

**Core Knowledge  
Mathematics for Teachers II:  
Basic Algebra and Geometry Concepts  
Syllabus**

This is a syllabus for the second half of a two-semester course in mathematics for prospective K-8 teachers. Because this course builds on knowledge acquired in Mathematics for Teachers I and uses many of the same books, that course should be a prerequisite for this one, and the two should be offered in consecutive semesters.

The syllabus assumes that 13 full weeks are available and provides a breakdown of topics by week. This scheduling of topics is necessarily approximate since different classes of students proceed at different paces, sometimes more quickly one week and more slowly during another.

The syllabus is arranged so that instructors may use one of two widely used textbooks for these courses (but not both). They are:

*A Problem Solving Approach to Mathematics for Elementary School Teachers* (Seventh Edition) by Billstein, Libeskind, Lott, published by Addison Wesley Longman.

*Mathematics for elementary School Teachers: A Contemporary Approach* (Fifth Edition), by Musser, Burger, and Peterson, published by Wiley & Sons, Inc.

In addition, this syllabus relies heavily on *The Winning Equation* for classroom presentations. *The Winning Equation* was created by the Curriculum and Instruction Steering Committee (CISC) of the California County Superintendents Educational Services Association (CCSESA). This material consists of master copies of overhead slides, and facilitator's notes, and is part of a California State approved training for grade four through seven teachers, under California's Assembly Bill 1331. *The Winning Equation* modules require an overhead projector, but otherwise they are easy to use. The slides are generally self-explanatory and develop their respective topics coherently and gradually. Instructors need only project them in order, comment on them, and use them to stimulate classroom discussion. The facilitator notes are generally not needed, but they are available to assist instructors. Include time for worksheets, with labels like H-2, H-20, etc. "H" stands for "Handout" and "T" stands for "Transparency." It is not necessary to use the pre- or post-tests from this series, but they are available if the instructor wishes to use them or adapt them. The relevant portions of *The Winning Equation* for this semester are available on Professor David Klein's website (<http://www.csun.edu/~vcmath00m/WE.html>).

The second semester course also relies on the following reference available in popular bookstores:

*Algebra Success in 20 Minutes A Day*, by Barbara Jund from Learning Express Skill Builders, copyright 2000 LearningExpress, LLC.

It is not essential that the instructor use this particular reference, and this one does have shortcomings, though the exercises are useful. Any material that provides students with practice and review of middle and high school algebra would suffice.

Finally, the syllabus makes reference to the following secondary materials, which may be used at the discretion of the instructor:

*Knowing and Teaching Elementary Mathematics*, by Liping Ma, published by Lawrence Erlbaum Associates, copyright 1999. This reference can be used for supplementary reading assignments made approximately a week in advance of associated material from the lectures. The syllabus includes suggestions for the use of this book.

*Algebra*, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993

*The Role of Long Division in the K-12 Curriculum*, by David Klein and R. James Milgram is posted on the "Mathematically Correct Website" or at:  
<http://www.mathematicallycorrect.com/> or the paper may be downloaded from:  
<ftp://math.stanford.edu/pub/papers/milgram/long-division/longdivisiondone.htm>

**General Instruction:** Calculators should not be used by students in the instructional components for this course, unless explicitly indicated in the syllabus.

**Use of this Syllabus:** This syllabus was developed by David Klein, professor of mathematics at California State University, Northridge, as part of *What Elementary Teachers Need to Know*, a teacher education initiative developed by the Core Knowledge Foundation. Although the syllabus is copyrighted by the foundation, and may not be marketed by third parties, anyone who wishes to use, reproduce, or adapt it for educational purposes is welcome to do so. However, we do ask individuals using this syllabus to notify us so we can assess the distribution and spread of the syllabi and serve as a repository of information about how they may be improved and more effectively used. Please contact us at <http://coreknowledge.org/CK/contact.htm>.

## **Week 1**

### Topic 1. Introduction to Geometry

Billstein et al: Assign Section 9.1, "Basic Notions" for review, but there should be no need to cover this in class, unless there are questions. Assign Section 9.2, "Polygons," but tell students that they may ignore material on curves, simple closed curves, and the oblique reference to the Jordan Curve Theorem in figure 9-14 on page 477. Be aware of the contradictory definitions of "trapezoid" on pages 480 and 481. Assign Section 9.3, "More About Angles." Assign representative problems from these sections.

Musser et al: Assign Section 12.2, "Analyzing Shapes." This section gives a general description of symmetry and can be covered lightly and quickly. Assign Section 12.3, "Properties of Geometric Shapes: Lines and Angles." This section culminates with a derivation of the sum of degrees in all three angles of a triangle.

For Presentation in Class:

Use the overhead transparency masters to make overhead slides from Module 9, (Measurement and Geometry) from "The Winning Equation." Use the slides numbered T-22 to T-27/H-27. These slides develop a proof that the sum of degrees in all three angles of a triangle is 180.

## **Week 2**

### Topic 2. Polygons and Dimensional Analysis

Billstein et al: Section 9.4, "Geometry in Three Dimensions," Section 11.1, "Linear Measure." Include an explanation of "dimensional analysis," that is explain how units may be multiplied or cancelled as are numerators and denominators in fractions. For example, miles/foot x feet/second x seconds = miles by cancellation of units.

Musser et al: Section 12.4, "Regular Polygons and Tessellations." This section gives the sum of measures of angles as  $(n - 2) 180$  in a regular polygon with  $n$  sides. The instructor should explain that this result also holds for convex polygons in general. Assign also Section 12.5, "Describing Three Dimensional Shapes." Include Section 13.1, "Measurement with nonstandard and Standard Units," but focus on the subsection, "Dimensional Analysis."

Supplemental Reading Assignment for the next topic: Chapter 4 of "Knowing and Teaching Elementary Mathematics," by Liping Ma.

## **Week 3**

### Topic 3. Perimeter, Circumference, Area, and the Pythagorean Theorem.

Perimeters, Circumference, Areas of rectangles, parallelograms, triangles, trapezoid, circle, Pythagorean Theorem.

Billstein et al: Assign Section 11.2, "Areas of Polygons and Circles," Section 11. 3, "The Pythagorean Theorem," but postpone the subsections, "Converse of the Pythagorean Theorem" and "The Distance Formula: An Application of the Pythagorean Theorem" until week 8. Also assign Section 11. 4, "Surface Area."

Musser et al: Section 13.2, "Length and Area," Section 13.3, "Surface Area."

Use the overhead transparency masters to make overhead slides from Module 9, (Measurement and Geometry) from "The Winning Equation." Use the slides numbered T-1 to T-21/H-21. These slides develop concepts in perimeter, area, and the Pythagorean Theorem.

#### **Week 4**

Topic 4. Volume and Congruence.

Billstein et al: Section 10.1, "Congruence Through Constructions" and Section 11.5, "Volume, Mass, and Temperature," but postpone the subsection, "Temperature" until week 8.

Musser et al: Section 13.4, "Volume" and Section 14.1 "Congruence of Triangles"

Suggested Quiz 1: At the end of the week, give a 20 minute quiz on area, perimeter and the Pythagorean Theorem.

#### **Week 5**

Topic 5. Congruence and Similarity

Billstein et al: Section 10.2, "Other Congruence Properties" and Section 10.3, "Other Constructions"

Musser et al: Section 14.2, "Similarity of Triangles," Section 14.3, "Basic Euclidean Constructions."

Include a definition of the trigonometric functions, sine, cosine, and tangent, and give the sines and cosines for 0, 30, 45, 60, and 90 degrees. Explain how a scientific calculator may be used to find the approximate measure of an angle if its sine, cosine, or tangent is known.

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## Week 6

Topic 6. More Congruence and Similarity, Basic Algebra Skills

Billstein et al: Section 10.4, "Similar Triangles and Similar Figures"

Musser et al: Section 14.4, "Additional Euclidean Constructions."

As an additional topic, explain to students how to determine the image of points and triangles under translations, rotations, and reflections.

Assign from Learning Express *Algebra*:

Lesson 2 Working with Algebraic Expressions. This includes order of operations

Lesson 3 Combining Like Terms Supplement the material given by showing students how to combine terms using the distributive property. For example,

$$2x + 3x = (2 + 3)x = 5x \text{ by the distributive property}$$

Optional Reference to assist students: *Algebra*, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993

## Week 7

Topic 7. Solving Linear Equations

Assign from Learning Express *Algebra*:

Lesson 4: Solving Basic Equations

Lesson 5: Solving Multi-Step Equations

Lesson 6: Solving Equations with variables on both sides of an equation

Lesson 8: Assign all problems on page 55 for plotting points in the xy-plane

Addition references to Lesson 8:

Billstein et al: the first page of Section 10.5 of , "Lines in a Cartesian Coordinate System."

Musser et al: beginning of Section 9.3, "Functions and Their Graphs." See exercises 1 and 2 from that section for practice.

Optional Reference to assist students: *Algebra*, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993

## **Weeks 8, 9, and 10**

### Topic 8. Graphing Linear Equations

Use the overhead transparency masters to make overhead slides from Linear Equations and Inequalities from the Algebra I training of "The Winning Equation." This is likely to require at least 6 hours of class time.

Assign reading to students from these sections:

Billstein et al: Assign for reading the subsections of Section 11.3 entitled, "Converse of the Pythagorean Theorem" and "The Distance Formula: An Application of the Pythagorean Theorem" These are used in the module above from "The Winning Equation" to prove that the product of the slopes of two nonvertical lines is  $-1$  if and only if the lines are perpendicular. Assign for reading the subsection entitled "Temperature" from Section 11.5. Assign Section 10. 5, "Lines in a Cartesian Coordinate System" but the subsection "Fitting a Line to Data" is optional.

Musser et al: Section 15.1 "Distance and Slope in the Coordinate Plane," Section 15.2 "Equations and Coordinates" This section includes equations of lines and circles, simultaneous linear equations.

Optional Reference to assist students: *Algebra*, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993

## **Week 11**

Review previous material as needed and give a midterm on material covered in the course to this point.

## **Week 12**

### Topic 9. The Quadratic Formula

Derive the quadratic formula by completing the square, give exercises, and explain that the solutions correspond to the x-intercepts of the graph of a parabola.

Exercises are available in Lesson 20 of Learning Express *Algebra*.

Optional Reference to assist students: *Algebra*, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993. A derivation of the quadratic formula is given in this book.

## **Week 13**

Topic 10. Beginning Statistics

Statistics. Present material from:

Billstein et al: Chapter 8. Assign problems from that chapter. Cover material up through variance and standard deviation.

Musser et al: Chapter 10. Assign problems from that chapter. Cover material up through variance and standard deviation.