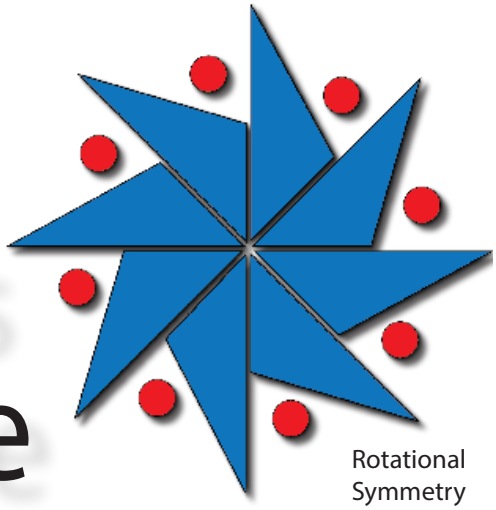
Core Knowledge® MATHEMATICS

Rigid Transformations and Congruence

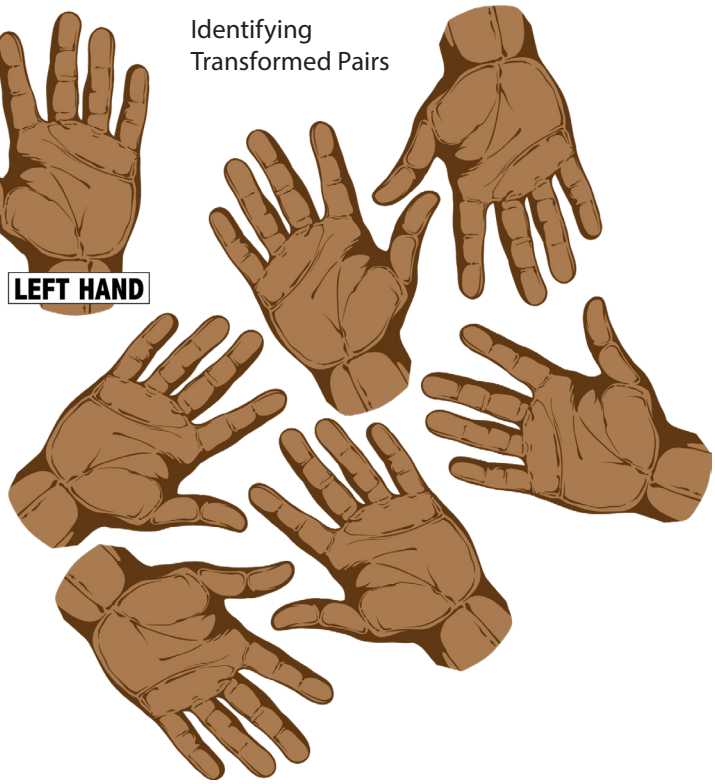


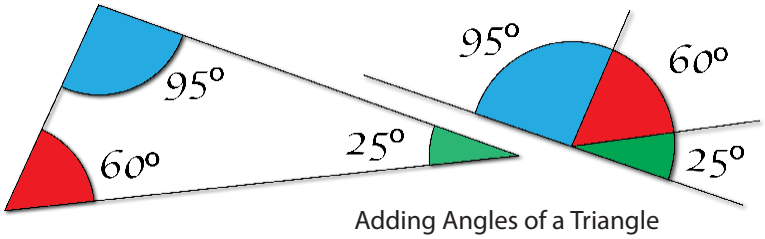
Rotational Symmetry

Student Workbook

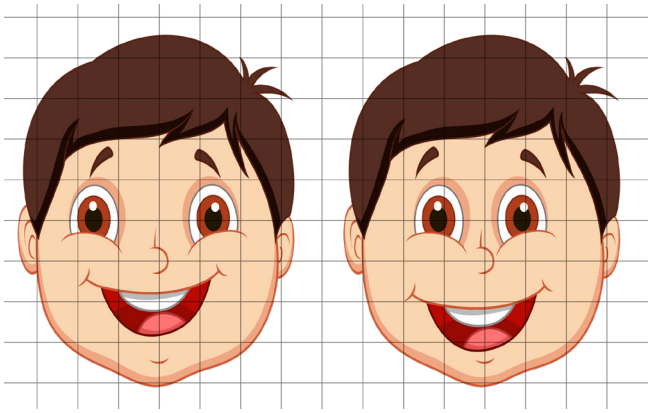
Describing Reflection and Movement

Identifying Transformed Pairs

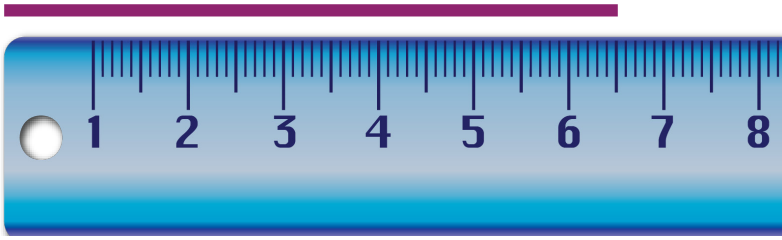




Adding Angles of a Triangle



Using Evidence to Determine Congruence



Measuring Segments

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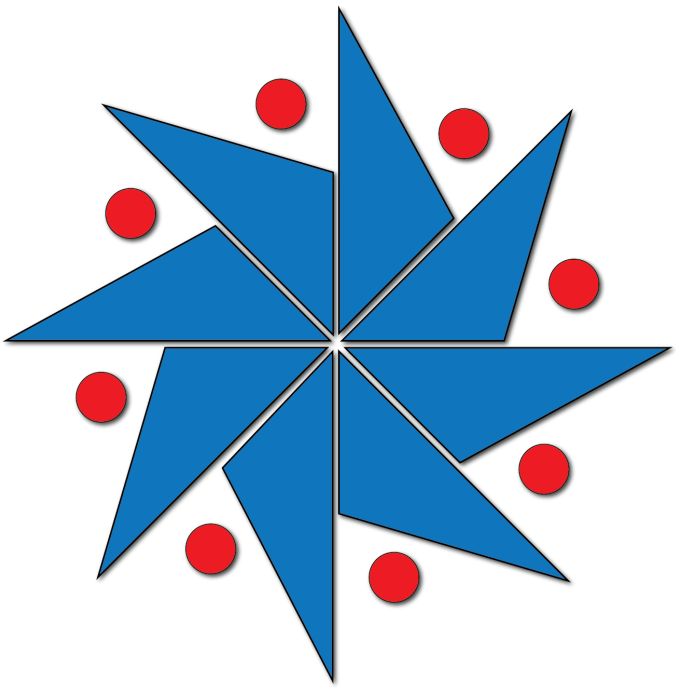
Rigid Transformations and Congruence

Table of Contents

Lesson 1	**Moving in the Plane**	1
Lesson 2	**Naming the Moves**	8
Lesson 3	**Grid Moves**	15
Lesson 4	**Making the Moves**	22
Lesson 5	**Coordinate Moves**	31
Lesson 6	**Describing Transformations**	38
Lesson 7	**No Bending or Stretching**	45
Lesson 8	**Rotation Patterns**	51
Lesson 9	**Moves in Parallel**	55
Lesson 10	**Composing Figures**	62
Lesson 11	**What is the Same?**	68
Lesson 12	**Congruent Polygons**	74
Lesson 13	**Congruence**	81
Lesson 14	**Alternate Interior Angles**	62
Lesson 15	**Adding the Angles in a Triangle**	68
Lesson 16	**Parallel Lines and Angles in a Triangle**	74
Lesson 17	**Rotate and Tessellate**	81



Rigid Transformations and Congruence

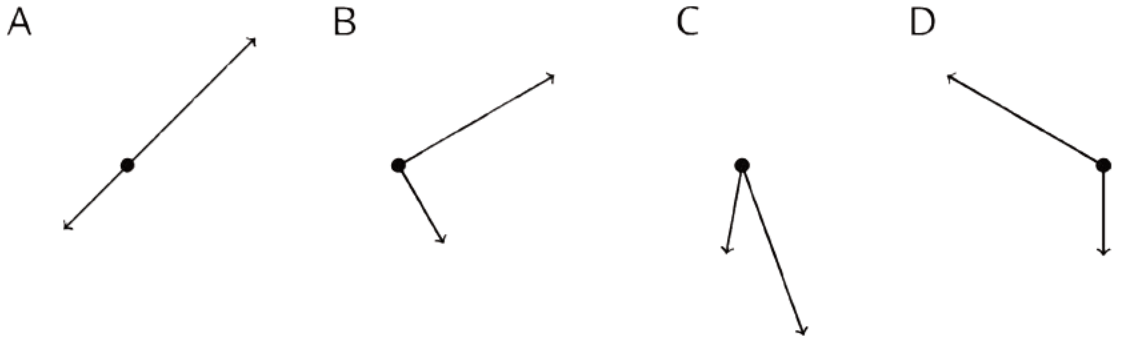
Student Workbook

Core Knowledge Mathematics™

Lesson 1: Moving in the Plane

1.1: Which One Doesn't Belong: Diagrams

Which one doesn't belong?



1.2: Triangle Square Dance

Your teacher will give you three pictures. Each shows a different set of dance moves.

1. Arrange the three pictures so you and your partner can both see them right way up. Choose who will start the game.
    - The starting player mentally chooses A, B, or C and describes the dance to the other player.
    - The other player identifies which dance is being talked about: A, B, or C.
2. After one round, trade roles. When you have described all three dances, come to an agreement on the words you use to describe the moves in each dance.
3. With your partner, write a description of the moves in each dance.

Are you ready for more?

We could think of each dance as a new dance by running it in reverse, starting in the 6th frame and working backwards to the first.

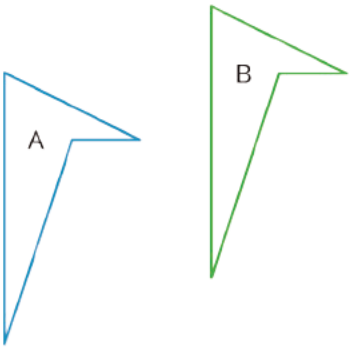
1. Pick a dance and describe in words one of these reversed dances.

2. How do the directions for running your dance in the forward direction and the reverse direction compare?

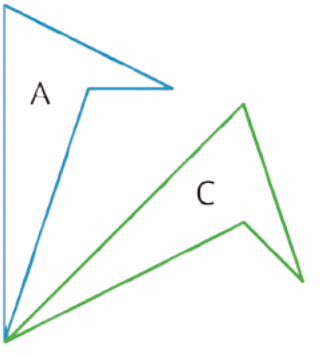
Lesson 1 Summary

Here are two ways for changing the position of a figure in a plane without changing its shape or size:

- Sliding or shifting the figure without turning it. Shifting Figure A to the right and up puts it in the position of Figure B.

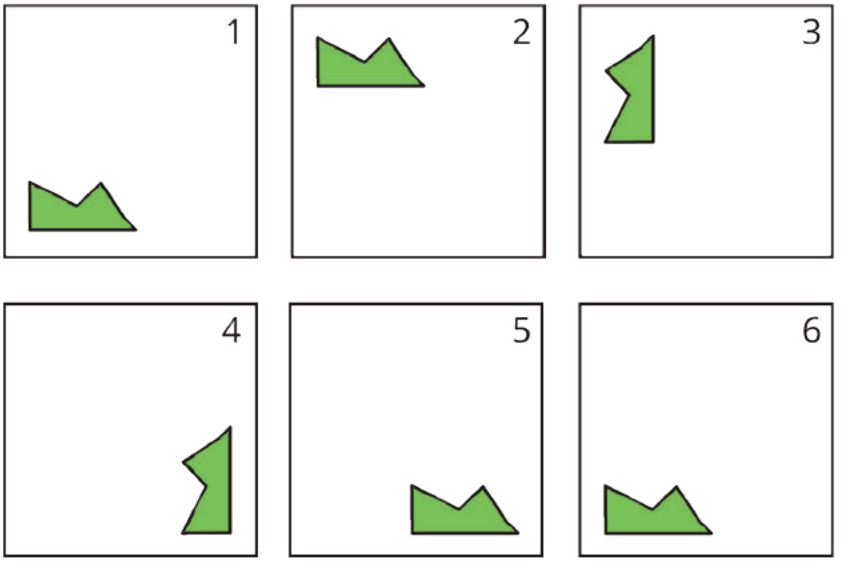


- Turning or rotating the figure around a point. Figure A is rotated around the bottom **vertex** to create Figure C.



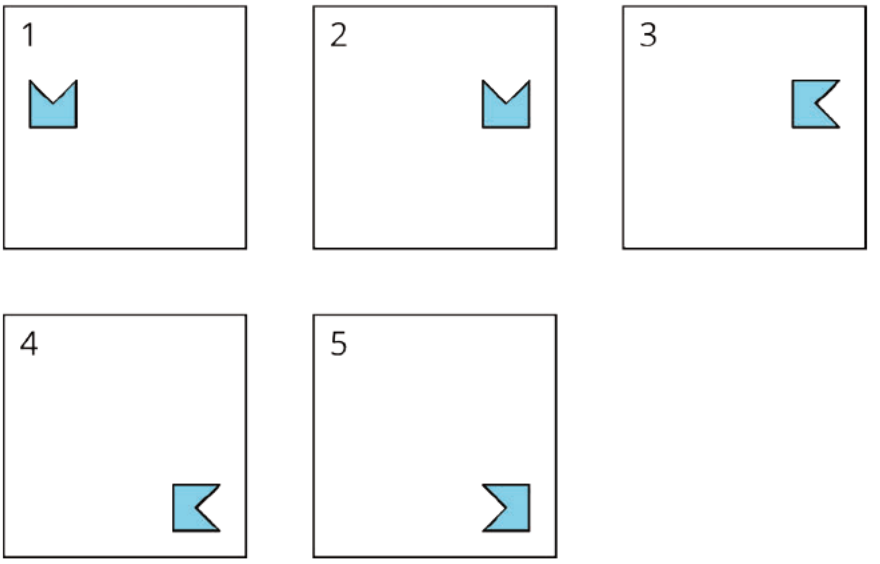
Unit 1 Lesson 1 Cumulative Practice Problems

1. The six frames show a shape's different positions.



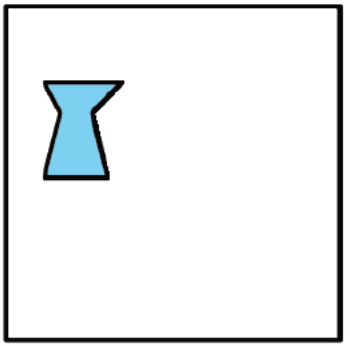
Describe how the shape moves to get from its position in each frame to the next.

2. These five frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

3. Diego started with this shape.

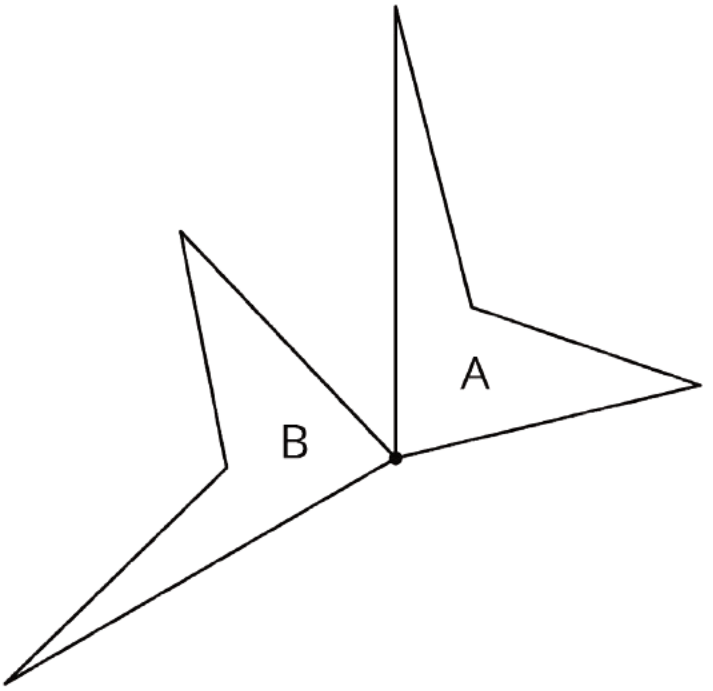


Diego moves the shape down, turns it 90 degrees clockwise, then moves the shape to the right. Draw the location of the shape after each move.

Lesson 2: Naming the Moves

2.1: A Pair of Quadrilaterals

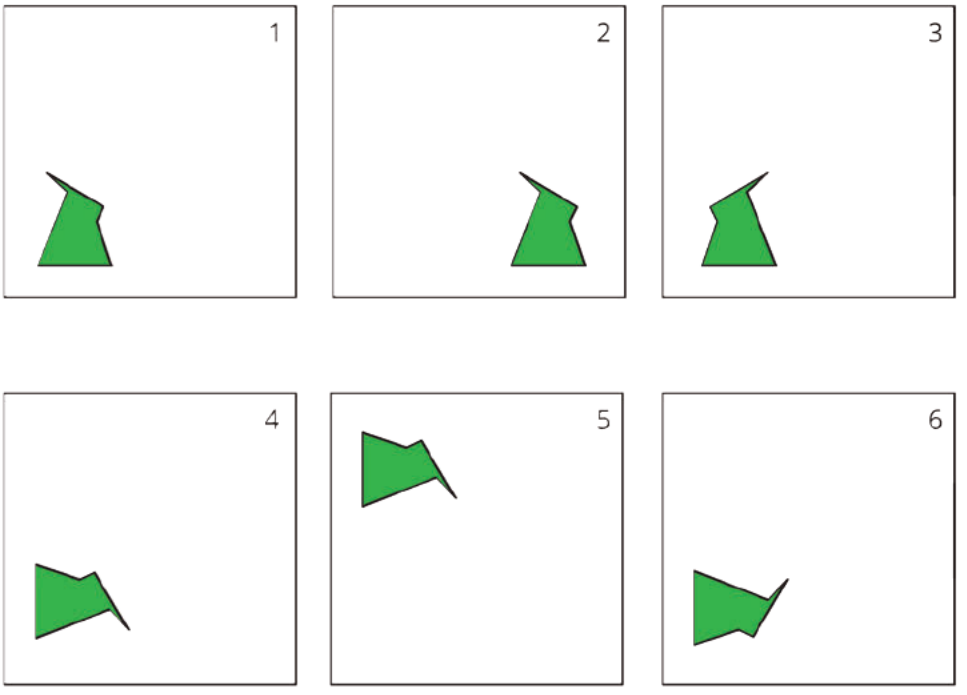
Quadrilateral A can be rotated into the position of Quadrilateral B.



Estimate the angle of rotation.

2.2: How Did You Make That Move?

Here is another set of dance moves.



1. Describe each move or say if it is a new move.

    a. Frame 1 to Frame 2.

    b. Frame 2 to Frame 3.

    c. Frame 3 to Frame 4.

    d. Frame 4 to Frame 5.

    e. Frame 5 to Frame 6.

2. How would you describe the new move?

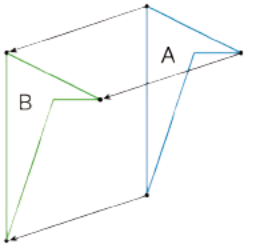
2.3: Card Sort: Move

Your teacher will give you a set of cards. Sort the cards into categories according to the type of move they show. Be prepared to describe each category and why it is different from the others.

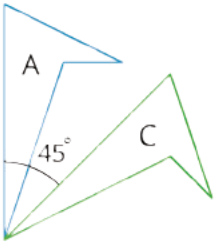
Lesson 2 Summary

Here are the moves we have learned about so far:

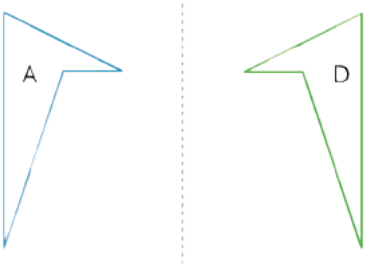
- A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



- A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



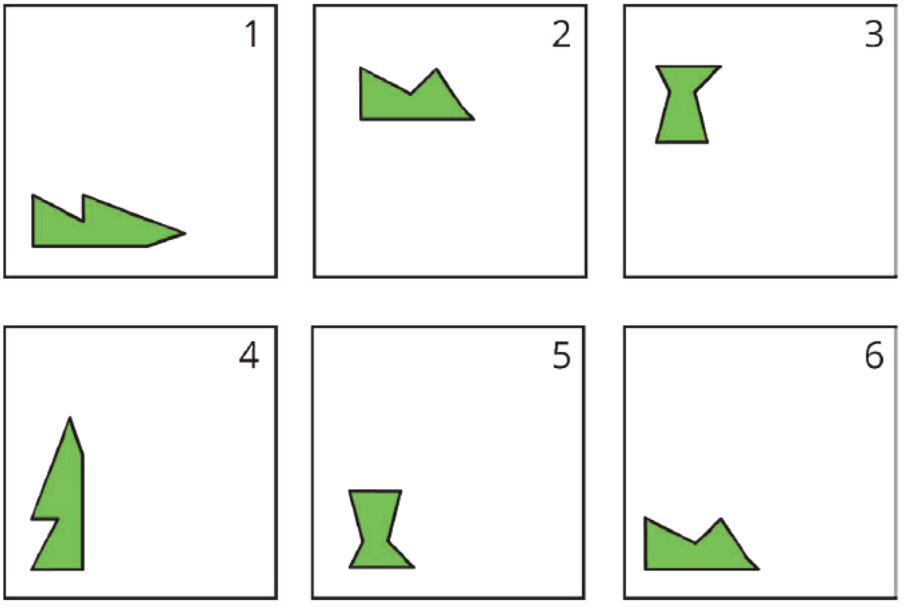
- A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



We use the word *image* to describe the new figure created by moving the original figure. If one point on the original figure moves to another point on the new figure, we call them *corresponding* points.

Unit 1 Lesson 2 Cumulative Practice Problems

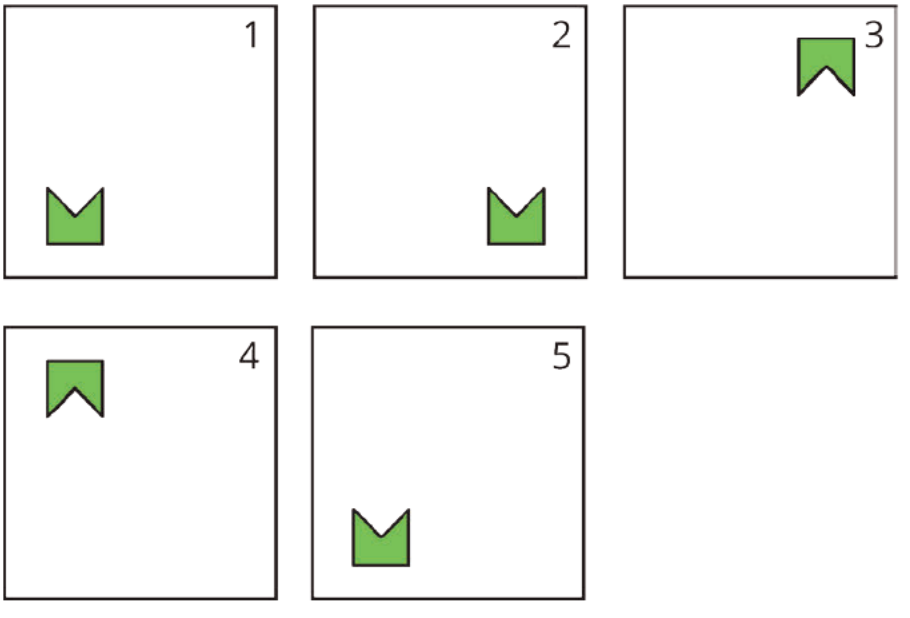
1. Each of the six cards shows a shape.



a. Which pair of cards shows a shape and its image after a rotation?

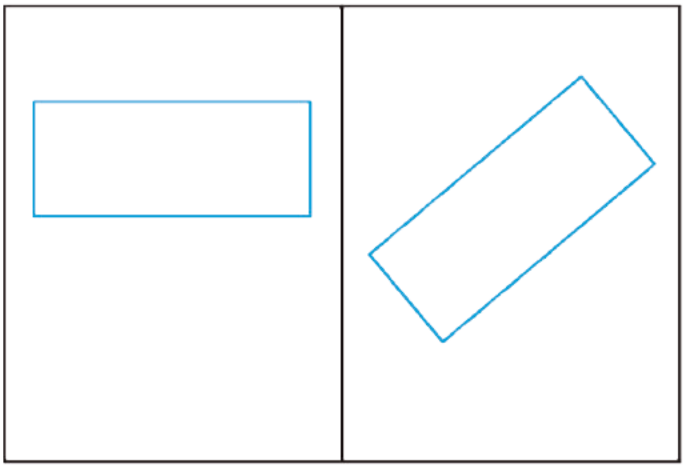
b. Which pair of cards shows a shape and its image after a reflection?

2. The five frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

3. The rectangle seen in Frame 1 is rotated to a new position, seen in Frame 2.



Select **all** the ways the rectangle could have been rotated to get from Frame 1 to Frame 2.

A. 40 degrees clockwise

B. 40 degrees counterclockwise

C. 90 degrees clockwise

D. 90 degrees counterclockwise

E. 140 degrees clockwise

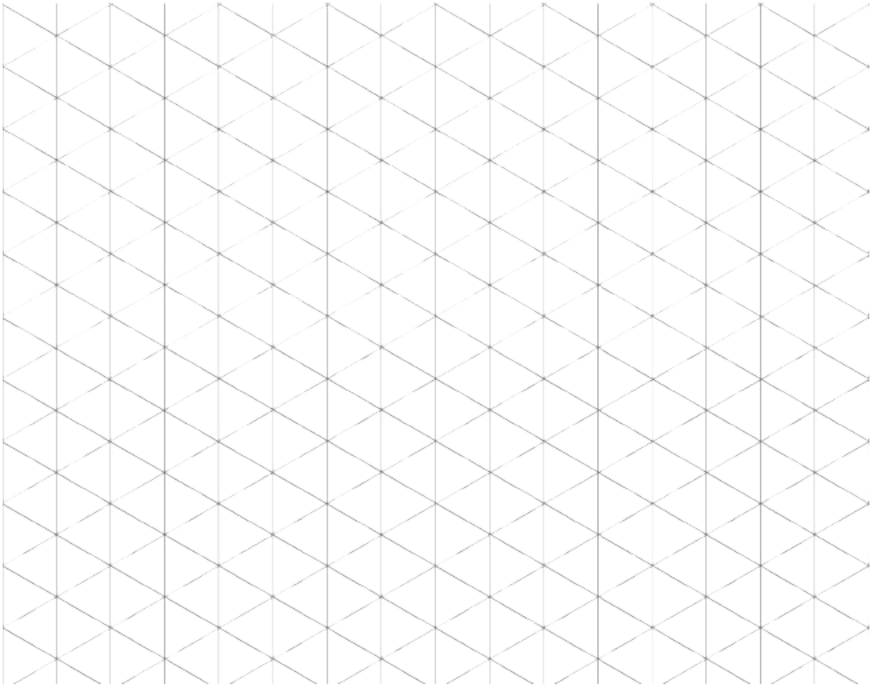
F. 140 degrees counterclockwise

(From Unit 1, Lesson 1.)

Lesson 3: Grid Moves

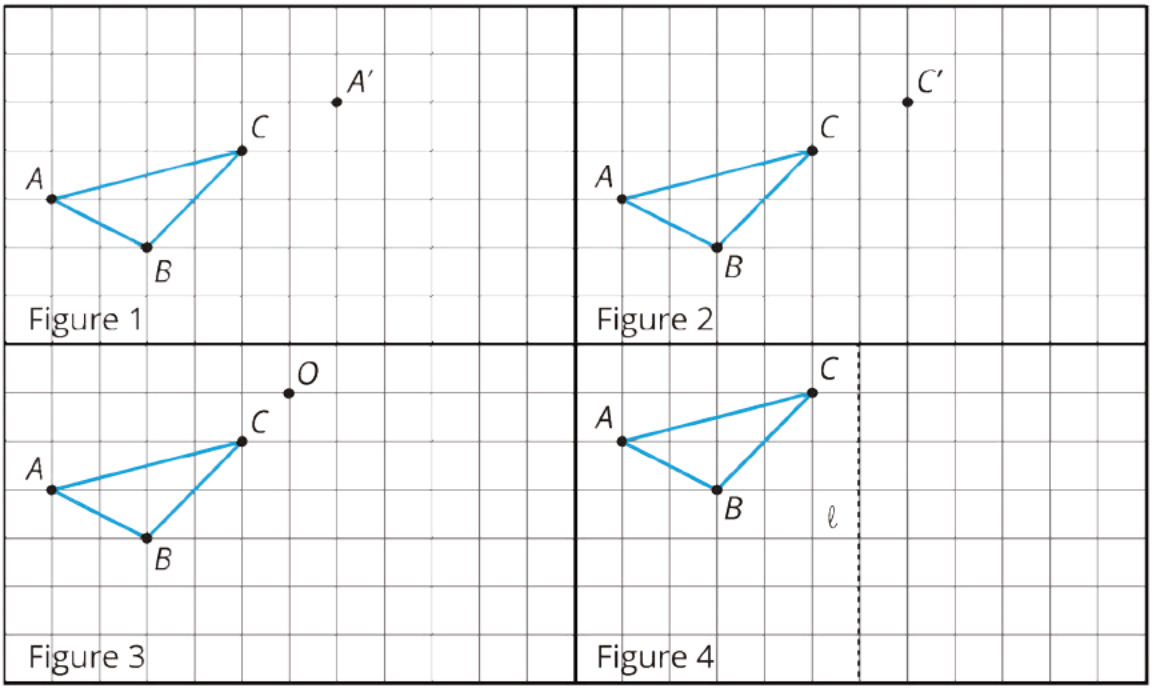
3.1: Notice and Wonder: The Isometric Grid

What do you notice? What do you wonder?



3.2: Transformation Information

Your teacher will give you tracing paper to carry out the moves specified. Use A', B', C', and D' to indicate vertices in the new figure that correspond to the points A, B, C, and D in the original figure.

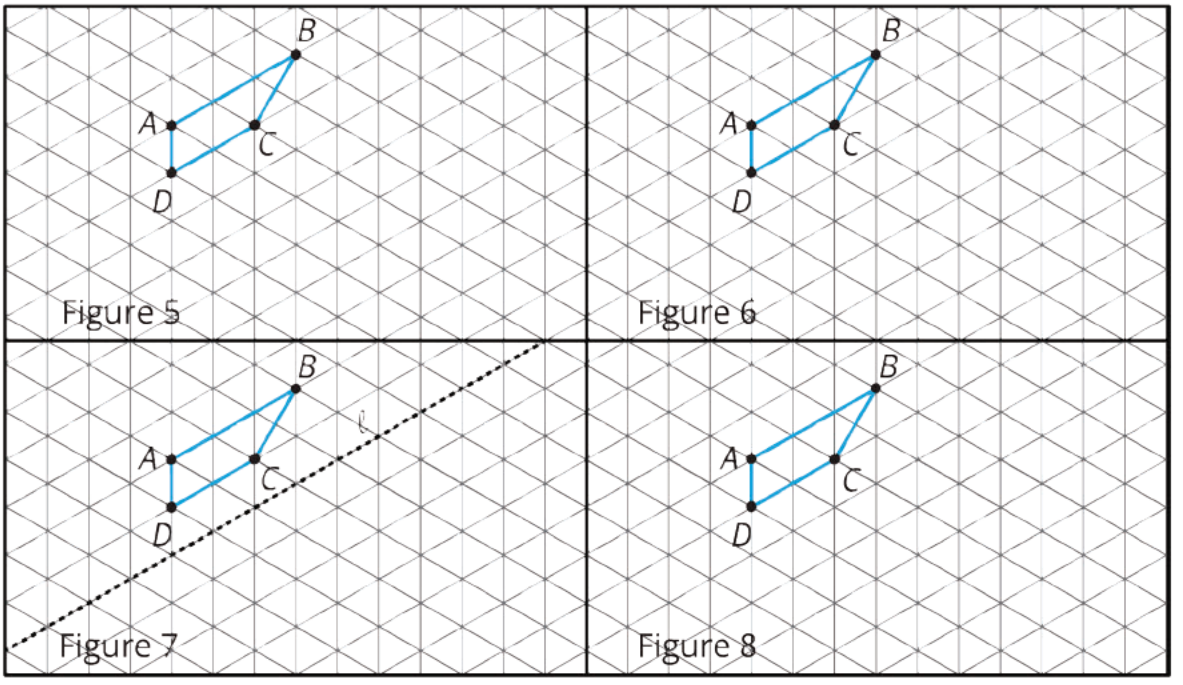


1. In Figure 1, translate triangle ABC so that A goes to A'.

2. In Figure 2, translate triangle ABC so that C goes to C'.

3. In Figure 3, rotate triangle ABC $90°$ counterclockwise using center O.

4. In Figure 4, reflect triangle ABC using line ℓ.



5. In Figure 5, rotate quadrilateral $ABCD$ $60°$ counterclockwise using center B.

6. In Figure 6, rotate quadrilateral $ABCD$ $60°$ clockwise using center C.

7. In Figure 7, reflect quadrilateral $ABCD$ using line ℓ.

8. In Figure 8, translate quadrilateral $ABCD$ so that A goes to C.

Are you ready for more?

The effects of each move can be "undone" by using another move. For example, to undo the effect of translating 3 units to the right, we could translate 3 units to the left. What move undoes each of the following moves?

1. Translate 3 units up

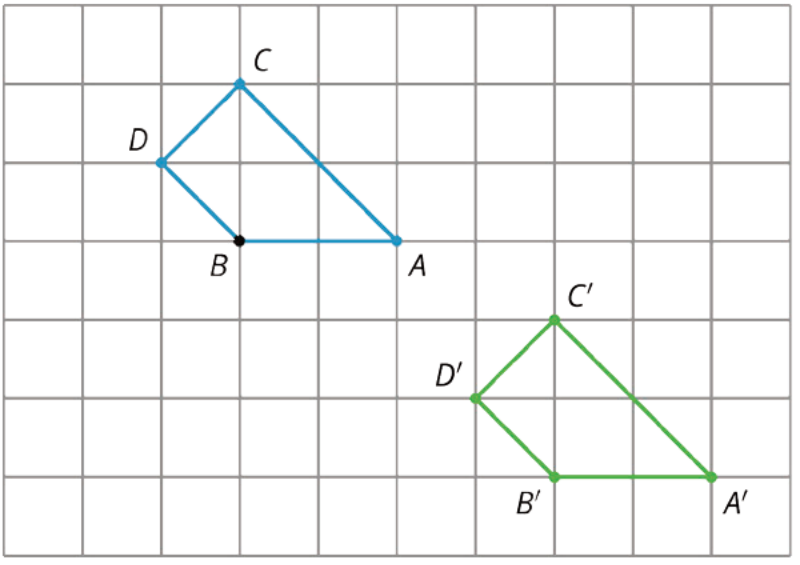
2. Translate 1 unit up and 1 unit to the left

3. Rotate 30 degrees clockwise around a point P

4. Reflect across a line ℓ

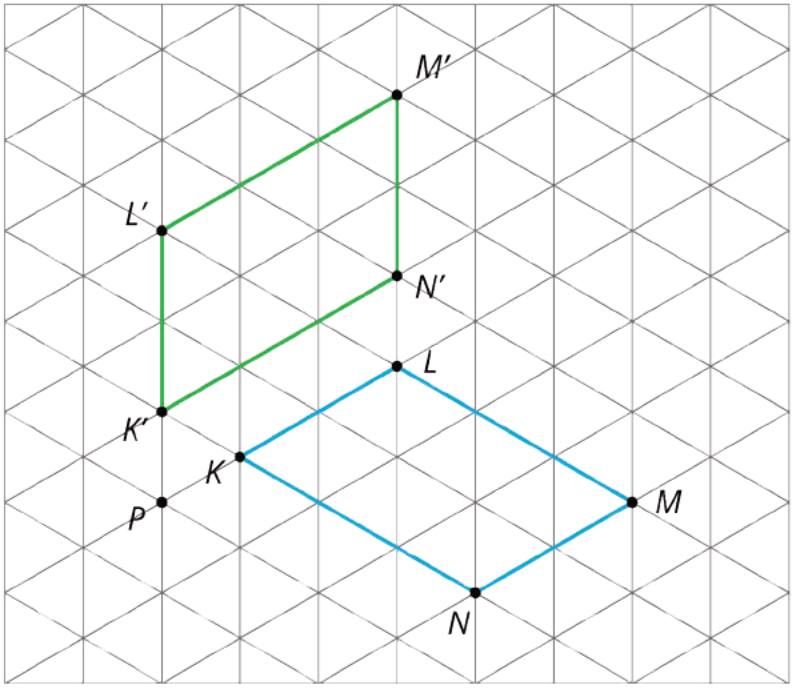
Lesson 3 Summary

When a figure is on a grid, we can use the grid to describe a transformation. For example, here is a figure and an **image** of the figure after a move.



Quadrilateral $ABCD$ is translated 4 units to the right and 3 units down to the position of quadrilateral $A'B'C'D'$.

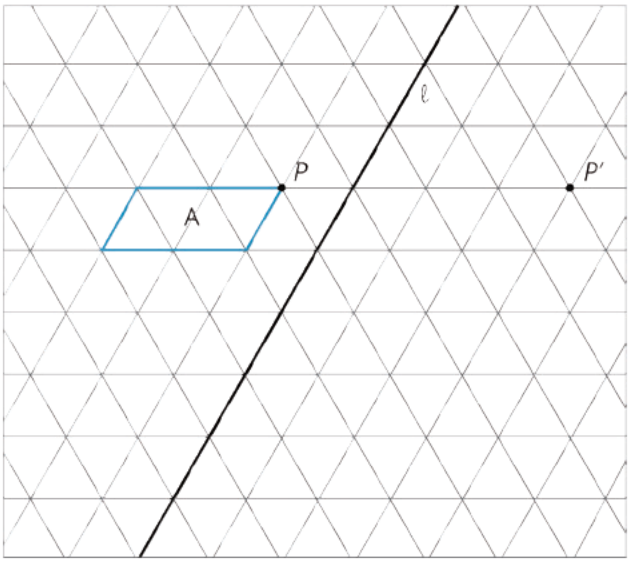
A second type of grid is called an *isometric grid*. The isometric grid is made up of equilateral triangles. The angles in the triangles all measure 60 degrees, making the isometric grid convenient for showing rotations of 60 degrees.



Here is quadrilateral $KLMN$ and its image $K'L'M'N'$ after a 60-degree counterclockwise rotation around a point P.

Unit 1 Lesson 3 Cumulative Practice Problems

1. Apply each transformation described to Figure A. If you get stuck, try using tracing paper.

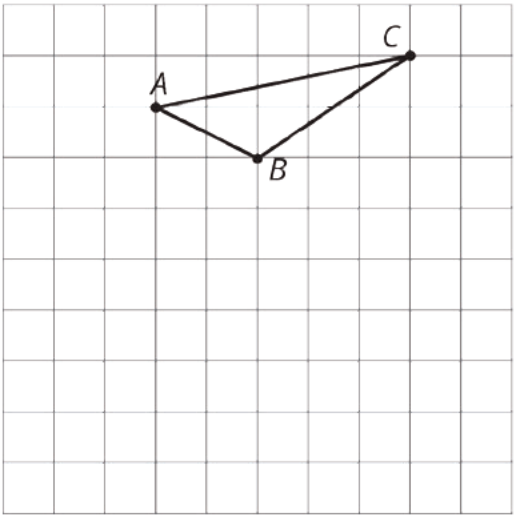


   a. A translation which takes P to P'

   b. A counterclockwise rotation of A, using center P, of 60 degrees

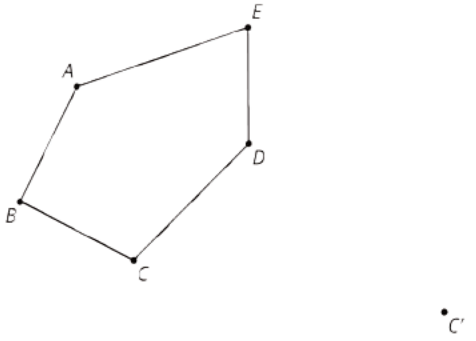
   c. A reflection of A across line ℓ

2. Here is triangle ABC drawn on a grid.

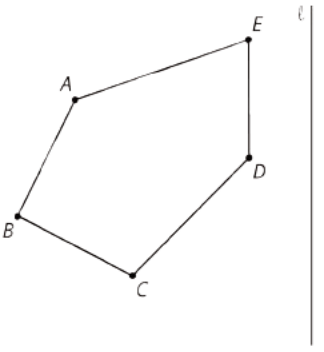


   On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

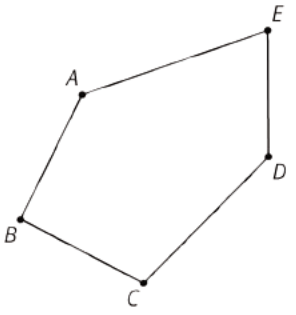
3. a. Draw the translated image of $ABCDE$ so that vertex C moves to C'. Tracing paper may be useful.



b. Draw the reflected image of Pentagon $ABCDE$ with line of reflection ℓ. Tracing paper may be useful.



c. Draw the rotation of Pentagon $ABCDE$ around C clockwise by an angle of 150 degrees. Tracing paper and a protractor may be useful.

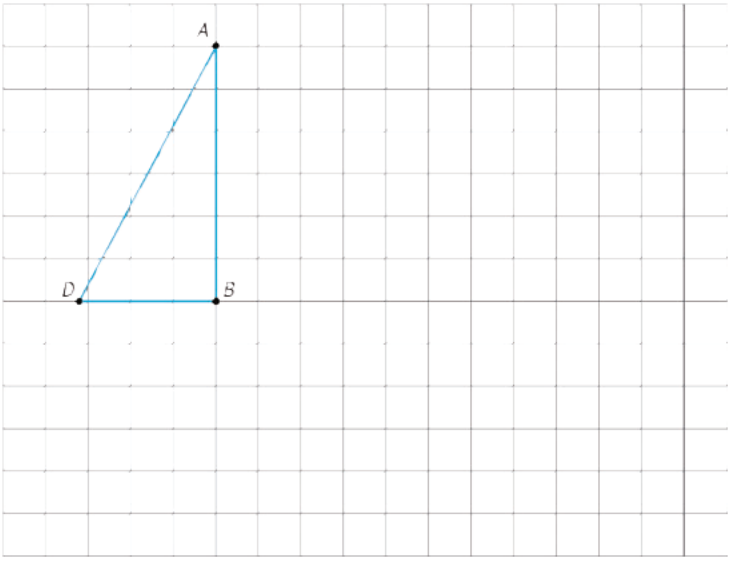


(From Unit 1, Lesson 2.)

Lesson 4: Making the Moves

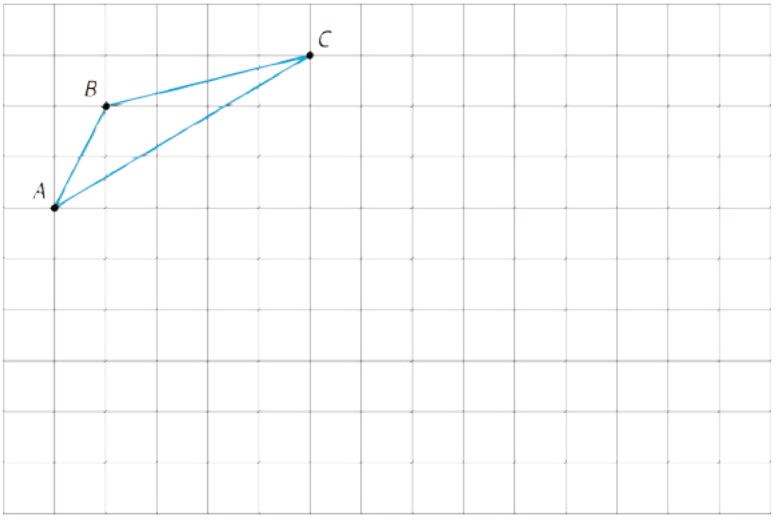
4.1: Reflection Quick Image

Here is an incomplete image. Your teacher will display the completed image twice, for a few seconds each time. Your job is to complete the image on your copy.



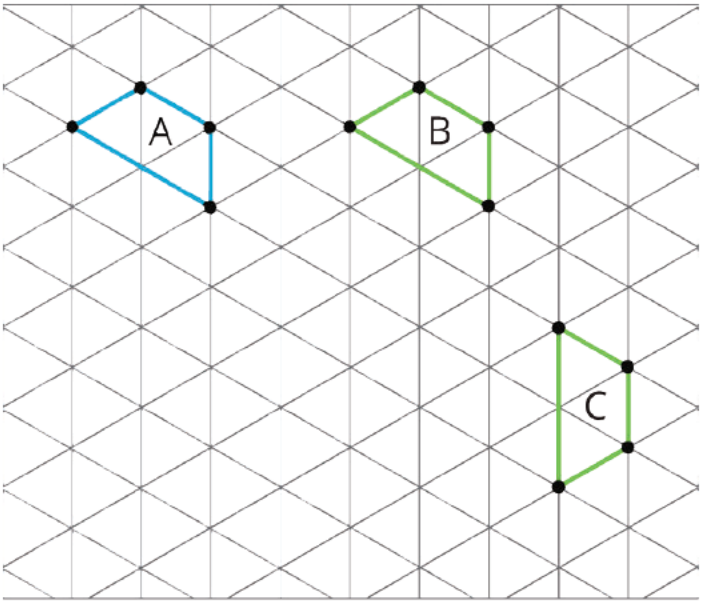
4.2: Make That Move

Your partner will describe the image of this triangle after a certain **transformation**. Sketch it here.



4.3: A to B to C

Here are some figures on an isometric grid.



1. Name a transformation that takes Figure A to Figure B. Name a transformation that takes Figure B to Figure C.

2. What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Are you ready for more?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with. . .

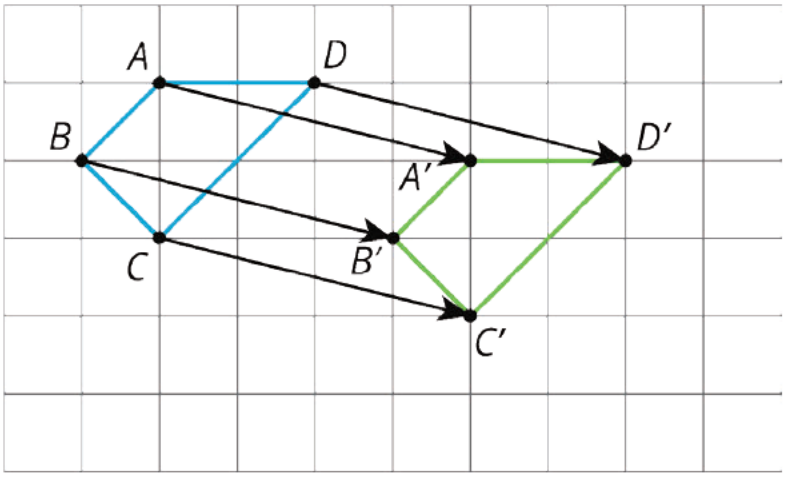
- No rotations?
- No reflections?
- No translations?

Lesson 4 Summary

A move, or combination of moves, is called a **transformation**. When we do one or more moves in a row, we often call that a **sequence of transformations**. To distinguish the original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol $'$ attached, as in A' (pronounced "A prime").

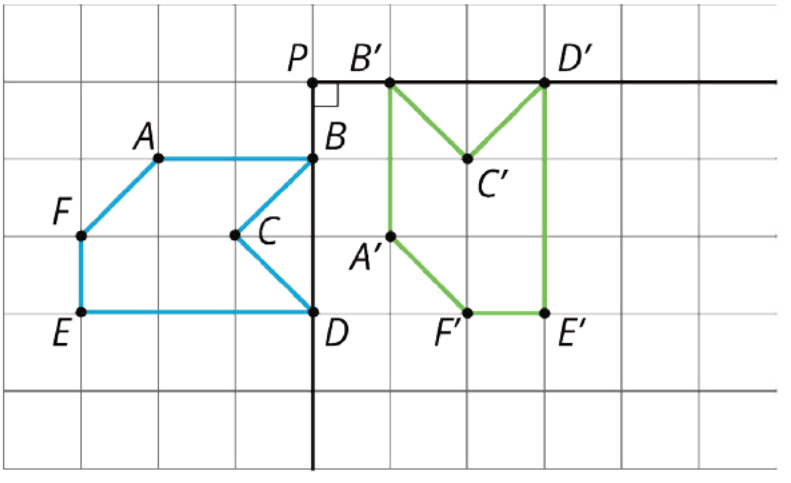
- A translation can be described by two points. If a translation moves point A to point A', it moves the entire figure the same distance and direction as the distance and direction from A to A'. The distance and direction of a translation can be shown by an arrow.

  For example, here is a translation of quadrilateral $ABCD$ that moves A to A'.



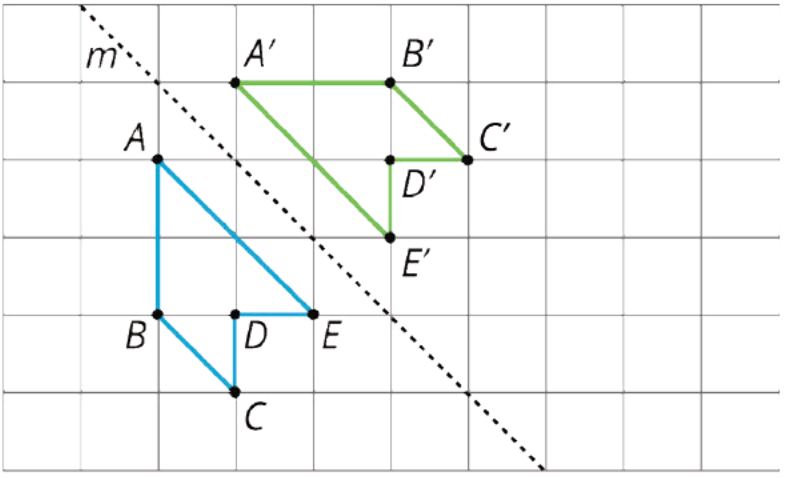
- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.

  For example, hexagon $ABCDEF$ is rotated $90°$ counterclockwise using center P.



- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.

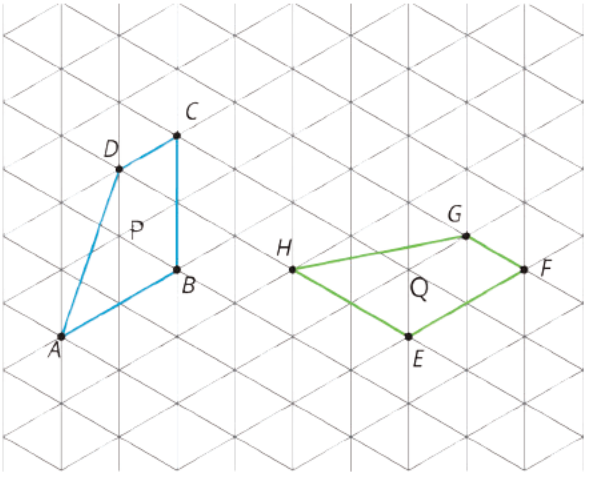
  For example, pentagon $ABCDE$ is reflected across line m.



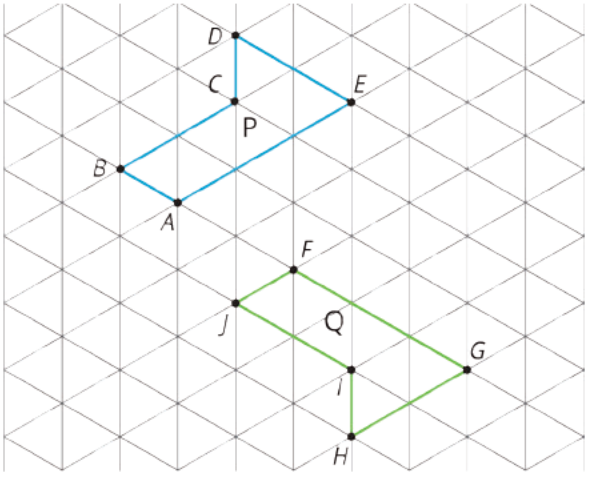
Unit 1 Lesson 4 Cumulative Practice Problems

1. For each pair of polygons, describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.

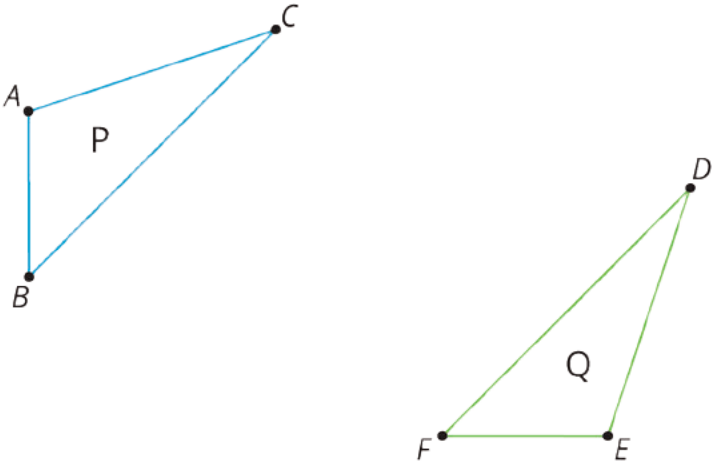
a.



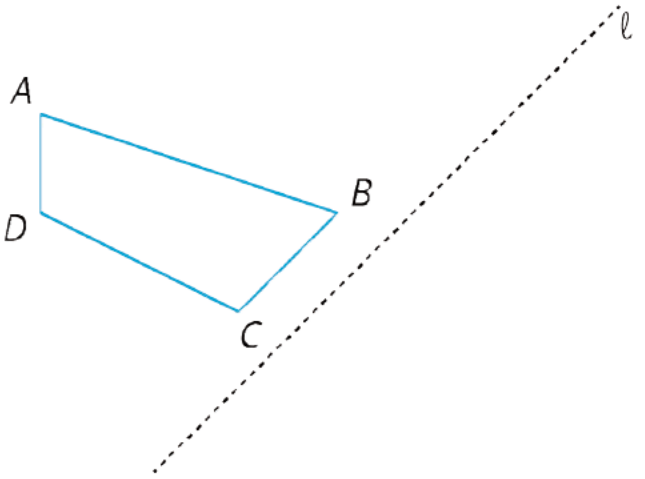
b.



c.



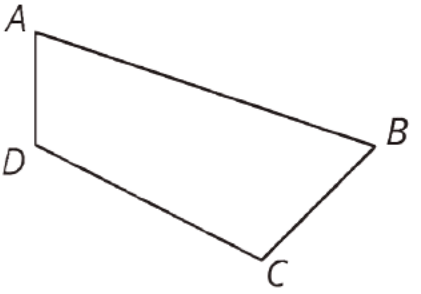
2. Here is quadrilateral $ABCD$ and line ℓ.



Draw the image of quadrilateral $ABCD$ after reflecting it across line ℓ.

(From Unit 1, Lesson 2.)

3. Here is quadrilateral $ABCD$.



Draw the image of quadrilateral $ABCD$ after each rotation using B as center.

a. 90 degrees clockwise

b. 120 degrees clockwise

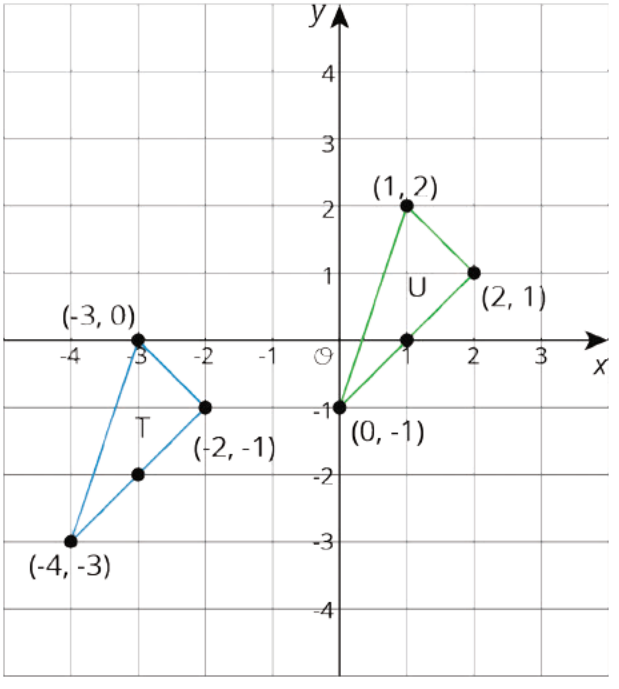
c. 30 degrees counterclockwise

(From Unit 1, Lesson 2.)

Lesson 5: Coordinate Moves

5.1: Translating Coordinates

Select all of the translations that take Triangle T to Triangle U. There may be more than one correct answer.



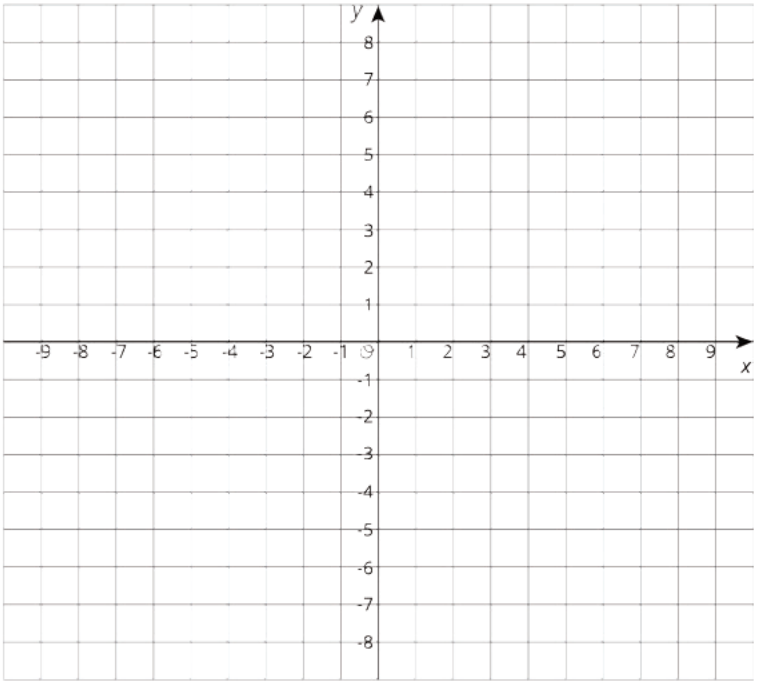
1. Translate $(-3, 0)$ to $(1, 2)$.

2. Translate $(2, 1)$ to $(-2, -1)$.

3. Translate $(-4, -3)$ to $(0, -1)$.

4. Translate $(1, 2)$ to $(2, 1)$.

5.2: Reflecting Points on the Coordinate Plane



1. Here is a list of points

   $A = (0.5, 4)$ $B = (-4, 5)$ $C = (7, -2)$ $D = (6, 0)$ $E = (0, -3)$

   On the **coordinate plane**:

   a. Plot each point and label each with its coordinates.

   b. Using the x-axis as the line of reflection, plot the image of each point.

   c. Label the image of each point with its coordinates.

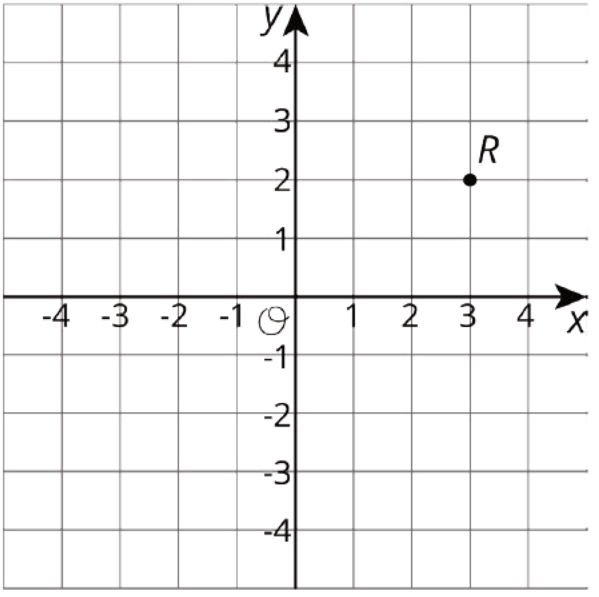
   d. Include a label using a letter. For example, the image of point A should be labeled A'.

2. If the point $(13, 10)$ were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? What about $(13, -20)$? $(13, 570)$? Explain how you know.

3. The point R has coordinates $(3, 2)$.

   a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

   b. Check your answer by finding the image of R on the graph.

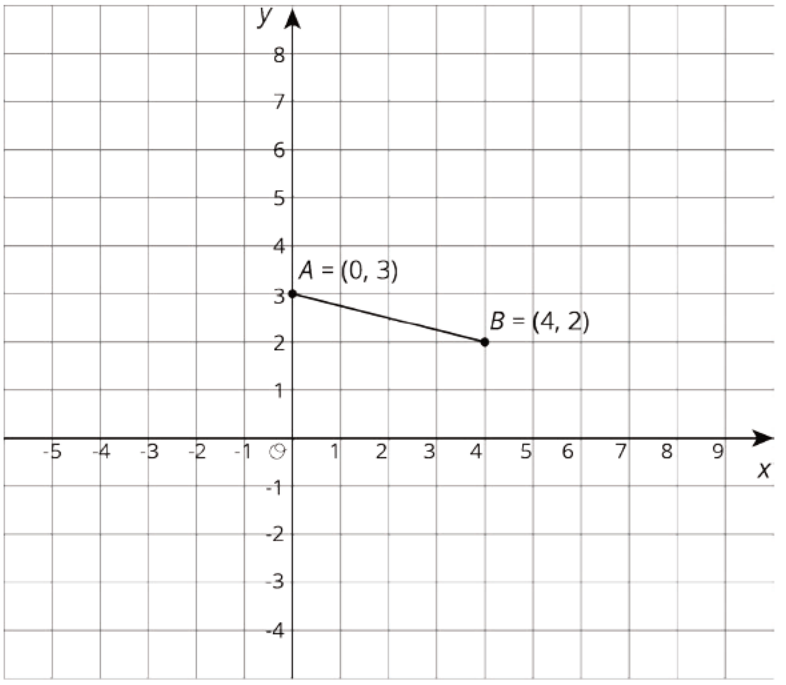


   c. Label the image of point R as R'.

   d. What are the coordinates of R'?

4. Suppose you reflect a point using the y-axis as line of reflection. How would you describe its image?

5.3: Transformations of a Segment



Apply each of the following transformations to segment AB.

1. Rotate segment AB 90 degrees counterclockwise around center B. Label the image of A as C. What are the coordinates of C?

2. Rotate segment AB 90 degrees counterclockwise around center A. Label the image of B as D. What are the coordinates of D?

3. Rotate segment AB 90 degrees clockwise around $(0, 0)$. Label the image of A as E and the image of B as F. What are the coordinates of E and F?

4. Compare the two 90-degree counterclockwise rotations of segment AB. What is the same about the images of these rotations? What is different?

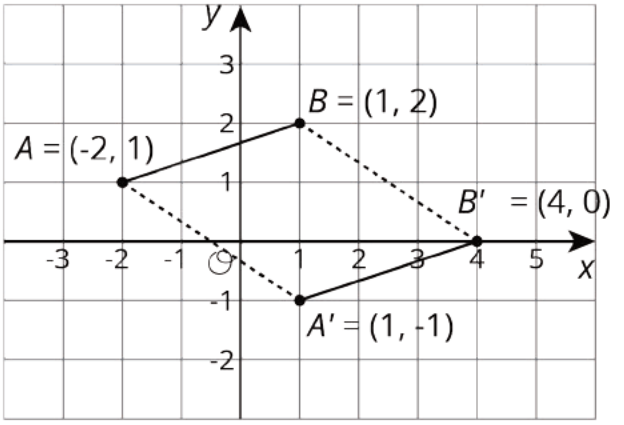
Are you ready for more?

Suppose EF and GH are line segments of the same length. Describe a sequence of transformations that moves EF to GH.

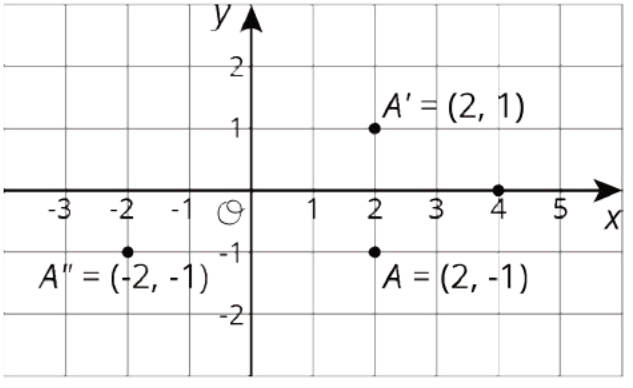
Lesson 5 Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.

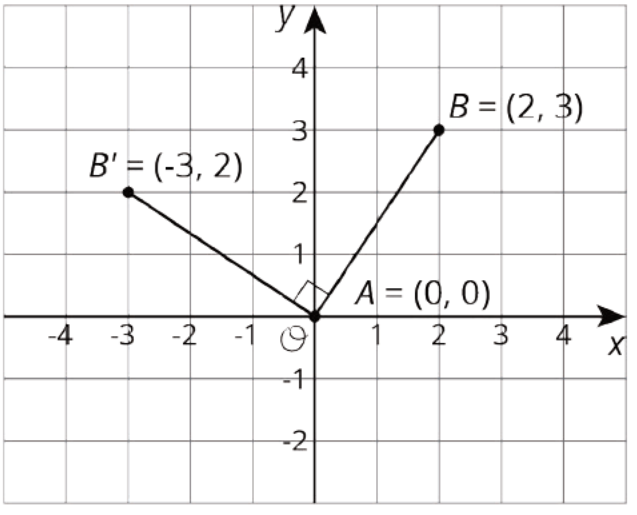


Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are $(2, -1)$ across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are $(2, 1)$. Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are $(-2, -1)$.



Reflections across other lines are more complex to describe.

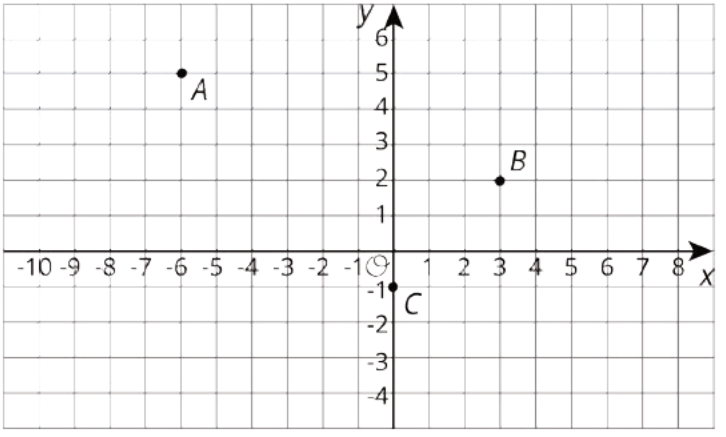
We don't have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a $90°$ rotation with center $(0, 0)$ in a counterclockwise direction.



Point A has coordinates $(0, 0)$. Segment AB was rotated $90°$ counterclockwise around A. Point B with coordinates $(2, 3)$ rotates to point B' whose coordinates are $(-3, 2)$.

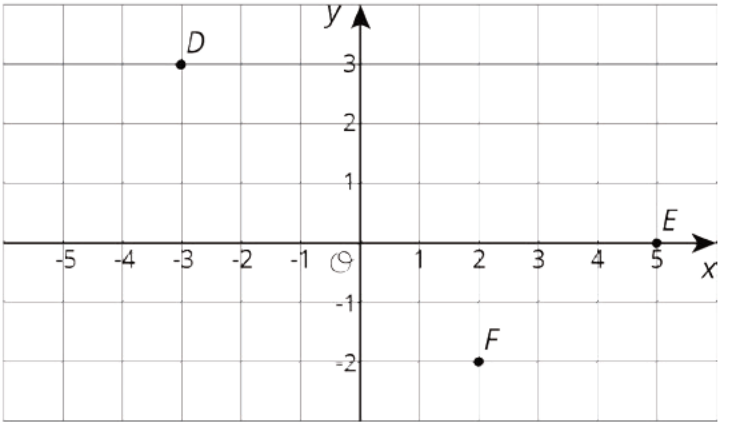
Unit 1 Lesson 5 Cumulative Practice Problems

1. a. Here are some points.



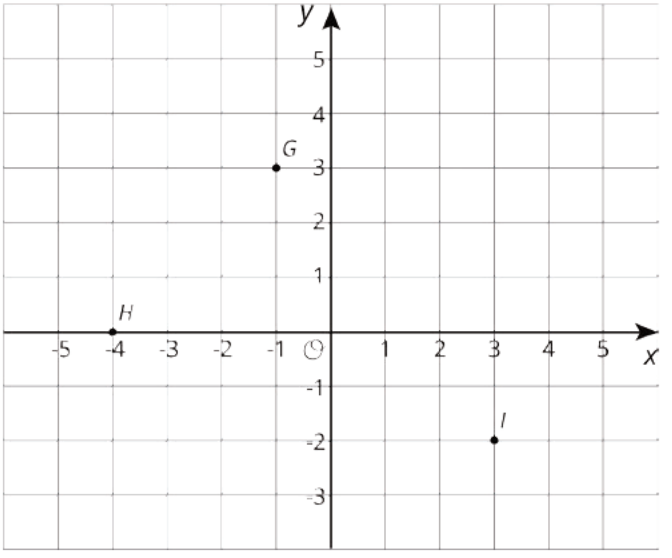
What are the coordinates of A, B, and C after a translation to the right by 4 units and up 1 unit? Plot these points on the grid, and label them A', B' and C'.

b. Here are some points.



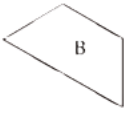
What are the coordinates of D, E, and F after a reflection over the y axis? Plot these points on the grid, and label them D', E' and F'.

c. Here are some points.



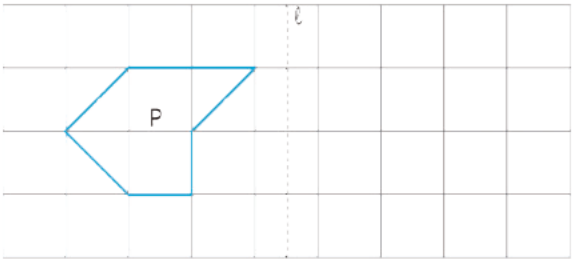
What are the coordinates of G, H, and I after a rotation about $(0, 0)$ by 90 degrees clockwise? Plot these points on the grid, and label them G', H' and I'.

2. Describe a sequence of transformations that takes trapezoid A to trapezoid B.



(From Unit 1, Lesson 4.)

3. Reflect polygon P using line ℓ.

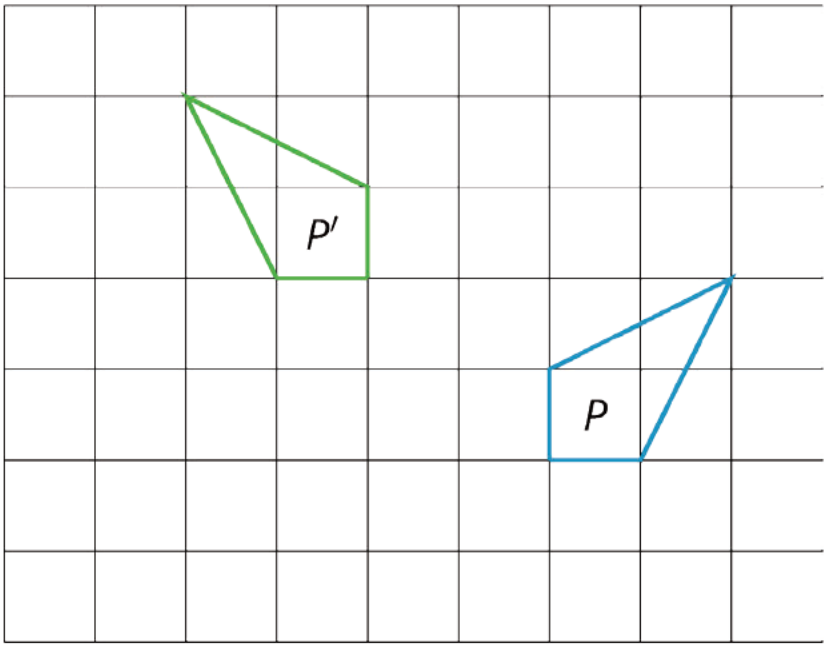


(From Unit 1, Lesson 3.)

Lesson 6: Describing Transformations

6.1: Finding a Center of Rotation

Andre performs a 90-degree counterclockwise rotation of Polygon P and gets Polygon P', but he does not say what the center of the rotation is. Can you find the center?



6.2: Info Gap: Transformation Information

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Are you ready for more?

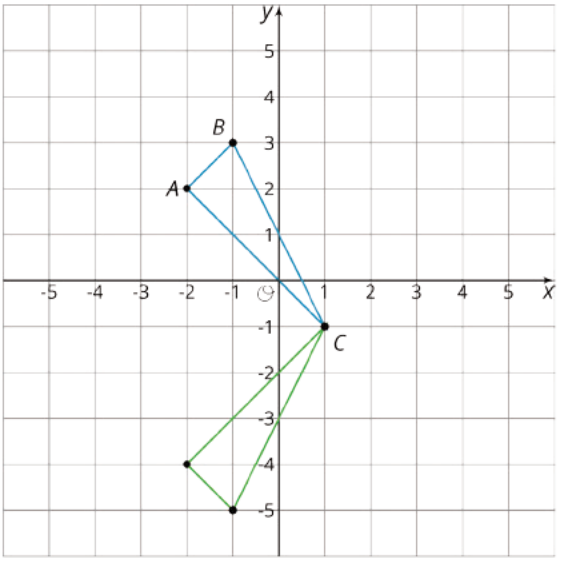
Sometimes two transformations, one performed after the other, have a nice description as a single transformation. For example, instead of translating 2 units up followed by translating 3 units up, we could simply translate 5 units up. Instead of rotating 20 degrees counterclockwise around the origin followed by rotating 80 degrees clockwise around the origin, we could simply rotate 60 degrees clockwise around the origin.

Can you find a simple description of reflecting across the x-axis followed by reflecting across the y-axis?

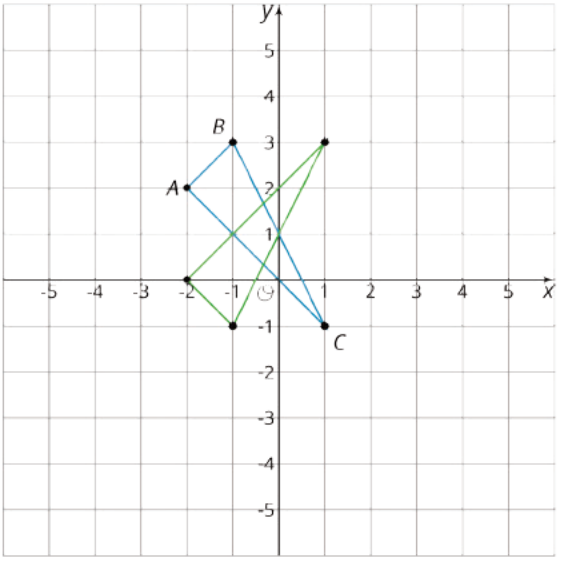
Lesson 6 Summary

The center of a rotation for a figure doesn't have to be one of the points on the figure. To find a center of rotation, look for a point that is the same distance from two corresponding points. You will probably have to do this for a couple of different pairs of corresponding points to nail it down.

When we perform a sequence of transformations, the order of the transformations can be important. Here is triangle ABC translated up two units and then reflected over the x-axis.



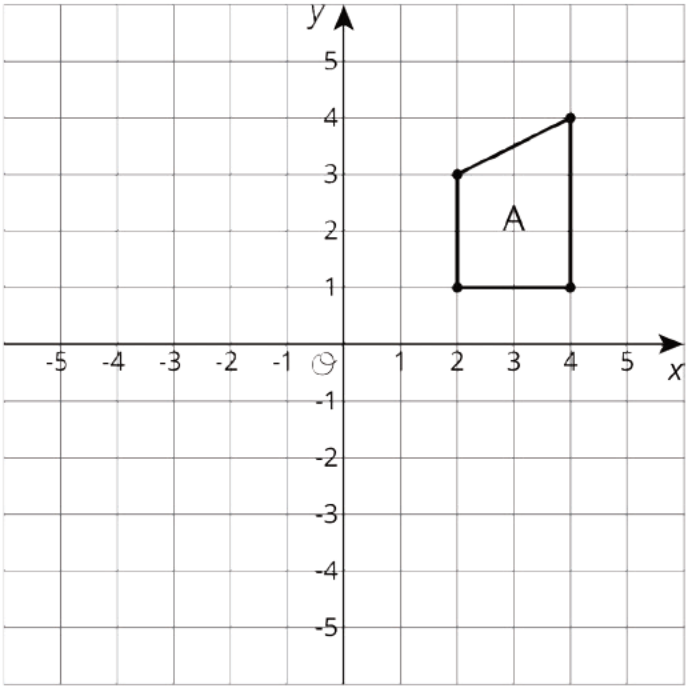
Here is triangle ABC reflected over the x-axis and then translated up two units.



Triangle ABC ends up in different places when the transformations are applied in the opposite order!

Unit 1 Lesson 6 Cumulative Practice Problems

1. Here is Trapezoid A in the coordinate plane:



   a. Draw Polygon B, the image of A, using the y-axis as the line of reflection.

   b. Draw Polygon C, the image of B, using the x-axis as the line of reflection.

   c. Draw Polygon D, the image of C, using the x-axis as the line of reflection.

2. The point $(-4, 1)$ is rotated 180 degrees counterclockwise using center $(-3, 0)$. What are the coordinates of the image?

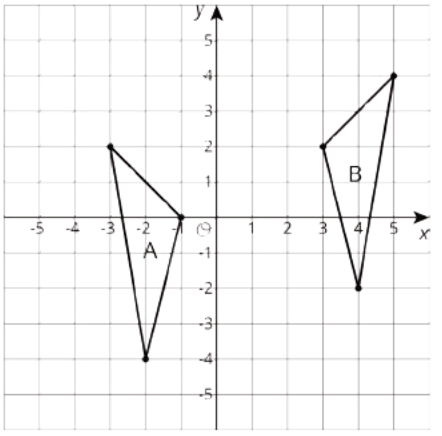
   A. $(-5, -2)$

   B. $(-4, -1)$

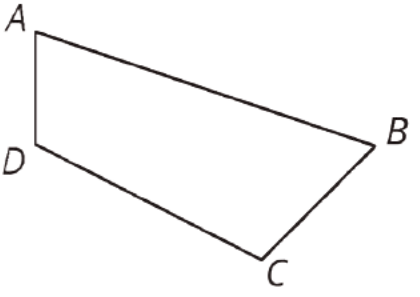
   C. $(-2, -1)$

   D. $(4, -1)$

3. Describe a sequence of transformations for which Triangle B is the image of Triangle A.



4. Here is quadrilateral $ABCD$.



Draw the image of quadrilateral $ABCD$ after each transformation.

a. The translation that takes B to D.

b. The reflection over segment BC.

c. The rotation about point A by angle DAB, counterclockwise.

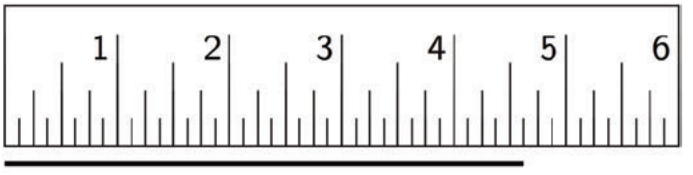
(From Unit 1, Lesson 2.)

Lesson 7: No Bending or Stretching

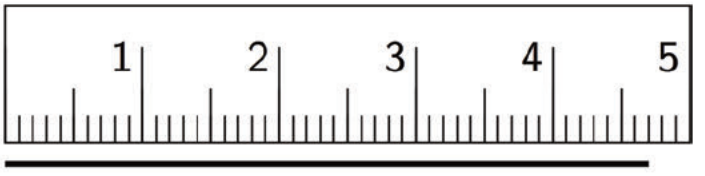
7.1: Measuring Segments

For each question, the unit is represented by the large tick marks with whole numbers.

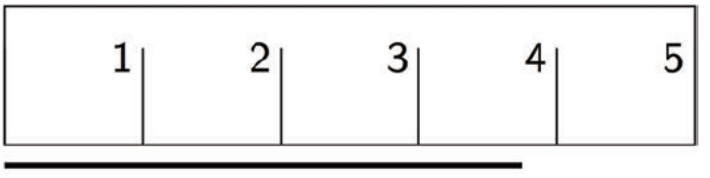
1. Find the length of this segment to the nearest $\frac{1}{8}$ of a unit.



2. Find the length of this segment to the nearest 0.1 of a unit.



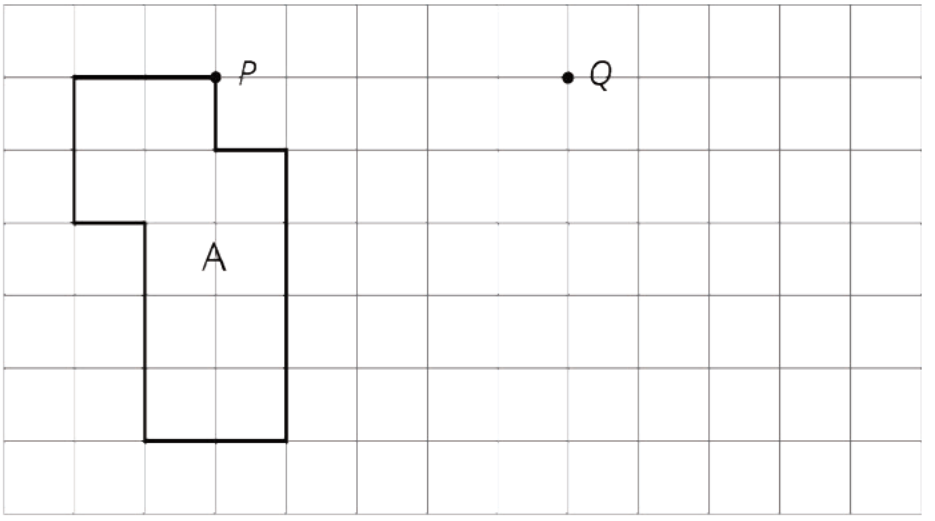
3. Estimate the length of this segment to the nearest $\frac{1}{8}$ of a unit.



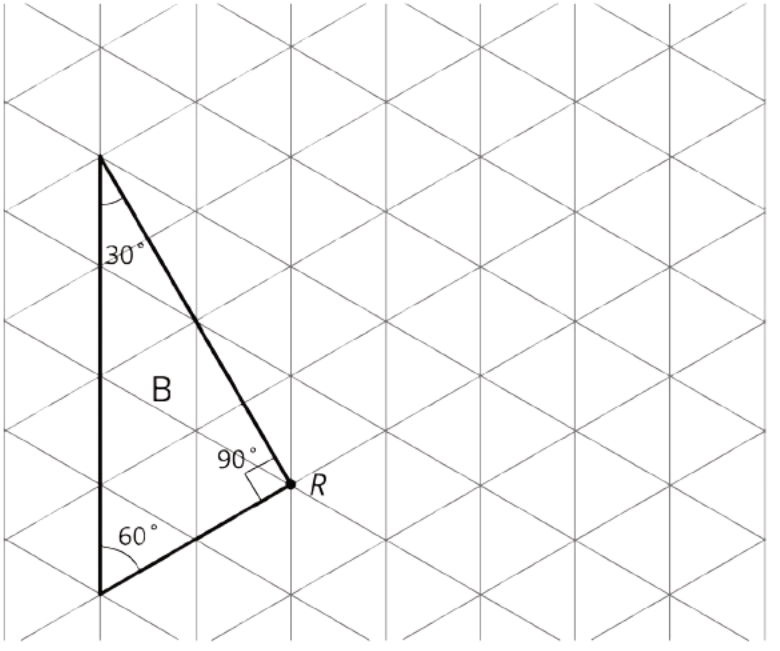
4. Estimate the length of the segment in the prior question to the nearest 0.1 of a unit.

7.2: Sides and Angles

1. Translate Polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.

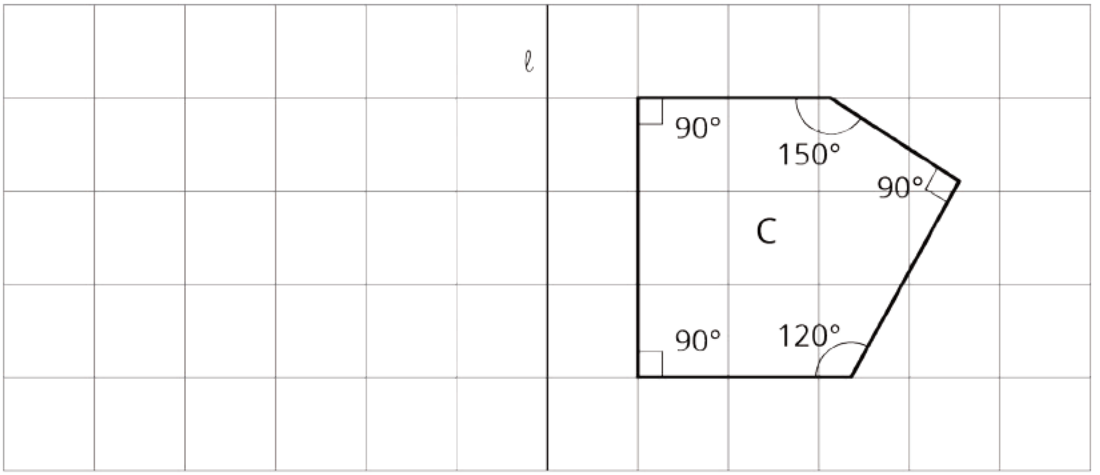


2. Rotate Triangle B 90 degrees clockwise using R as the center of rotation. In the image, write the measure of each angle in its interior.



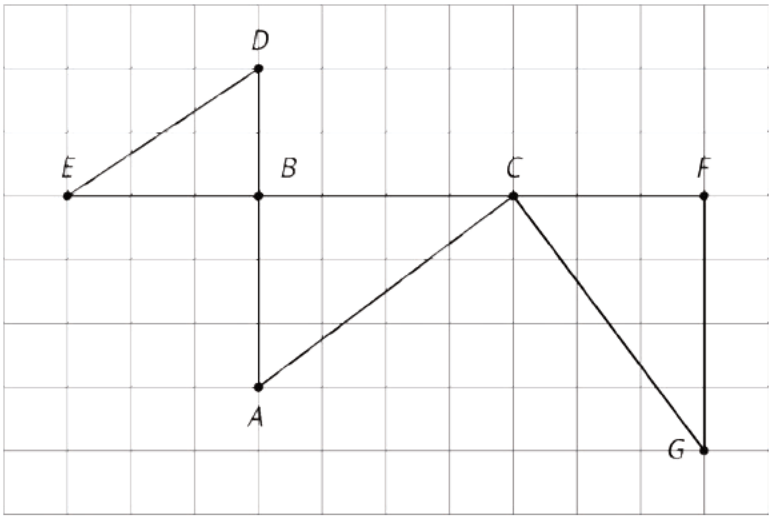
3. Reflect Pentagon C across line ℓ.
   a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

   b. In the image, write the measure of each angle in the interior.



7.3: Which One?

Here is a grid showing triangle ABC and two other triangles.



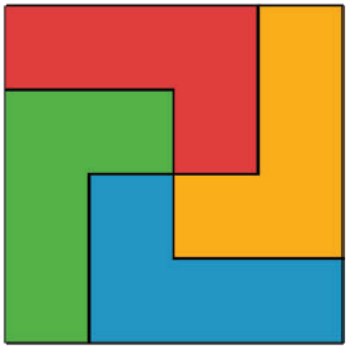
You can use a **rigid transformation** to take triangle ABC to *one* of the other triangles.

1. Which one? Explain how you know.

2. Describe a rigid transformation that takes ABC to the triangle you selected.

Are you ready for more?

A square is made up of an L-shaped region and three transformations of the region. If the perimeter of the square is 40 units, what is the perimeter of each L-shaped region?

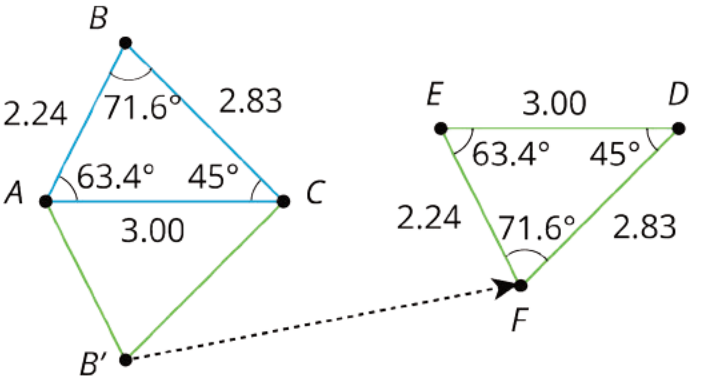


Lesson 7 Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

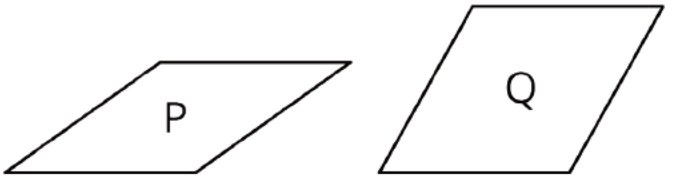
For example, triangle EFD was made by reflecting triangle ABC across a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same measures.



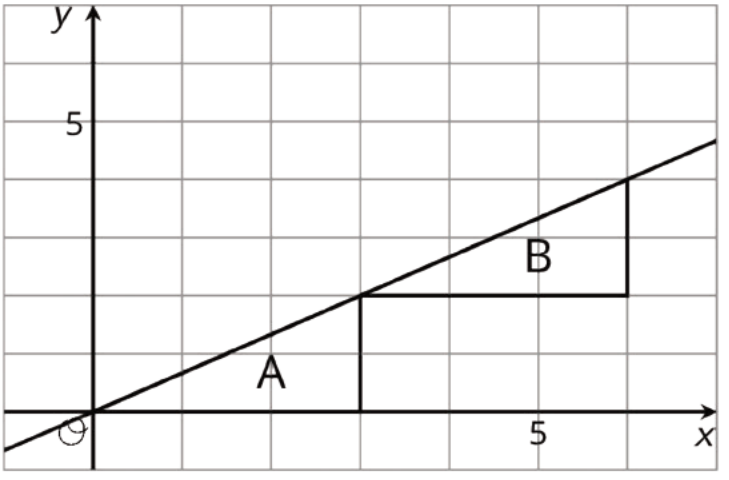
measurements in triangle ABC	corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
$m\angle ABC = 71.6^\circ$	$m\angle EFD = 71.6^\circ$
$m\angle BCA = 45.0^\circ$	$m\angle FDE = 45.0^\circ$
$m\angle CAB = 63.4^\circ$	$m\angle DEF = 63.4^\circ$

Unit 1 Lesson 7 Cumulative Practice Problems

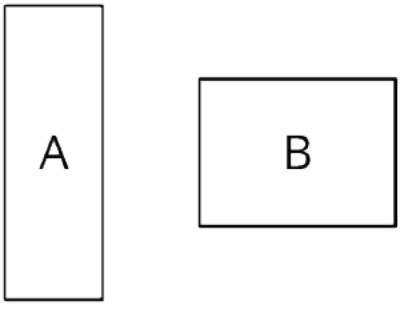
1. Is there a rigid transformation taking Rhombus P to Rhombus Q? Explain how you know.



2. Describe a rigid transformation that takes Triangle A to Triangle B.

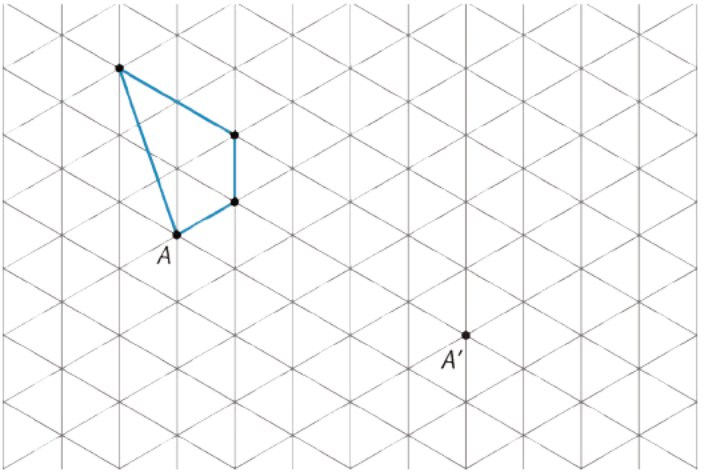


3. Is there a rigid transformation taking Rectangle A to Rectangle B? Explain how you know.

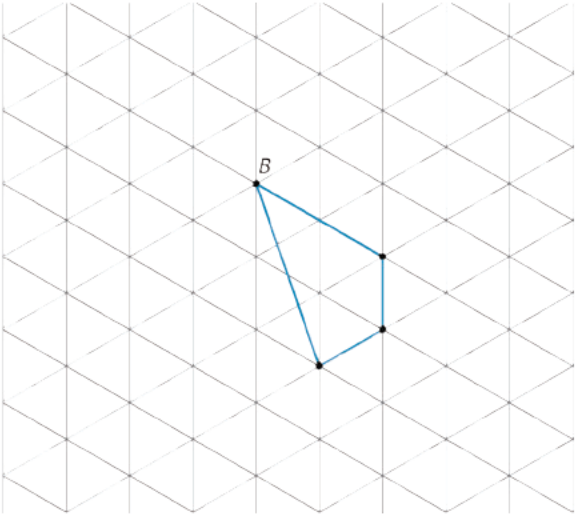


4. For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

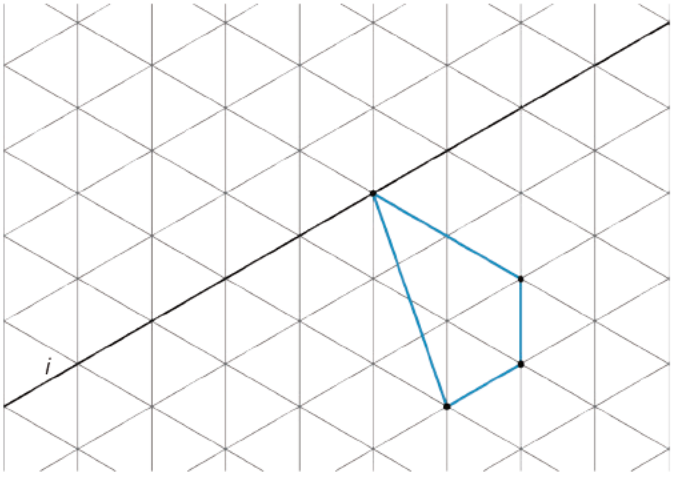
a. Translate the shape so that A goes to A'.



a. Rotate the shape 180 degrees counterclockwise around B.



a. Reflect the shape over the line shown.

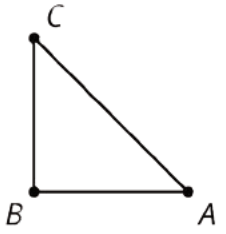


(From Unit 1, Lesson 4.)

Lesson 8: Rotation Patterns

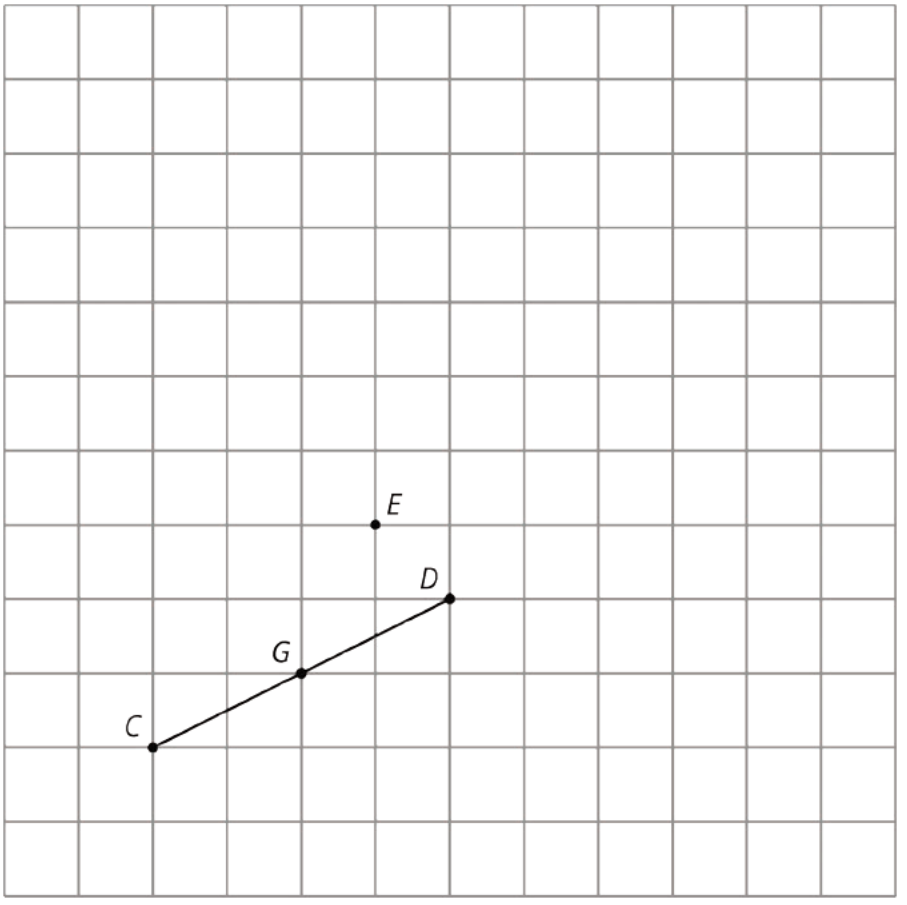
8.1: Building a Quadrilateral

Here is a right isosceles triangle:



1. Rotate triangle ABC 90 degrees clockwise around B.
2. Rotate triangle ABC 180 degrees clockwise round B.
3. Rotate triangle ABC 270 degrees clockwise around B.
4. What would it look like when you rotate the four triangles 90 degrees clockwise around B? 180 degrees? 270 degrees clockwise?

8.2: Rotating a Segment



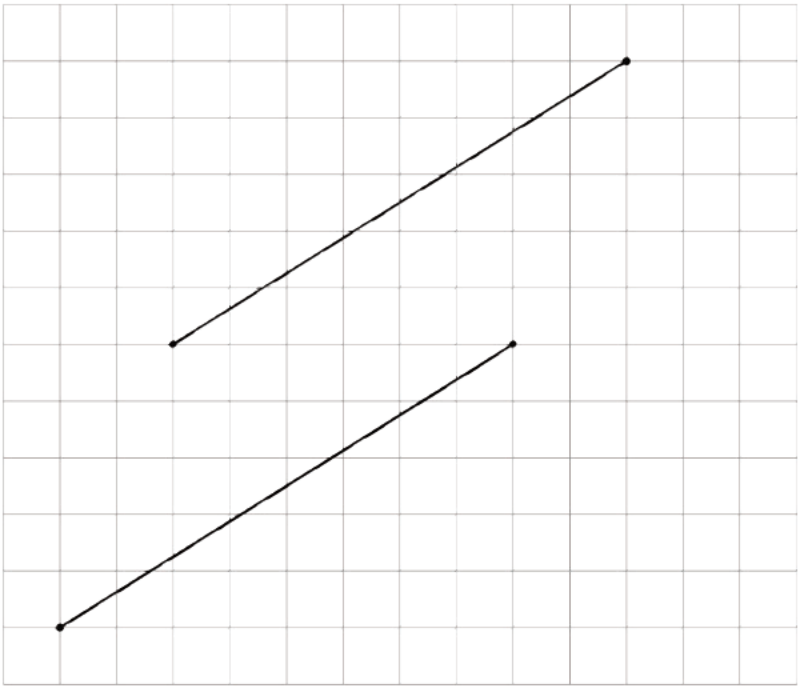
1. Rotate segment CD 180 degrees around point D. Draw its image and label the image of C as A.

2. Rotate segment CD 180 degrees around point E. Draw its image and label the image of C as B and the image of D as F.

3. Rotate segment CD 180 degrees around its midpoint, G. What is the image of C?

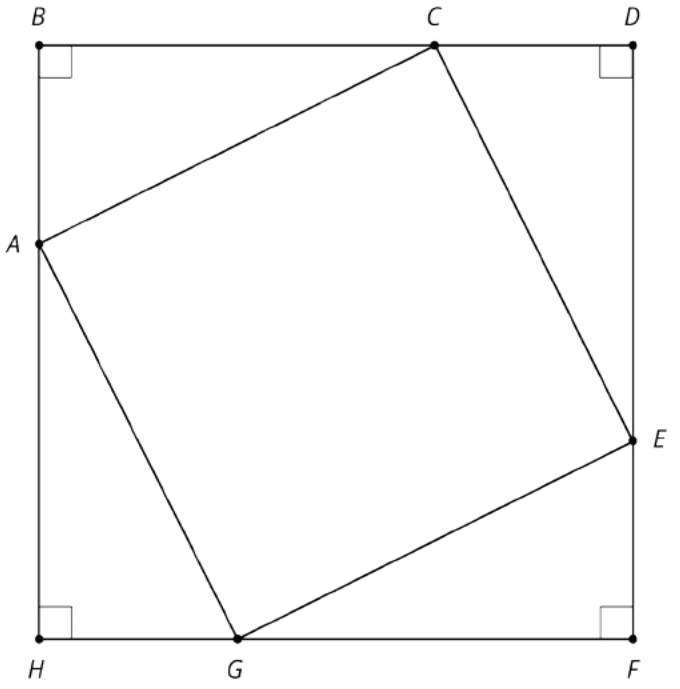
4. What happens when you rotate a segment 180 degrees around a point?

Are you ready for more?



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the center of such a rotation. If not, explain why not.

8.3: A Pattern of Four Triangles



You can use rigid transformations of a figure to make patterns. Here is a diagram built with three different transformations of triangle ABC.

1. Describe a rigid transformation that takes triangle ABC to triangle CDE.

2. Describe a rigid transformation that takes triangle ABC to triangle EFG.

3. Describe a rigid transformation that takes triangle ABC to triangle GHA.

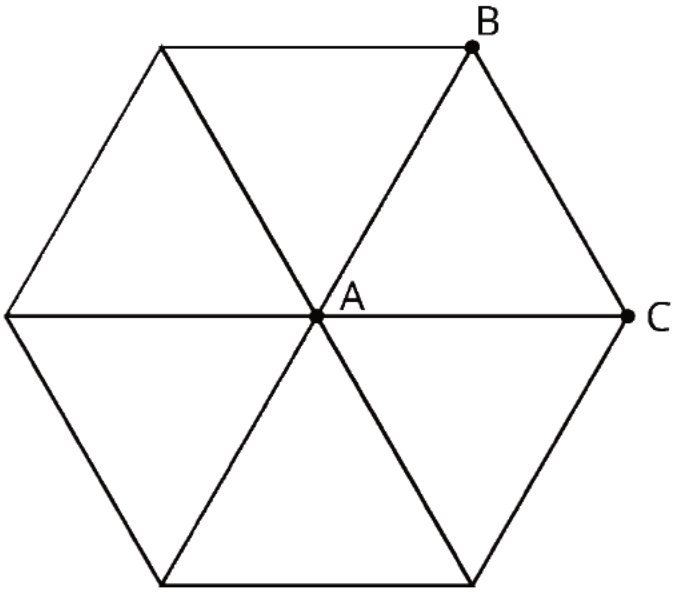
4. Do segments AC, CE, EG, and GA all have the same length? Explain your reasoning.

Lesson 8 Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

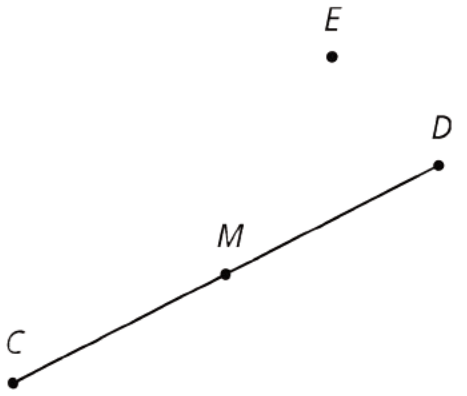
- The segment maps to itself (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment (if the center of rotation is *not* on the segment).

We can also build patterns by rotating a shape. For example, triangle ABC shown here has $m(\angle A) = 60$. If we rotate triangle ABC 60 degrees, 120 degrees, 180 degrees, 240 degrees, and 300 degrees clockwise, we can build a hexagon.



Unit 1 Lesson 8 Cumulative Practice Problems

1. For the figure shown here,

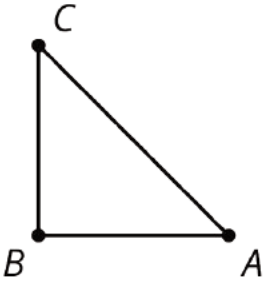


   a. Rotate segment CD 180° around point D.

   b. Rotate segment CD 180° around point E.

   c. Rotate segment CD 180° around point M.

2. Here is an isosceles right triangle:



   Draw these three rotations of triangle ABC together.

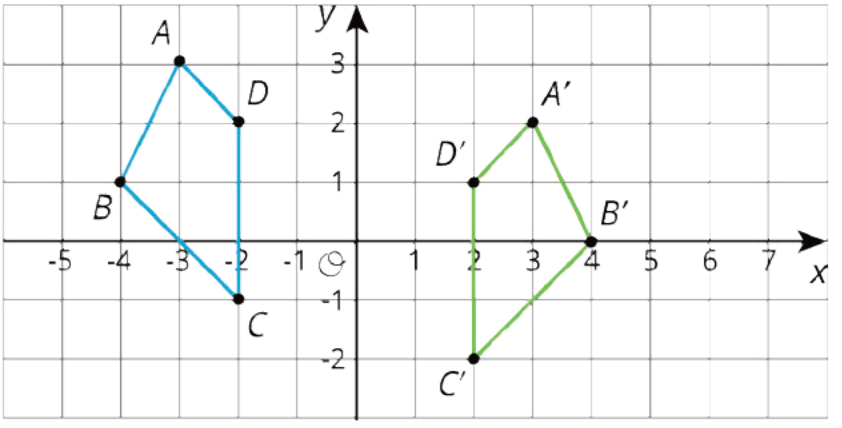
   a. Rotate triangle ABC 90 degrees clockwise around A.

   b. Rotate triangle ABC 180 degrees around A.

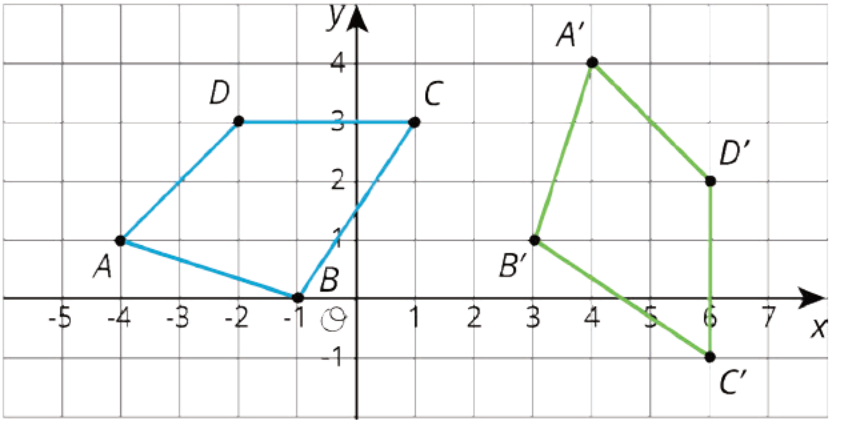
   c. Rotate triangle ABC 270 degrees clockwise around A.

3. Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.

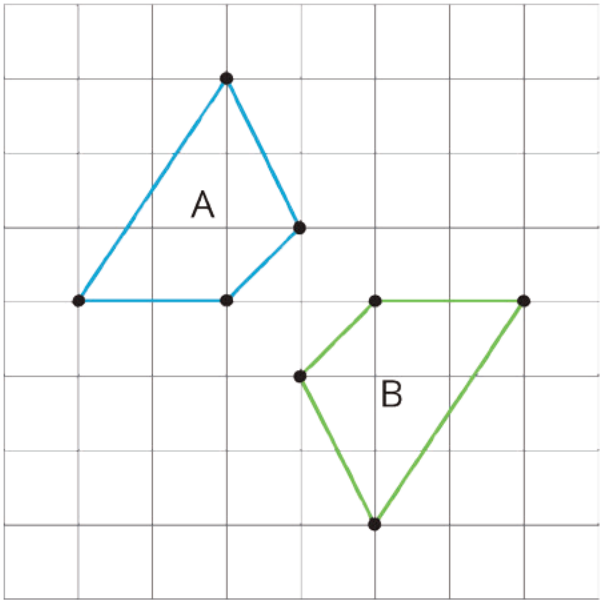


b.



(From Unit 1, Lesson 5.)

4. Lin says that she can map Polygon A to Polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.



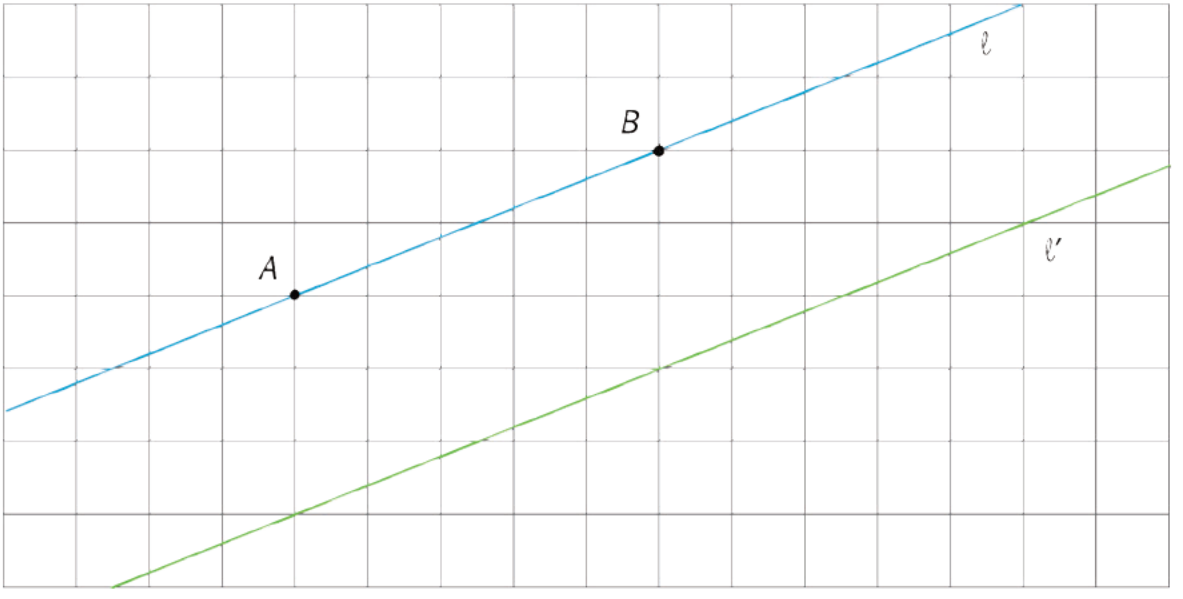
(From Unit 1, Lesson 4.)

Lesson 9: Moves in Parallel

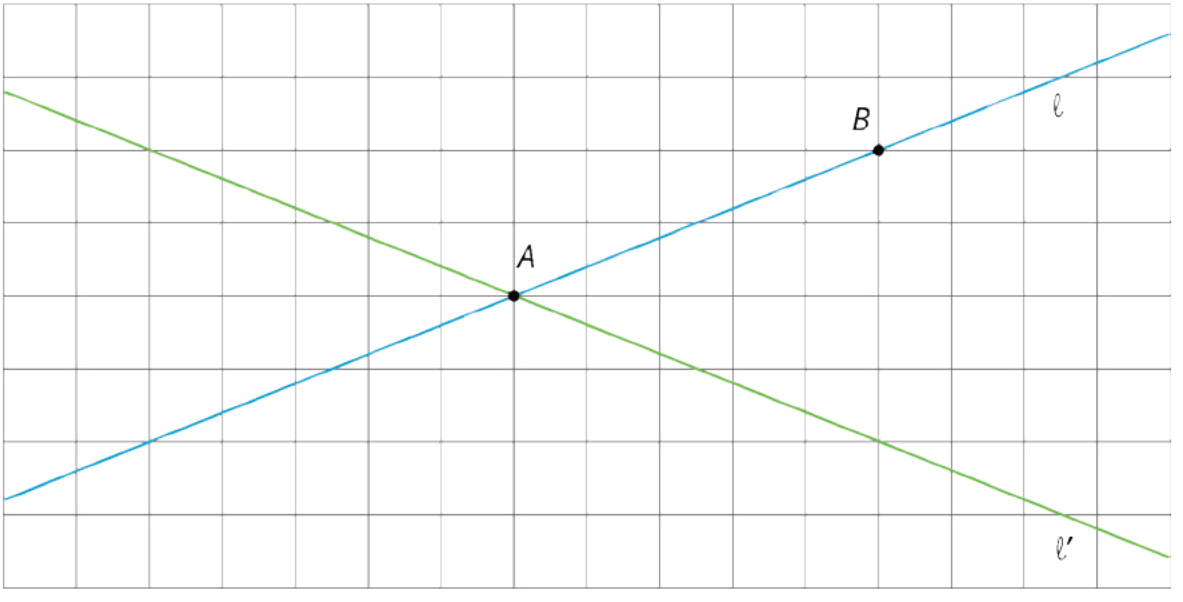
9.1: Line Moves

For each diagram, describe a translation, rotation, or reflection that takes line ℓ to line ℓ'. Then plot and label A' and B', the images of A and B.

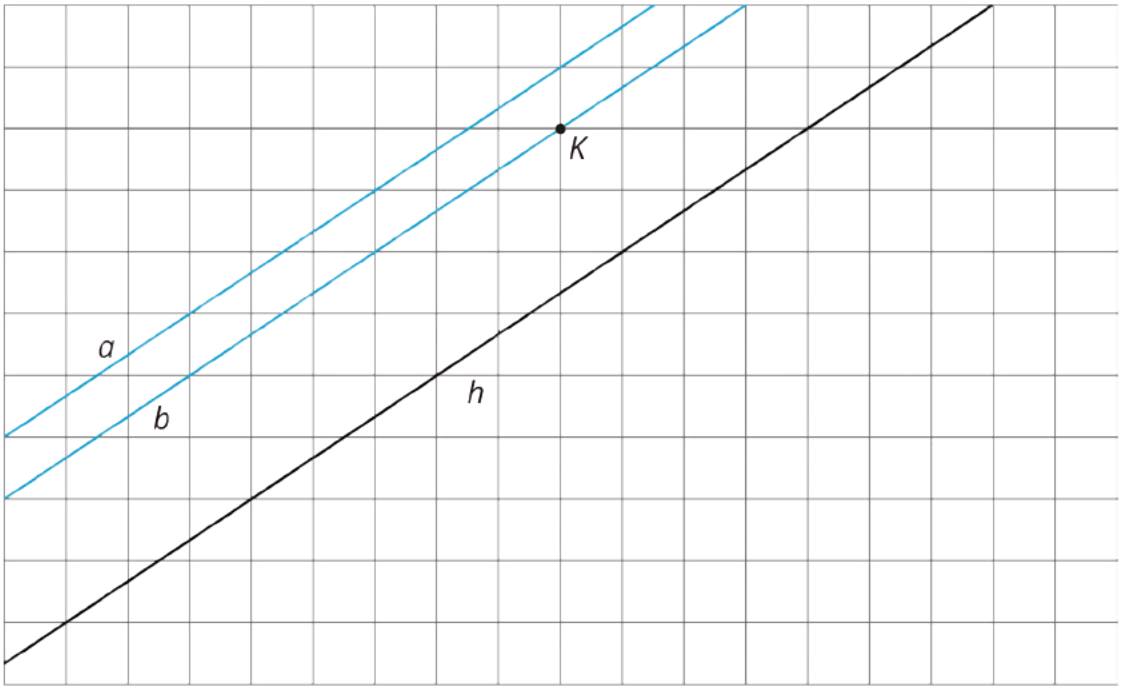
1.



2.



9.2: Parallel Lines



Use a piece of tracing paper to trace lines a and b and point K. Then use that tracing paper to draw the images of the lines under the three different transformations listed.

As you perform each transformation, think about the question:

What is the image of two parallel lines under a rigid transformation?

1. Translate lines a and b 3 units up and 2 units to the right.

   a. What do you notice about the changes that occur to lines a and b after the translation?

   b. What is the same in the original and the image?

2. Rotate lines a and b counterclockwise 180 degrees using K as the center of rotation.

   a. What do you notice about the changes that occur to lines a and b after the rotation?

   b. What is the same in the original and the image?

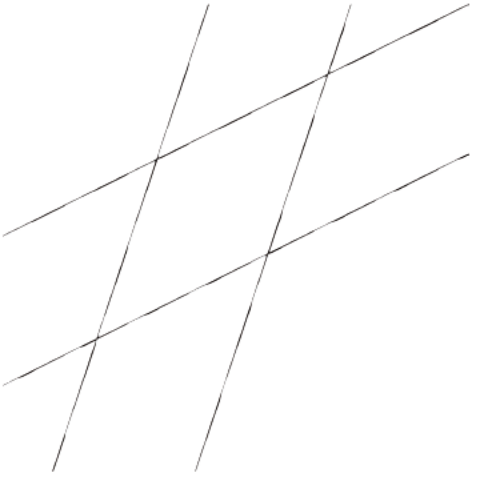
3. Reflect lines a and b across line h.

   a. What do you notice about the changes that occur to lines a and b after the reflection?

   b. What is the same in the original and the image?

Are you ready for more?

When you rotate two parallel lines, sometimes the two original lines intersect their images and form a quadrilateral. What is the most specific thing you can say about this quadrilateral? Can it be a square? A rhombus? A rectangle that isn't a square? Explain your reasoning.

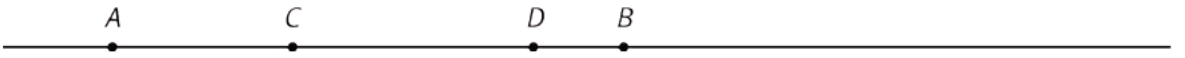


9.3: Let's Do Some 180's

1. The diagram shows a line with points labeled A, C, D, and B.
   a. On the diagram, draw the image of the line and points A, C, and B after the line has been rotated 180 degrees around point D.

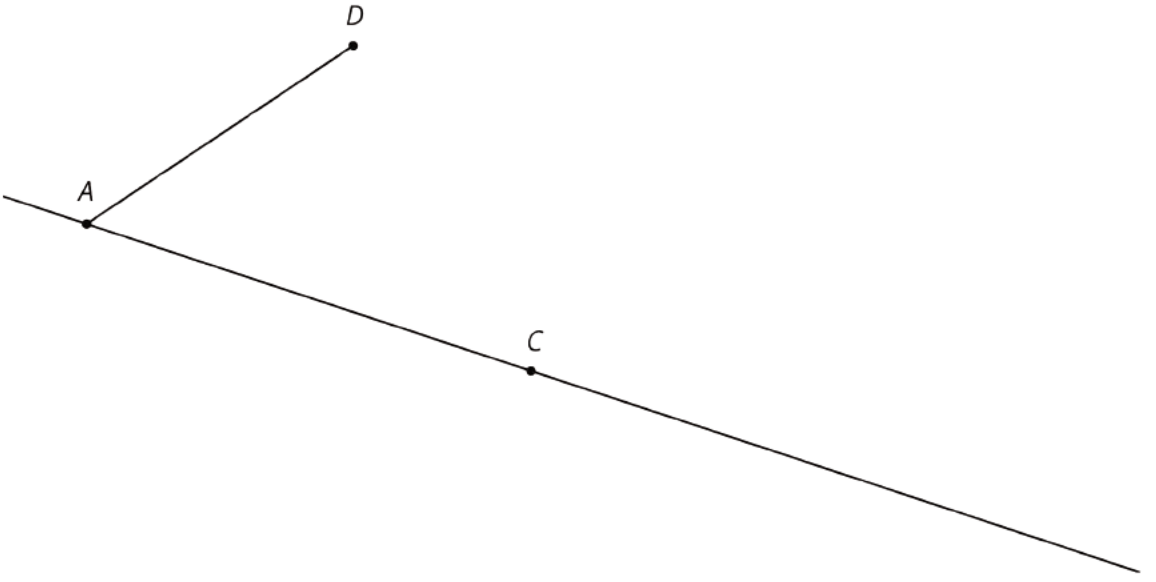
   b. Label the images of the points A', B', and C'.

   c. What is the order of all seven points? Explain or show your reasoning.



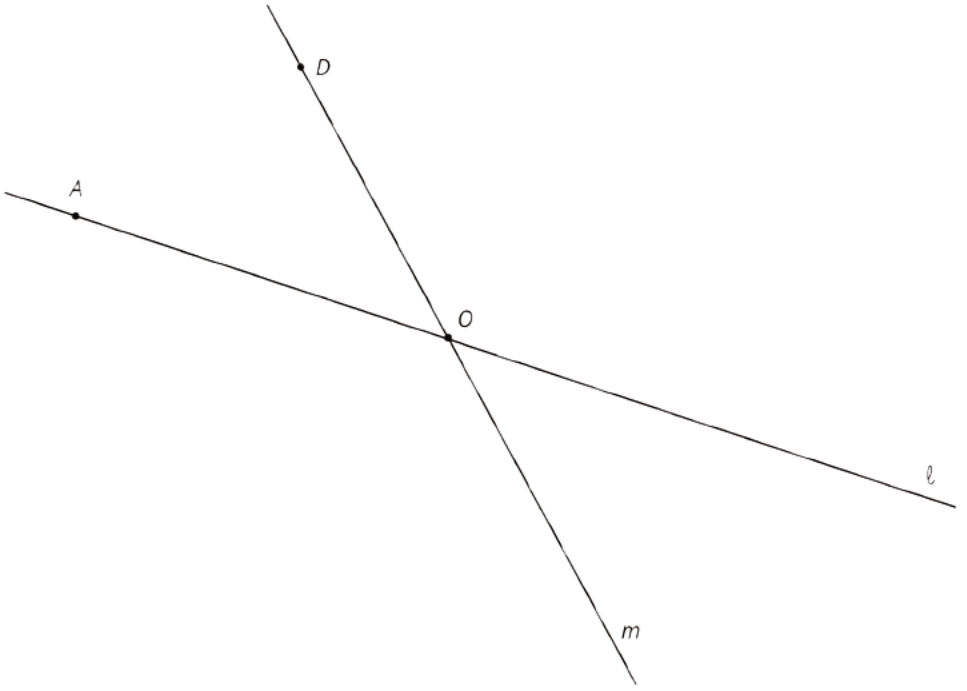
2. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.
   a. Rotate the figure 180 degrees about point C. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between angle CAD and angle $CA'D'$? Explain or show your reasoning.



3. The diagram shows two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.
   a. Rotate the figure 180 degrees around O. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.

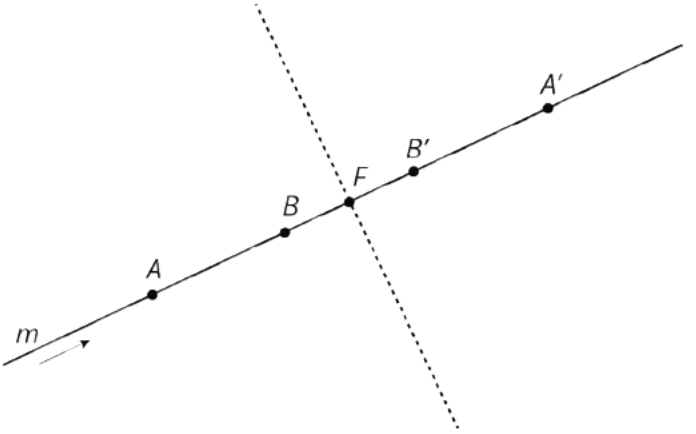


Lesson 9 Summary

Rigid transformations have the following properties:

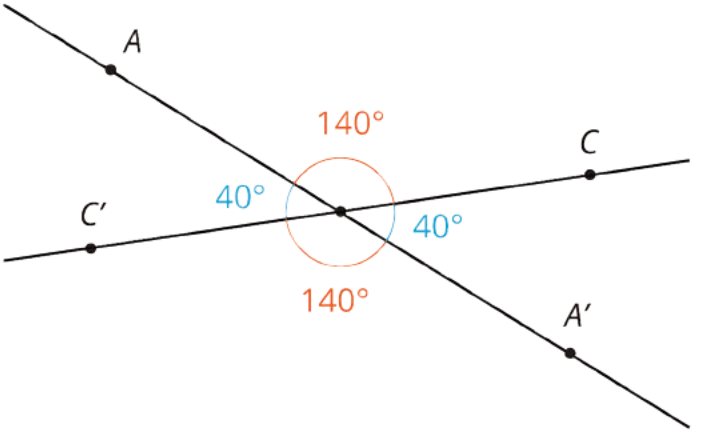
- A rigid transformation of a line is a line.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.

- Sometimes, a rigid transformation takes a line to itself. For example:



  - A translation parallel to the line. The arrow shows a translation of line m that will take m to itself.
  - A rotation by $180°$ around any point on the line. A $180°$ rotation of line m around point F will take m to itself.
  - A reflection across any line perpendicular to the line. A reflection of line m across the dashed line will take m to itself.

These facts let us make an important conclusion. If two lines intersect at a point, which we'll call O, then a $180°$ rotation of the lines with center O shows that **vertical angles** are congruent. Here is an example:



Rotating both lines by $180°$ around O sends angle AOC to angle $A'OC'$, proving that they have the same measure. The rotation also sends angle AOC' to angle $A'OC$.

Unit 1 Lesson 9 Cumulative Practice Problems

1. a. Draw parallel lines AB and CD.

   b. Pick any point E. Rotate AB 90 degrees clockwise around E.

   c. Rotate line CD 90 degrees clockwise around E.

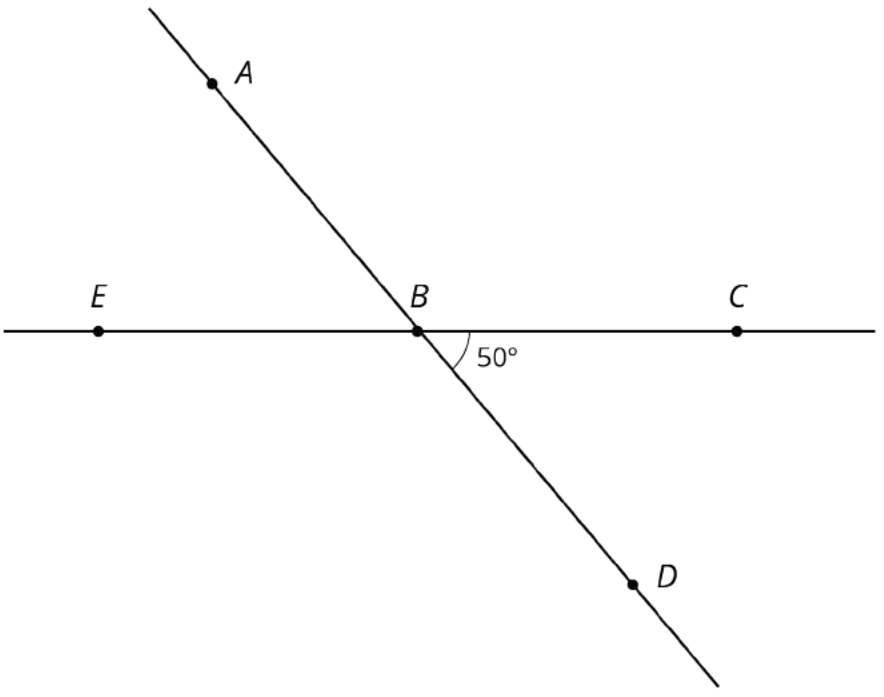
   d. What do you notice?

2. Use the diagram to find the measures of each angle. Explain your reasoning.

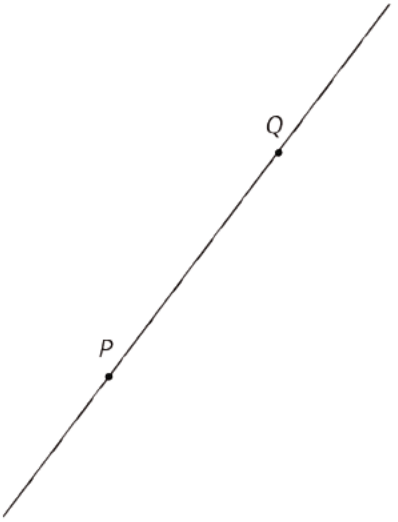
   a. $m\angle ABC$

   b. $m\angle EBD$

   c. $m\angle ABE$



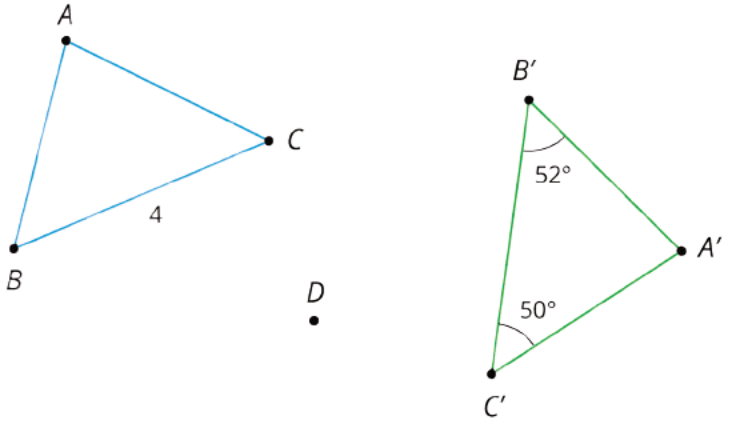
3. Points P and Q are plotted on a line.



a. Find a point R so that a 180-degree rotation with center R sends P to Q and Q to P.

b. Is there more than one point R that works for part a?

4. In the picture triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is D.



a. What is the length of side $B'C'$? Explain how you know.

b. What is the measure of angle B? Explain how you know.

c. What is the measure of angle C? Explain how you know.

(From Unit 1, Lesson 7.)

5. The point $(-4, 1)$ is rotated 180 degrees counterclockwise using center $(0, 0)$. What are the coordinates of the image?

   A. $(-1, -4)$

   B. $(-1, 4)$

   C. $(4, 1)$

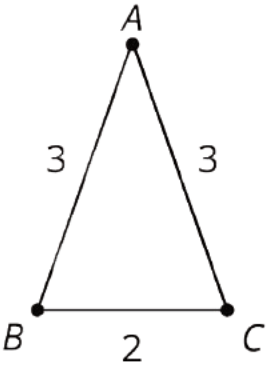
   D. $(4, -1)$

(From Unit 1, Lesson 6.)

Lesson 10: Composing Figures

10.1: Angles of an Isosceles Triangle

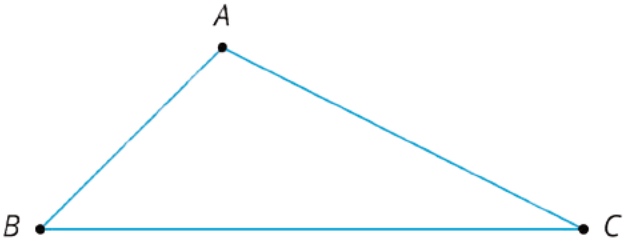
Here is a triangle.



1. Reflect triangle ABC over line AB. Label the image of C as C'.
2. Rotate triangle ABC' around A so that C' matches up with B.
3. What can you say about the measures of angles B and C?

10.2: Triangle Plus One

Here is triangle ABC.



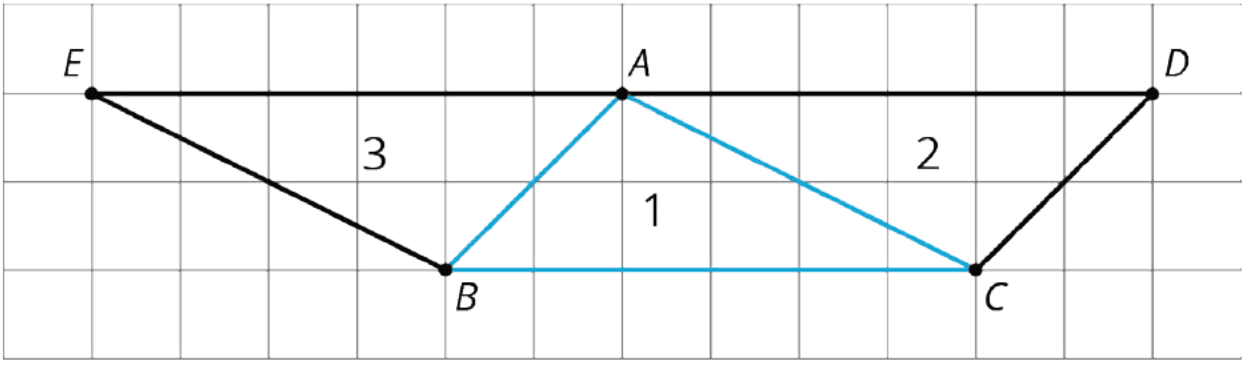
1. Draw midpoint M of side AC.
2. Rotate triangle ABC 180 degrees using center M to form triangle CDA. Draw and label this triangle.
3. What kind of quadrilateral is $ABCD$? Explain how you know.

Are you ready for more?

In the activity, we made a parallelogram by taking a triangle and its image under a 180-degree rotation around the midpoint of a side. This picture helps you justify a well-known formula for the area of a triangle. What is the formula and how does the figure help justify it?

10.3: Triangle Plus Two

The picture shows 3 triangles. Triangle 2 and Triangle 3 are images of Triangle 1 under rigid transformations.



1. Describe a rigid transformation that takes Triangle 1 to Triangle 2. What points in Triangle 2 correspond to points A, B, and C in the original triangle?

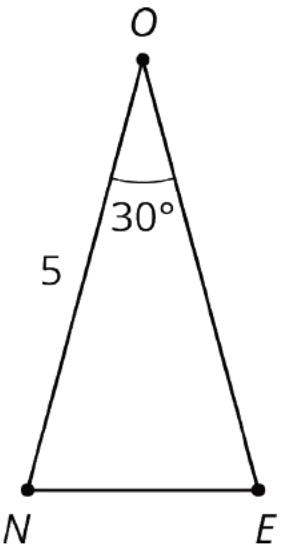
2. Describe a rigid transformation that takes Triangle 1 to Triangle 3. What points in Triangle 3 correspond to points A, B, and C in the original triangle?

3. Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

4. Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

10.4: Triangle ONE Plus

Here is isosceles triangle ONE. Its sides ON and OE have equal lengths. Angle O is 30 degrees. The length of ON is 5 units.

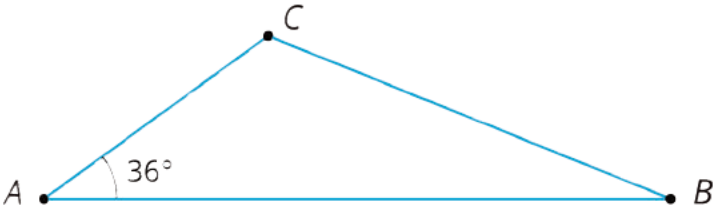


1. Reflect triangle ONE across segment ON. Label the new vertex M.
2. What is the measure of angle MON?
3. What is the measure of angle MOE?
4. Reflect triangle MON across segment OM. Label the point that corresponds to N as T.
5. How long is $\overline{OT}$? How do you know?
6. What is the measure of angle TOE?
7. If you continue to reflect each new triangle this way to make a pattern, what will the pattern look like?

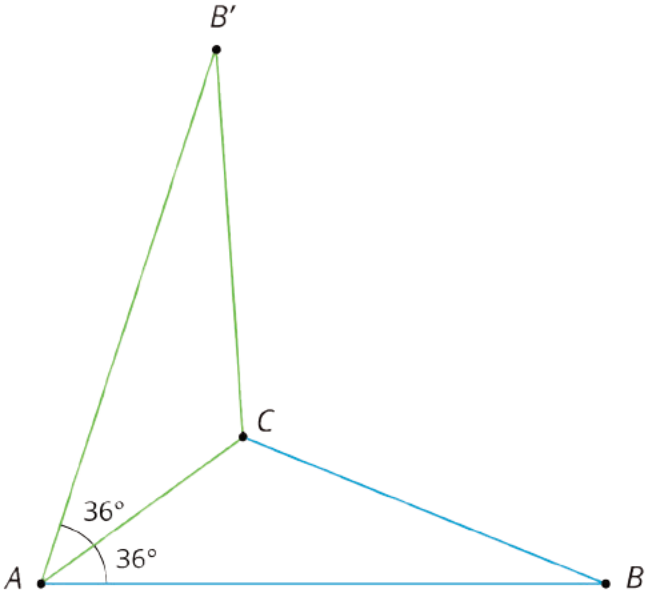
Lesson 10 Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.



We can reflect triangle ABC across side AC to form a new triangle:



Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures $36°$ because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

Unit 1 Lesson 10 Cumulative Practice Problems

1. Here is the design for the flag of Trinidad and Tobago.

Describe a sequence of translations, rotations, and reflections that take the lower left triangle to the upper right triangle.

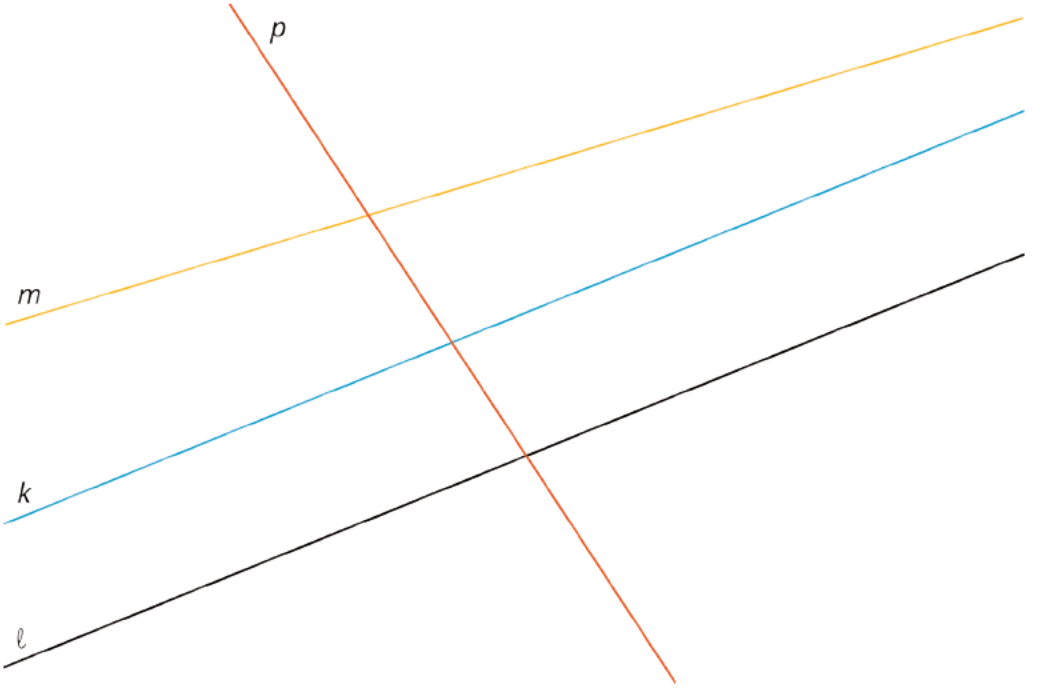
2. Here is a picture of an older version of the flag of Great Britain. There is a rigid transformation that takes Triangle 1 to Triangle 2, another that takes Triangle 1 to Triangle 3, and another that takes Triangle 1 to Triangle 4.

a. Measure the lengths of the sides in Triangles 1 and 2. What do you notice?

b. What are the side lengths of Triangle 3? Explain how you know.

c. Do all eight triangles in the flag have the same area? Explain how you know.

3. a. Which of the lines in the picture is parallel to line ℓ? Explain how you know.



b. Explain how to translate, rotate or reflect line ℓ to obtain line k.

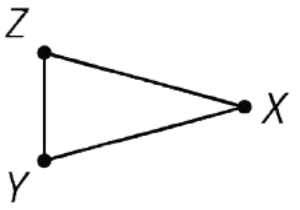
c. Explain how to translate, rotate or reflect line ℓ to obtain line p.

(From Unit 1, Lesson 9.)

4. Point A has coordinates $(3, 4)$. After a translation 4 units left, a reflection across the x-axis, and a translation 2 units down, what are the coordinates of the image?

(From Unit 1, Lesson 6.)

5. Here is triangle XYZ:



Draw these three rotations of triangle XYZ together.

a. Rotate triangle XYZ 90 degrees clockwise around Z.

b. Rotate triangle XYZ 180 degrees around Z.

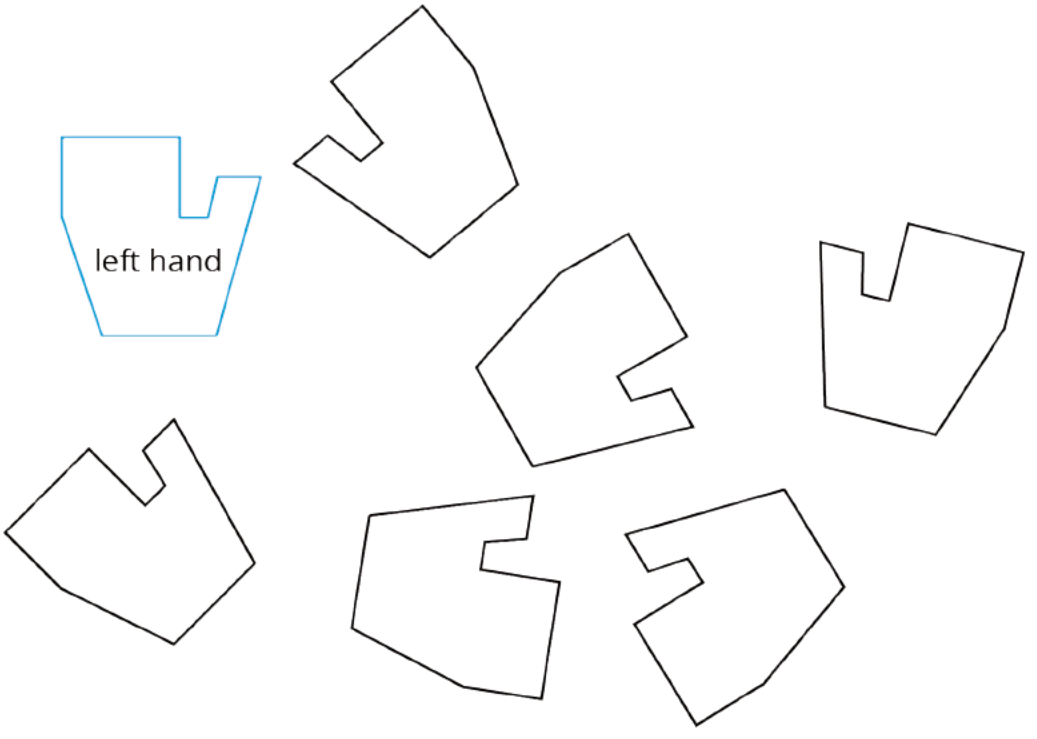
c. Rotate triangle XYZ 270 degrees clockwise around Z.

(From Unit 1, Lesson 8.)

Lesson 11: What Is the Same?

11.1: Find the Right Hands

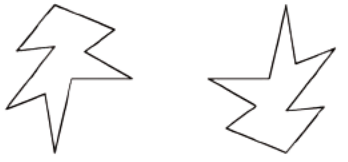
A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Shade all of the right hands.



11.2: Are They the Same?

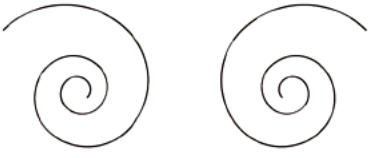
For each pair of shapes, decide whether or not they are the same.

A

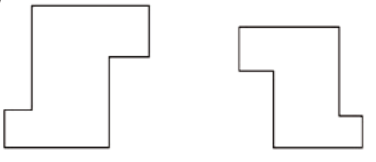


B

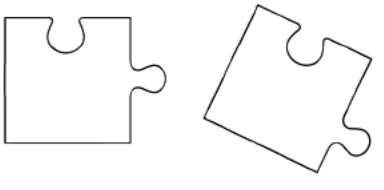
C



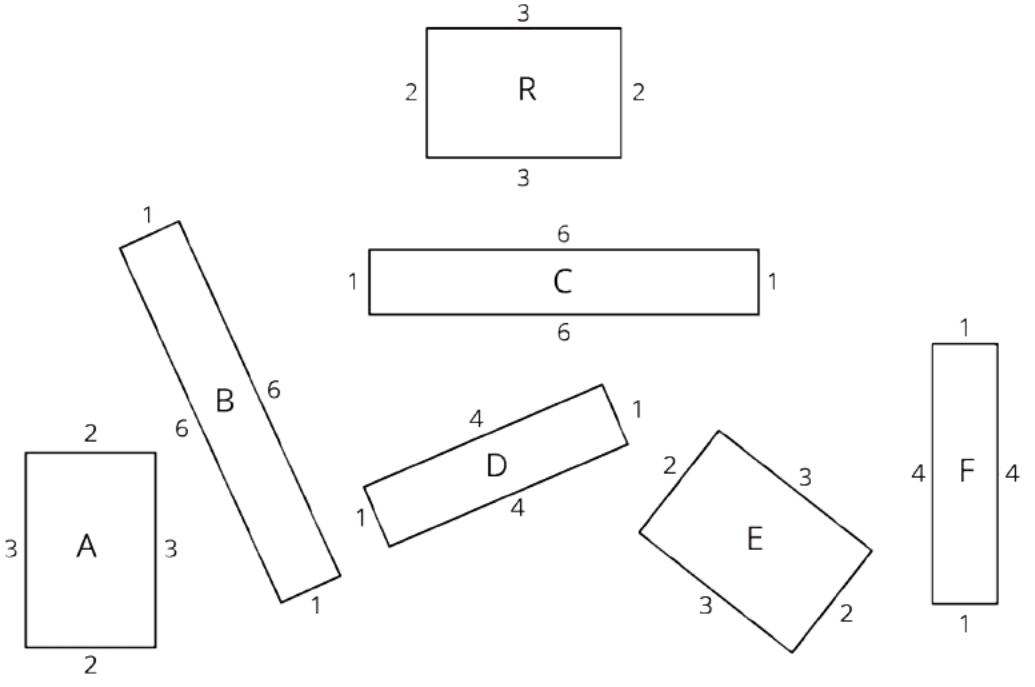
D



E



11.3: Area, Perimeter, and Congruence



1. Which of these rectangles have the same area as Rectangle R but different perimeter?

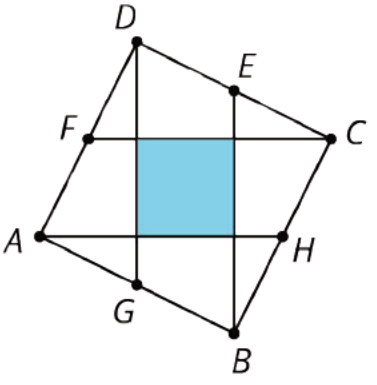
2. Which rectangles have the same perimeter as Rectangle R but different area?

3. Which have the same area *and* the same perimeter as Rectangle R?

4. Use materials from the geometry tool kit to decide which rectangles are **congruent**. Shade congruent rectangles with the same color.

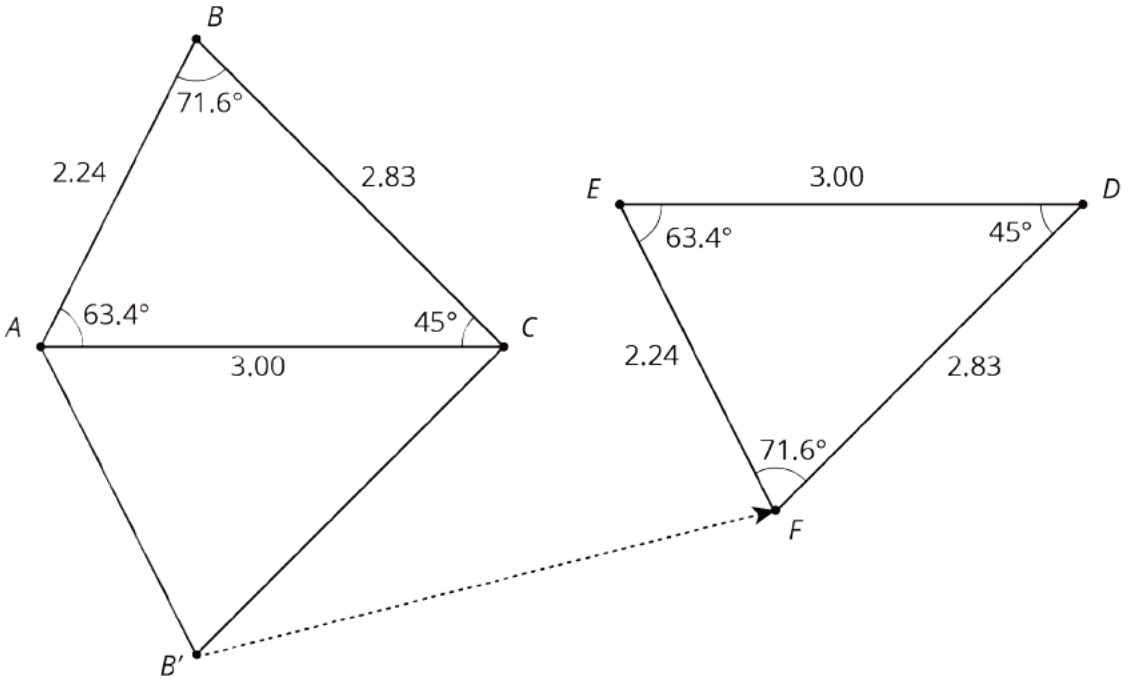
Are you ready for more?

In square $ABCD$, points E, F, G, and H are midpoints of their respective sides. What fraction of square $ABCD$ is shaded? Explain your reasoning.



Lesson 11 Summary

Congruent is a new term for an idea we have already been using. We say that two figures are congruent if one can be lined up exactly with the other by a sequence of rigid transformations. For example, triangle EFD is congruent to triangle ABC because they can be matched up by reflecting triangle ABC across AC followed by the translation shown by the arrow. Notice that all corresponding angles and side lengths are equal.



Here are some other facts about congruent figures:

- We don't need to check all the measurements to prove two figures are congruent; we just have to find a sequence of rigid transformations that match up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different perimeter or area.

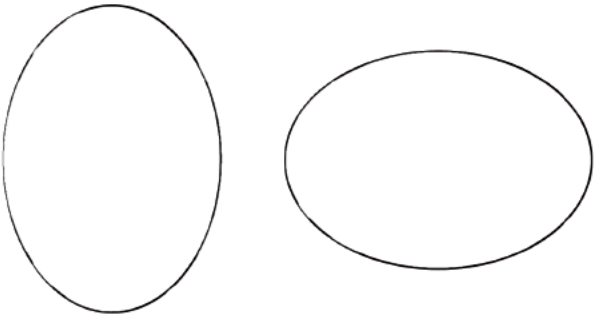
Unit 1 Lesson 11 Cumulative Practice Problems

1. If two rectangles have the same perimeter, do they have to be congruent? Explain how you know.

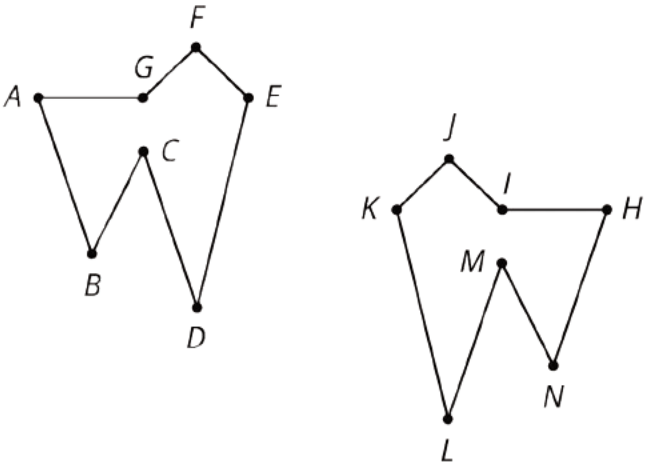
2. Draw two rectangles that have the same area, but are *not* congruent.

3. For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

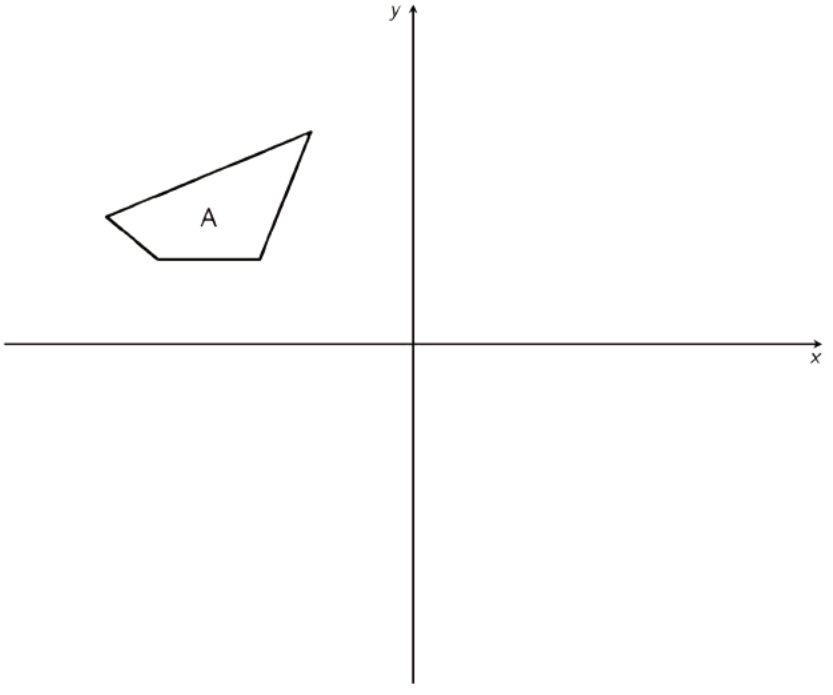
a.



b.



4. a. Reflect Quadrilateral A over the x-axis. Label the image quadrilateral B. Reflect Quadrilateral B over the y-axis. Label the image C.



b. Are Quadrilaterals A and C congruent? Explain how you know.

5. The point $(-2, -3)$ is rotated 90 degrees counterclockwise using center $(0, 0)$. What are the coordinates of the image?

A. $(-3, -2)$

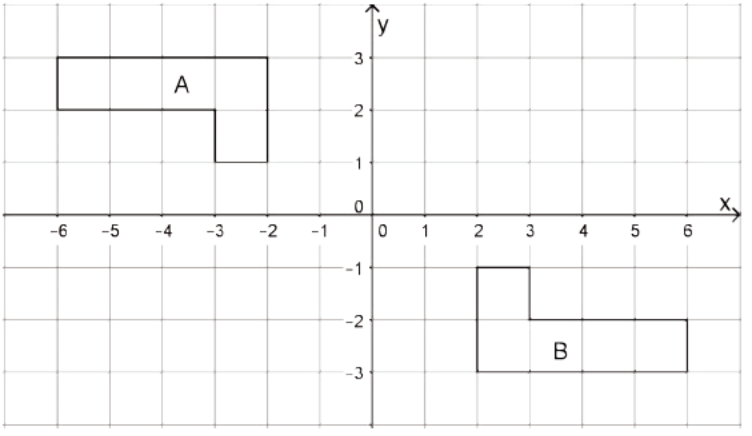
B. $(-3, 2)$

C. $(3, -2)$

D. $(3, 2)$

(From Unit 1, Lesson 6.)

6. Describe a rigid transformation that takes Polygon A to Polygon B.

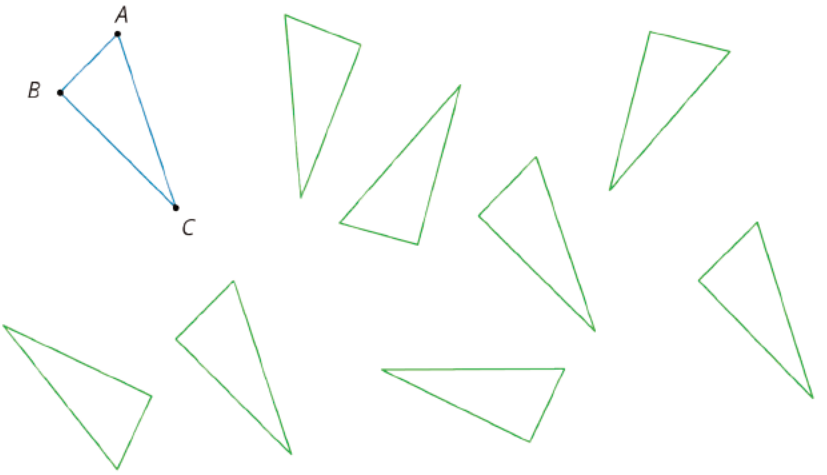


(From Unit 1, Lesson 7.)

Lesson 12: Congruent Polygons

12.1: Translated Images

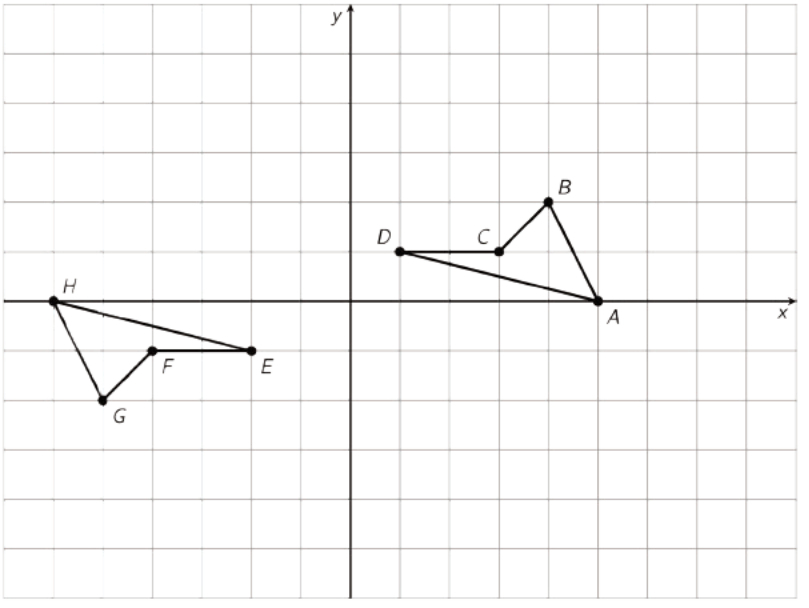
All of these triangles are congruent. Sometimes we can take one figure to another with a translation. Shade the triangles that are images of triangle ABC under a translation.



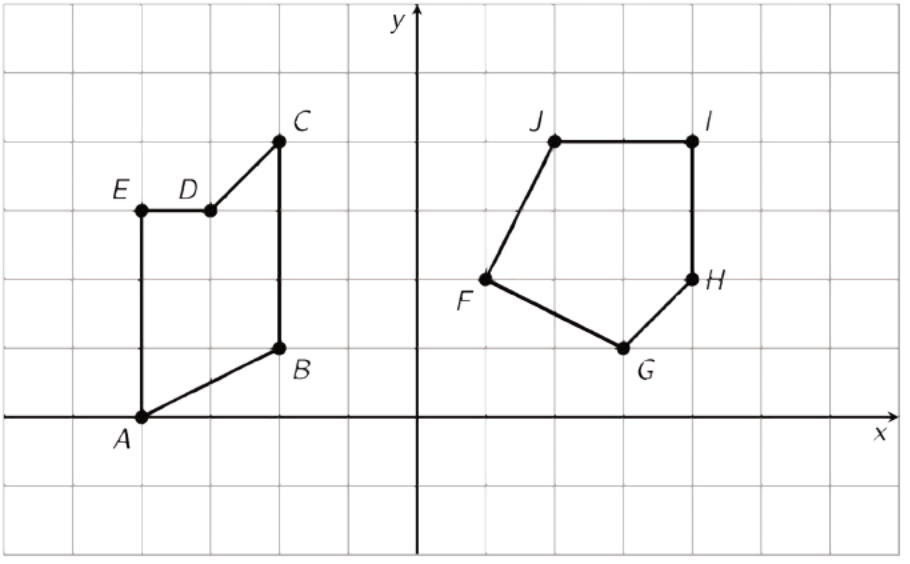
12.2: Congruent Pairs (Part 1)

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

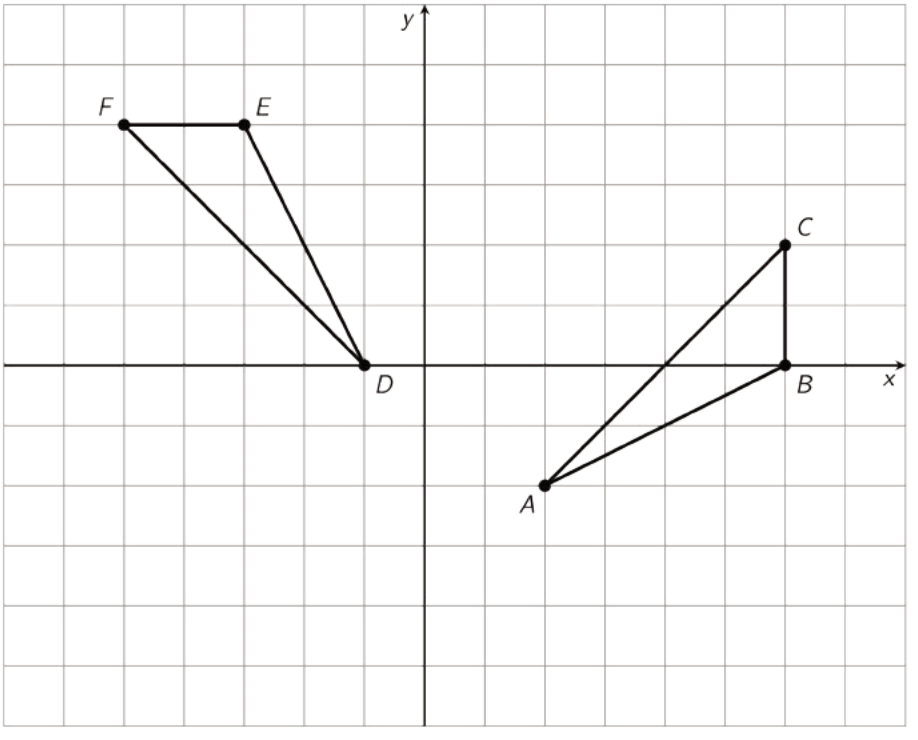
1.



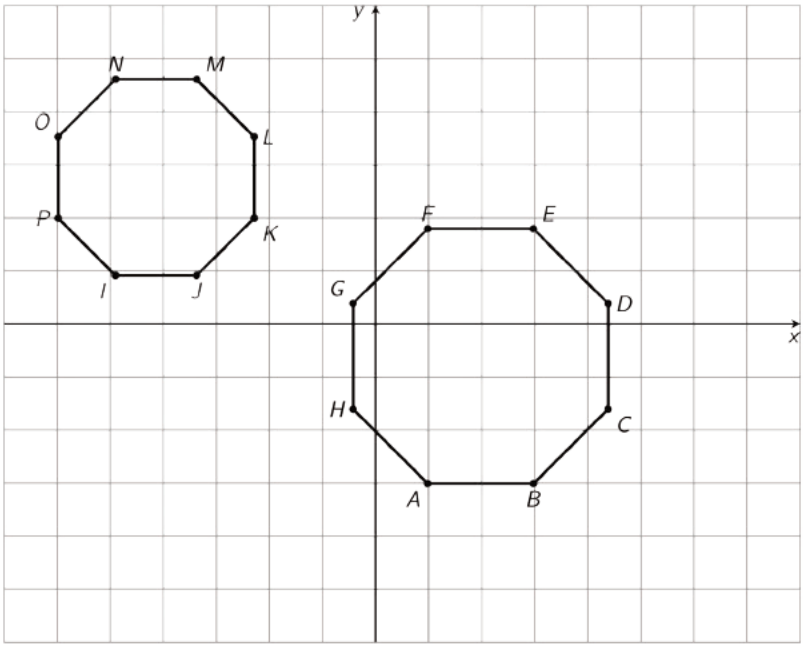
2.



3.



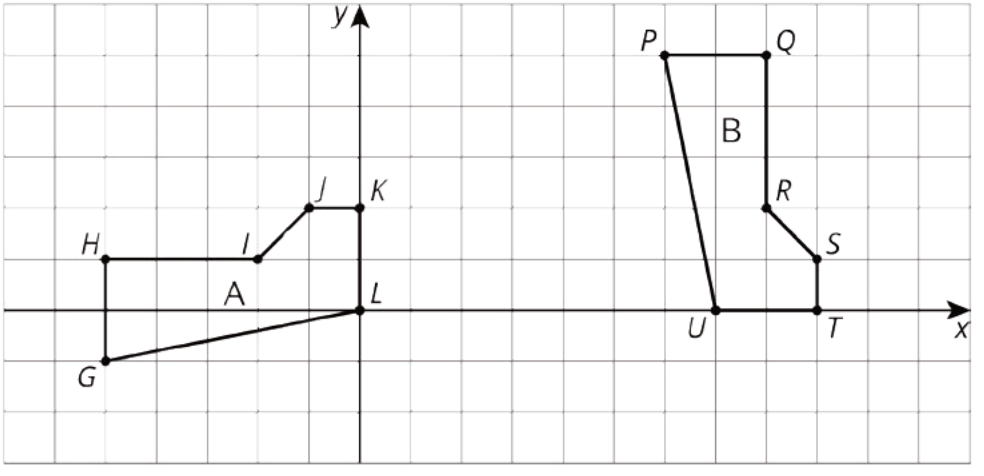
4.



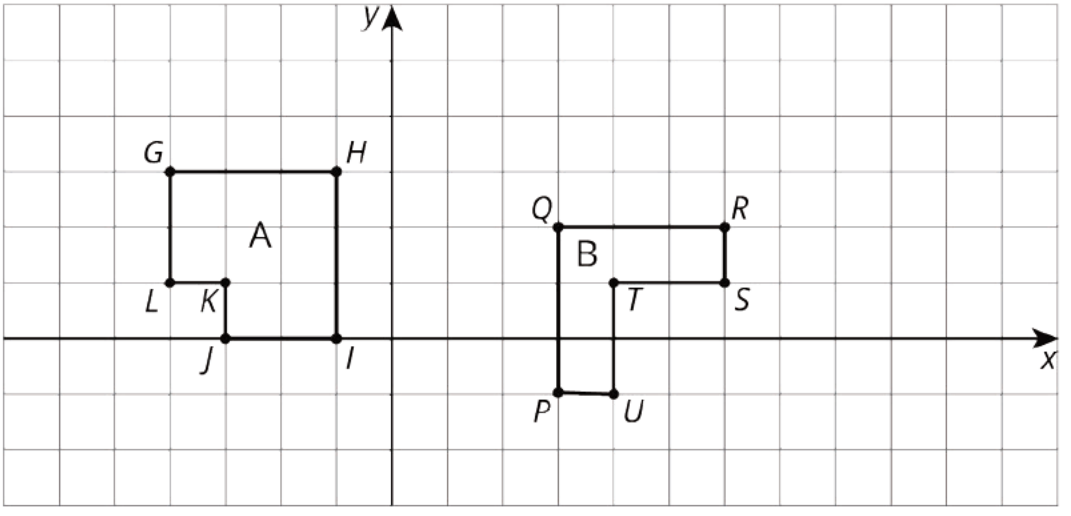
12.3: Congruent Pairs (Part 2)

For each pair of shapes, decide whether or not Shape A is congruent to Shape B. Explain how you know.

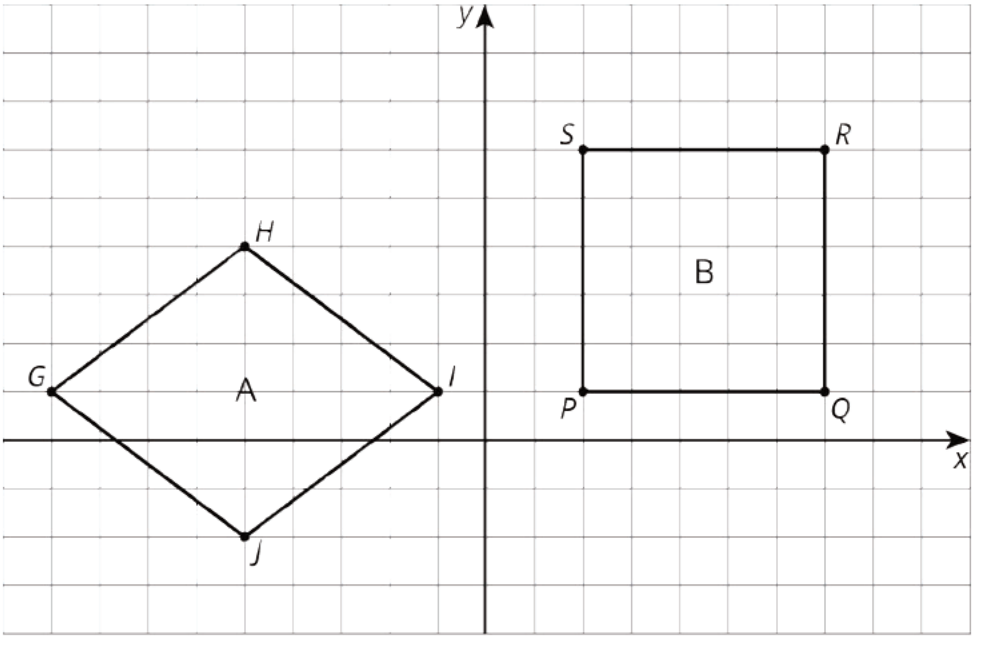
1.



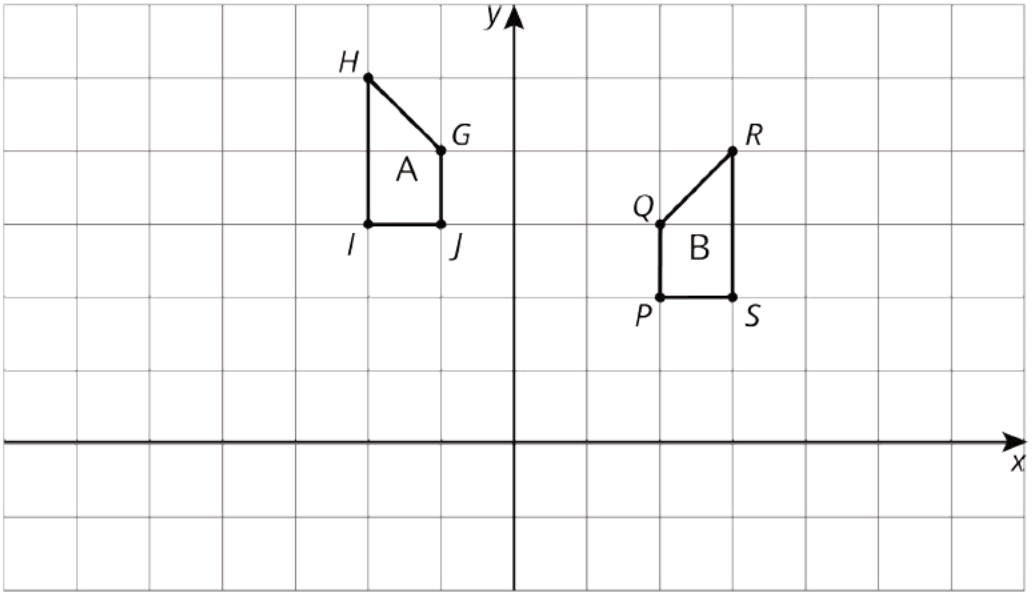
2.



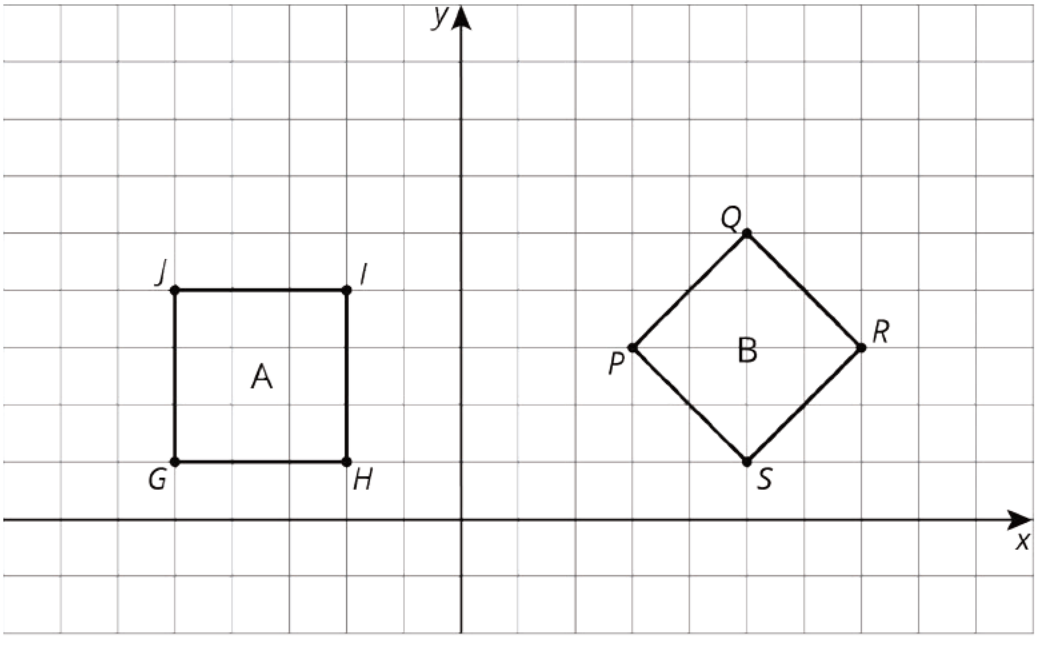
3.



4.



5.



Are you ready for more?

A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

1. Find a polygon with these properties.

2. Is there a second polygon, not congruent to your first, with these properties?

12.4: Building Quadrilaterals

Your teacher will give you a set of four objects.

1. Make a quadrilateral with your four objects and record what you have made.

2. Compare your quadrilateral with your partner's. Are they congruent? Explain how you know.

3. Repeat steps 1 and 2, forming different quadrilaterals. If your first quadrilaterals were not congruent, can you build a pair that is? If your first quadrilaterals were congruent, can you build a pair that is not? Explain.

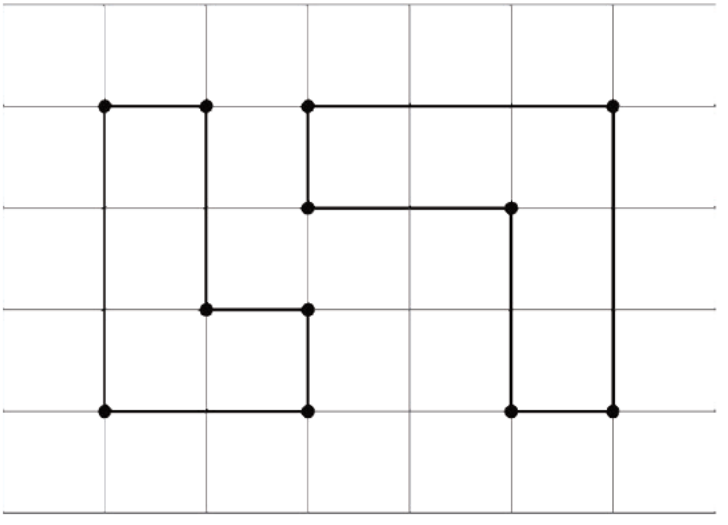
Lesson 12 Summary

How do we know if two figures are congruent?

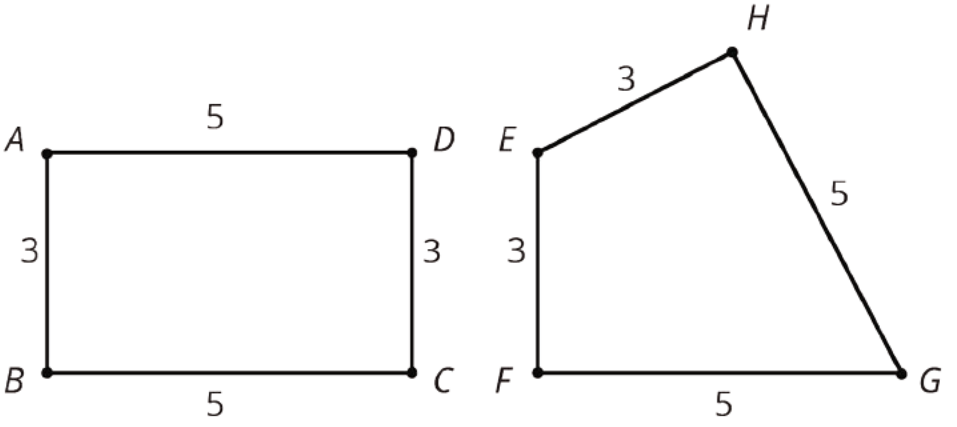
- If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- We can prove that two figures are congruent by describing a sequence of translations, rotations, and reflections that move one figure onto the other so they match up exactly.

How do we know that two figures are *not* congruent?

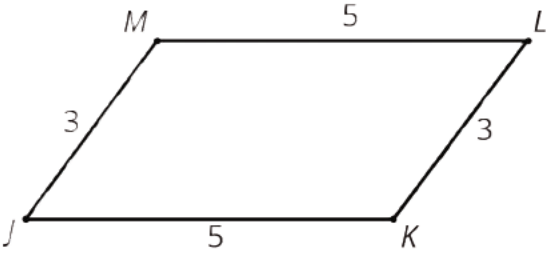
- If there is no correspondence between the figures where the parts have equal measure, that proves that the two figures are *not* congruent. In particular,
    - If two polygons have different sets of side lengths, they can't be congruent. For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.



- If two polygons have the same side lengths, but their orders can't be matched as you go around each polygon, the polygons can't be congruent. For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order. In $ABCD$, the order is 3, 5, 3, 5 or 5, 3, 5, 3; in $EFGH$, the order is 3, 3, 5, 5 or 3, 5, 5, 3 or 5, 5, 3, 3.



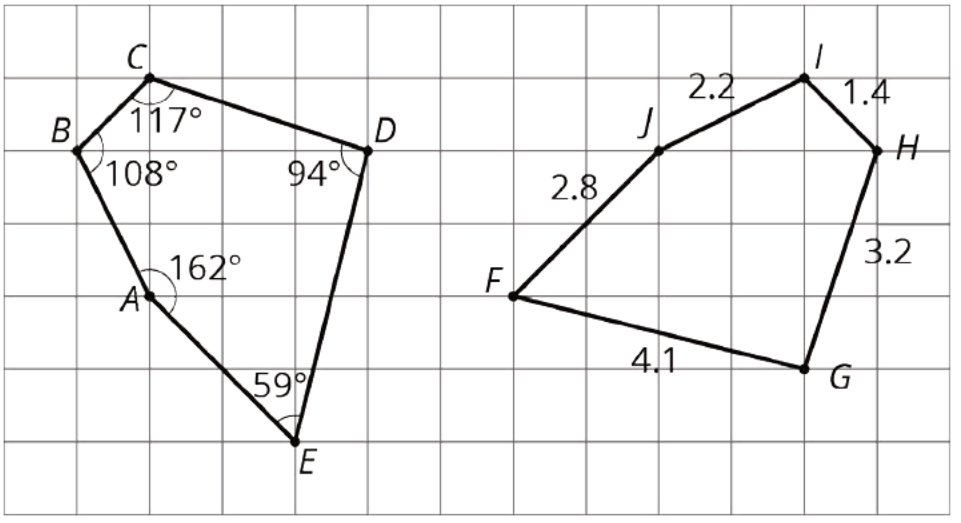
- If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent. For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are **right angles**. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.



Unit 1 Lesson 12 Cumulative Practice Problems

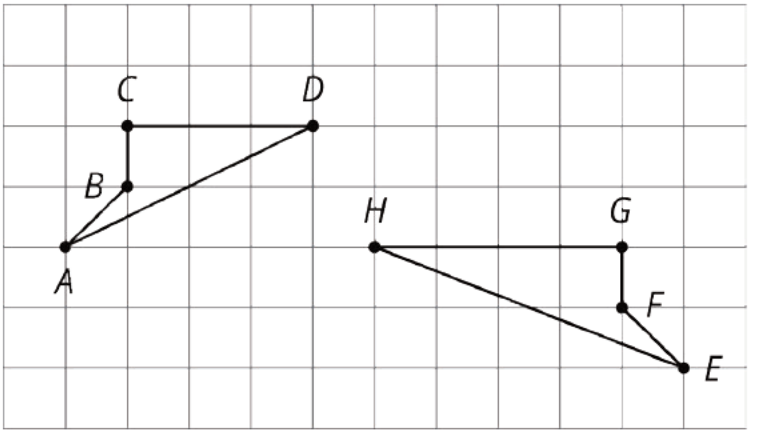
1. a. Show that the two pentagons are congruent.

   b. Find the side lengths of $ABCDE$ and the angle measures of $FGHIJ$.

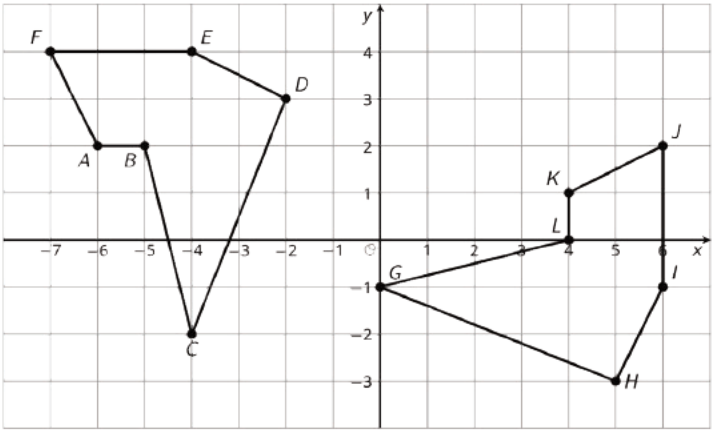


2. For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

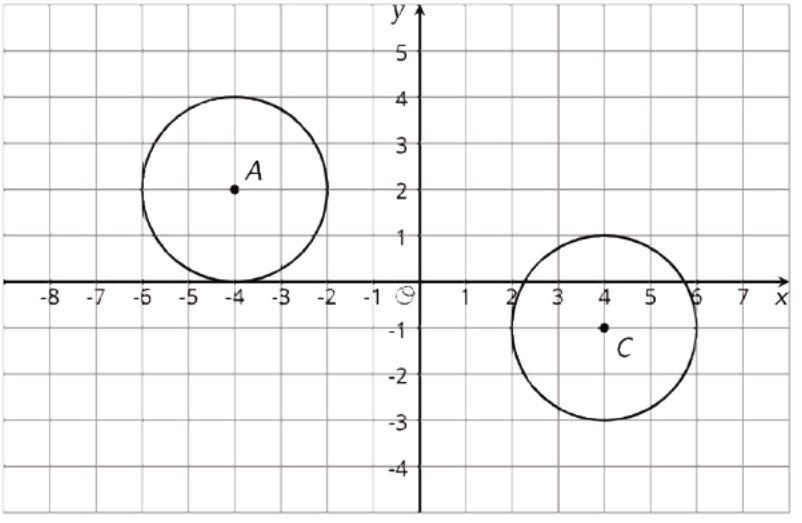
a.



b.



c.

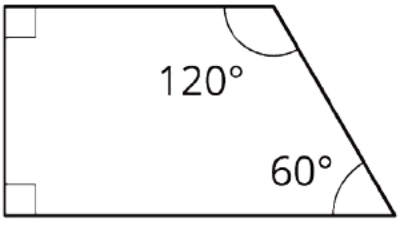


3. a. Draw segment PQ.

b. When PQ is rotated $180°$ around point R, the resulting segment is the same as PQ. Where could point R be located?

(From Unit 1, Lesson 8.)

4. Here is trapezoid $ABCD$.



Using rigid transformations on the trapezoid, build a pattern. Describe some of the rigid transformations you used.

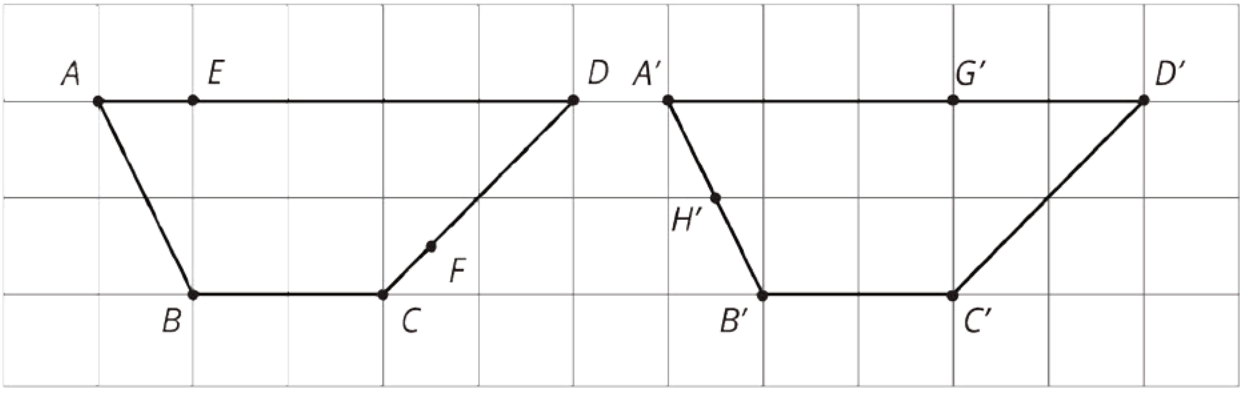
(From Unit 1, Lesson 10.)

Lesson 13: Congruence

13.1: Not Just the Vertices

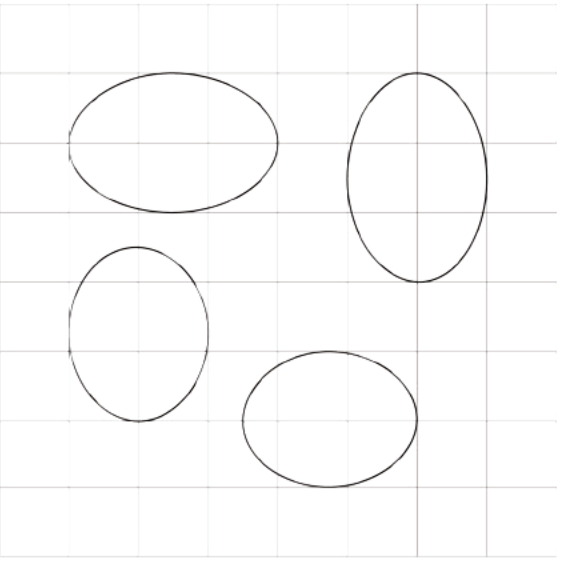
Trapezoids $ABCD$ and $A'B'C'D'$ are congruent.

- Draw and label the points on $A'B'C'D'$ that correspond to E and F.
- Draw and label the points on $ABCD$ that correspond to G' and H'.
- Draw and label at least three more pairs of corresponding points.



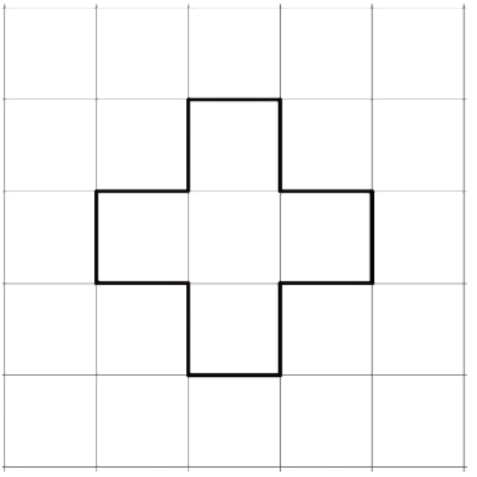
13.2: Congruent Ovals

Are any of the ovals congruent to one another? Explain how you know.



Are you ready for more?

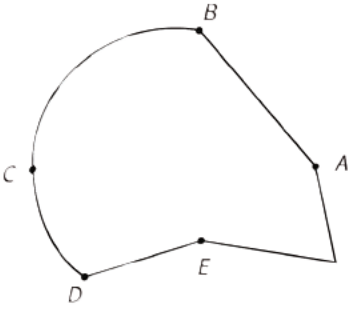
You can use 12 toothpicks to create a polygon with an area of five square toothpicks, like this:

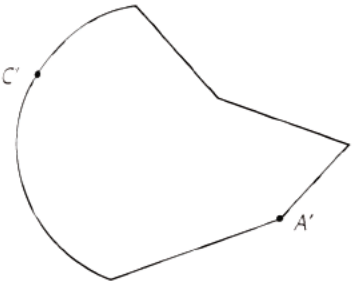


Can you use exactly 12 toothpicks to create a polygon with an area of four square toothpicks?

13.3: Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled.





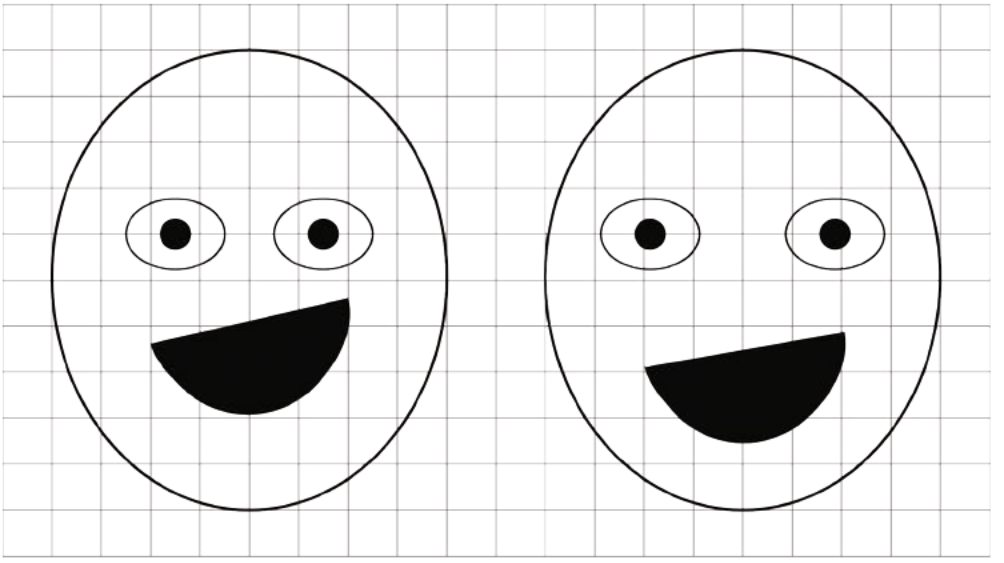
1. Draw the points corresponding to B, D, and E, and label them B', D', and E'.

2. Draw line segments AD and $A'D'$ and measure them. Do the same for segments BC and $B'C'$ and for segments AE and $A'E'$. What do you notice?

3. Do you think there could be a pair of corresponding segments with different lengths? Explain.

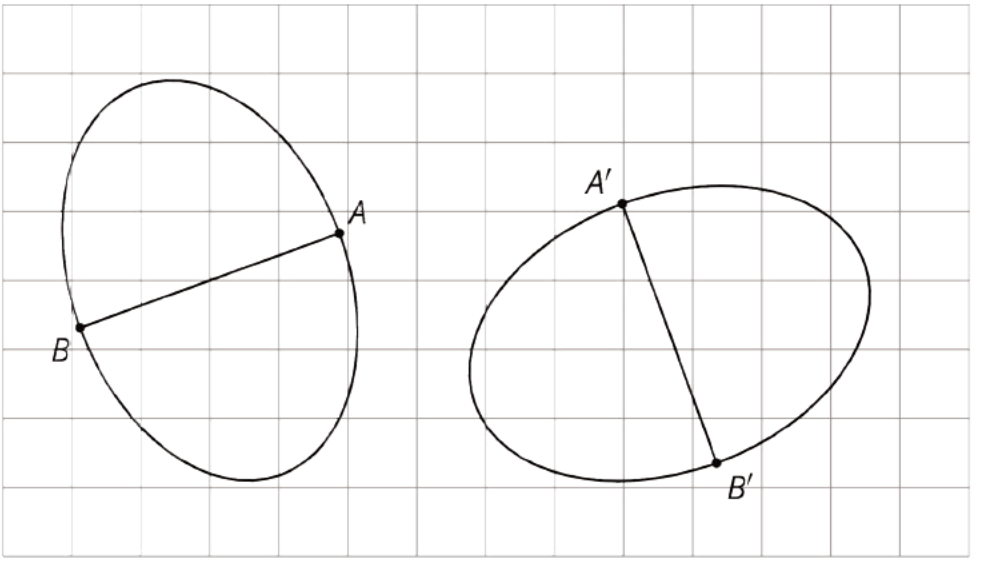
13.4: Astonished Faces

Are these faces congruent? Explain your reasoning.



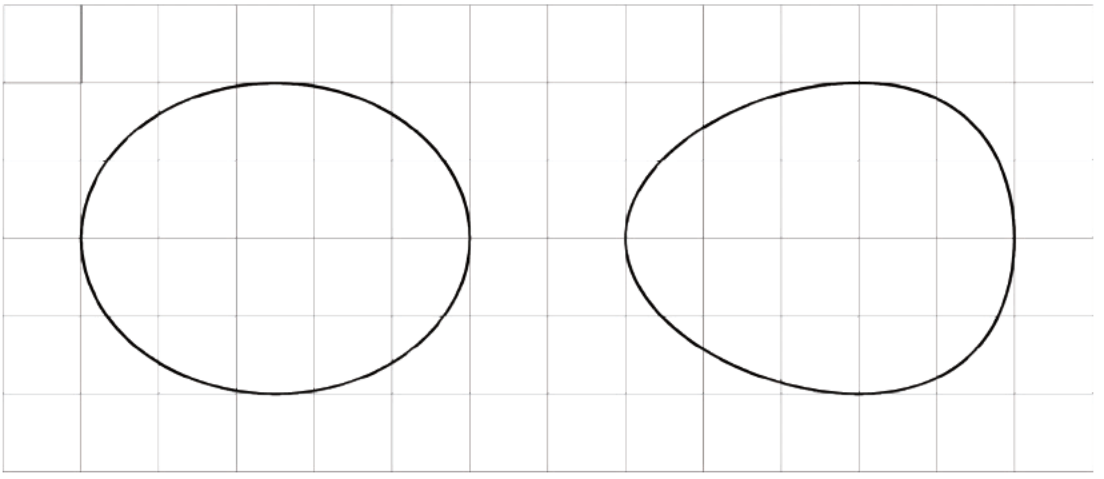
Lesson 13 Summary

To show two figures are congruent, you align one with the other by a sequence of rigid transformations. This is true even for figures with curved sides. Distances between corresponding points on congruent figures are always equal, even for curved shapes. For example, corresponding segments AB and $A'B'$ on these congruent ovals have the same length:

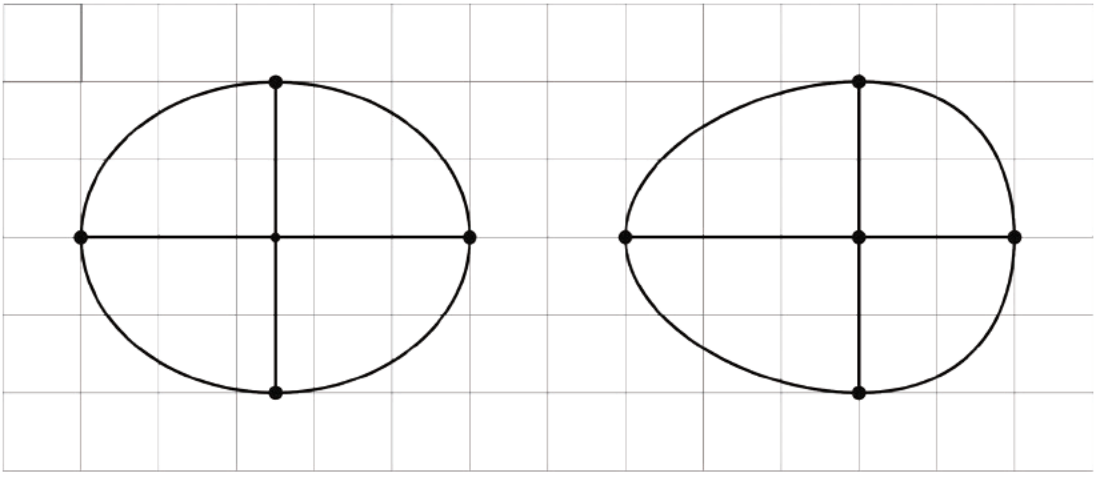


To show two figures are not congruent, you can find parts of the figures that should correspond but that have different measurements.

For example, these two ovals don't look congruent.

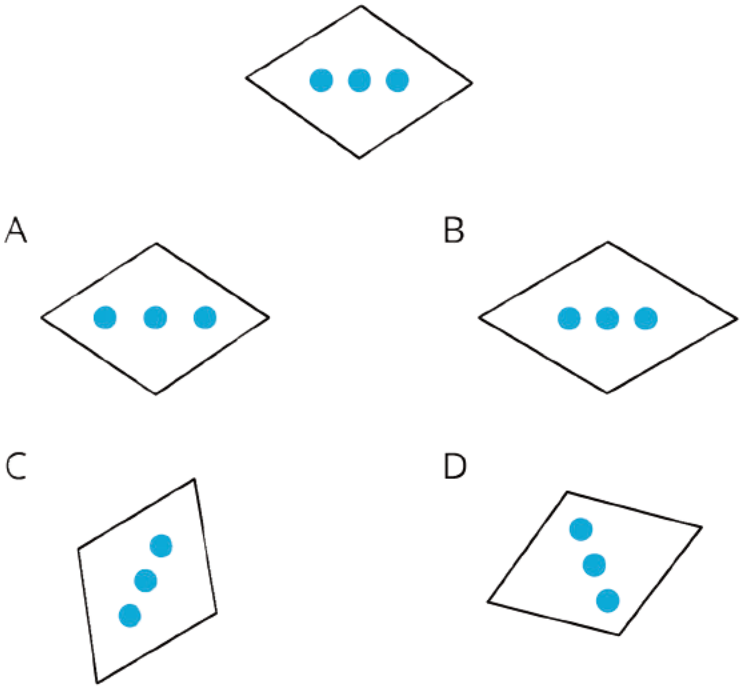


On both, the longest distance is 5 units across, and the longest distance from top to bottom is 4 units. The line segment from the highest to lowest point is in the middle of the left oval, but in the right oval, it's 2 units from the right end and 3 units from the left end. This proves they are not congruent.



Unit 1 Lesson 13 Cumulative Practice Problems

1. Which of these four figures are congruent to the top figure?



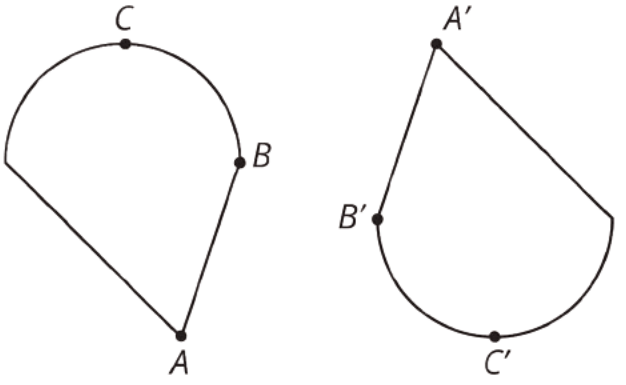
A. A

B. B

C. C

D. D

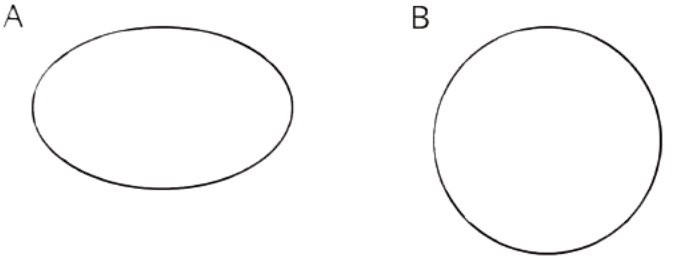
2. These two figures are congruent, with corresponding points marked.



a. Are angles ABC and $A'B'C'$ congruent? Explain your reasoning.

b. Measure angles ABC and $A'B'C'$ to check your answer.

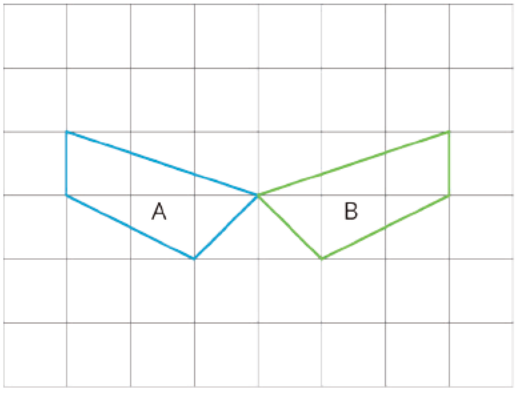
3. Here are two figures.



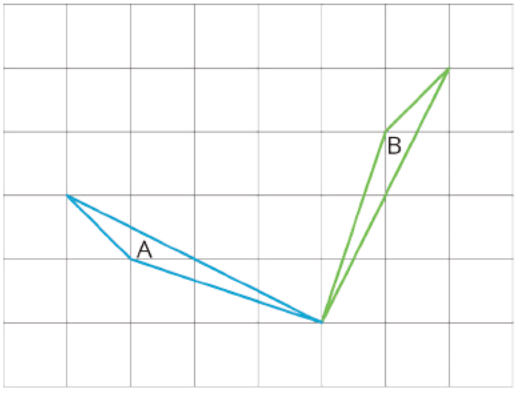
Show, using measurement, that these two figures are *not* congruent.

4. Each picture shows two polygons, one labeled Polygon A and one labeled Polygon B. Describe how to move Polygon A into the position of Polygon B using a transformation.

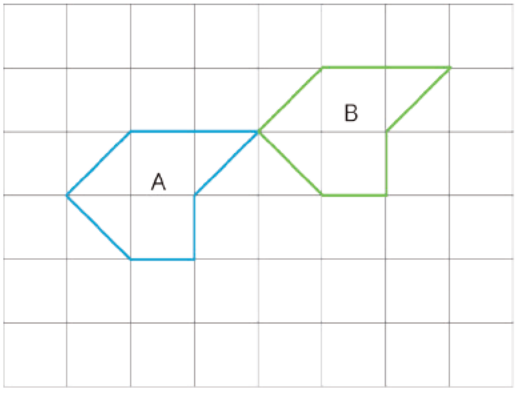
a.



b.



c.

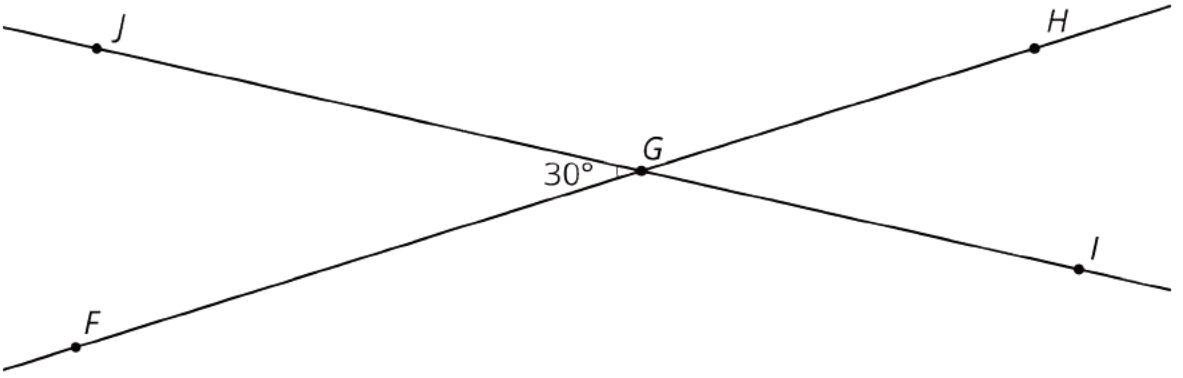


(From Unit 1, Lesson 3.)

Lesson 14: Alternate Interior Angles

14.1: Angle Pairs

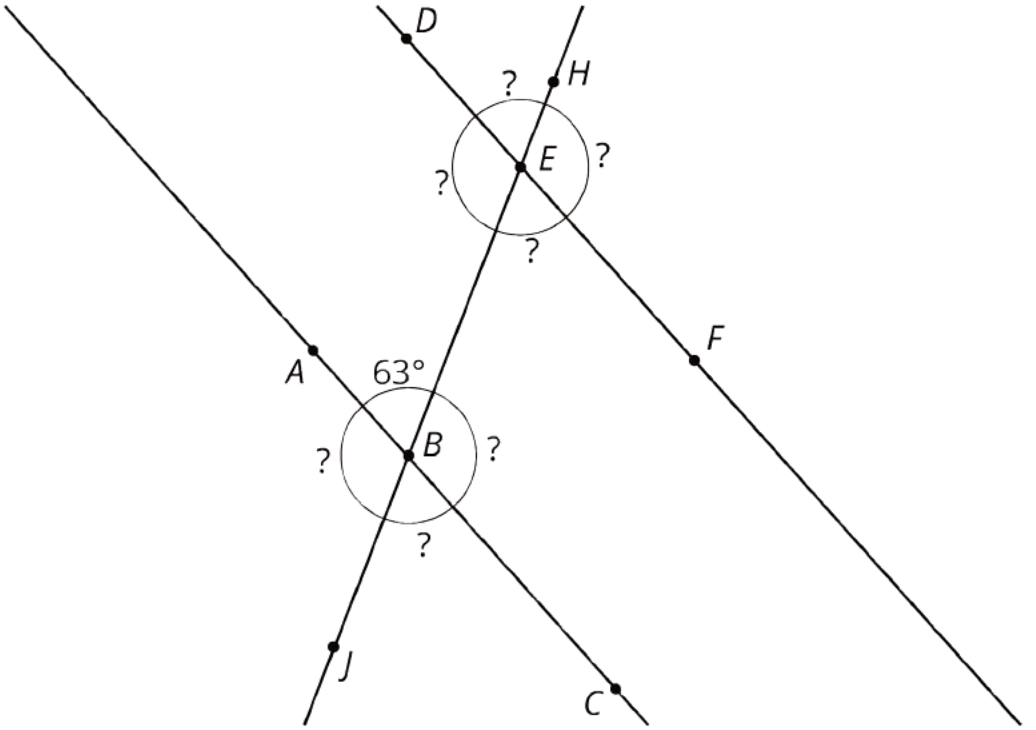
1. Find the measure of angle JGH. Explain or show your reasoning.



2. Find and label a second 30° degree angle in the diagram. Find and label an angle congruent to angle JGH.

14.2: Cutting Parallel Lines with a Transversal

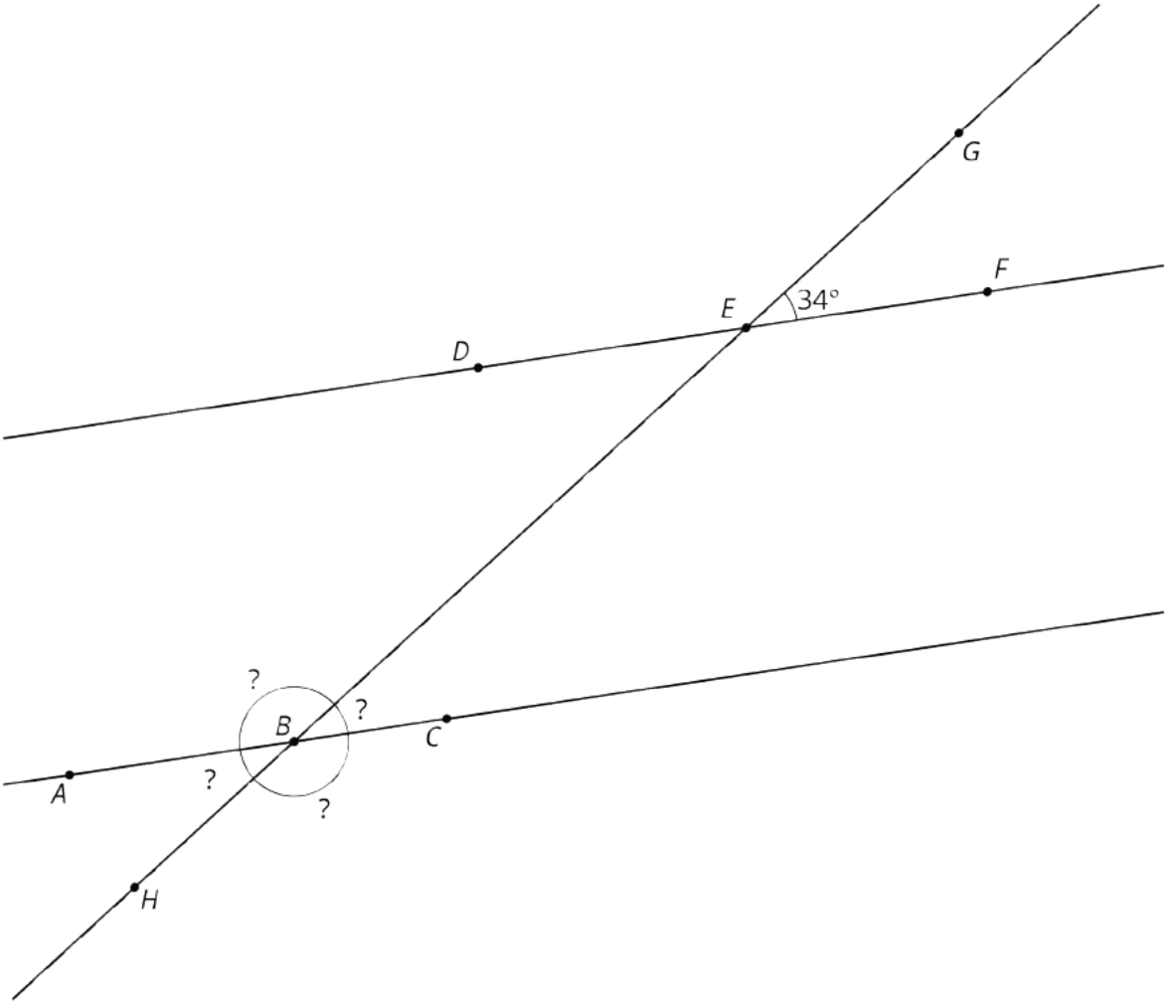
Lines AC and DF are parallel. They are cut by **transversal** HJ.



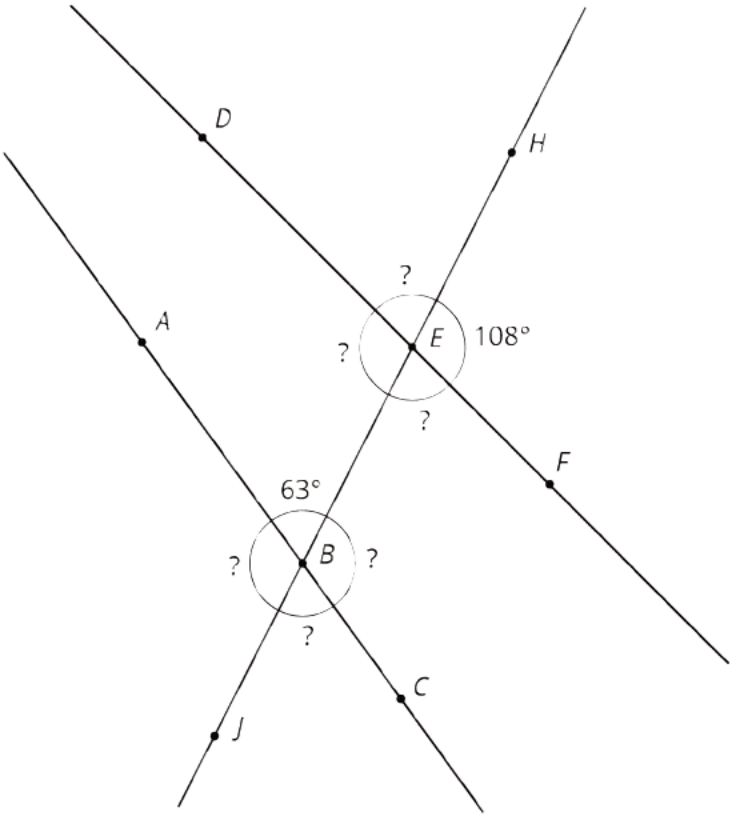
1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.

2. What do you notice about the angles with vertex B and the angles with vertex E?

3. Using what you noticed, find the measures of the four angles at point B in the second diagram. Lines AC and DF are parallel.

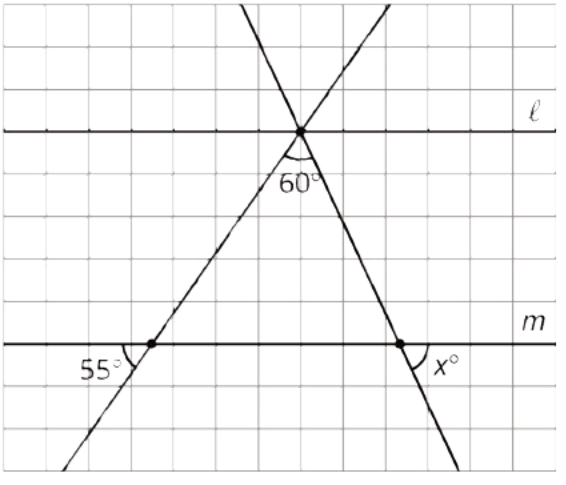


4. The next diagram resembles the first one, but the lines form slightly different angles. Work with your partner to find the six unknown angles with vertices at points B and E.



5. What do you notice about the angles in this diagram as compared to the earlier diagram? How are the two diagrams different? How are they the same?

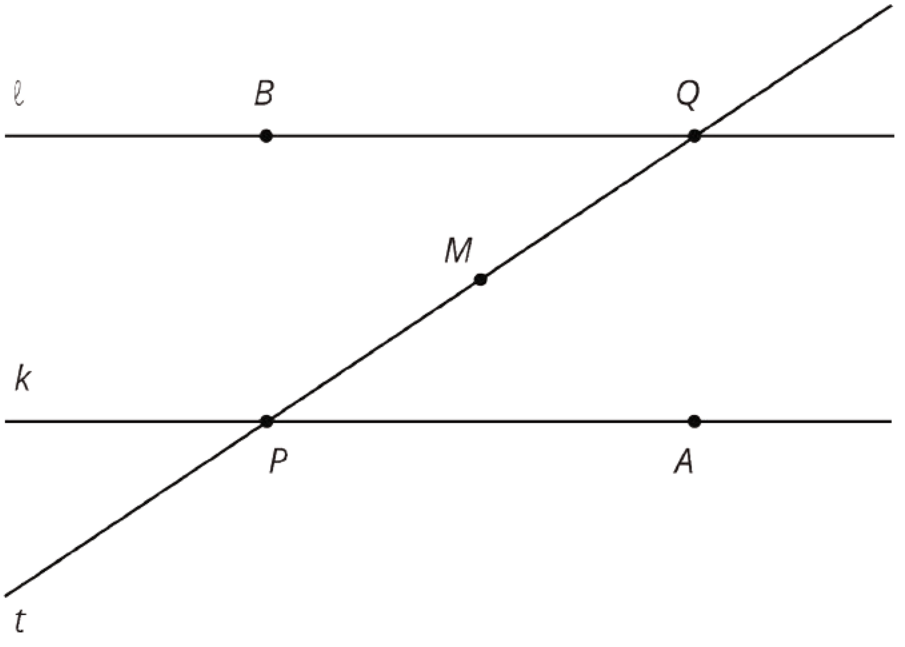
Are you ready for more?



Parallel lines ℓ and m are cut by two transversals which intersect ℓ in the same point. Two angles are marked in the figure. Find the measure x of the third angle.

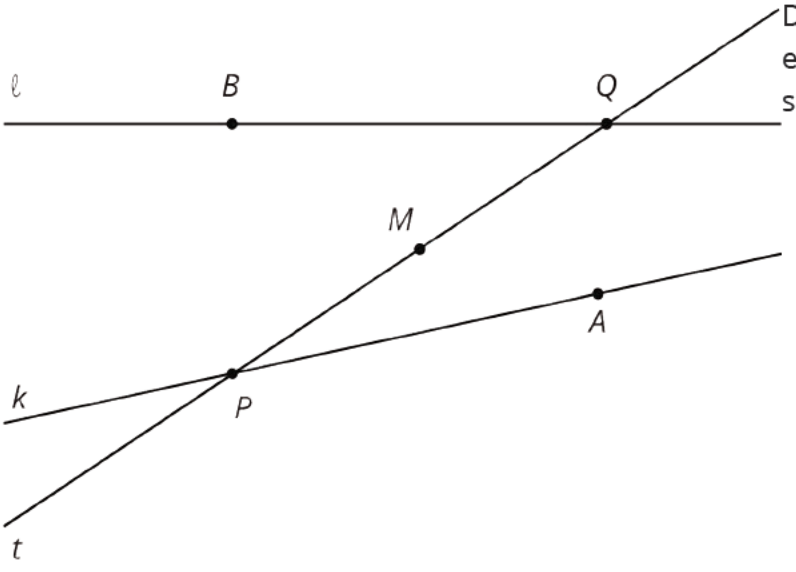
14.3: Alternate Interior Angles Are Congruent

1. Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.



Find a rigid transformation showing that angles MPA and MQB are congruent.

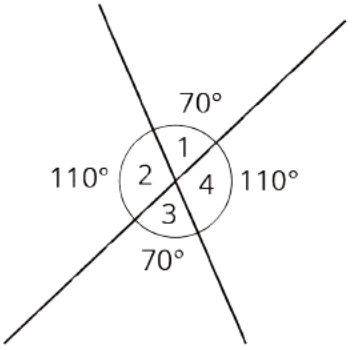
2. In this picture, lines ℓ and k are no longer parallel. M is still the midpoint of segment PQ.



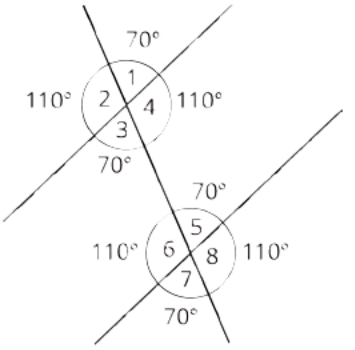
Does your argument in the earlier problem apply in this situation? Explain.

Lesson 14 Summary

When two lines intersect, vertical angles are equal and adjacent angles are supplementary, that is, their measures sum to 180°. For example, in this figure angles 1 and 3 are equal, angles 2 and 4 are equal, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a $180°$ rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections—can you see how rotating $180°$ about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is $70°$ we use vertical angles to see that angle 3 is $70°$, then we use alternate interior angles to see that angle 5 is $70°$, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is $110°$ since $180 - 70 = 110$. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure $70°$, and angles 2, 4, 6, and 8 measure $110°$.

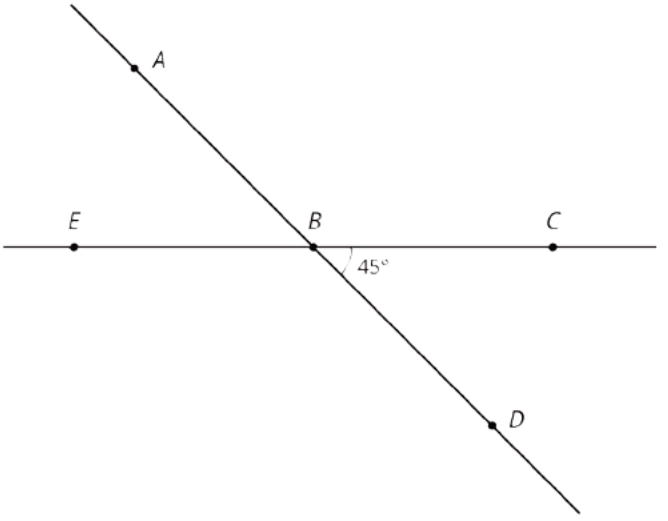
Unit 1 Lesson 14 Cumulative Practice Problems

1. Use the diagram to find the measure of each angle.

   a. $m\angle ABC$

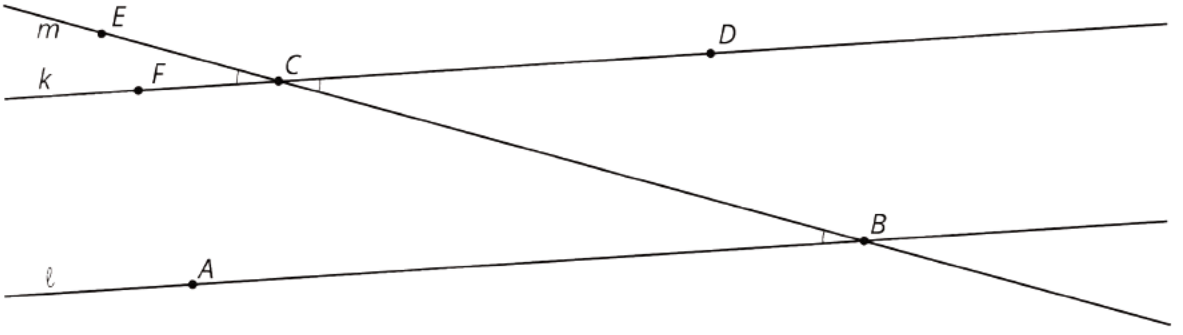
   b. $m\angle EBD$

   c. $m\angle ABE$



(From Unit 1, Lesson 9.)

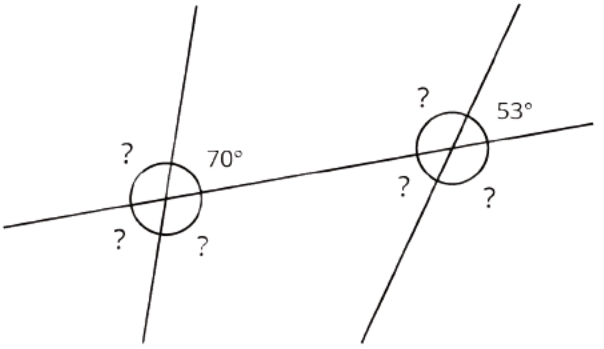
2. Lines k and ℓ are parallel, and the measure of angle ABC is 19 degrees.



   a. Explain why the measure of angle ECF is 19 degrees. If you get stuck, consider translating line ℓ by moving B to C.

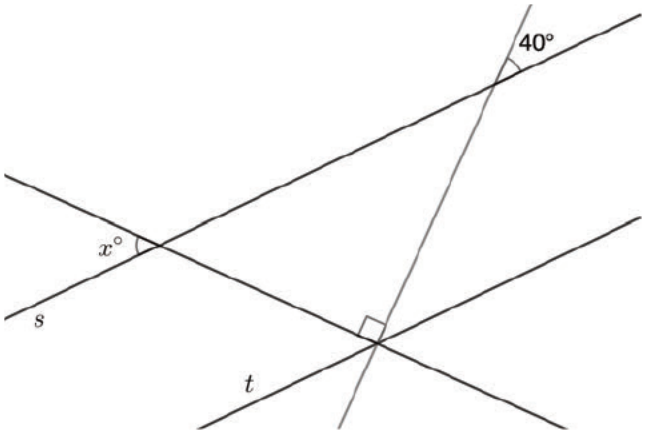
   b. What is the measure of angle BCD? Explain.

3. The diagram shows three lines with some marked angle measures.



Find the missing angle measures marked with question marks.

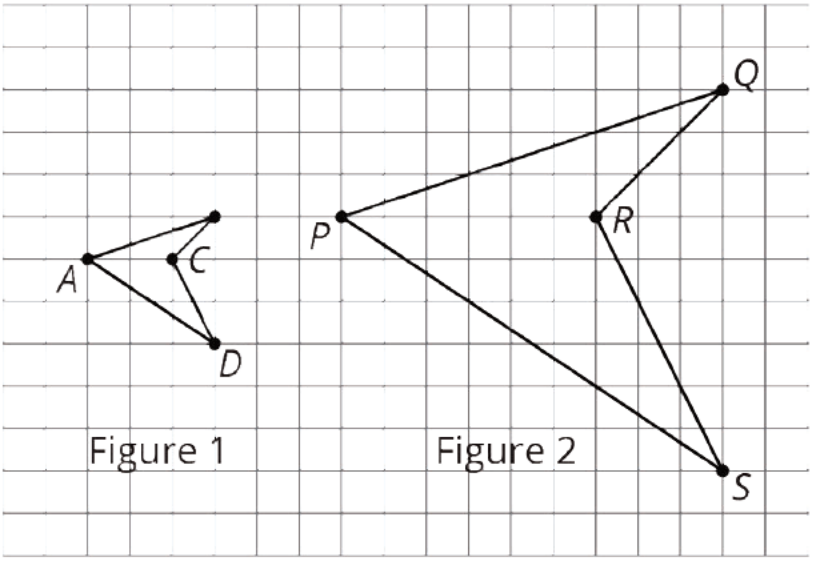
4. Lines s and t are parallel. Find the value of x.



5. The two figures are scaled copies of each other.

a. What is the scale factor that takes Figure 1 to Figure 2?

b. What is the scale factor that takes Figure 2 to Figure 1?



Lesson 15: Adding the Angles in a Triangle

15.1: Can You Draw It?

1. Complete the table by drawing a triangle in each cell that has the properties listed for its column and row. If you think you cannot draw a triangle with those properties, write "impossible" in the cell.

2. Share your drawings with a partner. Discuss your thinking. If you disagree, work to reach an agreement.

	acute (all angles acute)	right (has a right angle)	obtuse (has an obtuse angle)
scalene (side lengths all different)			
isosceles (at least two side lengthsare equal)			
equilateral (three side lengths equal)			

15.2: Find All Three

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Mentally estimate the measures of the other two angles.

2. Find two other students with triangles congruent to yours but with a different angle labeled. Confirm that the triangles are congruent, that each card has a different angle labeled, and that the angle measures make sense.

3. Enter the three angle measures for your triangle on the table your teacher has posted.

15.3: Tear It Up

Your teacher will give you a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

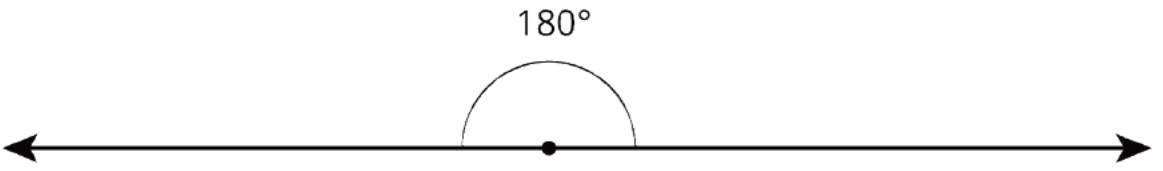
Are you ready for more?

1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?

2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

Lesson 15 Summary

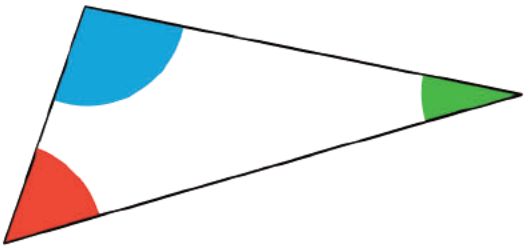
A 180° angle is called a **straight angle** because when it is made with two rays, they point in opposite directions and form a straight line.



If we experiment with angles in a triangle, we find that the sum of the measures of the three angles in each triangle is 180°—the same as a straight angle!

Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.



Unit 1 Lesson 15 Cumulative Practice Problems

1. In triangle ABC, the measure of angle A is 40°.

   a. Give possible measures for angles B and C if triangle ABC is isosceles.

   b. Give possible measures for angles B and C if triangle ABC is right.

2. For each set of angles, decide if there is a triangle whose angles have these measures in degrees:

   a. 60, 60, 60

   b. 90, 90, 45

   c. 30, 40, 50

   d. 90, 45, 45

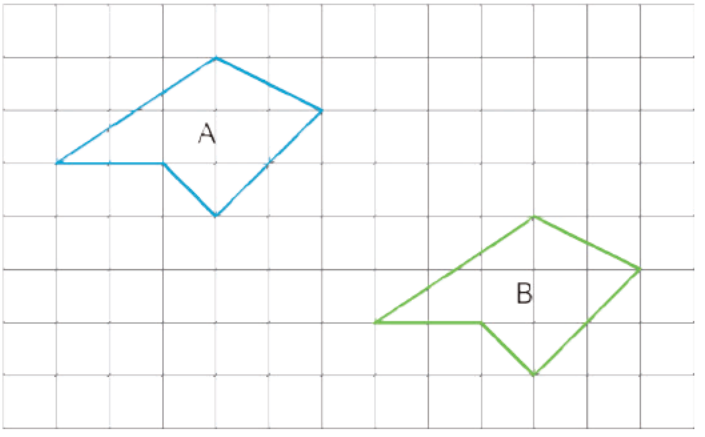
   e. 120, 30, 30

   If you get stuck, consider making a line segment. Then use a protractor to measure angles with the first two angle measures.

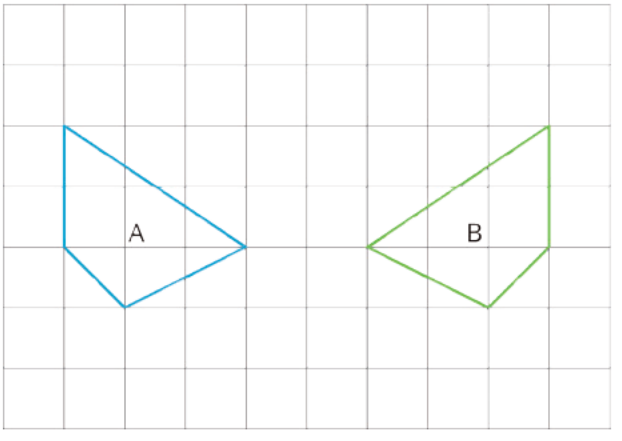
3. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your reasoning.

4. For each pair of polygons, describe the transformation that could be applied to Polygon A to get Polygon B.

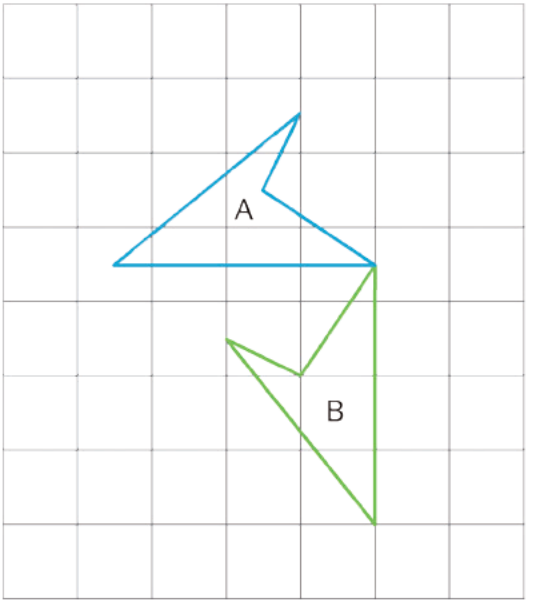
a.



b.

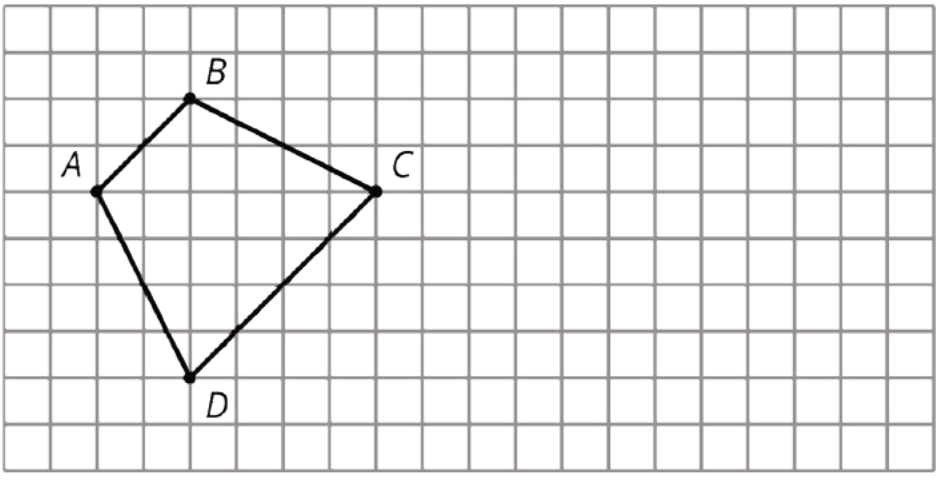


c.



(From Unit 1, Lesson 3.)

5. On the grid, draw a scaled copy of quadrilateral $ABCD$ using a scale factor of $\frac{1}{2}$.



(From Unit 1, Lesson 14.)

Lesson 16: Parallel Lines and the Angles in a Triangle

16.1: True or False: Computational Relationships

Is each equation true or false?

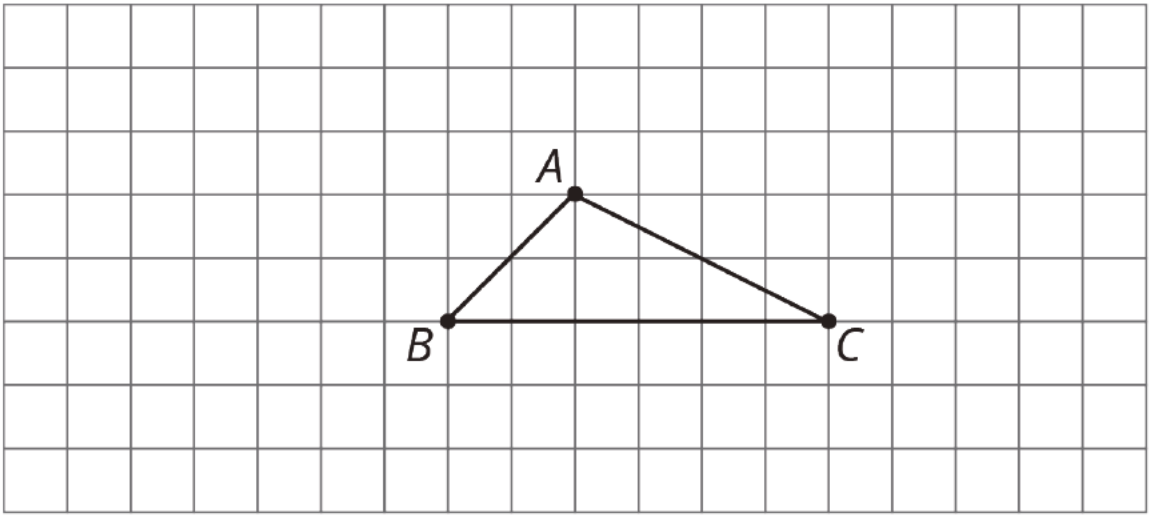
$62 - 28 = 60 - 30$

$3 \cdot -8 = (2 \cdot -8) - 8$

$\frac{16}{-2} + \frac{24}{-2} = \frac{40}{-2}$

16.2: Angle Plus Two

Here is triangle ABC.



1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.

2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.

3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.

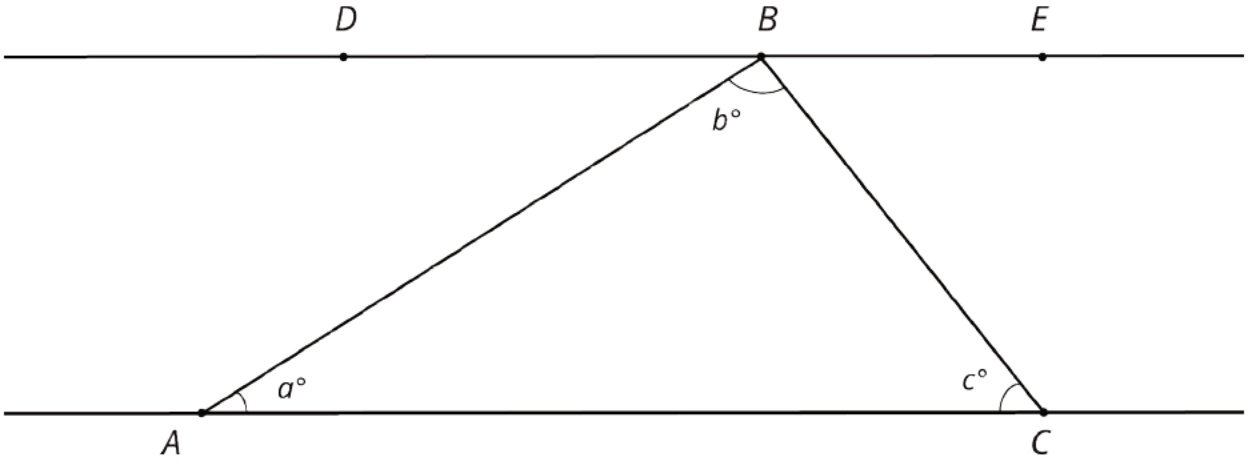
4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? If not, how do you know?

5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? If not, how do you know?

6. What is the sum of the measures of angles ABC, BAC, and ACB?

16.3: Every Triangle in the World

Here is $\triangle ABC$. Line DE is parallel to line AC.



1. What is $m\angle DBA + b + m\angle CBE$? Explain how you know.

2. Use your answer to explain why $a + b + c = 180$.

3. Explain why your argument will work for *any* triangle: that is, explain why the sum of the angle measures in *any* triangle is $180°$.

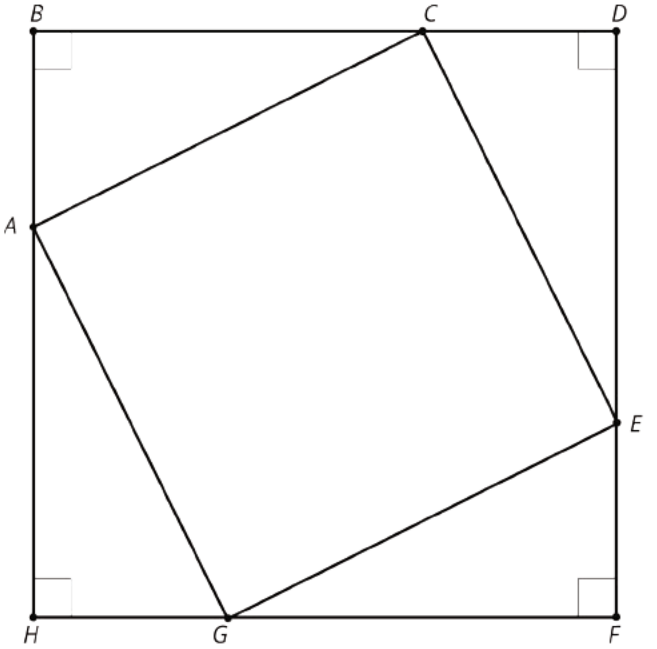
Are you ready for more?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?

2. Come up with an explanation for why anything you notice must be true (hint: draw one diagonal in each quadrilateral).

16.4: Four Triangles Revisited

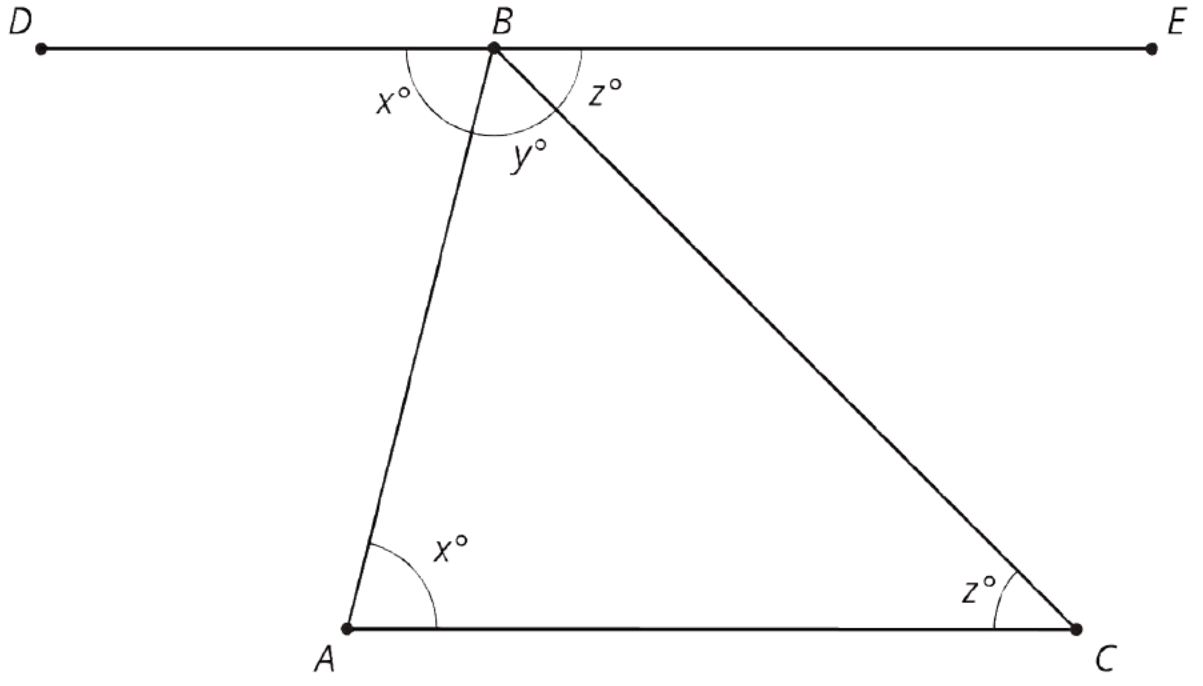
This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.



Given that angle BAC measures 53 degrees, find as many other angle measures as you can.

Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.



A 180 degree rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure: in the picture these angles are marked as x°. A 180 degree rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure: in the picture, these angles are marked as z°. Also, DBE is a straight line because 180 degree rotations take lines to parallel lines. So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in $\triangle ABC$ so the sum of the angles in a triangle is always 180°!

Unit 1 Lesson 16 Cumulative Practice Problems

1. For each triangle, find the measure of the missing angle.

Figure 3

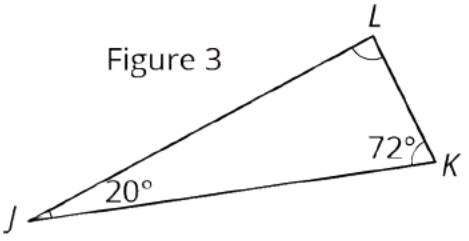


Figure 4

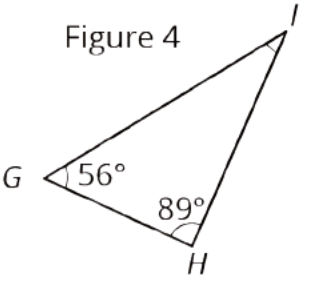


Figure 1

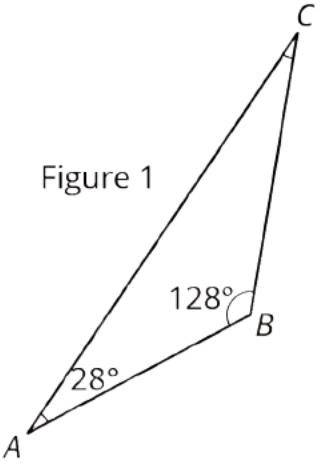
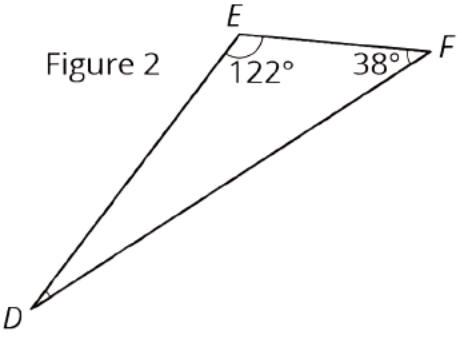
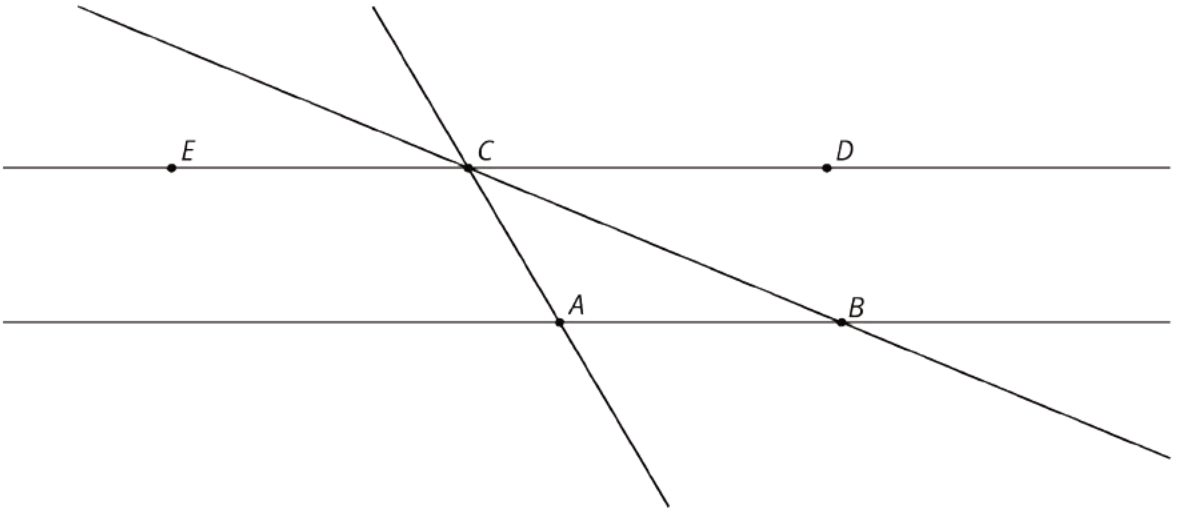


Figure 2



2. Is there a triangle with *two* right angles? Explain your reasoning.

3. In this diagram, lines AB and CD are parallel.



Angle ABC measures 35° and angle BAC measures 115°.

a. What is $m\angle ACE$?

b. What is $m\angle DCB$?

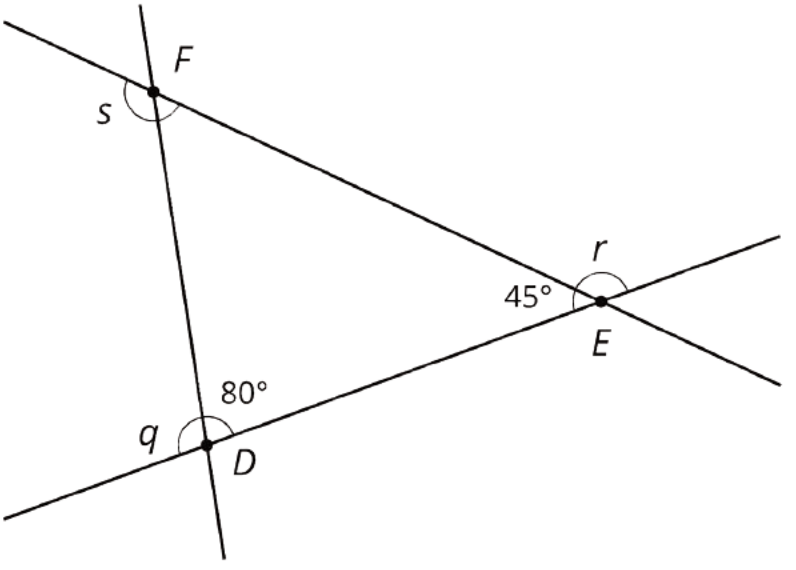
c. What is $m\angle ACB$?

4. Here is a diagram of triangle DEF.

a. Find the measures of angles q, r, and s.

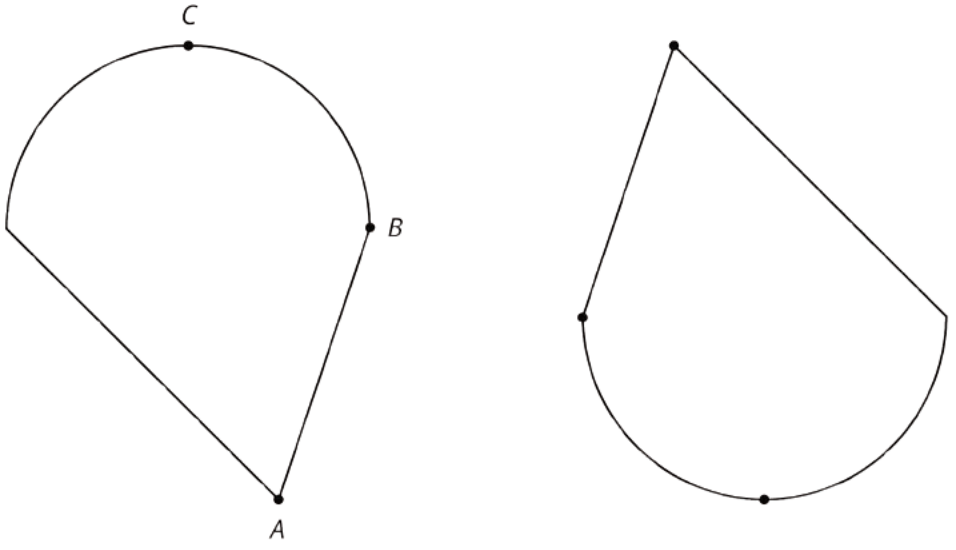
b. Find the sum of the measures of angles q, r, and s.

c. What do you notice about these three angles?



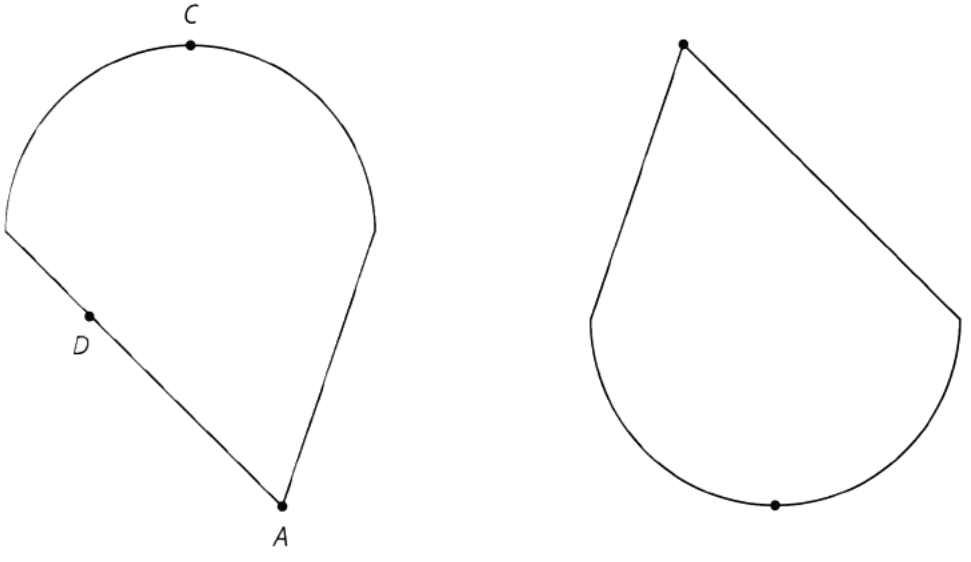
5. The two figures are congruent.

a. Label the points A', B' and C' that correspond to A, B, and C in the figure on the right.



b. If segment AB measures 2 cm, how long is segment $A'B'$? Explain.

c. The point D is shown in addition to A and C. How can you find the point D' that corresponds to D? Explain your reasoning.



(From Unit 1, Lesson 13.)

Lesson 17: Rotate and Tessellate

17.1: Deducing Angle Measures

Your teacher will give you some shapes.

1. How many copies of the equilateral triangle can you fit together around a single vertex, so that the triangles' edges have no gaps or overlaps? What is the measure of each angle in these triangles?

2. What are the measures of the angles in the

    a. square?

    b. hexagon?

    c. parallelogram?

    d. right triangle?

    e. octagon?

    f. pentagon?

17.2: Tessellate This

1. Design your own **tessellation**. You will need to decide which shapes you want to use and make copies. Remember that a tessellation is a repeating pattern that goes on forever to fill up the entire plane.

2. Find a partner and trade pictures. Describe a transformation of your partner's picture that takes the pattern to itself. How many different transformations can you find that take the pattern to itself? Consider translations, reflections, and rotations.

3. If there's time, color and decorate your tessellation.

17.3: Rotate That

1. Make a design with rotational symmetry.

2. Find a partner who has also made a design. Exchange designs and find a transformation of your partner's design that takes it to itself. Consider rotations, reflections, and translations.

3. If there's time, color and decorate your design.

Unit 1 Lesson 17 Cumulative Practice Problems

Credits

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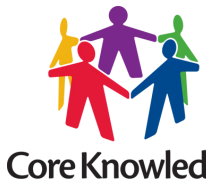

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