CKMath™ Core Knowledge MATHEMATICS Grade 6

Teacher Implementation Guide

Click on each topic to access resources that help to implement the Core Knowledge Mathematics™ curriculum. Some links provide access to files created by the Core Knowledge Foundation, including PDF documents that you can download and view with the appropriate software (such as Adobe Reader).

Grade 6

LEARN: About This Curriculum

The Core Knowledge Mathematics™ materials were created by Illustrative Mathematics in collaboration with Open Up Resources and originally published under a Creative Commons Attribution License: Creative Commons Attribution 4.0 International License (CC BY 4.0). The curriculum was piloted and revised in the 2016–2017 school year. Further revisions were made in 2019 by Illustrative Mathematics and the open-source curriculum has continued to be highly rated by reviewers such as EdReports.org. The Core Knowledge Mathematics materials include the Illustrative Math enhancements (also licensed under CC BY 4.0) such as additional English language learner supports, item-by-item guidance for how to use the *Check Your Readiness* pre-assessment results, and more. For a detailed list of the enhancements made to the Open Up Resources IM 6–8 curriculum (https://www.openupresources.org/math-curriculum/), please refer to this **document** [link].

Curriculum Overview

Each course in CKMath 6–8 contains nine units. Each of the first eight units are anchored by a few big ideas in grade-level mathematics. Units contain between 11 and 23 lesson plans. Each unit has a diagnostic assessment for the beginning of the unit (Check Your Readiness) and an end-of-unit assessment. Longer units also have a mid-unit assessment. The last unit in each course is structured differently, and contains optional lessons that help students apply and tie together big ideas from the year.

The time estimates in these materials refer to instructional time. Each lesson plan is designed to fit within a class period that is at least 45 minutes long. Some lessons contain optional activities that provide additional scaffolding or practice for teachers to use at their discretion.

Interacting with the Materials

There are two ways students can interact with these materials. Students can work solely with printed workbooks or PDFs. Alternatively, if all students have access to an appropriate device, students can look at the Student Workbook on that device and write their responses in a notebook. It is recommended that if students are to access the materials this way, they keep the notebook carefully organized so that they can go back to their work later. A key feature of the Core Knowledge Mathematics curriculum is distributed and repeated practice to build both conceptual understanding and procedural fluency.

Teachers can access the teacher materials either in print or in a browser. A classroom with a digital projector is recommended. Many activities are written in a card sort, matching, or information gap format that requires teachers to provide students with a set of cards or slips of paper that have been photocopied and cut up ahead of time. Teachers might stock up on two sizes of re-sealable plastic bags: sandwich size and gallon size. For a given activity, one set of cards can go in each small bag, and then the small bags for one class can be placed in a large bag. If these are labeled and stored in an organized manner, it can facilitate preparing ahead of time and re-using card sets between classes. Additionally, if possible, it is often helpful to print the slips for different parts of an activity on different color paper. This helps facilitate quickly sorting the cards between classes.

← Table of Contents The Design Principles of CKMath →

Design Principles of CKMath™

Developing Conceptual Understanding and Procedural Fluency

Each unit begins with a pre-assessment (Check Your Readiness) that helps teachers gauge what students know about both prerequisite and upcoming concepts and skills, so that teachers can gauge where students are and make adjustments accordingly. The initial lesson in a unit is designed to activate prior knowledge and provide an easy entry to point to new concepts, so that students at different levels of both mathematical and English language proficiency can engage productively in the work. As the unit progresses, students are systematically introduced to representations, contexts, concepts, language and notation. As their learning progresses, they make connections between different representations and strategies, consolidating their conceptual understanding, and see and understand more efficient methods of solving problems, supporting the shift towards procedural fluency. The distributed practice problems give students ongoing practice, which also supports developing procedural proficiency.

Applying Mathematics

Students have opportunities to make connections to real-world contexts throughout the materials. Frequently, carefully-chosen anchor contexts are used to motivate new mathematical concepts, and students have many opportunities to make connections between contexts and the concepts they are learning. Additionally, most units include a real-world application lesson at the end. In some cases, students spend more time developing mathematical concepts before tackling more complex application problems, and the focus is on mathematical contexts. The first unit on geometry is an example of this.

The Five Practices

Selected activities are structured using Five Practices for Orchestrating Productive Mathematical Discussions (Smith & Stein, 2011), also described in Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014), and Intentional Talk: How to Structure and Lead Productive Mathematical Discussions (Kazemi & Hintz, 2014). These activities include a presentation of a task or problem (may be print or other media) where student approaches are anticipated ahead of time. Students first engage in independent think-time followed by partner or small-group work on the problem. The teacher circulates as students are working and notes groups using different approaches. Groups or individuals are selected in a specific, recommended sequence to share their approach with the class, and finally the teacher leads a whole-class discussion to make connections and highlight important ideas.

Task Purposes

To learn more about the design of the student tasks, please refer to the **Student Task Design Principles**.

← Table of Contents   What is a "Problem-Based" Curriculum? →

Grade 6

What is a "Problem-Based" Curriculum?

What students should know and be able to do

Our ultimate purpose is to impact student learning and achievement. First, we define the attitudes and beliefs about mathematics and mathematics learning we want to cultivate in students, and what mathematics students should know and be able to do.

Attitudes and Beliefs We Want to Cultivate

Many people think that mathematical knowledge and skills exclusively belong to "math people." Yet research shows that students who believe that hard work is more important than innate talent learn more mathematics.[1] We want students to believe anyone can do mathematics and that persevering at mathematics will result in understanding and success. In the words of the NRC report Adding It Up, we want students to develop a "productive disposition—[the] habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy."[2]

[1] Uttal, D.H. (1997). Beliefs about genetic influences on mathematics achievement: a cross-cultural comparison. *Genetica*, 99(2-3), 165-172. doi.org/10.1023/A:1018318822120

[2] National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. doi.org/10.17226/9822

Knowledge

Conceptual understanding: Students need to understand the why behind the how in mathematics. Concepts build on experience with concrete contexts. Students should access these concepts from a number of perspectives in order to see math as more than a set of disconnected procedures.

Procedural fluency: We view procedural fluency as solving problems expected by the standards with speed, accuracy, and flexibility.

Application: Application means applying mathematical or statistical concepts and skills to a novel mathematical or real-world context.

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand and be able to do the mathematics.

A “Problem-Based” Curriculum continued

Mathematical Practices

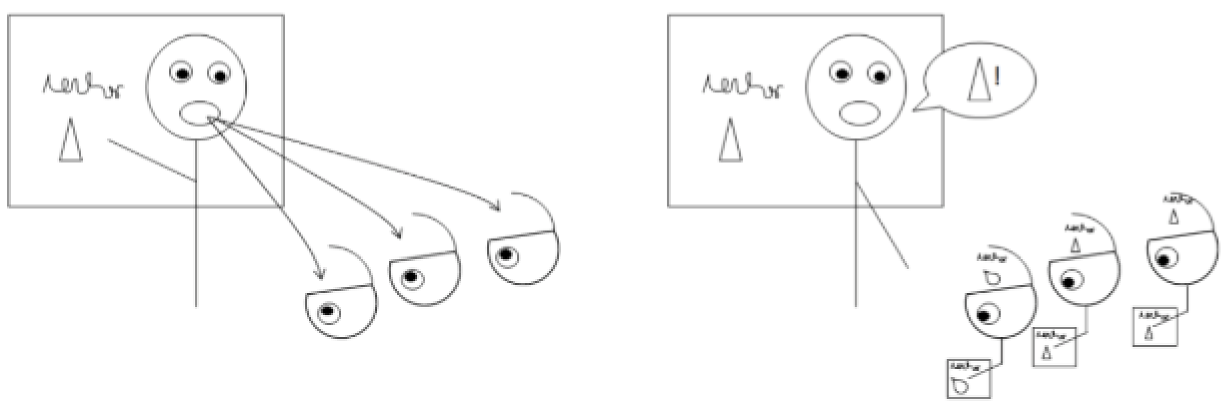
In a mathematics class, students should not just learn *about* mathematics, they should *do* mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

What teaching and learning should look like

How teachers should teach depends on what we want students to learn. To understand what teachers need to know and be able to do, we need to understand how students develop the different (but intertwined) strands of mathematical proficiency, and what kind of instructional moves support that development.

Principles for Mathematics Teaching and Learning

Active learning is best: Students learn best and retain what they learn better by solving problems. Often, mathematics instruction is shaped by the belief that if teachers tell students how to solve problems and then students practice, students will learn how to do mathematics.



Teacher tells
Students listen

Students practice
Teacher corrects

Decades of research tells us that the traditional model of instruction is flawed. Traditional instructional methods may get short-term results with procedural skills, but students tend to forget the procedural skills and do not develop problem solving skills, deep conceptual understanding, or a mental framework for how ideas fit together. They also don’t develop strategies for tackling non-routine problems, including a propensity for engaging in productive struggle to make sense of problems and persevere in solving them.

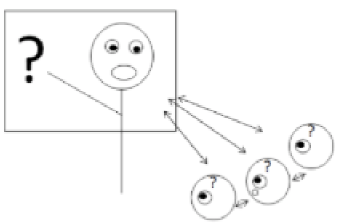
A “Problem-Based” Curriculum continued

In order to learn mathematics, students should spend time in math class *doing mathematics*.

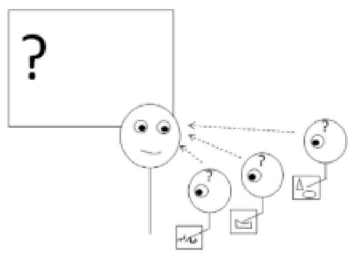
“Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience rather than the elements that must be taught before problem solving.”[3]

Students should take an active role, both individually and in groups of varied sizes, to see what they can figure out before having things explained to them or being told what to do. Teachers play a critical role in mediating student learning, but that role looks different than simply showing, telling, and correcting. The teacher’s role is to:

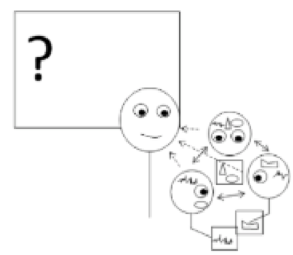
1. ensure students understand the context and what is being asked,
2. ask questions to advance students’ thinking in productive ways,
3. help students share their work and understand others’ work through orchestrating productive discussions, and
4. synthesize the learning with students at the end of activities and lessons.



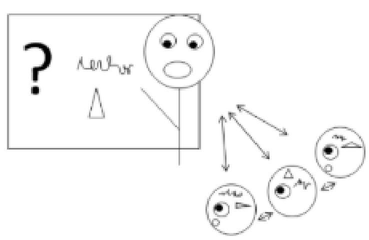
1. Teacher ensures students understand the question



2. Students work individually Teacher monitors, listens, questions



3. Students work in groups Teacher monitors, listens, and asks questions to understand students’ thinking



4. Teacher helps students synthesize their learning

Teachers should build on what students know: New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them.[4] In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students’ learning.

Good instruction starts with explicit learning goals: Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students’ understanding.

A "Problem-Based" Curriculum continued

Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different learning goals require different instructional moves: The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- Introduce students to a new topic of study and invite them to the mathematics
- Study new concepts and procedures deeply
- Integrate and connect representations, concepts, and procedures
- Work towards mastery
- Apply mathematics

Lessons should be designed based on what the intended learning outcomes are. This means that teachers should have a toolbox of instructional moves that they can use as appropriate.

Each and every student should have access to the mathematical work: With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional toolboxes should include knowledge of and skill in implementing supports for different learners.

[3] Hiebert, J., et. al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. *Educational Researcher* 25(4), 12-21. doi.org/10.3102/0013189X025004012

[4] National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. doi.org/10.17226/9822

Critical Practices

Intentional planning: Because different learning goals require different instructional moves, teachers need to be able to plan their instruction appropriately. While a high-quality curriculum does reduce the burden for teachers to create or curate lessons and tasks, it does not reduce the need to spend deliberate time planning lessons and tasks. Instead, teachers' planning time can shift to high-leverage practices (practices that teachers without a high-quality curriculum often report wishing they had more time for): reading and understanding the high-quality curriculum materials; identifying connections to prior and upcoming work; diagnosing students' readiness to do the work; leveraging instructional routines to address different student needs and differentiate instruction; anticipating student responses that will be important to move the learning forward; planning questions and prompts that will help students attend to, make sense of, and learn from each other's work; planning supports and extensions to give as many students as possible access to the main mathematical goals;

figuring out timing, pacing, and opportunities for practice; preparing necessary supplies; and the never-ending task of giving feedback on student work.

Establishing norms: Norms around doing math together and sharing understandings play an important role in the success of a problem-based curriculum. For example, students must feel safe taking risks, listen to each other, disagree respectfully, and honor equal air time when working together in groups. Establishing norms helps teachers cultivate a community of learners where making thinking visible is both expected and valued.

Building a shared understanding of a small set of instructional routines: Instructional routines allow the students and teacher to become familiar with the classroom choreography and what they are expected to do. This means that they can pay less attention to what they are supposed to do and more attention to the mathematics to be learned. Routines can provide a structure that helps strengthen students' skills in communicating their mathematical ideas.

Using high quality curriculum: A growing body of evidence suggests that using a high-quality, coherent curriculum can have a significant impact on student learning.[5] Creating a coherent, effective instructional sequence from the ground up takes significant time, effort, and expertise. Teaching is already a full-time job, and adding curriculum development on top of that means teachers are overloaded or shortchanging their students.

Ongoing formative assessment: Teachers should know what mathematics their students come into the classroom already understanding, and use that information to plan their lessons. As students work on problems, teachers should ask questions to better understand students' thinking, and use expected student responses and potential misconceptions to build on students' mathematical understanding during the lesson. Teachers should monitor what their students have learned at the end of the lesson and use this information to provide feedback and plan further instruction.

[5] Steiner, D. (2017). Curriculum research: What we know and where we need to go. *Standards Work*. Retrieved from https://standardswork.org/wp-content/uploads/2017/03/sw-curriculum-research-report-fnl.pdf

← Table of Contents A Typical CKMath Lesson →

CKMath™ Core Knowledge MATHEMATICS Grade 6

A Typical CKMath Lesson

A note about optional activities: A relatively small number of activities throughout the course have been marked "optional." Some common reasons an activity might be optional include:

- The activity addresses a concept or skill that is below grade level, but we know that it is common for students to need a chance to focus on it before encountering grade-level material. If the pre-unit diagnostic assessment ("*Check Your Readiness*") indicates that students don't need this review, an activity like this can be safely skipped.
- The activity addresses a concept or skill that goes beyond the requirements of a standard. The activity is nice to do if there is time, but students won't miss anything important if the activity is skipped.
- The activity provides an opportunity for additional practice on a concept or skill that we know many students (but not necessarily all students) need. Teachers should use their judgment about whether class time is needed for such an activity.

A typical lesson has four phases:

1. **A warm-up**
2. **One or more instructional activities**
3. **The lesson synthesis**
4. **A cool-down**

The Warm-up

The first event in every lesson is a warm-up. A warm-up either:

- helps students get ready for the day's lesson, or
- gives students an opportunity to strengthen their number sense or procedural fluency.

A warm-up that helps students get ready for today's lesson might serve to remind them of a context they have seen before, get them thinking about where the previous lesson left off, or preview a calculation that will happen in the lesson so that the calculation doesn't get in the way of learning new mathematics.

A warm-up that is meant to strengthen number sense or procedural fluency asks students to do mental arithmetic or reason numerically or algebraically. It gives them a chance to make deeper connections or become more flexible in their thinking.

Four instructional routines frequently used in warm-ups are Number Talks, Notice and Wonder, Which One Doesn't Belong, and True or False. In addition to the mathematical purposes, these routines serve the additional purpose of strengthening students' skills in listening and speaking about mathematics.

A Typical CKMath Lesson continued

Once students and teachers become used to the routine, warm-ups should take 5–10 minutes. If warm-ups frequently take much longer than that, the teacher should work on concrete moves to more efficiently accomplish the goal of the warm-up.

At the beginning of the year, consider establishing a small, discreet hand signal students can display to indicate they have an answer they can support with reasoning. This signal could be a thumbs up, or students could show the number of fingers that indicate the number of responses they have for the problem. This is a quick way to see if students have had enough time to think about the problem and keeps them from being distracted or rushed by classmates' raised hands.

Classroom Activities (Student Tasks)

After the warm-up, lessons consist of a sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.

An activity can serve one or more of many purposes.

- Provide experience with a new context.
- Introduce a new concept and associated language.
- Introduce a new representation.
- Formalize a definition of a term for an idea previously encountered informally.
- Identify and resolve common mistakes and misconceptions that people make.
- Practice using mathematical language.
- Work toward mastery of a concept or procedure.
- Provide an opportunity to apply mathematics to a modeling or other application problem.

The purpose of each activity is described in its Activity Narrative. Read more about how activities serve these different purposes in the section on Design Principles.

Lesson Synthesis

After the activities for the day are done, students should take time to synthesize what they have learned. This portion of class should take 5–10 minutes before students start working on the cool-down. Each lesson includes a Lesson Synthesis section that assists the teacher with ways to help students incorporate new insights gained during the activities into their big-picture understanding. Teachers can use this time in any number of ways, including posing questions verbally and calling on volunteers to respond, asking students to respond to prompts in a written journal, asking students to add on to a graphic organizer or concept map, or adding a new component to a persistent display like a word wall.

A Typical CKMath Lesson continued

The Cool-down

Each lesson includes a cool-down task to be given to students at the end of the lesson. Students are meant to work on the cool-down for about 5 minutes independently and turn it in. The cool-down serves as a brief formative assessment to determine whether students understood the lesson. Students' responses to the cool-down can be used to make adjustments to further instruction. You may read more about the cool-downs in the section titled Diagnostic Assessments.

← Table of Contents TEACH: How to Use These Materials →

CKMath™ Core Knowledge MATHEMATICS Grade 6

TEACH: How to Use These Materials

Each Lesson and Unit Tells a Story

The story of each grade is told in nine units. Each unit has a narrative that describes the mathematical work that will unfold in that unit. Each lesson in the unit also has a narrative. Lesson Narratives explain:

- The mathematical content of the lesson and its place in the learning sequence.
- The meaning of any new terms introduced in the lesson.
- How the mathematical practices come into play, as appropriate.

Activities within lessons also have narratives, which explain:

- The mathematical purpose of the activity and its place in the learning sequence.
- What students are doing during the activity.
- What teacher needs to look for while students are working on an activity to orchestrate an effective synthesis.
- Connections to the mathematical practices, when appropriate.

Launch - Work - Synthesize

Each classroom activity has three phases.

The Launch

During the launch, the teacher makes sure that students understand the context (if there is one) and *what the problem is asking them to do*. This is not the same as making sure the students know how to do the problem—part of the work that students should be doing for themselves is figuring out how to solve the problem.

Student Work Time

The launch for an activity frequently includes suggestions for grouping students. This gives students the opportunity to work individually, with a partner, or in small groups.

Activity Synthesis

During the activity synthesis, the teacher orchestrates some time for students to synthesize what they have learned. This time is used to ensure that all students have an opportunity to understand the mathematical punch line of the activity and situate the new learning within students' previous understanding.

How to Use These Materials continued

Practice Problems

Each lesson includes an associated set of practice problems. Teachers may decide to assign practice problems for homework or for extra practice in class; they may decide to collect and score it or to provide students with answers ahead of time for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Distributed practice (revisiting the same content over time) is more effective than massed practice (a large amount of practice on one topic, but all at once).

"Are You Ready For More?" Activities

Select classroom activities include an opportunity for differentiation for students ready for more of a challenge. We think of them as the "mathematical dessert" to follow the "mathematical entrée" of a classroom activity.

Every extension problem is made available to all students with the heading "Are You Ready for More?" These problems go deeper into grade-level mathematics and often make connections between the topic at hand and other concepts. Some of these problems extend the work of the associated activity, but some of them involve work from prior grades, prior units in the course, or reflect work that is related to the K–12 curriculum but a type of problem not required by the standards. They are not routine or procedural, and they are not just "the same thing again but with harder numbers."

They are intended to be used on an opt-in basis by students if they finish the main class activity early or want to do more mathematics on their own. It is not expected that an entire class engages in "Are You Ready for More?" problems, and it is not expected that any student works on all of them. These problems may also be good fodder for a *Problem of the Week* or similar structure.

Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These lesson plans include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

How to Use These Materials continued

Some of the instructional routines, known as Mathematical Language Routines (MLR), were developed by the Stanford University UL/SCALE team. The purpose of each MLR is described here, but you can read more about supports for students with emerging English language proficiency in the Supporting English Language Learners section of this guide.

Please access this spreadsheet [link] to see all locations of each routine listed below in the curriculum.

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR4: Information Gap Cards
- MLR5: Co-Craft Questions
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn't Belong?

← Table of Contents Required Tools & Materials →

Grade 6

Required Tools & Materials for CKMath Grade 6

NOTE: The required materials for each unit are listed in the Introduction of each Teacher Guide.

Base-ten blocks

Beakers

Bingo chips

Blank paper

Colored pencils

Dot stickers

Small circular sticker useful for plotting points on a display.

Drink mix

A powder that is mixed with water to create a fruit-flavored or chocolate-flavored drink. Using a sugar-free drink mix is recommended, but *not* a mix that calls for adding a separate sweetener when mixing up the drink.

Empty containers

Food coloring

Four-function calculators

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Copies of blackline masters

Cuisenaire rods

Decks of playing cards

Demonstration nets with and without flaps

½-inch cubes

¼-inch graph paper

Gallon-sized jug

Required Tools & Materials continued

Glue or glue sticks

Graphing technology

Examples of graphing technology are: a handheld graphing calculator, a computer with a graphing calculator application installed, and an internet-enabled device with access to a site like desmos.com/calculator or geogebra.org/graphing. For students using the digital materials, a separate graphing calculator tool isn't necessary; interactive applets are embedded throughout, and a graphing calculator tool is accessible on the student digital toolkit page.

Graph paper

Grocery store circulars

Grocery store advertisements from the newspaper or that are picked up at the store. If students have Internet access, you could substitute an online version of this.

Household items

Inch cubes

Index cards

Internet-enabled device

Liter-sized bottle

Markers

Metal paper fasteners - brass brads

Meter sticks

Nets of polyhedra

Pre-assembled or commercially produced polyhedra

Pre-assembled or commercially produced tangrams

Pre-assembled polyhedra

Graduated cylinders

Masking tape

Materials assembled from the blackline master

Measuring tapes

Origami paper

Paper cups

Pattern blocks

Required Tools & Materials continued

Pre-printed cards, cut from copies of the blackline master

Pre-printed slips, cut from copies of the blackline master

Quart-sized bottle

Rulers

Rulers marked with centimeters

Scale

a digital scale that can output in grams, kilograms, ounces, or pounds

Rulers marked with inches

Salt

Scissors

Snap cubes

Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

String

Students' collections of objects

Tape

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Tracing paper

Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

Sticky notes

Stopwatches

Teacher's collection of objects

Teaspoon

Required Tools & Materials continued

Tray

Water

Yardsticks

← Table of Contents Pacing & Sequence of Learning →

Grade 6

Pacing and Sequence of Learning

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.

(MA) means the unit has a mid-unit assessment.

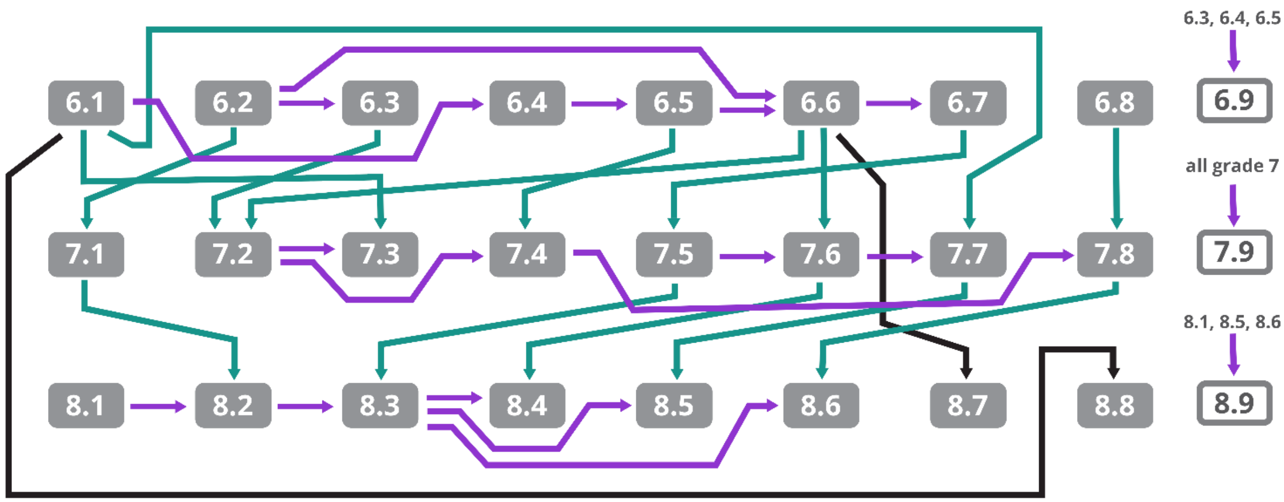
week 1, week 2, week 3, week 4, week 5, week 6, week 7, week 8, week 9, week 10, week 11, week 12, week 13, week 14, week 15, week 16, week 17, week 18, week 19, week 20, week 21, week 22, week 23, week 24, week 25, week 26, week 27, week 28, week 29, week 30, week 31, week 32, week 33, week 34, week 35, week 36

Grade 6	Grade 7	Grade 8
Unit 1 Area and Surface Area (21–22 days) (MA) Optional Lesson: 16	Unit 1 Scale Drawings (13–15 days) Optional Lessons: 8, 13	Unit 1 Rigid Transformations and Congruence (20 days) (MA) Optional Lessons: none
Unit 2 Introducing Ratios (19 days) Optional Lessons: none	Unit 2 Introducing Proportional Relationships (17 days) Optional Lessons: none	Unit 2 Dilations, Similarity, and Introducing Slope (15 days) Optional Lessons: none
Unit 3 Unit Rates and Percentages (18–19 days) Optional Lesson: 2	Unit 3 Measuring Circles (11–13 days) Optional Lessons: 5, 11	Unit 3 Linear Relationships (16 days) Optional Lessons: none
Unit 4 Dividing Fractions (20 days) (MA) Optional Lessons: none	Unit 4 Proportional Relationships and Percentages (17–18 days) Optional Lesson: 15	Unit 4 Linear Equations and Linear Systems (18 days) Optional Lessons: none
Unit 5 Arithmetic in Base Ten (16–18 days) (MA) Optional Lessons: 2, 15	Unit 5 Rational Number Arithmetic (19 days) Optional Lessons: none	Unit 5 Functions and Volume (22-25 days) (MA) Optional Lessons: 17, 18, 22
Unit 6 Expressions and Equations (20–22 days) (MA) Optional Lessons: 11, 18	Unit 6 Expressions, Equations, and Inequalities (26 days) (MA) Optional Lessons: none	Unit 6 Associations in Data (12–13 days) Optional Lesson: 11
Unit 7 Rational Numbers (21 days) Optional Lessons: none	Unit 7 Angles, Triangles, and Prisms (19 days) Optional Lessons: none	Unit 7 Exponents and Scientific Notation (18 days) Optional Lessons: none
Unit 8 Data Sets and Distributions (21 days) (MA) Optional Lessons: none	Unit 8 Probability and Sampling (21–23 days) (MA) Optional Lessons: 17, 20	Unit 8 Pythagorean Theorem and Irrational Numbers (18 days) Optional Lessons: none
Unit 9 Putting It All Together (0–18 days) Optional Lessons: all	Unit 9 Putting It All Together (0–13 days) Optional Lessons: all	Unit 9 Putting It All Together (0–10 days) Optional Lessons: all

The CKMath 6-8 Pacing Guide may also be downloaded as a separate file [here](#) [link].

Grade 6

Sequence of Learning: Unit Dependency Chart



In the above unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6, because students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8, because students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6, because students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

← Table of Contents

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans in the Teacher Guide for each unit. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson.

Learning Targets

Targets for each lesson are listed for the entire unit in the Introduction of each Teacher Guide. They describe, for a student audience, the mathematical goals of each lesson.

Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

How to Assess Progress

The materials contain many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.
- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.
- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.

For more information about assessing student progress and needs, please refer to the section on Supporting Diverse Learners.

← Table of Contents Grade-Level Glossary →

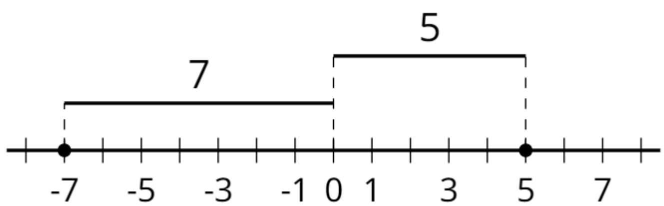
Grade 6

Grade 6 Glossary of Terms

The following glossary may be downloaded as a separate file **here** [link] for student use.

absolute value

The absolute value of a number is its distance from 0 on the number line.

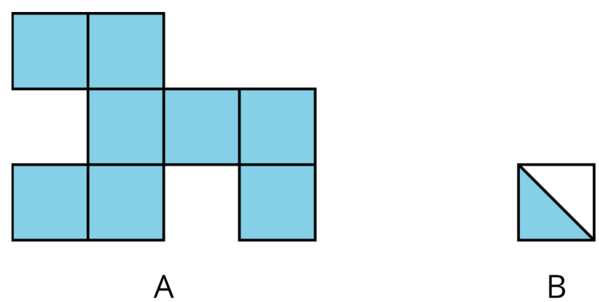


The absolute value of -7 is 7, because it is 7 units away from 0. The absolute value of 5 is 5, because it is 5 units away from 0.

area

Area is the number of square units that cover a two-dimensional region, without any gaps or overlaps.

For example, the area of region A is 8 square units. The area of the shaded region of B is ½ square unit.



average

The *average* is another name for the mean of a data set.

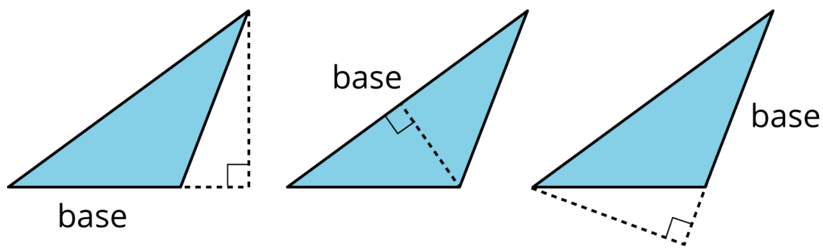
For the data set 3, 5, 6, 8, 11, 12, the average is 7.5.

3 + 5 + 6 + 8 + 11 + 12 = 45

45 ÷ 6 = 7.5

base (of a parallelogram or triangle)

We can choose any side of a parallelogram or triangle to be the shape's base. Sometimes we use the word *base* to refer to the length of this side.

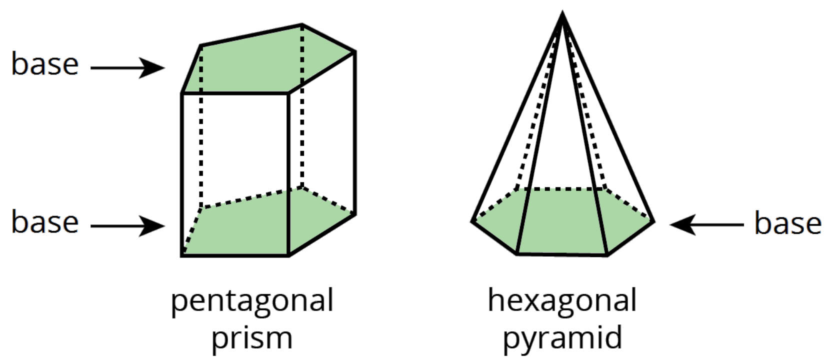


base (of a prism or pyramid)

The word *base* can also refer to a face of a polyhedron.

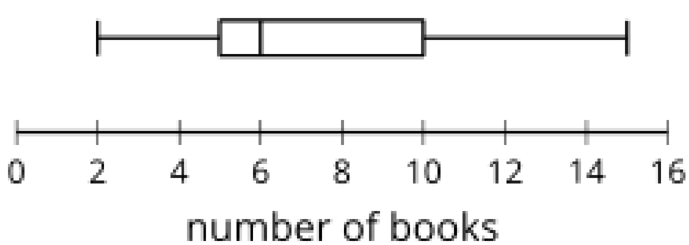
A prism has two identical bases that are parallel. A pyramid has one base.

A prism or pyramid is named for the shape of its base.



box plot

A *box plot* is a way to represent data on a number line. The data is divided into four sections. The sides of the box represent the first and third quartiles. A line inside the box represents the median. Lines outside the box connect to the minimum and maximum values.



For example, this box plot shows a data set with a minimum of 2 and a maximum of 15. The median is 6, the first quartile is 5, and the third quartile is 10.

Grade 6

categorical data

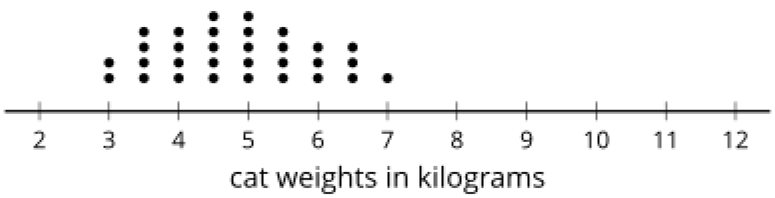
A set of categorical data has values that are words instead of numbers.

For example, Han asks 5 friends to name their favorite color. Their answers are: blue, blue, green, blue, orange.

center

The center of a set of numerical data is a value in the middle of the distribution. It represents a typical value for the data set.

For example, the center of this distribution of cat weights is between 4.5 and 5 kilograms.



coefficient

A coefficient is a number that is multiplied by a variable.

For example, in the expression $3x + 5$, the coefficient of x is 3. In the expression $y + 5$, the coefficient of y is 1, because $y = 1 \cdot y$.

common factor

A common factor of two numbers is a number that divides evenly into both numbers. For example, 5 is a common factor of 15 and 20, because $15 \div 5 = 3$ and $20 \div 5 = 4$. Both of the quotients, 3 and 4, are whole numbers.

- The factors of 15 are 1, 3, *5*, and 15.
- The factors of 20 are 1, 2, 4, *5*, 10, and 20.

common multiple

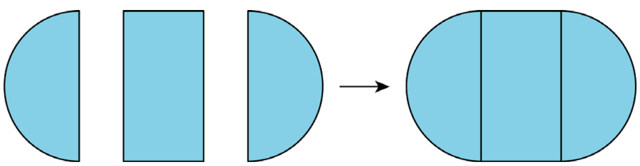
A common multiple of two numbers is a product you can get by multiplying each of the two numbers by some whole number. For example, 30 is a common multiple of 3 and 5, because $\mathbf{3 \cdot 10 = 30}$ and $\mathbf{5 \cdot 6 = 30}$. Both of the factors, 10 and 6, are whole numbers.

- The multiples of 3 are 3, 6, 9, 12, *15*, 18, 21, 24, 27, *30*, 33 . . .
- The multiples of 5 are 5, 10, *15*, 20, 25, *30*, 35, 40 . . .

The common multiples of 3 and 5 are 15, 30, 45, 60 . . .

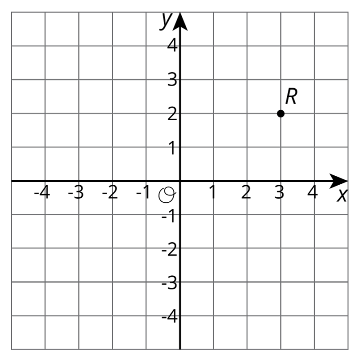
compose

Compose means "put together." We use the word *compose* to describe putting more than one figure together to make a new shape.



coordinate plane

The coordinate plane is a system for telling where points are. For example, point ***R*** is located at **(3, 2)** on the coordinate plane, because it is three units to the right and two units up.

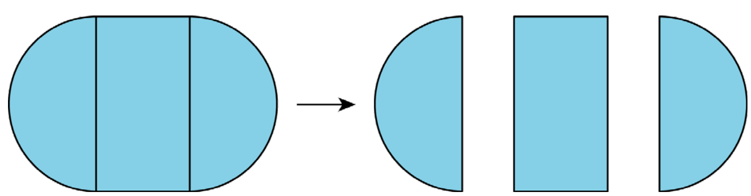


cubed

We use the word *cubed* to mean "to the third power." This is because a cube with side length $\boldsymbol{8}$ has a volume of $\boldsymbol{8 \cdot 8 \cdot 8}$, or $\boldsymbol{8^3}$.

decompose

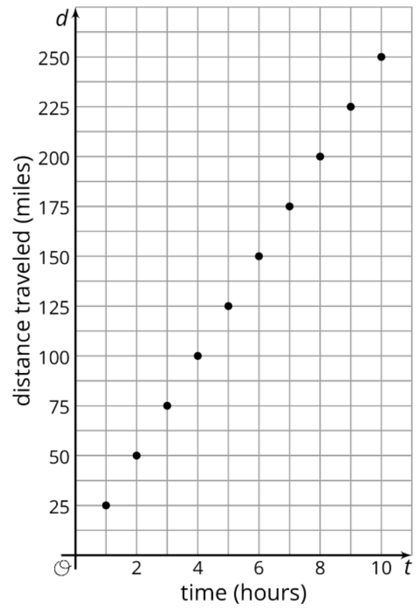
Decompose means "take apart." We use the word *decompose* to describe taking a figure apart to make more than one new shape.



dependent variable

The dependent variable is the result of a calculation.

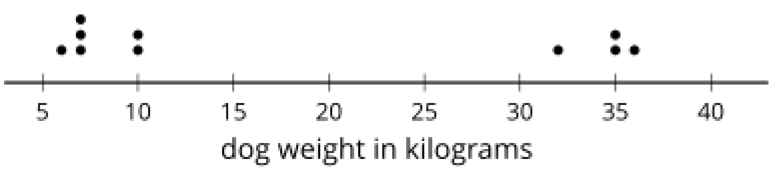
For example, a boat travels at a constant speed of 25 miles per hour. The equation $d = 25t$ describes the relationship between the boat's distance and time. The dependent variable is the distance traveled, because is the result of multiplying 25 by t.



distribution

The distribution tells how many times each value occurs in a data set. For example, in the data set blue, blue, green, blue, orange, the distribution is 3 blues, 1 green, and 1 orange.

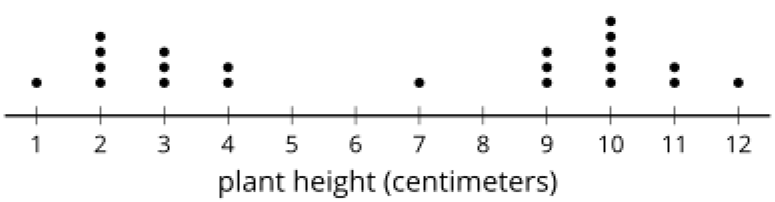
Here is a dot plot that shows the distribution for the data set 6, 10, 7, 35, 7, 36, 32, 10, 7, 35.



dot plot

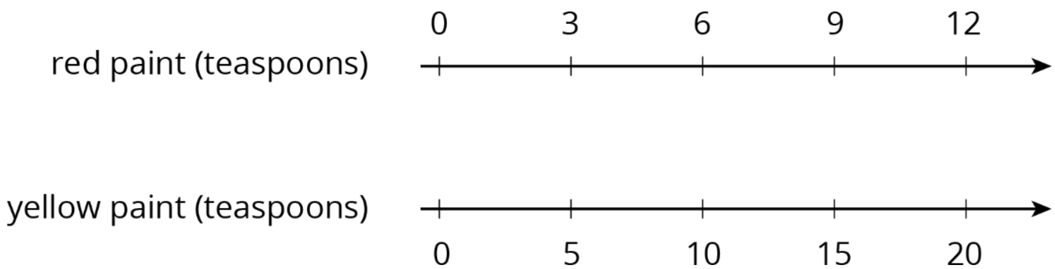
A *dot plot* is a way to represent data on a number line. Each time a value appears in the data set, we put another dot above that number on the number line.

For example, in this dot plot there are three dots above the 9. This means that three different plants had a height of 9 cm.



double number line diagram

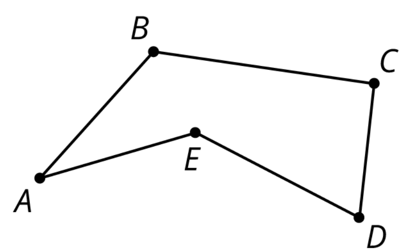
A double number line diagram uses a pair of parallel number lines to represent equivalent ratios. The locations of the tick marks match on both number lines. The tick marks labeled 0 line up, but the other numbers are usually different.



edge

Each straight side of a polygon is called an edge.

For example, the edges of this polygon are segments ***AB***, ***BC***, ***CD***, ***DE***, and ***EA***.



equivalent expressions

Equivalent expressions are always equal to each other. If the expressions have variables, they are equal whenever the same value is used for the variable in each expression.

For example, $3x + 4x$ is equivalent to $5x + 2x$. No matter what value we use for x, these expressions are always equal. When x is 3, both expressions equal 21. When x is 10, both expressions equal 70.

equivalent ratios

Two ratios are equivalent if you can multiply each of the numbers in the first ratio by the same factor to get the numbers in the second ratio. For example, $8 : 6$ is equivalent to $4 : 3$, because $8 \cdot \frac{1}{2} = 4$ and $6 \cdot \frac{1}{2} = 3$.

A recipe for lemonade says to use 8 cups of water and 6 lemons. If we use 4 cups of water and 3 lemons, it will make half as much lemonade. Both recipes taste the same, because $8 : 6$ and $4 : 3$ are equivalent ratios.

cups of water	number of lemons
8	6
4	3

exponent

In expressions like 5^3 and 8^2, the 3 and the 2 are called exponents. They tell you how many factors to multiply. For example, $5^3 = 5 \cdot 5 \cdot 5$, and $8^2 = 8 \cdot 8$.

face

Each flat side of a polyhedron is called a face. For example, a cube has 6 faces, and they are all squares.

frequency

The frequency of a data value is how many times it occurs in the data set.

For example, there were 20 dogs in a park. The table shows the frequency of each color of dog.

color	frequency
White	4
Brown	7
Black	3
multi-color	6

greatest common factor

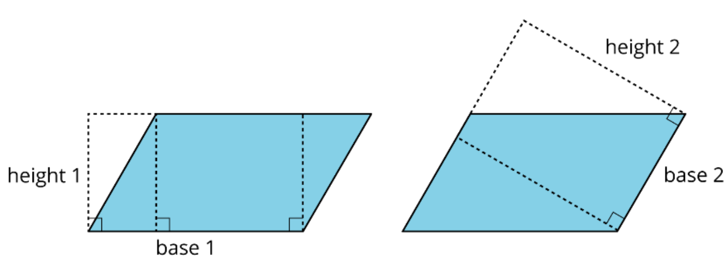
The greatest common factor of two numbers is the largest number that divides evenly into both numbers. Sometimes we call this the GCF. For example, 15 is the greatest common factor of 45 and 60.

- The factors of 45 are 1, 3, 5, 9, *15*, and 45.
- The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, *15*, 20, 30, and 60.

height (of a parallelogram or triangle)

The height is the shortest distance from the base of the shape to the opposite side (for a parallelogram) or opposite vertex (for a triangle).

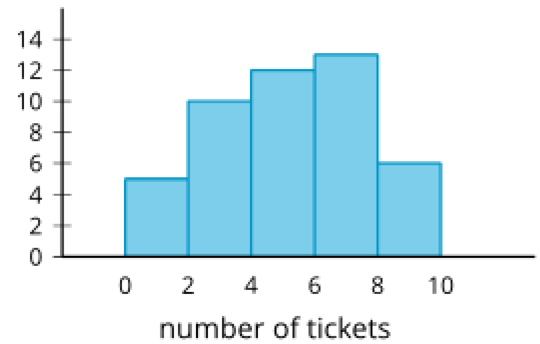
We can show the height in more than one place, but it will always be perpendicular to the chosen base.



histogram

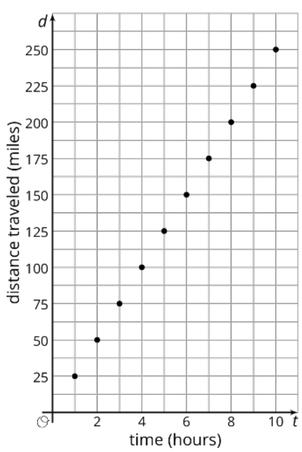
A histogram is a way to represent data on a number line. Data values are grouped by ranges. The height of the bar shows how many data values are in that group.

This histogram shows there were 10 people who earned 2 or 3 tickets. We can't tell how many of them earned 2 tickets or how many earned 3. Each bar includes the left-end value but not the right-end value. (There were 5 people who earned 0 or 1 tickets and 13 people who earned 6 or 7 tickets.)



independent variable

The independent variable is used to calculate the value of another variable.



For example, a boat travels at a constant speed of 25 miles per hour. The equation $d = 25t$ describes the relationship between the boat's distance and time. The independent variable is time, because t is multiplied by 25 to get d.

interquartile range (IQR)

The interquartile range is one way to measure how spread out a data set is. We sometimes call this the IQR. To find the interquartile range we subtract the first quartile from the third quartile.

For example, the IQR of this data set is 20 because $50 - 30 = 20$.

22	29	30	31	32	43	44	45	50	50	59
		Q1			Q2			Q3		

least common multiple

The least common multiple of two numbers is the smallest product you can get by multiplying each of the two numbers by some whole number. Sometimes we call this the LCM. For example, 30 is the least common multiple of 6 and 10.

- The multiples of 6 are 6, 12, 18, 24, *30*, 36, 42, 48, 54, 60 . . .
- The multiples of 10 are 10, 20, *30*, 40, 50, 60, 70, 80 . . .

long division

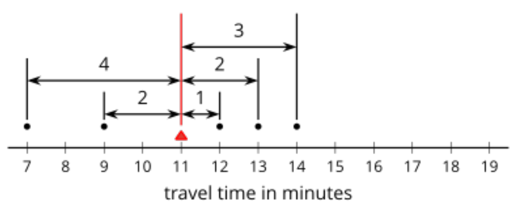
Long division is a way to show the steps for dividing numbers in decimal form. It finds the quotient one digit at a time, from left to right.

For example, here is the long division for $57 \div 4$.

$$\begin{array}{r} 14.25 \\ 4\overline{)57.00} \\ \underline{-4} \\ 17 \\ \underline{-16} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

mean

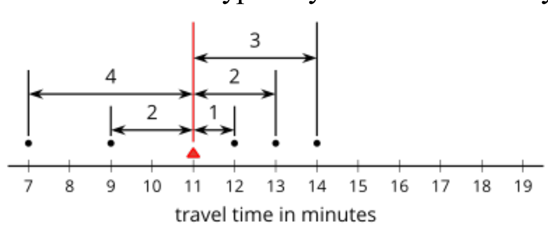
The mean is one way to measure the center of a data set. We can think of it as a balance point. For example, for the data set 7, 9, 12, 13, 14, the mean is 11.



To find the mean, add up all the numbers in the data set. Then, divide by how many numbers there are. $7 + 9 + 12 + 13 + 14 = 55$ and $55 \div 5 = 11$.

mean absolute deviation (MAD)

The mean absolute deviation is one way to measure how spread out a data set is. Sometimes we call this the MAD. For example, for the data set 7, 9, 12, 13, 14, the MAD is 2.4. This tells us that these travel times are typically 2.4 minutes away from the mean, which is 11.



To find the MAD, add up the distance between each data point and the mean. Then, divide by how many numbers there are. $4 + 2 + 1 + 2 + 3 = 12$ and $12 \div 5 = 2.4$

measure of center
A measure of center is a value that seems typical for a data distribution.

Mean and median are both measures of center.

median
The median is one way to measure the center of a data set. It is the middle number when the data set is listed in order.

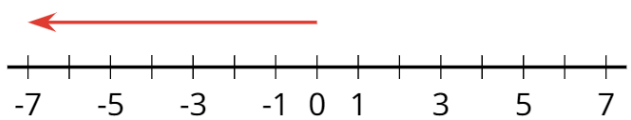
For the data set 7, 9, 12, 13, 14, the median is 12.

For the data set 3, 5, 6, 8, 11, 12, there are two numbers in the middle. The median is the average of these two numbers. $6 + 8 = 14$ and $14 \div 2 = 7$.

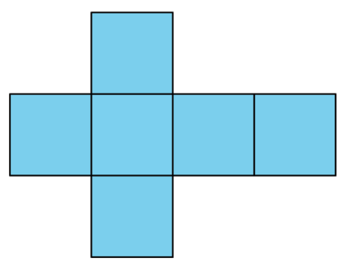
meters per second
Meters per second is a unit for measuring speed. It tells how many meters an object goes in one second.

For example, a person walking 3 meters per second is going faster than another person walking 2 meters per second.

negative number
A negative number is a number that is less than zero. On a horizontal number line, negative numbers are usually shown to the left of 0.



net
A net is a two-dimensional figure that can be folded to make a polyhedron. Here is a net for a cube.



numerical data

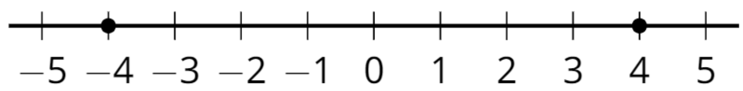
A set of numerical data has values that are numbers.

For example, Han lists the ages of people in his family: 7, 10, 12, 36, 40, 67.

opposite

Two numbers are opposites if they are the same distance from 0 and on different sides of the number line.

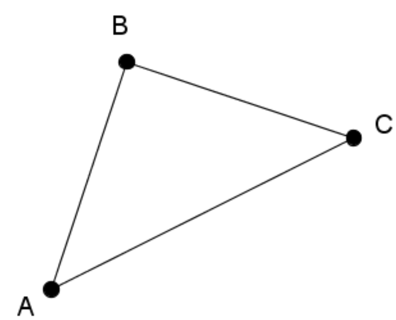
For example, 4 is the opposite of -4, and -4 is the opposite of 4. They are both the same distance from 0. One is negative, and the other is positive.



opposite vertex

For each side of a triangle, there is one vertex that is not on that side. This is the opposite vertex.

For example, point *A* is the opposite vertex to side *BC*.



pace

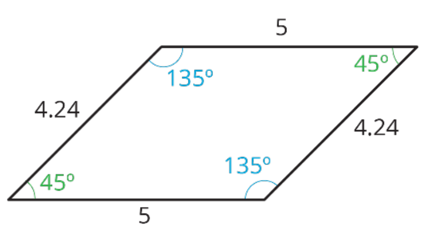
Pace is one way to describe how fast something is moving. Pace tells how much time it takes the object to travel a certain distance.

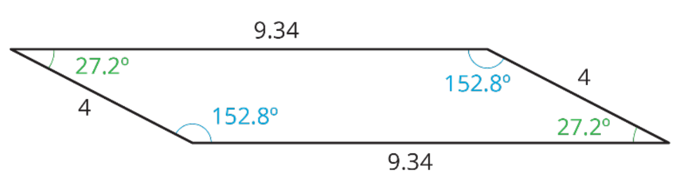
For example, Diego walks at a pace of 10 minutes per mile. Elena walks at a pace of 11 minutes per mile. Elena walks slower than Diego, because it takes her more time to travel the same distance.

parallelogram

A parallelogram is a type of quadrilateral that has two pairs of parallel sides.

Here are two examples of parallelograms.





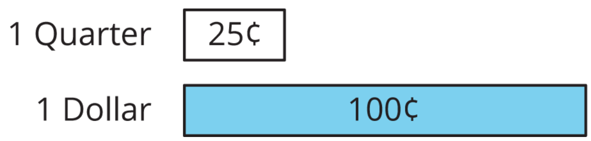
per

The word *per* means "for each." For example, if the price is $5 per ticket, that means you will pay $5 *for each* ticket. Buying 4 tickets would cost $20, because $4 \cdot 5 = 20$.

percent

The word *percent* means "for each 100." The symbol for percent is %.

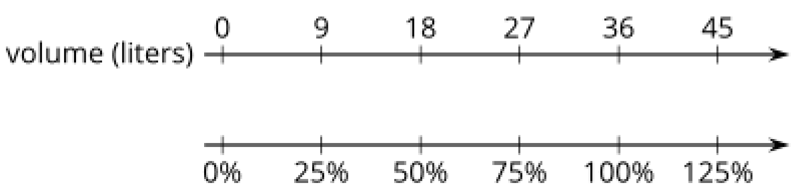
For example, a quarter is worth 25 cents, and a dollar is worth 100 cents. We can say that a quarter is worth 25% of a dollar.



percentage

A percentage is a rate per 100.

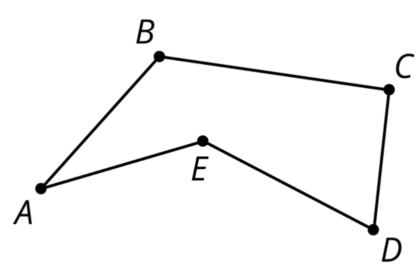
For example, a fish tank can hold 36 liters. Right now there are 27 liters of water in the tank. The percentage of the tank that is full is 75%.



polygon

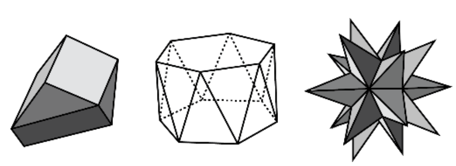
A polygon is a closed, two-dimensional shape with straight sides that do not cross each other.

Figure ***ABCDE*** is an example of a polygon.



polyhedron

A polyhedron is a closed, three-dimensional shape with flat sides. When we have more than one polyhedron, we call them polyhedra. Here are some drawings of polyhedra.

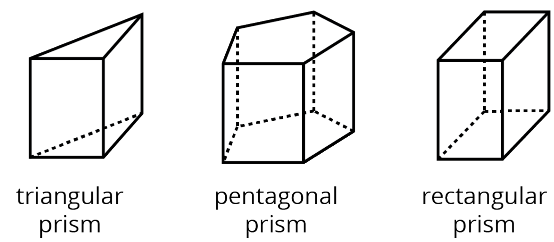


positive number

A positive number is a number that is greater than zero. On a horizontal number line, positive numbers are usually shown to the right of 0.

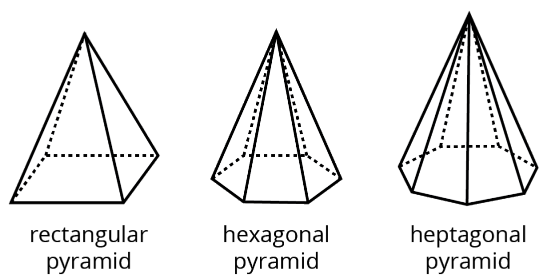
prism

A prism is a type of polyhedron that has two bases that are identical copies of each other. The bases are connected by rectangles or parallelograms. Here are some drawings of prisms.



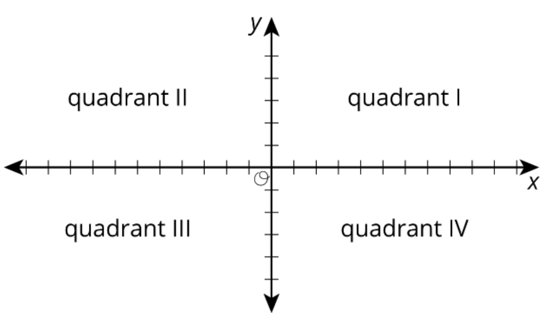
pyramid

A pyramid is a type of polyhedron that has one base. All the other faces are triangles, and they all meet at a single vertex. Here are some drawings of pyramids.



quadrant

The coordinate plane is divided into 4 regions called quadrants. The quadrants are numbered using Roman numerals, starting in the top right corner.



CKMath™ Core Knowledge MATHEMATICS Grade 6

quadrilateral

A quadrilateral is a type of polygon that has 4 sides. A rectangle is an example of a quadrilateral. A pentagon is not a quadrilateral, because it has 5 sides.

quartile

Quartiles are the numbers that divide a data set into four sections that each have the same number of values.

For example, in this data set the first quartile is 30. The second quartile is the same thing as the median, which is 43. The third quartile is 50.

22	29	30	31	32	43	44	45	50	50	59
		Q1			Q2			Q3		

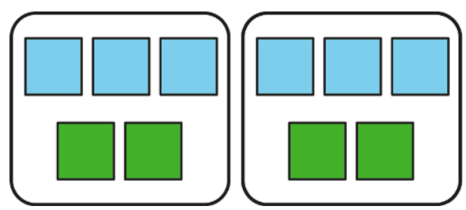
range

The range is the distance between the smallest and largest values in a data set. For example, for the data set 3, 5, 6, 8, 11, 12, the range is 9, because ***12 – 3 = 9***.

ratio

A ratio is an association between two or more quantities.

For example, the ratio ***3 : 2*** could describe a recipe that uses 3 cups of flour for every 2 eggs, or a boat that moves 3 meters every 2 seconds. One way to represent the ratio ***3 : 2*** is with a diagram that has 3 blue squares for every 2 green squares.



rational number

A rational number is a fraction or the opposite of a fraction.

For example, 8 and -8 are rational numbers because they can be written as 8/1 and -8/1

Also, 0.75 and -0.75 are rational numbers because they can be written as 75/100 and – 75/100.

reciprocal

Dividing 1 by a number gives the reciprocal of that number. For example, the reciprocal of 12 is 1/12, and the reciprocal of 2/5 is 5/2.

region

A region is the space inside of a shape. Some examples of two-dimensional regions are inside a circle or inside a polygon. Some examples of three-dimensional regions are the inside of a cube or the inside of a sphere.

same rate

We use the words *same rate* to describe two situations that have equivalent ratios.

For example, a sink is filling with water at a rate of 2 gallons per minute. If a tub is also filling with water at a rate of 2 gallons per minute, then the sink and the tub are filling at the same rate.

sign

The sign of any number other than 0 is either positive or negative.

For example, the sign of 6 is positive. The sign of -6 is negative. Zero does not have a sign, because it is not positive or negative.

solution to an equation

A solution to an equation is a number that can be used in place of the variable to make the equation true.

For example, 7 is the solution to the equation $m + 1 = 8$, because it is true that $7 + 1 = 8$. The solution to $m + 1 = 8$ is not 9, because $9 + 1 \neq 8$.

solution to an inequality

A solution to an inequality is a number that can be used in place of the variable to make the inequality true.

For example, 5 is a solution to the inequality $c < 10$, because it is true that $5 < 10$. Some other solutions to this inequality are 9.9, 0, and -4.

speed

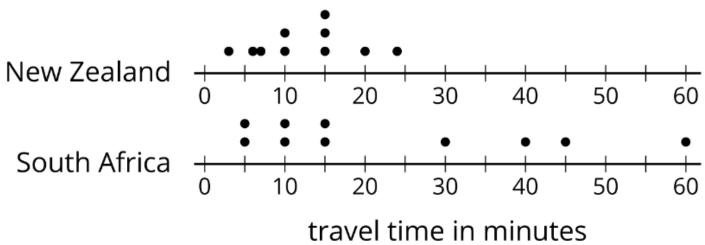
Speed is one way to describe how fast something is moving. Speed tells how much distance the object travels in a certain amount of time.

For example, Tyler walks at a speed of 4 miles per hour. Priya walks at a speed of 5 miles per hour. Priya walks faster than Tyler, because she travels more distance in the same amount of time.

spread

The spread of a set of numerical data tells how far apart the values are.

For example, the dot plots show that the travel times for students in South Africa are more spread out than for New Zealand.



squared

We use the word *squared* to mean "to the second power." This is because a square with side length has an area of $8 \cdot 8$, or 8^2.

statistical question

A statistical question can be answered by collecting data that has variability. Here are some examples of statistical questions:

- Who is the most popular musical artist at your school?
- When do students in your class typically eat dinner?
- Which classroom in your school has the most books?

surface area

The surface area of a polyhedron is the number of square units that covers all the faces of the polyhedron, without any gaps or overlaps.

For example, if the six faces of a cube each have an area of 9 cm^2, then the surface area of the cube is $6 \cdot 9$, or 54 cm^2.

table

A table organizes information into horizontal *rows* and vertical *columns*. The first row or column usually tells what the numbers represent.

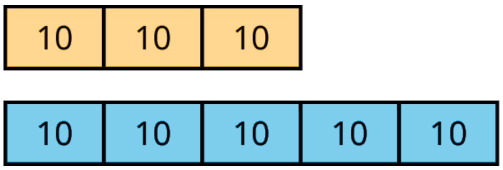
For example, here is a table showing the tail lengths of three different pets. This table has four rows and two columns.

pet	tail length (inches)
dog	22
cat	12
mouse	2

tape diagram

A tape diagram is a group of rectangles put together to represent a relationship between quantities.

For example, this tape diagram shows a ratio of 30 gallons of yellow paint to 50 gallons of blue paint.



If each rectangle were labeled 5, instead of 10, then the same picture could represent the equivalent ratio of 15 gallons of yellow paint to 25 gallons of blue paint.

term

A term is a part of an expression. It can be a single number, a variable, or a number and a variable that are multiplied together. For example, the expression $5x + 18$ has two terms. The first term is $5x$ and the second term is 18.

unit price

The unit price is the cost for one item or for one unit of measure. For example, if 10 feet of chain link fencing cost \$150, then the unit price is $150 \div 10$, or \$15 per foot.

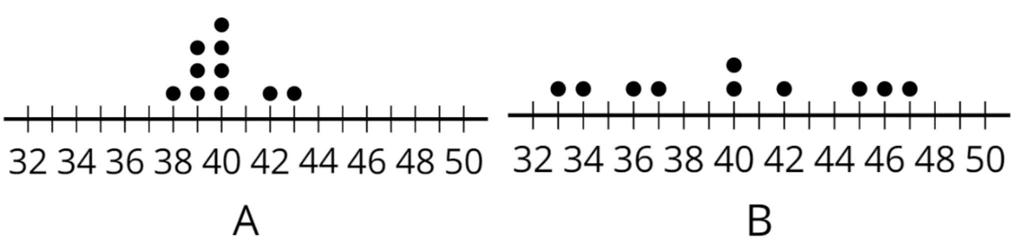
unit rate

A unit rate is a rate per 1.

For example, 12 people share 2 pies equally. One unit rate is 6 people per pie, because $12 \div 2 = 6$. The other unit rate is 1/6 of a pie per person, because $2 \div 12 = 1/6$.

variability

Variability means having different values. For example, data set B has more variability than data set A. Data set B has many different values, while data set A has more of the same values.



variable

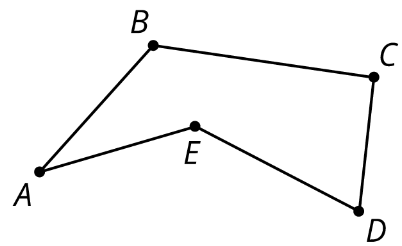
A variable is a letter that represents a number. You can choose different numbers for the value of the variable.

For example, in the expression $10 - x$, the variable is x. If the value of x is 3, then $10 - x = 7$, because $10 - 3 = 7$. If the value of x is 6, then $10 - x = 4$, because . $10 - 6 = 4$.

vertex

A vertex is a point where two or more edges meet. When we have more than one vertex, we call them vertices.

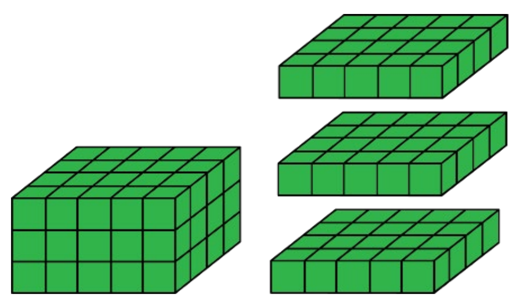
The vertices in this polygon are labeled ***A, B, C, D,*** and ***E***.



volume

Volume is the number of cubic units that fill a three-dimensional region, without any gaps or overlaps.

For example, the volume of this rectangular prism is 60 units^3, because it is composed of 3 layers that are each 20 units^3.



← Table of Contents ADAPT: Supporting Diverse Learners →

ADAPT: Supporting Diverse Learners

Pre-Unit Diagnostic Assessments

At the start of each unit is a pre-unit diagnostic assessment that is titled *Check Your Readiness*. These assessments vary in length. Most of the problems address prerequisite concepts and skills for the unit. Teachers can use these problems to identify students with particular below-grade needs, or topics to carefully address during the unit. *Check Your Readiness* also may include problems that assess what students already know of the upcoming unit's key ideas, which teachers can use to pace or tune instruction; in rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

What if a large number of students can't do the same pre-unit assessment problem? Teachers are encouraged to address below-grade skills while continuing to work through the on-grade tasks and concepts of each unit, instead of abandoning the current work in favor of material that only addresses below-grade skills. Look for opportunities within the upcoming unit where the target skill could be addressed in context. For example, an upcoming activity might require solving an equation in one variable. Some strategies might include:

- ask a student who can do the skill to present their method
- add additional questions to the warm-up with the purpose of revisiting the skill
- add to the activity launch a few related equations to solve, before students need to solve an equation while working on the activity
- pause the class while working on the activity to focus on the portion that requires solving an equation

Then, attend carefully to students as they work through the activity. If difficulty persists, add more opportunities to practice the skill, by adapting tasks or practice problems.

What if all students do really well on Check Your Readiness? That means they are ready for the work ahead, and special attention doesn't likely need to be paid to below-grade skills.

Cool-Downs

Each lesson includes a cool-down (also known as an exit slip or exit ticket) to be given to students at the end of the lesson. This activity serves as a brief checkpoint to determine whether students understood the main concepts of that lesson. Teachers can use this as a formative assessment to plan further instruction.

Supporting Diverse Learners continued

What if the feedback from a cool-down suggests students haven't understood a key concept? Choose one or more of these strategies:

- Look at the next few lessons to see if students have more opportunities to engage with the same topic. If so, plan to focus on the topic in the context of the new activities.
- During the next lesson, display the work of a few students on that cool-down. Anonymize their names, but show some correct and incorrect work. Ask the class to observe some things each student did well and could have done better.
- Give each student brief, written feedback on their cool-down that asks a question that nudges them to re-examine their work. Ask students to revise and resubmit.
- Look for practice problems that are similar to, or involve the same topic as the cool-down, then assign those problems over the next few lessons.

Here is an example. For a lesson in grade 6, unit 2, the learning goals are

- Understand that doubling, tripling, or halving a recipe yields something that tastes the same.
- Understand that "doubling, tripling, or halving the recipe" means "doubling, tripling, or halving each ingredient."

The cool-down reads:
Usually when Elena makes bird food, she mixes 9 cups of seeds with 6 tablespoons of maple syrup. However, today she is short on ingredients. Think of a recipe that would yield a smaller batch of bird food but still taste the same. Explain or show your reasoning.

A number of students responded with 8 cups of seeds and 5 tablespoons of maple syrup, and did not provide an explanation or show their reasoning. Here are some possible strategies:

- Notice that this lesson is the first of several that familiarize students with contexts where equivalent ratios carry physical meaning, for example, the taste of a recipe or the result of mixing paint colors. Over the next several lessons, there are more opportunities to reason about multiple batches of a recipe. When launching these activities, pause to assist students to interpret this correctly. Highlight the strategies of any students who use a discrete diagram or other representation to correctly represent multiple batches.
- Select the work of one student who answered correctly and one student whose work had the common error. In the next class, display these together for all to see (hide the students' names). Ask each student to decide which interpretation is correct, and defend their choice to their partner. Select students to share their reasoning with the class who have different ways of representing that ***9 : 6*** is equivalent to ***3 : 2***, ***6 : 4***, or ***4½ : 3***.
- Write feedback for each student along the lines of "If this recipe is 3 batches, how could you make 1 batch?" Allow students to revise and resubmit their work.
- Look for practice problems in upcoming lessons that require students to generate examples of different numbers of batches equivalent to a given ratio, and be sure to assign those problems.

CKMath™ Core Knowledge MATHEMATICS Grade 6

Summative Assessments

End-of-Unit Assessments

At the end of each unit is the *end-of-unit assessment*. These assessments have a specific length and breadth, with problem types that are intended to gauge students' understanding of the key concepts of the unit while also preparing students for new-generation standardized exams. Problem types include multiple-choice, multiple response, short answer, restricted constructed response, and extended response. Problems vary in difficulty and depth of knowledge.

Teachers may choose to grade these assessments in a standardized fashion, but may also choose to grade more formatively by asking students to show and explain their work on all problems. Teachers may also decide to make changes to the provided assessments to better suit their needs. If making changes, teachers are encouraged to keep the format of problem types provided, which helps students know what to expect and ensures each assessment will take approximately the same amount of time.

In longer units, a *mid-unit assessment* is also available. This assessment has the same form and structure as an end-of-unit assessment. In longer units, the end-of-unit assessment will include the breadth of all content for the full unit, with emphasis on the content from the second half of the unit.

All summative assessment problems include a complete solution and standard alignment. Multiple-choice and multiple response problems often include a reason for each potential error a student might make. Restricted constructed response and extended response items include a rubric.

Unlike formative assessments, problems on summative assessments generally do not prescribe a method of solution.

Design Principles for Summative Assessments

Students should get the correct answer on assessment problems for the right reasons, and get incorrect answers for the right reasons. To help with this, our assessment problems are targeted and short, use consistent, positive wording, and have clear, undebatable correct responses.

In multiple choice problems, distractors are common errors and misconceptions directly relating to what is being assessed, since problems are intended to test whether the student has proficiency on a specific skill. The distractors serve as a diagnostic, giving teachers the chance to quickly see which of the most common errors are being made. There are no "trick" questions, and the phrases "all of the above" and "none of the above" are never used, since they do not give useful information about the methods a student used.

Multiple response prompts always include the phrase "select **all**" to clearly indicate their type. Each part of a multiple response problem addresses a different piece of the same overall skill, again serving as a diagnostic for teachers to understand which common errors students are making.

Summative Assessments continued

Short answer, restricted constructed response, and extended response problems are careful to avoid the "double whammy" effect, where a part of the problem asks for students to use correct work from a previous part. This choice is made to ensure that students have all possible opportunities to show proficiency on assessments.

When possible, extended response problems provide multiple ways for students to demonstrate understanding of the content being assessed, through some combination of arithmetic or algebra, use of representations (tables, graphs, diagrams, expressions, and equations) and explanation.

Rubrics for Evaluating Student Answers

Restricted constructed response and extended response items have rubrics that can be used to evaluate the level of student responses.

Restricted Constructed Response

- *Tier 1 response*: Work is complete and correct.
- *Tier 2 response*: Work shows general conceptual understanding and mastery, with some errors.
- *Tier 3 response*: Significant errors in work demonstrate lack of conceptual understanding or mastery. Two or more error types from Tier 2 responsecan be given as the reason for a Tier 3 response instead of listing combinations.

Extended Response

- *Tier 1 response*: Work is complete and correct, with complete explanation or justification.
- *Tier 2 response*: Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- *Tier 3 response*: Work shows a developing but incomplete conceptual understanding, with significant errors.
- *Tier 4 response*: Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

Typically, sample errors are included. Acceptable errors can be listed at any Tier (as an additional bullet point), notably Tier 1, to specify exclusions.

Performance Tasks

Each unit has a culminating lesson where students have an opportunity to show off their problem-solving skills or apply the mathematics they have learned to a real-world problem. The end unit assessments, combined with students' work on the culminating lessons, will show a multi-faceted view of students' learning over the course of the unit.

English Language Supports and ELLs

Overview

This section is adapted with permission from work done by Understanding Language at Stanford University. For the original paper, *Guidance for Math Curricula Design and Development*, please visit https://ell.stanford.edu/content/mathematics-resources-additional-resources.

This curriculum builds on foundational principles for supporting language development for all students. This section aims to provide guidance to help teachers recognize and support students' language development in the context of mathematical sense-making. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

This table reflects the attention and support for language development at each level of the curriculum.

COURSE

- foundation of curriculum: theory of action and design principles that drive a continuous focus on language development
- student glossary of terms

UNIT

- unit-specific progression of language development included in each unit overview

LESSON

- language goals embedded in learning goals describe the language demands of the lesson
- definitions of new glossary terms

ACTIVITY

- additional supports for English language learners based on language demands of the activity
- math language routines

English Language Supports continued

Theory of Action

We believe that language development can be built into teachers' instructional practice and students' classroom experience through intentional design of materials, teacher commitments, administrative support, and professional development. Our theory of action is grounded in the interdependence of language learning and content learning, the importance of scaffolding routines that foster students' independent participation, the value of instructional responsiveness in the teaching process, and the central role of student agency in the learning process.

Mathematical understandings and language competence develop interdependently. Deep conceptual learning is gained through language. Ideas take shape through words, texts, illustrations, conversations, debates, examples, etc. Teachers, peers, and texts serve as language resources for learning. Instructional attention to academic language development, historically limited to vocabulary instruction, has now shifted to also include instruction around the demands of argumentation, explanation, generalization, analyzing the purpose and structure of text, and other mathematical discourse.

Scaffolding provides temporary supports that foster student autonomy. Learners with emerging language—at any level—can engage deeply with central mathematical ideas under specific instructional conditions. Mathematical language development occurs when students use their developing language to make meaning and engage with challenging problems that are beyond students' mathematical ability to solve independently and therefore require interaction with peers. However, these interactions should be structured with temporary supports that students can use to make sense of what is being asked of them, to help organize their own thinking, and to give and receive feedback.

Instruction supports learning when teachers respond to students' verbal and written work. Eliciting student thinking through language allows teachers and students to respond formatively to the language students generate. Formative peer and teacher feedback creates opportunities for revision and refinement of both content understandings and language.

Students are agents in their own mathematical and linguistic sense-making. Mathematical language proficiency is developed through the process of actively exploring and learning mathematics. Language is action: in the very doing of math, students have naturally occurring opportunities to need, learn, and notice mathematical ways of making sense and talking about ideas and the world. These experiences support learners in using as well as expanding their existing language toolkits.

English Language Supports continued

Design

The framework for supporting English language learners (ELLs) in this curriculum includes four design principles for promoting mathematical language use and development in curriculum and instruction. The design principles and related routines work to make language development an integral part of planning and delivering instruction while guiding teachers to amplify the most important language that students are expected to bring to bear on the central mathematical ideas of each unit.

Principle 1: SUPPORT SENSE-MAKING

Scaffold tasks and amplify language so students can make their own meaning. Students do not need to understand a language completely before they can engage with academic content in that language. Language learners of all levels can and should engage with grade-level content that is appropriately scaffolded. Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher.

Teachers can make language more accessible for students by amplifying rather than simplifying speech or text. Simplifying includes avoiding the use of challenging words or phrases. Amplifying means anticipating where students might need support in understanding concepts or mathematical terms, and providing multiple ways to access them. Providing visuals or manipulatives, demonstrating problem-solving, engaging in think-alouds, and creating analogies, synonyms, or context are all ways to amplify language so that students are supported in taking an active role in their own sense-making of mathematical relationships, processes, concepts, and terms.

Principle 2: OPTIMIZE OUTPUT

Strengthen opportunities and supports for students to describe their mathematical thinking to others, orally, visually, and in writing. Linguistic output is the language that students use to communicate their ideas to others (oral, written, visual, etc.), and refers to all forms of student linguistic expressions except those that include significant back-and-forth negotiation of ideas. (That type of conversational language is addressed in the third principle.) All students benefit from repeated, strategically optimized, and supported opportunities to articulate mathematical ideas into linguistic expression.

Opportunities for students to produce output should be strategically optimized for both (a) important concepts of the unit or course, and (b) important disciplinary language functions (for example, making conjectures and claims, justifying claims with evidence, explaining reasoning, critiquing the reasoning of others, making generalizations, and comparing approaches and representations). The focus for optimization must be determined, in part, by how students are currently using language to engage with important disciplinary concepts. When opportunities to produce output are optimized in these ways, students will get spiraled practice in making their thinking about important mathematical concepts stronger with more robust reasoning and examples, and making their thinking clearer with more precise language and visuals.

English Language Supports continued

Principle 3: CULTIVATE CONVERSATION

Strengthen opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class). Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated.

When students have a purpose for talking and listening to each other, communication is more authentic. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including when having conversations, making claims, justifying claims with evidence, making conjectures, communicating reasoning, critiquing the reasoning of others, engaging in other mathematical practices, and above all when making mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.

Principle 4: MAXIMIZE META-AWARENESS

Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language. Language is a tool that not only allows students to communicate their math understanding to others, but also to organize their own experiences, ideas, and learning for themselves. Meta-awareness is consciously thinking about one's own thought processes or language use. Meta-awareness develops when students and teachers engage in classroom activities or discussions that bring explicit attention to what students need to do to improve communication and reasoning about mathematical concepts. When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand—and be understood by—each other, they are motivated to attend to ways in which language can be both clarified and clarifying.

Meta-awareness in students can be strengthened when, for example, teachers ask students to explain to each other the strategies they brought to bear to solve a challenging problem. They might be asked, “How does yesterday’s method connect with the method we are learning today?” or, “What ideas are still confusing to you?” These questions are metacognitive because they help students to reflect on their own and others’ learning. Students can also reflect on their expanding use of language—for example, by comparing the language they used to clarify a mathematical concept with the language used by their peers in a similar situation. This is called metalinguistic awareness because students reflect on English as a language, their own growing use of that language, and the particular ways ideas are communicated in mathematics. Students learning English benefit from being aware of how language choices are related to the purpose of the task and the intended audience, especially if oral or written work is required. Both metacognitive and metalinguistic awareness are powerful tools to help students self-regulate their academic learning and language acquisition.

English Language Supports continued

These four principles are guides for curriculum development, as well as for planning and execution of instruction, including the structure and organization of interactive opportunities for students. They also serve as guides for and observation, analysis, and reflection on student language and learning. The design principles motivate the use of mathematical language routines, described in detail below, with examples. The eight routines included in this curriculum are:

- **MLR 1: Stronger and Clearer Each Time**
- **MLR 2: Collect and Display**
- **MLR 3: Clarify, Critique, Correct**
- **MLR 4: Information Gap**
- **MLR 5: Co-Craft Questions**
- **MLR 6: Three Reads**
- **MLR 7: Compare and Connect**
- **MLR 8: Discussion Supports**

When support beyond existing strategies embedded in the curriculum is required, additional supports for English language learners offer instructional strategies for teachers to meet the individual needs of a diverse group of learners. Lesson- and activity-level supports for English language learners stem from the design principles and are aligned to the language domains of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). These lesson-specific supports provide students with access to the mathematics by supporting them with the language demands of a specific activity without reducing the mathematical demand of the task. Using these supports will help maintain student engagement in mathematical discourse and ensure that the struggle remains productive. All of the supports are designed to be used as needed, and use should be faded out as students develop understanding and fluency with the English language. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

Example of an ELL Support Box from the Curriculum:

Support for English Language Learners

Writing, Conversing: MLR 1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners, to share their response to the question, “How do the two mixtures compare in taste?" Students should first check to see if they agree with each other about how Lin and Noah's mixtures compare. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, “How did you use double number lines to solve this problem?" or “Can you say more about what each ratio means?" Next, provide students with 3–4 minutes to revise their initial draft based on feedback from their peers. This will help strengthen students' understanding of how they determine whether the two situations involve equivalent ratios.
Design Principle(s): Support sense-making; Optimize output (for explanation)

From ***6.2 Lesson 10: Sparkling Orange Juice***

English Language Supports continued

A teacher who notices that students' written responses could get stronger and clearer with more opportunity to revise their writing could use this support to provide students with multiple opportunities to gain additional input, through direct and indirect feedback from their peers.

Based on their observations of student language, teachers can make adjustments to their teaching and provide additional language support where necessary. Teachers can select from the Supports for English language learners provided in the curriculum as appropriate. When selecting from these supports, teachers should take into account the language demands of the specific activity and the language needed to engage the content more broadly, in relation to their students' current ways of using language to communicate ideas as well as their students' English language proficiency.

Mathematical Language Routines

The mathematical language routines were selected because they are effective and practical for simultaneously learning mathematical practices, content, and language. A mathematical language routine is a structured but adaptable format for amplifying, assessing, and developing students' language. The routines emphasize uses of language that is meaningful and purposeful, rather than about just getting answers. These routines can be adapted and incorporated across lessons in each unit to fit the mathematical work wherever there are productive opportunities to support students in using and improving their English and disciplinary language use.

These routines facilitate attention to student language in ways that support in-the-moment teacher-, peer-, and self-assessment. The feedback enabled by these routines will help students revise and refine not only the way they organize and communicate their own ideas, but also ask questions to clarify their understandings of others' ideas.

Mathematical Language Routine 1: Stronger and Clearer Each Time

Adapted from Zwiers (2014)

Purpose

To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). This routine also provides a purpose for student conversation through the use of a discussion-worthy and iteration-worthy prompt. The main idea is to have students think and write individually about a question, use a structured pairing strategy to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original written response. Subsequent conversations and second drafts should naturally show evidence of incorporating or addressing new ideas and language. They should also show evidence of refinement in precision, communication, expression, examples, and reasoning about mathematical concepts.

English Language Supports continued

How it Happens

Prompt: This routine begins by providing a thought-provoking question or prompt. The prompt should guide students to think about a concept or big idea connected to the content goal of the lesson, and should be answerable in a format that is connected with the activity's primary disciplinary language function.

Response - First Draft: Students draft an initial response to the prompt by writing or drawing their initial thoughts in a first draft. Responses should attempt to align with the activity's primary language function. It is not necessary that students finish this draft before moving to the structured pair meetings step. However, students should be encouraged to write or draw something before meeting with a partner. This encouragement can come over time as class culture is developed, strategies and supports for getting started are shared, and students become more comfortable with the low stakes of this routine. (2–3 min)

Structured Pair Meetings: Next, use a structured pairing strategy to facilitate students having 2–3 meetings with different partners. Each meeting gives each partner an opportunity to be the speaker and an opportunity to be the listener. As the speaker, each student shares their ideas (without looking at their first draft, when possible). As a listener, each student should (a) ask questions for clarity and reasoning, (b) press for details and examples, and (c) give feedback that is relevant for the language goal. (1–2 min each meeting)

Response - Second Draft: Finally, after meeting with 2–3 different partners, students write a second draft. This draft should naturally reflect borrowed ideas from partners, as well as refinement of initial ideas through repeated communication with partners. This second draft will be stronger (with more or better evidence of mathematical content understanding) and clearer (more precision, organization, and features of disciplinary language function). After students are finished, their first and second drafts can be compared. (2–3 min)

Mathematical Language Routine 2: Collect and Display

Purpose

To capture a variety of students' oral words and phrases into a stable, collective reference. The intent of this routine is to stabilize the varied and fleeting language in use during mathematical work, in order for students' own output to become a reference in developing mathematical language. The teacher listens for, and scribes, the language students use during partner, small group, or whole class discussions using written words, diagrams and pictures. This collected output can be organized, revoiced, or explicitly connected to other language in a display that all students can refer to, build on, or make connections with during future discussion or writing. Throughout the course of a unit (and beyond), teachers can reference the displayed language as a model, update and revise the display as student language changes, and make bridges between prior student language and new disciplinary language (Dieckman, 2017). This routine provides feedback for students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

How it happens

Collect: During this routine, circulate and listen to student talk during paired, group, or as a whole-class discussion. Jot down the words, phrases, drawings, or writing students use. Capture a variety of uses of language that can be connected to the lesson content goals, as well as the relevant disciplinary language function(s). Collection can happen digitally, or with a clipboard, or directly onto poster paper; capturing on a whiteboard is not recommended due to risk of erasure.

Display: Display the language collected visually for the whole class to use as a reference during further discussions throughout the lesson and unit. Encourage students to suggest revisions, updates, and connections be added to the display as they develop—over time—both new mathematical ideas and new ways of communicating ideas. The display provides an opportunity to showcase connections between student ideas and new vocabulary. It also provides opportunity to highlight examples of students using disciplinary language functions, beyond just vocabulary words.

Mathematical Language Routine 3: Clarify, Critique, Correct

Purpose

To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written mathematical statement, and for students to improve upon the written work by correcting errors and clarifying meaning. Teachers can demonstrate how to effectively and respectfully critique the work of others with meta-think-alouds and pressing for details when necessary. This routine fortifies output and engages students in meta-awareness. More than just error analysis, this routine purposefully engages students in considering both the author's mathematical thinking as well as the features of their communication.

How it happens

Original Statement: Create or curate a written mathematical statement that intentionally includes conceptual (or common) errors in mathematical thinking as well as ambiguities in language. The mathematical errors should be driven by the content goals of the lesson and the language ambiguities should be driven by common or typical challenges with the relevant disciplinary language function.

English Language Supports continued

This mathematical text is read by the students and used as the draft, or "original statement," that students improve. (1–2 min)

Discussion with Partner: Next, students discuss the original statement in pairs. The teacher provides guiding questions for this discussion such as, "What do you think the author means?," "Is anything unclear?," or "Are there any reasoning errors?" In addition to these general guiding questions, 1–2 questions can be added that specifically address the content goals and disciplinary language function relevant to the activity. (2–3 min)

Improved Statement: Students individually revise the original statement, drawing on the conversations with their partners, to create an "improved statement." In addition to resolving any mathematical errors or misconceptions, clarifying ambiguous language, other requirements can be added as parameters for the improved response. These specific requirements should be aligned with the content goals and disciplinary language function of the activity. (3–5 min)

Mathematical Language Routine 4: Information Gap

Adapted from Zwiers 2004

Purpose

To create a need for students to communicate (Gibbons, 2002). This routine allows teachers to facilitate meaningful interactions by positioning some students as holders of information that is needed by other students. The information is needed to accomplish a goal, such as solving a problem or winning a game. With an information gap, students need to orally (or visually) share ideas and information in order to bridge a gap and accomplish something that they could not have done alone. Teachers should demonstrate how to ask for and share information, how to justify a request for information, and how to clarify and elaborate on information. This routine cultivates conversation.

How it happens

Problem/Data Cards: Students are paired into Partner A and Partner B. Partner A is given a card with a problem that must be solved, and Partner B has the information needed to solve it on a "data card." Data cards can also contain diagrams, tables, graphs, etc. Neither partner should read nor show their cards to their partners. Partner A determines what information they need, and prepares to ask Partner B for that specific information. Partner B should not share the information unless Partner A specifically asks for it and justifies the need for the information. Because partners don't have the same information, Partner A must work to produce clear and specific requests, and Partner B must work to understand more about the problem through Partner A's requests and justifications.

English Language Supports continued

Bridging the Gap

- Partner B asks “What specific information do you need?” Partner A asks for specific information from Partner B.
- Before sharing the requested information, Partner B asks Partner A for a justification: “Why do you need that information?”
- Partner A explains how they plan to use the information.
- Partner B asks clarifying questions as needed, and then provides the information.
- These four steps are repeated until Partner A is satisfied that they have information they need to solve the problem.

Solving the Problem

- Partner A shares the problem card with Partner B. Partner B does not share the data card.
- Both students solve the problem independently, then discuss their strategies. Partner B can share the data card after discussing their independent strategies.

Mathematical Language Routine 5: Co-craft Questions

Purpose

To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms and symbols can represent different situations. Through this routine, students are able to use conversation skills to generate, choose (argue for the best one), and improve questions and situations as well as develop meta-awareness of the language used in mathematical questions and problems.

How it happens

Hook: Begin by presenting students with a hook—a context or a stem for a problem, with or without values included. The hook can also be a picture, video, or list of interesting facts.

Students Write Questions: Next, students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math and could be questions about the situation, information that might be missing, and even about assumptions that they think are important. (1–2 minutes)

Students Compare Questions: Students compare the questions they generated with a partner (1–2 minutes) before sharing questions with the whole class. Demonstrate (or ask students to demonstrate) identifying specific questions that are aligned to the content goals of the lesson as well as the disciplinary language function. If there are no clear examples, teachers can demonstrate adapting a question or ask students to adapt questions to align with specific content or function goals. (2–3 minutes)

Actual Question(s) Revealed/Identified: Finally, the actual questions students are expected to work on are revealed or selected from the list that students generated.

Mathematical Language Routine 6: Three Reads

Purpose

To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to actively make sense of mathematical situations and information (Kelemanik, Lucenta, & Creighton, 2016). This routine supports reading comprehension, sense-making, and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner through mathematical conversation.

How it happens

In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. The intended question or main prompt is intentionally withheld until the third read so that students can concentrate on making sense of what is happening in the text before rushing to a solution or method.

Read #1: Shared Reading (one person reads aloud while everyone else reads with them) The first read focuses on the situation, context, or main idea of the text. After a shared reading, ask students "what is this situation about?" This is the time to identify and resolve any challenges with any non-mathematical vocabulary. (1 minute)

Read #2: Individual, Pairs, or Shared Reading After the second read, students list any quantities that can be counted or measured. Students are encouraged not to focus on specific values. Instead they focus on naming what is countable or measurable in the situation. It is not necessary to discuss the relevance of the quantities, just to be specific about them (examples: "number of people in her family" rather than "people," "number of markers after" instead of "markers"). Some of the quantities will be explicit (example: 32 apples) while others are implicit (example: the time it takes to brush one tooth). Record the quantities as a reference to use when solving the problem after the third read. (3–5 minutes)

English Language Supports continued

Read #3: Individual, Pairs, or Shared Reading During the third read, the final question or prompt is revealed. Students discuss possible solution strategies, referencing the relevant quantities recorded after the second read. It may be helpful for students to create diagrams to represent the relationships among quantities identified in the second read, or to represent the situation with a picture (Asturias, 2014). (1–2 minutes).

Mathematical Language Routine 7: Compare and Connect

Purpose

To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches and representations. This routine leverages the powerful mix of disciplinary representations available in mathematics as a resource for language development. In this routine, students make sense of mathematical strategies other than their own by relating and connecting other approaches to their own. Students should be prompted to reflect on, and linguistically respond to, these comparisons (for example, exploring why or when one might do or say something a certain way, identifying and explaining correspondences between different mathematical representations or methods, or wondering how a certain concept compares or connects to other concepts). Be sure to demonstrate asking questions that students can ask each other, rather than asking questions to "test" understanding. Use think alouds to demonstrate the trial and error, or fits and starts of sense-making (similar to the way teachers think aloud to demonstrate reading comprehension). This routine supports metacognition and metalinguistic awareness, and also supports constructive conversations.

How it Happens

Students Prepare Displays of their Work: Students are given a problem that can be approached and solved using multiple strategies, or a situation that can be modeled using multiple representations. Students are assigned the job of preparing a visual display of how they made sense of the problem and why their solution makes sense. Variation is encouraged and supported among the representations that different students use to show what makes sense.

Compare: Students investigate each others' work by taking a tour of the visual displays. Tours can be self-guided, a "travellers and tellers" format, or the teacher can act as "docent" by providing questions for students to ask of each other, pointing out important mathematical features, and facilitating comparisons. Comparisons focus on the typical structures, purposes, and affordances of the different approaches or representations: what worked well in this or that approach, or what is especially clear in this or that representation. During this discussion, listen for and amplify any comments about what might make this or that approach or representation more complete or easy to understand.

English Language Supports continued

Connect: The discussion then turns to identifying correspondences between different representations. Students are prompted to find correspondences in how specific mathematical relationships, operations, quantities, or values appear in each approach or representation. Guide students to refer to each other's thinking by asking them to make connections between specific features of expressions, tables, graphs, diagrams, words, and other representations of the same mathematical situation. During the discussion, amplify language students use to communicate about mathematical features that are important for solving the problem or modeling the situation. Call attention to the similarities and differences between the ways those features appear.

Mathematical Language Routine 8: Discussion Supports

Purpose

To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009). Rather than another structured format, the examples provided in this routine are instructional moves that can be combined and used together with any of the other routines. They include multimodal strategies for helping students make sense of complex language, ideas, and classroom communication. The examples can be used to invite and incentivize more student participation, conversation, and meta-awareness of language. Eventually, as teachers continue to demonstrate, students should begin using these strategies themselves to prompt each other to engage more deeply in discussions.

How it Happens

Unlike the other routines, this support is a collection of strategies and moves that can be combined and used to support discussion during almost any activity.

Examples of possible strategies:

- Revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task.

English Language Supports continued

- Demonstrate uses of disciplinary language functions such as detailing steps, describing and justifying reasoning, and questioning strategies.
- Give students time to make sure that everyone in the group can explain or justify each step or part of the problem. Then make sure to vary who is called on to represent the work of the group so students get accustomed to preparing each other to fill that role.
- Prompt students to think about different possible audiences for the statement, and about the level of specificity or formality needed for for a classmate vs. a mathematician, for example. [Convince Yourself, Convince a Friend, Convince a Skeptic (Mason, Burton, & Stacey, 2010)]

Sentence Frames

Sentence frames can support student language production by providing a structure to communicate about a topic. Helpful sentence frames are open-ended, so as to amplify language production, not constrain it. The table shows examples of generic sentence frames that can support common disciplinary language functions across a variety of content topics. Some of the lessons in these materials include suggestions of additional sentence frames that could support the specific content and language functions of that lesson.

language function

sample sentence frames

describe

- It looks like…
- I notice that…
- I wonder if…
- Let's try...
- A quantity that varies is _____.
- What do you notice?
- What other details are important?

explain

- First, I _____ because…
- Then/Next, I...
- I noticed _____ so I...
- I tried _____ and what happened was...
- How did you get…?
- What else could we do?

English Language Supports continued

justify

- I know _____ because...
- I predict _____ because…
- If _____ then _____ because…
- Why did you…?
- How do you know…?
- Can you give an example?

generalize

- _____ reminds me of _____ because…
- _____ will always _____ because…
- _____ will never _____ because…
- Is it always true that…?
- Is _____ a special case?

critique

- That could/couldn't be true because…
- This method works/doesn't work because…
- We can agree that...
- _____'s idea reminds me of…
- Another strategy would be _____ because…
- Is there another way to say/do...?

compare and contrast

- Both _____ and _____ are alike because…
- _____ and _____ are different because…
- One thing that is the same is…
- One thing that is different is…
- How are _____ and _____ different?
- What do _____ and _____ have in common?

represent

- _____ represents _____.
- _____ stands for _____.
- _____ corresponds to _____.
- Another way to show _____ is…
- How else could we show this?

English Language Supports continued

interpret

- We are trying to...
- We will need to know...
- We already know…
- It looks like _____ represents...
- Another way to look at it is…
- What does this part of _____ mean?
- Where does _____ show...?

References

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← Table of Contents Supporting Students with Disabilities →

Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. These materials empower students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Curriculum features that support access

Each lesson is carefully designed to maximize engagement and accessibility for all students. Purposeful design elements that support all learners, but that are especially helpful for students with disabilities, include:

Lesson Structures are Consistent

The structure of every lesson is the same: warm-up, activities, synthesis, cool-down. By keeping the components of each lesson similar from day to day, the flow of work in class becomes predictable for students. This reduces cognitive demand and enables students to focus on the mathematics at hand rather than the mechanics of the lesson.

Concepts Develop from Concrete to Abstract

Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time. Students begin with concrete examples, and transition to diagrams and tables before relying exclusively on symbols to represent the mathematics they encounter.

Individual to Pair, or Small Group to Whole Class Progression

Providing students with time to think through a situation or question independently before engaging with others allows students to carry the weight of learning, with supports arriving just in time from the community of learners. This progression allows students to first activate what they already know, and continue to build from this base with others.

Opportunities to Apply Mathematics to Real-World Contexts

Giving students opportunities to apply the mathematics they learn clarifies and deepens their understanding of core math concepts and skills and provides motivation and support. Mathematical modeling is a powerful activity for all students, but especially students with disabilities. Each unit has a culminating activity designed to explore, integrate, and apply all the big ideas of the unit. Centering instruction on these contextual situations can provide students with disabilities an anchor on which to base their mathematical understandings.

Supporting Students with Disabilities continued

Instructional strategies that support access

The following general instructional strategies can be used to make activities accessible to all students:

Eliminate Barriers

Eliminate any unnecessary barriers that students may encounter that prevent them from engaging with the important mathematical work of a lesson. This requires flexibility and attention to areas such as the physical environment of the classroom, access to tools, organization of lesson activities, and means of communication.

Processing Time

Increased time engaged in thinking and learning leads to mastery of grade-level content for all students, including students with disabilities. Frequent switching between topics creates confusion and does not allow for content to deeply embed in the mind of the learner. Mathematical ideas and representations are carefully introduced in the materials in a gradual, purposeful way to establish a base of conceptual understanding. Some students may need additional time, which should be provided as required.

Assistive Technology

Assistive technology can be a vital tool for students with learning disabilities, visual spatial needs, sensory integration, and students with autism. Assistive technology supports suggested in the materials are designed to either enhance or support learning, or to bypass unnecessary barriers.

Manipulatives

Physical manipulatives help students make connections between concrete ideas and abstract representations. Often, students with disabilities benefit from hands-on activities, which allow them to make sense of the problem at hand and communicate their own mathematical ideas and solutions.

Visual Aids

Visual aids such as images, diagrams, vocabulary anchor charts, color coding, or physical demonstrations are suggested throughout the materials to support conceptual processing and language development. Many students with disabilities have working memory and processing challenges. Keeping visual aids visible on the board allows students to access them as needed, so that they can solve problems independently. Leaving visual aids on the board especially benefits students who struggle with working or short-term memory issues.

Supporting Students with Disabilities continued

Graphic Organizers

Word webs, Venn diagrams, tables, and other metacognitive visual supports provide structures that illustrate relationships between mathematical facts, concepts, words, or ideas. Graphic organizers can be used to support students with organizing thoughts and ideas, planning problem solving approaches, visualizing ideas, sequencing information, and comparing and contrasting ideas.

Brain Breaks

Brain breaks are short, structured, 2–3 minute movement breaks taken between activities, or to break up a longer activity (approximately every 20–30 minutes during a class period). Brain breaks are a quick, effective way of refocusing and re-energizing the physical and mental state of students during a lesson. Brain breaks have also been shown to positively impact student concentration and stress levels, resulting in more time spent engaged in mathematical problem solving. This universal support is beneficial for all students, but especially those with ADHD.

Supports for Students with Disabilities

The additional supports for students with disabilities are activity-specific and provide teachers with strategies to increase access and eliminate barriers without reducing the mathematical demand of the task. Designed for students with disabilities, they are also appropriate for many students who need additional support to access rigorous, grade-level content. In addition to the guidance provided here, teachers should consider the individual needs of their students and use formative assessment to determine which supports to use and when.

Students' strengths and needs in the following areas of cognitive functioning are integral to learning mathematics (Brodesky et al., 2002) and provide an additional lens to help teachers select appropriate supports for specific types of learner needs.

- *Conceptual Processing* includes perceptual reasoning, problem solving, and metacognition.
- *Language* includes auditory and visual language processing and expression.
- *Visual-Spatial Processing* includes processing visual information and understanding relation in space of visual mathematical representations and geometric concepts.
- *Organization* includes organizational skills, attention, and focus.
- *Memory* includes working memory and short-term memory.
- *Attention* includes paying attention to details, maintaining focus, and filtering out extraneous information.
- *Social-Emotional Functioning* includes interpersonal skills and the cognitive comfort and safety required in order to take risks and make mistakes.
- *Fine Motor Skills* includes tasks that require small muscle movement and coordination such as manipulating objects (graphing, cutting with scissors, writing).

Supporting Students with Disabilities continued

The additional supports for students with disabilities were designed using the Universal Design for Learning Guidelines (http://udlguidelines.cast.org). Each support aligns to one of the three principles of UDL: engagement, representation, and action and expression.

Engagement:

Students' attitudes, interests, and values help to determine the ways in which they are most engaged and motivated to learn. Supports that align to this principle offer instructional strategies that provide students with multiple means of engagement and include suggestions that, help provide access by leveraging curiosity and students' existing interests, leverage choice around perceived challenge, encourage and support opportunities for peer collaboration; provide structures that help students maintain sustained effort and persistence during a task, and provide tools and strategies designed to help students self-motivate and become more independent.

Representation:

Teachers can reduce barriers and leverage students' individual strengths by inviting students to engage with the same content in different ways. Supports that align to this principle offer instructional strategies that provide students with multiple means of representation and include suggestions that offer alternatives for the ways information is presented or displayed, help develop students' understanding and use of mathematical language and symbols; illustrate connections between and across mathematical representations using color and annotations, identify opportunities to activate or supply background knowledge, and describe organizational methods and approaches designed to help students internalize learning.

Action and Expression:

Throughout the curriculum, students are invited to share both their understanding and their reasoning about mathematical ideas with others. Supports that align to this principle offer instructional strategies that provide students with multiple means of action and expression and include suggestions that encourage flexibility and choice with the ways students demonstrate their understanding; list sentence frames that support discourse or accompany writing prompts; indicate appropriate tools, templates, and assistive technologies; support the development of organizational skills in problem-solving; and provide checklists that enable students to monitor their own progress.

For additional information about the Universal Design for Learning framework, or to learn more about supporting students with disabilities, visit the Center for Applied Special Technology (CAST) at www.cast.org/udl.

Supporting Students with Disabilities continued

Supporting Students with Disabilities continued

References

- Brodesky et al. (2002). Accessibility strategies toolkit for mathematics. Education Development Center. http://www2.edc.org/accessmath/resources/strategiesToolkit.pdf
- CAST (2018). Universal design for learning guidelines version 2.2. Retrieved from http://udlguidelines.cast.org

Accessibility for Students with Visual Impairments

Features built into the materials that make them more accessible to students with visual impairments include:

1. A color palette using colors that are distinguishable to people with the most common types of color blindness.
2. Tasks and problems designed such that success does not depend on the ability to distinguish between colors.
3. Mathematical diagrams are presented in scalable vector graphic (SVG) format, which can be magnified without loss of resolution, and are possible to render in Braille.
4. Where possible, text associated with images is not part of the image file, but rather, included as an image caption that is accessible to screen readers.
5. Alt text on all images, to make the materials easier to interpret for users accessing the materials with a screen reader.

If students with visual impairments are accessing the materials using a screen reader, it is important to understand:

- All images in the curriculum have alt text: a very short indication of the image's contents, so that the screen reader doesn't skip over as if nothing is there.
- Some images have a longer description to help a student with a visual impairment recreate the image in their mind.

It is important for teachers to understand that students with visual impairments are likely to need help accessing images in lesson activities and assessments, and prepare appropriate accommodations. Be aware that mathematical diagrams are provided as scalable vector graphics (SVG format), because this format can be magnified without loss of resolution.

Accessibility experts who reviewed this curriculum recommended that students who would benefit should have access to a Braille version of the curriculum materials, because a verbal description of many of the complex mathematical diagrams would be inadequate for supporting their learning. All diagrams are provided in the SVG file type so that they can be rendered in Braille format.

CKMath™ Core Knowledge MATHEMATICS Grade 6

ACHIEVE: Standards Alignments for Grade 6

Standards by Lesson *(Jump to Lessons by Standard)*

Lesson	Standards Addressed
6.1.1	6.G.A.1
6.1.2	6.G.A, 6.G.A.1
6.1.3	6.G.A.1
6.1.4	6.G.A.1
6.1.5	6.EE.A.2.a, 6.EE.A.2.c, 6.G.A.1
6.1.6	6.EE.A.2.c, 6.G.A.1
6.1.7	6.G.A.1
6.1.8	6.G.A.1
6.1.9	6.EE.A.2.a, 6.EE.A.2.c, 6.G.A.1
6.1.10	6.EE.A.2.c, 6.G.A.1
6.1.11	6.G.A.1
6.1.12	6.G.A.4
6.1.13	6.G.A.4
6.1.14	6.G.A.4
6.1.15	6.G.A.2, 6.G.A.4
6.1.16	6.G.A.4
6.1.17	6.EE.A, 6.EE.A.1
6.1.18	6.EE.A.1, 6.EE.A.2.a, 6.G.A.4
6.1.19	6.G.A.1, 6.G.A.4

CKMath™ Core Knowledge MATHEMATICS Grade 6

Lesson	Standards Addressed
6.2.1	6.RP.A.1
6.2.2	6.RP.A.1
6.2.3	6.RP.A.1
6.2.4	6.RP.A.1
6.2.5	6.RP.A.1
6.2.6	6.RP.A.3
6.2.7	6.RP.A.3
6.2.8	6.RP.A.3.b
6.2.9	6.RP.A.3.b
6.2.10	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.b
6.2.11	6.RP.A.3.a
6.2.12	6.RP.A.3, 6.RP.A.3.a
6.2.13	6.RP.A.3, 6.RP.A.3.a
6.2.14	6.RP.A.3
6.2.15	6.RP.A.3
6.2.16	6.RP.A.3
6.2.17	6.RP.A, 6.RP.A.3
6.3.1	6.RP.A.2
6.3.2	6.RP.A.3.d
6.3.3	6.RP.A.3.d
6.3.4	6.RP.A.3.d
6.3.5	6.RP.A.2, 6.RP.A.3.b

Lesson	Standards Addressed
6.3.6	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.b
6.3.7	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.b
6.3.8	6.RP.A.3, 6.RP.A.3.b
6.3.9	6.RP.A.3, 6.RP.A.3.b, 6.RP.A.3.c, 6.RP.A.3.d
6.3.10	6.RP.A.3.c
6.3.11	6.RP.A.3.c
6.3.12	6.RP.A.3.c
6.3.13	6.RP.A.3.c
6.3.14	6.RP.A.3.c
6.3.15	6.RP.A.3, 6.RP.A.3.c
6.3.16	6.RP.A.3.c
6.3.17	6.G.A, 6.RP.A
6.4.1	6.NS.A
6.4.2	6.NS.A
6.4.3	6.NS.A.1
6.4.4	6.NS.A.1
6.4.5	6.NS.A.1
6.4.6	6.NS.A.1
6.4.7	6.NS.A.1
6.4.8	6.NS.A.1
6.4.9	6.NS.A.1
6.4.10	6.NS.A.1

CKMath™ Core Knowledge Mathematics Grade 6

Lesson	Standards Addressed
6.4.11	6.NS.A.1
6.4.12	6.NS.A.1
6.4.13	6.G.A.2, 6.NS.A.1
6.4.14	6.G.A.1, 6.G.A.2, 6.NS.A.1
6.4.15	6.G.A.2
6.4.16	6.NS.A.1
6.4.17	6.G.A.2, 6.NS.A.1
6.5.1	6.NS.B.3
6.5.2	6.NS.B.3
6.5.3	6.NS.B.3
6.5.4	6.NS.B.3
6.5.5	6.EE.A, 6.NS.B
6.5.6	6.NS.B
6.5.7	6.NS.B.3
6.5.8	6.EE.A, 6.NS.B.3
6.5.9	6.NS.B.2
6.5.10	6.EE.A, 6.NS.B.2
6.5.11	6.EE.A, 6.NS.B.2
6.5.12	6.EE.A, 6.NS.B.2, 6.NS.B.3
6.5.13	6.EE.A.4, 6.NS.B.2, 6.NS.B.3
6.5.14	6.NS.B.2, 6.NS.B.3
6.5.15	6.NS.B.3

Grade 6

Lesson	Standards Addressed
6.6.1	6.EE.B.5, 6.EE.B.6, 6.EE.B.7
6.6.2	6.EE.B.5, 6.EE.B.6
6.6.3	6.EE.B.5, 6.EE.B.6, 6.EE.B.7
6.6.4	6.EE.B, 6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.NS.B.3
6.6.5	6.EE.B, 6.EE.B.5, 6.EE.B.6, 6.EE.B.7
6.6.6	6.EE.A.2.a, 6.EE.A.2.c, 6.EE.B.6
6.6.7	6.EE.B.6, 6.EE.B.7, 6.RP.A.3.c
6.6.8	6.EE.A.4, 6.EE.B.5
6.6.9	6.EE.A.3, 6.EE.A.4
6.6.10	6.EE.A.2, 6.EE.A.3, 6.EE.A.4
6.6.11	6.EE.A.2, 6.EE.A.3, 6.EE.A.4
6.6.12	6.EE.A.1
6.6.13	6.EE.A.1
6.6.14	6.EE.A.1, 6.EE.A.2.c
6.6.15	6.EE.A.1, 6.EE.A.2.c, 6.EE.B.5
6.6.16	6.EE.C.9, 6.RP.A.1, 6.RP.A.3.a, 6.RP.A.3.b
6.6.17	6.EE.C.9, 6.RP.A.3.a, 6.RP.A.3.b
6.6.18	6.EE.C.9
6.6.19	6.EE.A.2, 6.EE.B.7, 6.EE.C.9
6.7.1	6.NS.C.5, 6.NS.C.6
6.7.2	6.NS.C.6, 6.NS.C.6.a, 6.NS.C.6.c
6.7.3	6.NS.C.7.a, 6.NS.C.7.b, 6.NS.C.7.d

CKMath™ Core Knowledge Mathematics Grade 6

Lesson	Standards Addressed
6.7.4	6.NS.C, 6.NS.C.6, 6.NS.C.6.a, 6.NS.C.7
6.7.5	6.NS.C.5
6.7.6	6.NS.C.7, 6.NS.C.7.c, 6.NS.C.7.d
6.7.7	6.NS.C.6, 6.NS.C.6.a, 6.NS.C.7, 6.NS.C.7.d
6.7.8	6.EE.B.6, 6.EE.B.8, 6.NS.C.7.b
6.7.9	6.EE.B.5, 6.EE.B.8, 6.NS.C.7.a
6.7.10	6.EE.A.2.b, 6.EE.B.5, 6.EE.B.6, 6.EE.B.8
6.7.11	6.NS.C.6, 6.NS.C.6.b, 6.NS.C.6.c, 6.NS.C.8
6.7.12	6.NS.C.6, 6.NS.C.6.c
6.7.13	6.NS.C.6.c, 6.NS.C.7.c, 6.NS.C.8
6.7.14	6.NS.C.6, 6.NS.C.6.b, 6.NS.C.8
6.7.15	6.G.A.3, 6.NS.C.6.c, 6.NS.C.8
6.7.16	6.NS.B.4
6.7.17	6.NS.B.4
6.7.18	6.NS.B.4
6.7.19	6.G.A.3, 6.NS.C.8
6.8.1	6.SP.B, 6.SP.B.4
6.8.2	6.SP.A, 6.SP.A.1, 6.SP.B, 6.SP.B.5.b
6.8.3	6.SP.A.1, 6.SP.A.2, 6.SP.B.4, 6.SP.B.5.a, 6.SP.B.5.b
6.8.4	6.SP.A.2, 6.SP.A.3, 6.SP.B, 6.SP.B.4, 6.SP.B.5.a, 6.SP.B.5.c
6.8.5	6.SP.A.2, 6.SP.A.3, 6.SP.B, 6.SP.B.4, 6.SP.B.5.b, 6.SP.B.5.c
6.8.6	6.SP.A.1, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5.b

Lesson	Standards Addressed
6.8.7	6.SP.A.1, 6.SP.A.2, 6.SP.B, 6.SP.B.4, 6.SP.B.5.b
6.8.8	6.SP.A.2, 6.SP.B.4, 6.SP.B.5.d
6.8.9	6.SP.A.3, 6.SP.B, 6.SP.B.5.c
6.8.10	6.SP.A.3, 6.SP.B.5.c
6.8.11	6.SP.A.2, 6.SP.A.3, 6.SP.B.5.c
6.8.12	6.NS.B.3, 6.SP.A.3, 6.SP.B.5.c, 6.SP.B.5.d
6.8.13	6.SP.B, 6.SP.B.5.c, 6.SP.B.5.d
6.8.14	6.SP.B.5.b, 6.SP.B.5.c, 6.SP.B.5.d
6.8.15	6.SP.B.4, 6.SP.B.5.c, 6.SP.B.5.d
6.8.16	6.SP.B.4, 6.SP.B.5.c, 6.SP.B.5.d
6.8.17	6.SP.A.1, 6.SP.B.4, 6.SP.B.5
6.8.18	6.SP.A.2, 6.SP.B, 6.SP.B.5.c, 6.SP.B.5.d
6.9.1	6.G.A, 6.NS.B, 6.RP.A
6.9.2	6.NS.B, 6.RP.A
6.9.3	6.NS.A
6.9.4	6.RP.A.1, 6.RP.A.3, 6.RP.A.3.c
6.9.5	6.RP.A.3
6.9.6	6.NS.B.3, 6.RP.A.2, 6.RP.A.3, 6.RP.A.3.c

CKMath™ Core Knowledge MATHEMATICS Grade 6

Lessons by Standard

(Jump to Standards by Lesson)

Standards	Aligned Lessons
6.EE.A	6.1.17, 6.5.5
6.EE.A.1	6.1.17, 6.1.18, 6.6.12, 6.6.13, 6.6.14, 6.6.15
6.EE.A.2	6.6.10, 6.6.11, 6.6.19
6.EE.A.2.a	6.1.5, 6.1.9, 6.1.18, 6.6.6
6.EE.A.2.b	6.7.10
6.EE.A.2.c	6.1.5, 6.1.6, 6.1.9, 6.1.10, 6.6.6, 6.6.14, 6.6.15
6.EE.A.3	6.6.10, 6.6.11
6.EE.A.4	6.5.13, 6.6.8, 6.6.10, 6.6.11
6.EE.B	6.6.4, 6.6.5
6.EE.B.5	6.6.2, 6.6.3, 6.6.4, 6.6.5, 6.6.8, 6.6.15, 6.7.9, 6.7.10
6.EE.B.6	6.6.1, 6.6.3, 6.6.4, 6.6.5, 6.6.6, 6.6.7, 6.7.8, 6.7.10
6.EE.B.7	6.6.3, 6.6.4, 6.6.5, 6.6.7, 6.6.19
6.EE.B.8	6.7.8, 6.7.9, 6.7.10
6.EE.C.9	6.6.16, 6.6.17, 6.6.18, 6.6.19
6.G.A	6.3.17, 6.9.1
6.G.A.1	6.1.2, 6.1.3, 6.1.4, 6.1.5, 6.1.6, 6.1.7, 6.1.8, 6.1.9, 6.1.10, 6.1.11, 6.1.19, 6.4.14
6.G.A.2	6.1.15, 6.4.14, 6.4.15, 6.4.17
6.G.A.3	6.7.15, 6.7.19
6.G.A.4	6.1.12, 6.1.13, 6.1.14, 6.1.15, 6.1.16, 6.1.18, 6.1.19
6.NS.A	6.9.3

Grade 6

Standards	Aligned Lessons
6.NS.A.1	6.4.3, 6.4.4, 6.4.5, 6.4.6, 6.4.7, 6.4.8, 6.4.9, 6.4.10, 6.4.11, 6.4.12, 6.4.13, 6.4.14, 6.4.16, 6.4.17
6.NS.B	6.5.5, 6.5.6, 6.9.1, 6.9.2
6.NS.B.2	6.5.9, 6.5.10, 6.5.11, 6.5.13, 6.5.14
6.NS.B.3	6.5.2, 6.5.3, 6.5.4, 6.5.7, 6.5.8, 6.5.12, 6.5.13, 6.5.14, 6.5.15, 6.6.4, 6.8.12, 6.9.6
6.NS.B.4	6.7.16, 6.7.17, 6.7.18
6.NS.C	6.7.4
6.NS.C.5	6.7.1, 6.7.5
6.NS.C.6	6.7.1, 6.7.2, 6.7.4, 6.7.7, 6.7.14
6.NS.C.6.a	6.7.2, 6.7.4, 6.7.7
6.NS.C.6.b	6.7.11, 6.7.14
6.NS.C.6.c	6.7.2, 6.7.11, 6.7.12, 6.7.13, 6.7.15
6.NS.C.7	6.7.4, 6.7.6, 6.7.7
6.NS.C.7.a	6.7.3, 6.7.9
6.NS.C.7.b	6.7.3, 6.7.8
6.NS.C.7.c	6.7.6, 6.7.13
6.NS.C.7.d	6.7.6, 6.7.7
6.NS.C.8	6.7.11, 6.7.13, 6.7.14, 6.7.15, 6.7.19
6.RP.A	6.2.17, 6.3.17, 6.9.1, 6.9.2
6.RP.A.1	6.2.1, 6.2.2, 6.2.3, 6.2.4, 6.2.5, 6.6.16, 6.9.4
6.RP.A.2	6.2.10, 6.3.1, 6.3.5, 6.3.6, 6.3.7, 6.9.6
6.RP.A.3	6.2.6, 6.2.7, 6.2.10, 6.2.12, 6.2.13, 6.2.14, 6.2.15, 6.2.16, 6.2.17, 6.3.6, 6.3.7, 6.3.8, 6.3.9, 6.3.15, 6.9.4, 6.9.5, 6.9.6

Grade 6

Standards	Aligned Lessons
6.RP.A.3.a	6.2.11, 6.2.12, 6.2.13, 6.6.16, 6.6.17
6.RP.A.3.b	6.2.8, 6.2.9, 6.2.10, 6.3.5, 6.3.6, 6.3.7, 6.3.8, 6.3.9, 6.6.16, 6.6.17
6.RP.A.3.c	6.3.10, 6.3.11, 6.3.12, 6.3.13, 6.3.14, 6.3.15, 6.3.16, 6.6.7, 6.9.4, 6.9.6
6.RP.A.3.d	6.3.3, 6.3.4, 6.3.9
6.SP.A	6.8.2
6.SP.A.1	6.8.2, 6.8.3, 6.8.6, 6.8.7, 6.8.17
6.SP.A.2	6.8.4, 6.8.5, 6.8.7, 6.8.8, 6.8.11, 6.8.18
6.SP.A.3	6.8.6, 6.8.9, 6.8.10, 6.8.11
6.SP.B	6.8.1, 6.8.2, 6.8.4, 6.8.5, 6.8.7, 6.8.9, 6.8.13, 6.8.18
6.SP.B.4	6.8.3, 6.8.4, 6.8.5, 6.8.6, 6.8.7, 6.8.8, 6.8.16, 6.8.17
6.SP.B.5	6.8.17
6.SP.B.5.a	6.8.3, 6.8.4
6.SP.B.5.b	6.8.2, 6.8.3, 6.8.5, 6.8.6, 6.8.7, 6.8.14
6.SP.B.5.c	6.8.9, 6.8.10, 6.8.11, 6.8.12, 6.8.13, 6.8.14, 6.8.15, 6.8.16, 6.8.18
6.SP.B.5.d	6.8.12, 6.8.14, 6.8.15, 6.8.16, 6.8.18

← Table of Contents