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# Scale Drawings

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Scale Drawings
Student Workbook
Core Knowledge Mathematics™
Lesson 1: What are Scaled Copies?

1.1: Printing Portraits

Here is a portrait of a student.

1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?

2. Some of the Portraits A–E are scaled copies of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

3. What do you think “scaled copy” means?
1.2: Scaling F

Here is an original drawing of the letter F and some other drawings.

1. Identify all the drawings that are scaled copies of the original letter F. Explain how you know.

2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?
3. On the grid, draw a different scaled copy of the original letter F.

1.3: Pairs of Scaled Polygons

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a pair of polygons that are scaled copies of one another.
   a. For each match you find, explain to your partner how you know it’s a match.
   b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.

2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. Select one pair of polygons to examine further. Draw both polygons on the grid. Explain or show how you know that one polygon is a scaled copy of the other.

Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.
Lesson 1 Summary
What is a scaled copy of a figure? Let's look at some examples.

The second and third drawings are both scaled copies of the original Y.

![Original Y](image)

However, here, the second and third drawings are not scaled copies of the original W.

![Original W](image)

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.
Unit 1 Lesson 1 Cumulative Practice Problems

1. Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.

![Figures A, 1, 2, 3, 4](image)

2. Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

Do you agree with Tyler? Explain your reasoning.

![Figures A and B](image)
3. Here is a picture of the Rose Bowl Stadium in Pasadena, CA.

Here are some copies of the picture. Select all the pictures that are scaled copies of the original picture.

A

B

C

D

4. Complete each equation with a number that makes it true.
   a. $5 \cdot \underline{\hspace{2cm}} = 15$
   b. $4 \cdot \underline{\hspace{2cm}} = 32$
   c. $6 \cdot \underline{\hspace{2cm}} = 9$
   d. $12 \cdot \underline{\hspace{2cm}} = 3$
Lesson 2: Corresponding Parts and Scale Factors

2.1: Number Talk: Multiplying by a Unit Fraction

Find each product mentally.

\[ \frac{1}{4} \cdot 32 \]

\[ (7.2) \cdot \frac{1}{9} \]

\[ \frac{1}{4} \cdot (5.6) \]
2.2: Corresponding Parts

Here is a figure and two copies, each with some points labeled.

1. Complete this table to show corresponding parts in the three figures.

<table>
<thead>
<tr>
<th>original</th>
<th>copy 1</th>
<th>copy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>point (P)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>segment (LM)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>segment (EF)</td>
<td></td>
<td>point (W)</td>
</tr>
<tr>
<td>angle (KLM)</td>
<td></td>
<td>angle (XYZ)</td>
</tr>
</tbody>
</table>

2. Is either copy a scaled copy of the original figure? Explain your reasoning.

3. Use tracing paper to compare angle \(KLM\) with its corresponding angles in Copy 1 and Copy 2. What do you notice?

4. Use tracing paper to compare angle \(NOP\) with its corresponding angles in Copy 1 and Copy 2. What do you notice?
2.3: Scaled Triangles

Here is Triangle O, followed by a number of other triangles.

Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.

2. As a group, identify all the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.

3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of Triangle O listed in each column.

<table>
<thead>
<tr>
<th>Triangle O</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>
4. Explain or show how each copy has been scaled from the original (Triangle O).

Are you ready for more?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.
Lesson 2 Summary

A figure and its scaled copy have corresponding parts, or parts that are in the same position in relation to the rest of each figure. These parts could be points, segments, or angles. For example, Polygon 2 is a scaled copy of Polygon 1.

![Polygon 1 and Polygon 2 diagrams]

- Each point in Polygon 1 has a corresponding point in Polygon 2. For example, point $B$ corresponds to point $H$ and point $C$ corresponds to point $I$.

- Each segment in Polygon 1 has a corresponding segment in Polygon 2. For example, segment $AF$ corresponds to segment $GL$.

- Each angle in Polygon 1 also has a corresponding angle in Polygon 2. For example, angle $DEF$ corresponds to angle $JKL$.

The scale factor between Polygon 1 and Polygon 2 is 2, because all of the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1. For example, the measure of angle $JKL$ is the same as the measure of angle $DEF$. 
Unit 1 Lesson 2 Cumulative Practice Problems

1. The second H-shaped polygon is a scaled copy of the first.

![Diagram of H-shaped polygons]

a. Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using colored pencils to highlight corresponding parts or labeling some of the vertices.

b. What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

2. Figure B is a scaled copy of Figure A. Select all of the statements that must be true:

A. Figure B is larger than Figure A.

B. Figure B has the same number of edges as Figure A.

C. Figure B has the same perimeter as Figure A.

D. Figure B has the same number of angles as Figure A.

E. Figure B has angles with the same measures as Figure A.
3. Polygon B is a scaled copy of Polygon A.
   
   a. What is the scale factor from Polygon A to Polygon B? Explain your reasoning.

   b. Find the missing length of each side marked with ? in Polygon B.

   c. Determine the measure of each angle marked with ? in Polygon A.

4. Complete each equation with a number that makes it true.
   
   a. \(8 \cdot _____ = 40\)
   
   b. \(8 + _____ = 40\)
   
   c. \(21 \div _____ = 7\)
   
   d. \(21 - _____ = 7\)
   
   e. \(21 \cdot _____ = 7\)
Lesson 3: Making Scaled Copies

3.1: More or Less?

For each problem, select the answer from the two choices.

1. The value of $25 \cdot (8.5)$ is:
   
   a. More than 205
   
   b. Less than 205

2. The value of $(9.93) \cdot (0.984)$ is:
   
   a. More than 10
   
   b. Less than 10

3. The value of $(0.24) \cdot (0.67)$ is:
   
   a. More than 0.2
   
   b. Less than 0.2
3.2: Drawing Scaled Copies

1. Draw a scaled copy of either Figure A or B using a scale factor of 3.

2. Draw a scaled copy of either Figure C or D using a scale factor of $\frac{1}{2}$. 
3.3: Which Operations? (Part 1)

Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.

Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.

1. What operation do you think Diego used to calculate the lengths for his drawing?

2. What operation do you think Jada used to calculate the lengths for her drawing?
3. Did each method produce a scaled copy of the polygon? Explain your reasoning.

3.4: Which Operations? (Part 2)

Andre wants to make a scaled copy of Jada’s drawing so the side that corresponds to 4 units in Jada’s polygon is 8 units in his scaled copy.

1. Andre says “I wonder if I should add 4 units to the lengths of all of the segments?”
   What would you say in response to Andre? Explain or show your reasoning.

2. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.

![Jada's drawing with measurements]

Jada's drawing
Are you ready for more?
The side lengths of Triangle B are all 5 more than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your reasoning.

Lesson 3 Summary
Creating a scaled copy involves multiplying the lengths in the original figure by a scale factor.

For example, to make a scaled copy of triangle $ABC$ where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle $DEF$, each side is 4 times as long as the corresponding side in triangle $ABC$. 

![Diagram](image.png)
Unit 1 Lesson 3 Cumulative Practice Problems

1. Here are 3 polygons.

[Diagram showing three polygons: A, B, and C]

- Draw a scaled copy of Polygon A using a scale factor of 2.
- Draw a scaled copy of Polygon B using a scale factor of $\frac{1}{2}$.
- Draw a scaled copy of Polygon C using a scale factor of $\frac{3}{2}$.

2. Quadrilateral A has side lengths 6, 9, 9, and 12. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side of length 2. What is the perimeter of Quadrilateral B?
3. Here is a polygon on a grid.

Draw a scaled copy of this polygon that has a perimeter of 30 units. What is the scale factor? Explain how you know.

4. Priya and Tyler are discussing the figures shown below. Priya thinks that B, C, and D are scaled copies of A. Tyler says B and D are scaled copies of A. Do you agree with Priya, or do you agree with Tyler? Explain your reasoning.
Lesson 4: Scaled Relationships

4.1: Three Quadrilaterals (Part 1)

Each of these polygons is a scaled copy of the others.

1. Name two pairs of corresponding angles. What can you say about the sizes of these angles?

2. Check your prediction by measuring at least one pair of corresponding angles using a protractor. Record your measurements to the nearest 5°.
4.2: Three Quadrilaterals (Part 2)

Each of these polygons is a scaled copy of the others. You already checked their corresponding angles.

1. The side lengths of the polygons are hard to tell from the grid, but there are other corresponding distances that are easier to compare. Identify the distances in the other two polygons that correspond to $DB$ and $AC$, and record them in the table.

<table>
<thead>
<tr>
<th>quadrilateral</th>
<th>distance that corresponds to $DB$</th>
<th>distance that corresponds to $AC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ABCD$</td>
<td>$DB = 4$</td>
<td>$AC = 6$</td>
</tr>
<tr>
<td>$EFGH$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$IJKL$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Look at the values in the table. What do you notice?

Pause here so your teacher can review your work.
3. The larger figure is a scaled copy of the smaller figure.

![Diagram: Two figures, one scaled to the other]

a. If $AE = 4$, how long is the corresponding distance in the second figure? Explain or show your reasoning.

b. If $IK = 5$, how long is the corresponding distance in the first figure? Explain or show your reasoning.
4.3: Scaled or Not Scaled?

Here are two quadrilaterals.

1. Mai says that Polygon \( ZSCH \) is a scaled copy of Polygon \( XJYN \), but Noah disagrees. Do you agree with either of them? Explain or show your reasoning.

2. Record the corresponding distances in the table. What do you notice?

<table>
<thead>
<tr>
<th>quadrilateral</th>
<th>horizontal distance</th>
<th>vertical distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( XJYN )</td>
<td>( XY = )</td>
<td>( JN = )</td>
</tr>
<tr>
<td>( ZSCH )</td>
<td>( ZC = )</td>
<td>( SH = )</td>
</tr>
</tbody>
</table>

3. Measure at least three pairs of corresponding angles in \( XJYN \) and \( ZSCH \) using a protractor. Record your measurements to the nearest 5°. What do you notice?
4. Do these results change your answer to the first question? Explain.

5. Here are two more quadrilaterals.

Kiran says that Polygon $EFGH$ is a scaled copy of $ABCD$, but Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

**Are you ready for more?**

All side lengths of quadrilateral $MNOP$ are 2, and all side lengths of quadrilateral $QRST$ are 3. Does $MNOP$ have to be a scaled copy of $QRST$? Explain your reasoning.
4.4: Comparing Pictures of Birds

Here are two pictures of a bird. Find evidence that one picture is not a scaled copy of the other. Be prepared to explain your reasoning.
Lesson 4 Summary

When a figure is a scaled copy of another figure, we know that:

- All distances in the copy can be found by multiplying the corresponding distances in the original figure by the same scale factor, whether or not the endpoints are connected by a segment.

For example, Polygon $STUVWX$ is a scaled copy of Polygon $ABCDEF$. The scale factor is 3. The distance from $T$ to $X$ is 6, which is three times the distance from $B$ to $F$.

- All angles in the copy have the same measure as the corresponding angles in the original figure, as in these triangles.

These observations can help explain why one figure is not a scaled copy of another.

For example, even though their corresponding angles have the same measure, the second rectangle is not a scaled copy of the first rectangle, because different pairs of corresponding lengths have different scale factors, $2 \cdot \frac{1}{2} = 1$ but $3 \cdot \frac{2}{3} = 2$. 
Unit 1 Lesson 4 Cumulative Practice Problems

1. Select all the statements that must be true for any scaled copy Q of Polygon P.

   A. The side lengths are all whole numbers.
   B. The angle measures are all whole numbers.
   C. Q has exactly 1 right angle.
   D. If the scale factor between P and Q is \(\frac{1}{5}\), then each side length of P is multiplied by \(\frac{1}{5}\) to get the corresponding side length of Q.
   E. If the scale factor is 2, each angle in P is multiplied by 2 to get the corresponding angle in Q.
   F. Q has 2 acute angles and 3 obtuse angles.

2. Here is Quadrilateral \(ABCD\).

   Quadrilateral \(PQRS\) is a scaled copy of Quadrilateral \(ABCD\). Point \(P\) corresponds to \(A\), \(Q\) to \(B\), \(R\) to \(C\), and \(S\) to \(D\).

   If the distance from \(P\) to \(R\) is 3 units, what is the distance from \(Q\) to \(S\)? Explain your reasoning.
3. Figure 2 is a scaled copy of Figure 1.

![Diagram showing Figure 1 and Figure 2]

Figure 1  Figure 2

a. Identify the points in Figure 2 that correspond to the points $A$ and $C$ in Figure 1. Label them $P$ and $R$. What is the distance between $P$ and $R$?

b. Identify the points in Figure 1 that correspond to the points $Q$ and $S$ in Figure 2. Label them $B$ and $D$. What is the distance between $B$ and $D$?

c. What is the scale factor that takes Figure 1 to Figure 2?

d. $G$ and $H$ are two points on Figure 1, but they are not shown. The distance between $G$ and $H$ is 1. What is the distance between the corresponding points on Figure 2?

4. To make 1 batch of lavender paint, the ratio of cups of pink paint to cups of blue paint is 6 to 5. Find two more ratios of cups of pink paint to cups of blue paint that are equivalent to this ratio.
Lesson 5: The Size of the Scale Factor

5.1: Number Talk: Missing Factor
Solve each equation mentally.

\[ 16x = 176 \]
\[ 16x = 8 \]
\[ 16x = 1 \]
\[ \frac{1}{3}x = 1 \]
\[ \frac{2}{3}x = 1 \]

5.2: Card Sort: Scaled Copies
Your teacher will give you a set of cards. On each card, Figure A is the original and Figure B is a scaled copy.

1. Sort the cards based on their scale factors. Be prepared to explain your reasoning.

2. Examine cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? What do you notice about the scale factors?

3. Examine cards 8 and 12 more closely. What do you notice about the figures? What do you notice about the scale factors?
Are you ready for more?

Triangle B is a scaled copy of Triangle A with scale factor $\frac{1}{2}$.

1. How many times bigger are the side lengths of Triangle B when compared with Triangle A?

2. Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to get Triangle C. How many times bigger will the side lengths of Triangle C be when compared with Triangle A?

3. Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale triangle A $n$ times to get Triangle N, always using a scale factor of $\frac{1}{2}$. How many times bigger will the side lengths of Triangle N be when compared with Triangle A?

5.3: Scaling A Puzzle

Your teacher will give you 2 pieces of a 6-piece puzzle.

1. If you drew scaled copies of your puzzle pieces using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original pieces? How do you know?

2. Create a scaled copy of each puzzle piece on a blank square, with a scale factor of $\frac{1}{2}$.

3. When everyone in your group is finished, put all 6 of the original puzzle pieces together like this:

```
1 2 3
4 5 6
```

Next, put all 6 of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem off? What might have caused those parts to be off?
4. Revise any of the scaled copies that may have been drawn incorrectly.

5. If you were to lose one of the pieces of the original puzzle, but still had the scaled copy, how could you recreate the lost piece?

5.4: Missing Figure, Factor, or Copy

1. What is the scale factor from the original triangle to its copy? Explain or show your reasoning.

![Original triangle and copy](image1)

2. The scale factor from the original trapezoid to its copy is 2. Draw the scaled copy.

![Original trapezoid and copy](image2)
3. The scale factor from the original figure to its copy is $\frac{3}{2}$. Draw the original figure.

4. What is the scale factor from the original figure to the copy? Explain how you know.

5. The scale factor from the original figure to its scaled copy is 3. Draw the scaled copy.
Lesson 5 Summary

The size of the scale factor affects the size of the copy. When a figure is scaled by a scale factor greater than 1, the copy is larger than the original. When the scale factor is less than 1, the copy is smaller. When the scale factor is exactly 1, the copy is the same size as the original.

Triangle $DEF$ is a larger scaled copy of triangle $ABC$, because the scale factor from $ABC$ to $DEF$ is $\frac{3}{2}$. Triangle $ABC$ is a smaller scaled copy of triangle $DEF$, because the scale factor from $DEF$ to $ABC$ is $\frac{2}{3}$.

This means that triangles $ABC$ and $DEF$ are scaled copies of each other. It also shows that scaling can be reversed using reciprocal scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

In other words, if we scale Figure A using a scale factor of 4 to create Figure B, we can scale Figure B using the reciprocal scale factor, $\frac{1}{4}$, to create Figure A.
Unit 1 Lesson 5 Cumulative Practice Problems

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor from one to the other is greater than 1, equal to 1, or less than 1.
   a. from P to Q
   b. from P to R
   c. from Q to S
   d. from Q to R
   e. from S to P
   f. from R to P
   g. from P to S

2. Triangle S and Triangle L are scaled copies of one another.
   a. What is the scale factor from S to L?
   b. What is the scale factor from L to S?
   c. Triangle M is also a scaled copy of S. The scale factor from S to M is \( \frac{3}{2} \). What is the scale factor from M to S?

3. Are two squares with the same side lengths scaled copies of one another? Explain your reasoning.
4. Quadrilateral A has side lengths 2, 3, 5, and 6. Quadrilateral B has side lengths 4, 5, 8, and 10. Could one of the quadrilaterals be a scaled copy of the other? Explain.

(From Unit 1, Lesson 2.)

5. Select all the ratios that are equivalent to the ratio 12 : 3.
   A. 6 : 1
   B. 1 : 4
   C. 4 : 1
   D. 24 : 6
   E. 15 : 6
   F. 1,200 : 300
   G. 112 : 13
Lesson 6: Scaling and Area

6.1: Scaling a Pattern Block

Your teacher will give you some pattern blocks. Work with your group to build the scaled copies described in each question.

![Diagram of pattern blocks: A (blue rhombus), B (green triangle), C (red trapezoid)]

1. How many blue rhombus blocks does it take to build a scaled copy of Figure A:
   a. Where each side is twice as long?
   b. Where each side is 3 times as long?
   c. Where each side is 4 times as long?

2. How many green triangle blocks does it take to build a scaled copy of Figure B:
   a. Where each side is twice as long?
   b. Where each side is 3 times as long?
   c. Using a scale factor of 4?

3. How many red trapezoid blocks does it take to build a scaled copy of Figure C:
   a. Using a scale factor of 2?
   b. Using a scale factor of 3?
   c. Using a scale factor of 4?
6.2: Scaling More Pattern Blocks

Your teacher will assign your group one of these figures.

D

E

F

1. Build a scaled copy of your assigned shape using a scale factor of 2. Use the same shape blocks as in the original figure. How many blocks did it take?

2. Your classmate thinks that the scaled copies in the previous problem will each take 4 blocks to build. Do you agree or disagree? Explain your reasoning.

3. Start building a scaled copy of your assigned figure using a scale factor of 3. Stop when you can tell for sure how many blocks it would take. Record your answer.

4. How many blocks would it take to build scaled copies of your figure using scale factors 4, 5, and 6? Explain or show your reasoning.

5. How is the pattern in this activity the same as the pattern you saw in the previous activity? How is it different?
Are you ready for more?

1. How many blocks do you think it would take to build a scaled copy of one yellow hexagon where each side is twice as long? Three times as long?

2. Figure out a way to build these scaled copies.

3. Do you see a pattern for the number of blocks used to build these scaled copies? Explain your reasoning.

6.3: Area of Scaled Parallelograms and Triangles

1. Your teacher will give you a figure with measurements in centimeters. What is the area of your figure? How do you know?

2. Work with your partner to draw scaled copies of your figure, using each scale factor in the table. Complete the table with the measurements of your scaled copies.

<table>
<thead>
<tr>
<th>scale factor</th>
<th>base (cm)</th>
<th>height (cm)</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Compare your results with a group that worked with a different figure. What is the same about your answers? What is different?

4. If you drew scaled copies of your figure with the following scale factors, what would their areas be? Discuss your thinking. If you disagree, work to reach an agreement. Be prepared to explain your reasoning.

<table>
<thead>
<tr>
<th>scale factor</th>
<th>area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{5} )</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 6 Summary

Scaling affects lengths and areas differently. When we make a scaled copy, all original lengths are multiplied by the scale factor. If we make a copy of a rectangle with side lengths 2 units and 4 units using a scale factor of 3, the side lengths of the copy will be 6 units and 12 units, because $2 \cdot 3 = 6$ and $4 \cdot 3 = 12$.

![Diagram of scaled rectangle]

The area of the copy, however, changes by a factor of $(\text{scale factor})^2$. If each side length of the copy is 3 times longer than the original side length, then the area of the copy will be 9 times the area of the original, because $3 \cdot 3$, or $3^2$, equals 9.

![Diagram of area comparison]

In this example, the area of the original rectangle is $8 \text{ units}^2$ and the area of the scaled copy is $72 \text{ units}^2$, because $9 \cdot 8 = 72$. We can see that the large rectangle is covered by 9 copies of the small rectangle, without gaps or overlaps. We can also verify this by multiplying the side lengths of the large rectangle: $6 \cdot 12 = 72$.

Lengths are one-dimensional, so in a scaled copy, they change by the scale factor. Area is two-dimensional, so it changes by the square of the scale factor. We can see this is true for a rectangle with length $l$ and width $w$. If we scale the rectangle by a scale factor of $s$, we get a rectangle with length $s \cdot l$ and width $s \cdot w$. The area of the scaled rectangle is $A = (s \cdot l) \cdot (s \cdot w)$, so $A = (s^2) \cdot (l \cdot w)$. The fact that the area is multiplied by the square of the scale factor is true for scaled copies of other two-dimensional figures too, not just for rectangles.
Unit 1 Lesson 6 Cumulative Practice Problems

1. On the grid, draw a scaled copy of Polygon Q using a scale factor of 2. Compare the perimeter and area of the new polygon to those of Q.

2. A right triangle has an area of 36 square units.

If you draw scaled copies of this triangle using the scale factors in the table, what will the areas of these scaled copies be? Explain or show your reasoning.

<table>
<thead>
<tr>
<th>scale factor</th>
<th>area (units$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3}$</td>
<td></td>
</tr>
</tbody>
</table>
3. Diego drew a scaled version of a Polygon P and labeled it Q. If the area of Polygon P is 72 square units, what scale factor did Diego use to go from P to Q? Explain your reasoning.

4. Here is an unlabeled polygon, along with its scaled copies Polygons A–D. For each copy, determine the scale factor. Explain how you know.

5. Solve each equation mentally.
   a. \( \frac{1}{7} \cdot x = 1 \)
   b. \( x \cdot \frac{1}{11} = 1 \)
   c. \( 1 + \frac{1}{5} = x \)

(From Unit 1, Lesson 2.)

(From Unit 1, Lesson 5.)
Lesson 7: Scale Drawings

7.1: What is a Scale Drawing?
Here are some drawings of a school bus, a quarter, and the subway lines around Boston, Massachusetts. The first three drawings are scale drawings of these objects.

The next three drawings are not scale drawings of these objects.

Discuss with your partner what a scale drawing is.
7.2: Sizing Up a Basketball Court

Your teacher will give you a scale drawing of a basketball court. The drawing does not have any measurements labeled, but it says that 1 centimeter represents 2 meters.

1. Measure the distances on the scale drawing that are labeled a–d to the nearest tenth of a centimeter. Record your results in the first row of the table.

2. The statement “1 cm represents 2 m” is the scale of the drawing. It can also be expressed as “1 cm to 2 m,” or “1 cm for every 2 m.” What do you think the scale tells us?

3. How long would each measurement from the first question be on an actual basketball court? Explain or show your reasoning.

<table>
<thead>
<tr>
<th>measurement</th>
<th>(a) length of court</th>
<th>(b) width of court</th>
<th>(c) hoop to hoop</th>
<th>(d) 3 point line to sideline</th>
</tr>
</thead>
<tbody>
<tr>
<td>scale drawing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>actual court</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. On an actual basketball court, the bench area is typically 9 meters long.

   a. Without measuring, determine how long the bench area should be on the scale drawing.

   b. Check your answer by measuring the bench area on the scale drawing. Did your prediction match your measurement?
7.3: Tall Structures

Here is a scale drawing of some of the world's tallest structures.

1. About how tall is the actual Willis Tower? About how tall is the actual Great Pyramid? Be prepared to explain your reasoning.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Explain or show your reasoning.

3. Measure the line segment that shows the scale to the nearest tenth of a centimeter. Express the scale of the drawing using numbers and words.

Are you ready for more?

The tallest mountain in the United States, Mount Denali in Alaska, is about 6,190 m tall. If this mountain were shown on the scale drawing, how would its height compare to the heights of the structures? Explain or show your reasoning.
Lesson 7 Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings. On a scale drawing:

- Every part corresponds to something in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A scale tells us how actual measurements are represented on the drawing. For example, if a map has a scale of “1 inch to 5 miles” then a \( \frac{1}{2} \)-inch line segment on that map would represent an actual distance of 2.5 miles.

Sometimes the scale is shown as a segment on the drawing itself. For example, here is a scale drawing of a stop sign with a line segment that represents 25 cm of actual length.

The width of the octagon in the drawing is about three times the length of this segment, so the actual width of the sign is about \( 3 \times 25 \), or 75 cm.

Because a scale drawing is two-dimensional, some aspects of the three-dimensional object are not represented. For example, this scale drawing does not show the thickness of the stop sign.

A scale drawing may not show every detail of the actual object; however, the features that are shown correspond to the actual object and follow the specified scale.
Unit 1 Lesson 7 Cumulative Practice Problems

1. The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.

Use the scales and scale drawings to approximate the actual lengths of:

a. the wingspan of the plane, to the nearest foot
b. the height of the plane, to the nearest foot
c. the length of the Lysander Mk. I, to the nearest meter

2. A blueprint for a building includes a rectangular room that measures 3 inches long and 5.5 inches wide. The scale for the blueprint says that 1 inch on the blueprint is equivalent to 10 feet in the actual building. What are the dimensions of this rectangular room in the actual building?
3. Here is a scale map of Lafayette Square, a rectangular garden north of the White House.

![Lafayette Square Map]

a. The scale is shown in the lower right corner. Find the actual side lengths of Lafayette Square in feet.

b. Use an inch ruler to measure the line segment of the graphic scale. About how many feet does one inch represent on this map?

4. Here is Triangle A. Lin created a scaled copy of Triangle A with an area of 72 square units.

![Scaled Copy of Triangle A]

a. How many times larger is the area of the scaled copy compared to that of Triangle A?

b. What scale factor did Lin apply to the Triangle A to create the copy?

c. What is the length of bottom side of the scaled copy?

(From Unit 1, Lesson 6.)
Lesson 8: Scale Drawings and Maps

8.1: A Train and a Car

Two cities are 243 miles apart.

- It takes a train 4 hours to travel between the two cities at a constant speed.
- A car travels between the two cities at a constant speed of 65 miles per hour.

Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

8.2: Driving on I-90

1. A driver is traveling at a constant speed on Interstate 90 outside Chicago. If she traveled from Point A to Point B in 8 minutes, did she obey the speed limit of 55 miles per hour? Explain your reasoning.

2. A traffic helicopter flew directly from Point A to Point B in 8 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.
8.3: Biking through Kansas

A cyclist rides at a constant speed of 15 miles per hour. At this speed, about how long would it take the cyclist to ride from Garden City to Dodge City, Kansas?

Are you ready for more?

Jada finds a map that says, “Note: This map is not to scale.” What do you think this means? Why is this information important?
Lesson 8 Summary
Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.

Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 miles, so we can estimate the distance between these cities is about 90 miles.

Since 90 miles in 1.5 hours is the same speed as 180 miles in 3 hours, the car is traveling about 60 miles per hour.

<table>
<thead>
<tr>
<th>time (hours)</th>
<th>distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
</tr>
</tbody>
</table>

Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 miles. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.
Unit 1 Lesson 8 Cumulative Practice Problems

1. Here is a map that shows parts of Texas and Oklahoma.

   - About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

   - Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.

2. A local park is in the shape of a square. A map of the local park is made with the scale 1 inch to 200 feet.

   - If the park is shown as a square on the map, each side of which is one foot long, how long is each side of the square park?

   - If a straight path in the park is 900 feet long, how long would the path be when represented on the map?
Lesson 9: Creating Scale Drawings

9.1: Number Talk: Which is Greater?

Without calculating, decide which quotient is larger.

11 ÷ 23 or 7 ÷ 13

0.63 ÷ 2 or 0.55 ÷ 3

15 ÷ \( \frac{1}{3} \) or 15 ÷ \( \frac{1}{4} \)

9.2: Bedroom Floor Plan

Here is a rough sketch of Noah’s bedroom (not a scale drawing).

Noah wants to create a floor plan that is a scale drawing.

1. The actual length of Wall C is 4 m. To represent Wall C, Noah draws a segment 16 cm long. What scale is he using? Explain or show your reasoning.
2. Find another way to express the scale.

3. Discuss your thinking with your partner. How do your scales compare?

4. The actual lengths of Wall A and Wall D are 2.5 m and 3.75 m. Determine how long these walls will be on Noah’s scale floor plan. Explain or show your reasoning.

Are you ready for more?
If Noah wanted to draw another floor plan on which Wall C was 20 cm, would 1 cm to 5 m be the right scale to use? Explain your reasoning.
9.3: Two Maps of Utah

A rectangle around Utah is about 270 miles wide and about 350 miles tall. The upper right corner that is missing is about 110 miles wide and about 70 miles tall.

1. Make a scale drawing of Utah where 1 centimeter represents 50 miles. Make a scale drawing of Utah where 1 centimeter represents 75 miles.

2. How do the two drawings compare? How does the choice of scale influence the drawing?
Lesson 9 Summary

If we want to create a scale drawing of a room’s floor plan that has the scale “1 inch to 4 feet,” we can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for our drawing.

Suppose the longest wall is 15 feet long. We should draw a line 3.75 inches long to represent this wall, because $15 \div 4 = 3.75$.

There is more than one way to express this scale. These three scales are all equivalent, since they represent the same relationship between lengths on a drawing and actual lengths:

- 1 inch to 4 feet
- $\frac{1}{2}$ inch to 2 feet
- $\frac{1}{4}$ inch to 1 foot

Any of these scales can be used to find actual lengths and scaled lengths (lengths on a drawing). For instance, we can tell that, at this scale, an 8-foot long wall should be 2 inches long on the drawing because $\frac{1}{4} \cdot 8 = 2$.

The size of a scale drawing is influenced by the choice of scale. For example, here is another scale drawing of the same room using the scale 1 inch to 8 feet.

Notice this drawing is smaller than the previous one. Since one inch on this drawing represents twice as much actual distance, each side length only needs to be half as long as it was in the first scale drawing.
Unit 1 Lesson 9 Cumulative Practice Problems

1. An image of a book shown on a website is 1.5 inches wide and 3 inches tall on a computer monitor. The actual book is 9 inches wide.
   a. What scale is being used for the image?
   b. How tall is the actual book?

2. The flag of Colombia is a rectangle that is 6 ft long with three horizontal strips.
   o The top stripe is 2 ft tall and is yellow.
   o The middle stripe is 1 ft tall and is blue.
   o The bottom stripe is also 1 ft tall and is red.
   a. Create a scale drawing of the Colombian flag with a scale of 1 cm to 2 ft.
   b. Create a scale drawing of the Colombian flag with a scale of 2 cm to 1 ft.
3. These triangles are scaled copies of each other.

For each pair of triangles listed, the area of the second triangle is how many times larger than the area of the first?

a. Triangle G and Triangle F
b. Triangle G and Triangle B
c. Triangle B and Triangle F
d. Triangle F and Triangle H
e. Triangle G and Triangle H
f. Triangle H and Triangle B

(From Unit 1, Lesson 6.)
4. Here is an unlabeled rectangle, followed by other quadrilaterals that are labeled.

a. Select all quadrilaterals that are scaled copies of the unlabeled rectangle. Explain how you know.

b. On graph paper, draw a different scaled version of the original rectangle.

(From Unit 1, Lesson 3.)
Lesson 10: Changing Scales in Scale Drawings

10.1: Appropriate Measurements

1. If a student uses a ruler like this to measure the length of their foot, which choices would be appropriate measurements? Select all that apply. Be prepared to explain your reasoning.

<table>
<thead>
<tr>
<th>Inches</th>
<th>Centimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

a. $9 \frac{1}{4}$ inches

b. $9 \frac{5}{64}$ inches

c. 23.47659 centimeters

d. 23.5 centimeters

e. 23.48 centimeters

2. Here is a scale drawing of an average seventh-grade student's foot next to a scale drawing of a foot belonging to the person with the largest feet in the world. Estimate the length of the larger foot.
10.2: Same Plot, Different Drawings

Here is a map showing a plot of land in the shape of a right triangle.

1. Your teacher will assign you a scale to use. On centimeter graph paper, make a scale drawing of the plot of land. Make sure to write your scale on your drawing.

2. What is the area of the triangle you drew? Explain or show your reasoning.

3. How many square meters are represented by 1 square centimeter in your drawing?

4. After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?
Are you ready for more?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1. Noah uses the scale 1 cm to 200 m.
2. Elena uses the scale 2 cm to 25 m.

10.3: A New Drawing of the Playground

Here is a scale drawing of a playground.

1. Make another scale drawing of the same playground at a scale of 1 centimeter to 20 meters.

2. How do the two scale drawings compare?
Lesson 10 Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 meters. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \times 10 = 30$ and $3 \times 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing, because $3^2 = 9$. 
Unit 1 Lesson 10 Cumulative Practice Problems

1. Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

   a. How long and how wide is the actual swimming pool?

   b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

   c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

2. A map of a park has a scale of 1 inch to 1,000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.
3. On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in$^2$. Han says that the actual area of the restaurant is 720 ft$^2$. Do you agree or disagree? Explain your reasoning.

4. If Quadrilateral $P$ is a scaled copy of Quadrilateral $Q$ created with a scale factor of 3, what is the perimeter of $Q$?

(From Unit 1, Lesson 3.)

5. Triangle $DEF$ is a scaled copy of triangle $ABC$. For each of the following parts of triangle $ABC$, identify the corresponding part of triangle $DEF$.

- angle $ABC$
- angle $BCA$
- segment $AC$
- segment $BA$

(From Unit 1, Lesson 2.)
Lesson 11: Scales without Units

11.1: One to One Hundred

A map of a park says its scale is 1 to 100.

1. What do you think that means?

2. Give an example of how this scale could tell us about measurements in the park.

11.2: Apollo Lunar Module

Your teacher will give you a drawing of the Apollo Lunar Module. It is drawn at a scale of 1 to 50.

1. The “legs” of the spacecraft are its landing gear. Use the drawing to estimate the actual length of each leg on the sides. Write your answer to the nearest 10 centimeters. Explain or show your reasoning.

2. Use the drawing to estimate the actual height of the Apollo Lunar Module to the nearest 10 centimeters. Explain or show your reasoning.
3. Neil Armstrong was 71 inches tall when he went to the surface of the Moon in the Apollo Lunar Module. How tall would he be in the drawing if he were drawn with his height to scale? Show your reasoning.

4. Sketch a stick figure to represent yourself standing next to the Apollo Lunar Module. Make sure the height of your stick figure is to scale. Show how you determined your height on the drawing.

**Are you ready for more?**

The table shows the distance between the Sun and 8 planets in our solar system.

<table>
<thead>
<tr>
<th>planet</th>
<th>average distance (millions of miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>35</td>
</tr>
<tr>
<td>Venus</td>
<td>67</td>
</tr>
<tr>
<td>Earth</td>
<td>93</td>
</tr>
<tr>
<td>Mars</td>
<td>142</td>
</tr>
<tr>
<td>Jupiter</td>
<td>484</td>
</tr>
<tr>
<td>Saturn</td>
<td>887</td>
</tr>
<tr>
<td>Uranus</td>
<td>1,784</td>
</tr>
<tr>
<td>Neptune</td>
<td>2,795</td>
</tr>
</tbody>
</table>

1. If you wanted to create a scale model of the solar system that could fit somewhere in your school, what scale would you use?

2. The diameter of Earth is approximately 8,000 miles. What would the diameter of Earth be in your scale model?
11.3: Same Drawing, Different Scales

A rectangular parking lot is 120 feet long and 75 feet wide.

- Lin made a scale drawing of the parking lot at a scale of 1 inch to 15 feet. The drawing she produced is 8 inches by 5 inches.

- Diego made another scale drawing of the parking lot at a scale of 1 to 180. The drawing he produced is also 8 inches by 5 inches.

1. Explain or show how each scale would produce an 8 inch by 5 inch drawing.

2. Make another scale drawing of the same parking lot at a scale of 1 inch to 20 feet. Be prepared to explain your reasoning.

3. Express the scale of 1 inch to 20 feet as a scale without units. Explain your reasoning.
Lesson 11 Summary

In some scale drawings, the scale specifies one unit for the distances on the drawing and a different unit for the actual distances represented. For example, a drawing could have a scale of 1 cm to 10 km.

In other scale drawings, the scale does not specify any units at all. For example, a map may simply say the scale is 1 to 1,000. In this case, the units for the scaled measurements and actual measurements can be any unit, so long as the same unit is being used for both. So if a map of a park has a scale 1 to 1,000, then 1 inch on the map represents 1,000 inches in the park, and 12 centimeters on the map represent 12,000 centimeters in the park. In other words, 1,000 is the scale factor that relates distances on the drawing to actual distances, and \( \frac{1}{1000} \) is the scale factor that relates an actual distance to its corresponding distance on the drawing.

A scale with units can be expressed as a scale without units by converting one measurement in the scale into the same unit as the other (usually the unit used in the drawing). For example, these scales are equivalent:

- 1 inch to 200 feet
- 1 inch to 2,400 inches (because there are 12 inches in 1 foot, and 200 \cdot 12 = 2,400)
- 1 to 2,400

This scale tells us that all actual distances are 2,400 times their corresponding distances on the drawing, and distances on the drawing are \( \frac{1}{2400} \) times the actual distances they represent.
Unit 1 Lesson 11 Cumulative Practice Problems

1. A scale drawing of a car is presented in the following three scales. Order the scale drawings from smallest to largest. Explain your reasoning. (There are about 1.1 yards in a meter, and 2.54 cm in an inch.)
   
   a. 1 in to 1 ft
   
   b. 1 in to 1 m
   
   c. 1 in to 1 yd

2. Which scales are equivalent to 1 inch to 1 foot? Select all that apply.
   
   A. 1 to 12
   
   B. $\frac{1}{12}$ to 1
   
   C. 100 to 0.12
   
   D. 5 to 60
   
   E. 36 to 3
   
   F. 9 to 108

3. A model airplane is built at a scale of 1 to 72. If the model plane is 8 inches long, how many feet long is the actual airplane?
4. Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of A with a shortest side length equal to 2. Jada says, “Since the side lengths go down by 1 in this scaling, the perimeter goes down by 4 in total.” Do you agree with Jada? Explain your reasoning.

(From Unit 1, Lesson 3.)

5. Polygon B is a scaled copy of Polygon A using a scale factor of 5. Polygon A’s area is what fraction of Polygon B’s area?

(From Unit 1, Lesson 6.)

6. Figures R, S, and T are all scaled copies of one another. Figure S is a scaled copy of R using a scale factor of 3. Figure T is a scaled copy of S using a scale factor of 2. Find the scale factors for each of the following:
   a. From T to S
   b. From S to R
   c. From R to T
   d. From T to R

(From Unit 1, Lesson 5.)
Lesson 12: Units in Scale Drawings

12.1: Centimeters in a Mile
There are 2.54 cm in an inch, 12 inches in a foot, and 5,280 feet in a mile. Which expression gives the number of centimeters in a mile? Explain your reasoning.

1. \[ \frac{2.54}{12 \cdot 5,280} \]

2. \[ 5,280 \cdot 12 \cdot (2.54) \]

3. \[ \frac{1}{5,280 \cdot 12 \cdot (2.54)} \]

4. \[ 5,280 + 12 + 2.54 \]

5. \[ \frac{5,280 \cdot 12}{2.54} \]

12.2: Card Sort: Scales
Your teacher will give you some cards with a scale on each card.

1. Sort the cards into sets of equivalent scales. Be prepared to explain how you know that the scales in each set are equivalent. Each set should have at least two cards.

2. Trade places with another group and check each other's work. If you disagree about how the scales should be sorted, work to reach an agreement.

Pause here so your teacher can review your work.

3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.
12.3: The World’s Largest Flag

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

<table>
<thead>
<tr>
<th></th>
<th>flag length</th>
<th>flag height</th>
<th>height of crescent moon</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual</td>
<td>396 m</td>
<td>99 m</td>
<td></td>
</tr>
<tr>
<td>at 1 to 2,000 scale</td>
<td>13.2 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

   a. 1 cm to _________ cm
   b. 1 cm to _________ m
   c. 1 cm to _________ km
   d. 2 m to _________ m
   e. 5 cm to _________ m
   f. _________ cm to 1,000 m
   g. _________ mm to 20 m

3. a. What is the area of the large flag?
   b. What is the area of the smaller flag?
   c. The area of the large flag is how many times the area of the smaller flag?
12.4: Pondering Pools

Your teacher will give you a floor plan of a recreation center.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.

2. Find the actual area of the large rectangular pool. Show your reasoning.

3. The kidney-shaped pool has an area of 3.2 cm$^2$ on the drawing. What is its actual area? Explain or show your reasoning.

Are you ready for more?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 units$^2$, what is the area of Square B?

2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 units$^3$, what is the volume of Cube B?

3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the “volume” of Hypercube A is 10 units$^4$, what do you think the “volume” of Hypercube B is?
Lesson 12 Summary

Sometimes scales come with units, and sometimes they don’t. For example, a map of Nebraska may have a scale of 1 mm to 1 km. This means that each millimeter of distance on the map represents 1 kilometer of distance in Nebraska. Notice that there are 1,000 millimeters in 1 meter and 1,000 meters in 1 kilometer. This means there are $1,000 \times 1,000$ or $1,000,000$ millimeters in 1 kilometer. So, the same scale without units is 1 to 1,000,000, which means that each unit of distance on the map represents 1,000,000 units of distance in Nebraska. This is true for any choice of unit to express the scale of this map.

Sometimes when a scale comes with units, it is useful to rewrite it without units. For example, let’s say we have a different map of Rhode Island, and we want to use the two maps to compare the size of Nebraska and Rhode Island. It is important to know if the maps are at the same scale. The scale of the map of Rhode Island is 1 inch to 10 miles. There are 5,280 feet in 1 mile, and 12 inches in 1 foot, so there are 63,360 inches in 1 mile (because $5,280 \times 12 = 63,360$). Therefore, there are 633,600 inches in 10 miles. The scale of the map of Rhode Island without units is 1 to 633,600. The two maps are not at the same scale, so we should not use these maps to compare the size of Nebraska to the size of Rhode Island.

Here is some information about equal lengths that you may find useful.

<table>
<thead>
<tr>
<th>Customary Units</th>
<th>Metric Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 foot (ft) = 12 inches (in)</td>
<td>1 meter (m) = 1,000 millimeters (mm)</td>
</tr>
<tr>
<td>1 yard (yd) = 36 inches</td>
<td>1 meter = 100 centimeters</td>
</tr>
<tr>
<td>1 mile = 5,280 feet</td>
<td>1 kilometer (km) = 1,000 meters</td>
</tr>
</tbody>
</table>

Equal Lengths in Different Systems

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch = 2.54 centimeters</td>
<td>1 centimeter $\approx$ 0.39 inch</td>
</tr>
<tr>
<td>1 foot $\approx$ 0.30 meter</td>
<td>1 meter $\approx$ 39.37 inches</td>
</tr>
<tr>
<td>1 mile $\approx$ 1.61 kilometers</td>
<td>1 kilometer $\approx$ 0.62 mile</td>
</tr>
</tbody>
</table>
Unit 1 Lesson 12 Cumulative Practice Problems

1. The Empire State Building in New York City is about 1,450 feet high (including the antenna at the top) and 400 feet wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an \( 8\frac{1}{2} \text{-inch-by-11-inch} \) piece of paper. Select a scale that you think is the most appropriate for the scale drawing. Explain your reasoning.
   a. 1 inch to 1 foot
   b. 1 inch to 100 feet
   c. 1 inch to 1 mile
   d. 1 centimeter to 1 meter
   e. 1 centimeter to 50 meters
   f. 1 centimeter to 1 kilometer

2. Elena finds that the area of a house on a scale drawing is 25 square inches. The actual area of the house is 2.025 square feet. What is the scale of the drawing?

3. Which of these scales are equivalent to 3 cm to 4 km? Select all that apply. Recall that 1 inch is 2.54 centimeters.
A. 0.75 cm to 1 km
B. 1 cm to 12 km
C. 6 mm to 2 km
D. 0.3 mm to 40 m
E. 1 inch to 7.62 km

4. These two triangles are scaled copies of one another. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Explain or show how you know.

5. Water costs $1.25 per bottle. At this rate, what is the cost of:
   a. 10 bottles?
   b. 20 bottles?
   c. 50 bottles?
6. The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

   a. Complete the table for 2, 3, and 4 batches.

   b. How much water and detergent is needed for 8 batches? Explain your reasoning.

<table>
<thead>
<tr>
<th>number of batches</th>
<th>cups of water</th>
<th>cups of detergent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 13: Draw It to Scale

13.1: Which Measurements Matter?
Which measurements would you need in order to draw a scale floor plan of your classroom? List which parts of the classroom you would measure and include in the drawing. Be as specific as possible.

13.2: Creating a Floor Plan (Part 1)
1. On a blank sheet of paper, make a rough sketch of a floor plan of the classroom. Include parts of the room that the class has decided to include or that you would like to include. Accuracy is not important for this rough sketch, but be careful not to omit important features like a door.

2. Trade sketches with a partner and check each other’s work. Specifically, check if any parts are missing or incorrectly placed. Return their work and revise your sketch as needed.

3. Discuss with your group a plan for measuring. Work to reach an agreement on:
   ○ Which classroom features must be measured and which are optional.
   ○ The units to be used.
   ○ How to record and organize the measurements (on the sketch, in a list, in a table, etc.).
   ○ How to share the measuring and recording work (or the role each group member will play).

4. Gather your tools, take your measurements, and record them as planned. Be sure to double-check your measurements.

5. Make your own copy of all the measurements that your group has gathered, if you haven’t already done so. You will need them for the next activity.

13.3: Creating a Floor Plan (Part 2)
Your teacher will give you several paper options for your scale floor plan.
1. Determine an appropriate scale for your drawing based on your measurements and your paper choice. Your floor plan should fit on the paper and not end up too small.

2. Use the scale and the measurements your group has taken to draw a scale floor plan of the classroom. Make sure to:
   - Show the scale of your drawing.
   - Label the key parts of your drawing (the walls, main openings, etc.) with their actual measurements.
   - Show your thinking and organize it so it can be followed by others.

**Are you ready for more?**

1. If the flooring material in your classroom is to be replaced with 10-inch by 10-inch tiles, how many tiles would it take to cover the entire room? Use your scale drawing to approximate the number of tiles needed.

2. How would using 20-inch by 20-inch tiles (instead of 10-inch by 10-inch tiles) change the number of tiles needed? Explain your reasoning.

**13.4: Creating a Floor Plan (Part 3)**

1. Trade floor plans with another student who used the same paper size as you. Discuss your observations and thinking.

2. Trade floor plans with another student who used a different paper size than you. Discuss your observations and thinking.

3. Based on your discussions, record ideas for how your floor plan could be improved.
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- Probability and Sampling
- Putting It All Together

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