking their work in the future.
Support for English Language Learners

Conversing: MLR7 Compare and Connect. During the analysis of Clare and Lin’s solutions, ask students first to identify what is similar and what is different about each of the approaches. Then ask students to connect the approaches by asking questions about the related mathematical operations (e.g., “Why does this approach include multiplication, and this one does not?”). Emphasize language used to make sense of strategies used to calculate lengths, areas, and volumes. This will help students make sense of different ways to solve the same equation that both lead to a correct solution.

Design Principle(s): Optimize output (for comparison); Cultivate conversation

4.3 Make Your Own Steps

15 minutes
The purpose of this lesson is to increase fluency in solving equations. Students will solve equations individually and then compare differing, though accurate, solution paths in order to compare their work with others. This will help students recognize that while the final solution will be the same, there is more than one path to the correct answer that uses principles of balancing equations learned in previous lessons.

Addressing
- 8.EE.C.7

Instructional Routines
- MLR2: Collect and Display

Launch
Arrange students in groups of 3–4. Give students quiet think time to complete the activity and then tell groups to share how they solved the equations for \( x \) and discuss the similarities and differences in their solution paths.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about solving equations. Encourage students to use previously solved equations as guidelines to determine appropriate steps.
Supports accessibility for: Memory; Conceptual processing
Support for English Language Learners

Representing, Conversing: MLR2 Collect and Display. As groups discuss their work, circulate and listen for the language students describe the similarities and differences in their solution paths. Write down different solution paths that led to the same result in a visual display. Consider grouping words and phrases used for each step in different areas of the display (e.g., “multiply first”, “divide first”, “subtract x from the right”, “add x from the left”). Continue to update the display as students move through the activity, and remind them to borrow from the display while discussing with their group. This will help students develop their mathematical language around explaining different solution paths to solving equations.

Design Principle(s): Maximize meta-awareness

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**Student Task Statement**
Solve these equations for $x$.

1. \[
\frac{12 + 6x}{3} = \frac{5 - 9}{2}
\]
2. \[
x - 4 = \frac{1}{3}(6x - 54)
\]
3. \[
-(3x - 12) = 9x - 4
\]

**Student Response**

1. $x = -3$. Solutions vary. Possible solution path:

\[
\frac{12 + 6x}{3} = \frac{5 - 9}{2}
\]

\[
4 + 2x = -2
\]

\[
2x = -6
\]

\[
x = -3
\]

2. $x = 14$. Solutions vary. Possible solution path:

\[
x - 4 = \frac{1}{3}(6x - 54)
\]

\[
x - 4 = 2x - 18
\]

\[
-4 = x - 18
\]

\[
14 = x
\]

3. $x = \frac{4}{3}$. Solutions vary. Possible solution path:
\[-(3x - 12) = 9x - 4\]
\[-3x + 12 = 9x - 4\]
\[12 = 12x - 4\]
\[16 = 12x\]
\[\frac{16}{12} = x\]
\[\frac{4}{3} = x\]

**Are You Ready for More?**
I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

**Student Response**

\[n + (n + 1) + (n + 2) = 24\] so \[n = 7\] therefore the cups have 7, 8, and 9 pencils in them.

**Activity Synthesis**

Students should take away from this activity the importance of using valid steps to solve an equation over following a specific solution path. Invite students to share what they discussed in their groups. Consider using some of the following prompts:

* “How many different ways did your group members solve each problem?”

* “When you compared solution paths, did you still come up with the same solution?” (Yes, even though we took different paths, we ended up with the same solutions.)

* “How can you make sure that the path you choose to solve an equation is a valid path?” (I can use the steps we discovered earlier when we were balancing: adding the same value to each side, multiplying (or dividing) by the same value to each side, distributing and collecting like terms whenever it is needed.)

* “What are some examples of steps that will not result in a valid solution?” (Performing an action to only one side of an equation and distributing incorrectly will give an incorrect solution.)

**Lesson Synthesis**

Display the following prompts one at a time and after each ask students if the move described maintains the equality of an equation:

* subtract a number from each side (maintains)
* subtract 4x from each side (maintains)
* dividing each side of the equation by 7 (maintains)
* adding 5x to one side and 10 to the other (maintains equality only if \(x = 2\))
• add 4 to one side and add 5 to the other (does not maintain equality)

Ask students to write an equation and a solution to the equation that contains an error. Then, tell students to swap with a partner and try to find the error in their partner’s solution.

### 4.4 Mis-Steps

**Cool Down: 5 minutes**

**Addressing**

• 8.EE.C.7

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**Student Task Statement**

Lin solved the equation $8(x - 3) + 7 = 2x(4 - 17)$ incorrectly. Find the errors in her solution. What should her answer have been?

Lin’s solution:

$8(x - 3) + 7 = 2x(4 - 17)$
$8(x - 3) + 7 = 2x(13)$
$8x - 24 + 7 = 26x$
$8x - 17 = 26x$
$-17 = 34x$
$\frac{1}{2} = x$

---

**Student Response**

Lin’s errors:

$8(x - 3) + 7 = 2x(4 - 17)$
$8(x - 3) + 7 = 2x(13)$
$8x - 24 + 7 = 26x$
$8x - 17 = 26x$
$-17 = 34x$

Lin should have subtracted $8x$ from each side.

Testing $x = \frac{1}{2}$ will show that the equation is not true.

---

Lin’s solution should have been $x = \frac{1}{2}$.

---

**Student Lesson Summary**

How do we make sure the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an $x$ from one line to the next—there are many possible mistakes to watch out for!
Fortunately, each step we take solving an equation results in a new equation with the same solution as the original. This means we can check our work by substituting the value of the solution into the original equation. For example, say we solve the following equation:

\[
2x = -3(x + 5) \\
2x = -3x + 15 \\
5x = 15 \\
x = 3
\]

Substituting 3 in place of x into the original equation,

\[
2(3) = -3(3 + 5) \\
6 = -3(8) \\
6 = -24
\]

we get a statement that isn't true! This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3. Fixing it, we now have

\[
2x = -3(x + 5) \\
2x = -3x - 15 \\
5x = -15 \\
x = -3
\]

Substituting -3 in place of x into the original equation to make sure we didn't make another mistake:

\[
2(-3) = -3(-3 + 5) \\
-6 = -3(2) \\
-6 = -6
\]

This equation is true, so \(x = -3\) is the solution.

**Lesson 4 Practice Problems**

**Problem 1**

**Statement**

Mai and Tyler work on the equation \(\frac{3}{2}b + 1 = -11\) together. Mai's solution is \(b = -25\) and Tyler's is \(b = -28\). Here is their work. Do you agree with their solutions? Explain or show your reasoning.
Mai:
\[
\frac{2}{3}b + 1 = -11
\]
\[
\frac{2}{3}b = -10
\]
\[
b = -10 \cdot \frac{3}{2}
\]
\[
b = -25
\]

Tyler:
\[
\frac{2}{3}b + 1 = -11
\]
\[
2b + 1 = -55
\]
\[
2b = -56
\]
\[
b = -28
\]

Solution
No, they both have errors in their solutions. Explanations vary. Sample response: Mai added -1 on the left side and 1 on the right side of the equation. Tyler multiplied both sides of the equation by 5 but forgot to multiply the 1 by 5.

Problem 2
Statement
Solve \(3(x - 4) = 12x\)

Solution
\[x = \frac{4}{3}\]. One way to solve is to distribute, subtract 3x from each side, and divide by 9. Another way is to first divide each side by 3, subtract x for each side, then divide each side by 3.

Problem 3
Statement
Describe what is being done in each step while solving the equation.

a. \(2(-3x + 4) = 5x + 2\)

b. \(-6x + 8 = 5x + 2\)

c. \(8 = 11x + 2\)

d. \(6 = 11x\)

e. \(x = \frac{6}{11}\)

Solution

a. Original equation

b. Distributive property

c. Add 6x to each side
d. Subtract 2 from each side

e. Multiply each side by $\frac{1}{11}$

**Problem 4**

**Statement**
Andre solved an equation, but when he checked his answer he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Andre's mistake and what is the solution to the equation?

- $-2(3x - 5) = 4(x + 3) + 8$
- $-6x + 10 = 4x + 12 + 8$
- $-6x + 10 = 4x + 20$
- $10 = -2x + 20$
- $-10 = -2x$
- $5 = x$

**Solution**
Andre's mistake occurred in the transition from the 3rd line to the 4th line. He added $6x$ on the left side but subtracted $6x$ on the right side. The correct solution is $x = -1$.

**Problem 5**

**Statement**
Choose the equation that has solutions $(5, 7)$ and $(8, 13)$.

A. $3x - y = 8$

B. $y = x + 2$

C. $y - x = 5$

D. $y = 2x - 3$

**Solution**
D
(From Unit 3, Lesson 12.)

**Problem 6**

**Statement**
A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the second piece, $x$, for each length of the first piece, $y$. 
a. How long is the ribbon?
   Explain how you know.

b. What is the slope of the line?

c. Explain what the slope of the line represents and why it fits the story.

Solution

a. 15 feet. Explanations vary. Sample response: When the second piece is 0 feet long, the first is 15 feet long, so that is the length of the ribbon.

b. -1

c. Answers vary. Sample response: The slope shows the change in length of one piece for every 1 foot increase in length of the other piece. If one piece is 1 foot longer, the other is 1 foot shorter because the total of the two lengths is constant.

(From Unit 3, Lesson 9.)
Lesson 5: Solving Any Linear Equation

Goals

• Calculate a value that is a solution to a linear equation in one variable, and explain (orally) the steps used to solve.

• Create an expression to represent a number puzzle, and justify (orally) that it is equivalent to another expression.

• Justify (orally) that each step used in solving a linear equation maintains equality.

Learning Targets

• I can solve an equation where the variable appears on both sides.

Lesson Narrative

The purpose of this lesson is to move towards a general method for solving linear equations. In the warm-up, students solve equations mentally, including equations with negative coefficients, prompting a discussion of multiplying or dividing each side of an equation by a negative number. In the first activity, students encounter several different structures of equations, and take turns suggesting moves for solving them. Then they apply their growing fluency in solving equations to constructing numbers puzzles of the sort they encountered in the first lesson in this unit.

As students explain their reasoning for choosing a particular move while solving equations and critiquing the choice of their partner, they engage in MP3.

Alignments

Addressing

• 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

• 8.EE.C.7: Solve linear equations in one variable.

Instructional Routines

• Algebra Talk

• MLR3: Clarify, Critique, Correct

• MLR7: Compare and Connect

• MLR8: Discussion Supports

Required Materials

Pre-printed cards, cut from copies of the blackline master
Required Preparation
Print and cut up the Trading Steps blackline master for the matching activity. Prepare one set of cards for every 2 students.

Student Learning Goals
Let's solve linear equations.

5.1 Equation Talk

Warm Up: 5 minutes
The purpose of this warm-up is to elicit students’ strategies for solving an equation for the value of \(x\). The negative integers and location of \(x\) in each equation were purposeful to spark a discussion about operations of integers and a negative coefficient of a variable.

Addressing
• 8.EE.C

Instructional Routines
• Algebra Talk

Launch
Tell students to close their books or devices and that they are going to see some equations they are to solve mentally. Display each problem one at a time for all to see. Give students 30 seconds of quiet think time for each equation. Ask students to share their strategies for finding the value of \(x\). Record and display their responses for all to see.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement
Solve each equation mentally.

\[5 - x = 8\]
\[-1 = x - 2\]
\[-3x = 9\]
\[-10 = -5x\]
Student Response

- \( x = -3 \)
- \( x = 1 \)
- \( x = -3 \)
- \( x = 2 \)

Activity Synthesis

Some students may reason about the value of \( x \) using logic. For example, in \(-3x = 9\), the \( x \) must be \(-3\) since \(-3 \cdot -3 = 9\). Other students may reason about the value of \( x \) by changing the value of each side of the equation equally by, for example, dividing each side of \(-3x = 9\) by \(-3\) to get the result \( x = -3\). Both of these strategies should be highlighted during the discussion where possible.

To involve more students in the conversation, consider asking as the students share their ideas:

- “Can you explain why you chose your strategy?”
- “Can anyone restate ___’s reasoning in a different way?”
- “Did anyone reason about the problem the same way but would explain it differently?”
- “Did anyone reason about the problem in a different way?”
- “Does anyone want to add on to ___’s strategy?”
- “Do you agree or disagree? Why?”

Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* Display sentence frames to support students when they explain their strategy. For example, “First, I ___ because…” or “I noticed ___ so I….” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

5.2 Trading Moves

20 minutes

The goal of this activity is for students to build fluency solving equations with variables on each side. Students describe each step in their solution process to a partner and justify how each of their changes maintains the equality of the two expressions (MP3).
Look for groups solving problems in different, but efficient, ways. For example, one group may distribute the \( \frac{1}{2} \) on the left side in problem 2 while another may multiply each side of the equation by 2 in order to re-write the equation with less factors on each side.

**Addressing**

- 8.EE.C.7

**Instructional Routines**

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

**Launch**

Arrange students in groups of 2. Instruct the class that they will receive 4 cards with problems on them and that their goal is to create a solution to the problems.

For the first two cards they draw, students will alternate solving by stating to their partner the step they plan to do to each side of the equation and why before writing down the step and passing the card. For the final two problem cards, each partner picks one and writes out its solution individually before trading to check each others’ work.

To help students understand how they are expected to solve the first two problems, demonstrate the trading process with a student volunteer and a sample equation. Emphasize that the “why” justification should include how their step maintains the equality of the equation. Use MLR 3 (Clarify, Critique, Correct) by reminding students to push each other to explain how their step guarantees that the equation is still balanced as they are working. For example, a student might say they are combining two terms on one side of the equation, which maintains the equality as the value of that side does not change, only the appearance.

Distribute 4 slips from the blackline master to each group. Give time for groups to complete the problems, leaving at least 5 minutes for a whole-class discussion. If any groups finish early, make sure they have checked their solutions and then challenge them to try and find a new solution to one of the problems that uses less steps than their first solution. Conclude with a whole-class discussion.

If time is a concern, give each group 2 cards rather than all 4 and have them only doing the trading steps portion of the activity, but make sure that all 4 cards are distributed throughout the class. Give 6–7 minutes for groups to complete their problems. Make sure each problem is discussed in a final whole-group discussion. Alternatively, extend the activity by selecting more problems for students to solve with their partners.

**Student Task Statement**

Your teacher will give you 4 cards, each with an equation.

1. With your partner, select a card and choose who will take the first turn.
2. During your turn, decide what the next move to solve the equation should be, explain your choice to your partner, and then write it down once you both agree. Switch roles for the next move. This continues until the equation is solved.

3. Choose a second equation to solve in the same way, trading the card back and forth after each move.

4. For the last two equations, choose one each to solve and then trade with your partner when you finish to check one another’s work.

**Student Response**

Answers vary. The solutions to the given equations are as follows:

1. \[-6x - 7 = 4x - 2 \quad (x = -0.5)\]
2. \[\frac{1}{2}(7x - 6) = 6x - 10 \quad (x = 2.8)\]
3. \[\frac{1}{2}x + 7 = x + 13 \quad (x = -12)\]
4. \[2(x + 7) = -4x + 14 \quad (x = 0)\]

Sample response: Students may decide the next step is to factor out a 2 from the right hand side of this equation \(2(x + 7) = -4x + 14\). So the next line would be \(2(x + 7) = 2(-2x + 7)\).

**Activity Synthesis**

Depending on your observations as students worked, you may wish to begin the discussion with a few common errors and ask students to explain why they are errors. For example, write out a solution to problem 2 where the second line has \(3.5x - 6\) instead of \(3.5x - 3\) and then ask students to find the error.

The goal of this discussion is for the class to see different, successful ways of solving the same equation. Record and display the student thinking that emerges during the discussion to help the class follow what is being said. To highlight some of the differences in solution paths, ask:

- “Did your partner ever make a move different than the one you expected them to? Describe it.”

- “For problem 4, could you start by halving the value of each side? Why might you want to do this?”

- “What's an arithmetic error you made but then caught when you checked your work?”
Support for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students cards a subset of the cards to start with and introduce the remaining cards once students have completed the initial set, or create an additional set of cards with equations that are more accessible.
*Supports accessibility for: Conceptual processing*

Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support students in producing statements about common errors in problem solving. Use the example offered, (the second line has \(3.5x - 6\) instead of \(3.5x - 3\) for problem 2), and provide sentence frames to support the discussion. For example, “The error this student made was... and I believe this happened because...” and “A different solution path could be...” Restate or revoice student language to demonstrate use of correct mathematical language to describe each move (e.g., “distribute the \(\frac{1}{2}\),” “combine like terms,” etc.), and include mathematical reasoning (e.g., “...because this maintains the equality”). Clarify explanations that detail differences in problem-solving strategies rather than errors, to help students see differences in solution paths. This will help students describe differences in solution paths and justify each step.
*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

5.3 A Puzzling Puzzle

10 minutes
In this activity, students investigate a number puzzle. After the puzzle is demonstrated, students are tasked with figuring out how it works and encouraged to create an algebraic representation of the puzzle. The goal of this activity is to build student fluency working with equations with complex structure. This activity also looks ahead to the future work on functions where students will revisit some of these ideas and learn the language of inputs and outputs. More immediately, this activity points to the study of equations that do not have a single answer, which students will learn about in more depth later in this unit.

While students work, identify those using expressions with different structures for their representation of the number puzzle to share during the whole-class discussion. For example, some students may use \(\frac{(3x - 7) - 2 - 22}{6}\) while others write \(\frac{1}{6} ((3x - 7) \cdot 2 - 22)\) and connecting these together with the outcome of the number puzzle, namely \(x - 6\), is the focus of the whole-class discussion.
Addressing
- 8.EE.C

Instructional Routines
- MLR7: Compare and Connect

Launch
Tell students to close their books or devices and choose a number (but not share the number with anyone else). Tell them they will perform a sequence of operations on their number and then tell you their final answer. Say each step of Tyler’s number puzzle, giving students time to calculate their new number after each step. Select 5–6 students to share their final number, and after each, tell them their original number as quickly as you can.

Pause here and ask students if they can tell how you are able to figure out their number so fast. If no students notice that each number you say is always 6 more than the number given at the end of the steps, you may wish to record and display the pairs of numbers for all students to see or call on more students so that everyone can hear more pairs of numbers.

Once the class agrees that you are able to figure out their original numbers by adding 6 to their final number, tell them that the number puzzle is really Tyler’s and that their task is to figure out how it works. Tell students to open their books or devices.

Give 3–4 minutes quiet work time for students to write their explanations, followed by a whole-class discussion.

Student Task Statement
Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to do the following:
- Triple the number
- Subtract 7
- Double the result
- Subtract 22
- Divide by 6

Clare says she now has a -3. Tyler says her original number must have been a 3. How did Tyler know that? Explain or show your reasoning. Be prepared to share your reasoning with the class.

Student Response
Answers vary. Possible solution: Following Tyler’s instructions for a number \( x \) results in the expression \( \frac{1}{6}((3x - 7) \cdot 2 - 22) \). Simplified, this expression is just \( x = 6 \). Tyler knows that to figure out Clare’s original number, 3, he only needs to solve the equation \( x - 6 = -3 \).
Activity Synthesis

Select previously identified students to share their representations with the class. Record and display the different ways Tyler’s number puzzle can be written as an expression. To highlight the connection between the different expressions, ask:

- “What do all these expressions have in common that make the number puzzle work?” (All of them are equivalent to \( x - 6 \).)
- “How would you justify that, for example, \( \frac{1}{6} \cdot (3x - 7 + 2 - 22) = x - 6 \)” (I would simplify the left side of the equation until it was \( x - 6 \).)
- “What does it mean if we have an equation that says \( x - 6 = x - 6 \)” (The two sides of this equation are always the same.)

Support for Students with Disabilities

**Representation:** Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

**Supports accessibility for:** Memory; Organization

Support for English Language Learners

**Representing:** MLR7 Compare and Connect. Use this routine when students write an expression to represent Tyler’s number puzzle. In pairs or groups, ask students to switch their expressions and compare them. Prompt students to share and explain their strategy and then discuss what is the same or different about their approaches. Monitor student discussion, then clarify how operations should be correctly arranged and grouped to help produce \( x - 6 \). Allow time for students to share with the whole class. This will help students make connections between different expressions, using appropriate mathematical language to detail their steps.

**Design Principle(s):** Optimize output (for explanation)

Lesson Synthesis

Give students 2–3 minutes to think about all the equations they solved in today’s lesson and to write down any errors they made or observed. Discuss and consider creating a permanent display showing:

- different approaches for different structures of equations
- types of errors to look out for
5.4 Check It

Cool Down: 5 minutes

Addressing

• 8.EE.C.7

Student Task Statement

Noah wanted to check his solution of $x = \frac{14}{5}$ for the equation $\frac{1}{2}(7x - 6) = 6x - 10$.

Substituting $\frac{14}{5}$ for $x$, he writes the following:

$$\frac{1}{2} \left(7 \left(\frac{14}{5}\right) - 6\right) = 6 \left(\frac{14}{5}\right) - 10$$
$$\left(\frac{14}{5}\right) - 6 = 12 \left(\frac{14}{5}\right) - 20$$
$$5 \left(\frac{14}{5}\right) - 6 = 5 \left(12 \left(\frac{14}{5}\right) - 20\right)$$
$$7 \cdot 14 - 6 = 12 \cdot 14 - 20$$
$$98 - 6 = 168 - 20$$
$$92 = 148$$

Find the incorrect step in Noah’s work and explain why it is incorrect.

Student Response

Noah made a mistake between these two lines:

$$5 \left(\frac{14}{5}\right) - 6 = 5 \left(12 \left(\frac{14}{5}\right) - 20\right)$$

$$7 \cdot 14 - 6 = 12 \cdot 14 - 20$$

Noah should have distributed the 5 through to each term inside the parentheses so that the second line read like this:

$$7 \cdot 14 - 30 = 12 \cdot 14 - 100$$

Then, the solution would finish like this:

$$98 - 30 = 168 - 100$$
$$68 = 68$$

Student Lesson Summary

When we have an equation in one variable, there are many different ways to solve it. We generally want to make moves that get us closer to an equation like

$$\text{variable} = \text{some number.}$$

For example, $x = 5$ or $t = \frac{7}{3}$. Since there are many ways to do this, it helps to choose moves that leave fewer terms or factors. If we have an equation like

$$3t + 5 = 7,$$
Adding -5 to each side will leave us with fewer terms. The equation then becomes

\[ 3t = 2. \]

Dividing each side of this equation by 3 will leave us with \( t \) by itself on the left and that

\[ t = \frac{2}{3}. \]

Or, if we have an equation like

\[ 4(5 - a) = 12, \]

dividing each side by 4 will leave us with fewer factors on the left,

\[ 5 - a = 3. \]

Some people use the following steps to solve a linear equation in one variable:

1. Use the distributive property so that all the expressions no longer have parentheses.
2. Collect like terms on each side of the equation.
3. Add or subtract an expression so that there is a variable on just one side.
4. Add or subtract an expression so that there is just a number on the other side.
5. Multiply or divide by a number so that you have an equation that looks like \( \text{variable} = \text{some number} \).

For example, suppose we want to solve \( 9 - 2b + 6 = -3(b + 5) + 4b \).

\[
\begin{align*}
9 - 2b + 6 &= -3b - 15 + 4b \\
15 - 2b &= b - 15 \\
15 &= 3b - 15 \\
30 &= 3b \\
10 &= b
\end{align*}
\]

Use the distributive property

Gather like terms

Add \( 2b \) to each side

Add 15 to each side

Divide each side by 3

Following these steps will always work, although it may not be the most efficient method. From lots of experience, we learn when to use different approaches.

**Glossary**

- term

**Lesson 5 Practice Problems**

**Problem 1**

**Statement**

Solve each of these equations. Explain or show your reasoning.

\[
\begin{align*}
2(x + 5) &= 3x + 1 \\
3y - 4 &= 6 - 2y \\
3(n + 2) &= 9(6 - n)
\end{align*}
\]
Solution
a. $x = 9$. Responses vary. Sample response: Distribute 2 on the left side, add -1 to each side, then add $-2x$ to each side.

b. $y = 2$. Responses vary. Sample response: Distribute 2 on the right side, add $2y$ to each side, add 4 to each side, then divide each side by 5.

c. $n = 4$. Responses vary. Sample response: Divide each side by 3, distribute 3 on the right side, subtract 2 from each side, add $3n$ to each side, then divide each side by 4.

Problem 2

Statement
Clare was solving an equation, but when she checked her answer she saw her solution was incorrect. She knows she made a mistake, but she can’t find it. Where is Clare’s mistake and what is the solution to the equation?

\[
12(5 + 2y) = 4y - (5 - 9y) \\
72 + 24y = 4y - 5 - 9y \\
72 + 24y = -5y - 5 \\
24y = -5y - 77 \\
29y = -77 \\
y = \frac{-77}{29}
\]

Solution
Clare’s mistakes occurred in the transition from the 1st line to the 2nd line. She wrote $4y - 9y$ as $4y - 9y$ instead of $4y + 9y$ and $12(5) = 72$ instead of $12(5) = 60$. The correct solution is $y = \frac{-65}{11}$.

Problem 3

Statement
Solve each equation, and check your solution.

\[
\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4) \\
-4(r + 2) = 4(2 - 2r) \\
12(5 + 2y) = 4y - (6 - 9y)
\]

Solution
a. $m = -7$

b. $r = 4$
c. $y = -6$

**Problem 4**

**Statement**
Here is the graph of a linear equation.
Select **all true statements** about the line and its equation.

A. One solution of the equation is (3, 2).
B. One solution of the equation is (-1, 1).
C. One solution of the equation is $(1, \frac{3}{2})$.
D. There are 2 solutions.
E. There are infinitely many solutions.
F. The equation of the line is $y = \frac{1}{4}x + \frac{3}{4}$.
G. The equation of the line is $y = \frac{5}{4}x + \frac{1}{4}$.

**Solution**

["A", "B", "C", "E", "F"]

(From Unit 3, Lesson 13.)

**Problem 5**

**Statement**
A participant in a 21-mile walkathon walks at a steady rate of 3 miles per hour. He thinks, “The relationship between the number of miles left to walk and the number of hours I already walked can be represented by a line with slope -3.” Do you agree with his claim? Explain your reasoning.
**Solution**

Yes. Explanations vary. Sample response: The walker completes 3 miles each hour, so 3 is subtracted for each 1 hour walked. Another sample response: Points on the graph of remaining miles (y) and hours walked (x) could be (0, 21), (1, 18), (2, 15), (3, 12), etc., so the line slopes down. Another sample response: The number of miles remaining decreases by 3 for every increase of 1 in the hours walked.

(From Unit 3, Lesson 9.)
Lesson 6: Strategic Solving

Goals

- Categorize (orally and in writing) linear equations in one variable based on their structure, and solve equations from each category.
- Describe (orally and in writing) features of linear equations that have one solution, no solution, or many solutions.
- Describe (orally) strategies for solving linear equations in one variable with different features or structures.

Learning Targets

- I can solve linear equations in one variable.

Lesson Narrative

In previous lessons students have started to acquire fluency with a general method of solving equations, and have seen that different solution paths are possible. In this lesson students learn to stop and think ahead strategically before plunging into a solution method. After a warm-up in which they construct their own equation to solve a problem, they look at equations with different structures and decide whether the solution will be positive, negative, or zero, without solving the equation. They judge which equations are likely to be easy to solve and which are likely to be difficult.

When students look for features of an equation that will tell them something about the solution or help them choose a solution path, they engage in MP7.

Alignments

Addressing

- 8.EE.C.7: Solve linear equations in one variable.
- 8.EE.C.7.b: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Poll the Class
- Think Pair Share
Student Learning Goals
Let’s solve linear equations like a boss.

6.1 Equal Perimeters

Warm Up: 5 minutes
The purpose of this activity is for students to begin building linear equations and solving them.

Addressing
- 8.EE.C.7

Instructional Routines
- Think Pair Share

Launch
Arrange students in groups of 2. Give students 2 minutes quiet think time, then 2 minutes to discuss their solutions with a partner. Instruct groups to explain to each other how they came up with expressions and an equation to represent the situation.

Student Task Statement
The triangle and the square have equal perimeters.

1. Find the value of x.

2. What is the perimeter of each of the figures?

Student Response
1. $x = 16$. Since the perimeters are equal, the perimeter of the triangle must equal the perimeter of the square. The perimeter of the triangle is $2x + 2x + (x - 8)$, which is $5x - 8$, and the perimeter of the square is 4 times the side length, or $4(x + 2)$.
   
   $5x - 8 = 4(x + 2)$
   $5x - 8 = 4x + 8$
   $5x = 4x + 16$
   $x = 16$

2. $P = 72$. Since the perimeters are equal, we can use either expression to find the perimeter.
   From the square: $4(16 + 2) = 72$.

Activity Synthesis
Ask groups to share their strategies for solving the question. Consider asking some of the following questions:
• “What expression represents the perimeter of the triangle? The perimeter of the square?” (The expression for perimeter of the triangle is $5x - 8$, and the perimeter of the square is $4(x + 2)$.)

• “What was your strategy in making an equation?” (If both perimeters are the same, we can say their expressions are equal.)

• “What does $x$ mean in the situation?” (It means an unknown value. None of the sides or perimeter is represented by $x$, so we cannot say it represents a specific thing on the figures.)

• “Looking at the figures, are there any values that $x$ could not be? Explain your reasoning.” (Since the triangles have sides that are $2x$, $x$ cannot be 0 or a negative value. Triangles cannot have sides with 0 or negative side lengths. Since the third side is $x = 8$, we can use this same reasoning to realize that $x$ must actually be greater than 8.)

• “How does this information help when solving?” (If I make a mistake in my solution and get a value of $x$ that is less than or equal to 8, then I know immediately that my answer is not reasonable and I can try to find my error.)

### 6.2 Predicting Solutions

10 minutes
The purpose of this activity is to shift the focus from solving an equation to thinking about what it means for a number to be a solution of an equation. Students inspect each equation, looking at the structure, the signs, and the operations in it to decide if the solution is positive, negative, or zero. Some questions are paired with another question (for example, the last two questions) so students can take advantage of their thinking from one to the next.

**Addressing**
- **8.EE.C.7**

**Instructional Routines**
- **MLR8: Discussion Supports**
- **Think Pair Share**

**Launch**
Arrange students in groups of 2.

Display the equation $5x = 6x$ for all to see.

Ask students, “How might we know whether $x$ is a positive number, negative number, or zero, without solving the equation?” (The variables can be combined into one term, but there are no constant terms. That means eventually the variable term has to equal 0, so $x$ must be 0.)

Display the equation $5x = -16.5$ for all to see and ask the same question. (Without solving, we can see that a positive number of $x$s has to equal a negative value, so $x$ must be a negative number.)
Instruct students to inspect each equation carefully and use reasoning to answer the questions in the activity rather than trying to solve each equation for a specific value. Give 5 minutes of quiet think time, and then ask students to compare their work with their partner. For any questions they disagree on, students should work to reach an agreement.

**Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge about operations with signed numbers.

Supports accessibility for: Memory; Conceptual processing

**Support for English Language Learners**

*Speaking: MLR8 Discussion Supports.* When partners are compare their work, provide sentence frames to help students describe reasons for whether an equation has a solution that is positive, negative, or zero. Displaying the following: “I know that equation _____will have a positive/negative/zero solution because _____.”, “Some features of equations with a positive/negative/zero solution are ___.”, and “When I look at structure of this equation I notice that _____.” This will help students practice talking about the structure of equations and the operations within them.

Design Principle(s): Cultivate conversation; Support sense-making

**Student Task Statement**

Without solving, identify whether these equations have a solution that is positive, negative, or zero.

1. \( \frac{x}{6} = \frac{3x}{4} \)
2. \( 7x = 3.25 \)
3. \( 7x = 32.5 \)
4. \( 3x + 11 = 11 \)
5. \( 9 - 4x = 4 \)
6. \( -8 + 5x = -20 \)
7. \( -\frac{1}{2}(8 + 5x) = -20 \)

**Student Response**

1. Zero. Sample reasoning: There are only \( x \) terms, so there is no constant term for the variable to equal, or the constant term is 0.
2. Positive. Sample reasoning: A positive amount of $x$s equals a positive value.

3. Positive. Sample reasoning: A positive amount of $x$s equals a positive value.

4. Zero. Sample reasoning: Since the constant terms on each side are equal, the final constant term is 0.

5. Positive. Sample reasoning: If you subtract 9 from each side, you will be left with a negative amount of $x$s equal to a negative number.

6. Negative. Sample reasoning: If you add 8 to each side, you will be left with a positive amount of $x$s equal to a negative number.

7. Positive. Sample reasoning: After distributing, we will have a negative amount of $x$s equal to a negative number.

**Activity Synthesis**

For each equation, invite groups to share how they decided if the solution was positive, negative, or zero. After each group shares, ask if any other group reasoned about the problem in a different way and invite them to share their reasoning.

The purpose of this discussion is for students to practice talking about equations, the operations within them, and use logical thinking. There is no need to try and formally generalize student thinking for all cases at this time. In later grades, students will continue the work started here looking for structure in equations.

### 6.3 Which Would You Rather Solve?

**20 minutes**

The purpose of this activity is for students to think about what they see as “least difficult” and “most difficult” when looking at equations and to practice solving equations. Students also discuss strategies for dealing with “difficult” parts of equations.

**Addressing**

- 8.EE.C.7.b

**Instructional Routines**

- MLR3: Clarify, Critique, Correct
- Poll the Class

**Launch**

Keep students in the same groups of 2. Give students 3–5 minutes quiet think time to get started and then 5–8 minutes to discuss and work with their partner. Leave ample time for a whole-class discussion.
Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement
Here are a lot of equations:

A. \(\frac{5}{6}(8 + 5b) = 75 + \frac{5}{3}b\)                      F. \(3(c - 1) + 2(3c + 1) = -(3c + 1)\)
B. \(-\frac{1}{2}(t + 3) - 10 = -6.5\)                          G. \(\frac{4m - 3}{4} = \frac{9 + 4m}{8}\)
C. \(\frac{10-n}{4} = 2(v + 17)\)                             H. \(p - 5(p + 4) = p - (8 - p)\)
D. \(2(4k + 3) - 13 = 2(18 - k) - 13\)                      I. \(2(2q + 1.5) = 18 - q\)
E. \(\frac{n}{7} - 12 = 5n + 5\)                              J. \(2r + 49 = -8(-r - 5)\)

1. Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.

2. Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Student Response
A. \(b = -14\)
B. \(t = -10\)
C. \(v = -14\)
D. \(k = 3\)
E. \(n = -3.5\)
F. \(c = 0\)
G. \(m = \frac{1}{4}\)
H. \(p = -2\)
I. \(q = 3\)
1. Answers vary. Sample response: Equation A would be difficult to solve because there are some fractions on each side of the equation. Equation C would be easy to solve because there is only one fraction on one side of the equation.

2. Answers vary.

**Are You Ready for More?**

Mai gave half of her brownies, and then half a brownie more, to Kiran. Then she gave half of what was left, and half a brownie more, to Tyler. That left her with one remaining brownie. How many brownies did she have to start with?

**Student Response**

7 brownies. An equation that represents this scenario is \( \frac{x + 1}{2} + \frac{x - \frac{x + 1}{2}}{2} = x - 1 \), where \( x \) is the number of brownies Mai started with.

**Activity Synthesis**

The purpose of this discussion is for students to discuss strategies for solving different types of equations. Some students may have thought that an equation was in the “least difficult” category, while others thought that the same equation was in the “most difficult” category. Remind students that once you feel confident about the strategies for solving an equation, it may move into the “least difficult” category, and recognizing good strategies takes practice and time.

Poll the class for each question as to whether they placed it in the “most difficult” category, “least difficult” category, or if it was somewhere in the middle. Record and display the results of the poll for all to see.

For questions with a split vote, have a group share something that was difficult about it and something that made it seem easy. If there are any questions that everyone thought would be more difficult or everyone thought would be less difficult, ask students why it seemed that way. Ask students, “Were there any equations that were more difficult to solve than you expected? Were there any that were less difficult to solve than you expected?”

Consider asking some of the following questions to further the discussion:

- “For equation A, what could we do to eliminate the fraction?” (Multiply each side by the common denominator of 6. Then the terms will all have integer coefficients.)

- “Which other equations could we use this strategy for?” (Any equations that had fractions, such as B, C, E, and G.)

- “What steps do you need to do to solve equation D? Which other equations are like this one?” (There is a lot of distributing and collecting like terms. F and H also have to distribute several times.)
• “What other strategies or steps did you use in solving the equations?”

Support for English Language Learners

Representing: MLR3 Clarify, Critique, Correct. After polling students about the difficulty of each problem, choose the question that was ranked as “most difficult” for students to solve, and display a possible incorrect solution. For example, display $2(2q + 1.5) = 4q + 1.5$, to invite discussion about the distributive property. Invite students to share what makes the question difficult and to name strategies or moves that can be used to solve the equation. Invite students to work with a partner to revise and correct the response. Let them know that they should each be prepared to explain their work. When pairs share their responses with the class, draw students' attention to the language used to describe the steps they took, to clarify their strategies. This will support student understanding of strategies they can use to deal with “difficult” parts of equations.

Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Instruct students to write an equation with a variable and a constant term on each side that they would look at and consider difficult to solve.

Select several students’ equations to display for all to see. Discuss with students:

• “What are some things these equations have in common that might be considered difficult to solve?” (Answers vary, but students may use fractions, decimals, distribution, and negatives to increase level of difficulty.)

• “What strategies do we know for solving equations that have each of these things?” (For fractions, we can find a common denominator and multiply each side of the equation by the denominator to eliminate the fractions. For distribution, we can make sure to collect like terms before we do other steps. For use of decimals and negatives, we can make sure we perform calculations carefully.)

Choose one of the displayed equations for students to solve. Have them compare solutions with a partner. Ask students, “Did any of you use different strategies for solving this equation than your partner? How many of you followed the same solution path?” Share a correct solution with the class so they can compare their solutions.

6.4 Think Before You Step

Cool Down: 5 minutes
Addressing
• 8.EE.C.7

Unit 4 Lesson 6
Student Task Statement
1. Without solving, identify whether this equation has a solution that is positive, negative, or zero:

   \[3x - 5 = -3\]

2. Solve the equation.

   \[x - 5(x - 1) = x - (2x - 3)\]

Student Response
1. If \(3x - 5 = -3\), then the \(x\) must be positive. If \(x\) is negative, then subtracting 5 from 3\(x\) would result in a number less than -3. For similar reasons, \(x\) cannot be zero.

2. \(x = \frac{2}{3}\)

Student Lesson Summary
Sometimes we are asked to solve equations with a lot of things going on on each side. For example,

\[x - 2(x + 5) = \frac{3(2x - 20)}{6}\]

This equation has variables on each side, parentheses, and even a fraction to think about. Before we start distributing, let’s take a closer look at the fraction on the right side. The expression \(2x - 20\) is being multiplied by 3 and divided by 6, which is the same as just dividing by 2, so we can re-write the equation as

\[x - 2(x + 5) = \frac{2x - 20}{2}\]

But now it’s easier to see that all the terms on the numerator of right side are divisible by 2, which means we can re-write the right side again as

\[x - 2(x + 5) = x - 10\]

At this point, we could do some distribution and then collect like terms on each side of the equation. Another choice would be to use the structure of the equation. Both the left and the right side have something being subtracted from \(x\). But, if the two sides are equal, that means the "something" being subtracted on each side must also be equal. Thinking this way, the equation can now be re-written with less terms as

\[2(x + 5) = 10\]

Only a few steps left! But what can we tell about the solution to this problem right now? Is it positive? Negative? Zero? Well, the 2 and the 5 multiplied together are 10, so that means the 2 and the \(x\) multiplied together cannot have a positive or a negative value. Finishing the steps we have:
\[ 2(x + 5) = 10 \]
\[ x + 5 = 5 \quad \text{Divide each side by 2} \]
\[ x = 0 \quad \text{Subtract 5 from each side} \]

Neither positive nor negative. Just as predicted.

**Lesson 6 Practice Problems**

**Problem 1**

**Statement**
Solve each of these equations. Explain or show your reasoning.

\[ 2b + 8 - 5b + 3 = -13 + 8b - 5 \]
\[ 2x + 7 - 5x + 8 = 3(5 + 6x) - 12x \]
\[ 2c - 3 = 2(6 - c) + 7c \]

**Solution**

a. \( b = \frac{29}{11} \). Responses vary. Sample response: Collect like terms on each side, add 18 to each side, add 3x to each side, then divide each side by 11.

b. \( x = 0 \). Responses vary. Sample response: Collect like terms on the left side, distribute and collect like terms on the right side, add 3x to each side, subtract 15 from each side, then divide each side by 9.

c. \( c = -5 \). Responses vary. Sample response: Distribute and collect like terms on each side, subtract 12 from each side, subtract 2x from each side, then divide each side by 3.

**Problem 2**

**Statement**
Solve each equation and check your solution.

\[ -3w - 4 = w + 3 \]
\[ 3(3 - 3x) = 2(x + 3) - 30 \]
\[ \frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z) \]

**Solution**

a. \( w = -\frac{7}{4} \)

b. \( x = 3 \)

c. \( z = 8 \)
Problem 3

Statement
Elena said the equation $9x + 15 = 3x + 15$ has no solutions because $9x$ is greater than $3x$. Do you agree with Elena? Explain your reasoning.

Solution
Elena is incorrect. Responses vary. Sample response: $9x > 3x$ when $x > 0$, but $9x < 3x$ when $x < 0$ and $9x = 3x$ when $x = 0$. The solution to the equation is $x = 0$.

Problem 4

Statement
The table gives some sample data for two quantities, $x$ and $y$, that are in a proportional relationship.

a. Complete the table.

b. Write an equation that represents the relationship between $x$ and $y$ shown in the table.

c. Graph the relationship. Use a scale for the axes that shows all the points in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>64</td>
<td></td>
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<tr>
<td></td>
<td>39</td>
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<td>1</td>
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</tbody>
</table>
Solution

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>14</td>
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<tr>
<td>64</td>
<td>96</td>
</tr>
<tr>
<td>26</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{2}$</td>
</tr>
</tbody>
</table>

a. $y = \frac{3}{2}x$ (or equivalent)

b. 

(From Unit 3, Lesson 3.)
Lesson 7: All, Some, or No Solutions

Goals
- Compare and contrast (orally and in writing) equations that have no solutions or infinitely many solutions.
- Create linear equations in one variable that have either no solutions or infinitely many solutions, using structure, and explain (orally) the solution method.

Learning Targets
- I can determine whether an equation has no solutions, one solution, or infinitely many solutions.

Lesson Narrative
In previous lessons, students have mostly worked with equations that have exactly one solution and have solved those equations by a sequence of steps that lead to an equation of the form \( x = \text{number} \). In this lesson, they encounter equations that have no solutions and equations for which every number is a solution. In the first case, when students try to solve the equation, they end up with false statements like \( 0 = 5 \). In the second case, they end up with a statement that is always true, such as \( 6x = 6x \). In preparation for the next lesson, where students will learn to predict the number of solutions from the structure of an equation, students complete equations in three different ways to make them have no solution, one solution, or infinitely many solutions.

Alignments

Addressing
- 8.EE.C.7.a: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a, a = a, \text{ or } a = b \) results (where \( a \) and \( b \) are different numbers).

Building Towards
- 8.EE.C.7.a: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a, a = a, \text{ or } a = b \) results (where \( a \) and \( b \) are different numbers).

Instructional Routines
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Which One Doesn’t Belong?
Student Learning Goals
Let’s think about how many solutions an equation can have.

7.1 Which One Doesn’t Belong: Equations

Warm Up: 5 minutes
The purpose of this warm-up is for students to think about equality and properties of operations when deciding whether equations are true. While there are many reasons students may decide one equation doesn’t belong, highlight responses that mention both sides of the equation being equal and ask students to explain how they can tell.

Building Towards
• 8.EE.C.7.a

Instructional Routines
• Which One Doesn't Belong?

Launch
Arrange students in groups of 2–4. Give students 1 minute of quiet think time. Ask students to indicate when they have noticed one equation that does not belong and can explain why not. Give students time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason each equation doesn’t belong.

Student Task Statement
Which one doesn’t belong?

1. $5 + 7 = 7 + 5$
2. $5 \cdot 7 = 7 \cdot 5$
3. $2 = 7 - 5$
4. $5 - 7 = 7 - 5$

Student Response
Answers vary. Possible solutions: 1 is different because it is the only one that involves addition explicitly. 2 is different because it is the only one that involves multiplication. 3 is different because it has a 2 in it and all the others only include 5 and 7. 3 is different because it has a single number on one side and all the others have two numbers on both sides. 4 is different because it is not true.

Activity Synthesis
After students have conferred in groups, invite each group to share one reason why a particular equation might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the
question of which equation does not belong, attend to students’ explanations and ensure the reasons given are correct.

If no students point out that 4 is not true, ask if all of the equations are true and to explain how they know.

**7.2 Thinking About Solutions**

15 minutes
Students who pause to think about the structure of a complex equation before taking steps to solve it can find the most efficient solution paths and, sometimes, notice that there is no single solution to be found. The goal of this lesson is to encourage students to make this pause part of their routine and to build their skill at understanding and manipulating the structure of equations through the study of two special types of equations: ones that are always true and ones that are never true.

Students begin the activity sorting a variety of equations into categories based on their number of solutions. The activity ends with students filling in the blank side of an equation to make an equation that is always true and then again to make an equation that is never true.

**Addressing**

- 8.EE.C.7.a

**Instructional Routines**

- MLR2: Collect and Display

**Launch**
Display the equation $2t + 5 = 2t + 5$ and ask students to find a value of $t$ that makes the equation true. After a brief quiet think time, record the responses of a few students next to the equation. Ask the class if they think there is any value of $t$ that doesn’t work and invite students to explain why or why not. If no students suggest seeing what happens if you try to solve for $t$, demonstrate that no matter what steps you take, the equation will always end with a statement that is always true such as $t = t$ or $5 = 5$.

Next, display the equation $n + 5 = n + 7$ and ask students to find a value of $n$ that makes the equation true. After a brief quiet think time, ask the class if they think there might be a value that works and select a few students to explain why or why not. While you can try and solve for $n$ here as with the previous example, encourage students to also use the logic that adding different values to the same value cannot result in two numbers that are the same.

Tell students that these are two special kinds of equations. The first equation has many solutions—it is true for all values of $t$. Remind students that they encountered this type of equation during the number trick activity where one side of the equation looked complicated but it was actually the same as a very simple expression, which is why the trick worked. The second equation has no solutions—it is not true for any values of $n$. 

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Arrange students in groups of 2. Give students 3–5 minutes quiet work time for the first problem, followed by partner discussion to share how they sorted the equations. Give time for partners to complete the remaining problems and follow with a whole-class discussion.

Support for English Language Learners

Representing: MLR2 Collect and Display. As groups of students discuss how they sorted the equations, circulate and record the language students use to justify their decisions on a visual display. Ask students to describe the reasons for their selection, and to name what these equations have in common. Listen for phrases such as “variables with the same coefficient” or “the variable was eliminated.” Consider dividing the display into sections labeled “true for all values” and “true for no values,” and group words and phrases in the appropriate area. Remind students to borrow language from the display as needed. This will help students use mathematical language to describe their reasoning and increase awareness about what these types of equations look like.

Design Principle(s): Support sense-making

Anticipated Misconceptions

For the last part of the activity, students may think any expression that is not equivalent to \(6u - 10\) is a good answer. Remind students that there is another possibility: that the equation will have one solution. For example, the expression \(3u + 5\) does allow for a solution.

Student Task Statement

\[
\begin{align*}
  n &= n \\
  5 - 9 + 3x &= -10 + 6 + 3x \\
  2t + 6 &= 2(t + 3) \\
  \frac{1}{2} + x &= \frac{1}{3} + x \\
  3(n + 1) &= 3n + 1 \\
  y \cdot -6 \cdot -3 &= 2 \cdot y \cdot 9 \\
  \frac{1}{4}(20d + 4) &= 5d \\
  v + 2 &= v - 2
\end{align*}
\]

1. Sort these equations into the two types: true for all values and true for no values.

2. Write the other side of this equation so that this equation is true for all values of \(u\).
   \(6(u - 2) + 2 = \)

3. Write the other side of this equation so that this equation is true for no values of \(u\).
   \(6(u - 2) + 2 = \)

Student Response

1. True for all values: \(n = n, y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9, 2t + 6 = 2(t + 3), 5 - 9 + 3x = -10 + 6 + 3x \) 
   True for no values: \(\frac{1}{2} + x = \frac{1}{3} + x, 3(n + 1) = 3n + 1, \frac{1}{4}(20d + 4) = 5d, v + 2 = v - 2\)

2. Answers vary. Sample response: \(6(u - 2) + 2 = 6u - 10\)
3. Answers vary. Sample response: \(6(u - 2) + 2 = 6u\)

**Are You Ready for More?**

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

How many sets of two or more consecutive positive integers can be added to obtain a sum of 100?

**Student Response**

There are two sets. The first has 8 consecutive integers: 9, 10, 11, 12, 13, 14, 15, 16. The second has five consecutive integers: 18, 19, 20, 21, 22.

**Activity Synthesis**

Display a list of the equations from the task with space to add student ideas next to the equations. The purpose of this discussion is for students to see multiple ways of thinking about and justifying the number of solutions an equation has.

Invite students to choose an equation, say if it is true for all values or true for no values, and then justify how they know. Continue until the solutions to all the equations are known. Record a summarized version of the student’s solution next to the equation.

Next, ask students for different ways to write the other side of the equation for the second problem and add these to the display. For example, students may have distributed \(6(u - 2) + 2\) to get \(6u - 12 + 2\) while others chose \(6u - 10\) or something with more terms, such as \(6(u - 2 + 1) - 4\).

End the discussion by asking students for different ways to write the other side of the incomplete equation in the last question. It is important to note, if no students point it out, that all solutions should be equivalent to \(6u + _\) where the blank represents any number other than -10.

**Support for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to determine whether an equation is true for all values or true for no values, and to include examples for each.

*Supports accessibility for: Conceptual processing; Organization*

**7.3 What's the Equation?**

15 minutes

In this activity, students are presented with three equations all with a missing term. They are asked to fill in the missing term to create equations with either no solution or infinitely many solutions,
building on the work begun in the previous activity. At the end, students summarize what they have learned about how to tell if an equation is true for all values of $x$ or no values of $x$.

**Addressing**
- 8.EE.C.7.a

**Instructional Routines**
- MLR8: Discussion Supports

**Launch**
Give students 3–5 minutes of quiet think time followed by 3–5 minutes of partner discussion. Follow with a whole-class discussion.

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**Support for Students with Disabilities**

*Action and Expression: Develop Expression and Communication.* To help get students started during partner discussion, display sentence frames such as “Equations that are always true for $x$ have ___”, “Equations which have no solution for any value of $x$ have ____ .

*Supports accessibility for: Language; Organization*

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**Student Task Statement**

1. Complete each equation so that it is true for all values of $x$.
   - $3x + 6 = 3(x + ___)$
   - $x - 2 = -(___ - x)$
   - $\frac{15x-10}{5} = ___ - 2$

2. Complete each equation so that it is true for no values of $x$.
   - $3x + 6 = 3(x + ___)$
   - $x - 2 = -(___ - x)$
   - $\frac{15x-10}{5} = ___ - 2$

3. Describe how you know whether an equation will be true for all values of $x$ or true for no values of $x$.

**Student Response**

1. a. 2
   b. 2
   c. 3x

2. a. Answers vary. Any number other than 2 will give an equation with no solution.
b. Answers vary. Any number other than 2 will give an equation with no solution.

c. Answers vary. Any expression of the form \((3x+a)\) a number other than 0 will give an equation with no solution. Note: A numerical answer will yield a linear equation of one variable which has one solution.

3. Explanations vary. Sample response: Equations which are always true for any value of \(x\) have equivalent expressions on each side. Equations which have no solution for any value of \(x\) simplify to a statement of two unequal numbers being equal, which is always false.

**Activity Synthesis**

Display each equation with a large space for writing. Under each equation, invite students to share what they used to make the equation be true for all values of \(x\) and record these for all to see. Ask:

- “What did all these answers have in common?” (There is only one possible answer for each equation that will make it be always true.)
- “What strategy did you use to figure out what that answer had to be?” (The solution had to be something that would make the right side equivalent to the left.)

Next, invite students to share what they used to make the equation true for no values of \(x\) and record these for all to see. Ask:

- “Why are there so many different solutions for these questions?” (As long as the answer wasn’t what we chose in part 1, then the equation will never have a solution.)
- “What was different about Equation C?” (We had to be careful to make sure that the variable coefficient was 3 and we added a constant so that the equation wouldn’t have a single solution.)

Ask students to share observations they made for the last question. If no student points it out, explain that an equation with no solution can always be rearranged or manipulated to say that two unequal values are equal (e.g., \(2=3\)), which means the equation is never true.

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**Support for English Language Learners**

**Speaking:** MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation that is shared for the last question, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. This will provide more students with an opportunity to describe what they have learned about how to tell if an equation is true for all values of \(x\) or no values of \(x\).

*Design Principle(s): Support sense-making*
Lesson Synthesis

Ask students to think about some ways they were able to determine how many solutions there were to the equations they solved today. Invite students to share some thing they did. For example, students may suggest:

- tested different values for the variable
- applied allowable moves to generate equivalent equations
- examined the structure of the equation

Ask students to write a short letter to someone taking the class next year about what they should look for when trying to decide how many solutions an equation has. Tell students to use examples, share any struggles they had in deciding on the number of solutions, and which strategies they prefer for figuring out the number of solutions.

7.4 Choose Your Own Solution

Cool Down: 5 minutes

Addressing

- 8.EE.C.7.a

Student Task Statement

$$3x + 8 = 3x + 8$$

What value could you write in after $$3x$$ that would make the equation true for:

1. no values of $$x$$?
2. all values of $$x$$?
3. just one value of $$x$$?

Student Response

1. Answers vary. Sample response: 7. The equation $$3x + 8 = 3x + 7$$ has no solutions. If you triple a number and add 8 to it, and triple the same number and add 7 to it, the results will never be equal, no matter what number you choose.

2. 8. The equation $$3x + 8 = 3x + 8$$ has many solutions. If you triple a number and add 8 to it, and triple the same number and add 8 to it, the results will always be equal, no matter what number you choose.

3. Answers vary. Sample response: x. The equation $$3x + 8 = 3x + x$$ has one solution. Students should add some variable term in order to create an equation with one solution.
Student Lesson Summary

An equation is a statement that two expressions have an equal value. The equation

\[ 2x = 6 \]

is a true statement if \( x \) is 3:

\[ 2 \cdot 3 = 6 \]

It is a false statement if \( x \) is 4:

\[ 2 \cdot 4 = 6 \]

The equation \( 2x = 6 \) has one and only one solution, because there is only one number (3) that you can double to get 6.

Some equations are true no matter what the value of the variable is. For example:

\[ 2x = x + x \]

is always true, because if you double a number, that will always be the same as adding the number to itself. Equations like \( 2x = x + x \) have an infinite number of solutions. We say it is true for all values of \( x \).

Some equations have no solutions. For example:

\[ x = x + 1 \]

has no solutions, because no matter what the value of \( x \) is, it can't equal one more than itself.

When we solve an equation, we are looking for the values of the variable that make the equation true. When we try to solve the equation, we make allowable moves assuming it has a solution. Sometimes we make allowable moves and get an equation like this:

\[ 8 = 7 \]

This statement is false, so it must be that the original equation had no solution at all.

Lesson 7 Practice Problems

Problem 1

Statement

For each equation, decide if it is always true or never true.

a. \( x - 13 = x + 1 \)

b. \( x + \frac{1}{2} = x - \frac{1}{2} \)
c. $2(x + 3) = 5x + 6 - 3x$

d. $x - 3 = 2x - 3 - x$

e. $3(x - 5) = 2(x - 5) + x$

**Solution**

a. Never true

b. Never true

c. Always true

d. Always true

e. Never true

**Problem 2**

**Statement**

Mai says that the equation $2x + 2 = x + 1$ has no solution because the left hand side is double the right hand side. Do you agree with Mai? Explain your reasoning.

**Solution**

Answers vary. Sample response: Mai is correct that $2x + 2 = 2(x + 1)$, so the left hand side in this equation is double the right hand side. But $-x$ and $-1$ can be added to both sides of the equation to get $x + 1 = 0$. So $x = -1$ is a solution. (This works because 0 is its own double, and it is the only number that is its own double.)

**Problem 3**

**Statement**

a. Write the other side of this equation so it’s true for all values of $x$: $\frac{1}{2}(6x - 10) - x = $

b. Write the other side of this equation so it’s true for no values of $x$: $\frac{1}{2}(6x - 10) - x = $

**Solution**

a. $2x - 5$ (or equivalent)

b. Answers vary. Sample response: $2x + 5$
Problem 4

Statement
Here is an equation that is true for all values of $x$: $5(x + 2) = 5x + 10$. Elena saw this equation and says she can tell $20(x + 2) + 31 = 4(5x + 10) + 31$ is also true for any value of $x$. How can she tell? Explain your reasoning.

Solution
Responses vary. Sample response: One could distribute the left side of the equation and show it is equal to the right side, but it is easier to see that each side of the original equation has been multiplied by 4 and added to 31. These moves keep both sides of the equation in balance, and so whatever values of $x$ make the first equation true also make the second equation true.

Problem 5

Statement
Elena and Lin are trying to solve $\frac{1}{2}x + 3 = \frac{7}{2}x + 5$. Describe the change they each make to each side of the equation.

a. Elena’s first step is to write $3 = \frac{7}{2}x - \frac{1}{2}x + 5$.

b. Lin’s first step is to write $x + 6 = 7x + 10$.

Solution

a. Elena subtracted $\frac{1}{2}x$ from each side.

b. Lin multiplied each side by 2.

(From Unit 4, Lesson 4.)

Problem 6

Statement
Solve each equation and check your solution.

\[3x - 6 = 4(2 - 3x) - 8x \quad \frac{1}{2}z + 6 = \frac{3}{2}(z + 6) \quad 9 - 7w = 8w + 8\]

Solution

a. $x = \frac{14}{23}$

b. $z = -3$
c. \( w = \frac{1}{15} \)

(From Unit 4, Lesson 6.)

**Problem 7**

**Statement**

The point \((-3, 6)\) is on a line with a slope of 4.

a. Find two more points on the line.

b. Write an equation for the line.

**Solution**

a. Answers vary. Sample response: \((-2, 10), (-1, 14)\)

b. \( y = 4x + 18 \) (or equivalent)

(From Unit 3, Lesson 12.)
Lesson 8: How Many Solutions?

Goals
- Describe (orally) a linear equation as having “one solution”, “no solutions”, or “an infinite number of solutions”, and solve equations in one variable with one solution.
- Describe (orally) features of linear equations with one solution, no solution, or an infinite number of solutions.

Learning Targets
- I can solve equations with different numbers of solutions.

Lesson Narrative
In the previous lesson, students learned that sometimes an equation has one solution, sometimes no solution, and sometimes infinitely many solutions. The purpose of this lesson is to help students identify structural features of an equation that tell them which of these outcomes will occur when they solve it. They also learn to stop solving an equation when they have reached a point where it is clear which of the outcomes will occur, for example when they reach an equation like $6x + 2 = 6x + 5$ (no solution) or $6x + 2 = 6x + 2$ (infinitely many solutions). When students monitor their progress in solving an equation by paying attention to the structure at each step, they engage in MP7.

Alignments

Addressing
- 8.EE.C.7.a: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a, a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).

Instructional Routines
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials
Pre-printed slips, cut from copies of the blackline master

Required Preparation
Make 1 copy of the Make Use of Structure blackline master for every 3 students, and cut them up ahead of time.

Student Learning Goals
Let’s solve equations with different numbers of solutions.
8.1 Matching Solutions

Warm Up: 5 minutes
Students extend their understanding from the previous lessons to recognize the structure of a linear equation for all possible types of solutions: one solution, no solution, or infinitely many solutions. Students are still using language such as “true for one value of \( x \),” “always true” or “true for any value of \( x \),” and “never true.” Students should be able to articulate that this depends both on the coefficient of the variable as well as the constant term on each side of the equation.

Addressing
- 8.EE.C.7.a

Launch
Give students 2–3 minutes of quiet think time followed by a whole-class discussion.

Student Task Statement
Consider the unfinished equation \( 12(x - 3) + 18 = \underline{\quad} \). Match the following expressions with the number of solutions the equation would have with that expression on the right hand side.

1. \( 6(2x - 3) \)  
   - one solution
2. \( 4(3x - 3) \)  
   - no solutions
3. \( 4(2x - 3) \)  
   - all solutions

Student Response
1. The equation \( 12(x - 3) + 18 = 6(2x - 3) \) has infinitely many solutions, or is true for any value of \( x \).
2. The equation \( 12(x - 3) + 18 = 4(3x - 3) \) has no solution.
3. The equation \( 12(x - 3) + 18 = 4(2x - 3) \) has one solution, \( x = \frac{3}{2} \).

Activity Synthesis
In order to highlight the structure of these equations, ask students:

- “What do you notice about equations with no solution?” (These equations have equal or equivalent coefficients for the variable, but unequal values for the constants on each side of the equation.)
- “What do you notice about equations that are always true?” (These equations have equivalent expressions on each side of the equation, so the coefficients are equal and the constants are equal or equivalent on each side.)
• “What do you notice about equations that have exactly one solution?” (These equations have different values for the coefficients on each side of the equation and it doesn’t matter what the constant term says.)

Display the equation $x = 12$ for all to see. Ask students how this fits with their explanations. (We can see that there is one solution. Another way to think of this is that the coefficient of $x$ is 1 on the left side of the equation, and the coefficient of $x$ is 0 on the right side of the equation. So the coefficients of $x$ are different, just like the explanation.)

8.2 Thinking About Solutions Some More

25 minutes
In this activity students solve a variety of equation types; both in form and number of solutions. After solving the 10 equations, groups sort them into categories of their choosing. The goal of this activity is to encourage students to look at the structure of equations before solving and to build fluency solving complex equations. For example, students who notice that equation D, $3 - 4x + 5 = 2(8 - 2x)$, has the same number of $x$s on each side but a different constant know that there are no values of $x$ that make the equation true. Similarly, equation J has the same number of $x$s on each side and the same constants on each side, meaning that all values of $x$ make the equation true. These up-front observations allow students to avoid spending time working out the steps to re-write the equation into a simpler form where the number of solutions to the equations is easier to see.

Addressing
- 8.EE.C.7.a

Instructional Routines
- MLR8: Discussion Supports

Launch
Arrange students in groups of 3. Distribute 10 pre-cut slips from the blackline master to each group. After groups have solved and sorted their equations, consider having groups switch to examine another group’s categories. Leave 3–4 minutes for a whole-class discussion.

Support for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial sort. Some students may benefit from an additional set of cards with more accessible values to start with.

*Supports accessibility for: Conceptual processing; Organization*
### Student Task Statement

Your teacher will give you some cards.

1. With your partner, solve each equation.
2. Then, sort them into categories.
3. Describe the defining characteristics of those categories and be prepared to share your reasoning with the class.

### Student Response

Answers vary. Possible solution:

- 5 equations of varying complexity, some including parentheses and some not, that have a single integer solution.
  - A $7(x - 5) = x + 13$ ($x = 8$)
  - C $-4(x - 2) = -2(x - \frac{17}{2})$ ($x = -4.5$)
  - G $3x + 9 = 2.5x + 14$ ($x = 10$)
  - H $7(x - 4) = 4x + 5$ ($x = 11$)
  - I $5x - 20 = -7x - 20$ ($x = 0$)

- 3 equations with no solution.
  - F $2x + 3 = 2x + 5$
  - B $-6x = -5(x - 1) - x$
  - D $3 - 4x + 5 = 2(8 - 2x)$

- 2 equations with an infinite number of solutions.
  - E $2x + 3 = 3 + 2x$
  - J $3(2x + 1) - 4x = 2x + 3$

### Activity Synthesis

Select 2–3 groups to share one of their categories’ defining characteristics and which equations they sorted into it. Given the previous activity, the categories “one solution, no solution, all solutions” are likely. Introduce students to the language “infinite number of solutions” if it has not already come up in discussion.

During the discussion, it is likely that students will want to refer to specific parts of an expression. Encourage students to use the words “coefficient” and “variable.” Define the word **constant term** as the term in an expression that doesn't change, or the term that does not have a variable part. For
example, in the expression $2x - 3$, the 2 is the coefficient of $x$, the 3 is a constant, and $x$ is the variable.

Support for English Language Learners

*Representing, Speaking: MLR8 Discussion Supports.* To support the whole-class discussion, provide the following sentence frames when groups share: “We grouped these equations into the category ___ because____.” and “Some characteristics of this category are ___.” Invite students to press for details by asking clarifying questions (e.g., “What other features do those equations have in common?” and “Can you explain how to create a different equation that would fall into that category?”). Emphasize language students use to define parts of an expression such as “variable” and “coefficient,” and use this opportunity to define the word “constant term.” This will help students generalize the categories of equations that have one solution, no solution, or “infinitely many solutions.”

*Design Principle(s): Support sense-making*

### 8.3 Make Use of Structure

**Optional: 15 minutes**
The purpose of this activity is so that students can compare the structure of equations that have no solution, one solution, and infinitely many solutions. This may be particularly useful for students needing more practice identifying which equations will have each of these types of solutions before attempting to solve.

**Addressing**

- 8.EE.C.7.a

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time

**Launch**

Arrange students in groups of 2. Give students 3–5 minutes of quiet think time followed by 2–3 minutes of partner discussion. Follow with a whole-class discussion.

**Anticipated Misconceptions**

On the second question, students may think that $x - 3 = 3 - x$ have the same coefficients of $x$. Recall that $x$ has a coefficient of 1, while $-x$ has a coefficient of -1.

**Student Task Statement**

For each equation, determine whether it has no solutions, exactly one solution, or is true for all values of $x$ (and has infinitely many solutions). If an equation has one solution, solve to find the value of $x$ that makes the statement true.
1.  a. $6x + 8 = 7x + 13$
   b. $6x + 8 = 2(3x + 4)$
   c. $6x + 8 = 6x + 13$

2.  a. $\frac{1}{4}(12 - 4x) = 3 - x$
   b. $x - 3 = 3 - x$
   c. $x - 3 = 3 + x$

3.  a. $-5x - 3x + 2 = -8x + 2$
   b. $-5x - 3x - 4 = -8x + 2$
   c. $-5x - 4x - 2 = -8x + 2$

4.  a. $4(2x - 2) + 2 = 4(x - 2)$
   b. $4x + 2(2x - 3) = 8(x - 1)$
   c. $4x + 2(2x - 3) = 4(2x - 2) + 2$

5.  a. $x - 3(2 - 3x) = 2(5x + 3)$
   b. $x - 3(2 + 3x) = 2(5x - 3)$
   c. $x - 3(2 - 3x) = 2(5x - 3)$

6. What do you notice about equations with one solution? How is this different from equations with no solutions and equations that are true for every $x$?

**Student Response**

1.  a. One solution. $x = -5$
   b. Infinitely many solutions, or true for every $x$.
   c. No solution.

2.  a. Infinitely many solutions, or true for every $x$.
   b. One solution. $x = 3$.
   c. No solution.

3.  a. Infinitely many solutions, or true for every $x$.
   b. No solutions.
   c. One solution. $x = -4$.

4.  a. One solution. $x = \frac{1}{2}$.
b. No solution.

c. Infinitely many solutions, or true for every $x$.

5. a. No solution.

b. One solution. $x = 0$.

c. Infinitely many solutions, or true for every $x$.

6. Answers vary. Sample response: Equations with only one solution have a different amount of $x$s on each side, or the coefficients of $x$ are not equal. Equations with no solution have the same coefficients of $x$ but a different constant on each side, while equations with infinitely many solutions have equivalent expressions on each side of the equation.

**Are You Ready for More?**

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

1. Choose any set of three consecutive numbers. Find their average. What do you notice?

2. Find the average of another set of three consecutive numbers. What do you notice?

3. Explain why the thing you noticed must always work, or find a counterexample.

**Student Response**

Explanations vary. Sample response: Any three consecutive numbers can be represented as $x$, $x + 1$, and $x + 2$. Then the average of the three numbers is $\frac{x + (x+1) + (x+2)}{3} = x + 1$.

**Activity Synthesis**

Review each of the equations and how many solutions it has. If students disagree, ask each to explain their thinking about the equation and work to reach agreement. Once students are satisfied with the solutions, display the following questions for all to see:

- “What do you notice about equations with one solution?”
- “What do you notice about equations with no solutions?”
- “What do you notice about equations with infinitely many solutions?”

Give students brief quiet think time and then ask them to share a response to at least one of the questions with a partner. After partners have shared, invite student to share something they noticed with the class. Record students responses for all to see next to the relevant question.
Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to determine whether an equation will have one solution, no solutions, or infinitely many solutions, and to include examples for each.

Supports accessibility for: Conceptual processing; Organization

Support for English Language Learners

*Writing, Conversing: MLR1 Stronger and Clearer Each Time.* Use this routine for students to respond in writing to one of the three questions for whole-class discussion. Divide the class into thirds and assign each group of students one of the questions. Give students 3 minutes of quiet time to write a response. Invite students to meet with at least 2 other students to share and get feedback on their writing. Students should first meet with a partner that responded to the same question they did, before meeting with a student from a different group. Encourage listeners to ask clarifying questions such as, “Can you describe that using a different example?” or “What is another feature of that type of equation?” Invite the students to write a final draft based on their peer feedback. This will help students solidify their understanding of the number of solutions in a given equation by conversing with their partners.

*Design Principle(s): Optimize output; Cultivate conversation*

Lesson Synthesis

Instruct students to write three equations with a variable term and a constant term on each side of the equation. Their equations should be one with no solution, one with infinitely many solutions, and one with exactly one solution. When they think they have three equations that meet these requirements, tell students to trade with a partner, then identify which equation is each type. Give partners 2–3 minutes to check their solutions and discuss how they came up with their equations.

Ask students, “How did you know how to make each type of equation?” (I knew that the single-solution equation should have different coefficients for the variable terms, I knew that the many-solution equation should have equivalent expressions on each side, and I knew that the no-solution equation should only differ by a constant term on each side.)

If time allows, consider making a poster for permanent display that shows an equation with coefficient, variable, and constant terms emphasized in different colors.

8.4 How Does She Know?

Cool Down: 5 minutes
Addressing
- 8.EE.C.7.a

Student Task Statement
Elena began to solve this equation:

\[
\frac{12x + 6(4x + 3)}{3} = 2(6x + 4) - 2
\]

\[
12x + 6(4x + 3) = 3(2(6x + 4) - 2)
\]

\[
12x + 6(4x + 3) = 6(6x + 4) - 6
\]

\[
12x + 24x + 18 = 36x + 24 - 6
\]

When she got to the last line she stopped and said the equation is true for all values of \( x \).
How could Elena tell?

Student Response
Answers vary. Sample response: Elena could see that there are the same number of \( x \)s and the same constants on each side of the equation.

Student Lesson Summary
Sometimes it’s possible to look at the structure of an equation and tell if it has infinitely many solutions or no solutions. For example, look at

\[
2(12x + 18) + 6 = 18x + 6(x + 7).
\]

Using the distributive property on the left and right sides, we get

\[
24x + 36 + 6 = 18x + 6x + 42.
\]

From here, collecting like terms gives us

\[
24x + 42 = 24x + 42.
\]

Since the left and right sides of the equation are the same, we know that this equation is true for any value of \( x \) without doing any more moves!

Similarly, we can sometimes use structure to tell if an equation has no solutions. For example, look at

\[
6(6x + 5) = 12(3x + 2) + 12.
\]

If we think about each move as we go, we can stop when we realize there is no solution:
\[
\frac{1}{6} \cdot 6(6x + 5) = \frac{1}{6} \cdot (12(3x + 2) + 12)
\]
Multiply each side by \(\frac{1}{6}\).

\[
6x + 5 = 2(3x + 2) + 2
\]
Distribute \(\frac{1}{6}\) on the right side.

\[
6x + 5 = 6x + 4 + 2
\]
Distribute 2 on the right side.

The last move makes it clear that the constant terms on each side, 5 and 4 + 2, are not the same. Since adding 5 to an amount is always less than adding 4 + 2 to that same amount, we know there are no solutions.

Doing moves to keep an equation balanced is a powerful part of solving equations, but thinking about what the structure of an equation tells us about the solutions is just as important.

**Glossary**
- coefficient
- constant term

**Lesson 8 Practice Problems**

**Problem 1**

**Statement**

Lin was looking at the equation \(2x - 32 + 4(3x - 2462) = 14x\). She said, "I can tell right away there are no solutions, because on the left side, you will have \(2x + 12x\) and a bunch of constants, but you have just \(14x\) on the right side." Do you agree with Lin? Explain your reasoning.

**Solution**

Lin is correct. Responses vary. Sample response: Ignoring everything but the terms with \(x\) on the left side, we have \(2x\) and \(4(3x)\). In total, this will give \(14x\). All of the constant terms on the left side are negative, so they won't cancel to 0. Therefore, we have \(14x + \text{non-zero stuff} = 4x\), which will have no solutions.

**Problem 2**

**Statement**

Han was looking at the equation \(6x - 4 + 2(5x + 2) = 16x\). He said, "I can tell right away there are no solutions, because on the left side, you will have \(6x + 10x\) and a bunch of constants, but you have just \(16x\) on the right side." Do you agree with Han? Explain your reasoning.
Solution

Han is incorrect. Responses vary. Sample response: Ignoring everything but the terms with $x$ on the left side, we have $6x$ and $2(5x)$. In total, this will give $16x$. Collecting all the constant terms on the left side will give $-4 + 2(2)$, which is 0. Therefore, we have $16x + 0 = 16x$, which is true for all values of $x$.

Problem 3

Statement

Decide whether each equation is true for all, one, or no values of $x$.

- a. $6x - 4 = -4 + 6x$
- b. $4x - 6 = 4x + 3$
- c. $-2x + 4 = -3x + 4$

Solution

- a. True for all values of $x$.
- b. True for no values of $x$.
- c. True for one value of $x$.

Problem 4

Statement

Solve each of these equations. Explain or show your reasoning.

- a. $3(x - 5) = 6$
- b. $2 \left( x - \frac{2}{3} \right) = 0$
- c. $4x - 5 = 2 - x$

Solution

- a. $x = 7$. Explanations vary. Sample response: Multiply both sides by $\frac{1}{3}$, then add 5.
- b. $x = \frac{2}{3}$. Explanations vary. Sample response: Multiply both sides by $\frac{1}{2}$, then add $\frac{2}{3}$.
- c. $\frac{7}{5}$. Explanations vary. Sample response: Add $x$ and 5 to both sides, then multiply by $\frac{1}{5}$.

(From Unit 4, Lesson 4.)
Problem 5

Statement

The points \((-2, 0)\) and \((0, -6)\) are each on the graph of a linear equation. Is \((2, 6)\) also on the graph of this linear equation? Explain your reasoning.

Solution

No. Answers vary. Sample response: If the two points are graphed with the line that goes through both of them, the line does not pass through the first quadrant where \((2, 6)\) is plotted.

(From Unit 3, Lesson 13.)

Problem 6

Statement

In the picture triangle \(A'B'C'\) is an image of triangle \(ABC\) after a rotation. The center of rotation is \(E\).

a. What is the length of side \(AB\)? Explain how you know.

b. What is the measure of angle \(D'\)? Explain how you know.

Solution

a. 9 units. Rotations preserve side lengths, and side \(A'B'\) corresponds to side \(AB\) under this rotation.

b. 45 degrees. Rotations preserve angle measures, and angles \(D\) and \(D'\) correspond to each other under this rotation.

(From Unit 1, Lesson 7.)
Lesson 9: When Are They the Same?

Goals

• Create an equation in one variable to represent a situation in which two conditions are equal.
• Interpret the solution of an equation in one variable in context.

Learning Targets

• I can use an expression to find when two things, like height, are the same in a real-world situation.

Lesson Narrative

In this lesson students apply their knowledge of solving equations by considering two real world situations: two tanks where one is filling and the other is emptying and two elevators traveling above and below ground level. Using the given expressions for each situation, students are asked to determine when the amount of water in the tanks or the travel time of elevators will be the same. It is the work of the student to recognize that they can set the two expressions equal and solve the equation for the unknown and this work sets up the concept of substitution for the coming section on systems of linear equations.

Alignments

Addressing

• 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
• 8.EE.C.7: Solve linear equations in one variable.
• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Building Towards

• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Instructional Routines

• MLR2: Collect and Display
• MLR6: Three Reads
• Think Pair Share

Student Learning Goals

Let’s use equations to think about situations.

9.1 Which Would You Choose?

Warm Up: 5 minutes
The purpose of this warm-up is for students to reason about two situations that can be represented with linear equations. Since the number of babysitting hours determines which situation would be most profitable, there is no one correct answer to the question. Students are asked to explain their reasoning.

**Addressing**
- 8.EE.C.8

**Launch**
Give students 2 minutes of quiet work time followed by a whole-class discussion.

**Student Task Statement**
If you were babysitting, would you rather

- Charge $5 for the first hour and $8 for each additional hour?

Or

- Charge $15 for the first hour and $6 for each additional hour?

Explain your reasoning.

**Student Response**
Answers vary. Students may choose to charge $15 for the first hour and $6 for each additional hour if they are only babysitting for up to 5 hours. Students may say it doesn’t matter which one they choose if they babysit for 6 hours (the first hour plus 5 additional hours) because the amount they will earn is the same. Students may choose $5 for the first hour and $8 for each additional hour if they are babysitting more than 6 hours.

**Activity Synthesis**
Poll the class on which situation they would choose. Invite students from each side to explain their reasoning. Record and display these ideas for all to see. If no one reasoned about babysitting for less than 5 hours and therefore chose the second option, mention this idea to students.

Students may not use linear equations or graphs to decide which situation they would choose. If there is time, ask students for the equation and graph we could use to model each scenario.

**9.2 Water Tanks**

10 minutes
The goal of this activity, and the two that follow, is for students to solve an equation in a real-world context while previewing some future work solving systems of equations. Here, students first make sense of the situation using a table of values describing the water heights of two tanks and then use the table to estimate when the water heights are equal. A key point in this activity is the next step: taking two expressions representing the water heights in two different tanks for a given time.
and recognizing that the equation created by setting the two expressions equal to one another has a solution that is the value for time, \( t \), when the water heights are equal.

**Addressing**
- 8.EE.C.7

**Building Towards**
- 8.EE.C.8

**Instructional Routines**
- MLR2: Collect and Display

**Launch**
Give students 2–3 minutes to read the context and answer the first problem. Select students to share their answer with the class, choosing students with different representations of the situation if possible. Give 3–4 minutes for the remaining problems followed by a whole-class discussion.

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**Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help illustrate the information provided.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

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**Support for English Language Learners**

*Representing, Conversing: MLR2 Collect and Display.* To begin the whole-class discussion, give students the opportunity to discuss their solutions to the first question in groups of 3–4. Circulate through the groups and record language students use to describe what is happening in each tank. Listen for language related to rate of change, differences between rates, initial amounts of water, etc. If groups are stuck, consider asking, “What happens to the amount of water in Tank 1 (or 2) as time goes on?” and “Can you draw a picture of Tank 1 (or 2) at 5 minutes and then another picture at 10 minutes? What do you notice?” Post the collected language in the front of the room so that students can refer to it throughout the rest of the activity and lesson. This will help students talk about the relationship between the two tanks prior to being asked to find the time when they are equal.

*Design Principle(s): Maximize meta-awareness*

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**Student Task Statement**

The amount of water in two tanks every 5 minutes is shown in the table.
<table>
<thead>
<tr>
<th>time (minutes)</th>
<th>tank 1 (liters)</th>
<th>tank 2 (liters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>175</td>
<td>900</td>
</tr>
<tr>
<td>10</td>
<td>325</td>
<td>800</td>
</tr>
<tr>
<td>15</td>
<td>475</td>
<td>700</td>
</tr>
<tr>
<td>20</td>
<td>625</td>
<td>600</td>
</tr>
<tr>
<td>25</td>
<td>775</td>
<td>500</td>
</tr>
<tr>
<td>30</td>
<td>925</td>
<td>400</td>
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<td>300</td>
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<tr>
<td>40</td>
<td>1225</td>
<td>200</td>
</tr>
<tr>
<td>45</td>
<td>1375</td>
<td>100</td>
</tr>
<tr>
<td>50</td>
<td>1525</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Describe what is happening in each tank. Either draw a picture, say it verbally, or write a few sentences.

2. Use the table to estimate when the tanks will have the same amount of water.

3. The amount of water (in liters) in tank 1 after \( t \) minutes is \( 30t + 25 \). The amount of water (in liters) in tank 2 after \( t \) minutes is \(-20t + 1000\). Find the time when the amount of water will be equal.

**Student Response**

1. Answers vary. Sample response: the water in Tank 1 is increasing while the water in Tank 2 is decreasing.

2. The tanks will have the same amount of water between 15 and 20 minutes, but closer to 20 minutes.

3. When the amounts of water in the two tanks are equal, \( 30t + 25 = -20t + 1000 \). This happens when \( t = 19.5 \), so 19 and a half minutes after the start.

**Activity Synthesis**

The purpose of this discussion is to elicit student thinking about why setting the two expressions in the task statement equal to one another is both possible and a way to solve the final problem.
Consider asking the following questions:

- “What does \( t \) represent in the first expression? The second?” (In each expression, \( t \) is the time in minutes since the tank’s water level started being recorded.)

- “After we substitute a time in for \( t \) and simplify one of the expressions to be a single number, what does that number represent? What units does it have?” (The number represents the amount of liters in the water tank.)

- “How accurate was your estimate about the water heights using the table?” (My estimate was within a few minutes of the actual answer.)

- “If you didn’t know which expression in the last problem belonged to which tank, how could you figure it out?” (One of the expressions is increasing as \( t \) increased, which means it must be Tank 1. The other is decreasing as \( t \) increases, so it must be Tank 2.)

- “How did you find the time the two water heights were equal using the expressions?” (Since each expression gives the height for a specific time, \( t \), and we want to know when the heights are equal, I set the two expressions equal to each other and then solved for the \( t \)-value that made the new equation true.)

### 9.3 Elevators

**15 minutes**

In this activity, students work with two expressions that represent the travel time of an elevator to a specific height. As with the previous activity, the goal is for students to work within a real-world context to understand taking two separate expressions and setting them equal to one another as a way to determine more information about the context.

**Addressing**

- 8.EE.C.7

**Building Towards**

- 8.EE.C.8

**Instructional Routines**

- MLR6: Three Reads
- Think Pair Share

**Launch**

Give students 1 minute to read the context and the problems. You may wish to share with the class that programming elevators in buildings to best meet the demands of the people in the building can be a complicated task depending on the number of floors in a building, the number of people, and the number of elevators. For example, many large buildings in cities have elevators...
programmed to stay near the ground floor in the morning when employees are arriving and then stay on higher floors in the afternoon when employees leave work.

Arrange students in groups of 2. Give 2–3 minutes of quiet work time for the first two question and then ask students to pause and discuss their solutions with their partner. Give 3–4 minutes for partners to work on the remaining questions followed by a whole-class discussion.

**Support for English Language Learners**

*Reading, Representing: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem, without solving it for students. Display only the diagram and the description of the situation, without revealing the questions. In the first read, students to read the problem with the goal of comprehending the situation (e.g., A building has two elevators that go above and below ground.). Use the second read to identify important quantities without yet focusing on specific values. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: height of elevator A, in meters; height of elevator B, in meters; elapsed time, in seconds. For the third read, reveal the questions and ask students to brainstorm possible ways to use the given expressions to determine when the elevators will reach ground level at the same time. This will help students connect the language in the word problem and the reasoning needed to solve the problem.

*Design Principle(s): Support sense-making*

**Anticipated Misconceptions**

Students may mix up height and time while working with these expressions. For example, they may think that at \( h = 0 \), the height of the elevators is 16 meters and 12 meters, respectively, instead of the correct interpretation that the elevators reach a height of 0 meters at 16 seconds and 12 seconds, respectively.

**Student Task Statement**

A building has two elevators that both go above and below ground.

At a certain time of day, the travel time it takes elevator A to reach height \( h \) in meters is \( 0.8h + 16 \) seconds.

The travel time it takes elevator B to reach height \( h \) in meters is \(-0.8h + 12\) seconds.

1. What is the height of each elevator at this time?
2. How long would it take each elevator to reach ground level at this time?
3. If the two elevators travel toward one another, at what height do they pass each other? How long would it take?

4. If you are on an underground parking level 14 meters below ground, which elevator would reach you first?

**Student Response**

1. Elevator A is 20 meters below ground. At \( t = 0 \), the initial height of Elevator A can be found by solving the equation \( 0 = 0.8h + 16 \). Elevator B is 15 meters above ground. At \( t = 0 \), the initial height of Elevator B can be found by solving the equation \( 0 = -0.8h + 12 \).

2. The elevator reaches the ground when \( h = 0 \). Elevator A reaches ground level after 16 seconds because \( 0.8(0) + 16 = 16 \). Elevator B reaches ground level after 12 seconds because \( -0.8(0) + 12 = 12 \).

3. At 14 seconds both elevators are 2.5 meters below ground. The solution can be found by solving the equation \( 0.8h + 16 = -0.8h + 12 \).

4. Elevator A will reach you first, in 4.8 seconds, because \( 0.8(-14) + 16 = 4.8 \). Elevator B will reach your level in 23.2 seconds because \( -0.8(-14) + 12 = 23.2 \).

**Are You Ready for More?**

1. In a two-digit number, the ones digit is twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the number.

2. The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 less than the original number. Find the number.

3. The sum of the digits in a two-digit number is 8. The value of the number is 4 less than 5 times the ones digit. Find the number.

**Student Response**

1. \( 10x + 2x + 36 = 10(2x) + x \) or equivalent, where \( x \) represents the tens digit. \( x = 4 \) so the number is 48.

2. \( 10x + (11 - x) - 45 = 10(11 - x) + x \) or equivalent, where \( x \) represents the tens digit. \( x = 8 \) so the number is 83.

3. \( 10x + (8 - x) = 5(8 - x) - 4 \) or equivalent, where \( x \) represents the tens digit. \( x = 2 \) so the number is 26.

**Activity Synthesis**

This discussion should focus on the act of setting the two expressions equal and what that means in the context of the situation.

Consider asking the following questions:
• "If someone thought that the height of Elevator A before we started timing was 16 meters because they substituted 0 for the variable of the expression $0.8h + 16$ and got 16, how would you help them correct their answer?" (I would remind them that $h$ is height and $0.8h + 16$ is the time, so when we start timing at 0 that means $0.8h + 16 = 0$, not that $h = 0$.)

• "Which of the elevators in the image is A and which is B? How do you know?" (A is the elevator on the right since at time 0 the height is negative, while B is the elevator on the left since at time 0 the height is positive.)

• "How did you find the height when the travel times are equal?" (Since each expression gives the time to travel to a height $h$, I solved the equation $0.8h + 16 = -0.8h + 12$, which gives the value of $h$ when the two expressions are equal.)

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Support for Students with Disabilities

*Representation: Internalize Comprehension.* Use color and annotations to illustrate student thinking. As students describe their strategies, use color and annotations to scribe their thinking on a display of the problem so that it is visible for all students.

*Supports accessibility for: Visual-spatial processing; Conceptual processing*

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Lesson Synthesis

The work in this lesson is a prelude to a simple form of a system of equations, where each equation can be written in the form $y = $ some expression (though students do not need to know the term "system of equations" at this point).

Arrange students in groups of 2. Ask partners to think of another situations where two quantities are changing and they want to know when the quantities are equal. Give groups time to to discuss and write down a few sentences explaining their situation. Invite groups to share their situation with the class. (For example, in a race where participants walk at steady rates but the slower person has a head start, when will they meet?) Consider allowing groups to share their situation by making a picture, a graph, in words, or by acting it out.

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9.4 Printers and Ink

**Cool Down: 5 minutes**

**Addressing**

• 8.EE.C

**Student Task Statement**

To own and operate a home printer, it costs $100 for the printer and an additional $0.05 per page for ink. To print out pages at an office store, it costs $0.25 per page. Let $p$ represent number of pages.
1. What does the equation $100 + 0.05p = 0.25p$ represent?

2. The solution to that equation is $p = 500$. What does the solution mean?

**Student Response**

1. The equation represents when the cost for owning and operating a home printer is equal to the cost for printing at an office store.

2. The solution of $p = 500$ means that the costs are equal for printing 500 pages.

Cost to own and operate a home printer: $100 + 0.05(500) = 125$ dollars.

Cost to print out pages at an office store: $0.25(500) = 125$ dollars.

**Student Lesson Summary**

Imagine a full 1,500 liter water tank that sprays a leak, losing 2 liters per minute. We could represent the number of liters left in the tank with the expression $-2x + 1,500$, where $x$ represents the number of minutes the tank has been leaking.

Now imagine at the same time, a second tank has 300 liters and is being filled at a rate of 6 liters per minute. We could represent the amount of water in liters in this second tank with the expression $6x + 300$, where $x$ represents the number of minutes that have passed.

Since one tank is losing water and the other is gaining water, at some point they will have the same amount of water—but when? Asking when the two tanks have the same number of liters is the same as asking when $-2x + 1,500$ (the number of liters in the first tank after $x$ minutes) is equal to $6x + 300$ (the number of liters in the second tank after $x$ minutes),

$$-2x + 1,500 = 6x + 300.$$

Solving for $x$ gives us $x = 150$ minutes. So after 150 minutes, the number of liters of the first tank is equal to the number of liters of the second tank. But how much water is actually in each tank at that time? Since both tanks have the same number of liters after 150 minutes, we could substitute $x = 150$ minutes into either expression.

Using the expression for the first tank, we get $-2(150) + 1,500$ which is equal to $-300 + 1,500$, or 1,200 liters.

If we use the expression for the second tank, we get $6(150) + 300$, or just $900 + 300$, which is also 1,200 liters. That means that after 150 minutes, each tank has 1,200 liters.
Lesson 9 Practice Problems

Problem 1

Statement

Cell phone Plan A costs $70 per month and comes with a free $500 phone. Cell phone Plan B costs $50 per month but does not come with a phone. If you buy the $500 phone and choose Plan B, how many months is it until your cost is the same as Plan A’s?

Solution

25 months

Problem 2

Statement

Priya and Han are biking in the same direction on the same path.

a. Han is riding at a constant speed of 16 miles per hour. Write an expression that shows how many miles Han has gone after $t$ hours.

b. Priya started riding a half hour before Han. If Han has been riding for $t$ hours, how long has Priya been riding?

c. Priya is riding at a constant speed of 12 miles per hour. Write an expression that shows how many miles Priya has gone after Han has been riding for $t$ hours.

d. Use your expressions to find when Han and Priya meet.

Solution

a. $16t$ miles

b. $t + \frac{1}{2}$ hours

c. $12(t + \frac{1}{2})$

d. $t = \frac{3}{2}$. To find when Han and Priya meet, set the two expressions equal to one another:

$16t = 12(t + \frac{1}{2})$. They meet after Han rides for one and a half hours and Priya rides for two hours.

Problem 3

Statement

Which story matches the equation $-6 + 3x = 2 + 4x$?
A. At 5 p.m., the temperatures recorded at two weather stations in Antarctica are -6 degrees and 2 degrees. The temperature changes at the same constant rate, \( x \) degrees per hour, throughout the night at both locations. The temperature at the first station 3 hours after this recording is the same as the temperature at the second station 4 hours after this recording.

B. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn \( x \) points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

Solution

A

Problem 4

Statement
For what value of \( x \) do the expressions \( \frac{2}{3}x + 2 \) and \( \frac{4}{3}x - 6 \) have the same value?

Solution
\[ x = 12 \]

Problem 5

Statement
Decide whether each equation is true for all, one, or no values of \( x \).

a. \( 2x + 8 = -3.5x + 19 \)

b. \( 9(x - 2) = 7x + 5 \)

c. \( 3(3x + 2) - 2x = 7x + 6 \)

Solution

a. True for one value of \( x \).

b. True for one value of \( x \).

c. True for all values of \( x \).

(From Unit 4, Lesson 8.)

Problem 6

Statement
Solve each equation. Explain your reasoning.
\[3d + 16 = -2(5 - 3d) \quad 2k - 3(4 - k) = 3k + 4\]
\[
\frac{3y - 6}{9} = \frac{4 - 2y}{-3}
\]

**Solution**

a. \(d = \frac{26}{3}\). Explanations vary. Sample response: Distribute on the right side of the equation, add 10 to each side, subtract \(3d\) from each side, then divide each side by 3.

b. \(k = 8\). Explanations vary. Sample response: Distribute and combine like terms on the left side, subtract \(3k\) on each side, add 12 to each side, and then divide each side by 2.

c. \(y = 2\). Explanations vary. Sample response: Multiply each side by 9, distribute -3 on the right side, subtract \(3y\) on each side, add 12 to each side, and then divide each side by 3.

*(From Unit 4, Lesson 6.)*

**Problem 7**

**Statement**

Describe a rigid transformation that takes Polygon A to Polygon B.

![Polygon A and B](image)

**Solution**

Answers vary. Sample response: Rotate Polygon A 180 degrees around \((0, 0)\).

*(From Unit 1, Lesson 7.)*
Section: Systems of Linear Equations
Lesson 10: On or Off the Line?

Goals

• Determine (in writing) a point that satisfies two relationships simultaneously, using tables or graphs.
• Interpret (orally and in writing) points that lie on one, both, or neither line on a graph of two simultaneous equations in context.

Learning Targets

• I can identify ordered pairs that are solutions to an equation.
• I can interpret ordered pairs that are solutions to an equation.

Lesson Narrative

This lesson builds upon earlier work with linear equations in two variables in two types of contexts: contexts like distance versus time, where there is an initial value and a rate of change, and contexts like budgets, where there is an equation constraining the possible combinations of two quantities. In this lesson, students consider pairs of linear equations in each type of context and interpret the meaning of points on the graphs of the equations.

In the first activity, students are given two constraints on the number of nickels and dimes in someone’s pocket: a constraint on the total value and a constraint on the total number of coins. In the second activity, students study two graphs that represent two different students producing locker signs at different rates. In each case students interpret the meaning of various points on and off the lines, including the point of intersection.

Alignments

Addressing

• 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Building Towards

• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Instructional Routines

• Anticipate, Monitor, Select, Sequence, Connect
• MLR5: Co-Craft Questions
• MLR8: Discussion Supports
• Which One Doesn’t Belong?

**Student Learning Goals**
Let’s interpret the meaning of points in a coordinate plane.

## 10.1 Which One Doesn’t Belong: Lines in the Plane

**Warm Up: 5 minutes**
The purpose of this warm-up is to elicit ways students can describe different characteristics that arise when more than one line is graphed in a coordinate plane.

**Addressing**
- 8.EE.C

**Building Towards**
- 8.EE.C.8

**Instructional Routines**
• Which One Doesn't Belong?

**Launch**
Arrange students in groups of 2–4. Display the image of all four graphs for all to see. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the groups to offer at least one reason why each graph doesn’t belong.

**Student Task Statement**
Which one doesn't belong? Explain your reasoning.
Student Response

Answers vary. Possible responses:

- Graph A is the only one with no intersection points (they are all parallel), or, graph A is the only one that appears to be the same line translated vertically in two different ways.

- Graph B is the only one with an intersection point that has a negative coordinate or the only one with two lines.

- Graph C is the only one with three lines through a single point.

- Graph D is the only one with multiple intersection points.

ActivitySynthesis

After students have conferred in groups, invite each group to share one reason why a particular graph might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which graph does not belong, attend to students’ explanations and ensure the reasons given are correct. Encourage students to use concepts and language introduced in previous lessons about lines such as slope and intercepts. In particular, draw students’ attention to any intersections of the lines.
10.2 Pocket Full of Change

15 minutes
In previous lessons, students have set two expressions equal to one another to find a common value where both expressions are true (if it exists). A system of two equations asks a similar question: at what common pair of values are both equations true? In this activity, students focus on a context involving coins and use multiple representations to think about the context in different ways. The goal of this activity is not for students to write equations or learn the language “system of equations,” but rather investigate the mathematical structure with two stated facts using familiar representations and context while reasoning about what must be true.

Monitor for students using different representations, such as a graph, table, or words, as they solve the final problem to share during the whole-class discussion.

Addressing
- 8.EE.C.8

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch
Before looking at the task, tell students, “I have $2 in my pocket. What might be in my pocket?” Students will likely guess that you have two $1 bills, but ask what else it might be. Some answers could be 8 quarters; 200 pennies; a $2 bill; or 20 nickels, 2 quarters, and 5 dimes.

Read the problem context together. Ensure students understand that we know that Jada has exactly $2 in her pocket, that she only has quarters and dimes, and that she has exactly 17 coins. Give 1–2 minutes for students to read and complete the first problem. Display the table for all to see and ask students for values to fill in the table.

Give students 5–7 minutes of quiet work time to finish the remaining problems followed by a whole-class discussion.

Anticipated Misconceptions
Students may be confused about the last row of the table. Tell them they can enter any values that make sense in the context and are not already on the table.

Student Task Statement
Jada told Noah that she has $2 worth of quarters and dimes in her pocket and 17 coins all together. She asked him to guess how many of each type of coin she has.

1. Here is a table that shows some combinations of quarters and dimes that are worth $2. Complete the table.
<table>
<thead>
<tr>
<th>number of quarters</th>
<th>number of dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

2. Here is a graph of the relationship between the number of quarters and the number of dimes when there are a total of 17 coins.
   a. What does Point A represent?
   b. How much money, in dollars, is the combination represented by Point A worth?

3. Is it possible for Jada to have 4 quarters and 13 dimes in her pocket? Explain how you know.

4. How many quarters and dimes must Jada have? Explain your reasoning.

**Student Response**
1. (0, 20); (4, 10); (8, 0); (6, 5); (2, 15)
2. a. A represents 8 quarters and 9 dimes.
   b. These coins would be worth $2.90, because $8 \cdot 0.25 + 9 \cdot 0.10 = 2.90$. 
3. No. Even though 4 quarters and 13 dimes is 17 coins, they are not worth $2. (They are worth $2.30.)

4. 2 quarters and 15 dimes. This is 17 coins that are worth $2 because
   \[ 2 \times 0.25 + 15 \times 0.10 = 2.00. \] It is the only combination of quarters and dimes that appears both in the table and as a point on the graph.

**Activity Synthesis**

The goal of this discussion is to focus students on what must be true based on the two facts they know: that the coins total $2 and that there are 17 coins. Begin the discussion by asking students about some things they know cannot be true about the coins in Jada's pocket and how they know. Students may respond that Jada cannot have 20 dimes and 0 quarters because that is not 17 coins or other variations where either the coins do not total $2, there are not exactly 17 coins, or neither are true.

Select previously identified students to share how they answered the last problem. If possible, begin a sequence with a student who added on to the table representation to figure out the solution, followed by one who added onto the graph. Include any students who wrote out their reasoning in words last. After each student shares, connect to the idea that their solution is one where both facts are true.

**Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using physical objects to represent abstract concepts. Students may benefit from access to coins (or paper copies of coins) to use as representations to visualize multiple combinations that add up to $2.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

**Support for English Language Learners**

*Speaking, Conversing: MLR8 Discussion Supports.* Use this routine to prepare students for the whole-class discussion. Give students 2 minutes of quiet think time to consider, “What are some things you know cannot be true about the coins in Jada's pocket? How do you know?” before inviting them to share their thinking with a partner. Encourage students to ask each other clarifying questions such as, “Can you describe that a different way?”, or “How do you know this cannot be true?” This will provide students with an opportunity to clarify their thinking before taking part in a whole-class discussion.

*Design Principle(s): Optimize output; Cultivate conversation*
10.3 Making Signs

10 minutes
In the previous activity, the system of equations was represented in words, a table, and a graph. In this activity, the system of equations is partially given in words, but key elements are only provided in the graph. Students have worked with lines that represent a context before. Now they must work with two lines at the same time to determine whether a point lies on one line, both lines, or neither line.

Addressing
- 8.EE.C.8

Instructional Routines
- MLR5: Co-Craft Questions

Launch
Arrange students in groups of 2. Give students 1 minute to read the problem and answer any questions they have about the context. Tell students to complete the table one row at a time with one person responding for Clare and the other responding for Andre. Give students 2–3 minutes to finish the table followed by whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students within the first 2–3 minutes of work time to ensure that they have understood the directions. If students are unsure how to begin, suggest that they consider each statement for Clare first, and then for Andre.

Supports accessibility for: Organization; Attention
Support for English Language Learners

Writing, Speaking: MLR5 Co-Craft Questions. Display the graph that shows Clare and Andre’s progress of making signs, without showing the table. Ask students to write down possible mathematical questions that can be asked about the situation, and then share with a partner. As students discuss, listen for and amplify questions that notice and wonder about the points A, B, C, and D; the location of these points; and why the points were possibly highlighted and what it means in context of the situation. Finally, reveal the table from the activity or use questions generated by the students and allow time to work on them. This will help students make sense of a graph with two lines at the same time by generating questions using mathematical language.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

Clare and Andre are making signs for all the lockers as part of the decorations for the upcoming spirit week. Yesterday, Andre made 15 signs and Clare made 5 signs. Today, they need to make more signs. Each person’s progress today is shown in the coordinate plane.

Based on the lines, mark the statements as true or false for each person.
<table>
<thead>
<tr>
<th>point</th>
<th>what it says</th>
<th>Clare</th>
<th>Andre</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>At 40 minutes, I have 25 signs completed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>At 75 minutes, I have 42 and a half signs completed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>At 0 minutes, I have 15 signs completed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>At 100 minutes, I have 60 signs completed.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Student Response**

**Are You Ready for More?**
- 4 toothpicks make 1 square
- 7 toothpicks make 2 squares
- 10 toothpicks make 3 squares

Do you see a pattern? If so, how many toothpicks would you need to make 10 squares according to your pattern? Can you represent your pattern with an expression?

**Student Response**
For each new square you will need 3 toothpicks. If you want to make x squares you will need 3x + 1 toothpicks. So 10 squares needs (3 \* 10) + 1 = 31 toothpicks.

**Activity Synthesis**
Display the graphs from the task statement. The goal of this discussion is for students to realize that points that lie on one line can be interpreted as statements that are true for Clare, and points that lie on the other line as statements that are true for Andre. Ask students:

- “What is true for Clare and Andre after 20 minutes?” (Clare has 15 signs completed and Andre has 20 signs completed.)
• “What, then, do you know about the point (20, 15) and the equation for Clare’s graph?” (The point (20, 15) is a solution to the equation for Clare’s graph.)

• “What do you know about the point (20, 20) and the equation for Andre’s graph?” (The point (20, 20) is a solution to the equation for Andre’s graph.)

Invite groups to share their reasoning about points A–D. Conclude by pointing out to students that, in this context, there are many points true for Clare and many points true for Andre but only one point true for both of them. Future lessons will be about how to figure out that point.

Lesson Synthesis

Tell students to think about how they found the ordered pair that makes two relationships true using tables and graphs today. Ask:

• “What are some advantages of tables? If you used two tables to describe the two relationships, how would you know whether a common point exists? If it did exist, how would you find it?” (Tables are good for knowing the exact values for individual points. If the common point is listed in each table, it may be easy to notice, but it may be missing from at least one table or difficult to find if the tables are large and unordered. If the common point is listed in each table, one row of the table should match in both columns.)

• “What are some advantages of graphs?” (Graphs give a better overall picture of the relationships and usually makes estimating (if not finding exactly) the common point easier.)

• “When using a graph, where are the points whose coordinates do not make a given relationship true? Do the coordinates of those points show up in a table of values?” (Points that are off of the line do not make the given relationship true. They can be above or below the line. The coordinates of these points do not show up in a table representing the given relationship.)

If time allows, invite students to make up their own stories with two quantities and two relationships to swap with a partner. Have each partner create either two tables of values, two graphs, or one of each to describe the situation and answer a question about the values of the two quantities that make both relationships true.

10.4 Another Pocket Full of Change

Cool Down: 5 minutes
This cool-down assesses whether students understand how to read graphs and interpret points on lines as pairs of values that make given relationships true.

Addressing

• 8.EE.C.8

Student Task Statement
On the coordinate plane shown, one line shows combinations of dimes and quarters that are worth $3. The other line shows combinations of dimes and quarters that total to 12 coins.
1. Name one combination of 12 coins shown on the graph.

2. Name one combination of coins shown on the graph that total to $3.

3. How many quarters and dimes would you need to have both 12 coins and $3 at the same time?

Student Response


3. 12 quarters and 0 dimes.

Student Lesson Summary

We studied linear relationships in an earlier unit. We learned that values of $x$ and $y$ that make an equation true correspond to points $(x, y)$ on the graph. For example, if we have $x$ pounds of flour that costs $0.80 per pound and $y$ pounds of sugar that costs $0.50 per pound, and the total cost is $9.00, then we can write an equation like this to represent the relationship between $x$ and $y$:

$$0.8x + 0.5y = 9$$

Since 5 pounds of flour costs $4.00 and 10 pounds of sugar costs $5.00, we know that $x = 5$, $y = 10$ is a solution to the equation, and the point $(5, 10)$ is a point on the graph. The line shown is the graph of the equation:
Notice that there are two points shown that are not on the line. What do they mean in the context? The point \((1, 14)\) means that there is 1 pound of flour and 14 pounds of sugar. The total cost for this is \(0.8 \cdot 1 + 0.5 \cdot 14\) or \$7.80. Since the cost is not \$9.00, this point is not on the graph. Likewise, 9 pounds of flour and 16 pounds of sugar costs \(0.8 \cdot 9 + 0.5 \cdot 16\) or \$15.20, so the other point is not on the graph either.

Suppose we also know that the flour and sugar together weigh 15 pounds. That means that

\[ x + y = 15 \]

If we draw the graph of this equation on the same coordinate plane, we see it passes through two of the three labeled points:

The point \((1, 14)\) is on the graph of \(x + y = 15\) because \(1 + 14 = 15\). Similarly, \(5 + 10 = 15\). But \(9 + 16 \neq 15\), so \((9, 16)\) is not on the graph of \(x + y = 15\). In general, if we have two lines in the coordinate plane,

- The coordinates of a point that is on both lines makes both equations true.
- The coordinates of a point on only one line makes only one equation true.
• The coordinates of a point on neither line make both equations false.

Lesson 10 Practice Problems
Problem 1

Statement

a. Match the lines \( m \) and \( n \) to the statements they represent:

\[ \begin{array}{c}
\text{i. A set of points where the coordinates of each point have a sum of 2} \\
\text{ii. A set of points where the } y \text{-coordinate of each point is 10 less than its } x \text{-coordinate}
\end{array} \]

b. Match the labeled points on the graph to statements about their coordinates:

\[ \begin{array}{c}
\text{i. Two numbers with a sum of 2} \\
\text{ii. Two numbers where the } y \text{-coordinate is 10 less than the } x \text{-coordinate} \\
\text{iii. Two numbers with a sum of 2 and where the } y \text{-coordinate is 10 less than the } x \text{-coordinate}
\end{array} \]

Solution

a. \( n \)

i. \( m \)

b. \( A, B, C \)

i. \( D, B, E \)

ii. \( B \)
Problem 2

Statement
Here is an equation: $4x - 4 = 4x + __$. What could you write in the blank so the equation would be true for:

- a. No values of $x$
- b. All values of $x$
- c. One value of $x$

Solution

a. Answers vary. Sample response: $19. 4x - 4 = 4x + 19$ has no solutions.

b. Answers vary. Sample response: $-4. 4x - 4 = 4x - 4$ is true for all values of $x$.

c. Answers vary. Sample response: $4x. 4x - 4 = 4x + 4x$ has one solution ($x = -1$).

(From Unit 4, Lesson 7)

Problem 3

Statement
Mai earns $7 per hour mowing her neighbors’ lawns. She also earned $14 for hauling away bags of recyclables for some neighbors.

Priya babysits her neighbor’s children. The table shows the amount of money $m$ she earns in $h$ hours. Priya and Mai have agreed to go to the movies the weekend after they have earned the same amount of money for the same number of work hours.

<table>
<thead>
<tr>
<th>$h$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8.40</td>
</tr>
<tr>
<td>2</td>
<td>$16.80</td>
</tr>
<tr>
<td>4</td>
<td>$33.60</td>
</tr>
</tbody>
</table>

a. How many hours do they each have to work before they go to the movies?

b. How much will each of them have earned?

c. Explain where the solution can be seen in tables of values, graphs, and equations that represent Priya’s and Mai’s hourly earnings.

Solution

a. 10 hours

b. $84
c. Explanations vary. Sample response: In a table of values for each person, we would see the same entry for h and m in both tables. In the graph, the solution is found in the coordinates of the point (h, m) where the graphs of the two relationships intersect. In the equations, it is the value of h when we set the two expressions for m equal to each other: 8.4h = 7h + 14.

Problem 4

Statement

For each equation, explain what you could do first to each side of the equation so that there would be no fractions. You do not have to solve the equations (unless you want more practice).

\[
\begin{align*}
\text{a. } & \frac{3x - 4}{8} = \frac{x + 2}{3} \\
\text{b. } & \frac{3(2 - r)}{4} = \frac{3 + r}{6}
\end{align*}
\]

Solution

a. Explanations vary. Sample response: If you multiply each side by 24 (the least common multiple of 8 and 3), then the equation becomes 3(3x - 4) = 8(x + 2). (The solution is x = 28, for those who go the extra mile)

b. Explanations vary. Sample response: If you multiply each side by 12 (the least common multiple of 6 and 4), then the equation becomes 9(2 - r) = 2(3 + r). (The solution is r = \frac{12}{11}, for those who go the extra mile)

c. Explanations vary. Sample response: If you multiply each side by 8 (the least common multiple of 8 and 4), then the equation becomes 4p + 3 = 2(p + 2). (The solution is p = \frac{1}{2}, for those who go the extra mile)

d. Explanations vary. Sample response: If you multiply each side by 30 (the least common multiple of 6 and 15), then the equation becomes 4(a - 7) = 5(a + 4). (The solution is a = -48, for those who go the extra mile)

(From Unit 4, Lesson 6.)

Problem 5

Statement

The owner of a new restaurant is ordering tables and chairs. He wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

a. Describe some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers. Explain how you found them.
b. Write an equation to represent the situation. What do the variables represent?

c. Create a graph to represent the situation.

\[
\begin{array}{c}
\text{y} \\
\hline
\text{x}
\end{array}
\]

d. What does the slope tell us about the situation?

e. Interpret the x and y intercepts in the situation.

**Solution**

a. No 2-seat and 30 4-seat, 10 2-seat and 25 4-seat, 40 2-seat and 10 4-seat. Explanations vary. Sample response: I decided on a number for the 2-seat tables, then figured out how many people that would be (multiply number of tables by 2) and subtracted that from 120. Then I divided by 4 to get the number of 4-seat tables needed for the remaining people.

b. Answers vary. Sample response: \(2x + 4y = 120\). \(x\) represents the number of 2-seat tables and \(y\) represents the number of 4-seat tables.

c. Graph is the line connecting \((0, 30)\) and \((60, 0)\).

d. Answers vary. Sample response. The slope is \(-\frac{1}{2}\). \(-\frac{1}{2}\) tells us that for every one fewer 4-seat table we can use 2 2-seat tables.

e. The intercepts are \((0, 30)\) and \((60, 0)\). They tell us how many tables there will be if only 4-seat tables are used (30) or only 2-seat tables are used (60).

(From Unit 3, Lesson 14.)
Lesson 11: On Both of the Lines

Goals

• Create a graph that represents two linear relationships in context, and interpret (orally and in writing) the point of intersection.

• Interpret a graph of two equivalent lines in context.

Learning Targets

• I can use graphs to find an ordered pair that two real-world situations have in common.

Lesson Narrative

For the next several lessons, students will study systems of linear equations where the context is of the distance-versus-time variety, where there is an initial value and a rate of change. The equations in the system are in the form $y = mx + b$. Such contexts are useful in thinking about the meaning of the solution to the system (the time when two quantities are equal). The purpose of this lesson is to introduce students to the graphical interpretation of such systems. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.

The first activity after the warm-up focuses on the solution by drawing a graph for a ladybug’s motion, marking the point on the graph where an ant and a ladybug meet, and asking the student to fill in the graph of the ant. The second activity draws attention to systems which have infinitely many solutions because the graphs of the equations are identical. This is interpreted as two runners staying together for the entire duration of a race.

Alignments

Addressing

• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Building Towards

• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Instructional Routines

• MLR2: Collect and Display

• MLR6: Three Reads

• Notice and Wonder

Required Materials

Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.
**Required Preparation**

Provide students with access to straightedges for drawing accurate lines.

**Student Learning Goals**

Let's use lines to think about situations.

### 11.1 Notice and Wonder: Bugs Passing in the Night

**Warm Up: 10 minutes**

The purpose of this warm-up is to get students to think about a context that will be explored in the following activity and to reason about the speed, distance, and time each animal is traveling in relation to one another. In the next activity, students will write equations for the bugs and graph these relationships.

**Building Towards**

- B.EE.C.8

**Instructional Routines**

- Notice and Wonder

**Launch**

Tell students they will see a picture that shows a ladybug and ant traveling for 6 seconds. Tell students to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something.

**Student Task Statement**

What do you notice? What do you wonder?

- Ladybug start
- 0 seconds
- 2 seconds
- 4 seconds
- 6 seconds
Student Response
Things students might notice:

• The ladybug is moving from left to right and the ant is moving from right to left.
• The ladybug and the ant are each moving at a constant speed.
• The ladybug is moving 8 units every 2 seconds and the ant is moving 16 units every 2 seconds.
• In the picture at 6 seconds, the ant is no longer visible in the picture.
• At some time in between the 2 second picture and the 4 second picture, they pass each other.

Things students might wonder:

• Where did the ant go in the last picture?
• At what time did they pass each other?
• At what tick mark did they pass each other?
• Did they wave as they passed each other?

Activity Synthesis
Ask students to share things they noticed and wondered. Record and display their responses for all to see. If any students remember a similar representation from an earlier unit where both the ladybug and the ant started on the same side, you may wish to include how this situation is similar and how it is different.

Important ideas to highlight during the discussion:

• The bugs are moving in opposite directions and at some time in between $t = 2$ and $t = 4$, they pass each other.
• The bugs are moving at a constant speed.
• The ant is moving faster than the ladybug.

11.2 Bugs Passing in the Night, Continued

10 minutes
In this task, students find and graph a linear equation given only the graph of another equation, information about the slope, and the coordinates where the lines intersect. The purpose of this task is to check student understanding about the point of intersection in relationship to the context while applying previously learned skills of equation writing and graphing.

Identify students who use different strategies to answer the first problem to share during the whole-class discussion. For example, some students may reason about the equation from an
algebraic perspective while others may start by drawing in the graph for the ant based on the provided information. Also, make note of what strategy is most common among students.

**Addressing**
- 8.EE.C.8

**Instructional Routines**
- MLR2: Collect and Display

**Launch**
Display the graph from the task statement. Tell students that this activity is about a different ant and ladybug from the warm-up, and we are going to think about their distances using a coordinate plane. Give 4–6 minutes for students to complete the problems followed by a whole-class discussion.

**Support for Students with Disabilities**

_Representation: Internalize Comprehension._ Activate or supply background knowledge about equation writing and graphing. Incorporate explicit opportunities for review and practice if necessary.  
_Supports accessibility for: Memory; Conceptual processing_

**Student Task Statement**
A different ant and ladybug are a certain distance apart, and they start walking toward each other. The graph shows the ladybug’s distance from its starting point over time and the labeled point (2.5, 10) indicates when the ant and the ladybug pass each other.
The ant is walking 2 centimeters per second.

1. Write an equation representing the relationship between the ant’s distance from the ladybug’s starting point and the amount of time that has passed.

2. If you haven’t already, draw the graph of your equation on the same coordinate plane.

**Student Response**

1. \( d = -2t + 15 \) or \( d = 15 - 2t \). In 2.5 seconds the ant will have walked 5 centimeters \((2 \cdot 2.5 = 5)\). Therefore, the ant started a distance of 15 centimeters away from the ladybug \((10 + 5 = 15)\). This yields the equation \( d = -2t + 15 \) or \( d = 15 - 2t \).

2.
**Activity Synthesis**

The purpose of this discussion is to ensure all students understand both how the labeled point in the task statement relates to the context and how to write and graph an equation from the given information.

Select previously identified students to share their strategy for the first problem, starting with the most common strategy used in the class. Record and display in one place the equations students write. While students may come up with equations like \(-2 = \frac{d-10}{t-2.5}\), let them know that this approach is valid, but that an equation of the form \(d = -2t + 15\) will be easier to work with today. Ask each student who shares how they knew to use the point \((2.5, 10)\) when making the equation for the ant.

If students struggled to graph the ant's path, you may wish to conclude the discussion by asking students for different ways to add the graph of the ant's distance onto the coordinate plane. For example, some students may say to use the equation figured out in the first problem to plot points and then draw a line through them. Other students may suggest starting from the known point, \((2.5, 10)\), and “working backwards” to figure out that 1 second earlier at 1.5 seconds, the ant would have to be 12 centimeters away since 10 + 2 = 12.
Support for English Language Learners

Speaking, Listening: MLR2 Collect and Display. Before the whole-class discussion, give groups the opportunity to discuss the first question. Circulate through the room and record the language students use to talk about the situation. Listen for words or phrases such as “rate of change,” “ordered pair,” “increasing/decreasing,” and “initial value.” Organize and group similar strategies in the display for students to reference throughout the lesson. For example, group strategies that reference the point (2.5, 10) and strategies that involved working backwards from the graph to write the equation. This will help students solidify their understanding about how to write an equation using the point of intersection on a graph.

Design Principle(s): Support sense-making

11.3 A Close Race

15 minutes
In previous lessons, students encountered equations with a single variable that had infinitely many solutions. In this activity, students interpret a situation with infinitely many solutions. A race is described using different representations (a table and a description in words). Students graph the relationships given by the descriptions and notice that the lines overlap so that both relationships are true for any pair of values along the graphed line.

Addressing
- 8.EE.C.8

Instructional Routines
- MLR6: Three Reads

Launch
Allow students 7–10 minutes of silent work time followed by a whole-class discussion.

Support for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of Elena and Jada’s bike race scenario with the table. In the first read, students read the problem with the goal of comprehending the situation (e.g., Elena and Jada are racing their bikes.). In the second read, ask students to look for quantities represented (e.g., total distance is 100 meters, Jada rode 36 meters in 6 seconds, etc.). In the third read, ask students to brainstorm possible strategies to answer the question, “Who won the race?” This will help students interpret a situation in which there are infinitely many solutions, given a description and table.

Design Principle(s): Support sense-making; Maximize meta-awareness
**Student Task Statement**

Elena and Jada were racing 100 meters on their bikes. Both racers started at the same time and rode at constant speed. Here is a table that gives information about Jada’s bike race:

<table>
<thead>
<tr>
<th>time from start (seconds)</th>
<th>distance from start (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>54</td>
</tr>
</tbody>
</table>

1. Graph the relationship between distance and time for Jada’s bike race. Make sure to label and scale the axes appropriately.

2. Elena traveled the entire race at a steady 6 meters per second. On the same set of axes, graph the relationship between distance and time for Elena’s bike race.

3. Who won the race?

**Student Response**

1. The line \( d = 6t \) should be graphed. The horizontal axis should have the title “time (seconds)” and the vertical axis should have the title “distance (meters).”

2. Elena’s graph is the same line as Jada’s.

3. The race ends in a tie since both racers traveled the same distance at the same speed.
Activity Synthesis

The key point for discussion is to connect what students observed about the graph they made to the concept of “infinitely many solutions” encountered in earlier lessons. Graphically, students see that there are situations where two lines align “on top of each other.” We can interpret each point on the line as representing a solution to both Elena’s and Jada’s equations. If $t$ is the time from start and $y$ is the distance from the start, the equation for Elena is $y = 6t$. The equation for Jada is also $y = 6t$. Every solution to Elena’s equation is also a solution to Jada’s equation, and every solution to Jada’s equation is also a solution to Elena’s equation. In this way, there are infinitely many points that are solutions to both equations at the same time.

Ensure students clearly understand that just because there are infinitely many points that are solutions, it does not mean that any pair of values will solve both Elena’s and Jada’s equations. In this example, the pair of values must still be related by the equation $y = 6x$. So, pairs of values like $(1, 6)$, $(10, 60)$, and $(\frac{1}{2}, 3)$ are all solutions, but $(1, 8)$ is not.

Lesson Synthesis

Display a set of axes for all to see. Ask each question one at a time, allowing students time to work through each problem. As students share their responses, add graphs of the lines described to the axes.

- “A line goes through the point $(2, 5)$ and has a slope of 1.5. What is an equation for this line?”
  ($y = 1.5x + 2$)
- “A second line goes through the point $(2, 5)$ and has a $y$-intercept of $(0, 10)$. What is an equation for this line?” ($y = -2.5x + 10$)
- “What does the point $(2, 5)$ represent for these lines?” (The pair of values that is true in both situations.)
- “A third line goes through this same point. How would that show up in a table representing the relationship for the third line?” (The number 2 would be in the $x$ column right next to the number 5 in the $y$ column.)

11.4 Saving Cash

Cool Down: 5 minutes

Addressing

- 8.EE.C.8

Student Task Statement

Andre and Noah started tracking their savings at the same time. Andre started with $15 and deposits $5 per week. Noah started with $2.50 and deposits $7.50 per week. The graph of Noah’s savings is given and his equation is $y = 7.5x + 2.5$, where $x$ represents the number of weeks and $y$ represents his savings.
Write the equation for Andre's savings and graph it alongside Noah's. What does the intersection point mean in this situation?

Student Response

In this situation, the intersection at (5, 40) means that after 5 weeks, Noah and Andre each have $40.
Student Lesson Summary

The solutions to an equation correspond to points on its graph. For example, if Car A is traveling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has traveled from the rest area after $t$ hours is

$$d = 75t$$

The point $(2, 150)$ is on the graph of this equation because $150 = 75 \cdot 2$: two hours after passing the rest area, the car has traveled 150 miles.

If you have two equations, you can ask whether there is an ordered pair that is a solution to both equations simultaneously. For example, if Car B is traveling towards the rest area and its distance from the rest area is

$$d = 14 - 65t$$

We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is “yes”, then the solution will correspond to a point that is on both lines.

Looking at the coordinates of the intersection point, we see that Car A and Car B will both be 7.5 miles from the rest area after 0.1 hours (which is 6 minutes).

Now suppose another car, Car C, had also passed the rest stop at time $t = 0$ and traveled in the same direction as Car A, also going 75 miles per hour. Its equation would also be $d = 75t$. Any solution to the equation for Car A would also be a solution for Car C, and any solution to the equation for Car C would also be a solution for Car A. The line for Car C would land right on top of the line for Car A. In this case, every point on the graphed line is a solution to both equations, so that there are infinitely many solutions to the question “when are Car A and Car C the same distance from the rest stop?” This would mean that Car A and Car C were side by side for their whole journey.

When we have two linear equations that are equivalent to each other, like $y = 3x + 2$ and $2y = 6x + 4$, we will get two lines that are “right on top” of each other. Any solution to one equation is also solution to the other, so these two lines intersect at infinitely many points.
Lesson 11 Practice Problems

Problem 1

Statement
Diego has $11 and begins saving $5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they have the same amount of money? How much do they have at that time?

Solution
After 7 weeks, $46

Problem 2

Statement
Use a graph to find \( x \) and \( y \) values that make both \( y = \frac{2}{3} x + 3 \) and \( y = 2x - 5 \) true.

Solution
\((3, 1)\)

Problem 3

Statement
The point where the graphs of two equations intersect has \( y \)-coordinate 2. One equation is \( y = -3x + 5 \). Find the other equation if its graph has a slope of 1.
Solution

\[ y = x + 1, 2 = -3x + 5 \] is true when \( x = 1 \), so the line needed has a slope of 1 and contains the point (1, 2).

Problem 4

Statement

A farm has chickens and cows. All the cows have 4 legs and all the chickens have 2 legs. All together, there are 82 cow and chicken legs on the farm. Complete the table to show some possible combinations of chickens and cows to get 82 total legs.

<table>
<thead>
<tr>
<th>number of chickens (x)</th>
<th>number of cows (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Here is a graph that shows possible combinations of chickens and cows that add up to 30 animals:
If the farm has 30 chickens and cows, and there are 82 chicken and cow legs all together, then how many chickens and how many cows could the farm have?

Solution

<table>
<thead>
<tr>
<th>number of chickens (x)</th>
<th>number of cows (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>21</td>
<td>10</td>
</tr>
<tr>
<td>19</td>
<td>11</td>
</tr>
<tr>
<td>31</td>
<td>5</td>
</tr>
</tbody>
</table>

The farm could have 19 chickens and 11 cows.

(From Unit 4, Lesson 10.)
Lesson 12: Systems of Equations

Goals

• Comprehend that solving a system of equations means finding values of the variables that makes both equations true at the same time.

• Coordinate (orally and in writing) graphs of parallel lines and a system of equations that has no solutions.

• Create a graph of two lines that represents a system of equations in context.

Learning Targets

• I can explain the solution to a system of equations in a real-world context.

• I can explain what a system of equations is.

• I can make graphs to find an ordered pair that two real-world situations have in common.

Lesson Narrative

This lesson formally introduces the concept of system of equations with different contexts. Students recognize that they have found solutions to systems of equations using graphing in the past few lessons by examining the intersection of graphed lines. The next activity introduces students to a system that has no solution and asks them to recognize this by connecting the concepts of parallel lines having no intersection points to the algebraic representations having no common solution.

Alignments

Addressing

• 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

• 8.EE.C.8.a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

• 8.EE.C.8.b: Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

Instructional Routines

• MLR2: Collect and Display

• MLR7: Compare and Connect

• Think Pair Share
Required Materials

Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation
Provide access to straightedges for students to accurately draw graphs of lines.

Student Learning Goals
Let’s learn what a system of equations is.

12.1 Milkshakes

Warm Up: 5 minutes
In the warm-up, students are given a situation and asked to describe the graph without actually graphing the lines. Identify students who correctly use mathematically correct terminology such as y-intercept, slope, x-intercept, and intersection to describe the graph.

Addressing
• 8.EE.C.8

Instructional Routines
• Think Pair Share

Launch
Arrange students in groups of 2. Give 3 minutes quiet work time followed by brief partner discussion for the last question.

Student Task Statement
Diego and Lin are drinking milkshakes. Lin starts with 12 ounces and drinks $\frac{1}{4}$ ounce per second. Diego starts with 20 ounces and drinks $\frac{2}{3}$ ounce per second.

1. How long will it take Lin and Diego to finish their milkshakes?

2. Without graphing, explain what the graphs in this situation would look like. Think about slope, intercepts, axis labels, units, and intersection points to guide your thinking.

3. Discuss your description with your partner. If you disagree, work to reach an agreement.

Student Response
1. Lin will take 48 seconds and Diego will take 30 seconds.

2. The horizontal axis is labeled “time (seconds)” and the vertical axis is labeled “amount of milkshake left (ounces).” The line for Lin’s graph starts at (0, 12) and decreases to the right to...
the point \((48, 0)\). The line for Diego’s graph starts at \((0, 20)\) and decreases more steeply than Lin’s line to the point \((30, 0)\). The two lines intersect somewhere.

3. No response needed.

**Activity Synthesis**

The purpose of the discussion is for students to practice describing graphs in words using correct mathematical terminology.

Select previously identified students to share their descriptions of the graphs. After each student shares, ask the class if the description is clear and to identify any vocabulary they heard that made the description precise or any vocabulary that is unclear. If any vocabulary needs to be reinforced for student understanding, this is a good time to discuss these words.

**12.2 Passing on the Trail**

20 minutes (there is a digital version of this activity)

In this activity, students start with an equation relating distance and time for Han’s hike and enough information to write a second equation relating distance and time for Jada’s hike. After writing Jada’s equation and graphing both lines, students then use the lines to identify the point of intersection and make sense of the point’s meaning in the context.

This activity is a culmination of student’s work writing, solving, and graphing equations along with the thinking they have done on what it means for an equation to be true. From this foundation, students are ready to understand solving systems of equations from an algebraic standpoint in the following lessons. Fluently solving systems algebraically is not expected at this time.

**Addressing**

- 8.EE.C.8.a

**Instructional Routines**

- MLR2: Collect and Display

**Launch**

Provide students with access to straightedges. Keep students in groups of 2.

Read the context of the problem with the students to help them understand the situation. Consider asking these questions to help them understand:

- “When \(t\) is zero, where is Han? Where is Jada?” (Han is at the lake. Jada is 0.6 miles from the parking lot.)

- “For times shortly after 0, is \(d\) decreasing or increasing for Han? Is \(d\) decreasing or increasing for Jada?” (Decreasing for Han and increasing for Jada.)
Give 2–3 minutes quiet work time and ask students to pause after they have completed the first problem to discuss their equation with a partner before starting to graph the equations. Give 5–7 minutes for students to complete the remaining problems with their partners followed by a whole-class discussion.

**Support for Students with Disabilities**

_Representation: Internalize Comprehension._ Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.

_supports accessibility for: Language; Conceptual processing_

**Student Task Statement**

There is a hiking trail near the town where Han and Jada live that starts at a parking lot and ends at a lake. Han and Jada both decide to hike from the parking lot to the lake and back, but they start their hikes at different times.

At the time that Han reaches the lake and starts to turn back, Jada is 0.6 miles away from the parking lot and hiking at a constant speed of 3.2 miles per hour towards the lake. Han’s distance, \( d \), from the parking lot can be expressed as \( d = -2.4t + 4.8 \), where \( t \) represents the time in hours since he left the lake.

1. What is an equation for Jada’s distance from the parking lot as she heads toward the lake?

2. Draw both graphs: one representing Han’s equation and one representing Jada’s equation. It is important to be very precise! Be careful, work in pencil, and use a ruler.
3. Find the point where the two graphs intersect each other. What are the coordinates of this point?

4. What do the coordinates mean in this situation?

5. What has to be true about the relationship between these coordinates and Jada's equation?

6. What has to be true about the relationship between these coordinates and Han's equation?

**Student Response**

1. $d = 3.2t + 0.6$ or equivalent.

2.
3. \((0.75, 3)\)

4. 0.75 hours or 45 minutes after Han left the lake, he passed Jada on the trail. This happened at a distance of 3 miles from the parking lot.

5. These values of \(t\) and \(d\) make Jada’s equation true.

6. These values of \(t\) and \(d\) also make Han’s equation true.

**Activity Synthesis**

The purpose of this discussion is to strengthen the connection between graphs and equations and formally introduce the vocabulary for *systems of equations*.

Begin the discussion by asking groups to share their responses for the last three questions. Ask students how they could check to make sure the coordinates they found with the graph are correct and give a brief quiet think time before selecting students to share their strategies. While students may suggest ideas like checking to make sure the lines are graphed correctly, it is important to point out that graphs are not always perfect. If not brought up by students, mention that a better strategy would be to substitute the coordinate values in for the variables to see if those values make both equations true.

Display a graph of the two equations for all to see alongside the system of equations:

\[
\begin{align*}
d &= -2.4t + 4.8 \\
d &= 3.2t + 0.6
\end{align*}
\]

Explain to students that this is called a *system of equations*, and “solving a system of equations” means to find the values of the variables that make both equations true at the same time. Point out that in this problem, the solution to the system of equations is the point where Han and Jada are at the same distance from the parking lot at the same time (point to \((0.75, 3)\)). Tell students that it is
also possible to solve a system of equations without graphing. To find when Jada and Han are the
same distance away from the parking lot, we can set the expression for Jada’s distance equal to the
expression for Han’s distance to get the equation \(-2.4t + 4.8 = 3.2t + 0.6\), which is an equation that
can be solved for \(t\). Ask students to solve this equation and confirm that \(t = 0.75\), the is the same
value they found earlier by carefully graphing the lines of each equation. Emphasize that the
intersection point gave a value of both \(t\) and \(d\), so it is important when solving algebraically to
substitute \(t\) back into one of the equations to find the value for \(d\). Since Han and Jada are at the
same distance from the parking lot when \(t = 0.75\), it doesn’t matter which equation is used to find
the value of \(d\).

**Support for English Language Learners**

*Speaking, Listening: MLR2 Collect and Display.* Listen for and record language students use to
discuss the question “How can you check to make sure the coordinates found are correct?”
Organize and group similar strategies in the display for students to refer back to throughout
the lesson. For instance, group strategies that focus on checking whether the lines are graphed
accurately and strategies that refer to substituting the coordinate points for the variables.
Emphasize words or phrases referring to these strategies such as “rate of change,” “slope,”
“initial value,” “y-intercept,” “substitute,” and “both equations are true.” Use this opportunity to
introduce the term “system of equations.” This will help students solidify their understanding
on how to check their solutions to a system of equations.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

### 12.3 Stacks of Cups

**10 minutes (there is a digital version of this activity)**

Students explore a system of equations with no solutions in the familiar context of cup stacking.
The context reinforces a discussion about what it means for a system of equations to have no
solutions, both in terms of a graph and in terms of the equations (MP2). Over the next few lessons,
the concept of one solution, no solutions, and infinitely many solutions will be abstracted to
problems without context. In those situations, it may be useful to refer back to the context in this
activity and others as a way to guide students towards abstraction.

**Addressing**

- 8.EE.C.8.b

**Instructional Routines**

- MLR7: Compare and Connect

**Launch**

5–7 minutes of quiet work time followed by a whole-class discussion.
Support for Students with Disabilities

*Representation: Internalize Comprehension.* Check in with students after the first 2–3 minutes of work time. Check to make sure students have attended to all parts of the graph and have selected an appropriate scale.

*Supports accessibility for: Conceptual processing; Organization*

**Student Task Statement**

A stack of $n$ small cups has a height, $h$, in centimeters of $h = 1.5n + 6$. A stack of $n$ large cups has a height, $h$, in centimeters of $h = 1.5n + 9$.

1. Graph the equations for each cup on the same set of axes. Make sure to label the axes and decide on an appropriate scale.

2. For what number of cups will the two stacks have the same height?
2. There is no number of cups for which the two stacks have the same height.

**Activity Synthesis**

The key point for discussion is to connect what students observed about the graph to the concept of “no solutions” from earlier lessons. Graphically, students see that the lines are parallel and always separated by a distance of 3 cm. This means the stack of \( n \) large cups will always be 3 cm taller than a stack of \( n \) small cups. Connecting this to the equations, this means that there is no value of \( n \) that is a solution to both \( 1.5n + 6 \) and \( 1.5n + 9 \) at the same time. By the end of the discussion, students should understand that the following are equivalent:

- The lines don’t intersect.
- The lines are parallel.
- There is no value of \( n \) for which the stacks have the same height.
- There is no value of \( n \) that makes \( 1.5n + 6 = 1.5n + 9 \) true.

Invite students to explain how they used the graph or equations to answer the second question. Ask other students if they answered the question with a different line of reasoning. If not brought up by students, demonstrate that setting the expression for the height of the large cup \( 1.5n + 9 \) equal to the expression for the height of the small cup \( 1.5n + 6 \) and subtracting \( 1.5n \) from both sides gives \( 6 = 9 \), which is false no matter what value of \( n \) is used.
Support for English Language Learners

* Representing, Conversing: MLR7 Compare and Connect. Before the whole-class discussion, use this routine to give students an opportunity to explain how they used the graph or equations to answer the question, "For what number of cups will the two stacks have the same height?" Invite students to demonstrate their strategy using a visual or numerical representation. Display one example of each representation to discuss. Invite students to compare their strategies with a partner. Ask students to discuss how their strategies are the same or different, and then share with the whole class. This will help students connect different strategies that lead to the same conclusion of "no solution" for a system of equations.
* Design Principle(s): Optimize output; Maximize meta-awareness

Lesson Synthesis

To highlight some of the main concepts from the lesson, ask:

- "Suppose Jada and Han had met up with another person at the exact same time they met each other along their hikes."
  - "What might the graph look like that represents that person's distance from the parking lot over time?" (There are an infinite number of lines, but they all pass must through the same intersection point as the lines for Jada and Han.)
  - "What information is known and what information might you need to write an equation representing their distance from the parking lot?" (I would need to know either something about the speed of the third person or their distance from the parking lot at another point in time.)

- "What is a system of equations?" (Two or more equations for which you want to find values for all of the variables so that all of the equations are true.)

- "What does the solution to a system of equations represent?" (The values for all of the variables that make all of the equations true.)

12.4 Milkshakes, Revisited

Cool Down: 5 minutes
This cool-down assesses student ability to graph lines based on descriptions and interpret any intersections in the context of the problem.

Addressing
- 8.EE.C.8
**Student Task Statement**

Determined to finish her milkshake before Diego, Lin now drinks her 12 ounce milkshake at a rate of $\frac{1}{3}$ an ounce per second. Diego starts with his usual 20 ounce milkshake and drinks at the same rate as before, $\frac{2}{3}$ an ounce per second.

1. Graph this situation on the axes provided.

![Graph showing milkshake consumption over time for Lin and Diego.](image)

2. What does the graph tell you about the situation and how many solutions there are?

**Student Response**

![Graph showing milkshake consumption over time for Lin and Diego.](image)

1.
2. There is one solution at (24, 4) meaning that after 24 seconds both of them had 4 ounces of milkshake left. Diego still finishes his milkshake first.

**Student Lesson Summary**

A **system of equations** is a set of 2 (or more) equations where the variables represent the same unknown values. For example, suppose that two different kinds of bamboo are planted at the same time. Plant A starts at 6 ft tall and grows at a constant rate of \( \frac{1}{4} \) foot each day. Plant B starts at 3 ft tall and grows at a constant rate of \( \frac{1}{2} \) foot each day. We can write equations \( y = \frac{1}{4} x + 6 \) for Plant A and \( y = \frac{1}{2} x + 3 \) for Plant B, where \( x \) represents the number of days after being planted, and \( y \) represents height. We can write this system of equations:

\[
\begin{align*}
    y &= \frac{1}{4} x + 6 \\
    y &= \frac{1}{2} x + 3
\end{align*}
\]

Solving a system of equations means to find the values of \( x \) and \( y \) that make both equations true at the same time. One way we have seen to find the solution to a system of equations is to graph both lines and find the intersection point. The intersection point represents the pair of \( x \) and \( y \) values that make both equations true. Here is a graph for the bamboo example:

![Graph showing the growth of two bamboo plants](image)

The solution to this system of equations is (12, 9), which means that both bamboo plants will be 9 feet tall after 12 days.

We have seen systems of equations that have no solutions, one solution, and infinitely many solutions.
• When the lines do not intersect, there is no solution. (Lines that do not intersect are parallel.)

• When the lines intersect once, there is one solution.

• When the lines are right on top of each other, there are infinitely many solutions.

In future lessons, we will see that some systems cannot be easily solved by graphing, but can be easily solved using algebra.

**Glossary**

• system of equations

**Lesson 12 Practice Problems**

**Problem 1**

**Statement**

Here is the graph for one equation in a system of equations:

![Graph](image)

a. Write a second equation for the system so it has infinitely many solutions.

b. Write a second equation whose graph goes through (0, 1) so the system has no solutions.

c. Write a second equation whose graph goes through (0, 2) so the system has one solution at (4, 1).

**Solution**

a. \( y = \frac{3}{4} x + 4 \)

b. \( y = \frac{3}{4} x + 1 \)
c. \( y = \frac{1}{4} x + 2 \)

**Problem 2**

**Statement**

Create a second equation so the system has no solutions.

\[
\begin{align*}
    y &= \frac{3}{4} x - 4
\end{align*}
\]

**Solution**

Answers vary. Any line of the form \( y = \frac{3}{4} x + b \) will make it so the system has no solutions.

**Problem 3**

**Statement**

Andre is in charge of cooking broccoli and zucchini for a large group. He has to spend all \$17 he has and can carry 10 pounds of veggies. Zucchini costs \$1.50 per pound and broccoli costs \$2 per pound. One graph shows combinations of zucchini and broccoli that weigh 10 pounds and the other shows combinations of zucchini and broccoli that cost \$17.

a. Name one combination of veggies that weighs 10 pounds but does not cost \$17.

b. Name one combination of veggies that costs \$17 but does not weigh 10 pounds.

c. How many pounds each of zucchini and broccoli can Andre get so that he spends all \$17 and gets 10 pounds of veggies?

**Solution**

a. Answers vary. Sample response: 4 pounds of zucchini and 6 pounds of broccoli weigh 10 pounds, but do not cost \$17 because (4, 6) is not on the line of combinations that cost \$17.

b. Answers vary. Sample response: 2 pounds of zucchini and 7 pounds of broccoli together cost \$17 because (2, 7) is on the \$17 line, but they only weigh 9 pounds.

c. 6 pounds of zucchini, and 4 pounds of broccoli

(From Unit 4, Lesson 10.)
Problem 4

Statement

The temperature in degrees Fahrenheit, $F$, is related to the temperature in degrees Celsius, $C$, by the equation

$$F = \frac{9}{5}C + 32$$

a. In the Sahara desert, temperatures often reach 50 degrees Celsius. How many degrees Fahrenheit is this?

b. In parts of Alaska, the temperatures can reach -60 degrees Fahrenheit. How many degrees Celsius is this?

c. There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. Use the expression from the equation, where $F$ is expressed in terms of $C$, to solve for this temperature.

Solution

a. 122 degrees Fahrenheit

b. $-51\frac{1}{9}$ degrees Celsius

c. $C = \frac{9}{5}C + 32$, $C = -40$

(From Unit 4, Lesson 9.)
Lesson 13: Solving Systems of Equations

Goals

- Coordinate (orally) the solution of an equation with variables on each side to the solution of a system of two linear equations.

- Create a graph of a system of equations, and identify (orally and in writing) the number of solutions of the system of equations.

Learning Targets

- I can graph a system of equations.

- I can solve systems of equations using algebra.

Lesson Narrative

In this lesson, students continue to explore systems where the equations are both of the form $y = mx + b$. They connect algebraic and graphical representations of systems, first by matching graphs to systems, then by drawing their own graphs from given systems. Additionally, students see how to see the number of solutions from both the graphical and algebraic representations. In the graphical representation the number of solutions is equal to the number of points where the graphs intersect. In the algebraic representation, two equations with the same rate of change can have 0 or infinitely many solutions depending on whether the initial values (y-intercepts) are the same or not. If the rates of change are different then there is a single solution, which can be interpreted physically as the point at which two quantities changing at different rates become equal.

Alignments

Addressing

- 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

- 8.EE.C.8.a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR8: Discussion Supports

- Think Pair Share
Required Materials
Copies of blackline master
Scissors
Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation
Print the Different Types of Systems blackline master. Prepare one set for every 2–3 students. Provide access to straightedges for drawing accurate graphs and scissors for groups that wish to cut apart the graphs on the blackline master.

Student Learning Goals
Let’s solve systems of equations.

13.1 True or False: Two Lines

Warm Up: 5 minutes
The purpose of this warm-up is to get students to reason about solutions to equations by looking at their structure and reading their graphs. While some students may solve each equation to find if it is true or false without relating it to the graphs, encourage all students to show why their answer is correct based on the graphs of the equations during the whole-class discussion.

Addressing
- 8.EE.C.8.a

Instructional Routines
- Think Pair Share

Launch
Arrange students in groups of 2. Display the image for all to see. Give students 2 minutes of quiet work time to begin the task individually and then 1 minute to discuss their responses with a partner followed by a whole-class discussion.
Student Task Statement

Use the lines to decide whether each statement is true or false. Be prepared to explain your reasoning using the lines.

1. A solution to \( y = -x + 10 \) is 2.
2. A solution to \( 2 = 2x + 4 \) is 8.
3. A solution to \(-x + 10 = 2x + 4\) is 8.
4. A solution to \(-x + 10 = 2x + 4\) is 2.
5. There are no values of \( x \) and \( y \) that make \( y = -x + 10 \) and \( y = 2x + 4 \) true at the same time.

Student Response

1. True because the line passes through point \((2, 8)\) where \( x = 2 \)
2. False
3. False
4. True
5. False

Activity Synthesis

Display the task image for all to see. Ask students to share their solutions and to reference the lines in their explanations. Emphasize the transitive property when students explain that since \( y = 8 \) at the point of intersection of \( y = 2x + 4 \) and \( y = -x + 10 \), then both \( 2x + 4 = 8 \) and \( -x + 10 = 8 \) are true, which leads to \(-x + 10 = 2x + 4\). If students do not mention this idea, bring it to their attention.
Ask students to solve \(-x + 10 = 2x + 4\) for \(x\) if they’ve not already done so and confirm that \(x = 2\) is the \(x\)-coordinate of the solution to the system of equations.

13.2 Matching Graphs to Systems

15 minutes (there is a digital version of this activity)
This activity represents the first time students solve a system of equations using algebraic methods. They first match systems of equations to their graphs and then calculate the solutions to each system. The purpose of matching is so students have a way to check that their algebraic solutions are correct, but not to shortcut the algebraic process since the graphs themselves do not include enough detail to accurately guess the coordinates of the solution.

Addressing
• 8.EE.C.8

Instructional Routines
• MLR8: Discussion Supports

Launch
Keep students in groups of 2. Give 2–3 minutes of quiet work time for the first problem and then ask students to pause their work. Select 1–2 students per figure to explain how they matched it to one of the systems of equations. For example, a student may identify the system matching Figure A as the only system with an equation that has negative slope. Give students 5–7 minutes of work time with their partner to complete the activity followed by a whole-class discussion. If students finish early and have not already done so on their own, ask them how they could check their solutions and encourage them to do so.

If using the digital activity, implement the lesson as indicated above. The only difference between the print and digital version is the digital lesson has an applet that will simulate the graphs so the students have another way of checking their solutions.

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. Invite students to illustrate connections between slopes and \(y\)-intercepts of each line to the corresponding parts of each equation using the same color.
Supports accessibility for: Visual-spatial processing
Support for English Language Learners

Conversing: **MLR8 Discussion Supports.** Use this routine to support small-group discussion as students describe the reasons for their matches. Arrange students in groups of 2. Invite Partner A to begin with this sentence frame: “Figure ___ matches with the system of equations ___ because ___.” Invite the listener, Partner B, to press for additional details referring to specific features of the graphs (e.g., positive slope, negative y-intercept, coordinates of the intersection point, etc). Students should switch roles for each figure. This will help students justify how features of the graph can be used to identify matching equations.

*Design Principle(s): Support sense-making; Cultivate conversation*

---

**Student Task Statement**

Here are three **systems of equations** graphed on a coordinate plane:

![Graphs A, B, and C](image)

1. Match each figure to one of the systems of equations shown here.
   - **a.** \[ \begin{align*} y &= 3x + 5 \\ y &= -2x + 20 \end{align*} \]
   - **b.** \[ \begin{align*} y &= 2x - 10 \\ y &= 4x - 1 \end{align*} \]
   - **c.** \[ \begin{align*} y &= 0.5x + 12 \\ y &= 2x + 27 \end{align*} \]

2. Find the solution to each system and check that your solution is reasonable based on the graph.

**Student Response**

1. a. Graph A since it is the only graph with a negative slope.
   
   b. Graph C since both graphs have a negative y-intercept.
   
   c. Graph B since both graphs have a positive y-intercept.
2. Graph A: (3, 14). Graph B: (-10, 7). Graph C: (-4.5, -19)

**Activity Synthesis**

The goal of this discussion is to deliberately connect the current topic of systems of equations to the previous topic of solving equations with variables on both sides. For each of the following questions, give students 30 seconds of quiet think time and then invite students per question to explain their answer. The final question looks ahead to the following activity.

- "Do you need to see the graphs of the equations in a system in order to solve the system?" (No, but the graphs made me feel more confident that my answer was correct.)
- "How do you know your solution doesn’t contain any errors?" (I know my solution does not have errors because I substituted my values for x and y into the equations and they made both equations true.)
- "How does solving systems of equations compare to solving equations with variables on both sides like we did in earlier lessons?" (They are very similar, only with a system of equations you are finding an x and a y to make both equations true and not just an x to make one equation true.)
- "When you solved equations with variables on both sides, some had one solution, some had no solutions, and some had infinite solutions. Do you think systems of equations can have no solutions or infinite solutions?" (Yes. We have seen some graphs of parallel lines where there were no solutions and some graphs of lines that are on top of one another where there are infinite solutions.)

### 13.3 Different Types of Systems

15 minutes (there is a digital version of this activity)

While students have encountered equations with different numbers of solutions in earlier activities, this is the first activity where students connect systems of equations with their previous thinking about equations that have no solution, one solution, or infinitely many solutions. The purpose of this activity is for students to connect the features of the graph of the equations of a system to the number of solutions of a system (MP7). While students are not asked to solve the systems of equations, they may choose to rewrite the equations in equivalent forms as they work to graph the lines.

Depending on instructional time available, you may wish to alter the activity and ask students to solve one or more of the systems of equations algebraically.

**Addressing**

- 8.EE.C.8

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time
Launch
Remind students of the activity they did sorting equations with a single variable where each equation had either one solution, no solution, or infinitely many solutions. Tell them that, just like one variable equations, systems of equations can also have either one solution, no solution, or infinitely many solutions. Point out that in the previous activity, each of the three systems of equations had one solution, which they found algebraically by solving the system, and so the graphs of the equations of the system showed one point where the lines intersected. Ask students what they think the graphs of equations from systems with no or infinitely many solutions might look like. Allow 30 seconds of quiet think time before inviting a few students to suggest possibilities for each type of system while recording and displaying their ideas for all to see. Remind students of the activities in previous lessons where they have seen these situations and their graphs (a bike race between Elena and Jada had infinite solutions and stacking different sized cups had none).

Arrange students in groups of 2–3. Provide each group with access to straightedges and scissors as well as one copy of the blackline master. Encourage partners to split the work by cutting apart the problems, each taking one to three graphs, and then trading pages within their group to check the work. Give 4–6 minutes for groups to complete the graphs and remind students to use straightedges for precision while graphing.

Before beginning the final problem, have each group trade their work with another group and place a question mark next to the graphs they are not sure are correct. Give groups 3–4 minutes to revise as needed and write their descriptions for the second problem followed by a whole-class discussion.

If using the digital activity, use the discussion structure above. The digital applet will make the graphing and solving of systems go quickly so students can spend more time analyzing the solutions. Using technology to graph allows students to focus on the main purpose of the lesson and also recognize the value in technology when solving systems in addition to appreciating when the graphing method is efficient. In this activity, one of the main purposes is to notice what is common among systems with the same number of solutions. Therefore, it may be useful to ask students to justify why the lines graphed with no obvious intersections are actually parallel.

Student Task Statement
Your teacher will give you a page with some systems of equations.

1. Graph each system of equations carefully on the provided coordinate plane.

2. Describe what the graph of a system of equations looks like when it has . . .
   a. 1 solution
   b. 0 solutions
c. infinitely many solutions

Student Response

1.
2. a. Two lines that cross at a single point, which is the only solution they have in common. Graphs A and B have one solution.

b. Two distinct lines that are parallel. They have no solutions in common. Graphs C and D have no solution.

c. A single line since both equations have the same set of solutions. Graph E has infinitely many solutions.

**Are You Ready for More?**
The graphs of the equations \(Ax + By = 15\) and \(Ax - By = 9\) intersect at \((2, 1)\). Find \(A\) and \(B\). Show or explain your reasoning.

**Student Response**
\(A = 6, \ B = 3.\)

Explanations vary. Sample response: If the lines intersect at \((2, 1)\) then that point is on both lines. So we can substitute \(x = 2, \ y = 1\) into both equations and write \(2A + B = 15, \ 2A - B = 9.\) Then there are different ways we can solve.

- Since \(2A - B\) has the same value as 9, we can add \(2A - B\) to one side of the first equation and 9 to the other.

- We can rewrite the second equation as \(B = 2A - 9\) and substitute this expression for \(B\) into the first equation.
Activity Synthesis

The goal of this discussion is for students to draw conclusions about the relationship between the number of solutions a system of equations has and the appearance of the graphs of the equations in the system. Select 2–3 students to share and explain their answers to the second problem.

If no students mention it, bring in slope language and how inspecting the slopes of the equations before graphing or solving can give clues to the possible number of solutions the system has. In particular, students should notice that systems with lines that have different slopes have a single solution, lines that have the same slope and different $y$-intercept have no solution, and lines that have the same slope and $y$-intercept will have infinitely many solutions.

Assign a number of solutions (one, none, or infinite) to each group and ask them to write a system of equations that would have that number of solutions. Have a few groups share their systems and describe how the graphs of the systems would look. In particular, ask each group to describe how the slope and $y$-intercept of their written lines would be seen in the graph and how the number of solutions would appear on the graph. Following the description, display the graph of the system using a digital resource, if possible, or a general sketch on a set of displayed axes.

Support for English Language Learners

Writing: MLR1 Stronger and Clearer Each Time. Use this routine to give students an opportunity to revise and improve their response to the final question, “Describe what the graph of a system of equations looks like when it has one solution, zero solutions, and infinitely many solutions.” Give students time to meet with 2–3 partners, to share and get feedback on their response. Encourage the listener to press for supporting details and evidence by asking, “How do the slopes compare?” “How do the $y$-intercepts compare?” or “What do you notice about the slopes and the $y$-intercepts?” Students can borrow ideas and language from each partner to strengthen the final product. This will help students produce a written generalization for how to identify the number of solutions for a system of equations by using the features of a graph. 

Design Principle(s): Optimize output (for generalization)

Lesson Synthesis

To highlight the connection between the number of solutions to a system of equations and features of its graph and equations, ask:

- “How can you know the number of solutions for a system of equations from its graph?” (If the two lines intersect at a point, there is one solution. If the two lines are parallel and do not intersect, there are no solutions. If the two lines are drawn through the same points, there are infinitely many solutions.)

- “How can you know the number of solutions for a system of equations from their equations?” (If the two equations have different slopes, there is one solution. If the two equations have the
same slope and different y-intercepts, there are no solutions. If the two equations have the same slope and the same y-intercept, there are infinitely many solutions.)

If students do not make the connection themselves, remind them of their earlier conclusions about the number of solutions and equation in one variable has.

## 13.4 Two Lines

**Cool Down: 5 minutes**
This cool-down asks students to write equations that could match a given graph and then to identify the number of solutions in the system shown.

**Addressing**
- 8.EE.C.8

### Student Task Statement

1. Given the lines shown here, what are two possible equations for this system of equations?

2. How many solutions does this system of equations have? Explain your reasoning.

### Student Response

1. Correct answers should have the same positive slope for each linear equation. One equation should have a negative y-intercept and the other should have a positive y-intercept. Sample response:
   \[
   \begin{align*}
   y &= \frac{1}{4}x - 10 \\
   y &= \frac{1}{4}x + 1
   \end{align*}
   \]

2. 0. Since the lines are parallel and do not intersect, there are no solutions to the system of equations.
Student Lesson Summary

Sometimes it is easier to solve a system of equations without having to graph the equations and look for an intersection point. In general, whenever we are solving a system of equations written as

\[
\begin{align*}
  y &= \text{[some stuff]} \\
  y &= \text{[some other stuff]}
\end{align*}
\]

we know that we are looking for a pair of values \((x, y)\) that makes both equations true. In particular, we know that the value for \(y\) will be the same in both equations. That means that

\[
\text{[some stuff]} = \text{[some other stuff]}
\]

For example, look at this system of equations:

\[
\begin{align*}
  y &= 2x + 6 \\
  y &= -3x - 4
\end{align*}
\]

Since the \(y\) value of the solution is the same in both equations, then we know

\[2x + 6 = -3x - 4\]

We can solve this equation for \(x\):

\[
\begin{align*}
  2x + 6 &= -3x - 4 \\
  5x + 6 &= -4 \\
  5x &= -10 \\
  x &= -2
\end{align*}
\]

add 3x to each side
subtract 6 from each side
divide each side by 5

But this is only half of what we are looking for: we know the value for \(x\), but we need the corresponding value for \(y\). Since both equations have the same \(y\) value, we can use either equation to find the \(y\)-value:

\[y = 2(-2) + 6\]

Or

\[y = -3(-2) - 4\]
In both cases, we find that $y = 2$. So the solution to the system is $(-2, 2)$. We can verify this by graphing both equations in the coordinate plane.

In general, a system of linear equations can have:

- No solutions. In this case, the lines that correspond to each equation never intersect.
- Exactly one solution. The lines that correspond to each equation intersect in exactly one point.
- An infinite number of solutions. The graphs of the two equations are the same line!

**Lesson 13 Practice Problems**

**Problem 1**

**Statement**

a. Write equations for the lines shown.

b. Describe how to find the solution to the corresponding system by looking at the graph.

c. Describe how to find the solution to the corresponding system by using the equations.
Solution

a. \( y = 3x + 2 \) and \( y = 8 - 3x \)

b. The point where the two lines meet, \((1, 5)\)

c. Set the two expressions for \( y \) equal to each other and solve: \( 3x + 2 = 8 - 3x \), \( 6x = 6 \), \( x = 1 \), \( y = 3(1) + 2 = 5 \)

Problem 2

Statement
The solution to a system of equations is \((5, -19)\). Choose two equations that might make up the system.

A. \( y = -3x - 6 \)
B. \( y = 2x - 23 \)
C. \( y = -7x + 16 \)
D. \( y = x - 17 \)
E. \( y = -2x - 9 \)

Solution
["C", "E"]

Problem 3

Statement
Solve the system of equations: \[
\begin{cases}
  y = 4x - 3 \\
  y = -2x + 9
\end{cases}
\]

Solution
\((2, 5)\)

Problem 4

Statement
Solve the system of equations: \[
\begin{cases}
  y = \frac{5}{4}x - 2 \\
  y = \frac{-3}{4}x + 19
\end{cases}
\]
Solution

\((14, 15 \frac{1}{2})\)

**Problem 5**

**Statement**

Here is an equation: \(\frac{15(x-3)}{5} = 3(2x - 3)\)

a. Solve the equation by using the distributive property first.

b. Solve the equation without using the distributive property.

c. Check your solution.

**Solution**

a. \(x = 0\). Responses vary. Sample response:

\[
\frac{15(x-3)}{5} = 3(2x - 3)
\]

\[
15x - 45 = 6x - 9\]

distributive property on each side

\[
3x - 9 = 6x - 9\]

divide each term in the numerator of the left side by 5

\[
x = 6x\]

add 9 to each side

\[
0 = 3x\]

subtract 3x on each side

\[
0 = x\]

divide each side by 3

b. \(x = 0\). Responses vary. Sample response:

\[
\frac{15(x-3)}{5} = 3(2x - 3)
\]

\[
15(x - 3) = 15(2x - 3)\]

multiply each side by 5

\[
x - 3 = 2x - 3\]

divide each side by 15

\[
x = 2x\]

add 3 to each side

\[
0 = x\]

subtract \(x\) on each side

c. \$

This equation is true, so \(x = 0\) is the solution.

(From Unit 4, Lesson 6.)
Lesson 14: Solving More Systems

Goals

- Calculate values that are a solution for a system of equations, and explain (orally) the solution method.
- Generalize (orally) a process for solving systems of equations using substitution.
- Justify (orally and in writing) that a particular system of equations has no solutions using the structure of the equations.

Learning Targets

- I can use the structure of equations to help me figure out how many solutions a system of equations has.

Lesson Narrative

In previous lessons, students have worked with contexts where two quantities are changing at different (or possibly the same) rate, and they must find when they are equal. Such systems are represented by equations of the form $y = mx + b$ and are solved by setting the two expressions for $y$ equal to each other.

In this lesson, students graduate to other types of systems with different structures. They learn that examining structure is a good first step since it is sometimes possible to recognize an efficient method for solving the system through observation. They see that if at least one of the equations has a single variable isolated, then that expression can be substituted into the other equation in place of $y$ or $x$ to get a single equation in one variable that can be solved. Finally, students use the structure of a system of equations to reason about its lack of solutions.

When students look at the structure of a system before starting to solve it in order to develop a good approach to solving, they engage in MP7.

Alignments

Building On

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Addressing

- 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Instructional Routines

- Algebra Talk
• MLR1: Stronger and Clearer Each Time
• MLR2: Collect and Display
• MLR8: Discussion Supports

Student Learning Goals
Let’s solve systems of equations.

14.1 Algebra Talk: Solving Systems Mentally

Warm Up: 5 minutes
The purpose of this warm-up is to encourage students to use substitution to solve equations mentally and see that sometimes they cannot use this method.

Building On
• 6.EE.B.5

Instructional Routines
• Algebra Talk
• MLR8: Discussion Supports

Launch
Display each problem one at a time. Give students 30 seconds of quiet think time followed by a whole-class discussion. Leave each problem displayed throughout the talk.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement
Solve these without writing anything down:

\[
\begin{align*}
&\begin{cases}
x = 5 \\
y = x - 7
\end{cases} \\
&\begin{cases}
y = 4 \\
y = x + 3
\end{cases} \\
&\begin{cases}
x = 8 \\
y = -11
\end{cases}
\end{align*}
\]
Student Response

- $x = 5$ and $y = -2$
- $x = 1$ and $y = 4$
- $x = 8$ and $y = -11$

Activity Synthesis

After each problem, ask students to share their solutions. Record and display their responses for all to see. As students share their strategies, highlight the term substitution as a strategy to solve an equation. For the third question, ask students why they could not use the same strategy as they did in the earlier questions.

Support for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

14.2 Challenge Yourself

15 minutes
In this activity, students solve systems of linear equations that lend themselves to substitution. There are 4 kinds of systems presented: one kind has both equations given with the $y$ value isolated on one side of the equation, another kind has one of the variables given as a constant, a third kind has one variable given as a multiple of the other, and the last kind has one equation given as a linear combination. This progression of systems nudges students towards the idea of substituting an expression in place of the variable it is equal to.

Notice which kinds of systems students think are least and most difficult to solve.

In future grades, students will manipulate equations to isolate one of the variables in a linear system of equations. For now, students do not need to solve a system like $x + 2y = 7$ and $2x - 2y = 2$ using this substitution method.

Addressing

- 8.EE.C.8

Instructional Routines

- MLR2: Collect and Display
Launch

Arrange students in groups of 2. Give students 10 minutes of quiet work time. Encourage students to check in with their partner between questions. Tell students that if there is disagreement, they should work to reach an agreement. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may have trouble transitioning from systems where both equations are given with one variable isolated to other kinds of systems. Ask these students to look at a system where one of the variables is given as a constant. For example, ask them to look at equation B:

\[
\begin{align*}
  y &= 7 \\
  x &= 3y - 4
\end{align*}
\]

Ask, “If \( y \) is equal to 7, then what is \( 3y \) equal to?” If a student continues to struggle, refer them back to this example and then ask, “In this new problem, do we know what expression \( y \) (or \( x \)) is equal to? Then whenever we see \( y \) (or \( x \)), what can we replace it with instead?”

Student Task Statement

Here are a lot of systems of equations:

A \[
\begin{align*}
  y &= 4 \\
  x &= -5y + 6
\end{align*}
\]

B \[
\begin{align*}
  y &= 7 \\
  x &= 3y - 4
\end{align*}
\]

C \[
\begin{align*}
  y &= \frac{3}{2}x + 7 \\
  x &= -4
\end{align*}
\]

D \[
\begin{align*}
  y &= -3x + 10 \\
  y &= -2x + 6
\end{align*}
\]

E \[
\begin{align*}
  y &= -3x - 5 \\
  y &= 4x + 30
\end{align*}
\]

F \[
\begin{align*}
  y &= 3x - 2 \\
  y &= -2x + 8
\end{align*}
\]

G \[
\begin{align*}
  y &= 3x \\
  x &= -2y + 56
\end{align*}
\]

H \[
\begin{align*}
  x &= 2y - 15 \\
  y &= -2x
\end{align*}
\]

I \[
\begin{align*}
  3x + 4y &= 10 \\
  x &= 2y
\end{align*}
\]

J \[
\begin{align*}
  y &= 3x + 2 \\
  2x + y &= 47
\end{align*}
\]

K \[
\begin{align*}
  y &= -2x + 5 \\
  2x + 3y &= 31
\end{align*}
\]

L \[
\begin{align*}
  x + y &= 10 \\
  x &= 2y + 1
\end{align*}
\]

1. Without solving, identify 3 systems that you think would be the least difficult to solve and 3 systems that you think would be the most difficult to solve. Be prepared to explain your reasoning.

2. Choose 4 systems to solve. At least one should be from your ”least difficult” list and one should be from your ”most difficult” list.

Student Response

1. Answers vary. Sample response: A, B, and C seem easy since one of the variable solutions is already given. J, K, and L seem the most difficult since there are multiple terms and the variables are on the same side of the equation in some of the equations.

2. Four of these answers:
   a. \( x = -14, y = 4 \)
b. \( x = 17, y = 7 \)
c. \( x = -4, y = 1 \)
d. \( x = 4, y = -2 \)
e. \( x = -5, y = 10 \)
f. \( x = 2, y = 4 \)
g. \( x = 8, y = 24 \)
h. \( x = -3, y = 6 \)
i. \( x = 2, y = 1 \)
j. \( x = 9, y = 29 \)
k. \( x = -4, y = 13 \)
l. \( x = 7, y = 3 \)

**Activity Synthesis**

This discussion has a two main takeaways. The first is to formalize the idea of substitution in a system of equations. Another is to recognize systems where both equations are written with one variable isolated are actually special cases of substitution.

Invite students to share which systems they thought would be easiest to solve and which would be hardest. To involve more students in the conversation, consider asking:

- “Did you change your mind about any of the systems being more or less difficult after you solved them?”
- “What was similar in these problems? What was different?” (The systems vary slightly in how they are presented, but all of the problems can be solved by replacing a variable with an expression it is equal to.)
- “Will your strategy work for the other systems in this list?” (Yes, substitution works in all the given problems.)

Tell students that the key underlying concept for all of these problems is that it is often helpful to replace a variable with the expression it is equal to, and that this “replacing” is called “substitution.” Point out that even setting the expressions for \( y \) in the first two problems equal to each other is really substituting \( y \) in one equation with the expression it is equal to as given by the other equation. It may be helpful for students to hear language like, “Since \( y \) is equal to \(-2x\), that means wherever I see \( y \), I can substitute in \(-2x\).”
Support for Students with Disabilities

*Representation: Internalize Comprehension.* Use color and annotations to illustrate student thinking. As students describe their strategies and the relationships they noticed, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

*Supports accessibility for:* Visual-spatial processing; Conceptual processing

Support for English Language Learners

*Representing, Speaking, Listening: MLR2 Collect and Display.* As students discuss which systems they thought would be easiest to solve and which would be hardest, create a table with the headings “least difficult” and “most difficult” in the two columns. Circulate through the groups and record student language in the appropriate column. Look for phrases such as “different variables on the same side,” “variables already isolated,” and “various terms.” Invite students to share strategies they can use to address the features that make these systems of equations more difficult to solve. This will help students begin to generalize and make sense of the structures of equations for substitution.

*Design Principle(s):* Support sense-making

14.3 Five Does Not Equal Seven

15 minutes

In this activity, students are asked to make sense of Tyler’s justification for the number of solutions to the system of equations (MP3). This activity continues the thread of reasoning about the structure of an equation (MP7) and the focus should be on what, specifically, in the equations students think Tyler sees that makes him believe the system has no solutions.

**Addressing**

- 8.EE.C.8

**Instructional Routines**

- MLR1: Stronger and Clearer Each Time

**Launch**

Give students 2–3 minutes of quiet think time to read the problem and decide if they agree or disagree with Tyler. Use the remaining time for a whole-class discussion.
Student Task Statement

Tyler was looking at this system of equations:

\[
\begin{align*}
    x + y &= 5 \\
    x + y &= 7
\end{align*}
\]

He said, "Just looking at the system, I can see it has no solution. If you add two numbers, that sum can't be equal to two different numbers."

Do you agree with Tyler?

Student Response

Yes. Explanations vary. Sample explanation: The sum of two numbers cannot be equal to both 5 and 7. Students may choose to make a graph to show that the lines will never cross.

Are You Ready for More?

In rectangle $ABCD$, side $AB$ is 8 centimeters and side $BC$ is 6 centimeters. $F$ is a point on $BC$ and $E$ is a point on $AB$. The area of triangle $DFC$ is 20 square centimeters, and the area of triangle $DEF$ is 16 square centimeters. What is the area of triangle $AED$?

Student Response

$\frac{48}{5}$ square centimeters or equivalent. Since the area of triangle $DFC$ is 20 and $DC = 8, CF = 5$ and hence $FB = 1$. The area of rectangle $ABCD$ is 48 square centimeters. Summing the areas of the four triangles, we get $48 = 20 + \frac{1}{2} \cdot 6 \cdot AE + \frac{1}{2} \cdot BE$. We also have $AE + EB = 8$. This is a system of equations where one solution is $AE = \frac{16}{5}$ leading to the area of triangle $AED$ is $\frac{48}{5}$.

It may simplify the work to use a variable to represent the lengths of $AE$ and $EB$. 
**Activity Synthesis**

The goal of this discussion is to look at one way to reason about the structure of a system of equations in order to determine the solution and then have students make their own reasoning about a different, but similar, system of equations.

Poll the class to see how many students agree with Tyler and how many students disagree with Tyler. If possible, invite students from each side to explain their reasoning. As students explain, it should come out that Tyler is correct and, if no student brings up the idea, make sure to point out that we can also visualize this by graphing the equations in the system and noting that the lines look parallel and will never cross.

In the previous activity, students noticed that if they knew what one variable was equal to, they could substitute that value or expression into another equation in the same problem. Point out that, in this problem, the expression \((x + y)\) is equal to 5 in the first equation. If, in the second equation, we replace \((x + y)\) with 5, the resulting equation is \(5 = 7\) which cannot be true regardless of the choice of \(x\) and \(y\).

Display the following system and ask students how many solutions they think it has and to give a signal when they think they know:

\[
\begin{align*}
4x + 2y &= 8 \\
2x + y &= 5
\end{align*}
\]

Once the majority of the class signals they have an answer, invite several students to explain their thinking. There are multiple ways students might use to reason about the number of solutions this system has. During the discussion, encourage students to use the terms *coefficient* and *constant term* in their reasoning. Introduce these terms if needed to help students recall their meanings. Bring up these possibilities if no students do so in their explanations:

- “Re-write the second equation to isolate the \(y\) variable and substitute the new expression into the first equation in order to find that the system of equations has no solutions.”
• “Notice that both equations are lines with the same slope but different y-intercepts, which means the system of equations has no solutions.”

• “Notice that $4x + 2y$ is double $2x + y$, but 8 is not 5 doubled, so the system of equations must have no solution.”

Support for English Language Learners

Writing: MLR1 Stronger and Clearer Each Time. If time allows, use this routine to give students an opportunity to summarize the whole-class discussion. Display the prompt, “How can you use the structure of a system of equations to determine when there are no solutions?” Give students 3–4 minutes to write a response, then invite students to meet with 2–3 partners, to share and get feedback on their writing. Encourage listeners to ask their partner clarifying questions such as, “What do you notice about the coefficients and constant terms in the equations?”, “What do the equations tell us about the rates of change and initial values?”, “Can you use the example of Tyler’s problem to explain that more?” Students can borrow ideas and language from each partner to strengthen their final product. This will help students solidify their understanding of how to use the structure of the system of equations to determine the number of solutions.

*Design Principle(s): Optimize output; Cultivate conversation*

Lesson Synthesis

To emphasize the concepts from this lesson, consider displaying the three systems and asking these discussion questions:

\[
\begin{align*}
\begin{cases}
  x = 2 \\
  y = 3x - 1
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
  x = 2y + 4 \\
  x = 9 - 3y
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
  x = 2y + 3 \\
  y = 2x - 9
\end{cases}
\end{align*}
\]

• “What is the first step you would take to solve the first system?” (Since we already know the $x$ value of the solution, we only need to find the $y$ value. Substituting 2 in for $x$ in the other equation should help us solve for the $y$ value that makes both equations true when $x$ is 2.)

• “What steps would you take to solve the second system?” (Since we know two expressions that are equal to $x$, we can set those expressions equal to one another. Therefore, we know that $2y + 4 = 9 - 3y$ which can be solved using the techniques to solve equations with a single variable. Once we know the value for $y$, we can find the value for $x$ from either of the original equations from the system.)
• “For the third system, a student begins the substitution method by writing $y = 2 \cdot 2y + 3 - 9 \quad \text{then} \quad y = 4y - 6$. What has this student done wrong?” (When substituting for $x$, the student did not multiply the entire expression by 2.)

### 14.4 Solve It

**Cool Down: 5 minutes**

This cool-down asks students to solve a system of equations presented in an algebraic form. Although no method is specified, the main ideas from this lesson as well as a lack of a coordinate plane may lead students to use a substitution method which is both efficient and effective on this system.

**Addressing**

- 8.EE.C.8

#### Student Task Statement

Solve this system of equations:

\[
\begin{align*}
y &= 2x \\
x &= -y + 6
\end{align*}
\]

#### Student Response

(2, 4). Sample response: Students may use the substitution method to rewrite the system as the one variable equation $x = -(2x) + 6$, then solve.

### Student Lesson Summary

When we have a system of linear equations where one of the equations is of the form $y = \text{[stuff]}$ or $x = \text{[stuff]}$, we can solve it algebraically by using a technique called substitution. The basic idea is to replace a variable with an expression it is equal to (so the expression is like a substitute for the variable). For example, let’s start with the system:

\[
\begin{align*}
y &= 5x \\
2x - y &= 9
\end{align*}
\]

Since we know that $y = 5x$, we can substitute $5x$ for $y$ in the equation $2x - y = 9$,

\[
2x - (5x) = 9,
\]

and then solve the equation for $x$,

\[
x = -3.
\]

We can find $y$ using either equation. Using the first one: $y = 5 \cdot -3$. So
is the solution to this system. We can verify this by looking at the graphs of the equations in the system:

Sure enough! They intersect at (-3, -15).

We didn’t know it at the time, but we were actually using substitution in the last lesson as well. In that lesson, we looked at the system

\[
\begin{align*}
  y &= 2x + 6 \\
  y &= -3x - 4
\end{align*}
\]

and we substituted \(2x + 6\) for \(y\) into the second equation to get \(2x + 6 = -3x - 4\). Go back and check for yourself!

**Lesson 14 Practice Problems**

**Problem 1**

**Statement**

Solve: \[
\begin{align*}
  y &= 6x \\
  4x + y &= 7
\end{align*}
\]

**Solution**

\(\left(\frac{7}{10}, \frac{21}{5}\right)\)

**Problem 2**

**Statement**

Solve: \[
\begin{align*}
  y &= 3x \\
  x &= -2y + 70
\end{align*}
\]

**Solution**

\((10, 30)\)
Problem 3

Statement
Which equation, together with $y = -1.5x + 3$, makes a system with one solution?

A. $y = -1.5x + 6$
B. $y = -1.5x$
C. $2y = -3x + 6$
D. $2y + 3x = 6$
E. $y = -2x + 3$

Solution
E

Problem 4

Statement
The system $x - 6y = 4$, $3x - 18y = 4$ has no solution.

a. Change one constant or coefficient to make a new system with one solution.

b. Change one constant or coefficient to make a new system with an infinite number of solutions.

Solution
a. Answers vary. Sample response: $2x - 6y = 4$

b. Answers vary. Sample response: $3x - 18y = 12$

Problem 5

Statement
Match each graph to its equation.
a. $y = 2x + 3$

b. $y = -2x + 3$

c. $y = 2x - 3$

d. $y = -2x - 3$

**Solution**

a. A

b. C

c. B

d. D

(From Unit 3, Lesson 11.)

**Problem 6**

**Statement**

Here are two points: (-3, 4), (1, 7). What is the slope of the line between them?
A. $\frac{4}{3}$
B. $\frac{3}{4}$
C. $\frac{1}{6}$
D. $\frac{2}{3}$

Solution

B

(From Unit 3, Lesson 10.)
Lesson 15: Writing Systems of Equations

Goals

- Categorize (in writing) systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety of strategies.

- Create a system of equations that represents a situation and interpret (orally and in writing) the solution in context.

Learning Targets

- I can write a system of equations from a real-world situation.

Lesson Narrative

Previously, students have been given systems of equations to interpret and solve. In this lesson, they learn to write their own systems representing different contexts, and to interpret the solutions for those systems. Different contexts can lead to systems in different forms, so students also continue to practice looking at different systems and thinking ahead about how to solve them. When students represent a real-world problem with a system, they develop an important skill for mathematical modeling (MP4).

Alignments

Addressing

- 8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

- 8.EE.C.8.b: Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, \(3x + 2y = 5\) and \(3x + 2y = 6\) have no solution because \(3x + 2y\) cannot simultaneously be 5 and 6.

- 8.EE.C.8.c: Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Building Towards

- 8.EE.C.8.c: Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR3: Clarify, Critique, Correct

- MLR4: Information Gap Cards
• MLR8: Discussion Supports
• Think Pair Share

Required Materials
Pre-printed slips, cut from copies of the blackline master

Required Preparation
You will need the Info Gap: Racing and Play Tickets blackline master for this lesson. Make 1 copy for every 4 students, and cut them up ahead of time.

Student Learning Goals
Let’s write systems of equations from real-world situations.

15.1 How Many Solutions? Matching

Warm Up: 5 minutes
This warm-up asks students to connect the algebraic representations of systems of equations to the number of solutions. Efficient students will recognize that this can be done without solving the system, but rather using slope, $y$-intercept, or other methods for recognizing the number of solutions.

Monitor for students who use these methods:

1. Solve the systems to find the number of solutions.
2. Use the slope and $y$-intercept to determine the number of solutions.
3. Manipulate the equations into another form, then compare the equations.
4. Notice that the left side of the second equation in system C is double the left side of the first equation, but the right side is not.

Addressing
• 8.EE.C.8.b

Instructional Routines
• Anticipate, Monitor, Select, Sequence, Connect

Launch
Arrange students in groups of 2. Tell students that each number can be used more than once. Allow students 2 minutes of work time followed by a whole-class discussion.

Student Task Statement
Match each system of equations with the number of solutions the system has.
1. \[ \begin{cases} y = -\frac{4}{3} x + 4 \\ y = -\frac{4}{3} x - 1 \end{cases} \]
2. \[ \begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases} \]
3. \[ \begin{cases} 2x + 3y = 8 \\ 4x + 6y = 17 \end{cases} \]
4. \[ \begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases} \]

**Activity Synthesis**

The purpose of the discussion is to bring out any methods students used to find the number of solutions for the systems.

Select previously identified students to share their methods for finding the number of solutions in the sequence given in the narrative. After each student shares their method, ask the class which method they preferred to answer the given question. Connect each problem to the concepts learned in the previous lesson by asking students to describe how the graphs of the lines for each system might intersect.

### 15.2 Situations and Systems

**10 minutes**

In this activity, students are presented with a number of scenarios that could be solved using a system of equations. Students are not asked to solve the systems of equations, since the focus at this time is for students to understand how to set up the equations for the system and to understand what the solution represents in context.

**Building Towards**
- 8.EE.C.8.c

**Instructional Routines**
- MLR3: Clarify, Critique, Correct
- Think Pair Share
Launch

Arrange students in groups of 2. Suggest that groups split up the problems so that one person works on the first and third problem while their partner works on the second and fourth. Students may work with their partners to get help when they are stuck, but are encouraged to try to set up the equations on their own first. Partners should discuss their systems and interpretation of the solution after each has had a chance to work on their own.

Allow students 5–7 minutes of partner work time followed by a whole-class discussion.

Support for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select 2–3 of the situations to complete.

Supports accessibility for: Organization; Attention; Social-emotional skills

Student Task Statement

For each situation:

- Create a system of equations.
- Then, without solving, interpret what the solution to the system would tell you about the situation.

1. Lin’s family is out for a bike ride when her dad stops to take a picture of the scenery. He tells the rest of the family to keep going and that he’ll catch up. Lin’s dad spends 5 minutes taking the photo and then rides at 0.24 miles per minute until he meets up with the rest of the family further along the bike path. Lin and the rest were riding at 0.18 miles per minute.

2. Noah is planning a kayaking trip. Kayak Rental A charges a base fee of $15 plus $4.50 per hour. Kayak Rental B charges a base fee of $12.50 plus $5 per hour.

3. Diego is making a large batch of pastries. The recipe calls for 3 strawberries for every apple. Diego used 52 fruits all together.

4. Flour costs $0.80 per pound and sugar costs $0.50 per pound. An order of flour and sugar weighs 15 pounds and costs $9.00.

Student Response

1. \[
\begin{cases}
    d = 0.24t \\
    d = 0.18r + 5 \cdot 0.18
\end{cases}
\]

The solution would represent the time (t) it would take for Lin’s dad to catch up with the rest of the family.
2. \( \begin{align*}
  y &= 15 + 4.5x \\
  y &= 12.5 + 5x
\end{align*} \). The solution would represent the amount of time \((x)\) spent with the kayak so that the cost \((y)\) would be the same from each rental company.

3. \( \begin{align*}
  s &= 3a \\
  a + s &= 52
\end{align*} \). The solution would represent the number of apples \((a)\) and the number of strawberries \((s)\) that Diego used to make the large batch of pastries.

4. \( \begin{align*}
  0.8f + 0.5s &= 9 \\
  f + s &= 15
\end{align*} \). The solution would represent the number of pounds of flour \((f)\) and the number of pounds of sugar \((s)\) purchased in this order.

**Activity Synthesis**

The focus of the discussion should be on making sense of the context and interpreting the solutions within the context of the problems.

Invite groups to share their systems of equations and interpretation of the solution for each problem. As groups share, record their systems of equations for all to see. When necessary, ask students to explain the meaning of the variables they used. For example, \(t\) represents the number of minutes the family rides after Lin's dad starts riding again after taking the picture.

To highlight the connections between the situations and the equations that represent them, ask:

- "How many solutions will each of these systems of equations have?" (Each system has exactly one solution. I can tell this because the slopes of each pair of equations are different.)

- "If Lin's dad biked 0.17 miles per minute instead of 0.24 miles per minute, how would that change the system of equations?" (The first equation would be \(d = 0.17t\).)
  - "How many solutions would there be for this new system where Lin's dad rides slower?" (Based on the equations there should still be one solution.)
  - "Would Lin's dad ever catch up with the family?" (He would not. He started farther back and rides slower than the family. The solution to the system would have a negative value for time which does not make sense in the context of the problem.)

If students disagree that there is a solution to the modified first problem in which Lin's dad rides slower than the family, you may display the graph of the modified system and point out the point where the lines intersect. So, although the system has a solution, it is disregarded in this context since it does not make sense.
Support for English Language Learners

Representing, Conversing: MLR3 Clarify, Critique, Correct. To help students make sense of the solution to a system of equations, offer an incorrect or ambiguous response for the problem about Noah’s kayaking trip. After revealing the correct system of equations, display this ambiguous statement: “This solution represents the same amount of money for both rental places.” Ask pairs of students to clarify the meaning of this response and then critique it. Invite pairs to offer a correct response by asking “What language might you add or change to make this statement more accurate?” Improved responses should include a reference to both variables (e.g. the amount of time and cost). This will help students interpret the solution to a system of equations in a specific context.

Design Principle(s): Maximize meta-awareness; Support sense-making

15.3 Info Gap: Racing and Play Tickets

20 minutes

In this activity students have an opportunity to apply what they know about systems of linear equations to solve a problem about a real-world situation. One equation for each situation is given. Students may choose to write another equation to create a system that represents the constraints in the problem, and then solve the system algebraically or by graphing. Another possible strategy would be to pull quantities out of the given equation and solve the problem arithmetically. Monitor for students who use each of these strategies to share during the whole-class discussion.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

**Problem Card 1**

Prize and Lin are having a race. The equation \( y = 9.5x \) represents one person's progress.

If one of them had a head start, how long is it until the other person catches up?

**Problem Card 2**

A school sells adult tickets and student tickets for the drama play. One equation that represents the situation is \( x + y = 115 \).

How many of each type of ticket did they sell?

**Data Card 1**

- The equation \( y = 0.5x \) represents Lin's progress, where \( y \) is her distance in feet, \( x \) is the time in seconds, and \( y \) is the time she has been running.
- Prize has the head start. She was 14 feet in front of the starting line when Lin started.
- Prize runs at a constant 6 feet per second.

**Data Card 2**

- The equation \( y = 115 - x \) represents how many tickets were sold, where \( x \) is student tickets and \( y \) is adult tickets.
- The school made $720 total from ticket sales.

Unit 4  Lesson 15

201
Addressing
- 8.EE.C.8.c

Instructional Routines
- MLR4: Information Gap Cards

Launch
Arrange students in groups of 2. Provide access to geometry toolkits. In each group, distribute the first problem card to one student and a data card to the other student. After debriefing on the first problem, distribute the cards for the second problem, in which students switch roles.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Support for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve a problem about a real-world situation by applying what they know about systems of linear equations. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)?”, and “Why do you need to know . . . (that piece of information)?”
Design Principle(s): Cultivate Conversation

Student Task Statement
Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.
If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner "What specific information do you need?" and wait for them to ask for information.
3. Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

**Student Response**

1. 12 seconds
2. 40 student tickets and 75 adult tickets

**Activity Synthesis**

Select students with different strategies to share their approaches to each question, starting with less efficient methods and ending with more efficient methods.

**15.4 Solving Systems Practice**

Optional: 10 minutes

In this activity, students solve a variety of systems of equations, some involving fractions, some involving substitution, and some involving inspection. This gives students a chance to practice using the methods they have learned in this section for solving systems of equations to solidify that learning. Some of the systems listed are ones students could have used in an earlier activity in this lesson, to describe the situations there. In the discussion, students compare the systems here to the ones they wrote in that activity and interpret the answer in that context.

**Addressing**

- 8.EE.C.8
Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Allow students 5-7 minutes of partner work time followed by a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Here are a lot of systems of equations:

- \[ \begin{cases} y = -2x + 6 \\ y = x - 3 \end{cases} \]
- \[ \begin{cases} y = 5x - 4 \\ y = 4x + 12 \end{cases} \]
- \[ \begin{cases} y = \frac{2}{3}x - 4 \\ y = -\frac{4}{3}x + 9 \end{cases} \]
- \[ \begin{cases} 4y + 7x = 6 \\ 4y + 7x = -5 \end{cases} \]
- \[ \begin{cases} y = x - 6 \\ x = 6 + y \end{cases} \]
- \[ \begin{cases} y = 0.24x \\ y = 0.18x + 0.9 \end{cases} \]
- \[ \begin{cases} y = 4.5x + 15 \\ y = 5x + 12.5 \end{cases} \]
- \[ \begin{cases} y = 3x \\ x + y = 52 \end{cases} \]

1. Without solving, identify 3 systems that you think would be the least difficult for you to solve and 3 systems you think would be the most difficult. Be prepared to explain your reasoning.

2. Choose 4 systems to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Student Response

1. Answers vary.

2. Possible solutions.
   a. \( x = 3, y = 0 \)
   b. \( x = 16, y = 76 \)
   c. \( x = 6\frac{1}{2}, y = \frac{1}{3} \)
d. No solution

e. Infinitely many solutions

f. \( x = 15, y = 3.6 \)
g. \( x = 5, y = 37.5 \)
h. \( x = 13, y = 39 \)

**Activity Synthesis**

There are two key takeaways from this discussion. The first is to reinforce that some systems can be solved by reasoning whether its possible for a solution to one equation to also be a solution to another. The second takeaway is that there are some systems that students will only be able to solve after learning techniques in future grades.

For each problem, ask students to indicate if they identified the system as least or most difficult. Record the responses for all to see.

Bring students' attention to this system:

\[
\begin{align*}
4y + 7x &= 6 \\
4y + 7x &= -5
\end{align*}
\]

Ask students what the two equations in the system have in common to each other and to think about whether a solution to the first equation could also be a solution to the second. One can reason there is no solution because \(4y + 7x\) cannot be equal to both 6 and -5.

Ask students to return to the earlier activity and see if they can find any of those systems in these problems. (Lin’s family ride is the sixth system. Noah’s kayaking trip is the seventh system. Diego’s baking is the eighth system.) After students notice the connection, invite students who chose those systems to solve to interpret the numerical solutions in the contexts from the earlier activity.

Students may be tempted to develop the false impression that all systems where both equations are given as linear combinations can be solved by inspection. Conclude the discussion by displaying this system that students defined in the last activity about sugar and flour:

\[
\begin{align*}
0.8x + 0.5y &= 9 \\
x + y &= 15
\end{align*}
\]

Tell students that this system has one solution and they will learn more sophisticated techniques for solving systems of equations like this in future grades.
Support for English Language Learners

Speaking, Listening, Conversing: MLR8 Discussion Supports. Before the whole-class discussion, use this routine to give students an opportunity to discuss their reasons for labeling systems of equations as “least difficult” or “most difficult.” Provide the following sentence frame: “____ is the least/most difficult to solve because ____.” Encourage listeners to press for detail by asking questions such as “What would make that problem easier to solve?”; “What do your least/most difficult problems have in common?” This will help students make connections between the structures of given systems of equations, and possible strategies they can use to solve them.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

To wrap up the lessons on solving systems of equations, consider displaying the three systems of equations and asking students how they might begin to solve the systems.

\[
\begin{align*}
\begin{cases} 
 y &= 2x + 1 \\
 y &= \frac{1}{2}x + 10 
\end{cases} & \text{(Both graphing and substitution methods work well)} \\
\begin{cases} 
 x &= 5 - 2y \\
 2x + 6y &= 16 
\end{cases} & \text{(Substitution works best)} \\
\begin{cases} 
 5x + 4y &= 20 \\
 10x + 8y &= 60 
\end{cases} & \text{(Inspection may work best)}
\end{align*}
\]

If there is time, consider assigning each system to small groups for them to solve, then share their solutions with the class.

15.5 Solve This

Cool Down: 5 minutes

This cool-down asks students to solve a system of equations that has rational numbers in the system equation as well as in the solution. Watch for students who use the substitution method correctly even if they get stuck working with fractions.

Addressing

- 8.EE.C.8

Student Task Statement

Solve
\[
\begin{cases}
y = \frac{3}{4}x \\
\frac{5}{2}x + 2y = 5
\end{cases}
\]

**Student Response**

\(x = \frac{5}{4}, y = \frac{15}{16}\)

**Student Lesson Summary**

We have learned how to solve many kinds of systems of equations using algebra that would be difficult to solve by graphing. For example, look at

\[
\begin{cases}
y = 2x - 3 \\
x + 2y = 7
\end{cases}
\]

The first equation says that \(y = 2x - 3\), so wherever we see \(y\), we can substitute the expression \(2x - 3\) instead. So the second equation becomes \(x + 2(2x - 3) = 7\).

We can solve for \(x\):

\[
\begin{align*}
x + 4x - 6 &= 7 \\
5x - 6 &= 7 \\
5x &= 13 \\
x &= \frac{13}{5}
\end{align*}
\]

We know that the \(y\) value for the solution is the same for either equation, so we can use either equation to solve for it. Using the first equation, we get:

\[
\begin{align*}
y &= 2(\frac{13}{5}) - 3 \\
&= \frac{26}{5} - 3 \\
&= \frac{26}{5} - \frac{15}{5} \\
y &= \frac{11}{5}
\end{align*}
\]

If we substitute \(x = \frac{13}{5}\) into the other equation, \(x + 2y = 7\), we get the same \(y\) value. So the solution to the system is \(\left(\frac{13}{5}, \frac{11}{5}\right)\).

There are many kinds of systems of equations that we will learn how to solve in future grades, like

\[
\begin{cases}
2x + 3y = 6 \\
-x + 2y = 3
\end{cases}
\]

Or even

\[
\begin{cases}
y = x^2 + 1 \\
y = 2x + 3
\end{cases}
\]
Lesson 15 Practice Problems

Problem 1

Statement
Kiran and his cousin work during the summer for a landscaping company. Kiran's cousin has been working for the company longer, so his pay is 30% more than Kiran's. Last week his cousin worked 27 hours, and Kiran worked 23 hours. Together, they earned $493.85. What is Kiran's hourly pay? Explain or show your reasoning.

Solution
$8.50. Explanations vary. Sample response: \( n \) = Kiran’s hourly wage and \( c \) = Kiran’s cousin’s hourly wage. \( c = 1.3n \) and \( 27c + 23n = 493.85 \). Substituting 1.3\( n \) for \( c \) yields the equation 
\( 27(1.3n) + 23n = 493.85 \).

Problem 2

Statement
Decide which story can be represented by the system of equations \( y = x + 6 \) and \( x + y = 100 \). Explain your reasoning.

a. Diego’s teacher writes a test worth 100 points. There are 6 more multiple choice questions than short answer questions.

b. Lin and her younger cousin measure their heights. They notice that Lin is 6 inches taller, and their heights add up to exactly 100 inches.

Solution
The second story. Explanations vary. Sample response: In the first story, \( y = x + 6 \) can be written where \( x \) and \( y \) represent the number of questions of each type, but the other fact is about points, so \( x + y = 100 \) does not make sense. In the second story, Lin’s height can be represented by \( y \), and her younger cousin’s height can be represented by \( x \).

Problem 3

Statement
Clare and Noah play a game in which they earn the same number of points for each goal and lose the same number of points for each penalty. Clare makes 6 goals and 3 penalties, ending the game with 6 points. Noah earns 8 goals and 9 penalties and ends the game with -22 points.

a. Write a system of equations that describes Clare and Noah’s outcomes. Use \( x \) to represent the number of points for a goal and \( y \) to represent the number of points for a penalty.
b. Solve the system. What does your solution mean?

Solution
a. Clare: $6x + 3y = 6$, Noah: $8x + 9y = -22$

b. $(4, -6)$. A goal earns 4 points and a penalty earns -6 points.

Problem 4
Statement
Solve: \[
\begin{align*}
  y &= 6x - 8 \\
  y &= -3x + 10
\end{align*}
\]

Solution
$(2, 4)$. First solve $6x - 8 = -3x + 10$ for $x$ and substitute that value into either of the original equations to solve for $y$.

(From Unit 4, Lesson 14.)

Problem 5
Statement
a. Estimate the coordinates of the point where the two lines meet.

b. Choose two equations that make up the system represented by the graph.

i. $y = \frac{5}{4}x$

ii. $y = 6 - 2.5x$
iii. \( y = 2.5x + 6 \)
iv. \( y = 6 - 3x \)
v. \( y = 0.8x \)

c. Solve the system of equations and confirm the accuracy of your estimate.

**Solution**

a. Answers vary. Sample response: \((1.8, 1.4)\)

b. b. and e

c. \( x \approx 1.82, y \approx 1.46 \) (the exact values are \( x = \frac{20}{11} \) and \( y = \frac{16}{11} \)). Find the \( x \) coordinate of the intersection point by solving \( 6 - 2.5x = 0.8x \). To find the \( y \) coordinate, substitute this value of \( x \) into either equation.

(From Unit 4, Lesson 13.)
Section: Let's Put It to Work

Lesson 16: Solving Problems with Systems of Equations

Goals

- Calculate the solution to a system of equations in context, and present (using words and other representations) the solution method.
- Create a system of equations to solve a problem in context.
- Critique (orally) peer solutions to a system of equations.

Learning Targets

- I can use a system of equations to represent a real-world situation and answer questions about the situation.

Lesson Narrative

In this final lesson on systems of equations, students work in groups as they apply what they have learned to solve three problems with different structures and then create a new problem similar in structure to one of the ones they solved. Groups trade problems, prepare well-explained solutions, and take turns sharing their solutions with the class. While groups share, ask other students to interpret particular aspects of the presentation, such as the slope of a graph, the coefficient of a variable, or the solution to a system, in terms of the context of the problem. When students present solutions and critique other students’ solutions, they engage in MP3.

Alignments

Building On

- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time.

Addressing

- 8.EE.C.8.c: Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Instructional Routines
- Group Presentations
- MLR8: Discussion Supports

Required Materials

Tools for creating a visual display  
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation
Provide access to tools for making a visual display to each group of 2.

Student Learning Goals
Let's solve some gnarly problems.

16.1 Are We There Yet?

Warm Up: 5 minutes
The purpose of this warm-up is to get students to reason about representing a context about distance as an expression. For students who use the equation $d = rt$ to choose their answer, encourage them to explain how each part of the expression matches the context (MP3).

Building On
- 6.EE.C.9

Launch
Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement
A car is driving towards home at 0.5 miles per minute. If the car is 4 miles from home at $t = 0$, which of the following can represent the distance that the car has left to drive?

- $0.5t$
- $4 + 0.5t$
- $4 - 0.5t$
- $4 \cdot (0.5t)$

Student Response
The expression $4 - 0.5t$ represents the distance the car has left to drive towards home. Since $0.5t$ represents the distance traveled toward home, subtracting that from the 4 miles the car is from home will give us the distance left to drive.
Activity Synthesis
Poll students on which expression they chose. Ask students who chose any expression, right or wrong, to explain their reasoning. After each explanation ask the rest of the class if they agree or disagree and how the context is represented in the expression.

16.2 Cycling, Fundraising, Working, and ___?
35 minutes
In this activity, students reason about situations involving two different relationships between the same two quantities. Then they invent their own problem of the same type. While students are encouraged by the language of the activity to use a system of equations to solve the problems, they may elect to use a different representation to explain their thinking (MP1, MP3).

As students work through the first three problems, notice the ways students reason about the problems with and without systems of equations. Identify some groups with particularly compelling or clear reasoning to share later.

Addressing
- 8.EE.C.8.c

Instructional Routines
- Group Presentations
- MLR8: Discussion Supports

Launch
Arrange students in groups of 2. Provide tools for creating a visual display.

Once students have completed the first three problems, select previously identified groups to share their solutions. Bring out why these solutions are particularly good (i.e., clarity, efficiency), and discuss the connections between them, particularly the connections between groups that did and did not use systems of equations where possible. Next, have students begin the second part of the activity and write their own problem to trade with another group.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2–3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far, as well as any questions they have, before continuing.

Supports accessibility for: Organization; Attention
Support for English Language Learners

*Representing, Conversing: MLR8 Discussion Supports.* Use this routine to help students consider audience when preparing a visual display of their work. When students create their displays for the problems they invent, invite them to consider how to display their strategies so that another student can interpret them. For example, students may wish to add notes or details to their equations to help communicate their thinking. When students receive the problem created by another group, provide 2–3 minutes of quiet think time for students to read and interpret the display before they begin their small-group discussion. During whole-group discussion, ask students to share the kinds of details that were most helpful when interpreting another group’s display.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

Anticipated Misconceptions

If students struggle to write a system of equations, ask them to identify any unknown quantities in the problem and assign variables to them. Then ask them if there are ways to describe the relationships between the variables. If students still struggle to think about the relationships, ask them about some possible values for each of the variables including some that make sense (such as 20 grapefruits) and some that do not (such as 1,000 grapefruits). Have students explain why some values are not possible to help them understand the relationships between variables.

Student Task Statement

Solve each problem. Explain or show your reasoning.

1. Two friends live 7 miles apart. One Saturday, the two friends set out on their bikes at 8 am and started riding towards each other. One rides at 0.2 miles per minute, and the other rides at 0.15 miles per minute. At what time will the two friends meet?

2. Students are selling grapefruits and nuts for a fundraiser. The grapefruits cost $1 each and a bag of nuts cost $10 each. They sold 100 items and made $307. How many grapefruits did they sell?

3. Jada earns $7 per hour mowing her neighbors’ lawns. Andre gets paid $5 per hour for the first hour of babysitting and $8 per hour for any additional hours he babysits. What is the number of hours they both can work so that they get paid the same amount?

4. Pause here so your teacher can review your work. Then, invent another problem that is like one of these, but with different numbers. Solve your problem.

5. Create a visual display that includes:
   - The new problem you wrote, without the solution.
   - Enough work space for someone to show a solution.
6. Trade your display with another group, and solve each other's new problem. Make sure that you explain your solution carefully. Be prepared to share this solution with the class.

7. When the group that got the problem you invented shares their solution, check that their answer is correct.

**Student Response**

1. The friends will meet at 8:20. Students may write and solve this system: \[ \begin{align*}
    y &= 0.2x \\
    y &= -0.15x + 7
\end{align*} \]

2. They sold 77 grapefruit. Students may write and solve this system: \[ \begin{align*}
    x + 10y &= 307 \\
    x + y &= 100
\end{align*} \]

3. Jada and Andre will both earn 21 dollars for working 3 hours. Students may write and solve this system: \[ \begin{align*}
    y &= 7x \\
    y &= 5 + 8(x - 1)
\end{align*} \]

4. Answers vary.

5. Answers vary.

6. Answers vary.

7. Answers vary.

**Are You Ready for More?**

On a different Saturday, two friends set out on bikes at 8:00 am and met up at 8:30 am. (The same two friends who live 7 miles apart.) If one was riding at 10 miles per hour, how fast was the other riding?

**Student Response**

The other friend is riding 4 miles per hour.

Sample response: 10 miles an hour is equivalent to 5 miles in a half hour. The first friend will be at mile 2 in a half hour because \( 2 = -10(0.5) + 7 \). The second friend will arrive at mile 2 in a half hour which is equivalent to 4 miles an hour. This situation can be represented by the system:

\[ \begin{align*}
    y &= -10x + 7 \\
    y &= 4x
\end{align*} \]

Students may use a graph:
**Activity Synthesis**

Most of the discussions happen within and between groups, but the last question requires a whole-class discussion. Have each group share the peer-generated question they were assigned and the solution. Though the group that wrote the question will be responsible for confirming the answer, encourage all to listen to the reasoning each group used.

Alternatively, after groups have checked the work of the group that solved their problem, have students complete a gallery walk to see all the created problem. Ask students to look for situations similar to theirs and to identify the most common solution methods used. After the gallery, select a few groups to share a problem and how they solved it.
Family Support Materials
Family Support Materials

Linear Equations and Linear Systems

Here are the video lesson summaries for Grade 8, Unit 4: Linear Equations and Linear Systems. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

<table>
<thead>
<tr>
<th>Grade 8, Unit 4: Linear Equations and Linear Systems</th>
<th>Vimeo</th>
<th>YouTube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video 1: Solving Linear Equations in One Variable (Lessons 1–4)</td>
<td>Link</td>
<td>Link</td>
</tr>
<tr>
<td>Video 2: Solving Any Linear Equation (Lessons 5–6)</td>
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<td>Video 3: Equations with Different Numbers of Solutions (Lessons 7–8)</td>
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<td>Video 4: Systems of Equations (Lessons 10–12)</td>
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Video 1

Video 'VLS G8U4V1 Solving Linear Equations in One Variable (Lessons 1–4)' available here: https://player.vimeo.com/video/481928840.
Video 2


Video 3


Video 4


Video 5

Puzzle Problems
Family Support Materials 1
This week your student will work on solving linear equations. We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on either side have equal value, just like a balanced hanger has equal weights on either side.

If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance.

\[ a = 3b \]

\[ a + 2b = 5b \]

We can do this with equations as well: adding or subtracting the same amount from both sides of an equation keeps the sides equal to each other. For example, if \(4x + 20\) and \(-6x + 10\) have equal value, we can write an equation \(4x + 20 = -6x + 10\). We could add -10 to both sides of the equation or divide both sides of the equation by 2 and keep the sides equal to each other. Using these moves in systematic ways, we can find that \(x = -1\) is a solution to this equation.

Here is a task to try with your student:

Elena and Noah work on the equation \(\frac{1}{2}(x + 4) = -10 + 2x\) together. Elena’s solution is \(x = 24\) and Noah’s solution is \(x = -8\). Here is their work:
Elena:
\[
\frac{1}{2}(x + 4) = -10 + 2x
\]
\[
x + 4 = -20 + 2x
\]
\[
x + 24 = 2x
\]
\[
24 = x
\]
\[
x = 24
\]

Noah:
\[
\frac{1}{2}(x + 4) = -10 + 2x
\]
\[
x + 4 = -20 + 4x
\]
\[
-3x + 4 = -20
\]
\[
-3x = -24
\]
\[
x = -8
\]

Do you agree with their solutions? Explain or show your reasoning.

Solution:

No, they both have errors in their solutions.

Elena multiplied both sides of the equation by 2 in her first step, but forgot to multiply the 2x by the 2. We can also check Elena’s answer by replacing x with 24 in the original equation and seeing if the equation is true.

\[
\frac{1}{2}(x + 4) = -10 + 2x
\]
\[
\frac{1}{2}(24 + 4) = -10 + 2(24)
\]
\[
\frac{1}{2}(28) = -10 + 48
\]
\[
14 = 38
\]

Since 14 is not equal to 38, Elena’s answer is not correct.

Noah divided both sides by -3 in his last step, but wrote -8 instead of 8 for \(-24 \div -3\). We can also check Noah’s answer by replacing x with -8 in the original equation and seeing if the equation is true. Noah’s answer is not correct.
Systems of Linear Equations

Family Support Materials 2

This week your student will work with systems of equations. A system of equations is a set of 2 (or more) equations where the letters represent the same values. For example, say Car A is traveling 75 miles per hour and passes a rest area. The distance in miles it has traveled from the rest area after \( t \) hours is \( d = 75t \). Car B is traveling toward the rest area and its distance from the rest area at any time is \( d = 14 - 65t \). We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is “yes,” then the solution will correspond to one point that is on both lines, such as the point \((0.1, 7.5)\) shown here. 0.1 hours after Car A passes the rest area, both cars will be 7.5 miles from the rest area.

We could also answer the question without using a graph. Since we are asking when the \( d \) values for each car will be the same, we are asking for what \( t \) value, if any, makes \( 75t = 14 - 65t \) true. Solving this equation for \( t \), we find that \( t = 0.1 \) is a solution and at that time the cars are 7.5 miles away since \( 75t = 75 \cdot 0.1 = 7.5 \). This finding matches the graph.

Here is a task to try with your student:

Lin and Diego are biking the same direction on the same path, but start at different times. Diego is riding at a constant speed of 18 miles per hour, so his distance traveled in miles can be represented by \( d \) and the time he has traveled in hours by \( t \), where \( d = 18t \). Lin started riding a quarter hour before Diego at a constant speed of 12 miles per hour, so her total distance traveled in miles can be represented by \( d \), where \( d = 12 \left( t + \frac{1}{4} \right) \). When will Lin and Diego meet?

Solution:
To find when Lin and Diego meet, that is, when they have traveled the same total distance, we can set the two equations equal to one another: \(18t = 12 \left( t + \frac{1}{4} \right)\). Solving this equation for \(t\),

\[
18t = 12t + 3 \\
6t = 3 \\
t = \frac{1}{2}
\]

They meet after Diego rides for one half hour and Lin rides for three quarters of an hour. The distance they each travel before meeting is 9 miles, since \(9 = 18 \cdot \frac{1}{2}\). Another way to find a solution would be to graph both \(d = 18t\) and \(d = 12 \left( t + \frac{1}{4} \right)\) on the same coordinate plane and interpret the point where these lines intersect.
Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B
Linear Equations and Linear Systems: Check Your Readiness (A)

1. Which of these expressions is equivalent to $3(x - 2)$?
   
   A. $3x - 6$
   
   B. $3x - 2$
   
   C. $3x + 2$
   
   D. $3x + 6$

2. Which of these expressions is equivalent to $-2(x - 5)$?
   
   A. $-2x - 5$
   
   B. $-2x + 5$
   
   C. $-2x + 10$
   
   D. $-2x - 10$

3. For each expression, combine like terms and write an equivalent expression with fewer terms.
   
   a. $4x + 3x$
   
   b. $3x + 5x - 1$
   
   c. $5 + 2x + 7 + 4x$
   
   d. $4 - 2x + 5x$
   
   e. $10x - 5 + 3x - 2$
4. For each equation, find a value for $x$ that makes the equation true.

a. $x \div 3 = 12$

b. $2x + 3 = 20$

c. $\frac{4}{3}x = \frac{10}{3}$

d. $-4x = -24$

e. $2(x - 4) = 10$

f. $-0.5x + 1.1 = -2.9$
5. For each equation, determine if \( x = 2 \) is a solution. Explain or show your reasoning.

a. \(-2(x - 4) = 4\)

b. \(\frac{26}{x} = 13\)

c. \(-3.8x = -7.4\)

d. \(4(x - 1) - 3(x - 2) = -8\)
Linear Equations and Linear Systems: Check Your Readiness (B)

1. Which expression is equivalent to $6(2y - 5)$?
   A. $12y - 5$
   B. $5 - 12y$
   C. $12y - 30$
   D. $30 - 12y$

2. Which of these expressions is equivalent to $-4(5 - 2x)$?
   A. $-20 + 8x$
   B. $-20 - 8x$
   C. $-20x + 8$
   D. $20x + 8$

3. For each expression, combine like terms and write an equivalent expression with fewer terms.
   a. $-2x + 4 - 5x$
   b. $16 + x + 5$
   c. $3x - 5x + 7 - 2$
   d. $1 - 7x - 3x - 1$
   e. $6x + 4x$
4. For each equation, find a value for $x$ that makes the equation true.

a. $65 = -5x$

b. $10 = 3x - 6$

c. $3(10 - x) = 6$

d. $\frac{5}{4}x = \frac{9}{4}$

e. $\frac{1}{2}(x - 6) = 7$

f. $1.5x - 2.4 = 0.6$
5. Why is -3 a solution to each of these questions?

\[
\frac{1}{3}x - 6 = -7 \\
2(x + 12) = 18
\]
Linear Equations and Linear Systems: End-of-Unit Assessment (A)

You may use graph paper and a four-function or scientific calculator, but not a graphing calculator.

1. Here is a balanced hanger diagram:

   A circle has a mass of 3 grams and a square has a mass of 2 grams. Which is the mass of a triangle?

   A. $\frac{9}{3}$ grams
   B. $\frac{11}{3}$ grams
   C. 3 grams
   D. 7 grams

2. Select all the systems of equations that have exactly one solution.

   A. \[
   \begin{align*}
   y &= 3x + 1 \\
   y &= -3x + 7
   \end{align*}
   \]
   B. \[
   \begin{align*}
   y &= 3x + 1 \\
   y &= x + 1
   \end{align*}
   \]
   C. \[
   \begin{align*}
   y &= 3x + 1 \\
   y &= 3x + 7
   \end{align*}
   \]
   D. \[
   \begin{align*}
   x + y &= 10 \\
   2x + 2y &= 20
   \end{align*}
   \]
   E. \[
   \begin{align*}
   x + y &= 10 \\
   x + y &= 12
   \end{align*}
   \]
3. Here is the graph for one of the equations in a system of two equations.

The solution to the system is (6, 2). Select all the equations that could be the other equation in the system.

A. $y = -3x$
B. $y = \frac{-3}{2}x + 6$
C. $y = \frac{-1}{6}x + 3$
D. $y = \frac{2}{3}x - 1$
E. $y = \frac{1}{2}x - 1$
F. $y = 4x - 2$

4. Solve this equation. Explain or show your reasoning.

$$\frac{1}{2}x - 7 = \frac{1}{3}(x - 12)$$
5. Solve this system of equations.

\[
\begin{align*}
3x + 4y &= 36 \\
y &= -\frac{1}{2}x + 8
\end{align*}
\]

6. Andre and Elena are each saving money. Andre starts with $100 in his savings account and adds $5 per week. Elena starts with $10 in her savings account and adds $20 each week.

a. After four weeks, who has more money in their savings account? Explain how you know.

b. After how many weeks will Andre and Elena have the same amount of money in their savings accounts?
7. 60 people attend a game night. Everyone chooses to play chess, a two-player game, or cribbage, a four-player game. All 60 people are playing either chess or cribbage.

a. Complete this table showing some possible combinations of the number of each type of game being played:

<table>
<thead>
<tr>
<th>chess games (x)</th>
<th>cribbage games (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

b. There are 3 more games of cribbage being played than games of chess being played. How many of each game are being played? Explain or show your reasoning.
Linear Equations and Linear Systems: End-of-Unit Assessment (B)

You may use graph paper and a four-function or scientific calculator, but not a graphing calculator.

1. Select all the equations that have no solution.
   A. \( x + 6 = 5 + x \)
   B. \(-2(x - 3) = -2x + 6\)
   C. \(4 - 4x = 3x + 2\)
   D. \(4(x + 1) = 3(x + 2)\)
   E. \(5 - 3x = -3x + 4\)

2. What is the solution to \(3x + 30 + x = 10 + 2x + 5x + 2\)?
   A. 18
   B. 6
   C. \(\frac{9}{2}\)
   D. -2

3. Which of the systems of equations has a solution of (-2, 6)?
   A. \[
   \begin{align*}
   y &= 2x + 10 \\
   y &= 3x - 6 \\
   \end{align*}
   \]
   B. \[
   \begin{align*}
   y - 2x &= 2 \\
   y + 2x &= 10 \\
   \end{align*}
   \]
   C. \[
   \begin{align*}
   x - y &= 9 \\
   y &= x + 9 \\
   \end{align*}
   \]
   D. \[
   \begin{align*}
   x + 2y &= 10 \\
   -4x - y &= 2 \\
   \end{align*}
   \]
4. Mai solved the equation below incorrectly. Identify Mai’s error and then solve the equation correctly.

\[
\frac{1}{3} \left( \frac{1}{2} x - 9 \right) = \frac{1}{2} (x + 18)
\]

\[
\frac{1}{6} x - 3 = \frac{1}{2} x + 9
\]

\[
\frac{1}{4} x - 3 = 9
\]

\[
\frac{1}{4} x = 12
\]

\[
x = 48
\]

5. Solve this system of equations. \[
\begin{align*}
\frac{1}{2} x + 6y &= 12 \\
y &= x + 15
\end{align*}
\]
6. Lin and Han’s families provide money for their school lunch accounts.

- Han starts with $100 in his account. He spends $15 each week on lunches.
- Lin starts with $25 in her account and does not spend any of it. Her family adds $10 each week to her account.

a. At the end of 4 weeks, who has more money in their account?

b. After how many weeks will Han and Lin have the same amount of money in their lunch accounts?
7. There are 50 athletes signed up for a neighborhood basketball competition. Players can select to play in the 6-player games (“3 on 3”) or the 2-player games (“1 on 1”).

a. All 50 athletes sign up for only one kind of game. Complete the table to show different combinations of games that could be played.

<table>
<thead>
<tr>
<th>number of 6-player games</th>
<th>number of 2-player games</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

b. If the neighborhood holds 13 total games and all 50 athletes participate, how many 6-player games and how many 2-player games are played?
CKMath™
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Assessment Answer Keys
Check Your Readiness A and B
End-of-Unit Assessment A and B
Assessments

Assessment: Check Your Readiness (A)

Teacher Instructions
In many ways, this unit is a continuation of the previous unit. This pre-unit diagnostic assessment is shorter than usual, since much of what students need to know has been recently assessed.

Problem 1
The content assessed in this problem is first encountered in Lesson 3: Balanced Moves.

The distributive property will prove to be an important tool in solving linear equations.

If most students struggle with this item, plan to revisit it before Activity 3 to review using the distributive property. Use hanger diagrams as a context for reviewing the distributive property. Monitor student strategies during the Activity for students who need more practice writing equivalent expressions using the distributive property. Another opportunity to practice using the distributive property appears in Lesson 7.

Statement
Which of these expressions is equivalent to $3(x - 2)$?

A. $3x - 6$
B. $3x - 2$
C. $3x + 2$
D. $3x + 6$

Solution
A

Aligned Standards
6.EE.A.3

Problem 2
The content assessed in this problem is first encountered in Lesson 6: Strategic Solving.

Like the previous question, this question assesses the distributive property. This time, the work involves more difficult multiplication by negative numbers.
If most students struggle with this item, plan to revisit it with students before Activity 2, or use this item as the Warm-Up. Ask students to choose a few values to substitute for $x$ to verify their choice of equivalent expressions. (Note: This method is not sufficient for judging equivalent expressions, but will give students practice with operations with rational numbers.)

**Statement**

Which of these expressions is equivalent to $-2(x - 5)$?

A. $-2x - 5$
B. $-2x + 5$
C. $-2x + 10$
D. $-2x - 10$

**Solution**

C

**Aligned Standards**

6.EE.A.3

**Problem 3**

The content assessed in this problem is first encountered in Lesson 2: Keeping the Equation Balanced.

Parts a through c of this problem simply involve recognizing which are “like terms.” Parts d and e require understanding to which terms the negative signs apply. Students will need to be comfortable combining like terms when solving linear equations.

If most students struggle with this item, plan to ask students to write expressions in Activity 3 in different ways, using the hanger diagram to emphasize that $x + 4x$ is the same as $5x$. Each hanger diagram in the lessons offers an opportunity to write equivalent expressions that support students in combining like terms.

**Statement**

For each expression, combine like terms and write an equivalent expression with fewer terms.

1. $4x + 3x$
2. $3x + 5x - 1$
3. $5 + 2x + 7 + 4x$
4. $4 - 2x + 5x$

5. $10x - 5 + 3x - 2$

**Solution**

1. $7x$

2. $8x - 1$

3. $6x + 12$

4. $4 + 3x$

5. $13x - 7$

**Aligned Standards**

6.EE.A.3

**Problem 4**

The content assessed in this problem is first encountered in Lesson 3: Balanced Moves.

In part d, students may not know how to handle the two negative signs. Part e requires either distributing the left side or dividing each side by 2 as the first step. Part f requires attention to detail with both the decimals and the negative numbers.

This is also an opportunity to check in with students’ calculations on decimals and fractions. Encourage students to work on these problems without a calculator.

If most students struggle with this item, plan to revisit this item or one-and two-step equations before this lesson and after students have worked with the hanger diagrams. Work with the hangers should support students with making decisions about equation-solving moves.

**Statement**

For each equation, find a value for $x$ that makes the equation true.

1. $x \div 3 = 12$

2. $2x + 3 = 20$

3. $\frac{4}{3}x = \frac{10}{3}$

4. $-4x = -24$

5. $2(x - 4) = 10$

Unit 4: Linear Equations and Linear Systems
6. \(-0.5x + 1.1 = -2.9\)

**Solution**

1. \(x = 36\)
2. \(x = \frac{17}{2}\) or equivalent
3. \(x = \frac{5}{2}\) or equivalent
4. \(x = 6\)
5. \(x = 9\)
6. \(x = 8\)

**Aligned Standards**

6.EE.B.7, 7.EE.B.4.a

**Problem 5**

The content assessed in this problem is first encountered in Lesson 2: Keeping the Equation Balanced.

Before beginning work with systems of equations, students must have a solid handle on what it means for a value to be a solution to an equation. For each of these questions, it is most efficient to substitute 2 for \(x\) and see if the result is a true equation. Some students may instead take algebraic steps to solve each equation. Make sure that students who do the latter are exposed to the substitution strategy, whether through class discussion or individual conversation.

The equation in part c is close: \(2 \cdot (-3.8) = -7.6\), not -7.4.

Students may think that \(x = 2\) is a solution to the equation in part d if they solve the equation algebraically and make a sign error distributing the -3.

If most students struggle with this item, plan to have students check solutions they find by substituting values back into equations beginning in this lesson, emphasizing that a solution to an equation is a value for the variable that makes the equation true.

**Statement**

For each equation, determine if \(x = 2\) is a solution. Explain or show your reasoning.

1. \(-2(x - 4) = 4\)
2. \(\frac{26}{x} = 13\)
3. \(-3.8x = -7.4\)
4. \(4(x - 1) - 3(x - 2) = -8\)
Solution
1. Yes, because \(-2(2 - 4) = 4\).
2. Yes, because \(\frac{26}{2} = 13\).
3. No, because \((-3.8) \cdot 2 \neq -7.4\).
4. No, because \(4(2 - 1) - 3(2 - 2) \neq -8\).

Aligned Standards
6.EE.B.5
Assessment: Check Your Readiness (B)

Teacher Instructions

In many ways, this unit is a continuation of the previous unit. This pre-unit diagnostic assessment is shorter than usual, since much of what students need to know has been recently assessed.

Problem 1

The content assessed in this problem is first encountered in Lesson 3: Balanced Moves.

The distributive property will prove to be an important tool in solving linear equations.

If most students struggle with this item, plan to revisit it before Activity 3 to review using the distributive property. Use hanger diagrams as a context for reviewing the distributive property. Monitor student strategies during the Activity for students who need more practice writing equivalent expressions using the distributive property. Another opportunity to practice using the distributive property appears in Lesson 7.

Statement

Which expression is equivalent to $6(2y - 5)$?

A. $12y - 5$
B. $5 - 12y$
C. $12y - 30$
D. $30 - 12y$

Solution

C

Aligned Standards

6.EE.A.3

Problem 2

The content assessed in this problem is first encountered in Lesson 6: Strategic Solving.

Like the previous question, this question assesses the distributive property. This time, the work involves more difficult multiplication by negative numbers.

If most students struggle with this item, plan to revisit it with students before Activity 2, or use this item as the Warm-Up. Ask students to choose a few values to substitute for $x$ to verify their choice of equivalent expressions. (Note: This method is not sufficient for judging equivalent expressions, but will give students practice with operations with rational numbers.)
Statement
Which of these expressions is equivalent to \(-4(5 - 2x)\)?

A. \(-20 + 8x\)
B. \(-20 - 8x\)
C. \(-20x + 8\)
D. \(20x + 8\)

Solution
A

Aligned Standards
6.EE.A.3

Problem 3
The content assessed in this problem is first encountered in Lesson 2: Keeping the Equation Balanced.

Parts b and e of this problem simply involve recognizing which are “like terms.” Parts a, c, and d require understanding which terms the negative signs apply to. Students will need to be comfortable combining like terms when solving linear equations.

If most students struggle with this item, plan to ask students to write expressions in Activity 3 in different ways, using the hanger diagram to emphasize that \(x+4x\) is the same as \(5x\). Each hanger diagram in the lessons offers an opportunity to write equivalent expressions that support students in combining like terms.

Statement
For each expression, combine like terms and write an equivalent expression with fewer terms.

1. \(-2x + 4 - 5x\)
2. \(16 + x + 5\)
3. \(3x - 5x + 7 - 2\)
4. \(1 - 7x - 3x - 1\)
5. \(6x + 4x\)

Solution
1. \(-7x + 4\)
2. \( x + 21 \)
3. \(-2x + 5\)
4. \(-10x\)
5. \(10x\)

(Equivalent expressions should also be accepted; for example, \(21 + x\) instead of \(x + 21\).)

**Aligned Standards**

6.EE.A.3

**Problem 4**

The content assessed in this problem is first encountered in Lesson 3: Balanced Moves.

Part a and e requires either distributing the left side or dividing each side by the coefficient as the first step. Part f requires attention to detail with both the decimals and the negative numbers. This is also an opportunity to check in with students’ calculations on decimals and fractions. Encourage students to work on these problems without a calculator.

If most students struggle with this item, plan to revisit this item or one-and two-step equations before this lesson and after students have worked with the hanger diagrams. Work with the hangers should support students with making decisions about equation-solving moves.

**Statement**

For each equation, find a value for \( x \) that makes the equation true.

1. \(65 = -5x\)
2. \(10 = 3x - 6\)
3. \(3(10 - x) = 6\)
4. \(\frac{5}{4}x = \frac{9}{4}\)
5. \(\frac{1}{2}(x - 6) = 7\)
6. \(1.5x - 2.4 = 0.6\)

**Solution**

1. \(x = -13\)
2. \(x = \frac{16}{3}\) or equivalent
3. \(x = 8\)
4. $x = \frac{9}{5}$ or equivalent
5. $x = 20$
6. $x = 2$

**Aligned Standards**
6.EE.B.7, 7.EE.B.4.a

**Problem 5**
The content assessed in this problem is first encountered in Lesson 2: Keeping the Equation Balanced.

Before beginning work with systems of equations, students must have a solid handle on what it means for a value to be a solution to an equation. For each of these questions, it is most efficient to substitute -3 for $x$ and see if the result is a true equation. Some students may instead take algebraic steps to solve each equation. Make sure that students who do the latter are exposed to the substitution strategy, whether through class discussion or individual conversation.

If most students struggle with this item, plan to have students check solutions they find by substituting values back into equations beginning in this lesson, emphasizing that a solution to an equation is a value for the variable that makes the equation true.

**Statement**
Why is -3 a solution to each of these questions?

\[
\frac{1}{3}x - 6 = -7 \quad \quad \quad 2(x + 12) = 18
\]

**Solution**
Answers vary. Sample response:

1. $\frac{1}{3} \cdot -3 - 6 = -7, -1 - 6 = -7$
2. $2(-3 + 12) = 18, 2(9) = 18$

**Aligned Standards**
6.EE.B.5

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Unit 4: Linear Equations and Linear Systems
Assessment : End-of-Unit Assessment (A)

Teacher Instructions
Graphing calculators should not be used. Use of a four-function or scientific calculator is acceptable. Provide access to graph paper.

Student Instructions
You may use graph paper and a four-function or scientific calculator, but not a graphing calculator.

Problem 1
Students might tally the masses of all the shapes on each side and solve the equation $2x + 16 = 7 + 5x$. Alternatively, they might first cross out equal numbers of matching shapes on each side before writing an equation, which leads to the equation $9 = 3x$.

Students selecting A may have forgotten to account for the triangles on the left side of the hanger, leading to the equation $16 = 7 + 5x$ or $9 = 5x$. Students selecting B have probably switched the weights of the square and triangle, leading to the equation $2x + 19 = 8 + 5x$ or $11 = 3x$. Students selecting D may have counted how many triangles there are.

Statement
Here is a balanced hanger diagram:

A circle has a mass of 3 grams and a square has a mass of 2 grams. Which is the mass of a triangle?

A. $\frac{9}{5}$ grams
B. $\frac{11}{3}$ grams
C. 3 grams
D. 7 grams

Solution
C

Aligned Standards
8.EE.C
Problem 2

The systems here are deliberately simple, in case students decide to solve all the systems instead of using their knowledge to decide how many solutions they have.

A student who fails to select A may have missed the negative sign in front of \(-3x\). A student who fails to select B has a general misunderstanding of the concept. A student selecting C may not recognize that the lines have the same slope and are therefore parallel. A student selecting D may have found one solution by inspection without noticing there are more. A student selecting E may have found a different solution to each equation.

Statement

Select all the systems of equations that have exactly one solution.

A. \[
\begin{align*}
y &= 3x + 1 \\
y &= -3x + 7
\end{align*}
\]
B. \[
\begin{align*}
y &= 3x + 1 \\
y &= x + 1
\end{align*}
\]
C. \[
\begin{align*}
y &= 3x + 1 \\
y &= 3x + 7
\end{align*}
\]
D. \[
\begin{align*}
x + y &= 10 \\
2x + 2y &= 20
\end{align*}
\]
E. \[
\begin{align*}
x + y &= 10 \\
x + y &= 12
\end{align*}
\]

Solution

["A", "B"]

Aligned Standards

8.EE.C.8.b

Problem 3

There are two approaches to this problem that students are likely to take. One way is to check whether the ordered pair \((6, 2)\) satisfies each equation. Another is to graph each line and check whether it passes through the point \((6, 2)\).

Students selecting A may have probably made a sign error when substituting into the equation. Students selecting B may have drawn in the line but used a slope of \(\frac{2}{3}\) instead of \(\frac{3}{2}\). Students selecting D have selected a line parallel to the given line, but with a different y-intercept: this is a more serious error than the others, since students have learned that parallel lines will never intersect in one point. In choice F, the point \((2, 6)\) is on the line. Students selecting F may have
reversed the coordinates when substituting them into the equation. Students failing to select C or E may either have made a substitution error or graphed the lines incorrectly.

**Statement**

Here is the graph for one of the equations in a system of two equations.

![Graph](image)

The solution to the system is (6, 2). Select all the equations that could be the other equation in the system.

A. \( y = -3x \)
B. \( y = \frac{3}{2} x + 6 \)
C. \( y = \frac{1}{6} x + 3 \)
D. \( y = \frac{2}{3} x - 1 \)
E. \( y = \frac{1}{2} x - 1 \)
F. \( y = 4x - 2 \)

**Solution**

["C", "E"]

**Aligned Standards**

8.EE.C.8.a

**Problem 4**

Watch for students having difficulty with the step of combining terms. Students may be having difficulty understanding that the same steps that work for integers also work for fractions and decimals.
Statement

Solve this equation. Explain or show your reasoning.

\[
\frac{1}{2}x - 7 = \frac{1}{3}(x - 12)
\]

Solution

\(x = 18\).

Use the distributive property to rewrite the equation as \(\frac{1}{2}x - 7 = \frac{1}{3}x - 4\). Then, subtract \(\frac{1}{3}x\) from each side: \(\frac{1}{6}x - 7 = -4\). Add 7 to each side: \(\frac{1}{6}x = 3\).

Then \(x = 3 \div \frac{1}{6} = 3 \cdot 6 = 18\).

Minimal Tier 1 response:

- Work is complete and correct.

Sample:

\[
\begin{align*}
\frac{1}{2}x - 7 &= \frac{1}{3}x - 4 \\
\frac{1}{6}x - 7 &= -4 \\
\frac{1}{6}x &= 3
\end{align*}
\]

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: algebra mistakes not directly related to the work of this unit: incorrectly subtracting or dividing fractions; incorrectly adding or subtracting integers.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: solution given with no work shown; algebra mistakes that are pertinent to the work of this unit: dividing both sides of the equation by \(\frac{1}{3}\) initially, but dividing only \(\frac{1}{2}x\) or -7 by \(\frac{1}{3}\); incorrect use of the distributive property; failure to use inverse operations.

Aligned Standards

8.EE.C.7.b

Problem 5

This system is most likely solved by substitution, but can also be solved by inspection after graphing.

Unit 4: Linear Equations and Linear Systems
Statement

Solve this system of equations.

\[
\begin{align*}
3x + 4y &= 36 \\
y &= -\frac{1}{2}x + 8
\end{align*}
\]

Solution

\[x = 4, \ y = 6\]

Aligned Standards

8.EE.C.7.b, 8.EE.C.8

Problem 6

While the most likely technique is to set up and solve a one-variable equation, it is also possible for students to solve this problem through graphing, or through making a table of values for each person.

Statement

Andre and Elena are each saving money. Andre starts with $100 in his savings account and adds $5 per week. Elena starts with $10 in her savings account and adds $20 each week.

1. After four weeks, who has more money in their savings account? Explain how you know.

2. After how many weeks will Andre and Elena have the same amount of money in their savings accounts?

Solution

1. Andre has more money. Andre has $120, because $120 = 100 + 4 \cdot 5$. Elena has $90, because $90 = 10 + 4 \cdot 20$.

2. 6 weeks. After \( n \) weeks, Andre has \( (100 + 5n) \) dollars in his account, and Elena has \( (10 + 20n) \) dollars in her account. The number of weeks is the solution to \( 100 + 5n = 10 + 20n \). The solution is \( n = 6 \). After 6 weeks, Andre and Elena each have $130 in their savings accounts.

Minimal Tier 1 response:

- Work is complete and correct.

Sample:

1. \( 100 + 4 \cdot 5 = 120 \) and \( 10 + 4 \cdot 20 = 90 \), so Andre has more.

2. Solve \( 100 + 5n = 10 + 20n \). \( n = 6 \), so 6 weeks.

Tier 2 response:
• Work shows general conceptual understanding and mastery, with some errors.
• Sample errors: work in part a is set up correctly but contains arithmetic mistakes; equations in part b are set up correctly but contain algebra mistakes; work contains a correct solution to the equation in part b, but the explanation does not answer the question, “how many weeks?”; in a graphical approach to part b, the equations are graphed correctly but there is an error in spotting the intersection point; in a tabular approach to part b, small arithmetic errors result in an incorrect intersection point or no intersection point.

Tier 3 response:
• Significant errors in work demonstrate lack of conceptual understanding or mastery.
• Sample errors: no sensible approach to calculating the amount of money after 4 weeks; no reasonable equations, tables, or graphs to represent amount of money earned after n weeks; no reasonable method for finding the number of weeks after which Andre and Elena will have the same amount of money.

**Aligned Standards**
8.EE.C.7

**Problem 7**
Most likely, students will solve this system of equations by substitution, but the system can also be solved by graphing or by making a detailed table.

**Statement**
60 people attend a game night. Everyone chooses to play chess, a two-player game, or cribbage, a four-player game. All 60 people are playing either chess or cribbage.

1. Complete this table showing some possible combinations of the number of each type of game being played:

<table>
<thead>
<tr>
<th>chess games (x)</th>
<th>cribbage games (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2. There are 3 more games of cribbage being played than games of chess being played. How many of each game are being played? Explain or show your reasoning.

**Unit 4: Linear Equations and Linear Systems**
Solution

1. Here is the completed table:

<table>
<thead>
<tr>
<th>chess games ((x))</th>
<th>cribbage games ((y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>

2. There were 8 chess games, and 11 cribbage games. Sample explanation: the solution solves the system of equations \(2x + 4y = 60\) and \(y = x + 3\). Solve by substitution:
\[
2x + 4(x + 3) = 60
\]
Then \(x = 8\), and \(y = 11\) because \(y = x + 3\).

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Solutions that simply involve filling out more rows of the table are acceptable.
- Sample:
  
  1. See table.
  2. Solve the system \(2x + 4y = 60\) and \(y = x + 3\).
     
     \[
     2x + 4(x + 3) = 60
     \]
     \[
     2x + 4x + 12 = 60
     \]
     \[
     6x = 48
     \]
     \[
     x = 8
     \]
     \[
     y = 11.
     \]
     8 chess games, 11 cribbage.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one row of the table is incorrect; system of equations is present but work to solve those equations contains algebra errors; equation to represent “3 more games of cribbage than chess” actually represents “3 more games of chess than cribbage.”

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
• Acceptable errors: one of the equations in part b is incorrect because of an error filling out the table in part a.

• Sample errors: several rows of the table are incorrect; a system of equations is present and their solution is correct, but the equations do not come close to representing the situation; a correct system of equations is present but the work is incorrect and does not involve the substitution method.

Tier 4 response:

• Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

• Sample errors: incorrect or missing answer to part b with no system of equations written; misinterpretation of the 4-player game/2-player game constraint means that the table is filled out completely incorrectly.

**Aligned Standards**

8.EE.C.8.c
Assessment : End-of-Unit Assessment (B)

Teacher Instructions
Graphing calculators should not be used. Use of a four-function or scientific calculator is acceptable. Provide access to graph paper.

Student Instructions
You may use graph paper and a four-function or scientific calculator, but not a graphing calculator.

Problem 1
The equations here are deliberately simple, in case students decide to solve all the equations instead of using their knowledge to decide which equations have no solution. Students failing to select A and E may not know that it’s impossible to add different numbers to the same term and get equal results. Choices B and D involve using the distributive property.

Statement
Select all the equations that have no solution.

A. \( x + 6 = 5 + x \)
B. \( -2(x - 3) = -2x + 6 \)
C. \( 4 - 4x = 3x + 2 \)
D. \( 4(x + 1) = 3(x + 2) \)
E. \( 5 - 3x = -3x + 4 \)

Solution
["A", "E"]

Aligned Standards
None

Problem 2
A student who selects A may have subtracted \( x \) from both \( 3x \) and \( x \), as well as subtracted \( 2x \) from both \( 2x \) and \( 5x \). A student who selects C may think that \( x \) represents \( 0x \) instead of \( 1x \). A student who selects D may have divided by 3 improperly.

Statement
What is the solution to \( 3x + 30 + x = 10 + 2x + 5x + 2 \)?
Problem 3

There are two approaches to this problem that students are likely to take. One way is to check whether the ordered pair (-2, 6) satisfies each equation. Another approach might be to solve each system of equations algebraically.

Statement

Which of the systems of equations has a solution of (-2, 6)?

A. \[
\begin{cases}
  y = 2x + 10 \\
  y = 3x - 6
\end{cases}
\]

B. \[
\begin{cases}
  y - 2x = 2 \\
  y + 2x = 10
\end{cases}
\]

C. \[
\begin{cases}
  x - y = 9 \\
  y = x + 9
\end{cases}
\]

D. \[
\begin{cases}
  x + 2y = 10 \\
  -4x - y = 2
\end{cases}
\]

Solution

D

Aligned Standards

8.EE.C.8.b

Problem 4

Watch for students having difficulty with the step of combining terms. Students may be having difficulty understanding that the same steps that work for integers also work for fractions and decimals.

Unit 4: Linear Equations and Linear Systems
Statement
Mai solved the equation below incorrectly. Identify Mai’s error and then solve the equation correctly.

\[
\frac{1}{3} \left( \frac{1}{2} x - 9 \right) = \frac{1}{2} (x + 18)
\]

\[
\frac{1}{6} x - 3 = \frac{1}{2} x + 9
\]

\[
\frac{1}{4} x - 3 = 9
\]

\[
\frac{1}{4} x = 12
\]

\[
x = 48
\]

Solution
Mai incorrectly combined the coefficient, \( \frac{1}{6} - \frac{1}{2} \neq \frac{1}{4} \).

Use the distributive property to rewrite the equation as \( \frac{1}{6} x - 3 = \frac{1}{2} x + 9 \). Then, subtract \( \frac{1}{6} x \) from each side: \(-3 = \frac{1}{3} x + 9\). Subtract 9 from each side: \(-12 = \frac{1}{3} x\). Then divide -12 by \( \frac{1}{3} \): \(-36 = x\).

Minimal Tier 1 Response:

- Mai incorrectly combined the coefficients.
- Work is complete and correct.
- Sample:

\[
\frac{1}{3} \left( \frac{1}{2} x - 9 \right) = \frac{1}{2} (x + 18)
\]

\[
\frac{1}{6} x - 3 = \frac{1}{2} x + 9
\]

\[-3 = \frac{1}{3} x + 9
\]

\[-12 = \frac{1}{3} x
\]

\[-36 = x
\]

Tier 2 Response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: algebra mistakes not directly related to the work of this unit: incorrectly subtracting or dividing fractions; incorrectly adding or subtracting integers.
Tier 3 Response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: solution given with no work shown; algebra mistakes that are pertinent to the work of this unit: dividing both sides of the equation by $\frac{1}{3}$ initially, but dividing only $x$ or 18; incorrect use of the distributive property; failure to use inverse operations.

**Aligned Standards**

8.EE.C.7.b

**Problem 5**

This system is most likely solved by substitution, but can also be solved by inspection after graphing.

**Statement**

Solve this system of equations. \[
\begin{align*}
\frac{1}{3}x + 6y &= 12 \\
y &= x + 15
\end{align*}
\]

**Solution**

$x = -12, y = 3$

**Aligned Standards**

8.EE.C.7.b, 8.EE.C.8

**Problem 6**

While the most likely technique is to set up and solve a one-variable equation, it is also possible for students to solve this problem through graphing, or through making a table of values for each person.

**Statement**

Lin and Han's families provide money for their school lunch accounts.

- Han starts with $100 in his account. He spends $15 each week on lunches.
- Lin starts with $25 in her account and does not spend any of it. Her family adds $10 each week to her account.

1. At the end of 4 weeks, who has more money in their account?

2. After how many weeks will Han and Lin have the same amount of money in their lunch accounts?

**Unit 4: Linear Equations and Linear Systems**
Solution

1. Lin will have more money in her account after 4 weeks. Let \( n \) represent time in weeks and \( y \) represent money in the account. Lin: \( y = 10n + 25 \), \( y = 10 \cdot 4 + 25 \), \( y = 65 \). Lin will have $65 in her account after 4 weeks. Han spends $15 each week on lunch, because \( 5 \cdot 3 = 15 \). Han: \( y = -15n + 100 \), \( y = -15 \cdot 4 + 100 \), \( y = -60 + 100 \), \( y = 40 \). Han will have $40 in his account after 4 weeks.

2. 3 weeks. After \( n \) weeks, Han has \( 100 - 15n \) dollars in his account, and Lin has \( 10n + 25 \) dollars in her account. The solution to \( 100 - 15n = 10n + 25 \) gives \( n \) when there is the same amount in each account. The solution is \( n = 3 \). After 3 weeks, Han and Lin each will have $55 in their lunch accounts.

Minimal Tier 1 response:

- Work is complete and correct.

- Sample:
  
  a. \( 100 - 4 \cdot 15 = 40 \) and \( 10 \cdot 4 + 25 = 65 \), so Lin has more.

  b. Solve \( 100 - 15n = 10n + 25 \). \( n = 3 \), so 3 weeks.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: work in part a is set up correctly but contains arithmetic mistakes; equations in part b are set up correctly but contain algebra mistakes; work contains a correct solution to the equation in part b, but the explanation does not answer the question, “how many weeks?”; in a graphical approach to part b, the equations are graphed correctly but there is an error in spotting the intersection point; in a tabular approach to part b, small arithmetic errors result in an incorrect intersection point or no intersection point.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: no sensible approach to calculating the amount of money after 4 weeks; no reasonable equations, tables, or graphs to represent amount of money earned after \( n \) weeks; no reasonable method for finding the number of weeks after which Han and Lin will have the same amount of money.

Aligned Standards

8.EE.C.7

Problem 7

Students may solve a system of equations by substitution, but the system can also be solved by graphing or by making a detailed table.
**Statement**

There are 50 athletes signed up for a neighborhood basketball competition. Players can select to play in the 6-player games ("3 on 3") or the 2-player games ("1 on 1").

1. All 50 athletes sign up for only one kind of game. Complete the table to show different combinations of games that could be played.

<table>
<thead>
<tr>
<th>number of 6-player games</th>
<th>number of 2-player games</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

2. If the neighborhood holds 13 total games and all 50 athletes participate, how many 6-player games and how many 2-player games are played?

**Solution**

1.

<table>
<thead>
<tr>
<th>number of 6-player games</th>
<th>number of 2-player games</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

2. There would need to be 6 six-player games and 7 two-player games.

Minimal Tier 1 response:
- Work is complete and correct, with complete explanation or justification.
- Solutions that simply involve filling out more rows of the table are acceptable.
- Sample:
  1. See table.
  2. Let \( x \) represent number of 6-player games and \( y \) represent number of 2-player games. Solve the system \( 6x + 2y = 50 \) and \( x + y = 13 \)

Unit 4: Linear Equations and Linear Systems
$6x + 2(-x + 13) = 50$
$6x - 2x + 26 = 50$
$4x + 26 = 50$
$4x = 24$
$x = 6$
$6 + y = 13$
$y = 7$

Tier 2 response:
- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one row of the table is incorrect; system of equations is present but work to solve those equations contains algebra errors such as $6x + 2y = 13$ or $x + y = 50$.

Tier 3 response:
- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Acceptable errors: one of the equations in part b is incorrect because of an error filling out the table in part a.
- Sample errors: several rows of the table are incorrect; a system of equations is present and their solution is correct, but the equations do not come close to representing the situation; a correct system of equations is present, but the work is incorrect and does not involve the substitution method.

Tier 4 response:
- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: incorrect or missing answer to part b with no system of equations written; misinterpretation of the "3 on 3" and "1 on 1" division games constraint means that the table is filled out completely incorrectly.

**Aligned Standards**

8.EE.C.8.c
Lesson
Cool Downs
Lesson 1: Number Puzzles

Cool Down: Seeing the Puzzle

Andre and Elena are reading the same book over the summer. Andre says he has read \( \frac{1}{5} \) of the book. Elena says she has read 20 more pages than Andre. If Elena is on page 55, how many pages are in the book?

Lin has drawn a diagram to solve this question. Find her error.

\[
\begin{array}{cccc}
\text{a} & 20 & 55 & 55 & 55 \\
\end{array}
\]

55
Lesson 2: Keeping the Equation Balanced

Cool Down: Changing Blocks

Here is a hanger that is in balance. We don’t know how much any of its shapes weigh. How could you change the number of shapes on it, but keep it in balance? Describe in words or draw a new diagram.
Lesson 3: Balanced Moves

Cool Down: More Matching Moves

Match these equation balancing steps with the description of what was done in each step.

Step 1: \(12x - 6 = 10\) \(6x - 3 = 5\)
Step 2: \(6x = 8\)
Step 3: \(x = \frac{4}{3}\)

Descriptions to match with each step:

A: Add 3 to both sides
B: Multiply both sides by \(\frac{1}{6}\)
C: Divide both sides by 2
Lesson 4: More Balanced Moves

Cool Down: Mis-Steps

Lin solved the equation $8(x - 3) + 7 = 2x(4 - 17)$ incorrectly. Find the errors in her solution. What should her answer have been?

Lin's solution:

\[ 8(x - 3) + 7 = 2x(4 - 17) \]
\[ 8(x - 3) + 7 = 2x(13) \]
\[ 8x - 24 + 7 = 26x \]
\[ 8x - 17 = 26x \]
\[ -17 = 34x \]
\[ \frac{1}{2} = x \]
Lesson 5: Solving Any Linear Equation

Cool Down: Check It

Noah wanted to check his solution of $x = \frac{14}{5}$ for the equation $\frac{1}{2}(7x - 6) = 6x - 10$.

Substituting $\frac{14}{5}$ for $x$, he writes the following:

\[
\begin{align*}
\frac{1}{2} \left(7 \left(\frac{14}{5}\right) - 6\right) &= 6 \left(\frac{14}{5}\right) - 10 \\
\left(7 \left(\frac{14}{5}\right) - 6\right) &= 12 \left(\frac{14}{5}\right) - 20 \\
5 \left(7 \left(\frac{14}{5}\right) - 6\right) &= 5 \left(12 \left(\frac{14}{5}\right) - 20\right) \\
7 \cdot 14 - 6 &= 12 \cdot 14 - 20 \\
98 - 6 &= 168 - 20 \\
92 &= 148
\end{align*}
\]

Find the incorrect step in Noah's work and explain why it is incorrect.
Lesson 6: Strategic Solving

Cool Down: Think Before You Step

1. Without solving, identify whether this equation has a solution that is positive, negative, or zero:

   \[ 3x - 5 = -3 \]

2. Solve the equation.

   \[ x - 5(x - 1) = x - (2x - 3) \]
Lesson 7: All, Some, or No Solutions

Cool Down: Choose Your Own Solution

\[3x + 8 = 3x+\]

What value could you write in after 3x that would make the equation true for:

1. no values of \(x\)?

2. all values of \(x\)?

3. just one value of \(x\)?
Lesson 8: How Many Solutions?

Cool Down: How Does She Know?

Elena began to solve this equation:

\[
\frac{12x + 6(4x + 3)}{3} = 2(6x + 4) - 2
\]

\[
12x + 6(4x + 3) = 3(2(6x + 4) - 2)
\]

\[
12x + 6(4x + 3) = 6(6x + 4) - 6
\]

\[
12x + 24x + 18 = 36x + 24 - 6
\]

When she got to the last line she stopped and said the equation is true for all values of \(x\). How could Elena tell?
Lesson 9: When Are They the Same?

Cool Down: Printers and Ink

To own and operate a home printer, it costs $100 for the printer and an additional $0.05 per page for ink. To print out pages at an office store, it costs $0.25 per page. Let \( p \) represent number of pages.

1. What does the equation \( 100 + 0.05p = 0.25p \) represent?

2. The solution to that equation is \( p = 500 \). What does the solution mean?
Lesson 10: On or Off the Line?

Cool Down: Another Pocket Full of Change

On the coordinate plane shown, one line shows combinations of dimes and quarters that are worth $3. The other line shows combinations of dimes and quarters that total to 12 coins.

1. Name one combination of 12 coins shown on the graph.

2. Name one combination of coins shown on the graph that total to $3.

3. How many quarters and dimes would you need to have both 12 coins and $3 at the same time?
Lesson 11: On Both of the Lines

Cool Down: Saving Cash

Andre and Noah started tracking their savings at the same time. Andre started with $15 and deposits $5 per week. Noah started with $2.50 and deposits $7.50 per week. The graph of Noah's savings is given and his equation is \( y = 7.5x + 2.5 \), where \( x \) represents the number of weeks and \( y \) represents his savings.

Write the equation for Andre's savings and graph it alongside Noah's. What does the intersection point mean in this situation?
Lesson 12: Systems of Equations

Cool Down: Milkshakes, Revisited

Determined to finish her milkshake before Diego, Lin now drinks her 12 ounce milkshake at a rate of $\frac{1}{3}$ an ounce per second. Diego starts with his usual 20 ounce milkshake and drinks at the same rate as before, $\frac{2}{3}$ an ounce per second.

1. Graph this situation on the axes provided.

2. What does the graph tell you about the situation and how many solutions there are?
Lesson 13: Solving Systems of Equations

Cool Down: Two Lines

1. Given the lines shown here, what are two possible equations for this system of equations?

2. How many solutions does this system of equations have? Explain your reasoning.
Lesson 14: Solving More Systems

Cool Down: Solve It

Solve this system of equations:

\[
\begin{align*}
y &= 2x \\
x &= -y + 6
\end{align*}
\]
Lesson 15: Writing Systems of Equations

Cool Down: Solve This

Solve

\[
\begin{align*}
  y &= \frac{3}{4}x \\
  \frac{5}{2}x + 2y &= 5
\end{align*}
\]
Instructional Masters
### Instructional Masters for Linear Equations and Linear Systems

<table>
<thead>
<tr>
<th>address</th>
<th>title</th>
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<th>written on?</th>
<th>requires cutting?</th>
<th>card stock recommended?</th>
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<td>Activity Grade8.4.5.2</td>
<td>Trading Moves</td>
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<td>no</td>
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<td>Activity Grade8.4.8.2</td>
<td>Thinking About Solutions Some More</td>
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<td>Activity Grade8.4.15.3</td>
<td>8.4.C15.BlacklineMaster.pdf</td>
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<td>Matching Equation Moves</td>
<td>Matching Equation Moves</td>
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<tr>
<td>1</td>
<td>3x + 7 = 5x</td>
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<tr>
<td></td>
<td>7 = 2x</td>
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<tr>
<td></td>
<td>Multiply each side by $-\frac{1}{3}$</td>
<td>A</td>
<td></td>
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<tr>
<td>2</td>
<td>12x + 3 = 6</td>
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<tr>
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<td>4x + 1 = 2</td>
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<tr>
<td></td>
<td>Add -3x to each side.</td>
<td>B</td>
<td></td>
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<tr>
<td>3</td>
<td>10 - 6x = 4 + 5x</td>
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<tr>
<td></td>
<td>7 - 6x = 1 + 5x</td>
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<tr>
<td></td>
<td>Add 3x to each side</td>
<td>C</td>
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<tr>
<td>4</td>
<td>$\frac{5x}{-3}$ = $\frac{12}{1}$</td>
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<tr>
<td></td>
<td>5x = -36</td>
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<td></td>
<td>Add -3 to each side.</td>
<td>D</td>
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<tr>
<td>5</td>
<td>-3(4x - 3) = -15</td>
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<tr>
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<td>4x - 3 = 5</td>
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<tr>
<td></td>
<td>Multiply each side by $\frac{1}{3}$.</td>
<td>E</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
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<tr>
<td></td>
<td>Multiply each side by -3.</td>
<td>F</td>
<td></td>
<td></td>
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<tr>
<td>Trading Moves</td>
<td>( -6x - 7 = 4x - 2 )</td>
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<tr>
<td><strong>2</strong></td>
<td>( \frac{1}{2}(7x - 6) = 6x - 10 )</td>
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<tr>
<td><strong>3</strong></td>
<td>( \frac{1}{2}x + 7 = x + 13 )</td>
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<tr>
<td><strong>4</strong></td>
<td>( 2(x + 7) = 4x + 14 )</td>
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</tr>
</tbody>
</table>
### Thinking About Solutions Some More

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7(x - 5) = x + 13$</td>
<td>$-6x = -5(x - 1) - x$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4(x - 2) = 2(x - \frac{17}{2})$</td>
<td>$3 - 4x + 5 = 2(8 - 2x)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 3 = 3 + 2x$</td>
<td>$2x + 3 = 2x + 5$</td>
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<table>
<thead>
<tr>
<th>G</th>
<th>H</th>
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<tbody>
<tr>
<td>$3x + 9 = 2.5x + 14$</td>
<td>$7(x - 4) = 4x + 5$</td>
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</table>

<table>
<thead>
<tr>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x - 20 = 7x - 20$</td>
<td>$3(2x + 1) - 4x = 2x + 3$</td>
</tr>
</tbody>
</table>
8.4.13.3 Different Types of Systems.

\[ A \begin{cases} 
  y = -4(x - 2) \\
  y = -2 \left( x - \frac{5}{2} \right) 
\end{cases} \]

One solution

\[ B \begin{cases} 
  y = 5(x - 3) \\
  y = 2x - 6 
\end{cases} \]

One solution

\[ C \begin{cases} 
  y = 2x + 3 \\
  y = 2x - 5 
\end{cases} \]

No solutions
8.4.13.3 Different Types of Systems.

\[ D \begin{cases} y = -6x \\ y = -5(x - 2) - x \end{cases} \]
No solutions

\[ E \begin{cases} y = 3(2x + 1) - 4x \\ y = 2x + 3 \end{cases} \]
Infinite solutions
**Problem Card 1**

Priya and Lin are having a race. The equation \( y = 9.5x \) represents one person's progress.

If one of them had a head start, how long is it until the other person catches up?

**Data Card 1**

- The equation \( y = 9.5x \) represents Lin's progress, where \( y \) is her distance, in feet, from the starting line, and \( x \) is the time, in seconds, that she has been running.
- Priya had the head start. She was 18 feet in front of the starting line when Lin started.
- Priya runs at a constant 8 feet per second.

**Problem Card 2**

A school sells adult tickets and student tickets for the drama play. One equation that represents the situation is \( x + y = 115 \).

How many of each type of ticket did they sell?

**Data Card 2**

- The equation \( x + y = 115 \) represents how many tickets were sold, where \( x \) is student tickets and \( y \) is adult tickets.
  
  This equation is equivalent to \( x = 115 - y \).
- Adult tickets cost $8 each.
- Student tickets cost $3 each.
- The school made $720 total from ticket sales.
Credits

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