Factors and Multiples

Teacher Guide

2 × 2 = 4
4 x 2 = 8
8 ÷ 4 = 2
4 - 2 = 2
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# Factors and Multiples

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Factors and Multiples
Teacher Guide
Core Knowledge Mathematics™
Introduction to the CKMath Program

Welcome to the Core Knowledge Math™ (CKMath) program, based on the carefully researched and designed Illustrative Math™ (IM) instructional materials. IM K-12 Math is a problem-based core curriculum that believes all students are able to understand and use mathematics. Students learn about math by doing math. They bring their current understanding of math and their world experiences to the classroom. In these lessons, students take an active role in the learning process by building on their previous knowledge, and by exploration to develop conceptual understanding instead of being told how to solve problems. Doing math includes: understanding problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning. Encouraging students to participate in mathematical practices with other students gives the opportunity for them to perceive themselves as mathematical thinkers and as part of a mathematical community. By observing students’ understanding of concepts and their thought processes, teachers are able to direct student learning and guide them to recognize the connection between concepts and procedures.

Organization of Units and Lessons

Each unit is divided into sections. Each section revolves around specific goals.

- The Section Overview identifies the learning goals for each section of the unit and describes how students will work towards these goals. Sections are labeled by letters; e.g. Section A, Section B, and so on. Each section uses scaffolding to identify the Common Core Standards that apply to that section. In Fourth Grade, there are five areas covered by the Common Core Standards. They include Operations and Algebraic Thinking (4.OA), Number and Operations in Base Ten (4.NBT), Number and Operations – Fractions (4.NF), Measuring and Data (4.MD), and Geometry (4.G).

The standards in each section are divided into three groups: Building On, Addressing, and Building Towards. A standard that reflects the work of prior grades and is being used to bridge to a grade-level standard is indicated as Building On. When the standard is focused on the grade-level work, the alignment is indicated as Addressing. A standard that is indicated as Building Towards means that the standard has not yet been achieved by the activities in that section.

- The Center Overview identifies the learning centers to be used in the unit. Each center has different stages, or levels. Students will progress through the stages as they master the objectives for each stage. Each center description includes the Common Core Standards that apply to that stage of the center, a stage narrative describing the activity with possible variations, and a list of materials needed for the center.

- The Standards for Mathematical Practice (MP) describe the types of thinking and behaviors students engage in as they are doing mathematics. Throughout the curriculum, the Teacher Guide identifies lessons and activities where different Mathematical Practices are likely to be observed.

Standards for Mathematical Practice Student Facing Learning Targets

**MP1 I Can Make Sense of Problems and Persevere in Solving Them**
- I can ask questions to make sure I understand the problem.
- I can say the problem in my own words.
- I can keep working when things aren’t going well and try again.
- I can show at least one try to figure out or solve the problem.
• I can check that my solution makes sense.

**MP2 I Can Reason Abstractly and Quantitatively**
• I can think about and show numbers in many ways.
• I can identify the things that can be counted in a problem.
• I can think about what the numbers in a problem mean and how to use them to solve the problem.
• I can make connections between real-world situations and objects, diagrams, numbers, expressions, or equations.

**MP3 I Can Construct Viable Arguments and Critique the Reasoning of Others**
• I can explain or show my reasoning in a way that makes sense to others.
• I can listen to and read the work of others and offer feedback to help clarify or improve the work.
• I can come up with an idea and explain whether that idea is true.

**MP4 I Can Model with Mathematics**
• I can wonder about what mathematics is involved in a situation.
• I can come up with mathematical questions that can be asked about a situation.
• I can identify what questions can be answered based on data I have.
• I can identify information I need to know and don’t need to know to answer a question.
• I can collect data or explain how it could be collected.
• I can model a situation using a representation such as a drawing, equation, line plot, picture graph, bar graph, or a building made of blocks.
• I can think about the real-world implications of my model.

**MP5 I Can Use Appropriate Tools Strategically**
• I can choose a tool that will help me make sense of a problem. These tools might include counters, base-ten blocks, tiles, a protractor, ruler, patty paper, graph, table, or external resources.
• I can use tools to help explain my thinking.
• I know how to use a variety of math tools to solve a problem.

**MP6 I Can Attend to Precision**
• I can use units or labels appropriately.
• I can communicate my reasoning using mathematical vocabulary and symbols.
• I can explain carefully so that others understand my thinking.
• I can decide if an answer makes sense for a problem.

**MP7 I Can Look for and Make Use of Structure**
• I can identify connections between problems I have already solved and new problems.
• I can compose and decompose numbers, expressions, and figures to make sense of the parts and of the whole.
• I can make connections between multiple mathematical representations.
• I can make use of patterns to help me solve a problem.

**MP8 I Can Look for and Express Regularity in Repeated Reasoning**
• I can identify and describe patterns and things that repeat.
• I can notice what changes and what stays the same when working with shapes, diagrams, or finding the value of expressions.
• I can use patterns to come up with a general rule.
• Each unit contains between 8 - 25 Lesson Plans.

Each lesson is designed to use 60 minutes. A typical lesson is divided into four phases; a warm-up activity, one or more instructional activities, the lesson synthesis, and a cool-down activity. Every activity within these phases is divided into three parts—the Launch, the Activity, and the Synthesis.

  o Warm-up Activity—The warm-up activity is designed to strengthen the idea of mathematical community. In these activities, students work with their peers. Students use their personal experiences and mathematical knowledge to develop ideas, ask questions, defend their responses, and evaluate the reasoning of others. A warm-up activity might review a context students have seen before, have them reflect on where the previous lesson left off, or preview a context or idea that will come up in that lesson.

There are several warm-up routines that are used during the lessons.

• Act It Out—This routine is for kindergarten and first grade students. It encourages young children to understand the relationship between words and numbers. It provides opportunities for students to make sense of story problems. In this routine, students listen to a story problem and act it out through movement, using their fingers, or objects to represent the action in the story.

• Choral Count—This routine encourages students to make predictions and think about patterns. It also provides opportunities for students to justify their reasoning. In this routine, students count aloud starting from a given number. The count might be forwards or backwards. The teacher records the numbers on a chart as students say them. Students then stop and look at the written numbers to make predictions and look for patterns.

• Estimation Exploration—Estimation Exploration encourages students to use what they know and what they can see to problem-solve for a rough evaluation of a quantity rather than giving a “wild guess.” The estimates can be in the context of measurement, computation, or numerosity—estimating about a large group of objects (MP2). In this routine, students make estimates in response to a question about an image. They first think about estimates that would be sensible, but too high or too low. Then they make a reasonable estimate and discuss why their estimate makes sense.

• How Many Do You See?—This routine encourages students to see groups when counting. Being able to see groups of objects in an organized way helps them visualize quantities and improves their ability to do mental computation. In this routine, students look at an image, which is typically an arrangement of dots or other shapes. Then students state how many dots or shapes they see. Also included in the discussion will be comments about the way they saw them or determined how many there were. This encourages students to see groups and patterns rather than count each item one by one.

• Notice and Wonder—This routine provides an opportunity for students to bring their understandings and experiences to a problem. They share their ideas and ask questions without any pressure to answer or solve a problem. This routine reinforces the importance of making sense of situations before solving a problem. In this routine, students look at an image related to the topic of the lesson and are asked, “What do you notice?” The teacher writes all comments on a chart. They are then asked, “What do you wonder?”, and their questions are also recorded on
• Number Talk—This routine provides an opportunity for students to practice mental math. It helps them solve problems and think about numbers in flexible ways. They not only justify their own reasoning, but critique the reasoning of others as they make sense of methods for solving problems. In this routine, a series of problems are presented one at a time. Students solve the problem in their head and signal when they have an answer. The teacher takes notes as they justify their answer and explain their method for solving.

• Questions About Us—This routine is used with kindergarten students. It provides them opportunities to learn more about their classmates and gives them practice asking questions, organizing quantities, counting, and analyzing data. In this routine, students ask their classmates a question with two choices. They keep track of the answers and count the responses. The teacher then asks follow up questions that students answer using the data that they collected.

• True or False?—This routine encourages students to make sense of equations, often without any computation. It provides another opportunity for students to justify their reasoning as they explain to others what they are thinking. In this routine, students are presented with a series of equations, one at a time. Some equations may be true, and some may be false. Students use what they know about place value, operations, and number relationships to decide if each is true or false. And then, students explain how they know.

• What Do You Know About _____?—This routine encourages students to share their experiences and understandings about a math topic. In this routine, students are presented with a number, expression, or are asked a general question about a math topic. They then list everything they know about that topic. The teacher writes what students say and then references the list later so that students can add more ideas.

• Which One Doesn’t Belong?—This routine provides an opportunity for students to reason about characteristics of shapes, math tools, or other images to decide which one doesn’t belong. Because any answer is correct, students are able to focus on communicating their reasoning and justifying their choice. In this routine, students are shown 4 different images, which may be numbers, equations, shapes, images, or diagrams. They decide which one doesn’t belong and explain why.

  o **Instructional Activities**—After the warm-up, lessons consist of one to three instructional activities.

  **Instructional Activities** include:

  • 5 Practices—Lessons that include this routine are designed to allow students to solve problems in ways that make sense to them. During the activity, students engage in a problem in meaningful ways and teachers monitor to uncover and nurture conceptual understandings. During the activity synthesis, students collectively reveal multiple approaches to a problem and make connections between these approaches (MP3).

  • Card Sort—A card sorting task gives students opportunities to analyze representations, statements, and structures closely, and make connections (MP2 and MP7). As students
work, teachers monitor for the different ways groups choose their categories, and encourage increasingly precise mathematical language (MP6).

- **MLR1 Stronger and Clearer Each (MLR stands for Mathematics Learning Routine.)—** Provides students with a structured and interactive opportunity to revise and refine both their ideas and their verbal and written output. Embedded in grades 3–5.

- **MLR2 Collect and Display—** Captures a variety of students’ oral words and phrases into a stable, collective reference. Output can be organized, re-voiced, or explicitly connected to other languages in a display that all students can refer to, build on, or make connections with during future discussion or writing. Embedded in grades K–5.

- **MLR3 Clarify, Critique, Correct—** Gives students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. Embedded in grades 3–5.

- **MLR4 Information Gap—** Creates an authentic need for students to communicate. Partners or team members are given different pieces of necessary information that must be used together to solve a problem. Embedded in grades 3–5.

- **MLR5 Co-craft Questions—** Allows students to get inside a context before feeling pressure to produce answers, and creates opportunities for students to produce the language of mathematical questions. Embedded in grades 2–5.

- **MLR6 Three Reads—** Supports reading comprehension, sense-making, and meta-awareness of mathematical language. Students take time to understand mathematical situations and story problems, and plan their strategies before finding solutions. Embedded in grades K–5.

- **MLR7 Compare and Connect—** Fosters students’ meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. Embedded in grades K–5.

- **MLR8 Discussion Supports—** Includes a large variety of teacher moves that support rich discussions about mathematical ideas, representations, contexts, and strategies. Embedded in grades K–2.

  o **Lesson Synthesis—** After the instructional activities are completed, students take time to reflect on the knowledge they have gained during the instructional activities and incorporate this with their previous knowledge. The lesson synthesis activity should take 5–10 minutes. During this time, teachers help students with this process by asking questions verbally and having students respond orally or in a written journal, by asking students to add on to a graphic organizer or concept map, or some similar activity.

  o **Cool-down Activity—** The cool-down activity is given to students at the end of the lesson. This activity should take about 5 minutes. Students work on the cool-down independently and turn it in. The teacher uses the cool-down as a formative assessment to determine if students understand the lesson and to adjust further instruction. *Note: The Cool-down activity is identified in the introduction to the lesson plan and not at the end of the lesson.*

  o **Assessments—** There are several opportunities for assessment during each unit.

    - Pre-unit problems can be used as a pre-unit assessment.
    - Each instructional task includes expected student responses and suggestions to advance student thinking. Teachers will adjust their instruction depending on how the students respond to the task. Frequently there are suggested questions to help teachers better understand students’ thinking.
Practice problems are provided for each lesson that can be used for in-class practice, homework, or as a means to assess certain learning on a particular concept.

Each section has a checklist to indicate that students are meeting the section goals.

Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit.

**Unit Resources**

**Teacher Components**

**Teacher Guide:** The Teacher Guide for each unit contains an overview of the sections in which the unit is divided, a description of the centers students will use with the unit, detailed lesson plans, and teacher resources. Within the overview of the unit sections can be found suggested activities from each unit section that can be used as a PLC activity for teachers. PLCs, or Professional Learning Communities provide teachers the ability to work collaboratively in recurring cycles of collective inquiry and action research to achieve better results for students. PLCs give teachers the opportunity to discuss and plan instruction with peers.

The first few pages of each detailed lesson plan are directed to the teacher. Support notes to the teacher are in gray boxes throughout the lesson plan. On these first pages can be found:

- Alignment to the Common Core Standards
- Learning Goals
  - Teacher-facing learning goals appear at the top of lesson plans. They are directed to the teacher and describe the mathematical and pedagogical goals of the lesson.
  - Student-facing learning goals are directed to the student and start with the word "Let's." These learning goals can be written on the board before class begins. They are used to invite students into the work of that day without giving away too much and spoiling the problem-based instruction.
- Lesson Purpose
- Suggestions for instruction for English Learners and Students with Disabilities
- Instructional Routines
- List of materials needed for the lesson
- Lesson Timeline
- Description of the Cool-Down Activity
- Teacher Reflection Question – The purpose of this question is to provide a direction to the teachers to think critically about their teaching during the lesson.
- Sample Student Responses

At the back of the Teacher Guide are Teacher Resources for the unit.

- Family Support Materials
- Assessments
- Cool Downs
- Instructional Masters

These resources are also available for download from the CKF website and can be found in the section titled Individual Resources for each unit. Also included in the online resources are individual
Printable Lesson Plans and the PowerPoint Slides for the unit.

**PowerPoint Slides:** Slides are provided that can be used in the classroom. These slides include questions and directions to be used during the lesson activities, as well as vocabulary, visuals, and other support to be used during the lesson.

**Student Component**

**Activity Book:** The Activity Book is used by the students during the lessons. It coordinates with the lesson plans. It displays the student-facing learning goals for each lesson as well as activity sheets for some activities. Not all activities will use the Activity Book.

**Introduction to Grade 4**

The big ideas in grade 4 include: developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Grade 4 is divided into nine units:

1. Factors and Multiples
2. Fraction Equivalence and Comparison
3. Extending Operations to Fractions
4. From Hundredths to Hundred-thousands
5. Multiplicative Comparison and Measurement
6. Multiplying and Dividing Multi-digit Numbers
7. Angles and Angle Measurement
8. Properties of Two-dimensional Shapes
9. Putting it All Together
Unit 1: Factors and Multiples

At a Glance

Unit 1 is estimated to be completed in 8-10 days including 2 days for assessment.

This unit is divided into two sections including 6 lessons and 2 optional lessons.

- Section A—Understand Factors and Multiples (Lessons 1-4)
- Section B—Find Factor Pairs and Multiples (Lessons 5-8)

On page 6 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses five student centers.

- Can You Build It?
- Capture Squares
- Factors
- Five in a Row: Multiplication
- Secret Fraction
Unit 1: Factors and Multiples

Unit Learning Goals

- Students apply understanding of multiplication and area to work with factors and multiples.

In this unit, students extend their knowledge of multiplication, division, and the area of a rectangle to deepen their understanding of factors and to learn about multiples.

In grade 3, students learned that they can multiply the two side lengths of a rectangle to find its area, and divide the area by one side length to find the other side length.

To represent these ideas, they used area diagrams, wrote expressions and equations, and learned the terms “factors” and “products.”

In this unit, students return to the concept of area to make sense of factors and multiples of numbers. Given a rectangle with a particular area, students find as many pairs of whole-number side lengths as they can. They make sense of those side lengths as factor pairs of the whole-number area, and the area as a multiple of each side length.

Students also learn that a number can be classified as prime or composite based on the number of factor pairs it has.

Throughout the unit, students encounter various contexts related to school, gatherings, and celebrations. They are intended to invite conversations about students' lives and experiences. Consider them as opportunities to learn about students as individuals, to foster a positive learning community, and to shape each lesson based on insights about students.
Section A: Understand Factors and Multiples

Standards Alignments
Building On 3.MD.C, 3.MD.C.7.a, 3.OA.C.7
Addressing 4.OA.B.4, 4.OA.C.5
Building Towards 4.OA.B.4

Section Learning Goals

- Determine if a number is prime or composite.
- Explain what it means to be a factor or a multiple of a whole number.
- Relate the side lengths and area of a rectangle to factors and multiples

In this section, students revisit the ideas of area and factors from grade 3 and encounter the idea of multiples. They begin by building rectangles given specific side lengths and identifying possible areas when only one side length is known. Students use tiles and diagrams to build their understanding before learning new terminology.

Next, students build rectangles given a certain area. They see that the side lengths of the rectangles represent the factor pairs of the given area value. Students also observe the commutative property of multiplication when they see that rectangles with the same pair of side lengths have the same area, regardless of their orientation.

Build 5 different rectangles with the given width. Record the area of each rectangle in the table.

<table>
<thead>
<tr>
<th>area of rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 tiles wide</td>
</tr>
<tr>
<td>3 tiles wide</td>
</tr>
<tr>
<td>4 tiles wide</td>
</tr>
</tbody>
</table>

Students discover that for some whole-number values of area, only one rectangle can be built, and for other values, more than one rectangle is possible. Likewise, some numbers have only one factor pair (the number itself and 1) and other numbers have more than one factor pair. Students learn that we call the former “prime numbers” and the latter “composite numbers.”

The section closes with an optional game day, which is an opportunity to see students' fluency with multiplication and division within 100.

PLC: Lesson 1, Activity 2, What Areas Can You Build?
Suggested Centers

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Addressing)
- Can You Build It? (3–5), Stage 1: Rectangles (Supporting)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
- Find the Number (4), Stage 1: Factors (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)
Section B: Find Factor Pairs and Multiples

Standards Alignments
Building On 3.OA.B.5, 3.OA.C.7
Addressing 4.OA.A.3, 4.OA.B, 4.OA.B.4
Building Towards 4.OA.B.4

Section Learning Goals

- Apply multiplication fluency within 100 and the relationship between multiplication and division to find factor pairs and multiples.

In this section, students apply and deepen their understanding of the ideas of factors and multiples as they play games and solve problems in context. The activities prompt students to look for patterns in factors, multiples, and prime and composite numbers, and use them to make predictions and generalize their observations.

Twenty students are playing a game with 20 lockers in a row. The first student starts with the first locker and opens all the lockers. The second student starts at the second locker and shuts every other locker. The third student stops at every third locker and opens it if it is closed or closes it if it is open.

Which locker numbers does the third student touch?
How many students touch locker 17?

In the last lesson, students have a chance to use the ideas from this unit to create geometric art.

镥 PLC: Lesson 6, Activity 1, Questionable Lockers

Suggested Centers

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Addressing)
- Find the Number (4), Stage 2: Factors and Multiples (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)
Throughout the Unit

The warm-up activities in this unit allow students to build on multiplication and division fluency from grade 3 and prepare to work with factors and multiples. Number Talks are used to help further develop multiplication fluency and mental math strategies. These Number Talks focus on multiples of 6 and 7, and the factor 3, and allow students to use the distributive and associative properties to mentally find the value of expressions.

Here is a sampling of the Number Talk warm-ups in the unit.

<table>
<thead>
<tr>
<th>lesson 2</th>
<th>lesson 4</th>
<th>lesson 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 7</td>
<td>10 × 6</td>
<td>12 ÷ 3</td>
</tr>
<tr>
<td>4 × 7</td>
<td>3 × 6</td>
<td>30 ÷ 3</td>
</tr>
<tr>
<td>3 × 7</td>
<td>13 × 6</td>
<td>60 ÷ 3</td>
</tr>
<tr>
<td>7 × 7</td>
<td>12 × 4</td>
<td>72 ÷ 3</td>
</tr>
</tbody>
</table>
## Materials Needed

<table>
<thead>
<tr>
<th>LESSON</th>
<th>GATHER</th>
<th>COPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>• Inch tiles</td>
<td>• Centimeter Grid Paper - Standard (groups of 2)</td>
</tr>
</tbody>
</table>
| A.2    | • Glue or tape  
• Inch tiles  
• Scissors  
• Tools for creating a visual display | • Centimeter Grid Paper - Standard (groups of 2) |
| A.3    | • Grid paper  
• Inch tiles | • Card Sort: Area (groups of 2) |
| A.4    | • Centimeter cubes | • Card Sort: Multiplication (groups of 2)  
• Find the Number Stage 1 Directions and Gameboard (groups of 2) |
| B.5    | • none | • none |
| B.6    | • Coins  
• Index cards  
• Paper  
• Two-color counters | • none |
| B.7    | • Centimeter cubes | • Find the Number Stage 2 Directions and Gameboard (groups of 2) |
| B.8    | • Colored pencils, crayons, or markers  
• Glue or tape  
• Rulers or straightedges  
• Sticky notes | • Centimeter Grid Paper - Standard (groups of 2) |
Center: Can You Build It? (3–5)

Stage 1: Rectangles

Lessons
- Grade4.1.A1 (supporting)
- Grade4.1.A2 (supporting)

Stage Narrative
One partner builds a rectangle so that their partner can't see it. They describe the rectangle to their partner who tries to build the same rectangle. Students then compare their rectangles and switch roles. Students may choose to draw their rectangle on grid paper, rather than use inch tiles.

Variation:
Students may use more tiles to build their rectangles or may choose to build shapes made out of two rectangles.

Standards Alignments
Addressing 3.MD.C.5

Materials to Gather
Folders, Grid paper, Inch tiles

Materials to Copy
Can You Build It Stage 1 Directions (groups of 6)

Additional Information
Each group of 2 needs at least 48 inch tiles.

Stage 2: Multiple Rectangles

Lessons
- Grade4.1.A1 (addressing)
- Grade4.1.A2 (addressing)
- Grade4.1.B5 (addressing)
Stage Narrative

Before playing, students remove the cards that show 6 or higher and set them aside.

Students flip two number cards to get a number of tiles and then each partner tries to create as many rectangles as possible for that area. If both students each found all the rectangles, they get one point. A student gets two points for any rectangle they built that their partner did not. The player who gets the most points after eight rounds is the winner. Students may choose to draw their rectangle on grid paper, rather than use inch tiles.

This center stage is the first time Number Cards 0–10 are used in Grade 4, so they are provided as a Instructional master. Students will continue to use these throughout the year. Consider copying them on cardstock or laminating them and keeping them organized to be used repeatedly.

Standards Alignments

Addressing 4.OA.B.4

Materials to Gather

Folders, Grid paper, Inch tiles

Materials to Copy

Can You Build It Stage 2 Directions (groups of 6), Number Cards (0-10) (groups of 2)

Additional Information

Each group of 2 needs at least 120 inch tiles and a set of number cards.

Stages used in Grade 3

Stage 1

Addressing

• Grade3.2.A
Center: Capture Squares (1–3)

Stage 7: Multiply with 6–9

Lessons
- Grade4.1.A1 (supporting)
- Grade4.1.A2 (supporting)

Stage Narrative
Students roll a number cube and spin a spinner and find the product of the two numbers they generated. The spinner has the numbers 6–9.

Standards Alignments
Addressing 3.OA.C.7

Materials to Gather
Colored pencils or crayons, Number cubes, Paper clips

Materials to Copy
Capture Squares Stage 7 Gameboard (groups of 2), Capture Squares Stage 7 Spinner (groups of 2)

Additional Information
Each group of 2 needs one number cube.

Stages used in Grade 3

Stage 3
Supporting
- Grade3.1.A

Stage 4
Supporting
- Grade3.1.B

Stage 5
Addressing
- Grade3.1.B
- Grade3.1.C
Stage 6

Addressing

- Grade3.2.B

Supporting

- Grade3.3.C
- Grade3.4.A
- Grade3.4.B

Stage 7

Addressing

- Grade3.4.B
Center: Find the Number (4)

Stage 1: Factors

Lessons
- Grade4.1.A3 (addressing)
- Grade4.1.A4 (addressing)

Stage Narrative
Students find all the factors for a given number. One player chooses a number on the gameboard (1–36) and they get that many points. The other player covers all the factors of that number. Their score is the sum of all the covered factors. Students take turns choosing the starting number. If a player chooses a number that doesn't have any uncovered factors, they lose their next turn. When there are no numbers remaining with uncovered factors, the player with the most points wins.

Standards Alignments
Addressing 4.OA.B.4

Materials to Gather
Centimeter cubes

Materials to Copy
Find the Number Stage 1 Directions and Gameboard (groups of 2)

Additional Information
Each group of 2 needs 36 centimeter cubes.

Stage 2: Factors and Multiples

Lessons
- Grade4.1.B5 (addressing)
- Grade4.1.B6 (addressing)

Stage Narrative
Students find factors and multiples for a given number. To start, one player chooses an even number less than 50. The other player covers either a factor or multiple of that number. Students take turns covering either a multiple or factor of the previous number. When there are no factors or multiples left to cover, the player who covered the last number gets a point. Students take turns choosing the starting number. Students play 10 rounds, as time allows, and the player with the most points wins.

Standards Alignments
Addressing 4.OA.B.4
<table>
<thead>
<tr>
<th><strong>Materials to Gather</strong></th>
<th><strong>Materials to Copy</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Centimeter cubes</td>
<td>Find the Number Stage 2 Directions and Gameboard (groups of 2)</td>
</tr>
</tbody>
</table>

**Additional Information**

Each group of 2 needs 100 centimeter cubes.
Center: Five in a Row: Multiplication (3–5)

Stage 1: Factors 1–5 and 10

Lessons
- Grade 4.1.A3 (supporting)

Stage Narrative
Students multiply using factors of 1–5 and 10. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments
Addressing 3.OA.C.7

Materials to Gather
- Paper clips, Two-color counters

Materials to Copy
- Five in a Row Multiplication and Division Stage 1 Gameboard (groups of 2)

Additional Information
Each group of 2 needs 25 two-color counters and 2 paper clips.

Stage 2: Factors 1–9

Lessons
- Grade 4.1.A4 (addressing)
- Grade 4.1.B5 (addressing)
- Grade 4.1.B6 (addressing)
- Grade 4.1.B7 (addressing)
- Grade 4.1.B8 (addressing)

Stage Narrative
Students multiply using factors of 1–9. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments
Addressing 3.OA.C.7
Materials to Gather
Paper clips, Two-color counters

Materials to Copy
Five in a Row Multiplication and Division Stage 2 Gameboard (groups of 2)

Additional Information
Each group of 2 needs 25 two-color counters and 2 paper clips.

Stages used in Grade 3

Stage 1
Addressing
• Grade3.1.C

Supporting
• Grade3.2.A

Stage 2
Addressing
• Grade3.2.C

Supporting
• Grade3.3.B
• Grade3.3.D
• Grade3.4.A
• Grade3.4.B
• Grade3.5.D
Center: Secret Fraction (3)

Stage 1: Building Non-Unit Fractions

Lessons
- Grade4.1.A4 (supporting)
- Grade4.1.B5 (supporting)
- Grade4.1.B6 (supporting)
- Grade4.1.B7 (supporting)
- Grade4.1.B8 (supporting)

Stage Narrative
Students take turns trying to build secret fractions. On each turn students can choose to
- Pick up one unit fraction card.
- Trade both of your secret fractions for two new secret fractions from the stack.

Once students have enough unit fractions to make their secret fraction, they fill in the secret fraction on the gameboard.

Standards Alignments
Addressing 3.NF.A.1

Materials to Gather
Folders

Materials to Copy
Secret Fractions Stage 1 Cards (groups of 2), Secret Fractions Stage 1 Gameboard (groups of 2)

Stages used in Grade 3

Stage 1

Addressing
- Grade3.5.B
- Grade3.5.C
Section A: Understand Factors and Multiples

Lesson 1: Multiples of a Number

Standards Alignments
Building On 3.MD.C, 3.MD.C.7.a
Addressing 4.OA.B.4
Building Towards 4.OA.B.4

Teacher-facing Learning Goals

- Find areas of different rectangles with a given a side length.
- Understand that the area of a rectangle is a multiple of each of its side lengths.

Student-facing Learning Goals

- Let’s build some rectangles.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of area to explore multiples.

In grade 3, students learned how to find the area of a rectangle by tiling and found that multiplying the side lengths yields the same result.

The purpose of this lesson is for students to apply their understanding of area and multiplication to build rectangles and find their area. As students consider the areas of rectangles with a given side length, they explore the idea of multiples. Students learn that a multiple of a number is the result of multiplying that whole number by another.

While students are introduced to the term multiple in this lesson, they work more with it in upcoming lessons. They do not need to have a formal understanding of the term in this lesson. In upcoming lessons, students also explore and learn the terms factor and factor pair. In this lesson, they refer to them as side lengths within the context of area.

Math Community

Prepare a space, such as a piece of poster paper, titled “Mathematical Community” and a T-chart with the headers “Doing Math” and “Norms” as shown here.
The two sections encourage the students and teacher to be mindful that both respective parties are responsible for the way math is done in the classroom.

**Access for:**

- **Students with Disabilities**
  - Representation (Activity 2)

- **English Learners**
  - MLR2 (Activity 1)

**Instructional Routines**

*Which One Doesn't Belong? (Warm-up)*

**Materials to Gather**

- Inch tiles: Activity 1, Activity 2

**Materials to Copy**

- Centimeter Grid Paper - Standard (groups of 2): Activity 2

**Lesson Timeline**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>20 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>15 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

**Teacher Reflection Question**

In grade 3, students fluently multiplied and divided within 100 and related area to multiplication and addition. How is that prior knowledge supporting students in understanding multiples in this lesson?
**Cool-down** (to be completed at the end of the lesson)  

Area and Multiples

**Standards Alignments**  
Addressing: 4.OA.B.4

**Student-facing Task Statement**  
If a rectangle is 6 tiles wide, what could be its area? Name three possibilities. Explain or show your reasoning.

**Student Responses**  
Sample response: 12, 18, and 24, because $6 \times 2 = 12$, $6 \times 3 = 18$, and $6 \times 4 = 24$.

---

**Begin Lesson**

---

**Warm-up**  

Which One Doesn't Belong: All Kinds of Area

**Standards Alignments**  
Building On: 3.MD.C

This warm-up prompts students to carefully analyze and compare the area of different figures. In making comparisons, students have a reason to use language precisely (MP6) as they describe the area of different figures. It also enables the teacher to hear the terminologies students know and how they talk about characteristics of shapes that help them find different areas.

For all warm-up routines, consider establishing a small, discreet hand signal that students can display to indicate they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or a different subtle signal. This is a quick way to see if students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

**Math Community**
After the warm-up, ask students to reflect on both individual and group actions while considering the questions, “What does it look and sound like to do math together as a mathematical community? What am I doing? What are you doing?”

Record and display their responses under the “Doing Math” header. Students might mention things such as: we talked to each other and to the teacher, we had quiet time to think, we shared our ideas, we thought about the math ideas and words we knew, you were writing down our answers, you were waiting until we gave the answers.

**Instructional Routines**

**Which One Doesn't Belong?**

**Student-facing Task Statement**

Which one doesn't belong?

A

B

C

D

**Launch**

- Groups of 2
- Display the image.
- “Pick one that doesn't belong. Be ready to share why it doesn't belong.”
- 1 minute: quiet think time

**Activity**

- “Discuss your thinking with your partner.”
- 2–3 minutes: partner discussion
- Share and record responses.

**Synthesis**

- “How could we determine the area of each figure?” (We can use multiplication for most of them or count the units in one of them.)
- Consider saying: “Let’s find at least one reason why each one doesn't belong.”

**Student Responses**

Sample responses:

- A is the only figure that doesn't have a horizontal side of 4 units. It is the only one whose vertical sides are not longer than the
horizontal side.
- B is the only one that doesn't have side lengths labeled.
- C is the only one that doesn't have an area of 20 square units.
- D is the only figure that doesn't have exactly four sides or isn't a rectangle.

Activity 1

Build Rectangles and Find Area

Standards Alignments

Building On 3.MD.C.7.a
Building Towards 4.OA.B.4

The purpose of this activity is for students to find the area of a rectangle by tiling and to recall that the area can also be found by multiplying the side lengths. Students use inch tiles to build rectangles with a given side length and find the area of those rectangles. They work together to compare and explain the strategies used to find the area of rectangles and make connections between strategies. Students observe how the area of rectangles with a given width varies as the length changes and make predictions about what areas are possible with the given widths (MP7).

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they build rectangles. On a visible display, record words and phrases such as: “row,” “column,” “area,” “length,” “width,” “wide.” Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading.

Materials to Gather

Inch tiles

Required Preparation

- Each group of 2 needs at least 36 tiles.
Student-facing Task Statement

1. Build 5 different rectangles with each of the given widths. Record the area of each rectangle in the table.

<table>
<thead>
<tr>
<th>area of rectangle</th>
<th>2 tiles wide</th>
<th>3 tiles wide</th>
<th>4 tiles wide</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 tiles wide</td>
<td>12 6 8 4 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 tiles wide</td>
<td>6 18 12 15 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 tiles wide</td>
<td>8 12 20 24 16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Discuss with a partner what you notice about the areas in each row of the table.

3. Predict the area of another rectangle that has each width. Explain your reasoning.

○ 2 tiles:

○ 3 tiles:

○ 4 tiles:

Student Responses

Sample responses:

1. We can skip-count by the width to get the area.

3. ○ 2 tiles wide: 16. I added 4 to the largest area I found earlier.

○ 3 tiles wide: 24. If I build 3 rows of 8, the rectangle would have 24 squares.

○ 4 tiles wide: 40. I know that $4 \times 10$ is 40.

Launch

- Display the rectangle in the book.
- “Look at the rectangle on your page and describe it to a partner.” (It has 6 units. There are 2 rows and 3 columns.)
- Give each group 10 tiles.
- “Build all the rectangles you can using all 10 tiles. Describe them to a partner.”
- 2 minutes: partner discussion
- “Who has a rectangle that is 2 tiles wide, 5 tiles wide, 10 tiles wide?”
- Draw each rectangle as students share their responses.

Activity

- Give students more inch tiles.
- “Now build five different rectangles that are each 2 tiles wide. Record the area of each rectangle in the table.”
- “Repeat with rectangles that are each 3 tiles and 4 tiles wide.”
- 5–7 minutes: partner work time
- Monitor for students who:

  ○ build one row or column and repeat the same number of tiles over again to build the area
  ○ skip-count or multiply to determine the area of each rectangle
  ○ combine skip-counting with another counting strategy

Synthesis

- Collect predictions for areas of rectangles with a width of 2. (18, 14, 20, 30)
- “For rectangles that are 2 tiles wide, how can we tell if our area predictions are true without building each rectangle?” (Each area is an even number. It is what we say...
Progress Check

- “How can we check our predictions for rectangles that are 3 or 4 tiles wide?” (The predictions are numbers we get when we multiply a number by 3 or 4.)

Advancing Student Thinking

Students may count tiles by 1 to determine the area of the rectangles they build. Consider asking, “Do you see groups of tiles that could help you count?”

Activity 2

What Areas Can You Build?

Standards Alignments

Addressing 4.OA.B.4

The purpose of this activity is for students to explore the idea of multiples through an area context. Students learn that a multiple of a number is the result of multiplying any whole number by another whole number. As students build and find the area of rectangles given one side length, they see that every area is a multiple of each of the side lengths of a rectangle.

Access for Students with Disabilities

Represetation: Develop Language and Symbols. Synthesis: Maintain a visible display to record new vocabulary. Invite students to suggest details (words, pictures, or equations) that will help them remember the meaning of the terms. In this lesson, include the terms “multiple,” “even,” and “odd.” Throughout the unit, add the terms “factor,” “factor pair,” “composite,” and “prime.” Supports accessibility for: Language, Memory

Materials to Gather

- Inch tiles

Materials to Copy

- Centimeter Grid Paper - Standard (groups of 2)
Required Preparation

- Each group of 2 needs at least 36 tiles from the previous activity.

Student-facing Task Statement

1. Elena is building rectangles with a width of 3 units and an area of 30 square units or less.
   a. Build the rectangles Elena could make and draw the rectangles on grid paper. Label the area and the side lengths of each rectangle.
   b. What is the area of each rectangle you built?
   c. What do you notice about the areas?
2. Why is 28 square units not a possible area for a rectangle with a width of 3 units?
3. If the area of the rectangle can be more than 30 square units, find 2 other areas it could have. Explain or show your reasoning.
4. What is an area that is not possible for a rectangle with a width of 3 units? Explain or show your reasoning.

Student Responses

1. a. There are ten possible rectangles, each having one side length that is 3 units and the other side length being a whole number from 1 to 10.
   b. 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30 square units
   c. Sample responses:
      ■ The areas went up by 3 for each rectangle when we added a number to the length.
      ■ The areas were alternating: odd, even, odd, even.
2. Sample response: There is no number we can multiply 3 by to get 28.
3. Sample responses:

Launch

- Groups of 2
- Give each group inch tiles and access to grid paper.
- “I am thinking of a rectangle that is 2 tiles wide. What could be the area of my rectangle?”
- 1 minute: partner discussion
- Share and record responses.
- “How do we know all of these are possible areas?” (We can multiply another number by 2 to get those numbers.)

Activity

- “In this activity, we are going to think about what the area of a rectangle could be if we only knew one side length. Work with your partner to answer these questions.”
- 5–7 minutes: partner work time
- Monitor for students who notice that areas you can build are a result of multiplying 3 by another possible side length.

Synthesis

- Display a set of student-generated areas that are less than 30 square units.
- “What did you notice about the areas you found?”
- Ask students to explain why the rectangle couldn't have an area of 28 square units.
- “For the last two questions, how did you know whether a rectangle with a width of 3 units could have that area?”
- “We can have an area of 12 square units when the width of the rectangle is 3 units. That is because 12 is a multiple of 3.”
○ 33 square units, since $33 \div 3 = 11$.
○ 45 square units, since $15 \times 3 = 45$.
○ 75 square units, since $3 \times 25 = 75$.

4. Sample responses:
○ 16 square units. We cannot multiply any number by 3 to get 16.
○ 35 square units. We cannot multiply any number by 3 to get 35.

• “A multiple of a number is the result of multiplying a number by a whole number.”
• “Look back at your work and discuss with your partner: Which numbers are multiples of 3?” (3, 6, 9, 12, 15, 18, 21, 24, 27, and 30)
• “Which numbers are not multiples of 3?” (29, 28, 26, 25, 23, 22, 20, 19, 17, 16, 14, 13, 11, 10, 8, 7, 5, 4, 2, 1)
• 2 minutes: partner discussion

Lesson Synthesis

“Today we built rectangles and learned about multiples of a number. A multiple of a number is the result of multiplying that number by a whole number.”

“How would you decide whether 28 is a multiple of 4?” (I think about whether there is a number I can multiply 4 by to get 28.)

“What is a number that would not be a multiple of 4? How do you know?” (Twenty-five is not a multiple of 4 because I cannot multiply 4 by any whole number to get 25.)

Math Community

After the cool-down, revisit the “Doing Math” list of actions. Ask students to discuss with a partner where they saw evidence of the actions during the rest of the lesson. As a whole group, add any missing actions and revise earlier ideas.

Suggested Centers

• Can You Build It? (3–5), Stage 2: Multiple Rectangles (Addressing)
• Can You Build It? (3–5), Stage 1: Rectangles (Supporting)
• Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)
Response to Student Thinking

Students create rectangles but do not relate them to multiplication.

Next Day Support

- After the warm-up, ask students to work with a partner to discuss and add to their responses to this cool-down.

Prior Unit Support

Grade 3, Unit 2, Section A: Concepts of Area Measurement
Lesson 2: Factor Pairs

Standards Alignments
Building On 3.OA.C.7
Addressing 4.OA.B.4

Teacher-facing Learning Goals
- Find side lengths of different rectangles with a given area.
- Understand that each side length of a rectangle is a factor of its area.

Student-facing Learning Goals
- Let's learn about factor pairs.

Lesson Purpose
The purpose of this lesson is for students to learn the meaning of factor pairs by building rectangles with a specified area.

In grade 3, students learned that a factor is a number being multiplied by another number. For instance, when we multiply 3 and 5 to find the total in 3 groups of 5, or to find the area of a rectangle that is 3 units by 5 units, the 3 and 5 are factors. In this lesson, students learn that a factor pair of a number $n$ is a pair of whole numbers that multiply to result in $n$. For example, 3 and 5 a factor pair of 15.

Previously, students made sense of multiples of a number in the context of area: they built and drew rectangles with given a side length and reasoned about their area. Here, they use the same context to make sense of factor pairs. Students build and draw rectangles with a given area and reason about their side lengths. Students then analyze the rectangles that the class has drawn in a gallery walk. They make observations about the side lengths of the rectangles and consider whether all possible rectangles have been drawn for each area. In these activities, a rectangle with 3 rows and 2 columns is considered the same as a rectangle with 2 rows and 3 columns.

Math Community
Tell students they will have an opportunity to revise their “Mathematical Community” ideas at the end of this lesson, so as they work today they should think about actions that may be missing from the current list.

Access for:

Students with Disabilities
- Engagement (Activity 1)
Instructional Routines
MLR7 Compare and Connect (Activity 1, Activity 2), Number Talk (Warm-up)

Materials to Gather
- Glue or tape: Activity 1
- Inch tiles: Activity 1
- Scissors: Activity 1
- Tools for creating a visual display: Activity 1

Materials to Copy
- Centimeter Grid Paper - Standard (groups of 2): Activity 1

Lesson Timeline

<table>
<thead>
<tr>
<th>Warm-up</th>
<th>10 min</th>
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<tbody>
<tr>
<td>Activity 1</td>
<td>20 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>15 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question
Which question asked during the synthesis gave the most information about students' understanding of the learning goal for the lesson? What did you hear or see that made you feel this way?

Cool-down (to be completed at the end of the lesson)

The Side Lengths of Rectangles

Standards Alignments
Addressing 4.OA.B.4

Student-facing Task Statement

1. What are all of the possible side lengths of a rectangle with an area of 21 square units?
2. What are all of the possible side lengths of a rectangle with an area of 50 square units?

Student Responses
1. 1 and 21, 3 and 7
2. 1 and 50, 2 and 25, and 5 and 10
Warm-up

Number Talk: Multiplication

Standards Alignments
Building On 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying single-digit numbers. These understandings help students develop fluency and will be helpful later in this lesson when students find factor pairs of numbers.

As students use earlier problems to find the new products, they look for and make use of structure (MP7) and use repeated reasoning (MP8).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $2 \times 7$
- $4 \times 7$
- $3 \times 7$
- $7 \times 7$

Student Responses

- 14: I just know it. It's 2 groups of 7.
- 28: Four is double 2, so $4 \times 7$ is double $2 \times 7$ or double 14, which is 28.
- 21: It's one less group of 7 than $4 \times 7$, or 7 less than 28.
- 49: Seven is $4 + 3$, so I could add the answers to the previous two problems: $28 + 21 = 49$.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How did the first three expressions help you find $7 \times 7$?” (The 7 breaks apart into 3 and 4, so I could multiply in parts and add them.)
- Consider asking:
  - “Who can restate _____’s reasoning in a different way?”
Did anyone have the same strategy but would explain it differently?
Did anyone approach the expression in a different way?
Does anyone want to add on to ____’s strategy?

Activity 1

How Many Rectangles?

Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is for students to find all the possible pairs of whole-number side lengths given the area of a rectangle. Each group is assigned 2 areas for which they find all the possible rectangles. They draw and cut out the possible rectangles with that area. In the next activity, they will display the rectangles in a gallery walk. To find all possible rectangles with a given area, students may use tiles but they may also start to observe patterns such as if there are an even number of rows then the number of tiles in the rectangle is an even number (MP7).

Areas to assign (in square units):

Group A: 11, 27  Group B: 25, 5  Group C: 16, 8  Group D: 9, 18
Group E: 24, 12  Group F: 14, 28  Group G: 15, 30  Group H: 19, 20

This activity uses MLR7 Compare and Connect. Advances: representing, conversing.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Students may benefit from 2–3 minutes of independent work time to make sense of and begin the task before joining a group.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Instructional Routines

MLR7 Compare and Connect
Materials to Gather
Glue or tape, Inch tiles, Scissors, Tools for creating a visual display

Required Preparation
• Each of the 8 groups needs tools for creating a visual display.

Student-facing Task Statement
Your teacher will assign 2 numbers to your group. Each number represents the area of a rectangle.

1. On grid paper:
   ○ Draw all the possible rectangles that have the given area.
   ○ Label the area and the side lengths.
   ○ Use each pair of side lengths only once.

   (For example, if you draw a rectangle with 4 units across and 6 units down, you don’t need to also draw a rectangle with 6 units across and 4 units down because they have the same pair of side lengths.)

2. When you think you've drawn all the possible rectangles for both areas, cut out your rectangles and put them on a poster for each area you were assigned.

3. Display your poster for all to see.

Student Responses
Answers vary. Students create posters that show each possible rectangle for their given

Launch
• 8 groups
• Give each group access to inch tiles, grid paper, poster paper, scissors, and glue.

Activity
MLR7 Compare and Connect
• “You are going to be given 2 numbers. Each number represents the area of a rectangle. With your group, draw all the possible rectangles with that area and create a poster for each area. Inch tiles are available if you find them helpful.”

• “Your poster should show the rectangles with each of your assigned areas. Include details such as area and side lengths to help others understand your thinking.”

• Assign each group 2 area values.
• 15 minutes: small-group work time

Synthesis
• “Before we walk around and look at all the posters, take a minute to reflect on the numbers you worked with. What did you notice and wonder as you worked on this activity?”

• 1–2 minutes: quiet think time
Advancing Student Thinking

Students may list only some of the factor pairs for a given number. Consider asking: “Are there any other rectangles you can draw?” or “How can you be sure that you have drawn all of the possible rectangles?”

Activity 2

How Many Rectangles: Gallery Walk

Standards Alignments
Addressing 4.OA.B.4

In this activity, students examine the rectangles drawn by their classmates and learn the term factor pairs. Students recognize the side lengths of each rectangle as a factor pair of its area.

This activity uses MLR7 Compare and Connect. Advances: representing, conversing

Instructional Routines
MLR7 Compare and Connect

Student-facing Task Statement
As you visit each poster, discuss with your partner:

1. What do you notice? Use the following sentence frames when you share:
   a. “I notice that some of the posters . . . .”
   b. “I notice the posters for numbers ____ and ____ are alike because . . . .”
2. How do you know that all possible rectangles with a given area have been found.

Launch
• Groups of 2

Activity
• 5–7 minutes; gallery walk
• Monitor for different explanations students offer for how they know whether all possible rectangles with a given area have been found.
rectangles were found for the given area?

**Student Responses**

1. Sample responses:
   a. I notice some posters only have 2 rectangles.
   b. I notice posters for even numbers are alike because they each have 2 as a side length.
2. Sample response: We know that there are no more rectangles for a given area because there are no more numbers that multiply together to give that area as their product.

**Synthesis**

**MLR7 Compare and Connect**

- “What is the same and what is different between the rectangles on the posters?”
- 30 seconds: quiet think time
- 1 minute: partner discussion
- “How do you know that all possible rectangles have been found for the given area?” (We could not find any other numbers that multiply together to make the area.)
- Display the rectangles for 21: 1 by 21 and 3 by 7.
- “Are there any more rectangles we can draw? Why or why not?” (No, there are no more whole-number factors of 21. Or, no, because to get 21 we can multiply only 1 and 21, 3 and 7, 7 and 3, and 21 and 1.)
- “We call 1 and 21 a factor pair of 21 because each of them is a factor of 21 and multiplying them gives 21. Another factor pair of 21 is 3 and 7.”
- “Work with your partner to write down the factor pairs for the areas you were assigned.”
- 2 minutes: partner work time

**Lesson Synthesis**

“Today we learned that a factor pair of a whole number is a pair of whole numbers that multiply to result in that number. For example, 5 and 4 are a factor pair of 20.”

“What are the factors pairs of 24?” (1 and 24, 2 and 12, 3 and 8, and 4 and 6)

“How do we know if we have found all of the factor pairs of 24?” (We went in order. When we reached 4 and 6, there are no more pairs between 4 and 6, so we can stop there. Or, we used multiplication to see how many facts we could pair to make 24. Or, we used division, and these were all of the numbers that we could divide equally.)
“Can you use the same strategies to find all of the factor pairs of 45?” (Yes, 1 and 45, 3 and 15, 5 and 9. There are no more factors between 5 and 9, so I have found all of the factor pairs.)

“Can you use these strategies to find the factor pairs of any whole number?”

**Math Community**

After the cool-down, give students 2–3 minutes to discuss any revisions to the “Doing Math” actions in small groups. Share ideas as a whole group and record any revisions.

**Suggested Centers**

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Addressing)
- Can You Build It? (3–5), Stage 1: Rectangles (Supporting)
- Capture Squares (1–3), Stage 7: Multiply with 6–9 (Supporting)

---

**Response to Student Thinking**

Students do not list all of the factor pairs for a given number.

The work in this lesson builds from concepts of area developed in a prior unit.

**Next Day Support**

- During the launch of the first activity in the next lesson, remind students about available math tools that can help them reason about the side lengths and area of rectangles.

**Prior Unit Support**

Grade 3, Unit 2, Section A: Concepts of Area Measurement
Lesson 3: Prime and Composite Numbers

Standards Alignments
Addressing 4.OA.B.4, 4.OA.C.5

Teacher-facing Learning Goals
- Determine whether a given whole number in the range 1–100 is prime or composite.
- Find the factor pairs of a given whole number 1–100.

Student-facing Learning Goals
- Let’s identify prime and composite numbers.

Lesson Purpose
The purpose of this lesson is for students to identify factor pairs and determine whether a given whole number in the range 1–100 is prime or composite.

In previous lessons, students were introduced to the terms “multiples” and “factor pairs.” In this lesson, they learn that whole numbers can be classified as prime or composite based on the number of factor pairs they have.

Students reason about these numbers in terms of the area and pairs of side lengths of rectangles. They learn that a prime number has exactly 1 factor pair—1 and the number itself, and that a composite number has more than 1 factor pair. They relate “prime” to a number that could represent the area of only one rectangle (with only one pair of side length) and “composite” to a number that could represent the area of multiple rectangles (with multiple pairs of side lengths).

Math Community
Tell students that, at the end of the lesson, they will be asked to identify specific actions they personally experienced from their “Doing Math” list (both teacher and student sections).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities
- Engagement (Activity 1)

English Learners
- MLR8 (Activity 1)

Instructional Routines
Card Sort (Activity 1), Choral Count (Warm-up)
Materials to Gather
- Grid paper: Activity 1, Activity 2
- Inch tiles: Activity 1, Activity 2

Materials to Copy
- Card Sort: Area (groups of 2): Activity 1

Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>15 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>20 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question
Which voices went unheard in math class today? How might you leverage each student’s ideas and support them being heard and seen in tomorrow’s lesson?

Cool-down (to be completed at the end of the lesson)
Prime or Composite?

Standards Alignments
Addressing 4.OA.B.4

Student-facing Task Statement
1. a. What are the factor pairs of 40?
   b. Is 40 a prime or composite number? Explain or show your reasoning.
2. Is 17 a prime or composite number? Explain or show your reasoning.

Student Responses
1. a. 1 and 40, 2 and 20, 4 and 10, 5 and 8.
   b. Composite, because it has more than 1 factor pair.
2. Prime, because it has only one factor pair, 1 and 17.

Begin Lesson
**Warm-up**

Choral Count: Twos and Fives

**Standards Alignments**

Addressing 4.OA.C.5

The purpose of this Choral Count is to invite students to practice counting by 2 and 5 and notice patterns in the count. These understandings help students develop fluency and will be helpful later when students find factor pairs.

When students predict common multiples for 2 and 5 based on the numbers recorded from the count and what they know about multiplication, they look for and express regularity in repeated reasoning (MP8).

**Instructional Routines**

Choral Count

**Student Responses**

<table>
<thead>
<tr>
<th>count by 2</th>
<th>count by 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
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<tr>
<td>8</td>
<td>20</td>
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<td>10</td>
<td>25</td>
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<td>12</td>
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<td>14</td>
<td>35</td>
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<td>16</td>
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<td>18</td>
<td>45</td>
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<td>20</td>
<td>50</td>
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<td>22</td>
<td>55</td>
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<td>24</td>
<td>60</td>
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<tr>
<td>26</td>
<td>65</td>
</tr>
<tr>
<td>28</td>
<td>70</td>
</tr>
<tr>
<td>30</td>
<td>75</td>
</tr>
</tbody>
</table>

Sample responses:

- Pattern in counting by 2:
  - The digits in the ones place repeat 2, 4,

**Launch**

- “Count by 2, starting at 0."
- Record as students count.
- Stop counting and recording at 30.
- “Count by 5, starting at 0.”
- Record as students count.
- Stop counting and recording at 75.

**Activity**

- “What patterns do you see in the individual counts?”
- 1–2 minutes: quiet think time
- Record responses.
- “What patterns do you see between the two counts?”
- 1–2 minutes: quiet think time
- Record responses.
6, 8, 0, 2, 4, 6, 8, 0.

- Pattern in counting by 5:
  - The digits in the ones place alternate 5, 0, 5, 0.
- Patterns between the two counts:
  - Some numbers are on both lists, like 10 and 20.
  - It takes 5 twos to get to 10 and 2 fives to get to 10.
  - Between the columns we add 3 (2 + 3 = 5), then 6 (4 + 6 = 10), then 9 (6 + 9 = 15), and so on. The number added keeps going up by 3 for each row.

**Synthesis**

- If it doesn't come up in the student responses, ask: “How many twos did it take to get to 10? How many fives did it take to get to 10?” (It took 5 twos and 2 fives to get to 10.)
- “Ten is a multiple of 2 and 5. Do you notice any other multiples of both 2 and 5?” (20 and 30 are on both lists.)
- 1 minute: partner discussion
- Record responses.
- “If the counts continue, what other numbers would you see that are multiples of both 2 and 5?” (I think 40 would be the next common multiple because the multiples are going up by 10. I think 100 would be a common multiple because 2 × 50 = 100 and 5 × 20 = 100.)
- 2 minutes: partner discussion
- Record responses.

---

**Activity 1**

Card Sort: Area

**Standards Alignments**

Addressing 4.OA.B.4

The purpose of this activity is for students to learn about prime numbers and composite numbers. Students are given a set of cards with rectangles on them. They sort the rectangles by area and then attempt to draw an additional rectangle for each category. They notice that some areas can be represented by more than one rectangle and some areas can only be represented by one rectangle.

During the synthesis, highlight that the side lengths of each rectangle represent one factor pair (each pair of side lengths should be used only once), and that the area of each rectangle represents a multiple of each side length. Students learn that a number with only one factor pair—1 and the number itself—is a prime number, and a number with more than one factor pair
is a composite number.

Here is an image of the cards for reference.

![Image of cards for card sort activity]

**Access for English Learners**

*MLR8 Discussion Supports.* Invite students to take turns selecting a rectangle, and explaining how they should sort it to their partner. Display the following sentence frames: “This rectangle belongs with _____, because . . . .” Encourage students to challenge each other when they disagree.

*Advances: Conversing, Representing*

**Access for Students with Disabilities**

*Engagement: Develop Effort and Persistence.* Chunk this task into manageable parts to support organizational skills in problem solving. Some students may benefit from explicit guidance for how to begin. For example, before sorting, students can find the area of each rectangle.

*Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Organization*

**Instructional Routines**

*Card Sort*

**Materials to Gather**

Grid paper, Inch tiles

**Materials to Copy**

Card Sort: Area (groups of 2)
Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards to sort.

1. Sort the cards by area. Record your sorting results. Be prepared to explain your choices.
2. For each group of sorted cards, think of at least one more rectangle. Name its length and width. Be prepared to explain your reasoning.

Student Responses

1. ○ Area of 24 square units: A and E
   ○ Area of 36 square units: B, F, and H
   ○ Area of 18 square units: C and D
   ○ Area of 7 square units: G
2. Sample responses:
   ○ Area of 24 square units: 1 by 24 or 2 by 12
   ○ Area of 36 square units: 1 by 36, 2 by 18
   ○ Area of 18 square units: 9 by 2
   ○ Area of 7 square units: none

Launch

- Groups of 2
- Give each group a set of cards from the Instructional master.
- “Sort the cards into categories in any way that makes sense to you.”
- 2 minutes: partner work time
- Ask students to share ways in which they sorted.

Activity

- “If you did not already, sort the rectangles by their area.”
- 3–5 minutes: partner work time
- Ask students to check their work with another group to make sure the cards in each category match.
- “Now, create at least one rectangle to add to each category in your card sort.”
- 3–5 minutes: partner work time
- Observe the rectangles students add to each category. Monitor for students who notice that no new rectangles could be drawn for the area of 7 square units.

Synthesis

- Select 2–3 students to share the rectangles they added to each category.
- “Why were you able to create more rectangles for some areas and not others?” (Some of the numbers had more factor pairs. For some numbers, there was only one possible factor pair.)
- Revoice student reasoning. “Only one rectangle can be made for the area of 7. Numbers like this are called prime
Prime or Composite?

In this activity, students use area of rectangles to find all of the factor pairs of a given whole number and decide if the number is prime or composite. The synthesis focuses on finding all possible rectangles for a given area as a strategy to find all the factor pairs of a number. Students may notice that they do not need to find all possible rectangles to determine whether a number is prime or composite.

Materials to Gather

Grid paper, Inch tiles

Student-facing Task Statement

The table shows different areas. How many rectangles can be made for each area?

Complete the table and be prepared to explain or show your reasoning.

Launch

- Groups of 2
- Give each group access to inch tiles and grid paper.
- “If you were given a number that is the
Rectangles with the same pair of side lengths should be counted only once. For example, if you count a rectangle with 4 units across and 6 units down, you don’t need to also count a rectangle with 6 units across and 4 units down.

<table>
<thead>
<tr>
<th>area</th>
<th>how many rectangles?</th>
<th>prime or composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 square units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 square units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48 square units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 square units</td>
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</tr>
<tr>
<td>21 square units</td>
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<tr>
<td>31 square units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56 square units</td>
<td></td>
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</tr>
</tbody>
</table>

**Student Responses**

<table>
<thead>
<tr>
<th>area</th>
<th>number of rectangles</th>
<th>prime or composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 square units</td>
<td>1</td>
<td>prime</td>
</tr>
<tr>
<td>10 square units</td>
<td>2</td>
<td>composite</td>
</tr>
<tr>
<td>48 square units</td>
<td>5</td>
<td>composite</td>
</tr>
<tr>
<td>11 square units</td>
<td>1</td>
<td>prime</td>
</tr>
</tbody>
</table>

area of rectangle, how could you find out how many rectangles with that area can be made?” (Test it out with tiles. Think about factor pairs for the number.)

- 1 minute: partner discussion
- Share and record responses.

**Activity**

- “Work with your partner to complete this table. Inch tiles and grid paper are available if you’d like them.”
- 10 minutes: partner work time
- Monitor for different ways students find the number of rectangles, such as:
  - building the rectangles from inch tiles
  - drawing rectangles on grid paper
  - drawing rectangles freehand
  - listing the factor pairs of the number and knowing that one rectangle corresponds to each pair

**Synthesis**

- Invite 3–4 groups share their strategy for finding the number of rectangles for a given area.
- “How does the number of factor pairs relate to the number of rectangles?” (The side lengths of each rectangle is a factor pair. So finding all the rectangles would give us all the factor pairs. Or, finding all the factor pairs of the number would tell us how many rectangles have that number for their area.)
- “What are all of the prime numbers in our list? How do we know they are prime?” (2, 23, 31. They each only have one set of side lengths, 1 and the number itself.)
- “What do you notice about the prime numbers?” (They are odd numbers except
Lesson Synthesis

“Today we learned about prime and composite numbers.”

“How does finding all the rectangles with a certain area tell us if the value of the area is prime or composite?” (The side lengths of each rectangle are a factor pair of the area. If we can find more than one rectangle with that area, that means the number has more than one factor pair and is composite. If we can find only one rectangle, the number is prime.)

“What questions do you still have about these types of numbers?”

Suggested Centers

- Find the Number (4), Stage 1: Factors (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Supporting)

Student Section Summary

In this section, we used our understanding of the area of rectangles to learn about factors, multiples, factor pairs, prime numbers, and composite numbers.
If we know the side length of a rectangle, we can find the areas that the rectangle could have. For instance, a rectangle with a side length of 3 could have an area of 3, 6, 9, 12, 15, or other numbers that result from multiplying of a whole number and 3. We call these numbers multiples of 3.

If we know the area of a rectangle, we can find the side lengths that it could have. For example, a rectangle with an area of 24 square units can have side lengths of 1 and 24, 2 and 12, 3 and 8, or 4 and 6. We call these possible pairs of side lengths the factor pairs of 24.

We also learned that a number that has only one factor pair—1 and the number itself—is called a prime number. For instance, 5 is prime because its only factor pair is 1 and 5.

A number that has two or more factor pairs is a composite number. For instance, 15 is composite because its factor pairs are 1 and 15, and 3 and 5.

Response to Student Thinking

The student confuses vocabulary from the previous lessons: prime, composite, factor pairs, multiples.

Next Day Support

- When explaining how to play Find the Number game in the next lesson, connect key vocabulary with drawings of rectangles.
Lesson 4: Multiplication Practice (Optional)

Standards Alignments
Building On 3.OA.C.7
Addressing 4.OA.B.4

Teacher-facing Learning Goals
- Practice multiplication within 100.

Student-facing Learning Goals
- Let's practice multiplication.

Lesson Purpose
The purpose of this lesson is for students to practice multiplication within 100.

In grade 3, students fluently multiplied and divided within 100, including recalling single-digit multiplication facts from memory. The purpose of this optional lesson is for students to practice multiplication within 100 and review strategies for finding products they don't know.

Math Community
Before the lesson, explain to students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Tell students: “Some of these norms may apply to both you and me, however, there may be things you need me to do to support you in doing math each day.” Offer an example, such as: “It may help us share our ideas as a whole class if we have the norm ‘Listen as others share their ideas.’” Tell students you will pause at two different points of the lesson to identify norms that help everyone do math.

Access for:

Students with Disabilities
- Engagement (Activity 1)

English Learners
- MLR8 (Activity 1)

Instructional Routines
Number Talk (Warm-up)

Materials to Gather
- Centimeter cubes: Activity 2

Materials to Copy
- Card Sort: Multiplication (groups of 2): Activity 1
Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>15 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>20 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
<tr>
<td>Cool-down</td>
<td>5 min</td>
</tr>
</tbody>
</table>

Lesson Synthesis (groups of 2): Activity 2

Teacher Reflection Question

What new insight did you gain about how individual students think or reason about multiplication? How might you use what you learned about these students during tomorrow's lesson?

Cool-down (to be completed at the end of the lesson)

Reflect on Multiplication and Strategies

Standards Alignments

Building On 3.OA.C.7

Student-facing Task Statement

1. What multiplication facts do you want to keep practicing?
2. Describe a strategy you can use in the future to multiply two whole numbers.

Student Responses

Sample response:

1. I want to keep practicing products of 7 and 9 because I don’t know them right away yet and they take me longer to find.
2. When I multiply, I can see if there are facts I know that would help me figure out the product I’m working on.
Warm-up

Number Talk: Factors Over Ten

Standards Alignments

Building On 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying within 100 with one factor larger than 10. These understandings help students develop fluency and will be helpful when students find factor pairs of numbers later in the lesson.

In this activity, students have an opportunity to look for and make use of structure (MP7) as they use a combination of products of smaller factors to find products of larger factors.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 10 × 6
- 3 × 6
- 13 × 6
- 12 × 4

Student Responses

- 60: I just know it. It's 10 groups of 6.
- 18: I just know it.
- 78: 10 × 6 = 60 and 3 × 6 = 18, so I can just add 60 + 18 = 78, since that's 13 groups of 6.
- 48: I know that 10 × 4 = 40 and 2 × 4 = 8, so it's 40 + 8 = 48, I know that 12 × 2 = 24 and I can double that.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How can knowing the value of the first two expressions help you find the value of the third expression?” (I can multiply in parts and add the smaller parts together to find a larger product.)
Activity 1
Card Sort: Multiplication

Standards Alignments
Building On 3.OA.C.7

The purpose of this activity is for students to self-assess their own fluency with multiplication facts and practice the ones that are less familiar. Students are given a set of expressions. They sort them into categories of “know it right away,” “can find it quickly,” or “don’t know it yet.” Then, they identify products they don’t know (or don’t know well) yet, discuss strategies for finding those products, and practice using those strategies.

Access for English Learners
MLR8 Discussion Supports. Students who are working toward verbal output may benefit from access to mini-whiteboards, sticky notes, or spare paper to write down and show their responses to their partner.
Advances: Writing, Representing

Access for Students with Disabilities
Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set.
Supports accessibility for: Memory, Social-Emotional Functioning

Materials to Copy
Card Sort: Multiplication (groups of 2)

Required Preparation
- Create a set of multiplication fluency cards from the Instructional master for each group of 2.

Student-facing Task Statement
Take turns sorting the multiplication expressions into one of these groups:
- know it right away

Launch
- Groups of 1–2
- Give each group a set of fluency cards.
- “Take some time to sort the cards into one
can find it quickly
• don't know it yet

Multiplication expressions I'm going to practice:
A.
B.
C.
D.
E.

Student Responses
Answers vary.

Activity

• “Let's share strategies for finding products we don't know yet and practice with them.”
• “Choose 5 multiplication facts that you don't know yet and record the expressions. These are the products you will practice finding.”
• 1 minute: independent work time
• “Now, share your list with your partner. Discuss some strategies you could use to find the products quickly. Afterwards, take some time to practice finding the products you chose.”
• 3–5 minutes partner work time

Synthesis

• “What are some useful strategies for finding products you didn't know yet?”

(Think of a product I already know and use it to find the one I didn't know yet. Imagine equal-size groups. Break the expressions into smaller numbers, find those products, and then add them.)

Advancing Student Thinking

To support students in making connections between multiplication facts they know and those they are still working on, consider asking: “What do you notice about the numbers in the multiplication facts you are still working on?” or “Can you think of a related multiplication fact that might be helpful?”

Activity 2

Introduce Find the Number, Factors
Standards Alignments
Building On 3.OA.C.7
Addressing 4.OA.B.4

The purpose of this activity is for students to apply multiplication fluency within 100 to find factors of multiples within the range of 1–36. This game is Stage 1 of the center Find the Number. In this stage, students find all the factors for a given number.

One player chooses a number on the game board (1–36) and gets that many points. The second player covers all the factors of that number. The score of the second player is the sum of all of the covered factors.

Players take turns choosing the starting number. A player who chooses a number without any uncovered factors loses their next turn. When there are no numbers remaining with uncovered factors, the player with the most points wins.

Materials to Gather
Centimeter cubes

Materials to Copy
Find the Number Stage 1 Directions and Gameboard (groups of 2)

Launch
- Groups of 2
- Give each group a copy of the Instructional master and centimeter cubes.
- “We are going to play a game called Find the Number. Take a few minutes to read the directions.”
- 2 minutes: independent work time
- “What questions do you have about the game?”
- Consider playing a demonstration round or asking questions such as:
  - “If I covered the number 9, which factors would you cover?” (1, 3)
○ “What would your score be?” (4)
○ “What would my score be?” (9)

Activity
• 10–15 minutes: partner game time

Synthesis
• “What strategies were helpful for the game?” (Choose numbers with fewer factors to keep your opponent’s score low.)

Lesson Synthesis

“Today we practiced multiplication.”

“What strategies do you remember from grade 3? What new strategy did you learn today?”

Math Community

Ask students to reflect on both individual and group actions and consider the question: “As we did math together in our mathematical community, what norms, or expectations, did we keep in mind?”

Record and display their responses under the “Norms” header.

Suggested Centers
• Find the Number (4), Stage 1: Factors (Addressing)
• Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
• Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)
Response to Student Thinking

The work in this lesson builds from multiplication and division fluency developed in a prior unit.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division
Section B: Find Factor Pairs and Multiples

Lesson 5: More Multiples

Standards Alignments
Addressing 4.OA.A.3, 4.OA.B.4

Teacher-facing Learning Goals
• Apply understanding of multiplication and multiples in the range 1–100 to solve real-world problems.

Student-facing Learning Goals
• Let’s solve problems that involve factors and multiples.

Lesson Purpose
The purpose of this lesson is for students to use multiples of single-digit numbers to solve real-world problems. This includes situations with multiple solutions and situations with no solutions.

In previous lessons, students used the area of rectangles to develop an understanding of factors and multiples. They also worked on their fluency with multiplication facts. In this lesson, they apply these understandings to solve problems.

In some problems, the solutions are whole-number results of multiplying or dividing. For example: If eggs come in packages of 12, how many eggs are in 5 packages? (60 eggs). But in others, students need to make sense of products or quotients in terms of the situation. For instance: How many packages should we buy if we need exactly 50 eggs? Students reason that it is impossible to get exactly 50 eggs, since there are 48 eggs in 4 packages and 60 eggs in 5 packages.

As they examine the numbers in these situations, including interpreting remainders in division problems, students make sense of problems and persevere in solving them (MP1) and reason quantitatively and abstractly (MP2).

Math Community
Tell students that, at the end of the lesson, they will be asked to identify specific examples of norms they experienced as they did math.

Access for:

Students with Disabilities
• Representation (Activity 1)

English Learners
• MLR2 (Activity 1)
Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

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Teacher Reflection Question

What strategies do students use most often to decide if a number is a multiple of a given whole number?

Cool-down (to be completed at the end of the lesson)

Fourth-grade Party

Standards Alignments

Addressing 4.OA.B.4

Student-facing Task Statement

All of the fourth-grade classes are getting together for a party. They have tables where 6 people can sit and tables where 8 people can sit. There will be 72 students that need seats.

If you may only use one type of table, which type of table would you choose? Explain or show your reasoning.

Student Responses

Sample responses:

- I would choose the tables that seat 8 because 72 is a multiple of 8 and $8 \times 9 = 72$.
- I would choose the tables that seat 8 because 72 is a number I say when I skip count by 8.
- I would choose 12 tables that seat 6 because $12 \times 6 = 72$.
- I would choose the tables that seat 6 because 72 is a number I say when I skip count by 6.
**Warm-up**

Estimation Exploration: Banquet Seating

**Standards Alignments**

Addressing 4.OA.B.4

The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it. For example, students may reason that there are 10 chairs at each table and then make an estimate for the number of tables (MP3). Asking yourself “Does this make sense?” is a component of making sense of problems, and making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

**Instructional Routines**

Estimation Exploration

**Student-facing Task Statement**

About how many chairs are in the room?

Record an estimate that is:

- **too low**
- **about right**
- **too high**

**Launch**

- Groups of 2
- Display the image.

**Activity**

- “What is an estimate that's too high? Too low? About right?”
- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.

**Synthesis**

- “What part of the picture did you use to make your estimate?” (The number of seats at one table. The number of rows and tables in each row.)
- “Which estimates would have been unreasonable?” (353, 22, or 198, and other numbers not ending in a zero. Because there are ten chairs at a table, the total number has to be a multiple of 10 and end in a 0.)

**Student Responses**

- Too low: 200
- Too high: 500
- About right: 300–400
Activity 1
Choose the Right Tables

Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is to find multiples of two different numbers in context. Students decide possible table sizes for a party based on whether or not a given number of people is a multiple of 6, 8, both, or neither. In situations where a given number is not a multiple of 6 or 8, they reason about what it means in context (MP2).

Focus the synthesis on how the number of seats at the two table sizes relate to multiplication and what it means when the number of people is not a multiple of the number of seats at a table.

Access for English Learners
MLR2 Collect and Display. Collect the language students use as they discuss the different options for tables. Display words and phrases such as: no empty seats, 6 times a number, 8 times a number, multiplied by, multiples of 6 or 8, and ___ × 6 added to ___ × 8 for combinations of table sizes. During the synthesis, solicit ways to update the display: “What are some other words or phrases we should include?” etc. Invite students to borrow language from the display as needed.
Advances: Conversing

Access for Students with Disabilities
Representation: Access for Perception. Provide access to materials to help students represent the problem, such as inch tiles, connecting cubes, a blank or partially completed set diagram, and a blank or partially completed tape diagram. Invite students to identify correspondences between these representations, multiplication equations, and the term “multiple” throughout the lesson.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement
Students are preparing for a party. The school has tables where 6 people can sit and

Launch
- Groups of 2
- “Work with your partner to choose a type of table to seat different groups of students.”
tables where 8 people can sit.

The students can only choose one type of table and they want to avoid having empty seats.

1. Jada's class has 18 students. Which tables would you choose for Jada's class? Explain or show your reasoning.
2. Noah's class has 30 students. Which tables would you choose for Noah's class? Explain or show your reasoning.
3. Which tables would you choose for Noah's and Jada's classes together? Can you find more than one option? Explain or show your reasoning.
4. If you also want places for Noah's teacher and Jada's teacher to sit, which tables would you choose? Explain or show your reasoning.

Student Responses

1. 3 tables that seat 6 students, because $3 \times 6 = 18$.
2. 5 tables that seat 6 students, because $5 \times 6 = 30$.
3. 6 tables that seat 8, or 8 tables that seat 6, because $8 \times 6 = 48$ and both 6 and 8 are factors of 48.
4. Neither set of tables would have exactly enough seats for 31 or 19, because neither 31 nor 19 are multiples of 6 or 8.

Activity

- 8–10 minutes: partner work time
- Monitor for students who:
  - draw pictures to highlight why the number of students at tables of 6 is a multiple of 6 and that at tables of 8 is a multiple of 8
  - list the multiples of 6 and multiples of 8 to determine which table to use
  - use multiplication facts to explain their reasoning

Synthesis

- Invite students who used drawings, the idea of multiples, or multiplication facts to share while discussing the following questions.
- “Can Noah’s class fit exactly in tables for 6? How do you know?” (Yes. $30 = 5 \times 6$, so Noah’s class will fill 5 tables exactly.)
- “Can Noah’s class fit exactly in tables for 8? How do you know?” (No, 30 is not a multiple of 8. They would need at least 4 tables but that’s too many seats, so 2 of the seats would be empty.)
- “How do multiplication facts or the idea of multiples help us think about this problem?” (We know that 24 and 32 are products of 8 and another number, but 30 is not.)
- “For the problem with both classes and the teachers, could they all fit at tables of 6? Why or why not?” (Yes, they could get 9 tables of 6, but there will be 4 empty seats. If they don’t want empty seats, tables of 6 won’t work as 50 is not a multiple of 6.)
- “Could they all fit at tables of 8? Why or why not?” (Yes, they could get 7 tables of 8, but there will be 6 empty seats. If they don’t want empty seats, tables of 8 won’t work as 50 is not a multiple of 8.)
Activity 2  
Hot Dogs and Buns

Standards Alignments
Addressing 4.OA.A.3, 4.OA.B.4

In this activity, students solve problems that involve finding multiples that are shared by two different numbers: the number of hot dogs in a package and the number of hot dog buns in a package. They reason about how many of each package to get to make a certain number of servings of hot dogs. As multiple answers can be expected, the focus is on explaining why the solutions make sense (MP3).

To solve the problems, students may decontextualize the situation and reason about factors and multiples, and then recontextualize the solutions in terms of servings of hot dogs. As they do so, they practice reasoning quantitatively and abstractly (MP2). During the synthesis, analyze different solutions and discuss why numbers that are multiples both of 8 and 10 are useful in this situation.

Student-facing Task Statement
Each package of hot dogs has 10 hot dogs. Each package of hot dog buns has 8 buns.

1. Lin expects to need 50 hot dogs for a class picnic.
   a. How many packages of hot dogs should Lin get? Explain or show your reasoning.
   b. Can Lin get exactly 50 hot dog buns? How many packages of hot dog buns should Lin get? Explain or show your reasoning.

2. Diego expects to need 72 hot dogs for a class picnic.
   a. How many packages of hot dogs should Diego get? Explain or show your reasoning.

Launch

- Groups of 2–4
- Read the opening paragraph as a class.
- “What questions could you ask about the situation?”
- 1 minute: quiet think time
- Share responses.
- “Let’s solve problems about hot dogs and hot dog buns, which come in different-size packages.”
- “For each problem, look for possible numbers of packages that are needed to serve hot dogs at a picnic.”
- “Take a minute to read the problems and think about how you would solve them.”
- 1 minute: quiet think time
your reasoning.

b. How many packages of hot dog buns should Diego get? Explain or show your reasoning.

3. Is it possible to buy exactly the same number of hot dogs and buns? If you think so, what would that number be? If not, explain your reasoning.

Student Responses

1. Sample responses:
   a. 5, because 5 groups of 10 hot dogs is 50 hot dogs.
   b. No, because 50 is not a multiple of 8. If she gets 7 packages of buns that is $7 \times 8$ or 56 buns so if she needs 50 buns, she will have enough.

2. Sample responses:
   a. 7 packages would be 70 hot dogs, which is close to 72 but not quite enough. 8 packages would be 80, which is too many.
   b. 9 packages would be 72 buns, so that is perfect, but if he gets 8 packages of hot dogs he will need 10 packages of buns to have a bun for each hot dog.

3. Sample response: 40 hot dogs is 4 packages of 10 and 40 buns is 5 packages of 8. 80 hot dogs is 8 packages of 10 and 80 buns is 10 packages of 8.

Activity

- “Now work with your group to solve these problems.”
- 8–10 minutes: group work time
- Monitor for students who:
  - use the term “multiple” in their explanations
  - try different possibilities for the number of packages to buy
  - try to find numbers of packages that would give the exact (or close to the exact) numbers of servings

Synthesis

- Invite students to share their responses to the problem about 72 hot dogs, including those who get 70 hot dogs and 72 buns or 80 hot dogs and 80 buns. Encourage students to highlight how they are using multiples in their reasoning.
- “Why are there different answers to this question?” (The numbers do not work out exactly to get 72 hot dogs and 72 buns, so we need to look at different alternatives.)
- “Is it possible to buy exactly the same number of hot dogs and buns? If so, how many of each?” (Yes, 40, 80.)
- “Why is it possible to buy exactly the same number of hot dogs and buns when the packaging is different?” (Some multiples of 10 are also multiples of 8, such as 40 and 80.)

Advancing Student Thinking

If students are using guess and check, consider asking: “How are you choosing which numbers to try?” or “How can thinking about multiples help you solve the problem?”
Lesson Synthesis

Math Community

Create a T-chart titled “Ways to Disagree” and two columns labeled “sounds like” and “looks like”.

Review the list of what you are doing and what students are doing in math class.

“Is making mistakes a part of math class and doing math together?” (Yes.)

“What does it feel like when someone points out a mistake or disagrees with your idea?” (It’s embarrassing. It hurts my feelings.)

“What might it look like or sound like to challenge each other’s ideas without hurting feelings?” (Ask questions to better understand, say I disagree, ask “can you please explain what you mean by...?”)

Suggested Centers

- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Addressing)
- Find the Number (4), Stage 2: Factors and Multiples (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)

Response to Student Thinking

The work in this lesson builds from multiplication and division concepts and fluency developed in a prior unit.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division
Lesson 6: The Locker Problem

Standards Alignments
Addressing 4.OA.B, 4.OA.B.4

Teacher-facing Learning Goals
- Apply understanding of factors, multiples, and prime and composite numbers to solve problems.

Student-facing Learning Goals
- Let’s figure out what’s happening in a game about lockers.

Lesson Purpose
The purpose of this lesson is for students to examine factors of numbers from 1 to 20 and use them to solve problems.

In previous lessons, students used multiples to solve problems about equal-size groups (tables that accommodate certain numbers of seats and packages that contain certain numbers of items). In this lesson, students apply their knowledge of factors, multiples, prime numbers, and composite numbers to solve problems about a game involving opening and closing of lockers. Students look for patterns in the factors or multiples of numbers and use them to make predictions about the lockers that will have been touched after all 20 players of the game have a turn.

Math Community
Tell students they will reflect on the norms they identified at the end of this lesson.

Access for:

- Students with Disabilities
  - Representation (Activity 2)

- English Learners
  - MLR7 (Activity 2)

Instructional Routines
Choral Count (Warm-up)

Materials to Gather
- Coins: Activity 1
- Index cards: Activity 1
Lesson Timeline

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Teacher Reflection Question

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student’s ideas are a part of the collective learning?

Cool-down (to be completed at the end of the lesson)

Reflect on Problem Solving

Standards Alignments

Addressing 4.OA.B.4

Student-facing Task Statement

Reflect on your work today. How did you organize your thinking? How did you adjust your work and thinking along the way? What was helpful to you?

Student Responses

Sample response: I started trying to write everything in one color but then I saw that if I used different colors, it was easier to keep track.
Standards Alignments
Addressing 4.OA.B

The purpose of this Choral Count is for students to practice counting by 2 and 4 and notice common multiples of the two numbers in the count. Students may notice that the count by 4 is every other number in the count by 2. The relationship between the two counts can also be described using multiplication and this is the goal of the synthesis.

Instructional Routines
Choral Count

Student Responses
Sample responses:

- Pattern in counting by 2s:
  - The numbers in the ones place repeat 2, 4, 6, 8, 0, 2, 4, 6, 8, 0.
- Pattern in counting by 4s:
  - The numbers in the ones place repeat 4, 8, 2, 6, 0, 4, 8, 2, 6, 0.
- Patterns between the two counts:
  - The second list is included in the first list.
  - Every other number in the first list is in the second list.
  - Each number in the second list is twice the corresponding number in the first list.
  - Each number in the first list is half the corresponding number in the second list.

Launch
- “Count by 2, starting at 0.”
- Record as students count.
- Stop counting and recording at 30.
- Count by 4, starting at 0.
- Record as students count.
- Stop counting and recording at 60.

Activity
- “What patterns do you see?”
- 1–2 minutes: quiet think time
- Record responses.

Synthesis
- “How can I get the number in the second list from the corresponding number in the first list?” (Double it by adding it to itself or multiply the number by 2.)
- “How can I get the number in the first list from the corresponding number in the second list?” (I can take half of it or divide the number by 2.)
Activity 1

Questionable Lockers

Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is for students to visualize and make sense of the context of problems they will solve in the next activity. They will also consider representations that can be used to model the quantities and actions in the situation (MP4) and try creating them. Students will not have time to make a complete representation, so the focus is on making sense of the situation and persevering in finding effective representations (MP1).

Materials to Gather
Coins, Index cards, Paper, Two-color counters

Student-facing Task Statement

The picture shows lockers in a school hallway.

The 20 students in Tyler's fourth-grade class are playing a game in a hallway that is lined with 20 lockers in a row.

1. The first student starts with the first locker and goes down the hallway and opens all the lockers.
2. The second student starts with the second locker and goes down the hallway and shuts every other locker.
3. The third student stops at every third locker and opens the locker if it is closed or

Launch

- Groups of 3 or 4
- Display the image of the lockers.
- “In your small group, discuss what you notice and wonder about the picture.”
- 1 minute: quiet think time
- 1 minute: group discussion
- Share and record responses. If students are not familiar with lockers, explain that lockers are used to hold students’ coats, books, and backpacks in some schools.

Activity

- Read the problem aloud to students.
- “Take some time on your own to make sense of the problem. You may want to try to visualize it or reread the problem.”
- 2 minutes: quiet think time
- Give students access to two-color counters,
closes the locker if it is open.

This process continues until all 20 students in the class have touched the lockers.

Create a representation to show what you understand about this problem. Consider:

- How does your representation show lockers?
- How does your representation keep track of students who touch lockers?
- How does your representation show which lockers are open or closed?

**Student Responses**

Students may:

- put the numbers 1–20 on index cards
- use two-color counters and 10-frames
- use the head and tail sides of coins
- use of symbols or colors to indicate whether a locker is closed or open, for example X or O
- list of each student number by row and an indication of how their action changes the symbols or colors

index cards, coins, and other materials that may be helpful to them.

- “Based on what you know, create a representation of the problem.”
- 5 minutes: independent work time
- Monitor for and select 3 different representations that:
  - represent lockers in different ways
  - show instances of contact with lockers (or of opening and closing)
  - show correspondence between each student and the lockers touched

**Synthesis**

While discussing each representation, ask students:

- “Where are the lockers in this representation?” (Numbers or pictures used to indicate lockers)
- “How can we tell if a locker is open or closed in this representation?” (Symbols or colors used to indicate open or closed)
- “How does this representation show which student is touching the lockers?” (Students and their actions listed in a table or a T-chart)

- “In the next activity, you will answer questions about the game. You may use your representation or one shared by a classmate to help you think through the problems.”

### Activity 2

**An Open and Shut Case**
Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is for students to solve problems about a locker game by using the ideas of factors and multiples and by observing patterns in the numbers. The situation offers many possible explorations, but the questions around which lockers are touched and by how many people are designed to elicit understandings about multiplication.

Discussion about which lockers will be open or closed or by whom will likely move students away from thinking about factors and multiples and should not be the focus. The final question challenges students who are ready for more to look for additional patterns and generalize their observations. It is not expected that all students complete it. Students who are ready may simulate opening and closing the lockers in different ways, such as:

- using actual lockers, if available
- using a printed set of squares labeled 1 through 20
- using two color counters and 10-frames, if available
- using the head and tail sides of coins

Access for English Learners

MLR7 Compare and Connect. Synthesis: Invite groups to prepare a visual display that shows the strategy they used to determine which lockers will be open. Encourage students to include details that will help others interpret their thinking. For example, specific language, using different colors, shading, arrows, labels, notes, diagrams or drawings. Give students time to investigate each other’s work. During the whole-class discussion, ask students, “Which students open or close locker 10? What about locker 20? What patterns do you notice about the number of factors for a locker number and whether it is open or closed?”

Advances: Representing, Conversing

Access for Students with Disabilities

Representation: Internalize Comprehension. Support students in transferring their knowledge to a new context. Begin by asking, “Does this problem remind anyone of the concepts we have been studying?” Invite students to reference the vocabulary display, and continue to refer them to it throughout the lesson.

Supports accessibility for: Conceptual Processing, Language, Attention
Student-facing Task Statement

The 20 students in Tyler's fourth-grade class are playing a game in a hallway with 20 lockers in a row.

Your goal is to find out which lockers will be touched as all 20 students take their turn touching lockers.

1. Which locker numbers does the 3rd student touch?
2. Which locker numbers does the 5th student touch?
3. How many students touch locker 17? Explain or show how you know.
4. Which lockers are only touched by 2 students? Explain or show how you know.
5. Which lockers are touched by only 3 students? Explain or show how you know.
6. Which lockers are touched the most? Explain or show how you know.

If you have time: Which lockers are still open at the end of the game? Explain or show how you know.

Student Responses

1. 3, 6, 9, 12, 15, 18
2. 5, 10, 15, 20
3. 2. Sample responses:
   - I made a diagram and saw that the only students who touch locker 17 are 1 and 17.
   - I didn't say 17 when I was counting by any number other than 1 or 17.
4. 2, 3, 5, 7, 11, 13, 17, 19. Sample response:

Launch

- Groups of 3–4
- “Let’s solve some problems about a game you read about earlier, where students take turns opening and closing lockers.”
- “Silently read and think about each question.”
- 1 minute: quiet think time
- “Work in your group to answer each question. Consider the representations you created or saw earlier to help you think about the problems.”

Activity

- 15–20 minutes: group work time
- Monitor for students who:
  - represent each contact with the lockers in a systematic way
  - represent the open and closed lockers in various ways
  - reason about factors and multiples and use the vocabulary in their explanations
- Encourage students who use a pattern to answer the final question to share their explanation. Invite them to consider a situation with more lockers such as 50 or 100.

Synthesis

- “Take a few minutes to share your answers with students in a different group.”
- Invite students to share how they kept track of the number of touches for each locker.
- See lesson synthesis.
Each of these numbers only have 2 factors, one of which is 1.

5. 4, 9. Sample response: These numbers have 3 factors. The factors of 4 are 1, 2, and 4. Those of 9 are 1, 3, and 9.

6. 12, 18, and 20 are each touched by 6 different students. They are also the numbers that have the most factors.

If you have time: The lockers that are open are 1, 4, 9, and 16. Sample responses:

- These numbers have an odd number of factors, so the lockers changed an odd number of times. When we list their factor pairs, one pair is the same number twice. For example, \(4 = 1 \times 4\) and \(4 = 2 \times 2\), so 4 has three factors: 1, 2, 4. So the first, second, and fourth students touch locker number 4, and the pattern is open-close-open. For 16, which has five factors, the pattern is open-close-open-close-open.

- All other numbers have an even number of factors so the lockers will be closed because they are touched an even number of times (open-close-open-close . . .).

**Lesson Synthesis**

“Today, we looked at a game about lockers being opened and closed.”

“Which lockers were only touched by 2 students? What do those numbers have in common?” (2,3,5,7,11,13,17,19, They are all prime numbers.)

“Which lockers were only touched by 3 students? What do those numbers have in common?” (4, 9, 16. They all have an odd number of factors. One of their factor pairs involves the same number, like 3 and 3, or 4 and 4.)

“How do today's problems relate to what we've been studying?” (The factors of the locker numbers tell us how many and which students would touch the locker. If the locker number is a composite number, we know more than 2 students would touch the locker.)
Math Community

After the cool-down, ask students to individually reflect on the following question: “Which one of the norms did you feel was most important in your work today, and why?” Students can write their responses on the bottom of their cool-down paper, on a separate sheet of paper, or in a math journal.

Tell students that as their math community works together over the course of the year, the group will continually add to and revise its “Doing Math” and “Norms” actions and expectations.

Suggested Centers

- Find the Number (4), Stage 2: Factors and Multiples (Addressing)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)

Response to Student Thinking

Students share reflections (about organizing and adjusting their thinking) that may benefit the math community.

Next Day Support

- Before the warm-up, consider sharing reflections from the previous day and inviting students to keep these ideas in mind in their work today.

Prior Unit Support

Grade 3, Unit 4, Section B: Relate Multiplication and Division

The work in this lesson builds from multiplication and division concepts and fluency developed in a prior unit.
Lesson 7: Find Factors and Multiples

Standards Alignments
Building On 3.OA.B.5, 3.OA.C.7
Addressing 4.OA.B.4

Teacher-facing Learning Goals
- Determine whether a number from 1–100 is a multiple of another number.
- Find all factor pairs of a given whole number from 1–100.

Student-facing Learning Goals
- Let's find factors and multiples of whole numbers from 1–100.

Lesson Purpose
The purpose of this lesson is for students to find factors and multiples of a given whole number from 1–100.

In previous lessons, students learned about factor pairs, multiples, and prime and composite numbers.

The purpose of this lesson is for students to use the language of factors and multiples to describe numbers within 100. Students look for all factors of numbers and decide whether a given number is prime or composite. Students are encouraged to find patterns in composite numbers which help to identify a factor. For example, if the last digit of a number is 0 then 2, 5, and 10 are all factors of that number.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities
- Engagement (Activity 2)

English Learners
- MLR8 (Activity 1)

Instructional Routines
Number Talk (Warm-up)

Materials to Gather
- Centimeter cubes: Activity 2

Materials to Copy
- Find the Number Stage 2 Directions and
Lesson Timeline

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Teacher Reflection Question

As you finish up this section, reflect on the norms and activities that have supported each student in learning math. How have you seen each student grow as a young mathematician throughout this work? How have you seen yourself grow as a teacher?

Cool-down (to be completed at the end of the lesson)

Complete the Statements

Standards Alignments

Addressing 4.OA.B.4

Student-facing Task Statement

Complete the statements for each number.

<table>
<thead>
<tr>
<th>number</th>
<th>factor</th>
<th>multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>_____ is a factor of _____ because . . .</td>
<td>_____ is a multiple of _____ because . . .</td>
</tr>
<tr>
<td>24</td>
<td>_____ is a factor of _____ because . . .</td>
<td>_____ is a multiple of _____ because . . .</td>
</tr>
</tbody>
</table>

Student Responses

Sample responses:
<table>
<thead>
<tr>
<th>number</th>
<th>factor</th>
<th>multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>11 is a factor of 55 because $11 \times 5 = 55$.</td>
<td>11 is a multiple of 1 because $11 \times 1 = 11$.</td>
</tr>
<tr>
<td>24</td>
<td>8 is a factor of 24 because $8 \times 3 = 24$.</td>
<td>24 is a multiple of 8 because $8 \times 3 = 24$.</td>
</tr>
</tbody>
</table>

---

**Warm-up**

Number Talk: Division

**Standards Alignments**

Building On 3.OA.B.5, 3.OA.C.7

The purpose of this Number Talk is to elicit strategies and understandings students have for dividing within 100. These understandings help students develop fluency and will be helpful later in this lesson when students find factor pairs of numbers.

**Instructional Routines**

Number Talk

**Student-facing Task Statement**

Find the value of each expression mentally.

- $12 \div 3$
- $30 \div 3$
- $60 \div 3$
- $72 \div 3$

**Launch**

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

**Activity**

- Record answers and strategy.
Student Responses
- 4. I know 4 groups of 3 is 12.
- 10. I know that 10 groups of 3 is 30.
- 20. I know that $20 \times 3 = 60$.
- 24. 72 is 60 and 12 so that’s 20 and 4 groups or 24 groups of 3.

Keep expressions and work displayed.
Repeat for each expression.

Synthesis
- “How does knowing the first and third quotients help you find the last quotient?”
  (Since $12 + 60 = 72$, we can add the answers to those quotients to get the answer to the last problem.)
- Consider asking:
  - “Who can restate ____’s reasoning in a different way?”
  - “Did anyone have the same strategy but would explain it differently?”
  - “Did anyone approach the expression in a different way?”
  - “Does anyone want to add on to ____’s strategy?”

Activity 1

Factor and Multiple Statements

Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is for students to find factors and multiples of a given number and make statements that use the terms “factors” and “multiples.” This work prompts students to use language precisely (MP6).

Students can generate many different statements for each number and use the given number in either of the two blanks in the sentence stem. They then share their statements with their partner and explain why their sentences make sense. As they do so, students practice constructing viable arguments and attending to the reasoning of others (MP3).
Access for English Learners

MLR8 Discussion Supports. Use multi-modal examples to show the meaning of factor and multiple. Invite students to use verbal descriptions along with gestures, drawings, or concrete objects to show factors of 10 and multiples of 10.

Advances: Listening, Representing

Student-facing Task Statement

1. Complete a statement using the word “factor” and a statement using the word “multiple” for each number.

<table>
<thead>
<tr>
<th>number</th>
<th>factor</th>
<th>multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___ because . . .</td>
</tr>
<tr>
<td>7</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___ because . . .</td>
</tr>
<tr>
<td>50</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___ because . . .</td>
</tr>
<tr>
<td>16</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___ because . . .</td>
</tr>
<tr>
<td>35</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___ because . . .</td>
</tr>
<tr>
<td>20</td>
<td>____ is a factor of ___ because . . .</td>
<td>____ is a multiple of ___</td>
</tr>
</tbody>
</table>

Launch

- Groups of 2
- “We are going to practice using the words ‘factor’ and ‘multiple’ in preparation for a game we’ll play. Take some time to complete the statements on your own.”
- 3–5 minutes: independent work time

Activity

- “Now share your statements with your partner. Be sure to ask each other questions as you explain your statements.”
- 3–5 minutes: partner discussion

Synthesis

- “What was your favorite statement you came up with? Why was it your favorite?”
- “What did you and your partner notice and wonder about this activity?”
<table>
<thead>
<tr>
<th>number</th>
<th>factor</th>
<th>multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>____ is a factor of ____</td>
<td>____ is a multiple of ____</td>
</tr>
<tr>
<td>6</td>
<td>____ is a factor of ____</td>
<td>____ is a multiple of ____</td>
</tr>
</tbody>
</table>

2. As you compare statements with your partner, discuss one thing you notice and one thing you wonder.

**Student Responses**

1. Sample responses:
   - 10 is a factor of 20 because 
     \[ 10 \times 2 = 20. \] 10 is a multiple of 5 because we can count by 5 and say 10.
   - 5 is a factor of 10 because 
     \[ 5 \times 2 = 10. \] 30 is a multiple of 10 because \[ 10 \times 3 = 30. \]

2. Students may notice:
   - If a number was larger, we could think about what we could divide it by to find factors of the number.
   - If the number was smaller, we multiplied it by another number to find a multiple.
   - Prime numbers like 7 and 19 only have 2 factors.
   - 10 is a factor of a number and a number is a multiple of 10 when the last digit of the number is 0.
   - 5 is a factor of a number and a number is a multiple of 5 when the last digit of the number is 5 or 0.
Even numbers are multiples of 2. Students may wonder:
- Is every number a factor or multiple of another number?
- Is 1 a factor of every number?

Activity 2
Introduce Find the Number, Factors and Multiples

Standards Alignments
Addressing 4.OA.B.4

The purpose of this activity is for students to practice finding factors and multiples of numbers and using the vocabulary. Students can play multiple rounds of the game as time allows and should be encouraged to use the ideas from the previous activity, if needed. This game is Stage 2 of the center Find the Number. In this stage, students find factors and multiples for a given number. The gameboard is a square grid with the numbers 1–100.

To start, one player chooses an even number less than 50. The other player covers either a factor or multiple of that number. Students take turns covering either a multiple or factor of the previous number. When there are no factors or multiples left to cover, the player who covered the last number gets a point. Students take turns choosing the starting number. Students play 10 rounds, or as many rounds as time allows. The player with the most points wins.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations and possible language to use during group work, especially when playing a game that has a winner. Encourage students to discuss how they might support their partner’s learning or collaborate to find solutions, even though they are on opposing teams. Record responses on a display and keep visible during the activity.

Supports accessibility for: Language, Social-Emotional Functioning

Materials to Gather
Centimeter cubes

Materials to Copy
Find the Number Stage 2 Directions and
**Student Responses**

Sample round:
- Partner A chooses to cover 24.
- Partner B covers 12, because 12 is a factor of 24.
- Partner A covers 60, because 60 is a multiple of 12.
- Partner B covers 10, because 10 is a factor of 60.
- Partner A covers 5, because 5 is a factor of 10.
- Partner B claims that no other numbers can be covered.
- Partner A disagrees, saying, “We could select 15, 20, 25, or other multiples of 5, or we could choose 1 because it’s a factor of 5.”
- Partner A wins a point for the round.

**Launch**

- Groups of 2
- Give each group of 2 students the Instructional master and centimeter cubes.
- “We are going to play a game called Find the Number. Take a few minutes to read the directions.”
- 2 minutes: independent work time
- “What questions do you have about the game?”
- If needed, play an example round.

**Activity**

- 10–15 minutes: partner game time
- Monitor for students who strategically choose numbers to win the round.

**Synthesis**

- “What was your strategy for choosing numbers as you played the game?” (I wanted to keep the round going as long as possible, so I liked choosing numbers that I knew had a lot of factors. I tried to find a number that was prime that I could use so that my partner would have a harder time choosing a number.)

**Lesson Synthesis**

“In today’s lesson, we used the terms factors and multiples to describe numbers within 100.”

Display the following prompts:

- “How do you know if ____ is a factor of a number?”
- “How do you know if a number is a multiple of ____?”
“With your partner, take turns using each number 1, 2, 5, and 10 to ask and answer the prompts. For example: The first partner asks: ‘How do you know if 2 is a factor of a number?’ and the second partner responds. The second partner then asks: ‘How do you know if a number is a multiple of 2?’ and the first partner responds.”

Share and record responses. Highlight these observations:

- The number 1 is a factor of every number and every number is a multiple of 1.
- The number 2 is a factor of every even number and each even number is a multiple of 2.
- A number is a multiple of 5 when the last digit of the number is 5 or 0.
- The number 10 is a factor of a number when the last digit of the number is 0.
- A number is a multiple of 10 if its last digit is 0.

**Suggested Centers**

- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)

**Student Section Summary**

In this section, we used what we learned about factors, multiples, and prime and composite numbers between 1–100 to play games and solve problems.

We learned that numbers can share factors and multiples. For example:

- The number 2 is a factor of 6 and also a factor of 8.
- The number 24 is a multiple 6 and also a multiple of 8.

Knowing about factors and multiples helped us answer questions such as:

- “Can we put 24 chairs in 6 equal rows? What about 7 equal rows or 8 equal rows?”
- “If there are 20 lockers in a row and a student touches every fourth locker, how many lockers would they touch? Which locker numbers would they touch?”

---

**Complete Cool-Down**
Response to Student Thinking

Students confuse the terms “factor” and “multiple.”

Next Day Support

- Launch warm-up or activities by highlighting important vocabulary from previous lessons.
Lesson 8: Mondrian's Art (Optional)

Standards Alignments
Addressing  4.OA.B.4
Building Towards  4.OA.B.4

Teacher-facing Learning Goals
- Apply understanding of factors, multiples, prime and composite numbers to create a geometric design.

Student-facing Learning Goals
- Let’s make art with rectangles.

Lesson Purpose
The purpose of this lesson is for students to apply their understanding of the area of rectangles and factor pairs to create and analyze Mondrian-inspired art.

This lesson is optional because it does not address new mathematical content standards. It does give students an opportunity to develop mathematical modeling skills while applying the ideas of area, factors, multiples, prime numbers, and composite numbers.

Students create a piece of artwork that is based on area of rectangles and multiplication facts within 100. They begin by learning about Piet Mondrian and analyzing a number of his abstract paintings. They recognize that the paintings are composed of rectangles—some with the same area and some with different areas.

Students then outline their own composition, by dividing a 18-by-24 grid into rectangular spaces with certain requirements. Next, students examine a peer’s artwork. They identify rectangles with equal areas and those that represent prime or composite numbers.

When students isolate and describe the mathematical elements in art and adhere to mathematical constraints to create art, they model with mathematics (MP4).

This lesson may take more than 60 minutes, as students may need additional time to color, analyze, and present their designs. Consider modifying the activities or expanding the lesson across 2 days to meet students’ needs and any time constraints.

Access for:
- Students with Disabilities
  - Action and Expression (Activity 1)
- English Learners
  - MLR8 (Activity 2)
Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Colored pencils, crayons, or markers: Activity 1, Activity 2
- Glue or tape: Activity 2
- Rulers or straightedges: Activity 1
- Sticky notes: Activity 2

Materials to Copy

- Centimeter Grid Paper - Standard (groups of 2): Activity 1

Lesson Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm-up</td>
<td>10 min</td>
</tr>
<tr>
<td>Activity 1</td>
<td>20 min</td>
</tr>
<tr>
<td>Activity 2</td>
<td>20 min</td>
</tr>
<tr>
<td>Lesson Synthesis</td>
<td>10 min</td>
</tr>
</tbody>
</table>

Teacher Reflection Question

With which math ideas from today's lesson did students grapple most? Did this surprise you or was this what you expected?

Standards Alignments

Building Towards 4.OA.B.4

The purpose of this task is to introduce students to the artwork of Piet Mondrian. Students may notice that his paintings are composed of rectangles of various sizes. Students will create their own versions of Mondrian art in the first activity.

To show students additional artwork by Mondrian, consider visiting a virtual installation of Piet Mondrian's work on the website of Museum of Modern Art (MoMA) or visiting the website of the Tate Gallery.
Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Launch

- Groups of 2
- Display the images.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “These are digital copies of famous paintings by a Dutch artist named Piet Mondrian. He lived from 1872 to 1944. A little more than 100 years ago, he became known for painting in a style that relates to the math we have been studying. Many of his paintings hang in museums all around the world.”

- “How do you think his art connects with what we've been studying? Why are we looking at it during math class?” (He used a lot of rectangles. His art looks very precise. He seemed to have planned for the rectangles to have certain side lengths.)

- If not mentioned by students, highlight that some of the lines go from edge to edge of the painting while others are shorter, and that some rectangles seem to have the same area.

- Considering showing students additional artwork by Mondrian.

Student Responses

Students may notice:

- Lots of rectangles.
- The rectangles are all different sizes.
- The paintings weren't of anything.
- There are only 3 or 4 colors.
- There are black lines around all the rectangles.

Student may wonder:

- Are any of the rectangles the same size?
- What is the area of the white rectangles?
- Which colors fill more space on the paintings?
Activity 1
My Mondrian Outline

Standards Alignments
Building Towards 4.OA.B.4

The purpose of this activity is for students to create an outline for their artwork. In this activity, students draw lines on graph paper, marking out rectangular areas that will be the basis for their Mondrian-inspired artwork.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Keep a display of Mondrian's paintings visible throughout the activity.

Supports accessibility for: Memory, Organization

Materials to Gather
Colored pencils, crayons, or markers, Rulers or straightedges

Materials to Copy
Centimeter Grid Paper - Standard (groups of 2)

Required Preparation

- Each student will need a black marker or crayon.

Student-facing Task Statement

Create an outline for art in the Mondrian style, starting with an 18-by-24 grid.

Your artwork should:

- be partitioned into at least 12 rectangles
- include two different rectangles that have the same area
- include at least one rectangle whose area is a prime number

Launch

- “We are going to create our own art pieces that are inspired by Mondrian's work.”
- Read the activity statement as a class.
- Select a student to explain the task in their own words. Invite the class to ask clarifying questions.
- Give students a copy of the Instructional master, a straightedge, and black markers or crayons.
Try at least one of these challenges. Make a design where:

- all but two of the rectangles have a prime number for its area
- no two rectangles share a side entirely

**Student Responses**

Answers vary.

**Activity**

- “Use your straightedge and pencil to partition your grid. Try at least one of the challenges.”
- “Once you are happy with your design, trace it with a black marker or crayon.”
- 13–15 minutes: independent work time
- Monitor for students who attempt or accomplish one or more of the challenges.

**Synthesis**

- “Compare your work with a partner. What is alike or different about your outlines?” (They are alike because they show only rectangles, but the way they are arranged and their sizes are different.)
- “How can you determine if any of the rectangles have the same area?” (See if the sides are factor pairs of the same number. Use the side lengths and multiply them to see if the product is the same number.)

**Activity 2**

Analyze the Rectangles

**Standards Alignments**

Addressing 4.OA.B.4

In this activity, students use their understanding of factor pairs, prime, and composite numbers to analyze their peers’ artwork. They look for rectangles that have the same area and those with a prime number or a composite number for their area. Students practice communicating with precision as they identify rectangles and how they know the rectangles meet these conditions (MP6).
After students share their analyses with their partner and a brief class discussion, give students time to color their artwork and to prepare it for display.

Access for English Learners

*MLR8 Discussion Supports.* Synthesis: Display the following sentence frames to support whole-class discussion: “To find rectangles with the same area I looked for . . .”, “To find rectangles with an area that is prime, I looked for . . .”, and “To find rectangles with an area that is composite, I looked for . . .”

*Advances: Speaking, Representing*

Materials to Gather

Colored pencils, crayons, or markers, Glue or tape, Sticky notes

Student-facing Task Statement

Trade artwork with your partner.

Using your partner’s artwork, look for and describe each of the following:

1. Rectangles that have the same area
2. Rectangles with an area that is a prime number
3. Rectangles with an area that is a composite number
4. Which challenge they completed

Student Responses

Sample responses:

1. There is a rectangle that is 3 units by 10 units, and another that is 6 units by 5 units. The area of both is 30 square units.
2. There is one rectangle that is 11 units by 1 unit. The area is 11 square units.
3. There are many rectangles with a composite number for the area: 18 square units, 12 square units, 21 square units, and more.

Launch

- Groups of 2
- “Switch artwork with your partner.”
- “Look at your partner’s work and try to find three kinds of rectangles: rectangles with the same area, rectangles with an area that is a prime number, and rectangles with an area that is a composite number.”
- “Then, if your partner completed a challenge from the first activity, see which one they did.”

Activity

- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who consider factor pairs of a number as they look for rectangles with the same area in their partner’s artwork.

Synthesis

- Invite 1–2 previously selected students to share how they found rectangles with the
4. No two rectangles share a side entirely. (The sides are factor pairs of the same number.)

- “Now, take a moment to color your artwork with 3–4 colors. Later, you’ll display your work for others to see.”
- Give students colored pencils, crayons, or markers.
- 8–10 minutes: independent work time
- Give students materials needed for the gallery walk: glue or tape for displaying their artwork and sticky notes for writing comments or questions.

Lesson Synthesis

Direct students to display their artwork for all to see.

“You will now walk around and look at the art the class has made. As you do so, consider questions that you might ask the artists about their design. Choose at least one piece of artwork and write a question about it on a sticky note.”

5–6 minutes: gallery walk

Monitor for questions that are related to the artist's intent or mathematics in the design. Invite a couple of students to whom those questions are directed to answer questions about their art.

“Today you had a chance to create artwork and display it like in an art gallery.”

“What was the most challenging part about creating the artwork?” (I was limited to only using rectangles. I had to make sure two rectangles had the same area.)

“What connections do you see between the mathematics and art we experienced today?” (The art we made uses rectangles. We can use multiplication to figure out if the areas are the same or different. We had the same requirements, but our art was different because we chose different side lengths.)

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Addressing)
- Secret Fraction (3), Stage 1: Building Non-Unit Fractions (Supporting)
Family Support Materials
Family Support Materials

Factors and Multiples

In this unit, students learn about factors and multiples and apply their understanding of the area of rectangles. Students determine if a number between 1 and 100 is prime or composite.

Section A: Understand Factors and Multiples

In this section, students learn about the meaning of factors and multiples by relating them to the concept of area. They use square tiles to build rectangles with given length and width. Then, they find the area of the rectangles.

For example, this rectangle has an area of 14 square units with side lengths of 7 and 2.

We can say that 7 and 2 are a factor pair of 14, and that $7 \times 2 = 14$.

We can also say that 14 is a multiple of 7 and a multiple of 2.

Students discover that some numbers have many factor pairs and others have only one possible factor pair. They decide if a number is prime or composite based on how many rectangles can be made with that number as the area.

Section B: Find Factor Pairs and Multiples

In this section, students apply what they learned about factors and multiples to play games and solve problems in different contexts. Through the tasks, students look for patterns with factors and multiples. They find all of the factor pairs of a whole number between 1–100. They also decide if a whole number within 100 is a multiple of a given one-digit number.

Try it at home!

Complete the statements for each number. Explain your reasoning.

<table>
<thead>
<tr>
<th>number</th>
<th>factor</th>
<th>multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>_____ is a factor of _____ because . . .</td>
<td>_____ is a multiple of _____ because . . .</td>
</tr>
<tr>
<td>18</td>
<td>_____ is a factor of _____ because . . .</td>
<td>_____ is a multiple of _____ because . . .</td>
</tr>
</tbody>
</table>
Questions that may be helpful as they work:

- How did you know this was a factor of that number?
- How did you know this was a multiple of that number?
- How are factors related to multiples?
- Is the number prime or composite? How do you know?
Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment
Factors and Multiples: Section A Checkpoint

1. What are the possible side lengths of a rectangle with area 10 square units?

Draw an example of each possible rectangle on the grid.

2. Select all true statements.
   
   A. 5 is a factor of 35.
   
   B. 35 is a factor of 5.
   
   C. 5 is a multiple of 35.
   
   D. 35 is a multiple of 5.

3. Is 17 a prime number or a composite number? Explain how you know.
Factors and Multiples: Section B Checkpoint

1. Pencils come in packages of 10 and 12. Jada’s class needs 60 pencils. Which packages of pencils would you choose for Jada’s class? Explain or show your reasoning.

2. Find all of the factor pairs for each number.
   a. 13
   b. 16
   c. 24

3. Select all of the true statements.
   A. 19 is a prime number.
   B. The only factors of 9 are 1 and itself.
   C. 3 is a factor of 24.
   D. 56 is a multiple of 6.
Factors and Multiples: End-of-Unit Assessment

1. a. Is 27 a prime number or a composite number? Explain or show your reasoning.

b. Is 29 a prime number or a composite number? Explain or show your reasoning.

2. Select all true statements.

A. 15 is a multiple of 3.
B. 16 is a factor of 8.
C. 80 is a multiple of 4.
D. The only factor pair of 49 is 1 and 49.
E. The factor pairs of 12 are 1 and 12, 2 and 6, and 3 and 4.

3. Find all factor pairs of 84.
4. Han is playing a card game with friends. The number of cards never changes, but the number of players does.

   a. With 5 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

   b. With 3 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

   c. With 4 players, the cards can be divided equally between the players. How many cards could there be? Explain or show your reasoning.
4. Han is playing a card game with friends. The number of cards never changes, but the number of players does.

a. With 5 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

b. With 3 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

c. With 4 players, the cards can be divided equally between the players. How many cards could there be? Explain or show your reasoning.
Assessment Answer Keys
Assessment: Section A Checkpoint

Problem 1

Goals Assessed
- Relate the side lengths and area of a rectangle to factors and multiples

What are the possible side lengths of a rectangle with area 10 square units?

Draw an example of each possible rectangle on the grid.

Solution

2 units by 5 units, or 1 unit by 10 units
Problem 2

**Goals Assessed**
- Explain what it means to be a factor or a multiple of a whole number.

Select **all** true statements.

A. 5 is a factor of 35.
B. 35 is a factor of 5.
C. 5 is a multiple of 35.
D. 35 is a multiple of 5.

**Solution**

["A", "D"]

Problem 3

**Goals Assessed**
- Determine if a number is prime or composite.

Is 17 a prime number or a composite number? Explain how you know.

**Solution**

Prime, because 1 and 17 are its only factors.
Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Apply multiplication fluency within 100 and the relationship between multiplication and division to find factor pairs and multiples.

Pencils come in packages of 10 and 12. Jada’s class needs 60 pencils. Which packages of pencils would you choose for Jada’s class? Explain or show your reasoning.

Solution

Sample responses:

- I would get 6 packages of 10 pencils and that would give me 60 pencils exactly.
- I would get 5 packages of 12 pencils and that would give me 60 pencils exactly.

Problem 2

Goals Assessed

- Apply multiplication fluency within 100 and the relationship between multiplication and division to find factor pairs and multiples.

Find all of the factor pairs for each number.

a. 13
b. 16
c. 24

Solution

a. 1 and 13
b. 1 and 16, 2 and 8, 4 and 4
c. 1 and 24, 2 and 12, 3 and 8, 4 and 6
Problem 3

Goals Assessed

- Apply multiplication fluency within 100 and the relationship between multiplication and division to find factor pairs and multiples.

Select all of the true statements.

A. 19 is a prime number.
B. The only factors of 9 are 1 and itself.
C. 3 is a factor of 24.
D. 56 is a multiple of 6.

Solution

["A", "C"]
Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments
Addressing 4.OA.B.4

Narrative
Students determine whether a number is prime or composite. The number 27 has 1, 3, 9, and 27 as factors and \(3 \times 9 = 27\) should be a known fact from the previous grade. Since the only factors of 29 are 1 and 29, it is a prime number. Students can answer one or both problems incorrectly, but still understand the meaning of prime and composite, if they do not know their multiplication facts well.

a. Is 27 a prime number or a composite number? Explain or show your reasoning.
   b. Is 29 a prime number or a composite number? Explain or show your reasoning.

Solution

a. Composite. Sample response: 27 is a composite number because it has factor pairs 1 and 27 and also 3 and 9.
   b. Prime. Sample response: 29 is a prime number because it only has one factor pair, 1 and 29.

Problem 2

Standards Alignments
Addressing 4.OA.B.4

Narrative
This item assesses student understanding of the words factor and multiple. They may select B, and not select A, C, and E, if they confuse the meaning of factor and multiple. They may select D if they understand the meaning of factor but are not careful and forget the factor 7. Students may understand the meaning of factor but fail to select C if they do not see that \(80 = 20 \times 4\).

Select all true statements.
A. 15 is a multiple of 3.
B. 16 is a factor of 8.
C. 80 is a multiple of 4.
D. The only factor pair of 49 is 1 and 49.
E. The factor pairs of 12 are 1 and 12, 2 and 6, and 3 and 4.

Solution

["A", "C", "E"]

Problem 3

**Standards Alignments**
Addressing 4.OA.B.4

**Narrative**
Students find all factor pairs of a number within 100. The calculations on this item are more challenging than the previous items. If students omit one or more of the factor pairs, or choose incorrect factor pairs, they may need more practice with multiplication within 100.

Find all factor pairs of 84.

Solution

1 and 84, 2 and 42, 3 and 28, 4 and 21, 6 and 14, 7 and 12.

Problem 4

**Standards Alignments**
Addressing 4.OA.B.4

**Narrative**
Students examine multiples of different numbers. The first two questions give them some experience making the calculations they will need to solve the third problem but are not needed in order to work on the final problem. Students may realize that the product $3 \times 4 \times 5$ is a multiple of
3, 4, and 5 but more likely they will use a trial and error approach. This approach will work well unless they try, for example, all multiples of 3 in which case there are a lot of possibilities before reaching 60, the smallest possible answer and the only answer within the range from 0 to 100.

It is not essential that students completely solve the final problem. It is important that they demonstrate understanding of the relationship between the given number of players and the possible number of cards.

Han is playing a card game with friends. The number of cards never changes, but the number of players does.

a. With 5 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

b. With 3 players, the cards can be divided equally between the players. Could there be 50 cards? Explain or show your reasoning.

c. With 4 players, the cards can be divided equally between the players. How many cards could there be? Explain or show your reasoning.

Solution

a. Yes. 50 is a multiple of 5. Each player could get 10 cards.

b. No, 50 is not a multiple of 3 so you cannot divide 50 cards into 3 equal groups.

c. 60. I need a number that is a multiple of 3 (since the cards can be divided evenly between 3 players) and is also a multiple of 4 and 5. I know 20 is a multiple of 4 and 5, but it is not a multiple of 3. I tried 40, which is a multiple of 4 and 5, but it is not a multiple of 3. The number 60 is a multiple of 4, 5, and 3.
Lesson
Cool Downs
Lesson 1: Multiples of a Number

Cool Down: Area and Multiples

If a rectangle is 6 tiles wide, what could be its area? Name three possibilities. Explain or show your reasoning.
Lesson 2: Factor Pairs

Cool Down: The Side Lengths of Rectangles

1. What are all of the possible side lengths of a rectangle with an area of 21 square units?

2. What are all of the possible side lengths of a rectangle with an area of 50 square units?
Lesson 3: Prime and Composite Numbers

Cool Down: Prime or Composite?

1. a. What are the factor pairs of 40?

   b. Is 40 a prime or composite number? Explain or show your reasoning.

2. Is 17 a prime or composite number? Explain or show your reasoning.
Lesson 4: Multiplication Practice

Cool Down: Reflect on Multiplication and Strategies

1. What multiplication facts do you want to keep practicing?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

2. Describe a strategy you can use in the future to multiply two whole numbers.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
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Lesson 5: More Multiples

Cool Down: Fourth-grade Party

All of the fourth-grade classes are getting together for a party. They have tables where 6 people can sit and tables where 8 people can sit. There will be 72 students that need seats.

If you may only use one type of table, which type of table would you choose? Explain or show your reasoning.
Lesson 6: The Locker Problem

Cool Down: Reflect on Problem Solving

Reflect on your work today. How did you organize your thinking? How did you adjust your work and thinking along the way? What was helpful to you?
## Lesson 7: Find Factors and Multiples

### Cool Down: Complete the Statements

Complete the statements for each number.

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<th>multiple</th>
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<td>_____ is a multiple of _____ because . . .</td>
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Instructional Masters
# Instructional Masters for Factors and Multiples

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Directions:
- Partner A: Cover up a number. That is your score for this turn.
- Partner B: Cover up all the factors of that number. The sum of the factors are your score for this turn.
- Take turns choosing numbers and finding factors.
- If a player chooses a number with no uncovered factors remaining, that player loses a turn and does not get any points for the round.
- The game ends when there are no numbers left with uncovered factors. The player with the highest score wins.

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Find the Number Stage 1 Directions and Gameboard

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Find the Number Stage 2 Directions and Gameboard

Directions:
- Partner A: Cover up an even number that is less than 50.
- Partner B: Cover up a factor or multiple of the number your partner just covered.
- Partner A: Cover up a factor or multiple of the number your partner just covered.
- Take turns covering numbers.
- The round ends when there are no factors or multiples of the previous number left to cover. The last player to cover a number gets a point.
- The game ends after 10 rounds. The player with the most points wins.

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</table>
Card Sort: Multiplication

2 × 8
1 × 8
10 × 6
6 × 9

8 × 6
9 × 7
6 × 6
5 × 6
Card Sort: Multiplication

5 \times 7

6 \times 7

7 \times 7

8 \times 7

1 \times 7

2 \times 7

3 \times 7

4 \times 7
Card Sort: Multiplication

6 \times 10

6 \times 6

6 \times 8

6 \times 7
Can You Build It Stage 2 Directions

Directions:
- Take 2 number cards to make a two-digit number.
- Both partners build as many rectangles as they can with that area.
- When both players are finished, compare rectangles.
- Each player gets 1 point if they both have all the same rectangles.
- A player gets 2 points if they build a rectangle with the given area that their partner does not have.
- The player with the most points after 8 rounds wins the game.
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Number Cards (0-10)

4 5
6 7
8 9
Can You Build It Stage 1 Directions

Directions:
- **Partner A:**
  - Use 16–24 tiles to build a rectangle. Don't let your partner see it!
  - Describe it to your partner.
- **Partner B:** Build the rectangle your partner describes to you.
- Place the two rectangles next to each other and discuss what is the same and what is different about them.
- **Switch roles and repeat.**

Directions:
- **Partner A:**
  - Use 16–24 tiles to build a rectangle. Don't let your partner see it!
  - Describe it to your partner.
- **Partner B:** Build the rectangle your partner describes to you.
- Place the two rectangles next to each other and discuss what is the same and what is different about them.
- **Switch roles and repeat.**

Directions:
- **Partner A:**
  - Use 16–24 tiles to build a rectangle. Don't let your partner see it!
  - Describe it to your partner.
- **Partner B:** Build the rectangle your partner describes to you.
- Place the two rectangles next to each other and discuss what is the same and what is different about them.
- **Switch roles and repeat.**
Capture Squares Stage 7 Gameboard

Directions:
- On your turn:
  - Roll the number cube and spin the spinner. Find the product.
  - Choose a square on the gameboard that shows that number. Draw one line connecting any 2 dots around the number.
  - If you can't draw a line, roll and spin again.
  - If you draw a line that finishes a square around a number, shade in that box with your color.
- Take turns with your partner. The first player to shade in 3 boxes wins.
Directions:

- **Partner A:**
  - Put a paper clip on 2 numbers in the grey row.
  - Multiply the numbers.
  - Cover the product of the 2 numbers with a counter.

- **Partner B:**
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.

- Take turns. The first partner to cover 5 squares in a row wins.

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<td>50</td>
<td>4</td>
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### Five in a Row Multiplication and Division Stage 1 Gameboard
Directions:
● Partner A:
  ○ Put a paper clip on 2 numbers in the grey rows.
  ○ Multiply the numbers.
  ○ Cover the product of the 2 numbers with a counter.
● Partner B:
  ○ Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
● Take turns. The first partner to cover 5 squares in a row wins.

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Directions:

- Place the secret fraction cards in a stack face down.
- Place the unit fraction cards in a stack face down.
- Each partner:
  - Pick 2 secret fraction cards. These are the fractions you are trying to make with your unit fractions.
  - Partner A can:
    - Pick up one unit fraction card.
    - Trade both of your secret fractions for 2 new secret fractions from the pile.
  - When you have enough unit fractions to make your secret fraction, fill in your secret fraction on the gameboard and pick a new secret fraction.
- First partner to make 3 secret fractions wins.

Secret Fractions Stage 1 Gameboard
<table>
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**Core Knowledge Mathematics™**

units at this level include:

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- Fraction Equivalence and Comparison
- Extending Operations to Fractions
- From Hundredths to Hundred-thousands
- Multiplicative Comparison and Measurement
- Multiplying and Dividing Multi-digit Numbers
- Angles and Angle Measurement
- Properties of Two-dimensional Shapes
- Putting it All Together

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